Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

3-Logarithms/62-3.3-u-a+b-log-c-d+e-x-^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [547]. This is test number [62].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.27 (543)	0.73 (4)
Mathematica	99.27 (543)	0.73 (4)
Maple	62.89 (344)	37.11 (203)
Maxima	40.59 (222)	59.41 (325)
Fricas	40.40 (221)	59.60 (326)
Giac	39.49 (216)	60.51 (331)
Mupad	38.21 (209)	61.79 (338)
Sympy	30.35 (166)	69.65 (381)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

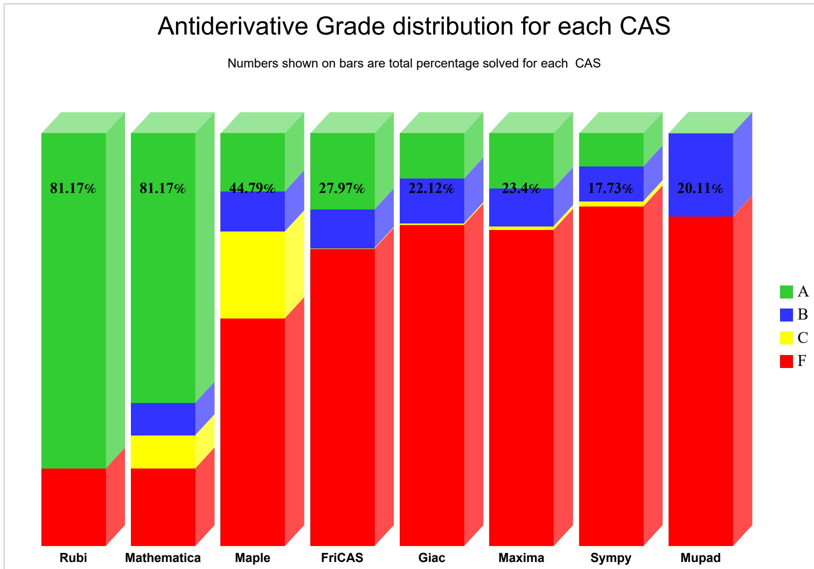
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

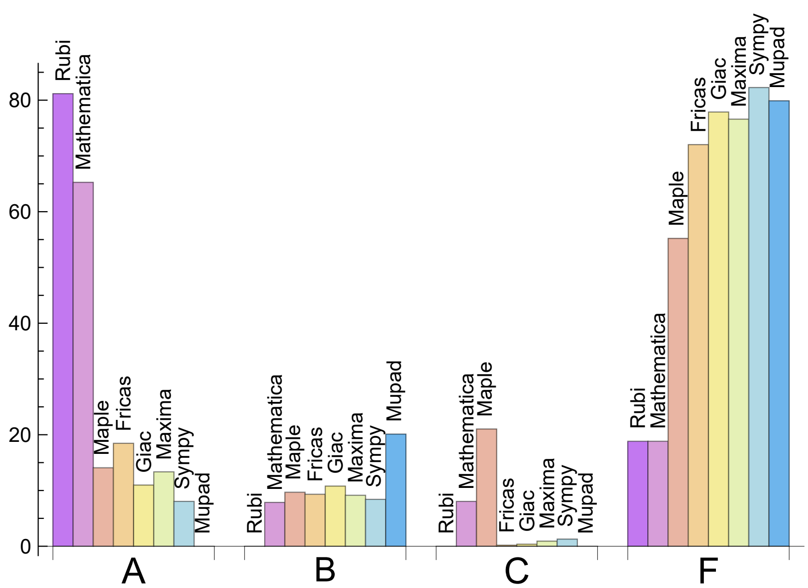
System	% A grade	% B grade	% C grade	% F grade
Rubi	80.256	0.914	0.000	18.830
Mathematica	65.265	7.861	8.044	18.830
Fricas	18.464	9.324	0.183	72.029
Maple	14.077	9.689	21.024	55.210
Maxima	13.346	9.141	0.914	76.600
Giac	10.969	10.786	0.366	77.879
Sympy	8.044	8.410	1.280	82.267
Mupad	0.000	20.110	0.000	79.890

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	4	100.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Maple	203	100.00	0.00	0.00
Fricas	326	73.01	0.00	26.99
Maxima	325	79.69	0.00	20.31
Giac	331	98.79	0.00	1.21
Mupad	338	0.00	100.00	0.00
Sympy	381	56.43	38.85	4.72

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.30
Giac	0.42
Rubi	0.75
Mathematica	0.84
Maxima	1.19
Mupad	1.34
Maple	2.96
Sympy	8.45

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	133.53	1.27	31.00	1.07
Maxima	205.23	3.25	85.50	1.20
Rubi	240.26	1.05	151.00	1.00
Sympy	257.43	2.13	44.00	1.13
Fricas	268.55	2.14	95.00	1.46
Mathematica	309.49	1.27	164.00	1.06
Maple	434.49	2.31	121.00	1.24
Giac	524.56	2.75	41.50	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

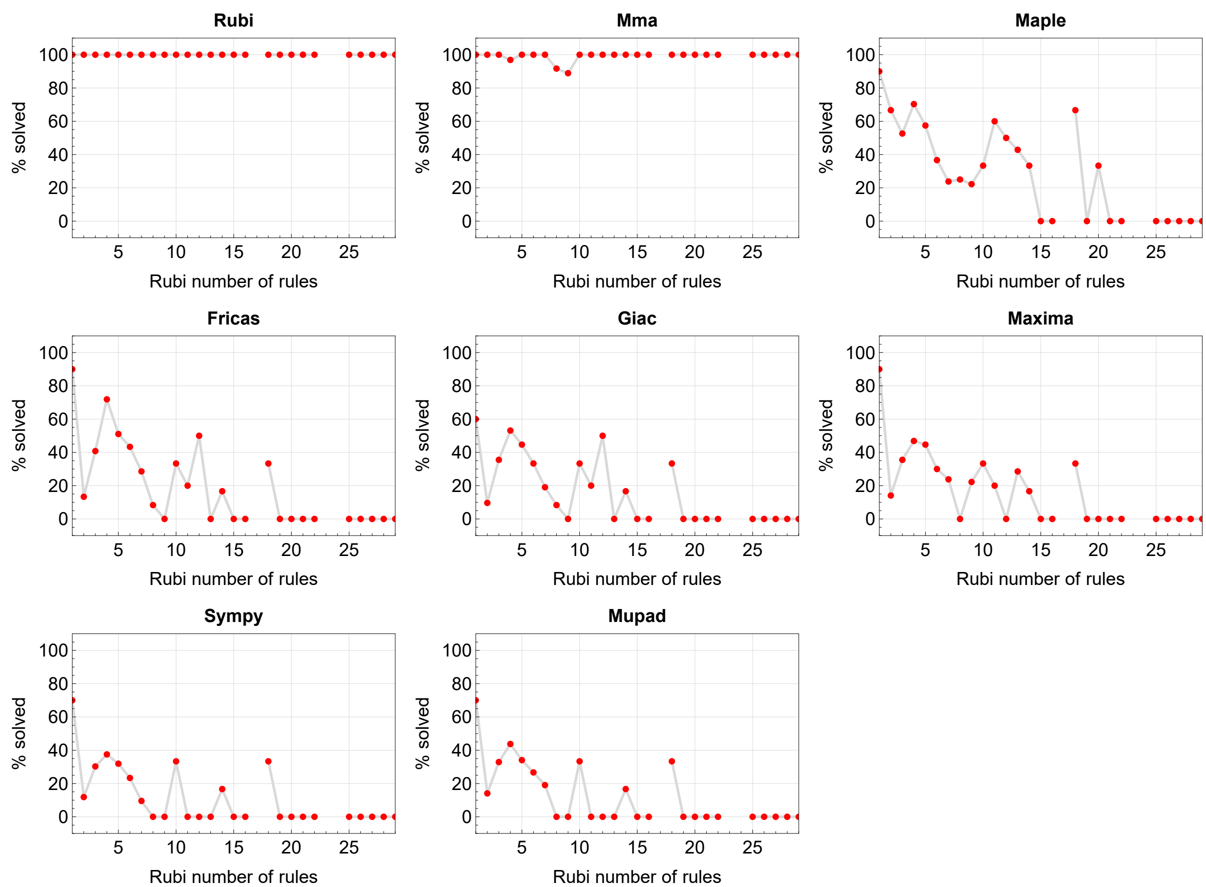


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

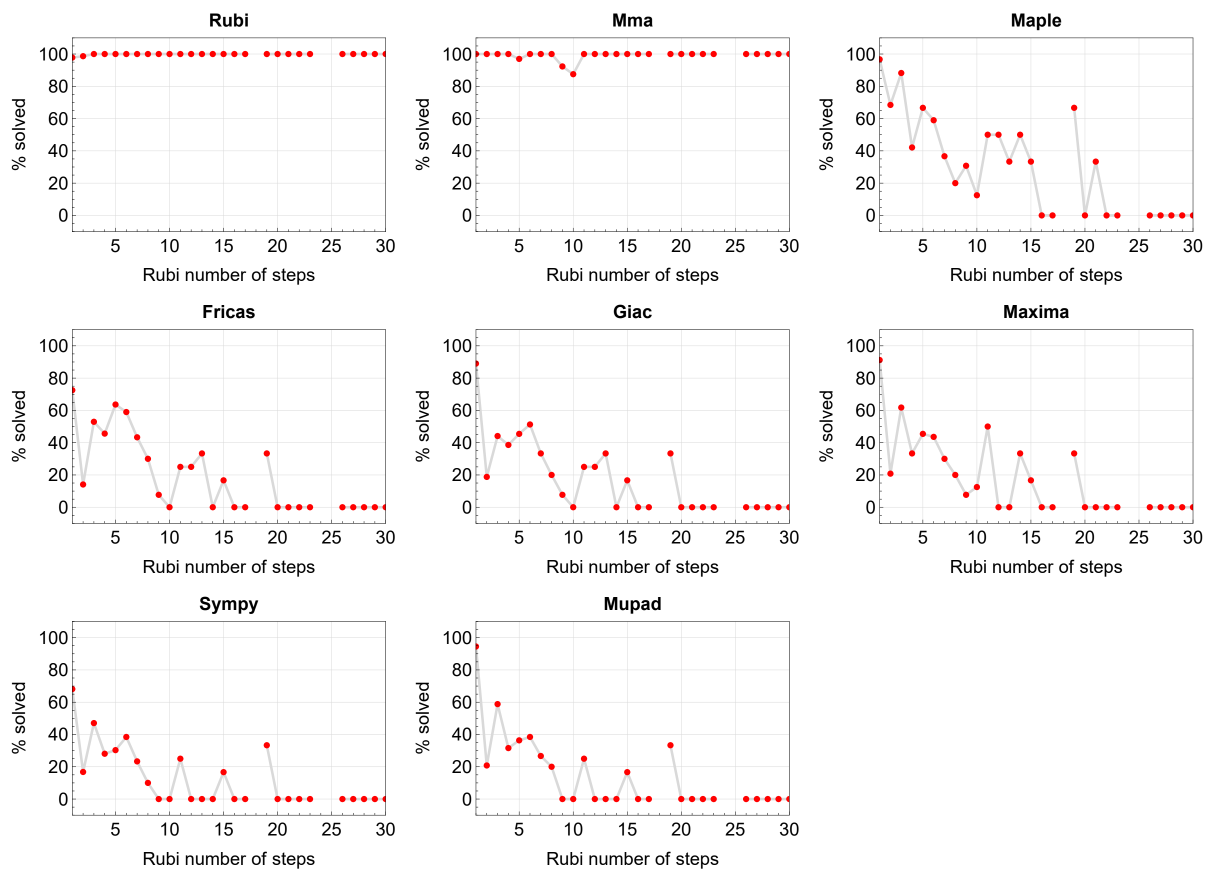


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

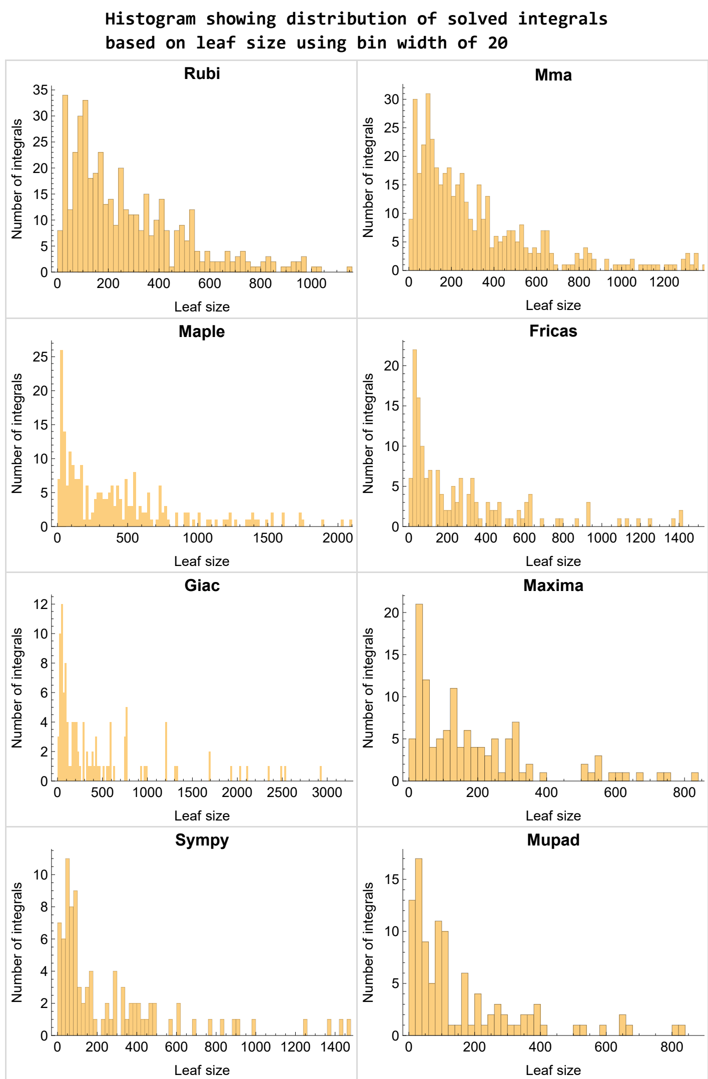


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

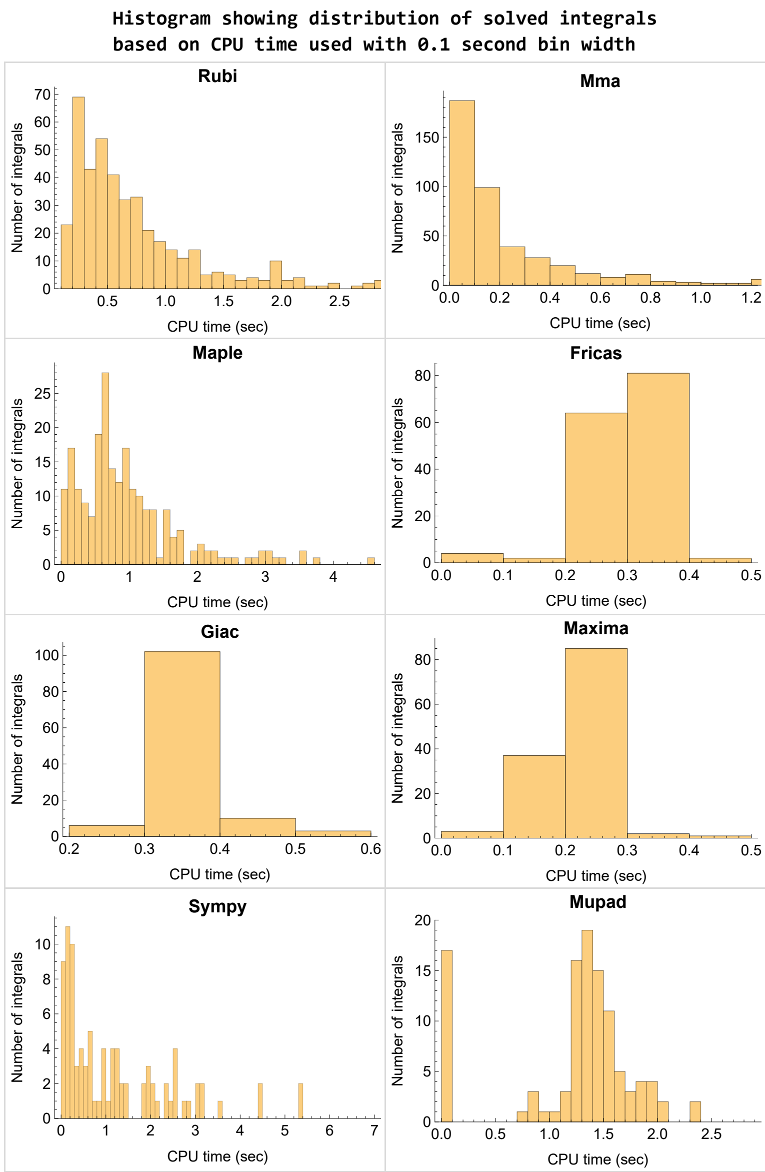


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

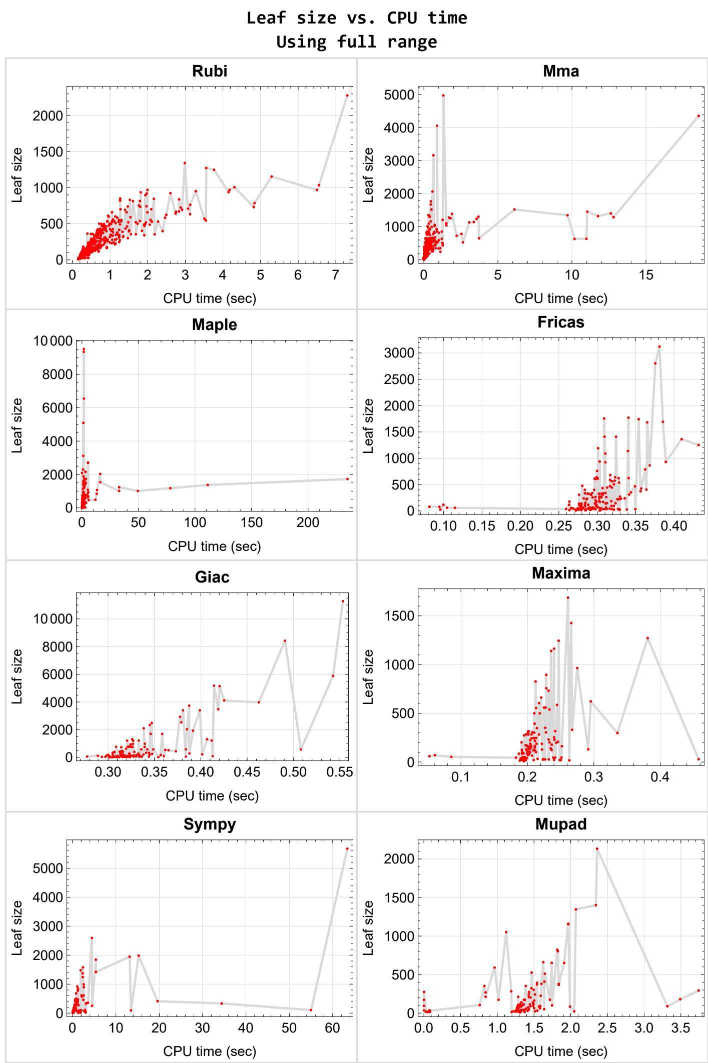


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{92, 93, 98, 99, 103, 104, 108, 109, 110, 114, 115, 116, 120, 121, 122, 127, 132, 137, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 174, 196, 197, 209, 214, 215, 216, 234, 235, 236, 237, 238, 239, 240, 241, 332, 346, 350, 375, 376, 377, 395, 396, 399, 400, 448, 449, 453, 454, 458, 459, 463, 464, 468, 469, 473, 477, 481, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 507, 508, 509, 510, 511, 512, 513, 517, 540, 541, 542, 543, 544, 545, 546, 547}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {16, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 428, 429, 431, 432, 434, 437, 438, 439, 440, 441, 442, 443, 444, 447, 451, 452, 455, 456, 457, 462, 467, 472, 475, 476, 478, 479, 480, 489, 490, 491, 492, 493, 494, 495, 516, 532, 537}

Mathematica {278, 522}

Maple {21, 22, 23, 40, 48, 49, 50, 51, 56, 57, 62, 63, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 217, 218, 219, 220, 221, 222, 223, 226, 227, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 306, 322, 331, 343, 344, 349, 358, 359, 360, 361, 362, 363, 364, 365, 366, 382, 383, 384, 385, 386, 387, 388, 389}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

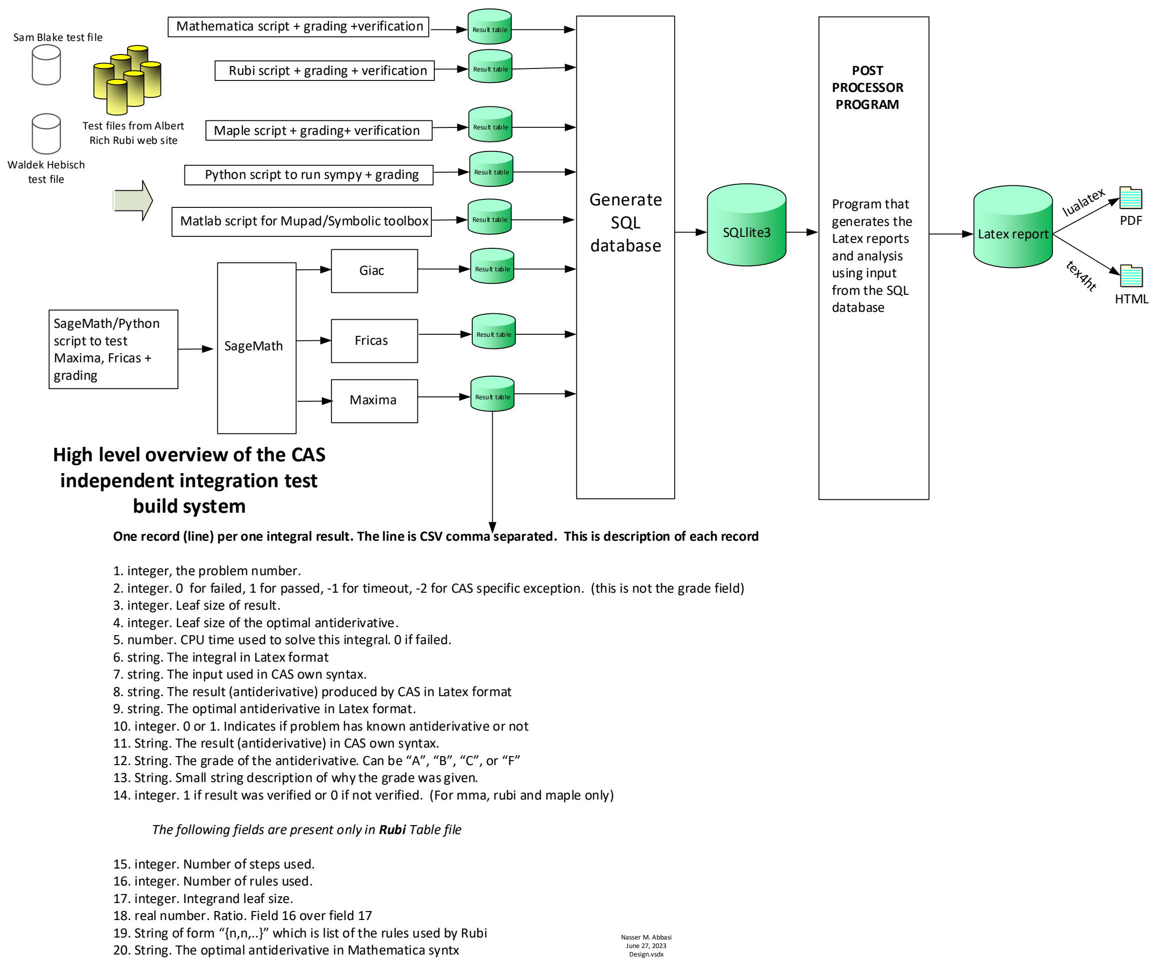
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 450, 451, 452, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

B grade { 100, 133, 134, 455, 478 }

C grade { }

F normal fail { 370, 371, 372, 374 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 134, 135, 136, 138, 139, 140, 141, 145, 146, 147, 148, 149, 162, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 198, 199, 200, 202, 204, 205, 206, 207, 210, 211, 212, 213, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 268, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 300, 302, 303, 304, 305, 306, 307, 308, 309, 331, 333, 334, 335, 336, 337, 338, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 437, 440, 441, 444, 445, 446, 447, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 475, 476, 478, 479, 480, 482, 483, 484, 485, 506, 514, 515, 516, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 539 }

B grade { 56, 57, 62, 63, 94, 95, 128, 129, 133, 225, 229, 230, 231, 339, 340, 341, 342, 343, 373, 394, 397, 398, 432, 438, 439, 442, 443, 450, 474, 489, 490, 491, 492, 493, 494, 495, 531, 532, 533, 535, 536, 537, 538 }

C grade { 142, 143, 144, 150, 151, 201, 203, 208, 266, 267, 269, 270, 293, 294, 295, 296, 297, 298, 299, 301, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 486, 487, 488 }

F normal fail { 191, 192, 276, 520 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 33, 38, 39, 41, 47, 64, 65, 66, 67, 68, 69, 70, 72, 75, 76, 77, 78, 81, 82, 83, 84, 85, 87, 140, 175, 176, 177, 178, 179, 184, 185, 186, 193, 194, 195, 213, 252, 279, 280, 281, 282, 303, 307, 309, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 354, 379, 380, 381, 402, 403, 407, 422, 423, 425, 444, 484 }

B grade { 17, 18, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 60, 61, 71, 73, 74, 79, 80, 180, 181, 182, 183, 187, 188, 191, 192, 304, 305, 308, 351, 352, 353, 355, 356, 357, 404, 405, 406, 420, 421, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441 }

C grade { 21, 22, 23, 40, 48, 49, 50, 51, 56, 57, 62, 63, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 204, 205, 206, 207, 208, 217, 218, 219, 220, 221, 222, 223, 226, 227, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 322, 331, 343, 344, 349, 358, 359, 360, 361, 362, 363, 364, 365, 366, 382, 383, 384, 385, 386, 387, 388, 389 }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 58, 59, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 189, 190, 198, 199, 200, 201, 202, 203, 210, 211, 212, 224, 225, 228, 229, 230, 233, 275, 276, 277, 278, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 347, 348, 367, 368, 369, 370, 371, 372, 373, 374, 378, 390, 391, 392, 393, 394, 397, 398, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 31, 33, 34, 38, 39, 41, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 81, 82, 85, 87, 88, 89, 90, 91, 94, 95, 96, 97, 138, 139, 140, 141, 173, 175, 176, 177, 178, 179, 183, 184, 185, 186, 191, 192, 193, 194, 195, 212, 213, 252, 268, 279, 281, 303, 304, 307, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 379, 380, 381, 402, 403, 407, 408, 409, 418, 422, 423, 425, 444, 445, 446, 447, 450, 451, 452, 483, 484, 485, 516 }

B grade { 17, 18, 19, 23, 35, 36, 37, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 100, 101, 102, 142, 143, 144, 187, 305, 308, 404, 405, 406, 410, 420, 421, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441, 455, 456, 457, 482, 486, 487, 488 }

C grade { 16 }

F normal fail { 32, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 73, 74, 77, 78, 79, 80, 83, 84, 86, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 180, 181, 182, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

F(-1) timedout fail { }

F(-2) exception fail { 9, 10, 11, 12, 13, 14, 15, 24, 25, 26, 27, 28, 29, 30, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 156, 157, 158, 159, 160, 161, 411, 412, 413, 414, 415, 416, 417, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 500, 501, 502, 503, 504, 505 }

2.1.5 Maxima

A grade { 3, 4, 5, 6, 7, 8, 13, 14, 15, 16, 20, 32, 37, 38, 39, 41, 42, 46, 64, 65, 66, 67, 68, 69, 70, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 195, 213, 252, 268, 303, 304, 307, 308, 345, 358, 359, 360, 361, 363, 364, 365, 366, 379, 380, 381, 402, 403, 406, 407, 419, 421, 422, 423, 425, 426, 429, 430, 431, 444 }

B grade { 1, 2, 17, 18, 19, 35, 36, 43, 44, 45, 47, 52, 53, 54, 55, 60, 61, 71, 75, 175, 176, 177, 178, 179, 183, 184, 185, 186, 187, 188, 189, 190, 279, 280, 305, 337, 338, 339, 340, 341, 342, 404, 405, 420, 427, 428, 435, 436, 437, 441 }

C grade { 9, 10, 11, 12, 309 }

F normal fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 72, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 162, 180, 181, 182, 191, 192, 193, 194, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336, 343, 344, 362, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 506, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

F(-1) timedout fail { }

F(-2) exception fail { 31, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 169, 170, 171, 172, 173, 174, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 329, 330, 331, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 377, 418, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 513, 514, 515, 516, 517 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 33, 39, 41, 42, 64, 65, 66, 67, 68, 69, 70, 76, 82, 85, 88, 89, 90, 91, 140, 141, 142, 175, 176, 177, 178, 179, 184, 185, 186, 195, 213, 252, 268, 307, 308, 381, 402, 403, 407, 408, 423, 425, 426, 444, 445, 446, 447, 484, 485, 486 }

B grade { 17, 18, 19, 22, 23, 35, 36, 37, 38, 43, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 87, 94, 95, 96, 97, 100, 101, 102, 183, 187, 303, 304, 305, 379, 380, 404, 405, 406, 409, 410, 420, 421, 422, 427, 428, 429, 430, 431, 435, 436, 437, 441, 450, 451, 452, 455, 456, 457, 487 }

C grade { 11, 12 }

F normal fail { 9, 10, 13, 14, 15, 16, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 83, 84, 86, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 180, 181, 182, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 411, 412, 413, 414, 415, 416, 417, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

F(-1) timedout fail { }

F(-2) exception fail { 169, 209, 215, 513 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 33, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 175, 176, 177, 178, 179, 183, 184, 185, 186, 187, 195, 213, 252, 268, 279, 281, 303, 304, 305, 307, 308, 337, 338, 379, 380, 381, 402, 403, 404, 405, 406, 407, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441, 444 }

C grade { }

F normal fail { }

F(-1) timedout fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 180, 181, 182, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 16, 20, 38, 39, 64, 65, 66, 67, 68, 69, 70, 76, 79, 80, 82, 83, 84, 85, 87, 176, 177, 178, 179, 186, 195, 303, 379, 380, 381, 402, 403, 407, 422, 423, 444 }

B grade { 11, 12, 13, 17, 18, 19, 35, 36, 37, 41, 42, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 175, 183, 184, 185, 187, 252, 304, 305, 307, 404, 405, 406, 420, 421, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441 }

C grade { 72, 73, 74, 75, 77, 78, 81 }

F normal fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 71, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 142, 145, 146, 147, 148, 170, 171, 172, 173, 180, 181, 182, 188, 189, 190, 191, 192, 193, 194, 200, 201, 205, 206, 212, 213, 217, 218, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 242, 243, 244, 245, 246, 248, 249, 250, 254, 255, 259, 275, 276, 277, 278, 279, 280, 281, 282, 306, 309, 313, 329, 330, 331, 337, 338, 343, 344, 345, 351, 352, 353, 354, 374, 382, 383, 384, 385, 408, 409, 410, 412, 413, 414, 415, 416, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 515, 516, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

F(-1) timedout fail { 9, 14, 15, 93, 114, 115, 116, 120, 121, 122, 137, 143, 144, 150, 151, 165, 198, 199, 204, 207, 208, 209, 210, 211, 215, 216, 222, 223, 228, 232, 233, 237, 247, 251, 253, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 308, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 339, 340, 341, 342, 347, 348, 349, 350, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 411, 417, 469, 481, 487, 488, 494, 495, 509, 514, 518 }

F(-2) exception fail { 43, 149, 162, 164, 166, 168, 169, 202, 203, 335, 336, 377, 401, 506, 508, 510, 512, 513 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	83	74	87	188	78	88	88	119
N.S.	1	1.02	0.91	1.07	2.32	0.96	1.09	1.09	1.47
time (sec)	N/A	0.243	0.007	0.132	0.204	0.278	0.121	0.413	1.494

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	64	57	67	125	60	68	68	88
N.S.	1	1.05	0.93	1.10	2.05	0.98	1.11	1.11	1.44
time (sec)	N/A	0.218	0.005	0.125	0.192	0.295	0.105	0.277	1.451

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	40	47	71	42	46	48	57
N.S.	1	1.10	0.98	1.15	1.73	1.02	1.12	1.17	1.39
time (sec)	N/A	0.189	0.004	0.119	0.201	0.289	0.084	0.307	1.389

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	26	21	26	31	25	26	31	25
N.S.	1	1.24	1.00	1.24	1.48	1.19	1.24	1.48	1.19
time (sec)	N/A	0.156	0.004	0.080	0.190	0.272	0.070	0.301	0.067

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	17	16	12	16	15
N.S.	1	1.00	1.00	1.47	1.13	1.07	0.80	1.07	1.00
time (sec)	N/A	0.159	0.008	0.253	0.263	0.264	0.244	0.302	1.373

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	34	36	43	20	47	29	37	36
N.S.	1	0.94	1.00	1.19	0.56	1.31	0.81	1.03	1.00
time (sec)	N/A	0.186	0.010	0.290	0.250	0.281	0.277	0.293	1.357

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	58	47	64	21	67	48	58	64
N.S.	1	0.92	0.75	1.02	0.33	1.06	0.76	0.92	1.02
time (sec)	N/A	0.217	0.012	0.282	0.240	0.290	0.273	0.344	1.501

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	82	57	87	20	90	71	78	67
N.S.	1	0.96	0.67	1.02	0.24	1.06	0.84	0.92	0.79
time (sec)	N/A	0.252	0.018	0.289	0.249	0.284	0.300	0.316	1.269

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	94	75	0	78	0	0	0	96
N.S.	1	0.96	0.77	0.00	0.80	0.00	0.00	0.00	0.98
time (sec)	N/A	0.324	0.011	0.000	0.203	0.000	0.000	0.000	1.320

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	71	63	0	65	0	105	0	82
N.S.	1	0.96	0.85	0.00	0.88	0.00	1.42	0.00	1.11
time (sec)	N/A	0.288	0.009	0.000	0.207	0.000	55.014	0.000	1.274

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	B	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	48	50	0	49	0	90	52	46
N.S.	1	0.96	1.00	0.00	0.98	0.00	1.80	1.04	0.92
time (sec)	N/A	0.246	0.007	0.000	0.193	0.000	0.902	0.307	1.281

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	B	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	25	0	63	25	45
N.S.	1	1.00	1.00	0.00	1.00	0.00	2.52	1.00	1.80
time (sec)	N/A	0.218	0.003	0.000	0.192	0.000	0.960	0.354	1.263

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	47	58	0	45	0	92	0	67
N.S.	1	0.96	1.18	0.00	0.92	0.00	1.88	0.00	1.37
time (sec)	N/A	0.254	0.068	0.000	0.239	0.000	13.486	0.000	1.290

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	73	72	0	45	0	0	0	113
N.S.	1	0.95	0.94	0.00	0.58	0.00	0.00	0.00	1.47
time (sec)	N/A	0.284	0.063	0.000	0.242	0.000	0.000	0.000	1.309

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	99	85	0	45	0	0	0	140
N.S.	1	0.98	0.84	0.00	0.45	0.00	0.00	0.00	1.39
time (sec)	N/A	0.324	0.066	0.000	0.251	0.000	0.000	0.000	1.326

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	41	45	0	53	27	54	0	45
N.S.	1	0.91	1.00	0.00	1.18	0.60	1.20	0.00	1.00
time (sec)	N/A	0.201	0.023	0.000	0.086	0.096	3.005	0.000	1.264

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	119	112	525	500	614	495	758	275
N.S.	1	0.91	0.85	4.01	3.82	4.69	3.78	5.79	2.10
time (sec)	N/A	0.307	0.036	1.359	0.210	0.297	0.903	0.339	1.433

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	92	85	322	282	324	294	399	172
N.S.	1	0.93	0.86	3.25	2.85	3.27	2.97	4.03	1.74
time (sec)	N/A	0.259	0.010	0.616	0.218	0.289	0.499	0.307	1.377

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	111	131	140	146	174	94
N.S.	1	1.00	0.91	1.71	2.02	2.15	2.25	2.68	1.45
time (sec)	N/A	0.214	0.007	0.348	0.198	0.297	0.265	0.299	1.297

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	36	40	40	41	45	35
N.S.	1	1.00	1.00	1.24	1.38	1.38	1.41	1.55	1.21
time (sec)	N/A	0.163	0.005	0.221	0.195	0.295	0.143	0.323	1.286

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	63	309	0	46	0	49	0
N.S.	1	1.00	1.00	4.90	0.00	0.73	0.00	0.78	0.00
time (sec)	N/A	0.252	0.070	0.868	0.000	0.268	0.000	0.297	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	96	94	123	456	0	117	0	286	0
N.S.	1	0.98	1.28	4.75	0.00	1.22	0.00	2.98	0.00
time (sec)	N/A	0.287	0.044	0.979	0.000	0.288	0.000	0.388	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	136	144	734	0	263	0	1218	0
N.S.	1	1.01	1.07	5.44	0.00	1.95	0.00	9.02	0.00
time (sec)	N/A	0.353	0.061	1.144	0.000	0.298	0.000	0.412	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	171	152	0	0	0	0	0	0
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.459	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	140	127	0	0	0	0	0	0
N.S.	1	0.98	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	109	106	0	0	0	0	0	0
N.S.	1	0.98	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	114	139	0	0	0	0	0	0
N.S.	1	0.98	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	158	163	0	0	0	0	0	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	202	203	0	0	0	0	0	0
N.S.	1	1.05	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	103	0	0	59	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.260	0.077	0.000	0.000	0.115	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	59	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.67	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.057	0.000	0.053	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	26	0	22	0	22	20
N.S.	1	1.00	1.00	1.30	0.00	1.10	0.00	1.10	1.00
time (sec)	N/A	0.246	0.023	1.503	0.000	0.322	0.000	0.324	1.249

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	27	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.299	0.099	0.000	0.000	0.339	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	160	315	589	393	471	568	1209	526
N.S.	1	0.90	1.77	3.31	2.21	2.65	3.19	6.79	2.96
time (sec)	N/A	0.319	0.203	1.615	0.211	0.317	2.168	0.321	1.469

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	136	226	428	284	340	410	770	352
N.S.	1	0.91	1.52	2.87	1.91	2.28	2.75	5.17	2.36
time (sec)	N/A	0.279	0.139	1.074	0.198	0.293	1.164	0.322	0.824

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	112	150	265	187	221	252	424	212
N.S.	1	0.93	1.25	2.21	1.56	1.84	2.10	3.53	1.77
time (sec)	N/A	0.256	0.099	0.853	0.214	0.286	0.665	0.314	0.840

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	88	95	98	102	119	134	183	104
N.S.	1	0.97	1.04	1.08	1.12	1.31	1.47	2.01	1.14
time (sec)	N/A	0.237	0.033	0.352	0.192	0.284	0.388	0.314	0.761

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	36	40	40	41	45	35
N.S.	1	1.00	1.00	1.24	1.38	1.38	1.41	1.55	1.21
time (sec)	N/A	0.163	0.005	0.061	0.200	0.297	0.150	0.311	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	62	217	0	0	0	0	0
N.S.	1	1.00	0.98	3.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.011	0.702	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	70	57	127	85	95	333	88	84
N.S.	1	0.95	0.77	1.72	1.15	1.28	4.50	1.19	1.14
time (sec)	N/A	0.198	0.046	0.810	0.187	0.299	3.116	0.301	1.325

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	95	83	283	167	274	1945	201	173
N.S.	1	0.85	0.74	2.53	1.49	2.45	17.37	1.79	1.54
time (sec)	N/A	0.263	0.066	1.076	0.199	0.303	13.160	0.333	1.017

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	121	110	455	301	507	0	401	283
N.S.	1	0.86	0.78	3.23	2.13	3.60	0.00	2.84	2.01
time (sec)	N/A	0.288	0.090	1.608	0.210	0.322	0.000	0.313	1.190

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	312	360	1343	827	1190	1241	2345	1051
N.S.	1	0.85	0.99	3.68	2.27	3.26	3.40	6.42	2.88
time (sec)	N/A	0.572	0.174	2.201	0.212	0.301	2.349	0.345	1.121

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	252	247	850	554	760	774	1315	591
N.S.	1	0.88	0.86	2.96	1.93	2.65	2.70	4.58	2.06
time (sec)	N/A	0.533	0.098	1.477	0.213	0.300	1.226	0.407	0.962

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	144	459	314	401	394	583	268
N.S.	1	1.00	0.77	2.47	1.69	2.16	2.12	3.13	1.44
time (sec)	N/A	0.396	0.052	0.612	0.202	0.284	0.690	0.384	0.840

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	111	131	140	146	174	94
N.S.	1	1.00	0.91	1.71	2.02	2.15	2.25	2.68	1.45
time (sec)	N/A	0.217	0.006	0.144	0.203	0.276	0.265	0.357	0.001

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	111	106	194	737	0	0	0	0	0
N.S.	1	0.95	1.75	6.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.111	1.270	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	118	126	537	0	0	0	0	0
N.S.	1	0.89	0.95	4.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	0.057	0.907	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	202	230	204	661	0	0	0	0	0
N.S.	1	1.14	1.01	3.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.736	0.123	1.346	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	317	380	302	755	0	0	0	0	0
N.S.	1	1.20	0.95	2.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.218	0.214	2.187	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	598	598	475	2719	1687	2802	2594	5182	2133
N.S.	1	1.00	0.79	4.55	2.82	4.69	4.34	8.67	3.57
time (sec)	N/A	0.973	0.266	5.422	0.261	0.376	4.445	0.414	2.359

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	333	1733	1140	1771	1578	2932	1157
N.S.	1	1.00	0.77	4.01	2.64	4.10	3.65	6.79	2.68
time (sec)	N/A	0.740	0.149	3.551	0.236	0.341	2.407	0.378	1.969

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	201	906	662	923	836	1321	511
N.S.	1	1.00	0.76	3.42	2.50	3.48	3.15	4.98	1.93
time (sec)	N/A	0.489	0.080	1.740	0.221	0.311	1.270	0.326	1.647

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	92	85	322	282	324	294	399	172
N.S.	1	0.93	0.86	3.25	2.85	3.27	2.97	4.03	1.74
time (sec)	N/A	0.258	0.011	0.257	0.201	0.305	0.499	0.328	0.002

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	158	149	335	1396	0	0	0	0	0
N.S.	1	0.94	2.12	8.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.564	0.184	1.988	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	190	161	410	1268	0	0	0	0	0
N.S.	1	0.85	2.16	6.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	0.246	2.022	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	342	325	620	0	0	0	0	0	0
N.S.	1	0.95	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.363	0.474	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	564	584	843	0	0	0	0	0	0
N.S.	1	1.04	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.525	0.705	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	258	1493	1163	1756	1372	2488	823
N.S.	1	1.00	0.76	4.39	3.42	5.16	4.04	7.32	2.42
time (sec)	N/A	0.586	0.143	3.524	0.240	0.309	2.313	0.347	1.816

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	119	112	525	500	614	495	758	275
N.S.	1	0.91	0.85	4.01	3.82	4.69	3.78	5.79	2.10
time (sec)	N/A	0.307	0.015	0.589	0.219	0.301	0.900	0.326	0.003

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	205	191	503	2172	0	0	0	0	0
N.S.	1	0.93	2.45	10.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	0.175	3.017	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	248	204	531	2156	0	0	0	0	0
N.S.	1	0.82	2.14	8.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.709	0.457	3.224	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	24	19	24	23	22	24	23	23
N.S.	1	1.26	1.00	1.26	1.21	1.16	1.26	1.21	1.21
time (sec)	N/A	0.165	0.005	0.076	0.191	0.275	0.067	0.316	0.070

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	41	36	40	27	36	42	44	48
N.S.	1	1.11	0.97	1.08	0.73	0.97	1.14	1.19	1.30
time (sec)	N/A	0.195	0.002	0.117	0.196	0.260	0.085	0.310	1.433

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	58	51	55	37	51	63	62	73
N.S.	1	1.05	0.93	1.00	0.67	0.93	1.15	1.13	1.33
time (sec)	N/A	0.219	0.007	0.120	0.190	0.274	0.104	0.330	1.419

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	32	25	32	34	30	36	34	31
N.S.	1	1.28	1.00	1.28	1.36	1.20	1.44	1.36	1.24
time (sec)	N/A	0.187	0.007	0.080	0.191	0.276	0.107	0.319	0.083

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	48	52	38	48	63	65	94
N.S.	1	1.08	0.98	1.06	0.78	0.98	1.29	1.33	1.92
time (sec)	N/A	0.223	0.006	0.139	0.192	0.269	0.135	0.333	1.488

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	67	71	51	67	95	91	138
N.S.	1	1.01	0.92	0.97	0.70	0.92	1.30	1.25	1.89
time (sec)	N/A	0.248	0.007	0.143	0.196	0.281	0.158	0.315	1.484

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	24	32	35	32	36	39	29
N.S.	1	1.21	1.00	1.33	1.46	1.33	1.50	1.62	1.21
time (sec)	N/A	0.173	0.006	0.226	0.197	0.285	0.141	0.323	0.070

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	54	102	27	0	0	23
N.S.	1	1.00	1.00	2.25	4.25	1.12	0.00	0.00	0.96
time (sec)	N/A	0.202	0.006	1.079	0.204	0.279	0.000	0.000	1.513

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	0	14	17	0	15
N.S.	1	1.00	1.00	1.07	0.00	0.93	1.13	0.00	1.00
time (sec)	N/A	0.184	0.004	0.173	0.000	0.284	3.014	0.000	0.082

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	33	20	0	87	0	18
N.S.	1	1.00	1.00	2.06	1.25	0.00	5.44	0.00	1.12
time (sec)	N/A	0.178	0.003	0.099	0.210	0.000	1.483	0.000	0.035

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	33	20	0	87	0	18
N.S.	1	1.00	1.00	2.06	1.25	0.00	5.44	0.00	1.12
time (sec)	N/A	0.178	0.003	0.107	0.189	0.000	1.472	0.000	0.031

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	19	7	10	0	18
N.S.	1	1.00	1.00	1.12	2.38	0.88	1.25	0.00	2.25
time (sec)	N/A	0.149	0.003	0.096	0.195	0.303	1.341	0.000	0.031

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.147	0.002	0.132	0.195	0.272	0.041	0.304	1.308

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	19	0	60	0	16
N.S.	1	1.00	1.00	0.85	0.95	0.00	3.00	0.00	0.80
time (sec)	N/A	0.191	0.003	0.095	0.197	0.000	1.942	0.000	0.030

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	27	19	20	0	102	0	18
N.S.	1	1.00	1.08	0.76	0.80	0.00	4.08	0.00	0.72
time (sec)	N/A	0.191	0.003	0.117	0.197	0.000	1.967	0.000	0.033

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	22	40	27	0	94	0	25
N.S.	1	1.00	1.05	1.90	1.29	0.00	4.48	0.00	1.19
time (sec)	N/A	0.189	0.003	0.207	0.224	0.000	1.992	0.000	1.235

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	22	40	27	0	94	0	25
N.S.	1	1.00	1.05	1.90	1.29	0.00	4.48	0.00	1.19
time (sec)	N/A	0.186	0.003	0.174	0.225	0.000	2.013	0.000	0.075

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	26	13	15	0	14
N.S.	1	1.00	1.00	1.07	1.86	0.93	1.07	0.00	1.00
time (sec)	N/A	0.177	0.003	0.179	0.232	0.285	1.844	0.000	0.059

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	15	16	14	14	14
N.S.	1	1.00	0.94	0.88	0.88	0.94	0.82	0.82	0.82
time (sec)	N/A	0.155	0.002	0.105	0.189	0.279	0.048	0.312	1.196

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	24	26	0	66	0	23
N.S.	1	1.00	1.04	0.92	1.00	0.00	2.54	0.00	0.88
time (sec)	N/A	0.199	0.003	0.217	0.223	0.000	2.717	0.000	1.293

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	26	27	0	109	0	25
N.S.	1	1.00	1.10	0.84	0.87	0.00	3.52	0.00	0.81
time (sec)	N/A	0.194	0.003	0.181	0.227	0.000	2.857	0.000	1.263

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	164	163	172	131	179	173	342	116
N.S.	1	0.88	0.87	0.92	0.70	0.96	0.93	1.83	0.62
time (sec)	N/A	0.371	0.037	0.444	0.198	0.264	0.765	0.305	1.354

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	186	172	483	150	0	0	0	0
N.S.	1	1.05	0.97	2.73	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	0.762	0.061	0.385	0.206	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	260	261	215	341	269	626	172
N.S.	1	1.00	0.91	0.92	0.75	1.20	0.94	2.20	0.60
time (sec)	N/A	0.506	0.052	0.831	0.211	0.311	1.374	0.319	1.361

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	266	9346	0	305	0	582	0
N.S.	1	1.00	0.89	31.26	0.00	1.02	0.00	1.95	0.00
time (sec)	N/A	0.751	0.625	1.542	0.000	0.292	0.000	0.349	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	219	219	197	1889	0	192	0	337	0
N.S.	1	1.00	0.90	8.63	0.00	0.88	0.00	1.54	0.00
time (sec)	N/A	0.572	0.168	1.340	0.000	0.303	0.000	0.341	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	139	126	937	0	105	0	159	0
N.S.	1	1.00	0.91	6.74	0.00	0.76	0.00	1.14	0.00
time (sec)	N/A	0.395	0.068	0.962	0.000	0.277	0.000	0.328	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	63	309	0	46	0	49	0
N.S.	1	1.00	1.00	4.90	0.00	0.73	0.00	0.78	0.00
time (sec)	N/A	0.243	0.010	0.098	0.000	0.278	0.000	0.318	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	31	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.29	0.83	1.08	1.08
time (sec)	N/A	0.189	0.160	0.045	0.263	0.284	1.183	0.326	1.208

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	55	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	2.29	0.00	1.08	1.08
time (sec)	N/A	0.192	0.354	0.013	0.266	0.317	0.000	0.320	1.237

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	339	585	1674	9517	0	681	0	3473	0
N.S.	1	1.73	4.94	28.07	0.00	2.01	0.00	10.24	0.00
time (sec)	N/A	1.194	0.463	1.702	0.000	0.317	0.000	0.419	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	259	425	1015	5089	0	433	0	2031	0
N.S.	1	1.64	3.92	19.65	0.00	1.67	0.00	7.84	0.00
time (sec)	N/A	0.882	0.251	1.273	0.000	0.307	0.000	0.385	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	257	208	2300	0	239	0	973	0
N.S.	1	1.45	1.18	12.99	0.00	1.35	0.00	5.50	0.00
time (sec)	N/A	0.705	0.138	1.008	0.000	0.282	0.000	0.327	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	96	94	123	456	0	117	0	286	0
N.S.	1	0.98	1.28	4.75	0.00	1.22	0.00	2.98	0.00
time (sec)	N/A	0.300	0.030	0.223	0.000	0.263	0.000	0.323	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	188	63	22	26	26
N.S.	1	1.00	1.08	1.00	7.83	2.62	0.92	1.08	1.08
time (sec)	N/A	0.187	0.232	0.015	0.278	0.297	2.767	0.370	1.283

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	289	104	24	26	26
N.S.	1	1.00	1.08	1.00	12.04	4.33	1.00	1.08	1.08
time (sec)	N/A	0.188	2.107	0.033	0.290	0.292	11.143	0.338	1.299

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	351	753	351	6545	0	1090	0	8422	0
N.S.	1	2.15	1.00	18.65	0.00	3.11	0.00	23.99	0.00
time (sec)	N/A	1.754	0.794	1.727	0.000	0.311	0.000	0.491	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	261	415	256	3114	0	588	0	4112	0
N.S.	1	1.59	0.98	11.93	0.00	2.25	0.00	15.75	0.00
time (sec)	N/A	1.254	0.211	1.247	0.000	0.317	0.000	0.425	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	136	144	734	0	263	0	1218	0
N.S.	1	1.01	1.07	5.44	0.00	1.95	0.00	9.02	0.00
time (sec)	N/A	0.354	0.056	0.451	0.000	0.312	0.000	0.333	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	703	95	22	26	26
N.S.	1	1.00	1.08	1.00	29.29	3.96	0.92	1.08	1.08
time (sec)	N/A	0.188	0.344	0.023	0.299	0.296	6.422	0.328	1.278

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	934	153	24	26	26
N.S.	1	1.00	1.08	1.00	38.92	6.38	1.00	1.08	1.08
time (sec)	N/A	0.193	2.719	0.026	0.302	0.313	35.271	0.346	1.292

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	404	404	374	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.061	0.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	235	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.637	0.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	109	106	0	0	0	0	0	0
N.S.	1	0.98	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	22	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.85	1.00	1.00
time (sec)	N/A	0.197	3.579	0.119	0.586	0.000	0.537	0.354	1.266

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	24	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.92	1.00	1.00
time (sec)	N/A	0.310	0.300	0.146	0.595	0.000	1.871	0.339	1.381

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	24	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.92	1.00	1.00
time (sec)	N/A	0.398	0.317	0.163	0.598	0.000	8.446	0.357	1.371

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	526	526	446	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.220	0.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	282	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.749	0.309	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	140	127	0	0	0	0	0	0
N.S.	1	0.98	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.209	2.519	0.121	0.613	0.000	0.000	0.463	1.363

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.313	0.534	0.124	0.626	0.000	0.000	0.386	1.414

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.394	0.532	0.122	0.654	0.000	0.000	0.424	1.473

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	660	660	511	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.432	1.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	413	413	326	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.900	0.496	0.000	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	171	152	0	0	0	0	0	0
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.209	2.293	0.118	0.618	0.000	0.000	0.581	1.339

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.322	7.919	0.117	0.646	0.000	0.000	0.448	1.405

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.421	5.384	0.115	0.666	0.000	0.000	0.886	1.487

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	383	331	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.089	0.315	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	252	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.832	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	164	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.536	0.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	24	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.92	1.00	1.00
time (sec)	N/A	0.202	0.067	0.150	0.601	0.000	1.266	0.375	1.367

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	422	735	1281	0	0	0	0	0	0
N.S.	1	1.74	3.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.829	1.696	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	325	533	828	0	0	0	0	0	0
N.S.	1	1.64	2.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.286	0.696	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	318	338	0	0	0	0	0	0
N.S.	1	1.45	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.001	0.492	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	114	139	0	0	0	0	0	0
N.S.	1	0.98	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.358	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	24	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.92	1.00	1.00
time (sec)	N/A	0.209	0.205	0.124	0.620	0.000	5.792	0.459	1.539

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	520	1341	1523	0	0	0	0	0	0
N.S.	1	2.58	2.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.009	6.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	421	924	527	0	0	0	0	0	0
N.S.	1	2.19	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.598	2.637	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	311	501	353	0	0	0	0	0	0
N.S.	1	1.61	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.740	1.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	158	163	0	0	0	0	0	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.120	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.210	0.416	0.145	0.635	0.000	0.000	0.577	1.691

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	161	137	0	0	538	0	0	0
N.S.	1	0.99	0.84	0.00	0.00	3.30	0.00	0.00	0.00
time (sec)	N/A	0.298	0.144	0.000	0.000	0.328	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	118	0	0	311	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.262	0.090	0.000	0.000	0.341	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	99	83	113	0	185	0	110	0
N.S.	1	1.02	0.86	1.16	0.00	1.91	0.00	1.13	0.00
time (sec)	N/A	0.237	0.056	0.862	0.000	0.318	0.000	0.319	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	224	0	101	0
N.S.	1	1.00	0.99	0.00	0.00	2.77	0.00	1.25	0.00
time (sec)	N/A	0.221	0.110	0.000	0.000	0.326	0.000	0.334	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	110	85	0	0	425	0	164	0
N.S.	1	0.96	0.75	0.00	0.00	3.73	0.00	1.44	0.00
time (sec)	N/A	0.244	0.031	0.000	0.000	0.357	0.000	0.331	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	146	78	0	0	789	0	0	0
N.S.	1	1.01	0.54	0.00	0.00	5.44	0.00	0.00	0.00
time (sec)	N/A	0.264	0.033	0.000	0.000	0.362	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	182	78	0	0	1252	0	0	0
N.S.	1	1.03	0.44	0.00	0.00	7.11	0.00	0.00	0.00
time (sec)	N/A	0.285	0.035	0.000	0.000	0.432	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	590	1007	854	0	0	0	0	0	0
N.S.	1	1.71	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.348	1.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	510	763	680	0	0	0	0	0	0
N.S.	1	1.50	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.148	0.756	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	418	553	566	0	0	0	0	0	0
N.S.	1	1.32	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.086	0.687	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	389	424	0	0	0	0	0	0
N.S.	1	1.25	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.370	0.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	423	529	557	0	0	0	0	0	0
N.S.	1	1.25	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.130	0.792	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	503	709	639	0	0	0	0	0	0
N.S.	1	1.41	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.136	1.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	583	940	728	0	0	0	0	0	0
N.S.	1	1.61	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.248	2.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	204	26	22	26	26
N.S.	1	1.00	1.08	0.92	7.85	1.00	0.85	1.00	1.00
time (sec)	N/A	0.198	0.439	0.158	0.315	0.291	21.145	0.375	1.223

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	174	26	22	26	26
N.S.	1	1.00	1.08	0.92	6.69	1.00	0.85	1.00	1.00
time (sec)	N/A	0.185	0.126	0.151	0.311	0.271	0.700	0.329	1.258

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	174	39	24	26	26
N.S.	1	1.00	1.08	0.92	6.69	1.50	0.92	1.00	1.00
time (sec)	N/A	0.193	0.434	0.129	0.314	0.294	1.845	0.394	1.210

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	166	63	24	26	26
N.S.	1	1.00	1.08	0.92	6.38	2.42	0.92	1.00	1.00
time (sec)	N/A	0.197	0.715	0.119	0.317	0.296	7.008	0.383	1.336

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	24	26	0	26	26	26
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	0.93
time (sec)	N/A	0.447	0.867	0.144	0.649	0.000	2.660	0.727	1.349

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	24	26	0	26	26	26
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	0.93
time (sec)	N/A	0.444	0.881	0.147	0.656	0.000	1.236	1.638	1.342

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	24	26	0	26	26	26
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	0.93
time (sec)	N/A	0.451	1.492	0.122	0.692	0.000	9.710	1.013	1.358

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	24	26	0	26	26	26
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	0.93
time (sec)	N/A	0.211	3.035	0.123	0.648	0.000	0.979	0.344	1.340

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	24	26	0	27	26	26
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.96	0.93	0.93
time (sec)	N/A	0.211	0.145	0.122	0.652	0.000	2.830	0.370	1.366

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	24	26	0	27	26	26
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.96	0.93	0.93
time (sec)	N/A	0.213	0.074	0.122	0.666	0.000	26.944	0.434	1.485

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	81	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.188	0.208	0.121	0.291	0.308	31.020	0.339	1.204

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	131	46	0	26	26
N.S.	1	1.00	1.08	1.00	5.46	1.92	0.00	1.08	1.08
time (sec)	N/A	0.190	5.811	0.006	0.313	0.302	0.000	0.416	1.307

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	48	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	1.85	0.00	1.00	1.00
time (sec)	N/A	0.212	9.498	0.148	0.672	0.302	0.000	1.856	1.409

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	26	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.209	0.047	0.129	0.659	0.307	0.000	0.780	1.314

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	26	24	26	26
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	1.00	1.00
time (sec)	N/A	0.205	7.629	0.125	0.630	0.315	4.596	0.387	1.353

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	62	0	26	26
N.S.	1	1.00	1.08	0.92	1.00	2.38	0.00	1.00	1.00
time (sec)	N/A	0.208	8.524	0.127	0.655	0.363	0.000	0.447	1.628

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	26	0	0	26
N.S.	1	1.00	1.08	1.00	0.00	1.08	0.00	0.00	1.08
time (sec)	N/A	0.184	0.256	0.151	0.000	0.343	0.000	0.000	1.285

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	474	474	343	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.999	0.991	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	348	262	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.749	0.244	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	181	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	103	0	0	60	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.256	0.043	0.000	0.000	0.105	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	26	20	26	26
N.S.	1	1.00	1.08	1.00	0.00	1.08	0.83	1.08	1.08
time (sec)	N/A	0.182	0.147	0.127	0.000	0.285	1.873	0.435	1.284

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	265	589	554	757	478	682	572	661
N.S.	1	0.84	1.87	1.76	2.40	1.52	2.17	1.82	2.10
time (sec)	N/A	0.494	0.338	0.895	0.229	0.281	0.844	0.508	1.627

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	207	375	356	539	308	427	369	393
N.S.	1	0.85	1.54	1.46	2.21	1.26	1.75	1.51	1.61
time (sec)	N/A	0.462	0.194	0.649	0.233	0.276	0.589	0.307	1.543

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	141	214	202	351	170	226	209	208
N.S.	1	0.90	1.36	1.29	2.24	1.08	1.44	1.33	1.32
time (sec)	N/A	0.409	0.102	0.648	0.227	0.303	0.402	0.313	1.430

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	70	66	92	201	71	85	92	100
N.S.	1	0.89	0.84	1.16	2.54	0.90	1.08	1.16	1.27
time (sec)	N/A	0.332	0.038	0.566	0.233	0.301	0.217	0.298	1.746

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	35	101	34	31	35	31
N.S.	1	1.00	1.00	1.30	3.74	1.26	1.15	1.30	1.15
time (sec)	N/A	0.209	0.004	0.504	0.213	0.298	0.067	0.306	1.413

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	91	177	0	0	0	0	0
N.S.	1	0.98	1.05	2.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.047	0.968	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	189	141	317	0	0	0	0	0
N.S.	1	1.25	0.93	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	0.104	1.072	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	335	226	510	0	0	0	0	0
N.S.	1	1.34	0.90	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.075	0.151	1.198	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	838	374	1135	1427	939	1479	1213	1346
N.S.	1	1.45	0.65	1.96	2.46	1.62	2.55	2.09	2.32
time (sec)	N/A	2.838	0.367	0.918	0.266	0.303	1.884	0.327	2.070

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	521	267	721	964	606	918	769	803
N.S.	1	1.12	0.58	1.55	2.08	1.31	1.98	1.66	1.73
time (sec)	N/A	1.685	0.208	0.679	0.275	0.298	1.100	0.309	1.826

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	262	171	397	586	336	473	420	408
N.S.	1	1.10	0.72	1.67	2.46	1.41	1.99	1.76	1.71
time (sec)	N/A	0.872	0.108	0.659	0.245	0.284	0.602	0.316	1.642

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	94	89	176	304	141	175	178	163
N.S.	1	0.83	0.79	1.56	2.69	1.25	1.55	1.58	1.44
time (sec)	N/A	0.435	0.043	0.575	0.228	0.279	0.338	0.315	1.503

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	54	128	53	51	59	50
N.S.	1	1.00	1.00	2.00	4.74	1.96	1.89	2.19	1.85
time (sec)	N/A	0.225	0.005	0.503	0.222	0.266	0.092	0.384	1.637

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	126	189	334	331	0	0	0	0
N.S.	1	0.89	1.33	2.35	2.33	0.00	0.00	0.00	0.00
time (sec)	N/A	0.578	0.172	0.942	0.267	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	275	360	0	622	0	0	0	0
N.S.	1	1.01	1.32	0.00	2.28	0.00	0.00	0.00	0.00
time (sec)	N/A	1.226	0.397	0.000	0.295	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	485	530	664	0	1271	0	0	0	0
N.S.	1	1.09	1.37	0.00	2.62	0.00	0.00	0.00	0.00
time (sec)	N/A	2.308	0.680	0.000	0.381	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	230	207	0	567	0	401	0	0	0
N.S.	1	0.90	0.00	2.47	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.719	0.000	3.017	0.000	0.296	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	177	160	0	361	0	260	0	0	0
N.S.	1	0.90	0.00	2.04	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.606	0.000	2.457	0.000	0.305	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	113	112	200	0	149	0	0	0
N.S.	1	0.91	0.90	1.61	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.512	0.238	2.124	0.000	0.313	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	69	88	0	70	0	0	0
N.S.	1	0.93	0.97	1.24	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.383	0.112	1.548	0.000	0.300	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	29	24	17	24	23
N.S.	1	1.00	1.00	1.04	1.26	1.04	0.74	1.04	1.00
time (sec)	N/A	0.223	0.038	0.524	0.458	0.290	0.087	0.309	2.048

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	68	99	34	34
N.S.	1	1.00	1.06	1.00	1.06	2.12	3.09	1.06	1.06
time (sec)	N/A	0.406	0.126	0.822	0.254	0.276	2.611	0.368	1.330

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	122	180	34	34
N.S.	1	1.00	1.06	1.00	1.06	3.81	5.62	1.06	1.06
time (sec)	N/A	0.472	0.160	0.618	0.248	0.282	4.979	0.370	1.493

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	485	969	818	0	0	0	0	0	0
N.S.	1	2.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.000	0.723	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	417	725	644	0	0	0	0	0	0
N.S.	1	1.74	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.836	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	519	534	0	0	0	0	0	0
N.S.	1	1.49	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.872	0.332	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	355	281	0	0	0	0	0	0
N.S.	1	1.39	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.140	0.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	491	526	0	0	0	0	0	0
N.S.	1	1.44	1.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.856	0.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	406	671	608	0	0	0	0	0	0
N.S.	1	1.65	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.775	0.509	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	695	556	416	0	0	0	0	0
N.S.	1	1.82	1.46	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.857	0.593	1.047	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	499	534	256	0	0	0	0	0
N.S.	1	1.54	1.65	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.909	0.232	0.899	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	345	369	155	0	0	0	0	0
N.S.	1	1.43	1.52	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.194	0.146	1.023	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	471	547	243	0	0	0	0	0
N.S.	1	1.49	1.73	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.855	0.250	0.901	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	641	568	370	0	0	0	0	0
N.S.	1	1.72	1.53	0.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.640	0.395	0.928	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	35	0	0	34
N.S.	1	1.00	1.06	1.00	1.06	1.09	0.00	0.00	1.06
time (sec)	N/A	0.258	0.172	0.236	0.254	0.312	0.000	0.000	1.630

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	305	288	242	0	0	0	0	0	0
N.S.	1	0.94	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	1.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	199	185	0	0	0	0	0	0
N.S.	1	0.95	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	0.375	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	110	114	0	0	118	0	0	0
N.S.	1	0.96	0.99	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.423	0.233	0.000	0.000	0.100	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	32	41	0	32	31
N.S.	1	1.00	1.00	1.03	1.03	1.32	0.00	1.03	1.00
time (sec)	N/A	0.238	0.010	0.680	0.215	0.293	0.000	0.320	1.546

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	43	37	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.34	1.16	1.06	1.06
time (sec)	N/A	0.267	0.190	0.282	0.280	0.285	154.494	0.349	1.454

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	68	0	0	34
N.S.	1	1.00	1.06	1.00	1.06	2.12	0.00	0.00	1.06
time (sec)	N/A	0.265	0.509	0.247	0.313	0.317	0.000	0.000	1.497

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	92	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	2.88	0.00	1.06	1.06
time (sec)	N/A	0.266	0.580	0.237	0.344	0.310	0.000	0.383	1.498

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	402	402	379	1208	0	0	0	0	0
N.S.	1	1.00	0.94	3.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.626	0.352	2.030	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	241	241	224	721	0	0	0	0	0
N.S.	1	1.00	0.93	2.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	0.172	1.244	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	119	110	394	0	0	0	0	0
N.S.	1	1.00	0.92	3.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.073	0.778	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	62	217	0	0	0	0	0
N.S.	1	1.00	0.98	3.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.003	0.566	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	111	378	0	0	0	0	0
N.S.	1	1.00	0.72	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	0.046	1.118	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	252	252	196	494	0	0	0	0	0
N.S.	1	1.00	0.78	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	0.115	1.724	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	402	402	311	771	0	0	0	0	0
N.S.	1	1.00	0.77	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.668	0.202	3.169	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	469	469	876	0	0	0	0	0	0
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.826	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	460	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.239	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	111	106	194	737	0	0	0	0	0
N.S.	1	0.95	1.75	6.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	0.042	0.732	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	264	264	353	1427	0	0	0	0	0
N.S.	1	1.00	1.34	5.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.600	0.175	2.905	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	427	630	0	0	0	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.807	0.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	660	660	1521	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.064	0.511	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	799	0	0	0	0	0	0
N.S.	1	1.00	2.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	0.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	158	149	335	1396	0	0	0	0	0
N.S.	1	0.94	2.12	8.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.563	0.072	1.056	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	372	372	599	2696	0	0	0	0	0
N.S.	1	1.00	1.61	7.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.784	0.256	5.395	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	602	602	1025	0	0	0	0	0	0
N.S.	1	1.00	1.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.104	0.724	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	37	24	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.28	0.83	1.07	1.07
time (sec)	N/A	0.387	0.099	0.046	0.294	0.281	2.601	0.336	1.313

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	31	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.29	0.83	1.08	1.08
time (sec)	N/A	0.199	0.007	0.010	0.260	0.318	1.232	0.442	1.260

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	61	26	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.97	0.84	1.06	1.06
time (sec)	N/A	0.378	0.531	0.244	0.271	0.290	2.778	0.323	1.283

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	111	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	3.58	0.00	1.06	1.06
time (sec)	N/A	0.423	1.502	0.034	0.279	0.308	0.000	0.331	1.256

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	223	69	26	31	31
N.S.	1	1.00	1.07	1.00	7.69	2.38	0.90	1.07	1.07
time (sec)	N/A	0.409	0.495	0.014	0.331	0.309	7.959	0.323	1.429

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	188	63	22	26	26
N.S.	1	1.00	1.08	1.00	7.83	2.62	0.92	1.08	1.08
time (sec)	N/A	0.187	0.030	0.013	0.282	0.296	2.785	0.327	1.200

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	488	118	27	33	33
N.S.	1	1.00	1.06	1.00	15.74	3.81	0.87	1.06	1.06
time (sec)	N/A	0.359	4.729	0.033	0.294	0.334	8.398	0.378	1.335

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	723	203	29	33	33
N.S.	1	1.00	1.06	1.00	23.32	6.55	0.94	1.06	1.06
time (sec)	N/A	0.426	9.877	122.391	0.314	0.332	42.429	0.416	1.323

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	281	281	241	507	0	0	0	0	0
N.S.	1	1.00	0.86	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	0.157	0.935	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	181	181	170	388	0	0	0	0	0
N.S.	1	1.00	0.94	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.088	0.805	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	104	104	95	284	0	0	0	0	0
N.S.	1	1.00	0.91	2.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.051	0.799	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	62	217	0	0	0	0	0
N.S.	1	1.00	0.98	3.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.003	0.571	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	107	85	276	0	0	0	0	0
N.S.	1	1.00	0.79	2.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.028	0.644	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	162	141	338	0	0	0	0	0
N.S.	1	1.00	0.87	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	0.057	0.813	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	250	208	429	0	0	0	0	0
N.S.	1	1.00	0.83	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	0.141	0.965	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	265	265	220	550	0	0	0	0	0
N.S.	1	1.00	0.83	2.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	0.179	0.945	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	186	186	153	435	0	0	0	0	0
N.S.	1	1.00	0.82	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	0.088	0.836	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	138	138	114	311	0	0	0	0	0
N.S.	1	1.00	0.83	2.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	0.060	0.795	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	70	57	127	85	95	333	88	84
N.S.	1	0.95	0.77	1.72	1.15	1.28	4.50	1.19	1.14
time (sec)	N/A	0.199	0.034	0.625	0.200	0.318	3.122	0.314	1.569

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	179	179	152	354	0	0	0	0	0
N.S.	1	1.00	0.85	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.069	0.735	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	240	240	199	437	0	0	0	0	0
N.S.	1	1.00	0.83	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.120	1.124	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	335	335	269	549	0	0	0	0	0
N.S.	1	1.00	0.80	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.248	1.658	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	397	397	331	549	0	0	0	0	0
N.S.	1	1.00	0.83	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	0.191	1.695	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	278	278	243	431	0	0	0	0	0
N.S.	1	1.00	0.87	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	0.106	0.938	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	203	203	172	356	0	0	0	0	0
N.S.	1	1.00	0.85	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.028	0.682	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	245	245	224	415	0	0	0	0	0
N.S.	1	1.00	0.91	1.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	0.060	0.663	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	331	331	279	499	0	0	0	0	0
N.S.	1	1.00	0.84	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	0.108	1.200	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	369	369	339	606	0	0	0	0	0
N.S.	1	1.00	0.92	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.632	0.197	1.168	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	276	276	263	491	0	0	0	0	0
N.S.	1	1.00	0.95	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.547	0.087	0.832	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	239	239	184	405	0	0	0	0	0
N.S.	1	1.00	0.77	1.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.034	0.750	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	290	290	280	492	0	0	0	0	0
N.S.	1	1.00	0.97	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	0.121	1.156	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	388	388	350	598	0	0	0	0	0
N.S.	1	1.00	0.90	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	0.232	1.752	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	417	417	530	624	0	0	0	0	0
N.S.	1	1.00	1.27	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.783	0.926	1.568	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	344	344	455	496	0	0	0	0	0
N.S.	1	1.00	1.32	1.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.683	0.728	1.177	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	120	165	969	130	373	0	194	366
N.S.	1	0.86	1.19	6.97	0.94	2.68	0.00	1.40	2.63
time (sec)	N/A	0.270	0.111	1.362	0.292	0.356	0.000	0.317	1.837

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	383	383	521	556	0	0	0	0	0
N.S.	1	1.00	1.36	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.747	0.817	0.910	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	460	460	596	656	0	0	0	0	0
N.S.	1	1.00	1.30	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.845	1.013	2.913	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	534	534	433	1619	0	0	0	0	0
N.S.	1	1.00	0.81	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.124	0.467	1.317	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	491	491	382	1521	0	0	0	0	0
N.S.	1	1.00	0.78	3.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.003	0.369	1.306	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	503	503	407	1406	0	0	0	0	0
N.S.	1	1.00	0.81	2.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.710	0.549	1.313	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	560	560	475	1619	0	0	0	0	0
N.S.	1	1.00	0.85	2.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.109	0.456	2.247	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	313	275	0	0	0	0	0	0
N.S.	1	0.96	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.865	0.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	506	349	0	0	0	0	0	0	0
N.S.	1	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.990	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	295	307	0	0	0	0	0	0
N.S.	1	1.06	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.864	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	510	336	631	0	0	0	0	0	0
N.S.	1	0.66	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.051	10.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	20	120	21	0	0	19
N.S.	1	1.00	1.12	0.83	5.00	0.88	0.00	0.00	0.79
time (sec)	N/A	0.193	0.006	0.642	0.209	0.329	0.000	0.000	1.321

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	81	62	119	0	0	0	0
N.S.	1	1.00	1.93	1.48	2.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.013	0.621	0.200	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	82	44	0	43	0	0	43
N.S.	1	1.00	2.00	1.07	0.00	1.05	0.00	0.00	1.05
time (sec)	N/A	0.257	0.025	0.665	0.000	0.316	0.000	0.000	1.368

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	80	85	0	0	0	0	0
N.S.	1	1.00	1.70	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.020	0.649	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	376	153	0	0	0	0	0
N.S.	1	1.00	1.01	0.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.742	0.175	0.647	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	297	77	0	0	0	0	0
N.S.	1	1.00	1.02	0.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	0.028	0.556	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	330	106	0	0	0	0	0
N.S.	1	1.00	1.02	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.051	0.667	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	405	186	0	0	0	0	0
N.S.	1	1.00	0.98	0.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.699	0.121	0.728	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	403	148	0	0	0	0	0
N.S.	1	1.00	0.97	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.826	0.227	0.580	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	369	127	0	0	0	0	0
N.S.	1	1.00	0.96	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.646	0.094	0.573	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	297	86	0	0	0	0	0
N.S.	1	1.00	0.83	0.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.547	0.052	0.563	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	294	94	0	0	0	0	0
N.S.	1	1.00	0.82	0.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.485	0.048	0.566	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	378	124	0	0	0	0	0
N.S.	1	1.00	0.95	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.099	0.638	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	371	150	0	0	0	0	0
N.S.	1	1.00	0.88	0.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	0.146	0.629	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	480	175	0	0	0	0	0
N.S.	1	1.00	0.96	0.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.979	0.151	0.753	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	383	85	0	0	0	0	0
N.S.	1	1.00	0.96	0.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.731	0.045	0.596	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	416	114	0	0	0	0	0
N.S.	1	1.00	0.96	0.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.834	0.063	0.651	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	530	484	163	0	0	0	0	0
N.S.	1	1.00	0.91	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.869	0.160	0.641	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	348	102	0	0	0	0	0
N.S.	1	1.00	0.74	0.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	0.055	0.611	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	506	158	0	0	0	0	0
N.S.	1	1.00	0.94	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.896	0.129	0.657	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	521	521	458	145	0	0	0	0	0
N.S.	1	1.00	0.88	0.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.961	0.148	0.656	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	464	94	0	0	0	0	0
N.S.	1	1.00	0.93	0.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.791	0.195	0.573	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	359	112	0	0	0	0	0
N.S.	1	1.00	0.72	0.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.688	0.066	0.582	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	536	132	0	0	0	0	0
N.S.	1	1.00	1.00	0.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.011	0.579	0.654	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	89	95	98	102	117	134	183	104
N.S.	1	0.98	1.04	1.08	1.12	1.29	1.47	2.01	1.14
time (sec)	N/A	0.260	0.040	0.335	0.200	0.299	0.636	0.300	1.282

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	113	150	265	187	219	252	424	212
N.S.	1	0.94	1.25	2.21	1.56	1.82	2.10	3.53	1.77
time (sec)	N/A	0.280	0.099	0.457	0.222	0.328	4.497	0.312	1.285

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	137	226	428	284	336	410	770	352
N.S.	1	0.92	1.52	2.87	1.91	2.26	2.75	5.17	2.36
time (sec)	N/A	0.294	0.142	0.721	0.206	0.307	19.603	0.325	1.388

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	62	217	0	0	0	0	0
N.S.	1	1.00	0.98	3.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.011	0.242	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	70	57	127	86	95	333	88	84
N.S.	1	0.95	0.77	1.72	1.16	1.28	4.50	1.19	1.14
time (sec)	N/A	0.216	0.045	0.351	0.192	0.322	34.401	0.360	1.990

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	96	83	283	169	272	0	201	173
N.S.	1	0.86	0.74	2.53	1.51	2.43	0.00	1.79	1.54
time (sec)	N/A	0.273	0.061	0.612	0.205	0.330	0.000	0.345	1.717

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	247	220	298	0	0	0	0
N.S.	1	1.00	1.00	0.89	1.21	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.126	0.362	0.336	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	831	831	862	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.455	0.887	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	499	499	637	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.942	0.485	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	317	464	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	0.318	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	397	397	576	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.863	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	551	551	811	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.191	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	701	701	821	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.205	1.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	623	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.849	0.687	0.000	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	485	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.475	0.000	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	461	461	668	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.854	0.529	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	694	694	930	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.329	0.863	0.000	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	936	936	1254	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.769	1.802	0.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	739	739	1103	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.392	1.512	0.000	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	430	409	590	1231	0	0	0	0	0
N.S.	1	0.95	1.37	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.769	0.493	1.542	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	814	814	1209	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.536	1.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	970	970	1391	0	0	0	0	0	0
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.906	1.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	897	897	1237	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.900	3.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	815	815	1132	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.723	3.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	821	821	1143	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.237	3.375	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	919	919	1304	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.840	3.683	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	477	477	754	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.796	0.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	347	347	488	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.579	0.253	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	178	392	0	0	0	0	0
N.S.	1	1.00	0.78	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	0.088	0.746	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	19	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.86	1.09	1.09
time (sec)	N/A	0.290	0.394	0.150	0.276	0.300	21.008	0.316	1.271

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	33	0	0	0
N.S.	1	1.00	1.07	1.00	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.296	0.019	2.548	0.000	0.350	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	33	0	0	0
N.S.	1	1.00	1.07	1.00	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.361	0.003	2.303	0.000	0.319	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	30	0	35	0	0	0
N.S.	1	1.00	1.11	1.07	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.299	0.018	2.878	0.000	0.328	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	30	0	35	0	0	0
N.S.	1	1.00	1.11	1.07	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.373	0.003	2.796	0.000	0.306	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	20	120	21	0	0	19
N.S.	1	1.00	1.12	0.83	5.00	0.88	0.00	0.00	0.79
time (sec)	N/A	0.210	0.005	0.713	0.199	0.304	0.000	0.000	1.284

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	20	120	21	0	0	19
N.S.	1	1.00	1.12	0.83	5.00	0.88	0.00	0.00	0.79
time (sec)	N/A	0.308	0.002	0.831	0.211	0.291	0.000	0.000	1.203

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	252	24	246	34	0	0	0
N.S.	1	1.00	6.81	0.65	6.65	0.92	0.00	0.00	0.00
time (sec)	N/A	0.182	0.143	0.925	0.213	0.327	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	37	252	24	246	34	0	0	0
N.S.	1	1.37	9.33	0.89	9.11	1.26	0.00	0.00	0.00
time (sec)	N/A	0.247	0.122	0.938	0.210	0.327	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	259	24	243	34	0	0	0
N.S.	1	1.00	9.59	0.89	9.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.166	0.199	0.921	0.210	0.312	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	37	259	24	243	34	0	0	0
N.S.	1	1.37	9.59	0.89	9.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.319	0.135	0.955	0.206	0.309	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	238	238	494	1756	0	0	0	0	0
N.S.	1	1.00	2.08	7.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.144	2.022	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	168	292	761	0	0	0	0	0
N.S.	1	1.00	1.74	4.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	0.086	1.280	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	98	156	123	0	0	0	0
N.S.	1	1.00	1.01	1.61	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.020	0.700	0.237	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	19	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.79	1.08	1.08
time (sec)	N/A	0.283	0.423	0.154	0.262	0.307	0.976	0.304	1.168

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	500	500	993	0	0	0	0	0	0
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	0.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	372	655	0	0	0	0	0	0
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	0.371	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	243	243	194	616	0	0	0	0	0
N.S.	1	1.00	0.80	2.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	0.157	1.289	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.362	0.620	0.623	0.257	0.290	0.000	0.338	1.200

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	464	766	0	0	0	0	0
N.S.	1	1.00	1.62	2.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	0.458	1.212	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	434	543	0	0	0	0	0
N.S.	1	1.00	1.85	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	0.221	1.101	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	210	361	0	0	0	0	0
N.S.	1	1.00	1.09	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	0.095	1.053	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	144	169	0	0	0	0	0
N.S.	1	1.00	0.94	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.051	1.094	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	227	370	0	0	0	0	0
N.S.	1	1.00	1.11	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.160	1.066	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	255	552	0	0	0	0	0
N.S.	1	1.00	1.02	2.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	0.265	1.178	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	311	782	0	0	0	0	0
N.S.	1	1.00	1.01	2.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	0.320	1.155	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	261	221	1180	231	0	0	0	0
N.S.	1	1.12	0.95	5.09	1.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	0.144	78.149	0.232	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	195	217	197	1012	207	0	0	0	0
N.S.	1	1.11	1.01	5.19	1.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	0.131	32.940	0.246	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	158	169	164	843	178	0	0	0	0
N.S.	1	1.07	1.04	5.34	1.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.101	13.040	0.237	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	99	112	116	657	139	0	0	0	0
N.S.	1	1.13	1.17	6.64	1.40	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.077	5.247	0.235	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	88	97	128	756	0	0	0	0	0
N.S.	1	1.10	1.45	8.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.073	4.598	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	102	99	111	709	162	0	0	0	0
N.S.	1	0.97	1.09	6.95	1.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.098	5.356	0.252	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	156	147	204	901	198	0	0	0	0
N.S.	1	0.94	1.31	5.78	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.104	5.392	0.243	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	194	240	1070	229	0	0	0	0
N.S.	1	1.01	1.24	5.54	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.720	0.114	13.378	0.241	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	230	241	273	1237	253	0	0	0	0
N.S.	1	1.05	1.19	5.38	1.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.967	0.118	32.955	0.240	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	705	845	976	0	0	0	0	0	0
N.S.	1	1.20	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.146	0.287	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	602	590	825	0	0	0	0	0	0
N.S.	1	0.98	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.493	0.170	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	301	457	0	0	0	0	0	0
N.S.	1	0.97	1.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	823	0	823	0	0	0	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	607	0	513	0	0	0	0	0	0
N.S.	1	0.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.471	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	939	0	781	0	0	0	0	0	0
N.S.	1	0.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	522	509	1163	0	0	0	0	0	0
N.S.	1	0.98	2.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.071	0.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	519	0	519	0	0	0	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.09
time (sec)	N/A	0.163	0.049	0.038	0.302	0.308	2.757	0.323	1.213

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	108	45	22	25	25
N.S.	1	1.00	1.09	1.00	4.70	1.96	0.96	1.09	1.09
time (sec)	N/A	0.164	0.346	0.033	0.384	0.297	31.823	0.311	1.235

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	0	25	0	25	25
N.S.	1	1.00	1.09	1.00	0.00	1.09	0.00	1.09	1.09
time (sec)	N/A	0.162	0.066	0.049	0.000	0.296	0.000	0.452	1.305

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	364	364	394	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	226	205	354	274	329	384	741	323
N.S.	1	0.88	0.79	1.37	1.06	1.28	1.49	2.87	1.25
time (sec)	N/A	0.510	0.128	0.655	0.195	0.287	1.175	0.314	1.466

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	177	152	284	224	256	296	467	203
N.S.	1	0.90	0.78	1.45	1.14	1.31	1.51	2.38	1.04
time (sec)	N/A	0.456	0.079	0.565	0.204	0.294	0.646	0.315	1.453

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	76	116	165	156	189	209	102
N.S.	1	1.06	0.69	1.05	1.50	1.42	1.72	1.90	0.93
time (sec)	N/A	0.427	0.017	0.399	0.211	0.285	0.299	0.338	1.343

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	158	197	227	626	0	0	0	0	0
N.S.	1	1.25	1.44	3.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	0.074	0.757	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	96	97	180	517	0	0	0	0	0
N.S.	1	1.01	1.88	5.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.025	0.373	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	156	163	251	589	0	0	0	0	0
N.S.	1	1.04	1.61	3.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	0.086	0.389	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	241	327	642	0	0	0	0	0
N.S.	1	1.03	1.40	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.054	0.117	0.441	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	742	850	605	2094	0	0	0	0	0
N.S.	1	1.15	0.82	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.255	0.715	0.065	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	558	612	492	1724	0	0	0	0	0
N.S.	1	1.10	0.88	3.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.960	0.501	234.845	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	397	412	341	1372	0	0	0	0	0
N.S.	1	1.04	0.86	3.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.742	0.329	111.258	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	249	329	1012	0	0	0	0	0
N.S.	1	1.07	1.42	4.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	0.139	49.443	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	637	553	605	0	0	0	0	0	0
N.S.	1	0.87	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.258	0.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	256	476	0	0	0	0	0	0
N.S.	1	0.95	1.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.612	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	421	374	765	0	0	0	0	0	0
N.S.	1	0.89	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	0.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1210	1273	2067	0	0	0	0	0	0
N.S.	1	1.05	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.506	0.582	0.000	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	649	702	1355	0	0	0	0	0	0
N.S.	1	1.08	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.033	0.318	0.000	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	168	93	0	36	36
N.S.	1	1.00	1.06	1.00	4.94	2.74	0.00	1.06	1.06
time (sec)	N/A	0.194	0.664	0.098	0.464	0.360	0.000	0.367	1.881

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	207	93	0	36	36
N.S.	1	1.00	1.06	1.00	6.09	2.74	0.00	1.06	1.06
time (sec)	N/A	0.198	0.814	0.187	0.515	0.325	0.000	0.505	2.328

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2050	2278	4971	0	0	0	0	0	0
N.S.	1	1.11	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	7.341	1.318	0.000	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1147	1248	3163	0	0	0	0	0	0
N.S.	1	1.09	2.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.712	0.645	0.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	292	135	0	36	36
N.S.	1	1.00	1.06	1.00	8.59	3.97	0.00	1.06	1.06
time (sec)	N/A	0.196	1.454	0.108	0.513	0.309	0.000	0.601	2.791

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	335	135	0	36	36
N.S.	1	1.00	1.06	1.00	9.85	3.97	0.00	1.06	1.06
time (sec)	N/A	0.196	2.144	0.120	0.558	0.316	0.000	0.594	2.745

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	64	62	0	0	0	0	0	0
N.S.	1	0.97	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	43	54	99	46	56	90	63
N.S.	1	1.05	0.47	0.59	1.08	0.50	0.61	0.98	0.68
time (sec)	N/A	0.289	0.070	0.577	0.204	0.288	0.135	0.324	1.583

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	108	58	70	159	61	75	113	92
N.S.	1	1.06	0.57	0.69	1.56	0.60	0.74	1.11	0.90
time (sec)	N/A	0.302	0.089	0.569	0.202	0.295	0.156	0.325	1.586

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	143	132	642	559	1409	609	1697	380
N.S.	1	0.89	0.82	4.01	3.49	8.81	3.81	10.61	2.38
time (sec)	N/A	0.495	0.051	5.581	0.227	0.324	2.560	0.346	1.838

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	110	100	386	317	639	360	772	242
N.S.	1	0.91	0.83	3.19	2.62	5.28	2.98	6.38	2.00
time (sec)	N/A	0.415	0.011	1.512	0.242	0.323	1.138	0.321	1.513

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	77	69	165	148	231	178	283	111
N.S.	1	0.99	0.88	2.12	1.90	2.96	2.28	3.63	1.42
time (sec)	N/A	0.335	0.009	0.417	0.211	0.331	0.516	0.333	1.361

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	42	45	50	53	59	41
N.S.	1	1.00	1.00	1.24	1.32	1.47	1.56	1.74	1.21
time (sec)	N/A	0.184	0.007	0.083	0.195	0.304	0.219	0.337	1.320

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	85	83	0	0	65	0	78	0
N.S.	1	1.02	1.00	0.00	0.00	0.78	0.00	0.94	0.00
time (sec)	N/A	0.407	0.094	0.000	0.000	0.311	0.000	0.335	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	124	163	0	0	171	0	582	0
N.S.	1	1.01	1.33	0.00	0.00	1.39	0.00	4.73	0.00
time (sec)	N/A	0.500	0.076	0.000	0.000	0.299	0.000	0.336	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	177	189	0	0	444	0	3401	0
N.S.	1	1.05	1.12	0.00	0.00	2.63	0.00	20.12	0.00
time (sec)	N/A	0.606	0.120	0.000	0.000	0.313	0.000	0.381	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	219	214	190	0	0	0	0	0	0
N.S.	1	0.98	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.824	0.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	177	160	0	0	0	0	0	0
N.S.	1	1.01	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.698	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	140	134	0	0	0	0	0	0
N.S.	1	1.01	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	106	104	0	0	0	0	0	0
N.S.	1	1.02	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	148	181	0	0	0	0	0	0
N.S.	1	1.01	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.646	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	203	211	0	0	0	0	0	0
N.S.	1	1.05	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	258	272	0	0	0	0	0	0
N.S.	1	1.09	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.907	0.279	0.000	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	135	131	0	0	80	0	0	0
N.S.	1	1.03	1.00	0.00	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.449	0.143	0.000	0.000	0.094	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	106	109	0	70	0	0	0	0
N.S.	1	0.97	1.00	0.00	0.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	0.147	0.000	0.061	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	141	232	466	304	405	457	987	370
N.S.	1	0.89	1.47	2.95	1.92	2.56	2.89	6.25	2.34
time (sec)	N/A	0.414	0.210	5.982	0.201	0.364	2.503	0.340	1.541

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	117	156	290	202	268	286	544	225
N.S.	1	0.91	1.22	2.27	1.58	2.09	2.23	4.25	1.76
time (sec)	N/A	0.371	0.127	1.970	0.191	0.305	1.229	0.362	1.592

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	93	113	173	112	148	156	236	113
N.S.	1	0.95	1.15	1.77	1.14	1.51	1.59	2.41	1.15
time (sec)	N/A	0.317	0.038	0.438	0.200	0.324	0.587	0.323	1.444

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	42	45	50	53	59	41
N.S.	1	1.00	1.00	1.24	1.32	1.47	1.56	1.74	1.21
time (sec)	N/A	0.183	0.007	0.102	0.183	0.305	0.208	0.322	1.347

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	67	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	75	69	139	90	113	357	96	89
N.S.	1	0.94	0.86	1.74	1.12	1.41	4.46	1.20	1.11
time (sec)	N/A	0.290	0.075	1.336	0.194	0.279	3.558	0.321	3.316

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	100	88	305	172	310	1984	221	180
N.S.	1	0.84	0.74	2.56	1.45	2.61	16.67	1.86	1.51
time (sec)	N/A	0.369	0.094	3.759	0.197	0.307	15.248	0.402	3.492

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	126	115	487	306	563	5673	443	293
N.S.	1	0.85	0.77	3.27	2.05	3.78	38.07	2.97	1.97
time (sec)	N/A	0.414	0.123	11.925	0.208	0.319	63.372	0.373	3.742

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	343	400	1537	895	1742	1421	3738	1154
N.S.	1	0.84	0.98	3.76	2.19	4.26	3.47	9.14	2.82
time (sec)	N/A	1.143	0.196	16.244	0.228	0.354	5.391	0.387	1.964

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	278	277	939	605	1137	894	2106	652
N.S.	1	0.86	0.86	2.91	1.87	3.52	2.77	6.52	2.02
time (sec)	N/A	1.002	0.110	5.678	0.218	0.340	2.594	0.338	1.741

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	164	540	348	622	466	939	302
N.S.	1	1.00	0.78	2.56	1.65	2.95	2.21	4.45	1.43
time (sec)	N/A	0.642	0.059	1.533	0.214	0.328	1.244	0.324	1.497

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	77	69	165	148	231	178	283	111
N.S.	1	0.99	0.88	2.12	1.90	2.96	2.28	3.63	1.42
time (sec)	N/A	0.326	0.013	0.423	0.202	0.287	0.470	0.315	1.309

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	117	324	0	0	0	0	0	0
N.S.	1	0.95	2.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.684	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	128	200	0	0	0	0	0	0
N.S.	1	0.89	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	0.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	247	316	0	0	0	0	0	0
N.S.	1	1.11	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.184	0.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	378	2034	1245	3121	1846	5146	1400
N.S.	1	1.00	0.77	4.13	2.53	6.34	3.75	10.46	2.85
time (sec)	N/A	1.258	0.193	16.190	0.247	0.381	5.381	0.420	2.342

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	231	1084	732	1692	991	2532	651
N.S.	1	1.00	0.75	3.54	2.39	5.53	3.24	8.27	2.13
time (sec)	N/A	0.824	0.093	5.224	0.232	0.385	2.585	0.379	1.911

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	110	100	386	317	639	360	772	242
N.S.	1	0.91	0.83	3.19	2.62	5.28	2.98	6.38	2.00
time (sec)	N/A	0.402	0.018	1.511	0.201	0.299	1.020	0.324	1.428

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	166	646	0	0	0	0	0	0
N.S.	1	0.94	3.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.903	0.228	0.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	177	444	0	0	0	0	0	0
N.S.	1	0.85	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.889	0.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	376	354	660	0	0	0	0	0	0
N.S.	1	0.94	1.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.204	0.555	0.000	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	143	132	642	559	1409	609	1697	380
N.S.	1	0.89	0.82	4.01	3.49	8.81	3.81	10.61	2.38
time (sec)	N/A	0.498	0.034	5.170	0.225	0.309	2.070	0.358	1.600

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	214	1095	0	0	0	0	0	0
N.S.	1	0.93	4.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.110	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	226	1301	0	0	0	0	0	0
N.S.	1	0.82	4.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.132	0.380	0.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	35	29	41	40	42	48	53	36
N.S.	1	1.21	1.00	1.41	1.38	1.45	1.66	1.83	1.24
time (sec)	N/A	0.243	0.006	0.086	0.194	0.312	0.207	0.328	0.077

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	252	0	0	243	0	517	0
N.S.	1	1.00	0.90	0.00	0.00	0.87	0.00	1.85	0.00
time (sec)	N/A	1.104	0.550	0.000	0.000	0.303	0.000	0.365	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	164	0	0	140	0	248	0
N.S.	1	1.00	0.92	0.00	0.00	0.78	0.00	1.39	0.00
time (sec)	N/A	0.725	0.117	0.000	0.000	0.311	0.000	0.348	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	85	83	0	0	65	0	78	0
N.S.	1	1.02	1.00	0.00	0.00	0.78	0.00	0.94	0.00
time (sec)	N/A	0.396	0.036	0.000	0.000	0.294	0.000	0.333	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	35	24	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.25	0.86	1.07	1.07
time (sec)	N/A	0.228	0.207	0.157	0.856	0.294	1.503	0.348	1.282

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	59	26	30	30
N.S.	1	1.00	1.07	1.00	1.07	2.11	0.93	1.07	1.07
time (sec)	N/A	0.225	0.478	0.188	0.858	0.303	3.772	0.347	1.244

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	538	1310	0	0	573	0	3975	0
N.S.	1	1.65	4.02	0.00	0.00	1.76	0.00	12.19	0.00
time (sec)	N/A	1.763	0.413	0.000	0.000	0.323	0.000	0.463	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	329	269	0	0	328	0	1930	0
N.S.	1	1.47	1.20	0.00	0.00	1.46	0.00	8.62	0.00
time (sec)	N/A	1.267	0.221	0.000	0.000	0.302	0.000	0.392	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	124	163	0	0	171	0	582	0
N.S.	1	1.01	1.33	0.00	0.00	1.39	0.00	4.73	0.00
time (sec)	N/A	0.495	0.074	0.000	0.000	0.308	0.000	0.342	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	267	71	26	30	30
N.S.	1	1.00	1.07	1.00	9.54	2.54	0.93	1.07	1.07
time (sec)	N/A	0.214	0.667	0.181	1.153	0.333	5.011	0.323	1.244

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	406	112	27	30	30
N.S.	1	1.00	1.07	1.00	14.50	4.00	0.96	1.07	1.07
time (sec)	N/A	0.217	8.431	0.177	1.136	0.318	31.297	0.359	1.270

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	432	951	438	0	0	1682	0	5889	0
N.S.	1	2.20	1.01	0.00	0.00	3.89	0.00	13.63	0.00
time (sec)	N/A	3.246	1.281	0.000	0.000	0.365	0.000	0.543	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	530	322	0	0	931	0	11278	0
N.S.	1	1.65	1.00	0.00	0.00	2.89	0.00	35.02	0.00
time (sec)	N/A	2.034	0.378	0.000	0.000	0.389	0.000	0.553	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	177	189	0	0	444	0	3401	0
N.S.	1	1.05	1.12	0.00	0.00	2.63	0.00	20.12	0.00
time (sec)	N/A	0.616	0.117	0.000	0.000	0.319	0.000	0.399	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1116	107	26	30	30
N.S.	1	1.00	1.07	1.00	39.86	3.82	0.93	1.07	1.07
time (sec)	N/A	0.219	0.553	0.173	1.570	0.319	17.311	0.348	1.298

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1486	165	27	30	30
N.S.	1	1.00	1.07	1.00	53.07	5.89	0.96	1.07	1.07
time (sec)	N/A	0.218	23.057	0.177	1.579	0.314	135.869	0.398	1.336

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	488	488	458	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.032	0.487	0.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	311	311	298	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.254	0.264	0.000	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	140	134	0	0	0	0	0	0
N.S.	1	1.01	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	0.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	26	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.87	1.00	1.00
time (sec)	N/A	0.249	4.541	0.161	11.956	0.000	0.608	0.429	1.360

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	27	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.90	1.00	1.00
time (sec)	N/A	0.304	0.257	0.164	10.690	0.000	2.502	0.402	1.416

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	625	625	545	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.474	0.936	0.000	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	348	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.498	0.386	0.000	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	177	160	0	0	0	0	0	0
N.S.	1	1.01	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.675	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	26	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.87	1.00	1.00
time (sec)	N/A	0.260	2.657	0.163	10.799	0.000	52.802	0.538	1.449

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	0	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.303	1.276	0.181	10.701	0.000	0.000	0.520	1.387

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	355	355	315	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.660	0.257	0.000	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	208	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.060	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	106	104	0	0	0	0	0	0
N.S.	1	1.02	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	27	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.90	1.00	1.00
time (sec)	N/A	0.258	0.079	0.191	10.492	0.000	1.499	0.490	1.351

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	404	666	1040	0	0	0	0	0	0
N.S.	1	1.65	2.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.805	1.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	402	435	0	0	0	0	0	0
N.S.	1	1.46	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.817	0.727	0.000	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	148	181	0	0	0	0	0	0
N.S.	1	1.01	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.646	0.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	27	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.90	1.00	1.00
time (sec)	N/A	0.281	0.323	0.191	10.750	0.000	24.357	0.511	1.565

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	514	1154	652	0	0	0	0	0	0
N.S.	1	2.25	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.179	3.733	0.000	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	380	632	491	0	0	0	0	0	0
N.S.	1	1.66	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.145	1.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	203	211	0	0	0	0	0	0
N.S.	1	1.05	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	0.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	0	0	30	30
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.275	0.568	0.185	12.525	0.000	0.000	0.835	1.702

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	166	153	0	0	624	0	0	0
N.S.	1	0.97	0.89	0.00	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.491	0.247	0.000	0.000	0.341	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	137	124	0	0	353	0	0	0
N.S.	1	0.99	0.89	0.00	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.404	0.127	0.000	0.000	0.344	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	104	89	118	0	201	0	124	0
N.S.	1	1.01	0.86	1.15	0.00	1.95	0.00	1.20	0.00
time (sec)	N/A	0.362	0.182	0.652	0.000	0.298	0.000	0.289	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	84	0	0	240	0	110	0
N.S.	1	1.00	0.98	0.00	0.00	2.79	0.00	1.28	0.00
time (sec)	N/A	0.335	0.113	0.000	0.000	0.336	0.000	0.310	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	115	91	0	0	467	0	184	0
N.S.	1	0.96	0.76	0.00	0.00	3.89	0.00	1.53	0.00
time (sec)	N/A	0.402	0.118	0.000	0.000	0.349	0.000	0.305	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	151	91	0	0	863	0	324	0
N.S.	1	0.99	0.60	0.00	0.00	5.68	0.00	2.13	0.00
time (sec)	N/A	0.437	0.063	0.000	0.000	0.368	0.000	0.344	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	187	91	0	0	1362	0	0	0
N.S.	1	1.02	0.49	0.00	0.00	7.40	0.00	0.00	0.00
time (sec)	N/A	0.499	0.070	0.000	0.000	0.410	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	635	1036	4354	0	0	0	0	0	0
N.S.	1	1.63	6.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.413	18.558	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	547	786	1323	0	0	0	0	0	0
N.S.	1	1.44	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.854	11.765	0.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	570	1407	0	0	0	0	0	0
N.S.	1	1.28	3.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.464	12.625	0.000	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	400	785	0	0	0	0	0	0
N.S.	1	1.21	2.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.421	2.540	0.000	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	546	1289	0	0	0	0	0	0
N.S.	1	1.22	2.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.522	12.812	0.000	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	537	732	1349	0	0	0	0	0	0
N.S.	1	1.36	2.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.756	9.704	0.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	625	969	1457	0	0	0	0	0	0
N.S.	1	1.55	2.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.398	11.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	30	26	30	30
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.87	1.00	1.00
time (sec)	N/A	0.240	0.524	0.191	0.935	0.302	33.766	0.399	1.242

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	30	26	30	30
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.87	1.00	1.00
time (sec)	N/A	0.227	0.161	0.158	0.938	0.291	0.933	0.376	1.221

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	43	27	30	30
N.S.	1	1.00	1.07	0.93	1.00	1.43	0.90	1.00	1.00
time (sec)	N/A	0.233	0.519	0.179	0.940	0.312	2.781	0.360	1.308

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	67	27	30	30
N.S.	1	1.00	1.07	0.93	1.00	2.23	0.90	1.00	1.00
time (sec)	N/A	0.240	0.799	0.187	0.930	0.313	15.332	0.488	1.222

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	30	0	29	30	30
N.S.	1	1.00	1.06	0.88	0.94	0.00	0.91	0.94	0.94
time (sec)	N/A	0.447	1.135	0.180	11.059	0.000	3.559	1.014	1.335

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	30	0	29	30	30
N.S.	1	1.00	1.06	0.88	0.94	0.00	0.91	0.94	0.94
time (sec)	N/A	0.430	1.201	0.162	11.047	0.000	1.381	2.238	1.329

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	30	0	29	30	30
N.S.	1	1.00	1.06	0.88	0.94	0.00	0.91	0.94	0.94
time (sec)	N/A	0.448	1.711	0.161	11.013	0.000	17.419	1.412	1.365

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	30	0	29	30	30
N.S.	1	1.00	1.06	0.88	0.94	0.00	0.91	0.94	0.94
time (sec)	N/A	0.261	3.996	0.176	11.081	0.000	1.061	0.369	1.314

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	30	0	31	30	30
N.S.	1	1.00	1.06	0.88	0.94	0.00	0.97	0.94	0.94
time (sec)	N/A	0.263	0.207	0.168	11.028	0.000	3.851	0.354	1.344

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	30	0	31	30	30
N.S.	1	1.00	1.06	0.88	0.94	0.00	0.97	0.94	0.94
time (sec)	N/A	0.266	0.090	0.188	11.197	0.000	51.955	0.418	1.403

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	86	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	24	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.86	1.07	1.07
time (sec)	N/A	0.216	0.234	0.195	0.930	0.273	13.569	0.352	1.182

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	176	54	0	30	30
N.S.	1	1.00	1.07	1.00	6.29	1.93	0.00	1.07	1.07
time (sec)	N/A	0.216	1.998	0.198	1.204	0.307	0.000	0.440	1.251

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	56	0	30	30
N.S.	1	1.00	1.07	0.93	1.00	1.87	0.00	1.00	1.00
time (sec)	N/A	0.259	15.262	0.204	11.313	0.296	0.000	3.232	1.326

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	30	0	30	30
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.241	0.056	0.177	11.183	0.305	0.000	0.989	1.255

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	30	27	30	30
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.90	1.00	1.00
time (sec)	N/A	0.247	10.606	0.207	10.975	0.332	5.938	0.348	1.273

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	74	0	30	30
N.S.	1	1.00	1.07	0.93	1.00	2.47	0.00	1.00	1.00
time (sec)	N/A	0.258	13.559	0.197	11.257	0.360	0.000	0.424	1.563

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	30	0	0	30
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.00	0.00	1.07
time (sec)	N/A	0.208	0.322	0.285	0.000	0.340	0.000	0.000	1.267

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	432	432	326	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.440	0.606	0.000	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	227	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.931	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	135	131	0	0	80	0	0	0
N.S.	1	1.03	1.00	0.00	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.442	0.097	0.000	0.000	0.082	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	30	24	30	30
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.86	1.07	1.07
time (sec)	N/A	0.218	0.233	0.213	0.000	0.351	6.080	0.554	1.302

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	190	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.706	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	318	284	0	0	0	0	0	0
N.S.	1	0.95	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.339	0.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	515	354	0	0	0	0	0	0	0
N.S.	1	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.537	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	300	257	0	0	0	0	0	0
N.S.	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.481	0.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	519	341	637	0	0	0	0	0	0
N.S.	1	0.66	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.876	10.981	0.000	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	427	386	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.002	0.581	0.000	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	258	258	231	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.734	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	120	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.533	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	67	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.003	0.000	0.000	0.000	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	117	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.666	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	225	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.814	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	425	425	363	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.083	0.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	519	519	927	0	0	0	0	0	0
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.483	0.405	0.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	852	0	0	0	0	0	0
N.S.	1	1.00	3.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.860	0.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	117	324	0	0	0	0	0	0
N.S.	1	0.95	2.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	652	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.153	0.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	463	463	654	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.448	0.408	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	742	742	4056	0	0	0	0	0	0
N.S.	1	1.00	5.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.984	0.889	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	1769	0	0	0	0	0	0
N.S.	1	1.00	5.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.145	0.475	0.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	166	646	0	0	0	0	0	0
N.S.	1	0.94	3.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.885	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	1350	0	0	0	0	0	0
N.S.	1	1.00	3.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.495	0.311	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	659	659	1057	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.035	0.733	0.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	41	27	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.24	0.82	1.06	1.06
time (sec)	N/A	0.394	0.135	0.180	1.550	0.309	3.554	0.465	1.344

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	35	24	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.25	0.86	1.07	1.07
time (sec)	N/A	0.223	0.011	0.026	0.905	0.295	1.485	0.295	1.224

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	65	29	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.86	0.83	1.06	1.06
time (sec)	N/A	0.415	0.614	0.330	0.884	0.343	3.728	0.291	1.241

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	115	31	37	37
N.S.	1	1.00	1.06	1.00	1.06	3.29	0.89	1.06	1.06
time (sec)	N/A	0.447	1.966	0.309	0.912	0.290	12.189	0.310	1.262

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	302	77	29	35	35
N.S.	1	1.00	1.06	1.00	9.15	2.33	0.88	1.06	1.06
time (sec)	N/A	0.420	1.360	0.171	2.060	0.295	16.904	0.306	1.479

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	267	71	26	30	30
N.S.	1	1.00	1.07	1.00	9.54	2.54	0.93	1.07	1.07
time (sec)	N/A	0.222	0.094	0.026	1.204	0.296	5.208	0.290	1.275

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	701	126	31	37	37
N.S.	1	1.00	1.06	1.00	20.03	3.60	0.89	1.06	1.06
time (sec)	N/A	0.408	13.246	0.319	1.269	0.330	16.727	0.375	1.347

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	1039	211	32	37	37
N.S.	1	1.00	1.06	1.00	29.69	6.03	0.91	1.06	1.06
time (sec)	N/A	0.433	19.109	0.323	1.272	0.330	112.691	0.446	1.419

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [145] had the largest ratio of [1.07692000000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.02	10	0.500
2	A	5	4	1.05	10	0.400
3	A	4	3	1.10	10	0.300
4	A	3	2	1.24	8	0.250
5	A	3	2	1.00	10	0.200
6	A	4	3	0.94	10	0.300
7	A	5	4	0.92	10	0.400
8	A	6	5	0.96	10	0.500
9	A	8	7	0.96	12	0.583
10	A	7	6	0.96	12	0.500
11	A	6	5	0.96	12	0.417
12	A	5	4	1.00	12	0.333
13	A	6	5	0.96	12	0.417
14	A	7	6	0.95	12	0.500
15	A	8	7	0.98	12	0.583
16	A	4	3	0.91	10	0.300
17	A	6	5	0.91	16	0.312
18	A	5	4	0.93	16	0.250
19	A	4	3	1.00	16	0.188
20	A	1	1	1.00	14	0.071
21	A	4	3	1.00	16	0.188
22	A	5	4	0.98	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	6	5	1.01	16	0.312
24	A	8	7	0.96	18	0.389
25	A	7	6	0.98	18	0.333
26	A	6	5	0.98	18	0.278
27	A	5	4	1.00	18	0.222
28	A	6	5	0.98	18	0.278
29	A	7	6	1.01	18	0.333
30	A	8	7	1.05	18	0.389
31	A	4	3	1.00	16	0.188
32	A	4	3	1.00	18	0.167
33	A	4	3	1.00	22	0.136
34	A	5	4	1.00	25	0.160
35	A	3	3	0.90	22	0.136
36	A	3	3	0.91	22	0.136
37	A	3	3	0.93	22	0.136
38	A	3	3	0.97	20	0.150
39	A	1	1	1.00	14	0.071
40	A	4	3	1.00	22	0.136
41	A	3	3	0.95	22	0.136
42	A	3	3	0.85	22	0.136
43	A	3	3	0.86	22	0.136
44	A	6	5	0.85	24	0.208
45	A	6	5	0.88	24	0.208
46	A	2	2	1.00	22	0.091
47	A	4	3	1.00	16	0.188
48	A	5	4	0.95	24	0.167
49	A	5	4	0.89	24	0.167
50	A	9	8	1.14	24	0.333
51	A	13	12	1.20	24	0.500
52	A	2	2	1.00	24	0.083
53	A	2	2	1.00	24	0.083
54	A	2	2	1.00	22	0.091
55	A	5	4	0.93	16	0.250
56	A	6	5	0.94	24	0.208

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
57	A	6	5	0.85	24	0.208
58	A	11	10	0.95	24	0.417
59	A	15	14	1.04	24	0.583
60	A	2	2	1.00	22	0.091
61	A	6	5	0.91	16	0.312
62	A	7	6	0.93	24	0.250
63	A	7	6	0.82	24	0.250
64	A	3	2	1.26	6	0.333
65	A	4	3	1.11	8	0.375
66	A	5	4	1.05	8	0.500
67	A	4	3	1.28	9	0.333
68	A	5	4	1.08	11	0.364
69	A	6	5	1.01	11	0.455
70	A	3	2	1.21	10	0.200
71	A	3	2	1.00	27	0.074
72	A	2	2	1.00	18	0.111
73	A	2	2	1.00	10	0.200
74	A	2	2	1.00	10	0.200
75	A	1	1	1.00	10	0.100
76	A	1	1	1.00	8	0.125
77	A	3	3	1.00	10	0.300
78	A	3	3	1.00	10	0.300
79	A	2	2	1.00	14	0.143
80	A	2	2	1.00	14	0.143
81	A	2	2	1.00	14	0.143
82	A	1	1	1.00	12	0.083
83	A	3	3	1.00	14	0.214
84	A	3	3	1.00	14	0.214
85	A	7	6	0.88	16	0.375
86	A	14	13	1.05	16	0.812
87	A	2	2	1.00	16	0.125
88	A	2	2	1.00	24	0.083
89	A	2	2	1.00	24	0.083
90	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	3	1.00	16	0.188
92	N/A	1	0	1.00	24	0.000
93	N/A	1	0	1.00	24	0.000
94	A	3	3	1.73	24	0.125
95	A	3	3	1.64	24	0.125
96	A	7	6	1.45	22	0.273
97	A	5	4	0.98	16	0.250
98	N/A	1	0	1.00	24	0.000
99	N/A	1	0	1.00	24	0.000
100	B	8	7	2.15	24	0.292
101	A	12	11	1.59	22	0.500
102	A	6	5	1.01	16	0.312
103	N/A	1	0	1.00	24	0.000
104	N/A	1	0	1.00	24	0.000
105	A	2	2	1.00	26	0.077
106	A	2	2	1.00	24	0.083
107	A	6	5	0.98	18	0.278
108	N/A	1	0	1.00	26	0.000
109	N/A	2	0	1.00	26	0.000
110	N/A	2	0	1.00	26	0.000
111	A	2	2	1.00	26	0.077
112	A	2	2	1.00	24	0.083
113	A	7	6	0.98	18	0.333
114	N/A	1	0	1.00	26	0.000
115	N/A	2	0	1.00	26	0.000
116	N/A	2	0	1.00	26	0.000
117	A	2	2	1.00	26	0.077
118	A	2	2	1.00	24	0.083
119	A	8	7	0.96	18	0.389
120	N/A	1	0	1.00	26	0.000
121	N/A	2	0	1.00	26	0.000
122	N/A	2	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	A	2	2	1.00	26	0.077
124	A	2	2	1.00	26	0.077
125	A	2	2	1.00	24	0.083
126	A	5	4	1.00	18	0.222
127	N/A	1	0	1.00	26	0.000
128	A	3	3	1.74	26	0.115
129	A	3	3	1.64	26	0.115
130	A	8	7	1.45	24	0.292
131	A	6	5	0.98	18	0.278
132	N/A	1	0	1.00	26	0.000
133	B	4	4	2.58	26	0.154
134	B	9	8	2.19	26	0.308
135	A	14	13	1.61	24	0.542
136	A	7	6	1.01	18	0.333
137	N/A	1	0	1.00	26	0.000
138	A	7	6	0.99	24	0.250
139	A	6	5	1.00	24	0.208
140	A	5	4	1.02	24	0.167
141	A	4	3	1.00	24	0.125
142	A	5	4	0.96	24	0.167
143	A	6	5	1.01	24	0.208
144	A	7	6	1.03	24	0.250
145	A	29	28	1.71	26	1.077
146	A	22	21	1.50	26	0.808
147	A	16	15	1.32	26	0.577
148	A	11	10	1.25	26	0.385
149	A	15	14	1.25	26	0.538
150	A	20	19	1.41	26	0.731
151	A	26	25	1.61	26	0.962
152	N/A	1	0	1.00	26	0.000
153	N/A	1	0	1.00	26	0.000
154	N/A	1	0	1.00	26	0.000
155	N/A	1	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	N/A	2	0	1.00	28	0.000
157	N/A	2	0	1.00	28	0.000
158	N/A	2	0	1.00	28	0.000
159	N/A	1	0	1.00	28	0.000
160	N/A	1	0	1.00	28	0.000
161	N/A	1	0	1.00	28	0.000
162	A	2	2	1.00	22	0.091
163	N/A	1	0	1.00	24	0.000
164	N/A	1	0	1.00	24	0.000
165	N/A	1	0	1.00	26	0.000
166	N/A	1	0	1.00	26	0.000
167	N/A	1	0	1.00	26	0.000
168	N/A	1	0	1.00	26	0.000
169	N/A	1	0	1.00	24	0.000
170	A	2	2	1.00	24	0.083
171	A	2	2	1.00	24	0.083
172	A	2	2	1.00	22	0.091
173	A	4	3	1.00	16	0.188
174	N/A	1	0	1.00	24	0.000
175	A	5	4	0.84	30	0.133
176	A	5	4	0.85	30	0.133
177	A	5	4	0.90	30	0.133
178	A	6	5	0.89	28	0.179
179	A	4	3	1.00	23	0.130
180	A	5	4	0.98	30	0.133
181	A	8	7	1.25	30	0.233
182	A	12	11	1.34	30	0.367
183	A	19	18	1.45	32	0.562
184	A	15	14	1.12	32	0.438
185	A	11	10	1.10	32	0.312
186	A	8	7	0.83	30	0.233
187	A	5	4	1.00	25	0.160

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	6	5	0.89	32	0.156
189	A	10	9	1.01	32	0.281
190	A	14	13	1.09	32	0.406
191	A	5	4	0.90	32	0.125
192	A	5	4	0.90	32	0.125
193	A	5	4	0.91	32	0.125
194	A	5	4	0.93	30	0.133
195	A	5	4	1.00	25	0.160
196	N/A	2	0	1.00	32	0.000
197	N/A	2	0	1.00	32	0.000
198	A	28	27	2.00	31	0.871
199	A	21	20	1.74	31	0.645
200	A	15	14	1.49	31	0.452
201	A	10	9	1.39	31	0.290
202	A	14	13	1.44	31	0.419
203	A	19	18	1.65	31	0.581
204	A	21	20	1.82	23	0.870
205	A	15	14	1.54	23	0.609
206	A	10	9	1.43	23	0.391
207	A	14	13	1.49	23	0.565
208	A	19	18	1.72	23	0.783
209	N/A	1	0	1.00	32	0.000
210	A	5	4	0.94	32	0.125
211	A	5	4	0.95	32	0.125
212	A	5	4	0.96	30	0.133
213	A	5	4	1.00	25	0.160
214	N/A	1	0	1.00	32	0.000
215	N/A	1	0	1.00	32	0.000
216	N/A	1	0	1.00	32	0.000
217	A	2	2	1.00	29	0.069
218	A	2	2	1.00	29	0.069
219	A	2	2	1.00	27	0.074
220	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
221	A	2	2	1.00	29	0.069
222	A	2	2	1.00	29	0.069
223	A	2	2	1.00	29	0.069
224	A	2	2	1.00	31	0.065
225	A	2	2	1.00	29	0.069
226	A	5	4	0.95	24	0.167
227	A	2	2	1.00	31	0.065
228	A	2	2	1.00	31	0.065
229	A	2	2	1.00	31	0.065
230	A	2	2	1.00	29	0.069
231	A	6	5	0.94	24	0.208
232	A	2	2	1.00	31	0.065
233	A	2	2	1.00	31	0.065
234	N/A	2	0	1.00	29	0.000
235	N/A	1	0	1.00	24	0.000
236	N/A	2	0	1.00	31	0.000
237	N/A	2	0	1.00	31	0.000
238	N/A	2	0	1.00	29	0.000
239	N/A	1	0	1.00	24	0.000
240	N/A	2	0	1.00	31	0.000
241	N/A	2	0	1.00	31	0.000
242	A	2	2	1.00	25	0.080
243	A	2	2	1.00	25	0.080
244	A	2	2	1.00	23	0.087
245	A	4	3	1.00	22	0.136
246	A	2	2	1.00	25	0.080
247	A	2	2	1.00	25	0.080
248	A	2	2	1.00	25	0.080
249	A	2	2	1.00	25	0.080
250	A	2	2	1.00	25	0.080
251	A	2	2	1.00	23	0.087
252	A	3	3	0.95	22	0.136
253	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
254	A	2	2	1.00	25	0.080
255	A	2	2	1.00	25	0.080
256	A	2	2	1.00	27	0.074
257	A	2	2	1.00	27	0.074
258	A	2	2	1.00	25	0.080
259	A	2	2	1.00	27	0.074
260	A	2	2	1.00	27	0.074
261	A	2	2	1.00	27	0.074
262	A	2	2	1.00	27	0.074
263	A	2	2	1.00	24	0.083
264	A	2	2	1.00	27	0.074
265	A	2	2	1.00	27	0.074
266	A	2	2	1.00	27	0.074
267	A	2	2	1.00	27	0.074
268	A	5	5	0.86	25	0.200
269	A	2	2	1.00	27	0.074
270	A	2	2	1.00	27	0.074
271	A	2	2	1.00	27	0.074
272	A	2	2	1.00	27	0.074
273	A	2	2	1.00	24	0.083
274	A	2	2	1.00	27	0.074
275	A	8	7	0.96	26	0.269
276	A	9	8	0.69	26	0.308
277	A	9	8	1.06	34	0.235
278	A	9	8	0.66	34	0.235
279	A	3	2	1.00	26	0.077
280	A	5	4	1.00	25	0.160
281	A	5	4	1.00	30	0.133
282	A	5	4	1.00	29	0.138
283	A	2	2	1.00	19	0.105
284	A	2	2	1.00	19	0.105
285	A	2	2	1.00	19	0.105
286	A	2	2	1.00	19	0.105
287	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	2	2	1.00	19	0.105
289	A	2	2	1.00	17	0.118
290	A	2	2	1.00	16	0.125
291	A	2	2	1.00	19	0.105
292	A	2	2	1.00	19	0.105
293	A	2	2	1.00	19	0.105
294	A	2	2	1.00	19	0.105
295	A	2	2	1.00	19	0.105
296	A	2	2	1.00	19	0.105
297	A	2	2	1.00	17	0.118
298	A	2	2	1.00	19	0.105
299	A	2	2	1.00	19	0.105
300	A	2	2	1.00	19	0.105
301	A	2	2	1.00	16	0.125
302	A	2	2	1.00	19	0.105
303	A	4	4	0.98	23	0.174
304	A	4	4	0.94	27	0.148
305	A	4	4	0.92	27	0.148
306	A	5	4	1.00	27	0.148
307	A	4	4	0.95	27	0.148
308	A	4	4	0.86	27	0.148
309	A	2	2	1.00	16	0.125
310	A	2	2	1.00	29	0.069
311	A	2	2	1.00	29	0.069
312	A	2	2	1.00	27	0.074
313	A	2	2	1.00	29	0.069
314	A	2	2	1.00	29	0.069
315	A	2	2	1.00	29	0.069
316	A	2	2	1.00	29	0.069
317	A	2	2	1.00	26	0.077
318	A	2	2	1.00	29	0.069
319	A	2	2	1.00	29	0.069
320	A	2	2	1.00	29	0.069
321	A	2	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
322	A	3	3	0.95	27	0.111
323	A	2	2	1.00	29	0.069
324	A	2	2	1.00	29	0.069
325	A	2	2	1.00	29	0.069
326	A	2	2	1.00	29	0.069
327	A	2	2	1.00	26	0.077
328	A	2	2	1.00	29	0.069
329	A	2	2	1.00	22	0.091
330	A	2	2	1.00	22	0.091
331	A	2	2	1.00	20	0.100
332	N/A	2	0	1.00	22	0.000
333	A	5	4	1.00	32	0.125
334	A	6	5	1.00	36	0.139
335	A	5	4	1.00	38	0.105
336	A	6	5	1.00	38	0.132
337	A	3	2	1.00	26	0.077
338	A	5	4	1.00	27	0.148
339	A	1	1	1.00	38	0.026
340	A	4	3	1.37	39	0.077
341	A	1	1	1.00	34	0.029
342	A	5	4	1.37	35	0.114
343	A	3	3	1.00	24	0.125
344	A	3	3	1.00	24	0.125
345	A	3	3	1.00	22	0.136
346	N/A	3	0	1.00	24	0.000
347	A	2	2	1.00	25	0.080
348	A	2	2	1.00	25	0.080
349	A	2	2	1.00	23	0.087
350	N/A	2	0	1.00	25	0.000
351	A	2	2	1.00	18	0.111
352	A	2	2	1.00	18	0.111
353	A	2	2	1.00	16	0.125
354	A	2	2	1.00	15	0.133

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
355	A	2	2	1.00	18	0.111
356	A	2	2	1.00	18	0.111
357	A	2	2	1.00	18	0.111
358	A	5	5	1.12	24	0.208
359	A	5	5	1.11	24	0.208
360	A	5	5	1.07	22	0.227
361	A	5	5	1.13	21	0.238
362	A	4	4	1.10	24	0.167
363	A	6	6	0.97	24	0.250
364	A	7	7	0.94	24	0.292
365	A	9	9	1.01	24	0.375
366	A	11	11	1.05	24	0.458
367	A	2	2	1.20	26	0.077
368	A	2	2	0.98	24	0.083
369	A	2	2	0.97	23	0.087
370	F	0	0	N/A	0.000	N/A
371	F	0	0	N/A	0.000	N/A
372	F	0	0	N/A	0.000	N/A
373	A	2	2	0.98	23	0.087
374	F	0	0	N/A	0.000	N/A
375	N/A	1	0	1.00	23	0.000
376	N/A	1	0	1.00	23	0.000
377	N/A	1	0	1.00	23	0.000
378	A	1	1	1.00	16	0.062
379	A	7	6	0.88	32	0.188
380	A	6	5	0.90	30	0.167
381	A	8	7	1.06	29	0.241
382	A	9	8	1.25	32	0.250
383	A	7	6	1.01	32	0.188
384	A	9	8	1.04	32	0.250
385	A	14	13	1.03	32	0.406
386	A	3	3	1.15	32	0.094
387	A	3	3	1.10	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
388	A	3	3	1.04	30	0.100
389	A	3	3	1.07	29	0.103
390	A	13	13	0.87	32	0.406
391	A	3	3	0.95	32	0.094
392	A	3	3	0.89	32	0.094
393	A	5	5	1.05	32	0.156
394	A	5	5	1.08	31	0.161
395	N/A	1	0	1.00	34	0.000
396	N/A	1	0	1.00	34	0.000
397	A	5	5	1.11	32	0.156
398	A	5	5	1.09	31	0.161
399	N/A	1	0	1.00	34	0.000
400	N/A	1	0	1.00	34	0.000
401	A	4	3	0.97	40	0.075
402	A	6	5	1.05	28	0.179
403	A	6	5	1.06	32	0.156
404	A	7	6	0.89	20	0.300
405	A	6	5	0.91	20	0.250
406	A	5	4	0.99	20	0.200
407	A	1	1	1.00	18	0.056
408	A	5	4	1.02	20	0.200
409	A	6	5	1.01	20	0.250
410	A	7	6	1.05	20	0.300
411	A	9	8	0.98	22	0.364
412	A	8	7	1.01	22	0.318
413	A	7	6	1.01	22	0.273
414	A	6	5	1.02	22	0.227
415	A	7	6	1.01	22	0.273
416	A	8	7	1.05	22	0.318
417	A	9	8	1.09	22	0.364
418	A	5	4	1.03	20	0.200
419	A	5	4	0.97	24	0.167
420	A	5	4	0.89	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
421	A	5	4	0.91	26	0.154
422	A	5	4	0.95	24	0.167
423	A	1	1	1.00	18	0.056
424	A	5	4	1.00	26	0.154
425	A	5	4	0.94	26	0.154
426	A	5	4	0.84	26	0.154
427	A	5	4	0.85	26	0.154
428	A	7	6	0.84	28	0.214
429	A	7	6	0.86	28	0.214
430	A	4	3	1.00	26	0.115
431	A	5	4	0.99	20	0.200
432	A	6	5	0.95	28	0.179
433	A	6	5	0.89	28	0.179
434	A	10	9	1.11	28	0.321
435	A	4	3	1.00	28	0.107
436	A	4	3	1.00	26	0.115
437	A	6	5	0.91	20	0.250
438	A	7	6	0.94	28	0.214
439	A	7	6	0.85	28	0.214
440	A	12	11	0.94	28	0.393
441	A	7	6	0.89	20	0.300
442	A	8	7	0.93	28	0.250
443	A	8	7	0.82	28	0.250
444	A	4	3	1.21	14	0.214
445	A	4	3	1.00	28	0.107
446	A	4	3	1.00	26	0.115
447	A	5	4	1.02	20	0.200
448	N/A	1	0	1.00	28	0.000
449	N/A	1	0	1.00	28	0.000
450	A	5	4	1.65	28	0.143
451	A	8	7	1.47	26	0.269
452	A	6	5	1.01	20	0.250
453	N/A	1	0	1.00	28	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
454	N/A	1	0	1.00	28	0.000
455	B	9	8	2.20	28	0.286
456	A	13	12	1.65	26	0.462
457	A	7	6	1.05	20	0.300
458	N/A	1	0	1.00	28	0.000
459	N/A	1	0	1.00	28	0.000
460	A	4	3	1.00	30	0.100
461	A	4	3	1.00	28	0.107
462	A	7	6	1.01	22	0.273
463	N/A	1	0	1.00	30	0.000
464	N/A	1	0	1.00	30	0.000
465	A	4	3	1.00	30	0.100
466	A	4	3	1.00	28	0.107
467	A	8	7	1.01	22	0.318
468	N/A	1	0	1.00	30	0.000
469	N/A	1	0	1.00	30	0.000
470	A	4	3	1.00	30	0.100
471	A	4	3	1.00	28	0.107
472	A	6	5	1.02	22	0.227
473	N/A	1	0	1.00	30	0.000
474	A	5	4	1.65	30	0.133
475	A	9	8	1.46	28	0.286
476	A	7	6	1.01	22	0.273
477	N/A	1	0	1.00	30	0.000
478	B	10	9	2.25	30	0.300
479	A	15	14	1.66	28	0.500
480	A	8	7	1.05	22	0.318
481	N/A	1	0	1.00	30	0.000
482	A	8	7	0.97	28	0.250
483	A	7	6	0.99	28	0.214
484	A	6	5	1.01	28	0.179
485	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
486	A	6	5	0.96	28	0.179
487	A	7	6	0.99	28	0.214
488	A	8	7	1.02	28	0.250
489	A	30	29	1.63	30	0.967
490	A	23	22	1.44	30	0.733
491	A	17	16	1.28	30	0.533
492	A	12	11	1.21	30	0.367
493	A	16	15	1.22	30	0.500
494	A	21	20	1.36	30	0.667
495	A	27	26	1.55	30	0.867
496	N/A	1	0	1.00	30	0.000
497	N/A	1	0	1.00	30	0.000
498	N/A	1	0	1.00	30	0.000
499	N/A	1	0	1.00	30	0.000
500	N/A	1	0	1.00	32	0.000
501	N/A	1	0	1.00	32	0.000
502	N/A	1	0	1.00	32	0.000
503	N/A	1	0	1.00	32	0.000
504	N/A	1	0	1.00	32	0.000
505	N/A	1	0	1.00	32	0.000
506	A	4	3	1.00	26	0.115
507	N/A	1	0	1.00	28	0.000
508	N/A	1	0	1.00	28	0.000
509	N/A	1	0	1.00	30	0.000
510	N/A	1	0	1.00	30	0.000
511	N/A	1	0	1.00	30	0.000
512	N/A	1	0	1.00	30	0.000
513	N/A	1	0	1.00	28	0.000
514	A	4	3	1.00	28	0.107
515	A	4	3	1.00	26	0.115
516	A	5	4	1.03	20	0.200

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
517	N/A	1	0	1.00	28	0.000
518	A	4	3	1.00	28	0.107
519	A	9	8	0.95	30	0.267
520	A	10	9	0.69	30	0.300
521	A	10	9	1.05	38	0.237
522	A	10	9	0.66	38	0.237
523	A	4	3	1.00	33	0.091
524	A	4	3	1.00	33	0.091
525	A	4	3	1.00	31	0.097
526	A	5	4	1.00	26	0.154
527	A	4	3	1.00	33	0.091
528	A	4	3	1.00	33	0.091
529	A	4	3	1.00	33	0.091
530	A	4	3	1.00	35	0.086
531	A	4	3	1.00	33	0.091
532	A	6	5	0.95	28	0.179
533	A	4	3	1.00	35	0.086
534	A	4	3	1.00	35	0.086
535	A	4	3	1.00	35	0.086
536	A	4	3	1.00	33	0.091
537	A	7	6	0.94	28	0.214
538	A	4	3	1.00	35	0.086
539	A	4	3	1.00	35	0.086
540	N/A	1	0	1.00	33	0.000
541	N/A	1	0	1.00	28	0.000
542	N/A	1	0	1.00	35	0.000
543	N/A	1	0	1.00	35	0.000
544	N/A	1	0	1.00	33	0.000
545	N/A	1	0	1.00	28	0.000
546	N/A	1	0	1.00	35	0.000
547	N/A	1	0	1.00	35	0.000

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \log^4(c(d+ex))\,dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \log^4(c(d+ex))\,dx = 24x-\frac{24(d+ex)\log(c(d+ex))}{e}+\frac{12(d+ex)\log^2(c(d+ex))}{e}-\frac{4(d+ex)\log^3(c(d+ex))}{e}+\frac{(d+ex)\log^4(c(d+ex))}{e}$$

output
```
24*x-24*(e*x+d)*ln(c*(e*x+d))/e+12*(e*x+d)*ln(c*(e*x+d))^2/e-4*(e*x+d)*ln(
c*(e*x+d))^3/e+(e*x+d)*ln(c*(e*x+d))^4/e
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \log^4(c(d+ex))\,dx = \frac{24ex-24(d+ex)\log(c(d+ex))+12(d+ex)\log^2(c(d+ex))-4(d+ex)\log^3(c(d+ex))+(d+ex)\log}{e}$$

input
```
Integrate[Log[c*(d + e*x)]^4,x]
```

output
```
(24*e*x - 24*(d + e*x)*Log[c*(d + e*x)] + 12*(d + e*x)*Log[c*(d + e*x)]^2
- 4*(d + e*x)*Log[c*(d + e*x)]^3 + (d + e*x)*Log[c*(d + e*x)]^4)/e
```

3.1.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2836, 2733, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^4(c(d+ex))\,dx$$

$$\Big\downarrow \ 2836$$

$$\frac{\int \log^4(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \ 2733$$

$$\frac{(d+ex)\log^4(c(d+ex))-4\int \log^3(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \ 2733$$

$$\frac{(d+ex)\log^4(c(d+ex))-4\left((d+ex)\log^3(c(d+ex))-3\int \log^2(c(d+ex))d(d+ex)\right)}{e}$$

$$\Big\downarrow \ 2733$$

$$\frac{(d+ex)\log^4(c(d+ex))-4\left((d+ex)\log^3(c(d+ex))-3\left((d+ex)\log^2(c(d+ex))-2\int \log(c(d+ex))d(d+ex)\right)\right.}{e}$$

$$\Big\downarrow \ 2732$$

$$\frac{(d+ex)\log^4(c(d+ex))-4\left((d+ex)\log^3(c(d+ex))-3\left((d+ex)\log^2(c(d+ex))-2\left((d+ex)\log(c(d+ex))-\right.\right.\right.}{e}$$

input
```
Int[Log[c*(d + e*x)]^4,x]
```

output
```
((d + e*x)*Log[c*(d + e*x)]^4 - 4*((d + e*x)*Log[c*(d + e*x)]^3 - 3*((d +
e*x)*Log[c*(d + e*x)]^2 - 2*(-d - e*x + (d + e*x)*Log[c*(d + e*x)]))))/e
```

3.1.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.1.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

method	result
risch	$\frac{(ex+d)\ln(c(ex+d))^4}{e} - \frac{4(ex+d)\ln(c(ex+d))^3}{e} + \frac{12(ex+d)\ln(c(ex+d))^2}{e} - 24x\ln(c(ex+d)) + 24x - \frac{2}{}$
derivativedivides	$\frac{\ln(cex+cd)^4(cex+cd)-4(cex+cd)\ln(cex+cd)^3+12(cex+cd)\ln(cex+cd)^2-24(cex+cd)\ln(cex+cd)+24cex+24cd}{ce}$
default	$\frac{\ln(cex+cd)^4(cex+cd)-4(cex+cd)\ln(cex+cd)^3+12(cex+cd)\ln(cex+cd)^2-24(cex+cd)\ln(cex+cd)+24cex+24cd}{ce}$
norman	$x\ln(c(ex+d))^4 + \frac{d\ln(c(ex+d))^4}{e} + 24x - 24x\ln(c(ex+d)) + 12x\ln(c(ex+d))^2 - 4x\ln$
parallelrisch	$\frac{x\ln(c(ex+d))^4e-4x\ln(c(ex+d))^3e+\ln(c(ex+d))^4d+12x\ln(c(ex+d))^2e-4\ln(c(ex+d))^3d-24\ln(c(ex+d))xe+12\ln(c(e}{e}$

input
```
int(ln(c*(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
(e*x+d)*ln(c*(e*x+d))^4/e-4*(e*x+d)*ln(c*(e*x+d))^3/e+12*(e*x+d)*ln(c*(e*x
+d))^2/e-24*x*ln(c*(e*x+d))+24*x-24*d/e*ln(e*x+d)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \log^4(c(d+ex))\,dx$$
$$= \frac{(ex+d)\log(cex+cd)^4 - 4\,(ex+d)\log(cex+cd)^3 + 12\,(ex+d)\log(cex+cd)^2 + 24\,ex - 24\,(ex+d)\,\mathrm{lo}}{e}$$

input
```
integrate(log(c*(e*x+d))^4,x, algorithm="fricas")
```

output
```
((e*x + d)*log(c*e*x + c*d)^4 - 4*(e*x + d)*log(c*e*x + c*d)^3 + 12*(e*x +
 d)*log(c*e*x + c*d)^2 + 24*e*x - 24*(e*x + d)*log(c*e*x + c*d))/e
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \log^4(c(d+ex))\,dx = 24e\left(-\frac{d\log(d+ex)}{e^2}+\frac{x}{e}\right) - 24x\log(c(d+ex)) + \frac{(-4d-4ex)\log(c(d+ex))^3}{e} + \frac{(d+ex)\log(c(d+ex))^4}{e} + \frac{(12d+12ex)\log(c(d+ex))^2}{e}$$

input
```
integrate(ln(c*(e*x+d))**4,x)
```

output
```
24*e*(-d*log(d + e*x)/e**2 + x/e) - 24*x*log(c*(d + e*x)) + (-4*d - 4*e*x)
*log(c*(d + e*x))**3/e + (d + e*x)*log(c*(d + e*x))**4/e + (12*d + 12*e*x)
*log(c*(d + e*x))**2/e
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(81) = 162.

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.32

$$\int \log^4(c(d+ex))\,dx = -4\,e\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)c\right)^3+x\log\left((ex+d)c\right)^4$$
$$+\left(e\left(\frac{4\left(d\log(ex+d)^3+3\,d\log(ex+d)^2-6\,ex+6\,d\log(ex+d)\right)\log\left((ex+d)c\right)}{e^3}-\frac{d\log(ex+d)^4+}{}\right.\right.$$

input
```
integrate(log(c*(e*x+d))^4,x, algorithm="maxima")
```

output
```
-4*e*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)*c)^3 + x*log((e*x + d)*c)^4
+ (e*(4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))
*log((e*x + d)*c)/e^3 - (d*log(e*x + d)^4 + 4*d*log(e*x + d)^3 + 12*d*log(
e*x + d)^2 - 24*e*x + 24*d*log(e*x + d))/e^3) - 6*(d*log(e*x + d)^2 - 2*e*
x + 2*d*log(e*x + d))*log((e*x + d)*c)^2/e^2)*e
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \log^4(c(d+ex))\,dx = \frac{(ex+d)\log\left((ex+d)c\right)^4}{e}-\frac{4\,(ex+d)\log\left((ex+d)c\right)^3}{e}+\frac{12\,(ex+d)\log\left((ex+d)c\right)^2}{e}-\frac{24\,(ex+d)\log\left((ex+d)c\right)}{e}+\frac{24\,(ex+d)}{e}$$

input
```
integrate(log(c*(e*x+d))^4,x, algorithm="giac")
```

output
```
(e*x + d)*log((e*x + d)*c)^4/e - 4*(e*x + d)*log((e*x + d)*c)^3/e + 12*(e*
x + d)*log((e*x + d)*c)^2/e - 24*(e*x + d)*log((e*x + d)*c)/e + 24*(e*x +
d)/e
```

3.1.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \log^4(c(d+ex))\,dx = 24\,x - 24\,x\,\ln\left(c\,d+c\,e\,x\right) + 12\,x\ln\left(c\,d+c\,e\,x\right)^2 - 4\,x\ln\left(c\,d+c\,e\,x\right)^3 + x\ln\left(c\,d+c\,e\,x\right)^4 + \frac{12\,d\ln\left(c\,d+c\,e\,x\right)^2}{e} - \frac{4\,d\ln\left(c\,d+c\,e\,x\right)^3}{e} + \frac{d\ln\left(c\,d+c\,e\,x\right)^4}{e} - \frac{24\,d\,\ln\left(d+e\,x\right)}{e}$$

input
```
int(log(c*(d + e*x))^4,x)
```

output
```
24*x - 24*x*log(c*d + c*e*x) + 12*x*log(c*d + c*e*x)^2 - 4*x*log(c*d + c*e
*x)^3 + x*log(c*d + c*e*x)^4 + (12*d*log(c*d + c*e*x)^2)/e - (4*d*log(c*d
+ c*e*x)^3)/e + (d*log(c*d + c*e*x)^4)/e - (24*d*log(d + e*x))/e
```

3.2 $\int \log^3(c(d+ex))\,dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \log^3(c(d+ex))\,dx = -6x+\frac{6(d+ex)\log(c(d+ex))}{e}-\frac{3(d+ex)\log^2(c(d+ex))}{e}+\frac{(d+ex)\log^3(c(d+ex))}{e}$$

output
```
-6*x+6*(e*x+d)*ln(c*(e*x+d))/e-3*(e*x+d)*ln(c*(e*x+d))^2/e+(e*x+d)*ln(c*(e
*x+d))^3/e
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \log^3(c(d+ex))\,dx = \frac{-6ex+6(d+ex)\log(c(d+ex))-3(d+ex)\log^2(c(d+ex))+(d+ex)\log^3(c(d+ex))}{e}$$

input
```
Integrate[Log[c*(d + e*x)]^3,x]
```

output
```
(-6*e*x + 6*(d + e*x)*Log[c*(d + e*x)] - 3*(d + e*x)*Log[c*(d + e*x)]^2 +
(d + e*x)*Log[c*(d + e*x)]^3)/e
```

3.2.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2836, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3(c(d+ex))\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^3(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^3(c(d+ex))-3\int \log^2(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^3(c(d+ex))-3\left((d+ex)\log^2(c(d+ex))-2\int \log(c(d+ex))d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(d+ex)\log^3(c(d+ex))-3\left((d+ex)\log^2(c(d+ex))-2((d+ex)\log(c(d+ex))-d-ex)\right)}{e}$$

input
```
Int[Log[c*(d + e*x)]^3,x]
```

output
```
((d + e*x)*Log[c*(d + e*x)]^3 - 3*((d + e*x)*Log[c*(d + e*x)]^2 - 2*(-d -
e*x + (d + e*x)*Log[c*(d + e*x)])))/e
```

3.2.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.2.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(ex+d)\ln(c(ex+d))^3}{e} - \frac{3(ex+d)\ln(c(ex+d))^2}{e} + 6x\ln\left(c(ex+d)\right) - 6x + \frac{6d\ln(ex+d)}{e}$
derivativedivides	$\frac{(cex+cd)\ln(cex+cd)^3-3(cex+cd)\ln(cex+cd)^2+6(cex+cd)\ln(cex+cd)-6cex-6cd}{ce}$
default	$\frac{(cex+cd)\ln(cex+cd)^3-3(cex+cd)\ln(cex+cd)^2+6(cex+cd)\ln(cex+cd)-6cex-6cd}{ce}$
norman	$x\ln\left(c(ex+d)\right)^3 + \frac{d\ln(c(ex+d))^3}{e} - 6x + 6x\ln\left(c(ex+d)\right) - 3x\ln\left(c(ex+d)\right)^2 + \frac{6d\ln(c(ex}{e}$
parallelrisch	$\frac{x\ln(c(ex+d))^3e-3x\ln(c(ex+d))^2e+\ln(c(ex+d))^3d+6\ln(c(ex+d))xe-3\ln(c(ex+d))^2d-6ex+6d\ln(c(ex+d))+6d}{e}$

input
```
int(ln(c*(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
(e*x+d)*ln(c*(e*x+d))^3/e-3*(e*x+d)*ln(c*(e*x+d))^2/e+6*x*ln(c*(e*x+d))-6*
x+6*d/e*ln(e*x+d)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \log^3(c(d+ex))\,dx = \frac{(ex+d)\log\left(cex+cd\right)^3 - 3\left(ex+d\right)\log\left(cex+cd\right)^2 - 6\,ex + 6\left(ex+d\right)\log\left(cex+cd\right)}{e}$$

input
```
integrate(log(c*(e*x+d))^3,x, algorithm="fricas")
```

output
```
((e*x + d)*log(c*e*x + c*d)^3 - 3*(e*x + d)*log(c*e*x + c*d)^2 - 6*e*x + 6
*(e*x + d)*log(c*e*x + c*d))/e
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \log^3(c(d+ex))\,dx = -6e\left(-\frac{d\log(d+ex)}{e^2}+\frac{x}{e}\right)+6x\log(c(d+ex)) + \frac{(-3d-3ex)\log(c(d+ex))^2}{e}+\frac{(d+ex)\log(c(d+ex))^3}{e}$$

input
```
integrate(ln(c*(e*x+d))**3,x)
```

output
```
-6*e*(-d*log(d + e*x)/e**2 + x/e) + 6*x*log(c*(d + e*x)) + (-3*d - 3*e*x)*
log(c*(d + e*x))**2/e + (d + e*x)*log(c*(d + e*x))**3/e
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(61) = 122.

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \log^3(c(d+ex))\,dx = -3\,e\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)c)^2+x\log((ex+d)c)^3 - e\left(\frac{3\left(d\log(ex+d)^2-2\,ex+2\,d\log(ex+d)\right)\log((ex+d)c)}{e^2}-\frac{d\log(ex+d)^3+3\,d\log(ex+d)^2-6$$

input
```
integrate(log(c*(e*x+d))^3,x, algorithm="maxima")
```

output
```
-3*e*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)*c)^2 + x*log((e*x + d)*c)^3
- e*(3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*log((e*x + d)*c)/e^2
- (d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))/e^2)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \log^3(c(d+ex))\,dx = \frac{(ex+d)\log\left((ex+d)c\right)^3}{e} - \frac{3\,(ex+d)\log\left((ex+d)c\right)^2}{e} + \frac{6\,(ex+d)\log\left((ex+d)c\right)}{e} - \frac{6\,(ex+d)}{e}$$

input
```
integrate(log(c*(e*x+d))^3,x, algorithm="giac")
```

output
```
(e*x + d)*log((e*x + d)*c)^3/e - 3*(e*x + d)*log((e*x + d)*c)^2/e + 6*(e*x
 + d)*log((e*x + d)*c)/e - 6*(e*x + d)/e
```

3.2.9 Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \log^3(c(d+ex))\,dx = 6\,x\,\ln\left(c\,d+c\,e\,x\right) - 6\,x - 3\,x\ln\left(c\,d+c\,e\,x\right)^2 + x\ln\left(c\,d+c\,e\,x\right)^3 - \frac{3\,d\ln\left(c\,d+c\,e\,x\right)^2}{e} + \frac{d\ln\left(c\,d+c\,e\,x\right)^3}{e} + \frac{6\,d\,\ln\left(d+e\,x\right)}{e}$$

input
```
int(log(c*(d + e*x))^3,x)
```

output
```
6*x*log(c*d + c*e*x) - 6*x - 3*x*log(c*d + c*e*x)^2 + x*log(c*d + c*e*x)^3
 - (3*d*log(c*d + c*e*x)^2)/e + (d*log(c*d + c*e*x)^3)/e + (6*d*log(d + e*
x))/e
```

3.3 $\int \log^2(c(d+ex))\,dx$

3.3.1 Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \log^2(c(d+ex))\,dx = 2x - \frac{2(d+ex)\log(c(d+ex))}{e} + \frac{(d+ex)\log^2(c(d+ex))}{e}$$

output
```
2*x-2*(e*x+d)*ln(c*(e*x+d))/e+(e*x+d)*ln(c*(e*x+d))^2/e
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \log^2(c(d+ex))\,dx = \frac{2ex - 2(d+ex)\log(c(d+ex)) + (d+ex)\log^2(c(d+ex))}{e}$$

input
```
Integrate[Log[c*(d + e*x)]^2,x]
```

output
```
(2*e*x - 2*(d + e*x)*Log[c*(d + e*x)] + (d + e*x)*Log[c*(d + e*x)]^2)/e
```

3.3.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2836, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2(c(d+ex))\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^2(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^2(c(d+ex))-2\int \log(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(d+ex)\log^2(c(d+ex))-2((d+ex)\log(c(d+ex))-d-ex)}{e}$$

input
```
Int[Log[c*(d + e*x)]^2,x]
```

output
```
((d + e*x)*Log[c*(d + e*x)]^2 - 2*(-d - e*x + (d + e*x)*Log[c*(d + e*x)]))
/e
```

3.3.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.3.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{(ex+d)\ln(c(ex+d))^2}{e} - 2x\ln\left(c(ex+d)\right) + 2x - \frac{2d\ln(ex+d)}{e}$	47
derivativedivides	$\frac{(cex+cd)\ln(cex+cd)^2 - 2(cex+cd)\ln(cex+cd) + 2cex + 2cd}{ce}$	57
default	$\frac{(cex+cd)\ln(cex+cd)^2 - 2(cex+cd)\ln(cex+cd) + 2cex + 2cd}{ce}$	57
norman	$x\ln\left(c(ex+d)\right)^2 + \frac{d\ln(c(ex+d))^2}{e} + 2x - 2x\ln\left(c(ex+d)\right) - \frac{2d\ln(c(ex+d))}{e}$	57
parallelrisch	$\frac{x\ln(c(ex+d))^2 e - 2\ln(c(ex+d))xe + \ln(c(ex+d))^2 d + 2ex - 2d\ln(c(ex+d)) - 2d}{e}$	61

input
```
int(ln(c*(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
(e*x+d)*ln(c*(e*x+d))^2/e-2*x*ln(c*(e*x+d))+2*x-2*d/e*ln(e*x+d)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \log^2(c(d+ex))\,dx = \frac{(ex+d)\log\left(cex+cd\right)^2 + 2\,ex - 2\,(ex+d)\log\left(cex+cd\right)}{e}$$

input
```
integrate(log(c*(e*x+d))^2,x, algorithm="fricas")
```

output
```
((e*x + d)*log(c*e*x + c*d)^2 + 2*e*x - 2*(e*x + d)*log(c*e*x + c*d))/e
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \log^2(c(d+ex))\,dx = 2e\left(-\frac{d\log{(d+ex)}}{e^2}+\frac{x}{e}\right)-2x\log{(c(d+ex))}+\frac{(d+ex)\log{(c(d+ex))}^2}{e}$$

input
```
integrate(ln(c*(e*x+d))**2,x)
```

output
```
2*e*(-d*log(d + e*x)/e**2 + x/e) - 2*x*log(c*(d + e*x)) + (d + e*x)*log(c*
(d + e*x))**2/e
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \log^2(c(d+ex))\,dx = -2\,e\left(\frac{x}{e}-\frac{d\log{(ex+d)}}{e^2}\right)\log{((ex+d)c)}+x\log{((ex+d)c)}^2-\frac{d\log{(ex+d)}^2-2\,ex+2\,d\log{(ex+d)}}{e}$$

input
```
integrate(log(c*(e*x+d))^2,x, algorithm="maxima")
```

output
```
-2*e*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)*c) + x*log((e*x + d)*c)^2 -
(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))/e
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \log^2(c(d+ex))\,dx = \frac{(ex+d)\log{((ex+d)c)}^2}{e}-\frac{2\,(ex+d)\log{((ex+d)c)}}{e}+\frac{2\,(ex+d)}{e}$$

input
```
integrate(log(c*(e*x+d))^2,x, algorithm="giac")
```

output
```
(e*x + d)*log((e*x + d)*c)^2/e - 2*(e*x + d)*log((e*x + d)*c)/e + 2*(e*x +
 d)/e
```

3.3.9 Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \log^2(c(d+ex))\,dx = 2\,x - 2\,x\,\ln(c\,d+c\,e\,x) + x\ln(c\,d+c\,e\,x)^2 + \frac{d\ln(c\,d+c\,e\,x)^2}{e} - \frac{2\,d\,\ln(d+e\,x)}{e}$$

input
```
int(log(c*(d + e*x))^2,x)
```

output
```
2*x - 2*x*log(c*d + c*e*x) + x*log(c*d + c*e*x)^2 + (d*log(c*d + c*e*x)^2)
/e - (2*d*log(d + e*x))/e
```

3.4 $\int \log(c(d+ex))\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \log(c(d+ex))\,dx = -x + \frac{(d+ex)\log(c(d+ex))}{e}$$

output
```
-x+(e*x+d)*ln(c*(e*x+d))/e
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \log(c(d+ex))\,dx = -x + \frac{(d+ex)\log(c(d+ex))}{e}$$

input
```
Integrate[Log[c*(d + e*x)],x]
```

output
```
-x + ((d + e*x)*Log[c*(d + e*x)])/e
```

3.4.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(c(d+ex))\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(d+ex)\log(c(d+ex))-d-ex}{e}$$

input
```
Int[Log[c*(d + e*x)],x]
```

output
```
(-d - e*x + (d + e*x)*Log[c*(d + e*x)])/e
```

3.4.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.4.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
risch	$x \ln \left(c(ex+d)\right)-x+\frac{d \ln(ex+d)}{e}$	26
norman	$x \ln \left(c(ex+d)\right)+\frac{d \ln(c(ex+d))}{e}-x$	28
parallelrisch	$\frac{\ln(c(ex+d))xe-ex+d\ln(c(ex+d))+d}{e}$	32
parts	$x \ln \left(c(ex+d)\right)-e\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^2}\right)$	33
derivativedivides	$\frac{(cex+cd)\ln(cex+cd)-cex-cd}{ce}$	36
default	$\frac{(cex+cd)\ln(cex+cd)-cex-cd}{ce}$	36

input
```
int(ln(c*(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(e*x+d))-x+d/e*ln(e*x+d)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \log(c(d+ex))\,dx = -\frac{ex-(ex+d)\log\left(cex+cd\right)}{e}$$

input
```
integrate(log(c*(e*x+d)),x, algorithm="fricas")
```

output
```
-(e*x - (e*x + d)*log(c*e*x + c*d))/e
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \log(c(d+ex))\,dx = -e\left(-\frac{d\log\left(d+ex\right)}{e^2}+\frac{x}{e}\right)+x\log\left(c(d+ex)\right)$$

input
```
integrate(ln(c*(e*x+d)),x)
```

output
```
-e*(-d*log(d + e*x)/e**2 + x/e) + x*log(c*(d + e*x))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \log(c(d+ex))\,dx = \frac{(ex+d)c\log\left((ex+d)c\right)-(ex+d)c}{ce}$$

input
```
integrate(log(c*(e*x+d)),x, algorithm="maxima")
```

output
```
((e*x + d)*c*log((e*x + d)*c) - (e*x + d)*c)/(c*e)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \log(c(d+ex))\,dx = \frac{(ex+d)c\log\left((ex+d)c\right)-(ex+d)c}{ce}$$

input
```
integrate(log(c*(e*x+d)),x, algorithm="giac")
```

output
```
((e*x + d)*c*log((e*x + d)*c) - (e*x + d)*c)/(c*e)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \log(c(d+ex))\,dx = x\,\ln\left(c\left(d+e\,x\right)\right)-x+\frac{d\,\ln\left(d+e\,x\right)}{e}$$

input
```
int(log(c*(d + e*x)),x)
```

output
```
x*log(c*(d + e*x)) - x + (d*log(d + e*x))/e
```

3.5 $\int \frac{1}{\log(c(d+ex))} \, dx$

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\log(c(d+ex))} \, dx = \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

output
```
Li(c*(e*x+d))/c/e
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(c(d+ex))} \, dx = \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

input
```
Integrate[Log[c*(d + e*x)]^(-1),x]
```

output
```
LogIntegral[c*(d + e*x)]/(c*e)
```

3.5.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2836, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log(c(d+ex))}\,dx$$
$$\Big\downarrow \text{2836}$$
$$\frac{\int \frac{1}{\log(c(d+ex))}d(d+ex)}{e}$$
$$\Big\downarrow \text{2735}$$
$$\frac{\text{LogIntegral}(c(d+ex))}{ce}$$

input
```
Int[Log[c*(d + e*x)]^(-1),x]
```

output
```
LogIntegral[c*(d + e*x)]/(c*e)
```

3.5.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.5.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{ce}$	22
default	$-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{ce}$	22
risch	$-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{ce}$	22

input
```
int(1/ln(c*(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
-1/c/e*Ei(1,-ln(c*e*x+c*d))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\mathrm{log\_integral}\,(cex+cd)}{ce}$$

input
```
integrate(1/log(c*(e*x+d)),x, algorithm="fricas")
```

output
```
log_integral(c*e*x + c*d)/(c*e)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\mathrm{li}\,(cd+cex)}{ce}$$

input
```
integrate(1/ln(c*(e*x+d)),x)
```

output
```
li(c*d + c*e*x)/(c*e)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\mathrm{Ei}(\log(cex+cd))}{ce}$$

input
```
integrate(1/log(c*(e*x+d)),x, algorithm="maxima")
```

output
```
Ei(log(c*e*x + c*d))/(c*e)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\mathrm{Ei}(\log((ex+d)c))}{ce}$$

input
```
integrate(1/log(c*(e*x+d)),x, algorithm="giac")
```

output
```
Ei(log((e*x + d)*c))/(c*e)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\mathrm{logint}(c\,(d+e\,x))}{c\,e}$$

input
```
int(1/log(c*(d + e*x)),x)
```

output
```
logint(c*(d + e*x))/(c*e)
```

3.6 $\int \frac{1}{\log^2(c(d+ex))} dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\log^2(c(d+ex))} dx = -\frac{d+ex}{e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

output
```
Li(c*(e*x+d))/c/e+(-e*x-d)/e/ln(c*(e*x+d))
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(c(d+ex))} dx = -\frac{d+ex}{e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

input
```
Integrate[Log[c*(d + e*x)]^(-2),x]
```

output
```
-((d + e*x)/(e*Log[c*(d + e*x)])) + LogIntegral[c*(d + e*x)]/(c*e)
```

3.6.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2836, 2734, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^2(c(d+ex))}dx$$
$$\Big\downarrow \text{ 2836}$$
$$\frac{\int \frac{1}{\log^2(c(d+ex))}d(d+ex)}{e}$$
$$\Big\downarrow \text{ 2734}$$
$$\frac{\int \frac{1}{\log(c(d+ex))}d(d+ex)-\frac{d+ex}{\log(c(d+ex))}}{e}$$
$$\Big\downarrow \text{ 2735}$$
$$\frac{\frac{\text{LogIntegral}(c(d+ex))}{c}-\frac{d+ex}{\log(c(d+ex))}}{e}$$

input
```
Int[Log[c*(d + e*x)]^(-2),x]
```

output
```
(-((d + e*x)/Log[c*(d + e*x)]) + LogIntegral[c*(d + e*x)]/c)/e
```

3.6.3.1 Defintions of rubi rules used

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.6.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{ex+d}{\ln(c(ex+d))e}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{ce}$	43
derivativedivides	$\frac{-\frac{cex+cd}{\ln(cex+cd)}-\mathrm{Ei}_1(-\ln(cex+cd))}{ce}$	45
default	$\frac{-\frac{cex+cd}{\ln(cex+cd)}-\mathrm{Ei}_1(-\ln(cex+cd))}{ce}$	45

input
```
int(1/ln(c*(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/ln(c*(e*x+d))/e*(e*x+d)-1/c/e*Ei(1,-ln(c*e*x+c*d))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = -\frac{cex+cd-\log(cex+cd)\,\mathrm{log\_integral}\,(cex+cd)}{ce\log(cex+cd)}$$

input
```
integrate(1/log(c*(e*x+d))^2,x, algorithm="fricas")
```

output
```
-(c*e*x + c*d - log(c*e*x + c*d)*log_integral(c*e*x + c*d))/(c*e*log(c*e*x
 + c*d))
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{-d-ex}{e\log\left(c\left(d+ex\right)\right)} + \frac{\operatorname{li}\left(cd+cex\right)}{ce}$$

input
```
integrate(1/ln(c*(e*x+d))**2,x)
```

output
```
(-d - e*x)/(e*log(c*(d + e*x))) + li(c*d + c*e*x)/(c*e)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{\Gamma(-1,-\log\left(cex+cd\right))}{ce}$$

input
```
integrate(1/log(c*(e*x+d))^2,x, algorithm="maxima")
```

output
```
gamma(-1, -log(c*e*x + c*d))/(c*e)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{\operatorname{Ei}(\log\left((ex+d)c\right))}{ce} - \frac{ex+d}{e\log\left((ex+d)c\right)}$$

input
```
integrate(1/log(c*(e*x+d))^2,x, algorithm="giac")
```

output
```
Ei(log((e*x + d)*c))/(c*e) - (e*x + d)/(e*log((e*x + d)*c))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{\mathrm{logint}(c\,(d+e\,x))}{c\,e} - \frac{d+e\,x}{e\,\ln\left(c\,\left(d+e\,x\right)\right)}$$

input
```
int(1/log(c*(d + e*x))^2,x)
```

output
```
logint(c*(d + e*x))/(c*e) - (d + e*x)/(e*log(c*(d + e*x)))
```

3.7 $\int \frac{1}{\log^3(c(d+ex))}\,dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = -\frac{d+ex}{2e\log^2(c(d+ex))}-\frac{d+ex}{2e\log(c(d+ex))}+\frac{\text{LogIntegral}(c(d+ex))}{2ce}$$

output
```
1/2*Li(c*(e*x+d))/c/e+1/2*(-e*x-d)/e/ln(c*(e*x+d))^2+1/2*(-e*x-d)/e/ln(c*(
e*x+d))
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{-\frac{(d+ex)(1+\log(c(d+ex)))}{\log^2(c(d+ex))}+\frac{\text{LogIntegral}(c(d+ex))}{c}}{2e}$$

input
```
Integrate[Log[c*(d + e*x)]^(-3),x]
```

output
```
(-(((d + e*x)*(1 + Log[c*(d + e*x)]))/Log[c*(d + e*x)]^2) + LogIntegral[c*
(d + e*x)]/c)/(2*e)
```

3.7.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2836, 2734, 2734, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^3(c(d+ex))} \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log^3(c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{1}{2}\int \frac{1}{\log^2(c(d+ex))}d(d+ex)-\frac{d+ex}{2\log^2(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{\log(c(d+ex))}d(d+ex)-\frac{d+ex}{\log(c(d+ex))}\right)-\frac{d+ex}{2\log^2(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2735}$$

$$\frac{\frac{1}{2}\left(\frac{\text{LogIntegral}(c(d+ex))}{c}-\frac{d+ex}{\log(c(d+ex))}\right)-\frac{d+ex}{2\log^2(c(d+ex))}}{e}$$

input
```
Int[Log[c*(d + e*x)]^(-3),x]
```

output
```
(-1/2*(d + e*x)/Log[c*(d + e*x)]^2 + (-((d + e*x)/Log[c*(d + e*x)]) + LogI
ntegral[c*(d + e*x)]/c)/2)/e
```

3.7.3.1 Defintions of rubi rules used

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{\ln(c(ex+d))xe+d\ln(c(ex+d))+ex+d}{2e\ln(c(ex+d))^{2}}-\frac{\mathrm{Ei}_{1}(-\ln(cex+cd))}{2ce}$	64
derivativedivides	$\frac{-\frac{cex+cd}{2\ln(cex+cd)^{2}}-\frac{cex+cd}{2\ln(cex+cd)}-\frac{\mathrm{Ei}_{1}(-\ln(cex+cd))}{2}}{ce}$	66
default	$\frac{-\frac{cex+cd}{2\ln(cex+cd)^{2}}-\frac{cex+cd}{2\ln(cex+cd)}-\frac{\mathrm{Ei}_{1}(-\ln(cex+cd))}{2}}{ce}$	66

input
```
int(1/ln(c*(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(c*(e*x+d))*x*e+d*ln(c*(e*x+d))+e*x+d)/e/ln(c*(e*x+d))^2-1/2/c/e*E
i(1,-ln(c*e*x+c*d))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = -\frac{cex - \log(cex+cd)^2 \operatorname{log\_integral}(cex+cd) + cd + (cex+cd)\log(cex+cd)}{2\,ce\log(cex+cd)^2}$$

input
```
integrate(1/log(c*(e*x+d))^3,x, algorithm="fricas")
```

output
```
-1/2*(c*e*x - log(c*e*x + c*d)^2*log_integral(c*e*x + c*d) + c*d + (c*e*x
+ c*d)*log(c*e*x + c*d))/(c*e*log(c*e*x + c*d)^2)
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{-d-ex+(-d-ex)\log(c(d+ex))}{2e\log(c(d+ex))^2} + \frac{\operatorname{li}(cd+cex)}{2ce}$$

input
```
integrate(1/ln(c*(e*x+d))**3,x)
```

output
```
(-d - e*x + (-d - e*x)*log(c*(d + e*x)))/(2*e*log(c*(d + e*x))**2) + li(c*
d + c*e*x)/(2*c*e)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = -\frac{\Gamma(-2, -\log(cex+cd))}{ce}$$

input
```
integrate(1/log(c*(e*x+d))^3,x, algorithm="maxima")
```

output
```
-gamma(-2, -log(c*e*x + c*d))/(c*e)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{\mathrm{Ei}(\log((ex+d)c))}{2\,ce} - \frac{ex+d}{2\,e\log((ex+d)c)} - \frac{ex+d}{2\,e\log((ex+d)c)^2}$$

input
```
integrate(1/log(c*(e*x+d))^3,x, algorithm="giac")
```

output
```
1/2*Ei(log((e*x + d)*c))/(c*e) - 1/2*(e*x + d)/(e*log((e*x + d)*c)) - 1/2*
(e*x + d)/(e*log((e*x + d)*c)^2)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{\mathrm{logint}(c\,(d+e\,x))}{2\,c\,e} - \frac{\frac{c\,d}{2} + \ln(c\,(d+e\,x))\,\left(\frac{c\,d}{2} + \frac{c\,e\,x}{2}\right) + \frac{c\,e\,x}{2}}{c\,e\ln(c\,(d+e\,x))^2}$$

input
```
int(1/log(c*(d + e*x))^3,x)
```

output
```
logint(c*(d + e*x))/(2*c*e) - ((c*d)/2 + log(c*(d + e*x))*((c*d)/2 + (c*e*
x)/2) + (c*e*x)/2)/(c*e*log(c*(d + e*x))^2)
```

3.8 $\int \frac{1}{\log^4(c(d+ex))}\,dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = -\frac{d+ex}{3e\log^3(c(d+ex))} - \frac{d+ex}{6e\log^2(c(d+ex))} - \frac{d+ex}{6e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{6ce}$$

output
```
1/6*Li(c*(e*x+d))/c/e+1/3*(-e*x-d)/e/ln(c*(e*x+d))^3+1/6*(-e*x-d)/e/ln(c*(
e*x+d))^2+1/6*(-e*x-d)/e/ln(c*(e*x+d))
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{-\frac{(d+ex)(2+\log(c(d+ex))+\log^2(c(d+ex)))}{\log^3(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{c}}{6e}$$

input
```
Integrate[Log[c*(d + e*x)]^(-4),x]
```

output
```
(-(((d + e*x)*(2 + Log[c*(d + e*x)] + Log[c*(d + e*x)]^2))/Log[c*(d + e*x)
]^3) + LogIntegral[c*(d + e*x)]/c)/(6*e)
```

3.8.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2836, 2734, 2734, 2734, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^4(c(d+ex))}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log^4(c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{1}{3}\int \frac{1}{\log^3(c(d+ex))}d(d+ex)-\frac{d+ex}{3\log^3(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{\log^2(c(d+ex))}d(d+ex)-\frac{d+ex}{2\log^2(c(d+ex))}\right)-\frac{d+ex}{3\log^3(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(\int \frac{1}{\log(c(d+ex))}d(d+ex)-\frac{d+ex}{\log(c(d+ex))}\right)-\frac{d+ex}{2\log^2(c(d+ex))}\right)-\frac{d+ex}{3\log^3(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2735}$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(\frac{\text{LogIntegral}(c(d+ex))}{c}-\frac{d+ex}{\log(c(d+ex))}\right)-\frac{d+ex}{2\log^2(c(d+ex))}\right)-\frac{d+ex}{3\log^3(c(d+ex))}}{e}$$

input
```
Int[Log[c*(d + e*x)]^(-4),x]
```

output
```
(-1/3*(d + e*x)/Log[c*(d + e*x)]^3 + (-1/2*(d + e*x)/Log[c*(d + e*x)]^2 +
(-((d + e*x)/Log[c*(d + e*x)]) + LogIntegral[c*(d + e*x)]/c)/2)/3)/e
```

3.8.3.1 Defintions of rubi rules used

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.8.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{cex+cd}{3\ln(cex+cd)^3}-\frac{cex+cd}{6\ln(cex+cd)^2}-\frac{cex+cd}{6\ln(cex+cd)}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{6}}{ce}$	87
default	$\frac{-\frac{cex+cd}{3\ln(cex+cd)^3}-\frac{cex+cd}{6\ln(cex+cd)^2}-\frac{cex+cd}{6\ln(cex+cd)}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{6}}{ce}$	87
risch	$-\frac{x\ln(c(ex+d))^2e+\ln(c(ex+d))^2d+\ln(c(ex+d))xe+d\ln(c(ex+d))+2ex+2d}{6e\ln(c(ex+d))^3}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{6ce}$	92

input
```
int(1/ln(c*(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/c/e*(-1/3*(c*e*x+c*d)/ln(c*e*x+c*d)^3-1/6*(c*e*x+c*d)/ln(c*e*x+c*d)^2-1/
6*(c*e*x+c*d)/ln(c*e*x+c*d)-1/6*Ei(1,-ln(c*e*x+c*d)))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\log(cex+cd)^3 \operatorname{log\_integral}(cex+cd) - 2\,cex - (cex+cd)\log(cex+cd)^2 - 2\,cd - (cex+cd)\log(cex-}{6\,ce\log(cex+cd)^3}$$

input
```
integrate(1/log(c*(e*x+d))^4,x, algorithm="fricas")
```

output
```
1/6*(log(c*e*x + c*d)^3*log_integral(c*e*x + c*d) - 2*c*e*x - (c*e*x + c*d
)*log(c*e*x + c*d)^2 - 2*c*d - (c*e*x + c*d)*log(c*e*x + c*d))/(c*e*log(c*
e*x + c*d)^3)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{-d-ex+\left(-\frac{d}{2}-\frac{ex}{2}\right)\log(c(d+ex))^2+\left(-\frac{d}{2}-\frac{ex}{2}\right)\log(c(d+ex))}{3e\log(c(d+ex))^3} + \frac{\operatorname{li}(cd+cex)}{6ce}$$

input
```
integrate(1/ln(c*(e*x+d))**4,x)
```

output
```
(-d - e*x + (-d/2 - e*x/2)*log(c*(d + e*x))**2 + (-d/2 - e*x/2)*log(c*(d +
 e*x)))/(3*e*log(c*(d + e*x))**3) + li(c*d + c*e*x)/(6*c*e)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\Gamma(-3,-\log(cex+cd))}{ce}$$

input
```
integrate(1/log(c*(e*x+d))^4,x, algorithm="maxima")
```

output
```
gamma(-3, -log(c*e*x + c*d))/(c*e)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\mathrm{Ei}(\log((ex+d)c))}{6\,ce} - \frac{ex+d}{6\,e\log((ex+d)c)} - \frac{ex+d}{6\,e\log((ex+d)c)^2} - \frac{ex+d}{3\,e\log((ex+d)c)^3}$$

input
```
integrate(1/log(c*(e*x+d))^4,x, algorithm="giac")
```

output
```
1/6*Ei(log((e*x + d)*c))/(c*e) - 1/6*(e*x + d)/(e*log((e*x + d)*c)) - 1/6*
(e*x + d)/(e*log((e*x + d)*c)^2) - 1/3*(e*x + d)/(e*log((e*x + d)*c)^3)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = -\frac{(d+e\,x)\left(\frac{1}{6\,\ln(c\,(d+e\,x))} + \frac{1}{6\ln(c\,(d+e\,x))^2} + \frac{1}{3\ln(c\,(d+e\,x))^3}\right)}{e} - \frac{\mathrm{expint}(-\ln(c\,(d+e\,x)))}{6\,c\,e}$$

input
```
int(1/log(c*(d + e*x))^4,x)
```

output
```
- ((d + e*x)*(1/(6*log(c*(d + e*x))) + 1/(6*log(c*(d + e*x))^2) + 1/(3*log
(c*(d + e*x))^3)))/e - expint(-log(c*(d + e*x)))/(6*c*e)
```

3.9 $\int \log^{\frac{5}{2}}(c(d+ex))\,dx$

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = -\frac{15\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{8ce} + \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e} - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e}$$

output
```
-5/2*(e*x+d)*ln(c*(e*x+d))^(3/2)/e+(e*x+d)*ln(c*(e*x+d))^(5/2)/e-15/8*erfi
(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e+15/4*(e*x+d)*ln(c*(e*x+d))^(1/2)/e
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \frac{-15\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right) + 2c(d+ex)\sqrt{\log(c(d+ex))}\left(15-10\log(c(d+ex))+4\log^2(c(d+ex))\right)}{8ce}$$

input
```
Integrate[Log[c*(d + e*x)]^(5/2),x]
```

output
```
(-15*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]] + 2*c*(d + e*x)*Sqrt[Log[c*(d +
 e*x)]]*(15 - 10*Log[c*(d + e*x)] + 4*Log[c*(d + e*x)]^2))/(8*c*e)
```

3.9.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2836, 2733, 2733, 2733, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^{\frac{5}{2}}(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))-\frac{5}{2}\int \log^{\frac{3}{2}}(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))-\frac{5}{2}\left((d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\int \sqrt{\log(c(d+ex))}d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))-\frac{5}{2}\left((d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{1}{2}\int \frac{1}{\sqrt{\log(c(d+ex))}}d(d+ex\right.\right.}{e}$$

$$\Big\downarrow \text{2736}$$

$$\frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))-\frac{5}{2}\left((d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{\int \frac{c(d+ex)}{\sqrt{\log(c(d+ex))}}d\log(c(d+ex))}{2c}\right.\right.}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))-\frac{5}{2}\left((d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{\int c(d+ex)d\sqrt{\log(c(d+ex))}}{c}\right)\right)}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex)) - \frac{5}{2}\left((d+ex)\log^{\frac{3}{2}}(c(d+ex)) - \frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))} - \frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2c}\right)\right)}{e}$$

input
```
Int[Log[c*(d + e*x)]^(5/2),x]
```

output
```
((d + e*x)*Log[c*(d + e*x)]^(5/2) - (5*((-3*(-1/2*(Sqrt[Pi]*Erfi[Sqrt[Log[
c*(d + e*x)]]])/c + (d + e*x)*Sqrt[Log[c*(d + e*x)]]))/2 + (d + e*x)*Log[c
*(d + e*x)]^(3/2)))/2)/e
```

3.9.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.9.4 Maple [F]

$$\int \ln \left(c(ex+d)\right)^{\frac{5}{2}} dx$$

input
```
int(ln(c*(e*x+d))^(5/2),x)
```

output
```
int(ln(c*(e*x+d))^(5/2),x)
```

3.9.5 Fricas [F(-2)]

Exception generated.

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(c*(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.9.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx$$
$$= \frac{2\,(cex+cd)\left(4\,\log\left(cex+cd\right)^{\frac{5}{2}}-10\,\log\left(cex+cd\right)^{\frac{3}{2}}+15\,\sqrt{\log\left(cex+cd\right)}\right)+15i\,\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{\log\left(cex+\right.}\right.}{8\,ce}$$

input
```
integrate(log(c*(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
1/8*(2*(c*e*x + c*d)*(4*log(c*e*x + c*d)^(5/2) - 10*log(c*e*x + c*d)^(3/2)
 + 15*sqrt(log(c*e*x + c*d))) + 15*I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d))
))/(c*e)
```

3.9.8 Giac [F]

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \int \log\left((ex+d)c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(log(c*(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate(log((e*x + d)*c)^(5/2), x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx$$
$$= \frac{\ln\left(c\,(d+e\,x)\right)^{5/2}\,\left(\frac{15\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\ln(c\,(d+e\,x))}\right)}{8}+c\,(d+e\,x)\,\left(\frac{15\,\sqrt{-\ln(c\,(d+e\,x))}}{4}+\frac{5\,(-\ln(c\,(d+e\,x)))^{3/2}}{2}+(-\ln\left(c\,(d\right.\right.\right.}{c\,e\,\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)^{5/2}}$$

input
```
int(log(c*(d + e*x))^(5/2),x)
```

output
```
(log(c*(d + e*x))^(5/2)*((15*pi^(1/2)*erfc((-log(c*(d + e*x)))^(1/2)))/8 +
 c*(d + e*x)*((15*(-log(c*(d + e*x)))^(1/2))/4 + (5*(-log(c*(d + e*x)))^(3
/2))/2 + (-log(c*(d + e*x)))^(5/2))))/(c*e*(-log(c*(d + e*x)))^(5/2))
```

3.9. $\int \log^{\frac{5}{2}}(c(d+ex))\,dx$

3.10 $\int \log^{\frac{3}{2}}(c(d+ex))\,dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{4ce} - \frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e}$$

output
```
(e*x+d)*ln(c*(e*x+d))^(3/2)/e+3/4*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e-3
/2*(e*x+d)*ln(c*(e*x+d))^(1/2)/e
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right) + 2c(d+ex)\sqrt{\log(c(d+ex))}(-3+2\log(c(d+ex)))}{4ce}$$

input
```
Integrate[Log[c*(d + e*x)]^(3/2),x]
```

output
```
(3*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]] + 2*c*(d + e*x)*Sqrt[Log[c*(d + e
*x)]]*(-3 + 2*Log[c*(d + e*x)]))/(4*c*e)
```

3.10.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2836, 2733, 2733, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^{\frac{3}{2}}(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\int \sqrt{\log(c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{1}{2}\int \frac{1}{\sqrt{\log(c(d+ex))}}d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{2736}$$

$$\frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{\int \frac{c(d+ex)}{\sqrt{\log(c(d+ex))}}d\log(c(d+ex))}{2c}\right)}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{\int c(d+ex)d\sqrt{\log(c(d+ex))}}{c}\right)}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))-\frac{3}{2}\left((d+ex)\sqrt{\log(c(d+ex))}-\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2c}\right)}{e}$$

input
```
Int[Log[c*(d + e*x)]^(3/2),x]
```

output
```
((-3*(-1/2*(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/c + (d + e*x)*Sqrt[Log[
c*(d + e*x)]]))/2 + (d + e*x)*Log[c*(d + e*x)]^(3/2))/e
```

3.10.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.10.4 Maple [F]

$$\int \ln \left(c(ex+d)\right)^{\frac{3}{2}} dx$$

input
```
int(ln(c*(e*x+d))^(3/2),x)
```

output
```
int(ln(c*(e*x+d))^(3/2),x)
```

3.10.5 Fricas [F(-2)]

Exception generated.

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(c*(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 55.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx$$
$$= \begin{cases} \tilde{\infty}x & \text{for } c=0 \\ x\log(cd)^{\frac{3}{2}} & \text{for } e=0 \\ \frac{\left(-\sqrt{-\log(cd+cex)}(cd+cex)\left(\log(cd+cex)-\frac{3}{2}\right)+\frac{3\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\log(cd+cex)}\right)}{4}\right)\log(cd+cex)^{\frac{3}{2}}}{ce(-\log(cd+cex))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(e*x+d))**(3/2),x)
```

output
```
Piecewise((zoo*x, Eq(c, 0)), (x*log(c*d)**(3/2), Eq(e, 0)), ((-sqrt(-log(c
*d + c*e*x))*(c*d + c*e*x)*(log(c*d + c*e*x) - 3/2) + 3*sqrt(pi)*erfc(sqrt
(-log(c*d + c*e*x)))/4)*log(c*d + c*e*x)**(3/2)/(c*e*(-log(c*d + c*e*x))**
(3/2)), True))
```

3.10.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \frac{2\,(cex+cd)\Big(2\,\log{(cex+cd)}^{\frac{3}{2}} - 3\,\sqrt{\log{(cex+cd)}}\Big) - 3i\,\sqrt{\pi}\operatorname{erf}\left(i\,\sqrt{\log{(cex+cd)}}\right)}{4\,ce}$$

input
```
integrate(log(c*(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*(c*e*x + c*d)*(2*log(c*e*x + c*d)^(3/2) - 3*sqrt(log(c*e*x + c*d)))
 - 3*I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d))))/(c*e)
```

3.10.8 Giac [F]

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \int \log\left((ex+d)c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(log(c*(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate(log((e*x + d)*c)^(3/2), x)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \frac{\ln\left(c\,(d+e\,x)\right)^{3/2}\left(\frac{3\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\ln(c\,(d+e\,x))}\right)}{4}+c\left(\frac{3\,\sqrt{-\ln(c\,(d+e\,x))}}{2}+\left(-\ln\left(c\,(d+e\,x)\right)\right)^{3/2}\right)\,(d+e\,x)\right)}{c\,e\,\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)^{3/2}}$$

input
```
int(log(c*(d + e*x))^(3/2),x)
```

output
```
(log(c*(d + e*x))^(3/2)*((3*pi^(1/2)*erfc((-log(c*(d + e*x)))^(1/2)))/4 +
c*((3*(-log(c*(d + e*x)))^(1/2))/2 + (-log(c*(d + e*x)))^(3/2))*(d + e*x))
)/(c*e*(-log(c*(d + e*x)))^(3/2))
```

3.10. $\int \log^{\frac{3}{2}}(c(d+ex))\,dx$

3.11 $\int \sqrt{\log(c(d+ex))}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2ce} + \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e}$$

output
```
-1/2*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e+(e*x+d)*ln(c*(e*x+d))^(1/2)/e
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2ce} + \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e}$$

input
```
Integrate[Sqrt[Log[c*(d + e*x)]],x]
```

output
```
-1/2*(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e) + ((d + e*x)*Sqrt[Log[c
*(d + e*x)]])/e
```

3.11.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2836, 2733, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\log (c(d+ex))} \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \sqrt{\log (c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\sqrt{\log (c(d+ex))}-\frac{1}{2}\int \frac{1}{\sqrt{\log (c(d+ex))}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2736}$$

$$\frac{(d+ex)\sqrt{\log (c(d+ex))}-\frac{\int \frac{c(d+ex)}{\sqrt{\log (c(d+ex))}}d\log (c(d+ex))}{2c}}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(d+ex)\sqrt{\log (c(d+ex))}-\frac{\int c(d+ex)d\sqrt{\log (c(d+ex))}}{c}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(d+ex)\sqrt{\log (c(d+ex))}-\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\log (c(d+ex))}\right)}{2c}}{e}$$

input
```
Int[Sqrt[Log[c*(d + e*x)]],x]
```

output
```
(-1/2*(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/c + (d + e*x)*Sqrt[Log[c*(d
+ e*x)]])/e
```

3.11.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.11.4 Maple [F]

$$\int \sqrt{\ln \left(c \left(ex+d\right)\right)}dx$$

input
```
int(ln(c*(e*x+d))^(1/2),x)
```

output
```
int(ln(c*(e*x+d))^(1/2),x)
```

3.11.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\log(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(c*(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.90 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \sqrt{\log(c(d+ex))}\,dx = \begin{cases} \tilde{\infty}x & \text{for } c=0 \\ x\sqrt{\log(cd)} & \text{for } e=0 \\ \frac{\left(\sqrt{-\log(cd+cex)}(cd+cex)+\frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-\log(cd+cex)}\right)}{2}\right)\sqrt{\log(cd+cex)}}{ce\sqrt{-\log(cd+cex)}} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(e*x+d))**(1/2),x)
```

output
```
Piecewise((zoo*x, Eq(c, 0)), (x*sqrt(log(c*d)), Eq(e, 0)), ((sqrt(-log(c*d
 + c*e*x))*(c*d + c*e*x) + sqrt(pi)*erfc(sqrt(-log(c*d + c*e*x)))/2)*sqrt(
log(c*d + c*e*x))/(c*e*sqrt(-log(c*d + c*e*x))), True))
```

3.11.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{-i\sqrt{\pi}\operatorname{erf}\left(i\sqrt{\log(cex+cd)}\right) - 2(cex+cd)\sqrt{\log(cex+cd)}}{2\,ce}$$

input
```
integrate(log(c*(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(-I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d))) - 2*(c*e*x + c*d)*sqrt(log
(c*e*x + c*d)))/(c*e)
```

3.11.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-i\sqrt{\log\left(cex+cd\right)}\right)}{2\,ce} + \frac{(cex+cd)\sqrt{\log\left(cex+cd\right)}}{ce}$$

input
```
integrate(log(c*(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(pi)*erf(-I*sqrt(log(c*e*x + c*d)))/(c*e) + (c*e*x + c*d)*sqrt(
log(c*e*x + c*d))/(c*e)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \sqrt{\log(c(d+ex))}\,dx = \frac{\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}\,\left(d+e\,x\right)}{e} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,c\,e}$$

input
```
int(log(c*(d + e*x))^(1/2),x)
```

output
```
(log(c*(d + e*x))^(1/2)*(d + e*x))/e + (pi^(1/2)*erf(log(c*(d + e*x))^(1/2
)*1i)*1i)/(2*c*e)
```

3.12 $\int \frac{1}{\sqrt{\log(c(d+ex))}} dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{1}{\sqrt{\log(c(d+ex))}} dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce}$$

output
```
erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e
```

3.12.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(c(d+ex))}} dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce}$$

input
```
Integrate[1/Sqrt[Log[c*(d + e*x)]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e)
```

3.12.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2836, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\log (c(d+ex))}} \, dx$$
$$\Big\downarrow \text{2836}$$
$$\frac{\int \frac{1}{\sqrt{\log (c(d+ex))}}d(d+ex)}{e}$$
$$\Big\downarrow \text{2736}$$
$$\frac{\int \frac{c(d+ex)}{\sqrt{\log (c(d+ex))}}d\log (c(d+ex))}{ce}$$
$$\Big\downarrow \text{2611}$$
$$\frac{2\int c(d+ex)d\sqrt{\log (c(d+ex))}}{ce}$$
$$\Big\downarrow \text{2633}$$
$$\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\log (c(d+ex))}\right)}{ce}$$

input
```
Int[1/Sqrt[Log[c*(d + e*x)]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e)
```

3.12.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.12.4 Maple [F]

$$\int \frac{1}{\sqrt{\ln \left(c \left(ex+d\right)\right)}}dx$$

input
```
int(1/ln(c*(e*x+d))^(1/2),x)
```

output
```
int(1/ln(c*(e*x+d))^(1/2),x)
```

3.12.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/log(c*(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \begin{cases} 0 & \text{for } c = 0 \\ \frac{x}{\sqrt{\log(cd)}} & \text{for } e = 0 \\ \frac{\sqrt{\pi}\sqrt{-\log(cd+cex)}\operatorname{erfc}\left(\sqrt{-\log(cd+cex)}\right)}{ce\sqrt{\log(cd+cex)}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/ln(c*(e*x+d))**(1/2),x)
```

output
```
Piecewise((0, Eq(c, 0)), (x/sqrt(log(c*d)), Eq(e, 0)), (sqrt(pi)*sqrt(-log
(c*d + c*e*x))*erfc(sqrt(-log(c*d + c*e*x)))/(c*e*sqrt(log(c*d + c*e*x))),
 True))
```

3.12.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = -\frac{i\sqrt{\pi}\operatorname{Erf}\left(i\sqrt{\log(cex+cd)}\right)}{ce}$$

input
```
integrate(1/log(c*(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
-I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d)))/(c*e)
```

3.12.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-i\sqrt{\log(cex+cd)}\right)}{ce}$$

input
```
integrate(1/log(c*(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*erf(-I*sqrt(log(c*e*x + c*d)))/(c*e)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{\sqrt{\pi}\,\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\,\mathrm{erfc}\left(\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\right)}{c\,e\,\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}}$$

input
```
int(1/log(c*(d + e*x))^(1/2),x)
```

output
```
(pi^(1/2)*(-log(c*(d + e*x)))^(1/2)*erfc((-log(c*(d + e*x)))^(1/2)))/(c*e*
log(c*(d + e*x))^(1/2))
```

3.13 $\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))} \, dx$

3.13.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))} \, dx = \frac{2\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce} - \frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}}$$

output
```
2*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e-2*(e*x+d)/e/ln(c*(e*x+d))^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))} \, dx = \frac{-2c(d+ex) + 2\Gamma\left(\frac{1}{2}, -\log(c(d+ex))\right)\sqrt{-\log(c(d+ex))}}{ce\sqrt{\log(c(d+ex))}}$$

input
```
Integrate[Log[c*(d + e*x)]^(-3/2),x]
```

output
```
(-2*c*(d + e*x) + 2*Gamma[1/2, -Log[c*(d + e*x)]]*Sqrt[-Log[c*(d + e*x)]])
/(c*e*Sqrt[Log[c*(d + e*x)]])
```

3.13.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2836, 2734, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{2\int \frac{1}{\sqrt{\log(c(d+ex))}}d(d+ex)-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}}{e}$$

$$\Big\downarrow \text{ 2736}$$

$$\frac{\frac{2\int \frac{c(d+ex)}{\sqrt{\log(c(d+ex))}}d\log(c(d+ex))}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}}{e}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{\frac{4\int c(d+ex)d\sqrt{\log(c(d+ex))}}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}}{e}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\frac{2\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}}{e}$$

input
```
Int[Log[c*(d + e*x)]^(-3/2),x]
```

output
```
((2*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/c - (2*(d + e*x))/Sqrt[Log[c*(d
 + e*x)]])/e
```

3.13.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.13.4 Maple [F]

$$\int \frac{1}{\ln \left(c \left(ex+d\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/ln(c*(e*x+d))^(3/2),x)
```

output
```
int(1/ln(c*(e*x+d))^(3/2),x)
```

3.13.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/log(c*(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 13.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = \begin{cases} 0 & \text{for } c = 0 \\ \frac{x}{\log(cd)^{\frac{3}{2}}} & \text{for } e = 0 \\ \frac{(-\log(cd+cex))^{\frac{3}{2}}\left(-2\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-\log(cd+cex)}\right)+\frac{2(cd+cex)}{\sqrt{-\log(cd+cex)}}\right)}{ce\log(cd+cex)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/ln(c*(e*x+d))**(3/2),x)
```

output
```
Piecewise((0, Eq(c, 0)), (x/log(c*d)**(3/2), Eq(e, 0)), ((-log(c*d + c*e*x
))**(3/2)*(-2*sqrt(pi)*erfc(sqrt(-log(c*d + c*e*x))) + 2*(c*d + c*e*x)/sqr
t(-log(c*d + c*e*x)))/(c*e*log(c*d + c*e*x)**(3/2)), True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = -\frac{\sqrt{-\log(cex+cd)}\,\Gamma\left(-\frac{1}{2}, -\log(cex+cd)\right)}{ce\sqrt{\log(cex+cd)}}$$

input
```
integrate(1/log(c*(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
-sqrt(-log(c*e*x + c*d))*gamma(-1/2, -log(c*e*x + c*d))/(c*e*sqrt(log(c*e*
x + c*d)))
```

3.13.8 Giac [F]

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = \int \frac{1}{\log\left((ex+d)c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/log(c*(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate(log((e*x + d)*c)^(-3/2), x)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = -\frac{2\,(d+e\,x)}{e\,\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}} - \frac{2\,\sqrt{\pi}\,\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)^{3/2}\,\mathrm{erfc}\left(\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\right)}{c\,e\,\ln\left(c\,\left(d+e\,x\right)\right)^{3/2}}$$

input
```
int(1/log(c*(d + e*x))^(3/2),x)
```

output
```
- (2*(d + e*x))/(e*log(c*(d + e*x))^(1/2)) - (2*pi^(1/2)*(-log(c*(d + e*x)
))^(3/2)*erfc((-log(c*(d + e*x)))^(1/2)))/(c*e*log(c*(d + e*x))^(3/2))
```

3.14 $\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))} dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))} dx = \frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{3ce} - \frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}}$$

output
```
-2/3*(e*x+d)/e/ln(c*(e*x+d))^(3/2)+4/3*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/
c/e-4/3*(e*x+d)/e/ln(c*(e*x+d))^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))} dx = -\frac{2\left(2\Gamma\left(\frac{1}{2}, -\log(c(d+ex))\right)(-\log(c(d+ex)))^{3/2} + c(d+ex)(1+2\log(c(d+ex)))\right)}{3ce\log^{\frac{3}{2}}(c(d+ex))}$$

input
```
Integrate[Log[c*(d + e*x)]^(-5/2),x]
```

output
```
(-2*(2*Gamma[1/2, -Log[c*(d + e*x)]]*(-Log[c*(d + e*x)])^(3/2) + c*(d + e*
x)*(1 + 2*Log[c*(d + e*x)])))/(3*c*e*Log[c*(d + e*x)]^(3/2))
```

3.14.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2836, 2734, 2734, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2}{3}\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}d(d+ex)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2}{3}\left(2\int \frac{1}{\sqrt{\log(c(d+ex))}}d(d+ex)-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2736}$$

$$\frac{\frac{2}{3}\left(\frac{2\int \frac{c(d+ex)}{\sqrt{\log(c(d+ex))}}d\log(c(d+ex))}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{2}{3}\left(\frac{4\int c(d+ex)d\sqrt{\log(c(d+ex))}}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2}{3}\left(\frac{2\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}}{e}$$

input
```
Int[Log[c*(d + e*x)]^(-5/2),x]
```

output
```
((2*((2*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/c - (2*(d + e*x))/Sqrt[Log[
c*(d + e*x)]]))/3 - (2*(d + e*x))/(3*Log[c*(d + e*x)]^(3/2)))/e
```

3.14.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.14.4 Maple [F]

$$\int \frac{1}{\ln\left(c\left(ex+d\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/ln(c*(e*x+d))^(5/2),x)
```

output
```
int(1/ln(c*(e*x+d))^(5/2),x)
```

3.14.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/log(c*(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \text{Timed out}$$

input
```
integrate(1/ln(c*(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = -\frac{(-\log(cex+cd))^{\frac{3}{2}}\,\Gamma\left(-\frac{3}{2}, -\log(cex+cd)\right)}{ce\log(cex+cd)^{\frac{3}{2}}}$$

input
```
integrate(1/log(c*(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
-(-log(c*e*x + c*d))^(3/2)*gamma(-3/2, -log(c*e*x + c*d))/(c*e*log(c*e*x +
 c*d)^(3/2))
```

3.14.8 Giac [F]

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \int \frac{1}{\log\left((ex+d)c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/log(c*(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate(log((e*x + d)*c)^(-5/2), x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \frac{4\sqrt{\pi}\left(-\ln\left(c\left(d+e\,x\right)\right)\right)^{5/2}\,\mathrm{erfc}\left(\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\right)}{3\,c\,e\ln\left(c\,\left(d+e\,x\right)\right)^{5/2}} - \frac{4\,d\ln\left(c\left(d+e\,x\right)\right)^2+2\,d\,\ln\left(c\left(d+e\,x\right)\right)+2\,e\,x\,\ln\left(c\left(d+e\,x\right)\right)+4\,e\,x\ln\left(c\left(d+e\,x\right)\right)^2}{3\,e\ln\left(c\,\left(d+e\,x\right)\right)^{5/2}}$$

input
```
int(1/log(c*(d + e*x))^(5/2),x)
```

output
```
(4*pi^(1/2)*(-log(c*(d + e*x)))^(5/2)*erfc((-log(c*(d + e*x)))^(1/2)))/(3*
c*e*log(c*(d + e*x))^(5/2)) - (4*d*log(c*(d + e*x))^2 + 2*d*log(c*(d + e*x
)) + 2*e*x*log(c*(d + e*x)) + 4*e*x*log(c*(d + e*x))^2)/(3*e*log(c*(d + e*
x))^(5/2))
```

3.15 $\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))} dx$

3.15.1 Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))} dx = \frac{8\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{15ce} - \frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} - \frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))} - \frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}}$$

output
```
-2/5*(e*x+d)/e/ln(c*(e*x+d))^(5/2)-4/15*(e*x+d)/e/ln(c*(e*x+d))^(3/2)+8/15
*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e-8/15*(e*x+d)/e/ln(c*(e*x+d))^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))} dx = \frac{8\Gamma\left(\frac{1}{2}, -\log(c(d+ex))\right)(-\log(c(d+ex)))^{5/2} - 2c(d+ex)\left(3+2\log(c(d+ex))+4\log^2(c(d+ex))\right)}{15ce\log^{\frac{5}{2}}(c(d+ex))}$$

input
```
Integrate[Log[c*(d + e*x)]^(-7/2),x]
```

output
```
(8*Gamma[1/2, -Log[c*(d + e*x)]]*(-Log[c*(d + e*x)])^(5/2) - 2*c*(d + e*x)
*(3 + 2*Log[c*(d + e*x)] + 4*Log[c*(d + e*x)]^2))/(15*c*e*Log[c*(d + e*x)]
^(5/2))
```

3.15.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2836, 2734, 2734, 2734, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{2}{5}\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}d(d+ex)-\frac{2(d+ex)}{5\log^{\frac{5}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{2}{5}\left(\frac{2}{3}\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}d(d+ex)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}\right)-\frac{2(d+ex)}{5\log^{\frac{5}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{2}{5}\left(\frac{2}{3}\left(2\int \frac{1}{\sqrt{\log(c(d+ex))}}d(d+ex)-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}\right)-\frac{2(d+ex)}{5\log^{\frac{5}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{ 2736}$$

$$\frac{\frac{2}{5}\left(\frac{2}{3}\left(\frac{2\int \frac{c(d+ex)}{\sqrt{\log(c(d+ex))}}d\log(c(d+ex))}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}\right)-\frac{2(d+ex)}{5\log^{\frac{5}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{\frac{2}{5}\left(\frac{2}{3}\left(\frac{4\int c(d+ex)d\sqrt{\log(c(d+ex))}}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}\right)-\frac{2(d+ex)}{5\log^{\frac{5}{2}}(c(d+ex))}}{e}$$

$$\Big\downarrow \; 2633$$

$$\frac{\frac{2}{5}\left(\frac{2}{3}\left(\frac{2\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{c}-\frac{2(d+ex)}{\sqrt{\log(c(d+ex))}}\right)-\frac{2(d+ex)}{3\log^{\frac{3}{2}}(c(d+ex))}\right)-\frac{2(d+ex)}{5\log^{\frac{5}{2}}(c(d+ex))}}{e}$$

input
```
Int[Log[c*(d + e*x)]^(-7/2),x]
```

output
```
((2*((2*((2*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/c - (2*(d + e*x))/Sqrt[
Log[c*(d + e*x)]]))/3 - (2*(d + e*x))/(3*Log[c*(d + e*x)]^(3/2))))/5 - (2*
(d + e*x))/(5*Log[c*(d + e*x)]^(5/2)))/e
```

3.15.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.15.4 Maple [F]

$$\int \frac{1}{\ln\left(c\left(ex+d\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/ln(c*(e*x+d))^(7/2),x)
```

output
```
int(1/ln(c*(e*x+d))^(7/2),x)
```

3.15.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/log(c*(e*x+d))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \text{Timed out}$$

input
```
integrate(1/ln(c*(e*x+d))**(7/2),x)
```

output
```
Timed out
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = -\frac{(-\log(cex+cd))^{\frac{5}{2}}\Gamma\left(-\frac{5}{2}, -\log(cex+cd)\right)}{ce\log(cex+cd)^{\frac{5}{2}}}$$

input
```
integrate(1/log(c*(e*x+d))^(7/2),x, algorithm="maxima")
```

output
```
-(-log(c*e*x + c*d))^(5/2)*gamma(-5/2, -log(c*e*x + c*d))/(c*e*log(c*e*x +
 c*d)^(5/2))
```

3.15.8 Giac [F]

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \int \frac{1}{\log((ex+d)c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/log(c*(e*x+d))^(7/2),x, algorithm="giac")
```

output
```
integrate(log((e*x + d)*c)^(-7/2), x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx =$$
$$-\frac{4\,d\ln(c\,(d+e\,x))^2+8\,d\ln(c\,(d+e\,x))^3+6\,d\,\ln(c\,(d+e\,x))+6\,e\,x\,\ln(c\,(d+e\,x))+4\,e\,x\ln(c\,(d+e}{15\,e\ln(c\,\,(d+e\,x))^{7/2}}$$
$$-\frac{8\sqrt{\pi}\,(-\ln(c\,(d+e\,x)))^{7/2}\,\mathrm{erfc}\left(\sqrt{-\ln(c\,\,(d+e\,x))}\right)}{15\,c\,e\ln(c\,\,(d+e\,x))^{7/2}}$$

input
```
int(1/log(c*(d + e*x))^(7/2),x)
```

output
```
- (4*d*log(c*(d + e*x))^2 + 8*d*log(c*(d + e*x))^3 + 6*d*log(c*(d + e*x))
+ 6*e*x*log(c*(d + e*x)) + 4*e*x*log(c*(d + e*x))^2 + 8*e*x*log(c*(d + e*x
))^3)/(15*e*log(c*(d + e*x))^(7/2)) - (8*pi^(1/2)*(-log(c*(d + e*x)))^(7/2
)*erfc((-log(c*(d + e*x)))^(1/2)))/(15*c*e*log(c*(d + e*x))^(7/2))
```

3.15. $\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx$

3.16 $\int \log^p(c(d+ex))\,dx$

3.16.1 Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \log^p(c(d+ex))\,dx = \frac{\Gamma(1+p,-\log(c(d+ex)))(-\log(c(d+ex)))^{-p}\log^p(c(d+ex))}{ce}$$

output
```
GAMMA(p+1,-ln(c*(e*x+d)))*ln(c*(e*x+d))^p/c/e/((-ln(c*(e*x+d)))^p)
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log^p(c(d+ex))\,dx = \frac{\Gamma(1+p,-\log(c(d+ex)))(-\log(c(d+ex)))^{-p}\log^p(c(d+ex))}{ce}$$

input
```
Integrate[Log[c*(d + e*x)]^p,x]
```

output
```
(Gamma[1 + p, -Log[c*(d + e*x)]]*Log[c*(d + e*x)]^p)/(c*e*(-Log[c*(d + e*x
)])^p)
```

3.16.3 Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2836, 2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^p(c(d+ex))\,dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \log^p(c(d+ex))d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2736}$$

$$\frac{\int c(d+ex)\log^p(c(d+ex))d\log(c(d+ex))}{ce}$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{(-d-ex)^{-p}\Gamma(p+1,-d-ex)\log^p(c(d+ex))}{ce}$$

input
```
Int[Log[c*(d + e*x)]^p,x]
```

output
```
(Gamma[1 + p, -d - e*x]*Log[c*(d + e*x)]^p)/(c*e*(-d - e*x)^p)
```

3.16.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.16.4 Maple [F]

$$\int \ln \left(c(ex+d)\right)^{p} dx$$

input
```
int(ln(c*(e*x+d))^p,x)
```

output
```
int(ln(c*(e*x+d))^p,x)
```

3.16.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int \log^p(c(d+ex))\,dx = \frac{e^{(-i\,\pi p)}\Gamma(p+1,-\log{(cex+cd)})}{ce}$$

input
```
integrate(log(c*(e*x+d))^p,x, algorithm="fricas")
```

output
```
e^(-I*pi*p)*gamma(p + 1, -log(c*e*x + c*d))/(c*e)
```

3.16.6 Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \log^p(c(d+ex))\,dx = \begin{cases} \tilde{\infty}^p x & \text{for}\ c=0 \\ x\log{(cd)}^p & \text{for}\ e=0 \\ \frac{(-\log{(cd+cex)})^{-p}\log{(cd+cex)}^p\Gamma(p+1,-\log{(cd+cex)})}{ce} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(e*x+d))**p,x)
```

output
```
Piecewise((zoo**p*x, Eq(c, 0)), (x*log(c*d)**p, Eq(e, 0)), (log(c*d + c*e*
x)**p*uppergamma(p + 1, -log(c*d + c*e*x))/(c*e*(-log(c*d + c*e*x))**p), T
rue))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \log^p(c(d+ex))\,dx = -\frac{(-\log(cex+cd))^{-p-1}\log(cex+cd)^{p+1}\Gamma(p+1,-\log(cex+cd))}{ce}$$

input
```
integrate(log(c*(e*x+d))^p,x, algorithm="maxima")
```

output
```
-(-log(c*e*x + c*d))^(-p - 1)*log(c*e*x + c*d)^(p + 1)*gamma(p + 1, -log(c
*e*x + c*d))/(c*e)
```

3.16.8 Giac [F]

$$\int \log^p(c(d+ex))\,dx = \int \log((ex+d)c)^p\,dx$$

input
```
integrate(log(c*(e*x+d))^p,x, algorithm="giac")
```

output
```
integrate(log((e*x + d)*c)^p, x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log^p(c(d+ex))\,dx = \frac{\ln(c\,(d+e\,x))^p\,\Gamma(p+1,-\ln(c\,(d+e\,x)))}{c\,e\,(-\ln(c\,(d+e\,x)))^p}$$

input
```
int(log(c*(d + e*x))^p,x)
```

output
```
(log(c*(d + e*x))^p*igamma(p + 1, -log(c*(d + e*x))))/(c*e*(-log(c*(d + e*
x)))^p)
```

3.17 $\int (a+b\log (c(d+ex)^n))^4 \, dx$

3.17.1 Optimal result

Integrand size = 16, antiderivative size = 131

$$\begin{aligned}\int (a+b\log (c(d+ex)^n))^4 \, dx = &-24ab^3n^3x+24b^4n^4x-\frac{24b^4n^3(d+ex)\log (c(d+ex)^n)}{e}\\&+\frac{12b^2n^2(d+ex)\,(a+b\log (c(d+ex)^n))^2}{e}\\&-\frac{4bn(d+ex)\,(a+b\log (c(d+ex)^n))^3}{e}\\&+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^4}{e}\end{aligned}$$

output
```
-24*a*b^3*n^3*x+24*b^4*n^4*x-24*b^4*n^3*(e*x+d)*ln(c*(e*x+d)^n)/e+12*b^2*n
^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e-4*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3
/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/e
```

3.17.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int (a+b\log (c(d+ex)^n))^4 \, dx$$

$$=\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^4-4bn\big((d+ex)\,(a+b\log (c(d+ex)^n))^3-3bn\big((d+ex)\,(a+b\log (c(d}{e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^4,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 - 4*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(
a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n]))))/e
```

3.17.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2836, 2733, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d+ex)^n))^4 \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log (c(d+ex)^n))^4 d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^4-4bn\int (a+b\log (c(d+ex)^n))^3 d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^4-4bn\left((d+ex) (a+b\log (c(d+ex)^n))^3-3bn\int (a+b\log (c(d+ex)^n))^2 d(d+$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^4-4bn\left((d+ex) (a+b\log (c(d+ex)^n))^3-3bn\left((d+ex) (a+b\log (c(d+ex)^n))\right.\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^4-4bn\left((d+ex) (a+b\log (c(d+ex)^n))^3-3bn\left((d+ex) (a+b\log (c(d+ex)^n))\right.\right.}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^4,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 - 4*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(a*(
d + e*x) - b*n*(d + e*x) + b*(d + e*x)*Log[c*(d + e*x)^n]))))/e
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(131) = 262$.

Time = 1.36 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.01

method	result
parallelrisch	$\frac{-12x\ln(c(ex+d)^n)a^2b^2e\,n^2+4x\ln(c(ex+d)^n)a^3ben+4x\ln(c(ex+d)^n)^3a\,b^3en-12x\ln(c(ex+d)^n)^2a\,b^3e\,n^2+24x\ln(c(ex+d)^n)}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))^4,x,method=_RETURNVERBOSE)
```

output
```
(-12*x*ln(c*(e*x+d)^n)*a^2*b^2*e*n^2+4*x*ln(c*(e*x+d)^n)*a^3*b*e*n+4*x*ln(
c*(e*x+d)^n)^3*a*b^3*e*n-12*x*ln(c*(e*x+d)^n)^2*a*b^3*e*n^2+24*x*ln(c*(e*x
+d)^n)*a*b^3*e*n^3+6*x*ln(c*(e*x+d)^n)^2*a^2*b^2*e*n+24*a*b^3*d*n^4-12*a^2
*b^2*d*n^3+4*a^3*b*d*n^2+ln(c*(e*x+d)^n)^4*b^4*d*n-4*ln(c*(e*x+d)^n)^3*b^4
*d*n^2+12*ln(c*(e*x+d)^n)^2*b^4*d*n^3-24*ln(c*(e*x+d)^n)*b^4*d*n^4+x*a^4*e
*n+24*x*b^4*e*n^5+12*x*a^2*b^2*e*n^3+4*ln(c*(e*x+d)^n)^3*a*b^3*d*n-12*ln(c
*(e*x+d)^n)^2*a*b^3*d*n^2+24*ln(c*(e*x+d)^n)*a*b^3*d*n^3-4*x*a^3*b*e*n^2+6
*ln(c*(e*x+d)^n)^2*a^2*b^2*d*n-12*ln(c*(e*x+d)^n)*a^2*b^2*d*n^2+4*ln(c*(e*
x+d)^n)*a^3*b*d*n+x*ln(c*(e*x+d)^n)^4*b^4*e*n-4*x*ln(c*(e*x+d)^n)^3*b^4*e*
n^2+12*x*ln(c*(e*x+d)^n)^2*b^4*e*n^3-24*x*ln(c*(e*x+d)^n)*b^4*e*n^4-24*x*a
*b^3*e*n^4-24*b^4*d*n^5-a^4*d*n)/e/n
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(131) = 262$.

Time = 0.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.69

$$\int (a + b\log(c(d+ex)^n))^4 \, dx$$
$$= \frac{b^4ex\log(c)^4 + (b^4en^4x + b^4dn^4)\log(ex+d)^4 - 4(b^4en - ab^3e)x\log(c)^3 - 4(b^4dn^4 - ab^3dn^3 + (b^4en^4 - \cdots}{\cdots}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="fricas")
```

output
```
(b^4*e*x*log(c)^4 + (b^4*e*n^4*x + b^4*d*n^4)*log(e*x + d)^4 - 4*(b^4*e*n
- a*b^3*e)*x*log(c)^3 - 4*(b^4*d*n^4 - a*b^3*d*n^3 + (b^4*e*n^4 - a*b^3*e*
n^3)*x - (b^4*e*n^3*x + b^4*d*n^3)*log(c))*log(e*x + d)^3 + 6*(2*b^4*e*n^2
 - 2*a*b^3*e*n + a^2*b^2*e)*x*log(c)^2 + 6*(2*b^4*d*n^4 - 2*a*b^3*d*n^3 +
a^2*b^2*d*n^2 + (b^4*e*n^2*x + b^4*d*n^2)*log(c)^2 + (2*b^4*e*n^4 - 2*a*b^
3*e*n^3 + a^2*b^2*e*n^2)*x - 2*(b^4*d*n^3 - a*b^3*d*n^2 + (b^4*e*n^3 - a*b
^3*e*n^2)*x)*log(c))*log(e*x + d)^2 - 4*(6*b^4*e*n^3 - 6*a*b^3*e*n^2 + 3*a
^2*b^2*e*n - a^3*b*e)*x*log(c) + (24*b^4*e*n^4 - 24*a*b^3*e*n^3 + 12*a^2*b
^2*e*n^2 - 4*a^3*b*e*n + a^4*e)*x - 4*(6*b^4*d*n^4 - 6*a*b^3*d*n^3 + 3*a^2
*b^2*d*n^2 - a^3*b*d*n - (b^4*e*n*x + b^4*d*n)*log(c)^3 + 3*(b^4*d*n^2 - a
*b^3*d*n + (b^4*e*n^2 - a*b^3*e*n)*x)*log(c)^2 + (6*b^4*e*n^4 - 6*a*b^3*e*
n^3 + 3*a^2*b^2*e*n^2 - a^3*b*e*n)*x - 3*(2*b^4*d*n^3 - 2*a*b^3*d*n^2 + a^
2*b^2*d*n + (2*b^4*e*n^3 - 2*a*b^3*e*n^2 + a^2*b^2*e*n)*x)*log(c))*log(e*x
 + d))/e
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(126) = 252$.

Time = 0.90 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.78

$$\int (a + b\log(c(d+ex)^n))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3bd\log(c(d+ex)^n)}{e} - 4a^3bnx + 4a^3bx\log(c(d+ex)^n) - \frac{12a^2b^2dn\log(c(d+ex)^n)}{e} + \frac{6a^2b^2d\log(c(d+ex)^n)^2}{e} + 1 \\ x(a+b\log(cd^n))^4 \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*d*log(c*(d + e*x)**n)/e - 4*a**3*b*n*x + 4*a*
*3*b*x*log(c*(d + e*x)**n) - 12*a**2*b**2*d*n*log(c*(d + e*x)**n)/e + 6*a*
*2*b**2*d*log(c*(d + e*x)**n)**2/e + 12*a**2*b**2*n**2*x - 12*a**2*b**2*n*
x*log(c*(d + e*x)**n) + 6*a**2*b**2*x*log(c*(d + e*x)**n)**2 + 24*a*b**3*d
*n**2*log(c*(d + e*x)**n)/e - 12*a*b**3*d*n*log(c*(d + e*x)**n)**2/e + 4*a
*b**3*d*log(c*(d + e*x)**n)**3/e - 24*a*b**3*n**3*x + 24*a*b**3*n**2*x*log
(c*(d + e*x)**n) - 12*a*b**3*n*x*log(c*(d + e*x)**n)**2 + 4*a*b**3*x*log(c
*(d + e*x)**n)**3 - 24*b**4*d*n**3*log(c*(d + e*x)**n)/e + 12*b**4*d*n**2*
log(c*(d + e*x)**n)**2/e - 4*b**4*d*n*log(c*(d + e*x)**n)**3/e + b**4*d*lo
g(c*(d + e*x)**n)**4/e + 24*b**4*n**4*x - 24*b**4*n**3*x*log(c*(d + e*x)**
n) + 12*b**4*n**2*x*log(c*(d + e*x)**n)**2 - 4*b**4*n*x*log(c*(d + e*x)**n
)**3 + b**4*x*log(c*(d + e*x)**n)**4, Ne(e, 0)), (x*(a + b*log(c*d**n))**4
, True))
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(131) = 262$.

Time = 0.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.82

$$\int (a+b\log(c(d+ex)^n))^4\,dx = b^4x\log((ex+d)^nc)^4 + 4\,ab^3x\log((ex+d)^nc)^3$$
$$-4\,a^3ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) + 6\,a^2b^2x\log((ex+d)^nc)^2 + 4\,a^3bx\log((ex+d)^nc)$$
$$-6\left(2\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d))n^2}{e}\right)a^2b^2$$
$$-4\left(3\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^2 - en\left(\frac{(d\log(ex+d)^3 + 3\,d\log(ex+d)^2 - 6\,ex + 6\,d\,\mathrm{l}}{e^2}\right.\right.$$
$$-\left(4\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^3 + \left(en\left(\frac{(d\log(ex+d)^4 + 4\,d\log(ex+d)^3 + 12\,d\log(ex}{e^3}\right.\right.\right.$$
$$+a^4x$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="maxima")
```

output
```
b^4*x*log((e*x + d)^n*c)^4 + 4*a*b^3*x*log((e*x + d)^n*c)^3 - 4*a^3*b*e*n*
(x/e - d*log(e*x + d)/e^2) + 6*a^2*b^2*x*log((e*x + d)^n*c)^2 + 4*a^3*b*x*
log((e*x + d)^n*c) - 6*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c
) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a^2*b^2 - 4*(3*e*
n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3
 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x +
 d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*a*b^3 - (4*e*
n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^3 + (e*n*((d*log(e*x + d)^
4 + 4*d*log(e*x + d)^3 + 12*d*log(e*x + d)^2 - 24*e*x + 24*d*log(e*x + d))
*n^2/e^3 - 4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x
+ d))*n*log((e*x + d)^n*c)/e^3) + 6*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*
x + d))*n*log((e*x + d)^n*c)^2/e^2)*e*n)*b^4 + a^4*x
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(131) = 262$.

Time = 0.34 (sec) , antiderivative size = 758, normalized size of antiderivative = 5.79

$$
\begin{aligned}
\int (a+b\log (c(d+ex)^n))^4\,dx = {} & \frac{(ex+d)b^4n^4\log (ex+d)^4}{e} - \frac{4\,(ex+d)b^4n^4\log (ex+d)^3}{e} \\
& + \frac{4\,(ex+d)b^4n^3\log (ex+d)^3\log (c)}{e} \\
& + \frac{12\,(ex+d)b^4n^4\log (ex+d)^2}{e} \\
& + \frac{4\,(ex+d)ab^3n^3\log (ex+d)^3}{e} \\
& - \frac{12\,(ex+d)b^4n^3\log (ex+d)^2\log (c)}{e} \\
& + \frac{6\,(ex+d)b^4n^2\log (ex+d)^2\log (c)^2}{e} \\
& - \frac{24\,(ex+d)b^4n^4\log (ex+d)}{e} \\
& - \frac{12\,(ex+d)ab^3n^3\log (ex+d)^2}{e} \\
& + \frac{24\,(ex+d)b^4n^3\log (ex+d)\log (c)}{e} \\
& + \frac{12\,(ex+d)ab^3n^2\log (ex+d)^2\log (c)}{e} \\
& - \frac{12\,(ex+d)b^4n^2\log (ex+d)\log (c)^2}{e} \\
& + \frac{4\,(ex+d)b^4n\log (ex+d)\log (c)^3}{e} \\
& + \frac{24\,(ex+d)b^4n^4}{e} + \frac{24\,(ex+d)ab^3n^3\log (ex+d)}{e} \\
& + \frac{6\,(ex+d)a^2b^2n^2\log (ex+d)^2}{e} - \frac{24\,(ex+d)b^4n^3\log (c)}{e} \\
& - \frac{24\,(ex+d)ab^3n^2\log (ex+d)\log (c)}{e} \\
& + \frac{12\,(ex+d)b^4n^2\log (c)^2}{e} \\
& + \frac{12\,(ex+d)ab^3n\log (ex+d)\log (c)^2}{e} \\
& - \frac{4\,(ex+d)b^4n\log (c)^3}{e} + \frac{(ex+d)b^4\log (c)^4}{e} \\
& - \frac{24\,(ex+d)ab^3n^3}{e} - \frac{12\,(ex+d)a^2b^2n^2\log (ex+d)}{e} \\
& + \frac{24\,(ex+d)ab^3n^2\log (c)}{e} \\
& + \frac{12\,(ex+d)a^2b^2n\log (ex+d)\log (c)}{e} \\
& - \frac{12\,(ex+d)ab^3n\log (c)^2}{e} + \frac{4\,(ex+d)ab^3\log (c)^3}{e} \\
& + \frac{12\,(ex+d)a^2b^2n^2}{e} + \frac{4\,(ex+d)a^3bn\log (ex+d)}{e}
\end{aligned}
$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="giac")
```

output
```
(e*x + d)*b^4*n^4*log(e*x + d)^4/e - 4*(e*x + d)*b^4*n^4*log(e*x + d)^3/e
+ 4*(e*x + d)*b^4*n^3*log(e*x + d)^3*log(c)/e + 12*(e*x + d)*b^4*n^4*log(e
*x + d)^2/e + 4*(e*x + d)*a*b^3*n^3*log(e*x + d)^3/e - 12*(e*x + d)*b^4*n^
3*log(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^4*n^2*log(e*x + d)^2*log(c)^2/e
- 24*(e*x + d)*b^4*n^4*log(e*x + d)/e - 12*(e*x + d)*a*b^3*n^3*log(e*x + d
)^2/e + 24*(e*x + d)*b^4*n^3*log(e*x + d)*log(c)/e + 12*(e*x + d)*a*b^3*n^
2*log(e*x + d)^2*log(c)/e - 12*(e*x + d)*b^4*n^2*log(e*x + d)*log(c)^2/e +
 4*(e*x + d)*b^4*n*log(e*x + d)*log(c)^3/e + 24*(e*x + d)*b^4*n^4/e + 24*(
e*x + d)*a*b^3*n^3*log(e*x + d)/e + 6*(e*x + d)*a^2*b^2*n^2*log(e*x + d)^2
/e - 24*(e*x + d)*b^4*n^3*log(c)/e - 24*(e*x + d)*a*b^3*n^2*log(e*x + d)*l
og(c)/e + 12*(e*x + d)*b^4*n^2*log(c)^2/e + 12*(e*x + d)*a*b^3*n*log(e*x +
 d)*log(c)^2/e - 4*(e*x + d)*b^4*n*log(c)^3/e + (e*x + d)*b^4*log(c)^4/e -
 24*(e*x + d)*a*b^3*n^3/e - 12*(e*x + d)*a^2*b^2*n^2*log(e*x + d)/e + 24*(
e*x + d)*a*b^3*n^2*log(c)/e + 12*(e*x + d)*a^2*b^2*n*log(e*x + d)*log(c)/e
 - 12*(e*x + d)*a*b^3*n*log(c)^2/e + 4*(e*x + d)*a*b^3*log(c)^3/e + 12*(e*
x + d)*a^2*b^2*n^2/e + 4*(e*x + d)*a^3*b*n*log(e*x + d)/e - 12*(e*x + d)*a
^2*b^2*n*log(c)/e + 6*(e*x + d)*a^2*b^2*log(c)^2/e - 4*(e*x + d)*a^3*b*n/e
 + 4*(e*x + d)*a^3*b*log(c)/e + (e*x + d)*a^4/e
```

3.17.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.10

$$\int (a+b\log(c(d+ex)^n))^4\,dx$$
$$= \ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{6\,(d\,a^2\,b^2-2\,d\,a\,b^3\,n+2\,d\,b^4\,n^2)}{e}+6\,b^2\,x\,(a^2-2\,a\,b\,n+2\,b^2\,n^2)\right)$$
$$+x\,(a^4-4\,a^3\,b\,n+12\,a^2\,b^2\,n^2-24\,a\,b^3\,n^3+24\,b^4\,n^4)+\ln\left(c\,(d+e\,x)^n\right)^4\left(b^4\,x+\frac{b^4\,d}{e}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)^3\left(\frac{4\,(a\,b^3\,d-b^4\,d\,n)}{e}+4\,b^3\,x\,(a-b\,n)\right)$$
$$-\frac{\ln(d+e\,x)\,\left(-4\,d\,a^3\,b\,n+12\,d\,a^2\,b^2\,n^2-24\,d\,a\,b^3\,n^3+24\,d\,b^4\,n^4\right)}{e}$$
$$+4\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right)$$

input
```
int((a + b*log(c*(d + e*x)^n))^4,x)
```

output
```
log(c*(d + e*x)^n)^2*((6*(a^2*b^2*d + 2*b^4*d*n^2 - 2*a*b^3*d*n))/e + 6*b^
2*x*(a^2 + 2*b^2*n^2 - 2*a*b*n)) + x*(a^4 + 24*b^4*n^4 - 24*a*b^3*n^3 + 12
*a^2*b^2*n^2 - 4*a^3*b*n) + log(c*(d + e*x)^n)^4*(b^4*x + (b^4*d)/e) + log
(c*(d + e*x)^n)^3*((4*(a*b^3*d - b^4*d*n))/e + 4*b^3*x*(a - b*n)) - (log(d
 + e*x)*(24*b^4*d*n^4 + 12*a^2*b^2*d*n^2 - 4*a^3*b*d*n - 24*a*b^3*d*n^3))/
e + 4*b*x*log(c*(d + e*x)^n)*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n)
```

3.17. $\int (a + b\log(c(d + ex)^n))^4 \, dx$

3.18 $\int (a+b\log(c(d+ex)^n))^3\,dx$

3.18.1 Optimal result

Integrand size = 16, antiderivative size = 99

$$\int (a+b\log(c(d+ex)^n))^3\,dx = 6ab^2n^2x-6b^3n^3x+\frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{e}-\frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}$$

output
```
6*a*b^2*n^2*x-6*b^3*n^3*x+6*b^3*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-3*b*n*(e*x+d
)*(a+b*ln(c*(e*x+d)^n))^2/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e
```

3.18.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int (a+b\log(c(d+ex)^n))^3\,dx$$
$$=\frac{(d+ex)(a+b\log(c(d+ex)^n))^3-3bn\big((d+ex)(a+b\log(c(d+ex)^n))^2-2bn(e(a-bn)x+b(d+ex)\ldots}{e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])))/e
```

3.18.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2836, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d+ex)^n))^3 \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log (c(d+ex)^n))^3d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^3-3bn\int (a+b\log (c(d+ex)^n))^2d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^3-3bn\left((d+ex) (a+b\log (c(d+ex)^n))^2-2bn\int (a+b\log (c(d+ex)^n))d(d+e$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex) (a+b\log (c(d+ex)^n))^3-3bn\left((d+ex) (a+b\log (c(d+ex)^n))^2-2bn(a(d+ex)+b(d+ex)\log (c(d+$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^2 - 2*b*n*(a*(d + e*x) - b*n*(d + e*x) + b*(d + e*x)*Log[c*(d
+ e*x)^n])))/e
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(99) = 198$.

Time = 0.62 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.25

method	result
parallelrisch	$\frac{x\ln(c(ex+d)^n)^3 b^3 en - 3x\ln(c(ex+d)^n)^2 b^3 e n^2 + 6x\ln(c(ex+d)^n) b^3 e n^3 - 6x b^3 e n^4 + 3x\ln(c(ex+d)^n)^2 a b^2 en - 6x\ln(c(ex+d)^n)}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
(x*ln(c*(e*x+d)^n)^3*b^3*e*n-3*x*ln(c*(e*x+d)^n)^2*b^3*e*n^2+6*x*ln(c*(e*x
+d)^n)*b^3*e*n^3-6*x*b^3*e*n^4+3*x*ln(c*(e*x+d)^n)^2*a*b^2*e*n-6*x*ln(c*(e
*x+d)^n)*a*b^2*e*n^2+6*x*a*b^2*e*n^3+ln(c*(e*x+d)^n)^3*b^3*d*n-3*ln(c*(e*x
+d)^n)^2*b^3*d*n^2+6*ln(c*(e*x+d)^n)*b^3*d*n^3+6*b^3*d*n^4+3*x*ln(c*(e*x+d
)^n)*a^2*b*e*n-3*x*a^2*b*e*n^2+3*ln(c*(e*x+d)^n)^2*a*b^2*d*n-6*ln(c*(e*x+d
)^n)*a*b^2*d*n^2-6*a*b^2*d*n^3+x*a^3*e*n+3*ln(c*(e*x+d)^n)*a^2*b*d*n+3*a^2
*b*d*n^2-a^3*d*n)/e/n
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.27

$$\int (a+b\log (c(d+ex)^n))^3 \, dx$$
$$= \frac{b^3ex\log (c)^3 + (b^3en^3x + b^3dn^3)\log (ex+d)^3 - 3\,(b^3en - ab^2e)x\log (c)^2 - 3\,(b^3dn^3 - ab^2dn^2 + (b^3en^3 -}{}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
(b^3*e*x*log(c)^3 + (b^3*e*n^3*x + b^3*d*n^3)*log(e*x + d)^3 - 3*(b^3*e*n
- a*b^2*e)*x*log(c)^2 - 3*(b^3*d*n^3 - a*b^2*d*n^2 + (b^3*e*n^3 - a*b^2*e*
n^2)*x - (b^3*e*n^2*x + b^3*d*n^2)*log(c))*log(e*x + d)^2 + 3*(2*b^3*e*n^2
 - 2*a*b^2*e*n + a^2*b*e)*x*log(c) - (6*b^3*e*n^3 - 6*a*b^2*e*n^2 + 3*a^2*
b*e*n - a^3*e)*x + 3*(2*b^3*d*n^3 - 2*a*b^2*d*n^2 + a^2*b*d*n + (b^3*e*n*x
 + b^3*d*n)*log(c)^2 + (2*b^3*e*n^3 - 2*a*b^2*e*n^2 + a^2*b*e*n)*x - 2*(b^
3*d*n^2 - a*b^2*d*n + (b^3*e*n^2 - a*b^2*e*n)*x)*log(c))*log(e*x + d))/e
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(95) = 190.

Time = 0.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.97

$$\int (a+b\log (c(d+ex)^n))^3 \, dx$$
$$= \begin{cases} a^3x + \frac{3a^2bd\log (c(d+ex)^n)}{e} - 3a^2bnx + 3a^2bx\log (c(d+ex)^n) - \frac{6ab^2dn\log (c(d+ex)^n)}{e} + \frac{3ab^2d\log (c(d+ex)^n)^2}{e} + 6ab \\ x(a+b\log (cd^n))^3 \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*d*log(c*(d + e*x)**n)/e - 3*a**2*b*n*x + 3*a*
*2*b*x*log(c*(d + e*x)**n) - 6*a*b**2*d*n*log(c*(d + e*x)**n)/e + 3*a*b**2
*d*log(c*(d + e*x)**n)**2/e + 6*a*b**2*n**2*x - 6*a*b**2*n*x*log(c*(d + e*
x)**n) + 3*a*b**2*x*log(c*(d + e*x)**n)**2 + 6*b**3*d*n**2*log(c*(d + e*x)
**n)/e - 3*b**3*d*n*log(c*(d + e*x)**n)**2/e + b**3*d*log(c*(d + e*x)**n)*
*3/e - 6*b**3*n**3*x + 6*b**3*n**2*x*log(c*(d + e*x)**n) - 3*b**3*n*x*log(
c*(d + e*x)**n)**2 + b**3*x*log(c*(d + e*x)**n)**3, Ne(e, 0)), (x*(a + b*l
og(c*d**n))**3, True))
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.

Time = 0.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.85

$$\int (a + b\log(c(d+ex)^n))^3 \, dx = b^3 x \log((ex+d)^n c)^3 - 3a^2 ben\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + 3ab^2 x\log((ex+d)^n c)^2 + 3a^2 bx\log((ex+d)^n c) - 3\left(2en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^n c) + \frac{(d\log(ex+d)^2 - 2ex + 2d\log(ex+d))n^2}{e}\right)ab^2 - \left(3en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^n c)^2 - en\left(\frac{(d\log(ex+d)^3 + 3d\log(ex+d)^2 - 6ex + 6d\log\right.\right.}{e^2} + a^3 x$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
b^3*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*n*(x/e - d*log(e*x + d)/e^2) + 3*a*
b^2*x*log((e*x + d)^n*c)^2 + 3*a^2*b*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e
- d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d
*log(e*x + d))*n^2/e)*a*b^2 - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x +
 d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log
(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log
((e*x + d)^n*c)/e^2))*b^3 + a^3*x
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.03

$$\int (a+b\log(c(d+ex)^n))^3\,dx = \frac{(ex+d)b^3n^3\log(ex+d)^3}{e} - \frac{3(ex+d)b^3n^3\log(ex+d)^2}{e} + \frac{3(ex+d)b^3n^2\log(ex+d)^2\log(c)}{e} + \frac{6(ex+d)b^3n^3\log(ex+d)}{e} + \frac{3(ex+d)ab^2n^2\log(ex+d)^2}{e} - \frac{6(ex+d)b^3n^2\log(ex+d)\log(c)}{e} + \frac{3(ex+d)b^3n\log(ex+d)\log(c)^2}{e} - \frac{6(ex+d)b^3n^3}{e} - \frac{6(ex+d)ab^2n^2\log(ex+d)}{e} + \frac{6(ex+d)b^3n^2\log(c)}{e} + \frac{6(ex+d)ab^2n\log(ex+d)\log(c)}{e} - \frac{3(ex+d)b^3n\log(c)^2}{e} + \frac{(ex+d)b^3\log(c)^3}{e} + \frac{6(ex+d)ab^2n^2}{e} + \frac{3(ex+d)a^2bn\log(ex+d)}{e} - \frac{6(ex+d)ab^2n\log(c)}{e} + \frac{3(ex+d)ab^2\log(c)^2}{e} - \frac{3(ex+d)a^2bn}{e} + \frac{3(ex+d)a^2b\log(c)}{e} + \frac{(ex+d)a^3}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
(e*x + d)*b^3*n^3*log(e*x + d)^3/e - 3*(e*x + d)*b^3*n^3*log(e*x + d)^2/e
+ 3*(e*x + d)*b^3*n^2*log(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^3*n^3*log(e*
x + d)/e + 3*(e*x + d)*a*b^2*n^2*log(e*x + d)^2/e - 6*(e*x + d)*b^3*n^2*lo
g(e*x + d)*log(c)/e + 3*(e*x + d)*b^3*n*log(e*x + d)*log(c)^2/e - 6*(e*x +
 d)*b^3*n^3/e - 6*(e*x + d)*a*b^2*n^2*log(e*x + d)/e + 6*(e*x + d)*b^3*n^2
*log(c)/e + 6*(e*x + d)*a*b^2*n*log(e*x + d)*log(c)/e - 3*(e*x + d)*b^3*n*
log(c)^2/e + (e*x + d)*b^3*log(c)^3/e + 6*(e*x + d)*a*b^2*n^2/e + 3*(e*x +
 d)*a^2*b*n*log(e*x + d)/e - 6*(e*x + d)*a*b^2*n*log(c)/e + 3*(e*x + d)*a*
b^2*log(c)^2/e - 3*(e*x + d)*a^2*b*n/e + 3*(e*x + d)*a^2*b*log(c)/e + (e*x
 + d)*a^3/e
```

3.18.9 Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int (a+b\log(c(d+ex)^n))^3 \, dx = x\left(a^3-3a^2bn+6ab^2n^2-6b^3n^3\right) + \ln\left(c\left(d+ex\right)^n\right)^3\left(b^3x+\frac{b^3d}{e}\right) + \ln\left(c\left(d+ex\right)^n\right)^2\left(\frac{3\left(ab^2d-b^3dn\right)}{e}+3b^2x\left(a-bn\right)\right) + \frac{\ln\left(d+ex\right)\left(3da^2bn-6dab^2n^2+6db^3n^3\right)}{e} + 3bx\ln\left(c\left(d+ex\right)^n\right)\left(a^2-2abn+2b^2n^2\right)$$

input
```
int((a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
x*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n) + log(c*(d + e*x)^n)^3*(b^3*
x + (b^3*d)/e) + log(c*(d + e*x)^n)^2*((3*(a*b^2*d - b^3*d*n))/e + 3*b^2*x
*(a - b*n)) + (log(d + e*x)*(6*b^3*d*n^3 + 3*a^2*b*d*n - 6*a*b^2*d*n^2))/e
 + 3*b*x*log(c*(d + e*x)^n)*(a^2 + 2*b^2*n^2 - 2*a*b*n)
```

3.19 $\int (a+b\log (c(d+ex)^n))^2 \, dx$

3.19.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int (a+b\log (c(d+ex)^n))^2 \, dx = -2abnx+2b^2n^2x-\frac{2b^2n(d+ex)\log (c(d+ex)^n)}{e}+\frac{(d+ex) (a+b\log (c(d+ex)^n))^2}{e}$$

output
```
-2*a*b*n*x+2*b^2*n^2*x-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e+(e*x+d)*(a+b*ln(c
*(e*x+d)^n))^2/e
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int (a+b\log (c(d+ex)^n))^2 \, dx = \frac{(d+ex) (a+b\log (c(d+ex)^n))^2}{e} - 2bn\left(ax-bnx+\frac{b(d+ex)\log (c(d+ex)^n)}{e}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - 2*b*n*(a*x - b*n*x + (b*(d +
e*x)*Log[c*(d + e*x)^n])/e)
```

3.19.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2836, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^2 \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^2 d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2-2bn\int (a+b\log(c(d+ex)^n))d(d+ex)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2-2bn(a(d+ex)+b(d+ex)\log(c(d+ex)^n)-bn(d+ex))}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(a*(d + e*x) - b*n*(d + e*
x) + b*(d + e*x)*Log[c*(d + e*x)^n]))/e
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.19.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

method	result
norman	$\left(2b^{2}n^{2}-2abn+a^{2}\right)x+b^{2}x\ln\left(c\,e^{n\ln(ex+d)}\right)^{2}+\left(-2b^{2}n+2ab\right)x\ln\left(c\,e^{n\ln(ex+d)}\right)+\frac{b^{2}d\ln\left(c\,e^{n\ln\ldots}\right.}{e}$
default	$a^{2}x+b^{2}x\ln\left(c\,e^{n\ln(ex+d)}\right)^{2}+\frac{b^{2}d\ln\left(c\,e^{n\ln(ex+d)}\right)^{2}}{e}+2b^{2}n^{2}x-2b^{2}nx\ln\left(c\,e^{n\ln(ex+d)}\right)-\frac{2n^{2}b^{2}d\ln(e\ldots}{e}$
parts	$a^{2}x+b^{2}x\ln\left(c\,e^{n\ln(ex+d)}\right)^{2}+\frac{b^{2}d\ln\left(c\,e^{n\ln(ex+d)}\right)^{2}}{e}+2b^{2}n^{2}x-2b^{2}nx\ln\left(c\,e^{n\ln(ex+d)}\right)-\frac{2n^{2}b^{2}d\ln(e\ldots}{e}$
parallelrisch	$\frac{x\ln(c(ex+d)^{n})^{2}b^{2}den-2x\ln(c(ex+d)^{n})b^{2}de\,n^{2}+2x\,b^{2}de\,n^{3}+2x\ln(c(ex+d)^{n})abden-2xabde\,n^{2}+\ln(c(ex+d)^{n})^{2}b^{2}d^{2}n-2\ln(\ldots}{end}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b^2*n^2-2*a*b*n+a^2)*x+b^2*x*ln(c*exp(n*ln(e*x+d)))^2+(-2*b^2*n+2*a*b)*
x*ln(c*exp(n*ln(e*x+d)))+b^2*d/e*ln(c*exp(n*ln(e*x+d)))^2+n*(-2*b^2*d*n+2*
a*b*d)/e*ln(e*x+d)
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.15

$$\int(a+b\log(c(d+ex)^{n}))^{2}\,dx$$
$$=\frac{b^{2}ex\log(c)^{2}+\left(b^{2}en^{2}x+b^{2}dn^{2}\right)\log(ex+d)^{2}-2\left(b^{2}en-abe\right)x\log(c)+\left(2\,b^{2}en^{2}-2\,aben+a^{2}e\right)x-2\,(b\ldots}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
(b^2*e*x*log(c)^2 + (b^2*e*n^2*x + b^2*d*n^2)*log(e*x + d)^2 - 2*(b^2*e*n
- a*b*e)*x*log(c) + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*e)*x - 2*(b^2*d*n^2 - a
*b*d*n + (b^2*e*n^2 - a*b*e*n)*x - (b^2*e*n*x + b^2*d*n)*log(c))*log(e*x +
 d))/e
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(63) = 126$.

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (a + b\log (c(d + ex)^n))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{2abd\log(c(d+ex)^n)}{e} - 2abnx + 2abx\log(c(d + ex)^n) - \frac{2b^2dn\log(c(d+ex)^n)}{e} + \frac{b^2d\log(c(d+ex)^n)^2}{e} + 2b^2n^2x - \\ x(a + b\log(cd^n))^2 \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*d*log(c*(d + e*x)**n)/e - 2*a*b*n*x + 2*a*b*x*lo
g(c*(d + e*x)**n) - 2*b**2*d*n*log(c*(d + e*x)**n)/e + b**2*d*log(c*(d + e
*x)**n)**2/e + 2*b**2*n**2*x - 2*b**2*n*x*log(c*(d + e*x)**n) + b**2*x*log
(c*(d + e*x)**n)**2, Ne(e, 0)), (x*(a + b*log(c*d**n))**2, True))
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int (a + b\log (c(d + ex)^n))^2 \, dx$$
$$= -2\,aben\left(\frac{x}{e} - \frac{d\log(ex + d)}{e^2}\right) + b^2x\log((ex + d)^nc)^2 + 2\,abx\log((ex + d)^nc)$$
$$- \left(2\,en\left(\frac{x}{e} - \frac{d\log(ex + d)}{e^2}\right)\log((ex + d)^nc) + \frac{(d\log(ex + d)^2 - 2\,ex + 2\,d\log(ex + d))n^2}{e}\right)b^2$$
$$+ a^2x$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-2*a*b*e*n*(x/e - d*log(e*x + d)/e^2) + b^2*x*log((e*x + d)^n*c)^2 + 2*a*b
*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*
c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2 + a^2*x
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.68

$$\int (a+b\log (c(d+ex)^n))^2\,dx = \frac{(ex+d)b^2n^2\log (ex+d)^2}{e} - \frac{2\,(ex+d)b^2n^2\log (ex+d)}{e} + \frac{2\,(ex+d)b^2n\log (ex+d)\log (c)}{e} + \frac{2\,(ex+d)b^2n^2}{e} + \frac{2\,(ex+d)abn\log (ex+d)}{e} - \frac{2\,(ex+d)b^2n\log (c)}{e} + \frac{(ex+d)b^2\log (c)^2}{e} - \frac{2\,(ex+d)abn}{e} + \frac{2\,(ex+d)ab\log (c)}{e} + \frac{(ex+d)a^2}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
(e*x + d)*b^2*n^2*log(e*x + d)^2/e - 2*(e*x + d)*b^2*n^2*log(e*x + d)/e +
2*(e*x + d)*b^2*n*log(e*x + d)*log(c)/e + 2*(e*x + d)*b^2*n^2/e + 2*(e*x +
 d)*a*b*n*log(e*x + d)/e - 2*(e*x + d)*b^2*n*log(c)/e + (e*x + d)*b^2*log(
c)^2/e - 2*(e*x + d)*a*b*n/e + 2*(e*x + d)*a*b*log(c)/e + (e*x + d)*a^2/e
```

3.19.9 Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int (a+b\log (c(d+ex)^n))^2\,dx = x\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)+\ln \left(c\,{\left(d+e\,x\right)}^n\right)^2\,\left(b^2\,x+\frac{b^2\,d}{e}\right) - \frac{\ln \left(d+e\,x\right)\,\left(2\,b^2\,d\,n^2-2\,a\,b\,d\,n\right)}{e} + 2\,b\,x\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\,\left(a-b\,n\right)$$

input
```
int((a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + log(c*(d + e*x)^n)^2*(b^2*x + (b^2*d)/e) -
 (log(d + e*x)*(2*b^2*d*n^2 - 2*a*b*d*n))/e + 2*b*x*log(c*(d + e*x)^n)*(a
- b*n)
```

3.20 $\int (a+b\log(c(d+ex)^n))\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (a+b\log(c(d+ex)^n))\,dx = ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}$$

output
```
a*x-b*n*x+b*(e*x+d)*ln(c*(e*x+d)^n)/e
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d+ex)^n))\,dx = ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}$$

input
```
Integrate[a + b*Log[c*(d + e*x)^n],x]
```

output
```
a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

3.20.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax + \frac{b(d+ex)\log(c(d+ex)^n)}{e} - bnx$$

input
```
Int[a + b*Log[c*(d + e*x)^n],x]
```

output
```
a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

method	result
default	$ax + b\ln(c(ex+d)^n)x - bnx + \frac{bnd\ln(ex+d)}{e}$
parts	$ax + b\ln(c(ex+d)^n)x - bnx + \frac{bnd\ln(ex+d)}{e}$
norman	$(-bn+a)x + bx\ln\left(c\,e^{n\ln(ex+d)}\right) + \frac{bnd\ln(ex+d)}{e}$
parallelrisch	$\frac{b(x\ln(c(ex+d)^n)den - xde\,n^2 + \ln(c(ex+d)^n)d^2n)}{den} + ax$
risch	$ax + bx\ln((ex+d)^n) - \frac{ib\pi x\ \text{csgn}(ic)\,\text{csgn}(i(ex+d)^n)\,\text{csgn}(ic(ex+d)^n)}{2} + \frac{ib\pi x\ \text{csgn}(ic)\,\text{csgn}(ic(ex+d)^n)^2}{2} + ib\pi$

input
```
int(a+b*ln(c*(e*x+d)^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*ln(c*(e*x+d)^n)*x-b*n*x+b/e*n*d*ln(e*x+d)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a + b\log{(c(d + ex)^n)})\, dx = \frac{bex\log{(c)} - (ben - ae)x + (benx + bdn)\log{(ex + d)}}{e}$$

input
```
integrate(a+b*log(c*(e*x+d)^n),x, algorithm="fricas")
```

output
```
(b*e*x*log(c) - (b*e*n - a*e)*x + (b*e*n*x + b*d*n)*log(e*x + d))/e
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a+b\log{(c(d+ex)^n)})\, dx = ax+b\left(\begin{cases} \frac{d\log{(c(d+ex)^n)}}{e} - nx + x\log{(c(d + ex)^n)} & \text{for}\: e \neq 0 \\ x\log{(cd^n)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*ln(c*(e*x+d)**n),x)
```

output
```
a*x + b*Piecewise((d*log(c*(d + e*x)**n)/e - n*x + x*log(c*(d + e*x)**n),
Ne(e, 0)), (x*log(c*d**n), True))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a + b\log{(c(d + ex)^n)})\, dx = -ben\left(\frac{x}{e} - \frac{d\log{(ex + d)}}{e^2}\right) + bx\log{((ex + d)^n c)} + ax$$

input
```
integrate(a+b*log(c*(e*x+d)^n),x, algorithm="maxima")
```

output
```
-b*e*n*(x/e - d*log(e*x + d)/e^2) + b*x*log((e*x + d)^n*c) + a*x
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int (a+b\log (c(d+ex)^n))\ dx = \left(\frac{(ex+d)n\log (ex+d)}{e} - \frac{(ex+d)n}{e} + \frac{(ex+d)\log (c)}{e}\right)b + ax$$

input
```
integrate(a+b*log(c*(e*x+d)^n),x, algorithm="giac")
```

output
```
((e*x + d)*n*log(e*x + d)/e - (e*x + d)*n/e + (e*x + d)*log(c)/e)*b + a*x
```

3.20.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int (a+b\log (c(d+ex)^n))\ dx = x\,(a-b\,n)+b\,x\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)+\frac{b\,d\,n\,\ln \left(d+e\,x\right)}{e}$$

input
```
int(a + b*log(c*(d + e*x)^n),x)
```

output
```
x*(a - b*n) + b*x*log(c*(d + e*x)^n) + (b*d*n*log(d + e*x))/e
```

3.21 $\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx$

3.21.1 Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

output
```
(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e/exp(a/b/n)/n/((c*(e*x+d)^n)^(1/n
))
```

3.21.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-1),x]
```

output
```
((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n
))*n*(c*(d + e*x)^n)^n^(-1))
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2836, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-1),x]
```

output
```
((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n
))*n*(c*(d + e*x)^n)^n^(-1))
```

3.21.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.21.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.90

method	result
risch	$-\frac{(ex+d)((ex+d)^n)^{-\frac{1}{n}} c^{-\frac{1}{n}} \mathrm{e}^{-\frac{-ib\pi\ \mathrm{csgn}(ic(ex+d)^n)\ \mathrm{csgn}(ic)\ \mathrm{csgn}(i(ex+d)^n)+i\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}(ic(ex+d)^n)^2 b+i\pi\ \mathrm{csgn}(i(ex+d)^n)\ \mathrm{csgn}(ic(ex-}{2bn}}}{}$

input
```
int(1/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/e/b/n*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e
*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*
b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^
3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)+1/2*I*(b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)-b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-b*Pi*csgn(I*(e*x+d)^n
)*csgn(I*c*(e*x+d)^n)^2+b*Pi*csgn(I*c*(e*x+d)^n)^3+2*I*b*ln(c)+2*I*b*(ln((
e*x+d)^n)-n*ln(e*x+d))+2*I*a)/b/n)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{\left(-\frac{b\log(c)+a}{bn}\right)}\ \mathrm{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)}{ben}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
e^(-(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)))
/(b*e*n)
```

3.21.6 Sympy [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+ex\right)^{n}\right)}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(1/(a + b*log(c*(d + e*x)**n)), x)
```

3.21.7 Maxima [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{1}{b\log\left((ex+d)^{n}c\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(1/(b*log((e*x + d)^n*c) + a), x)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log \left(c(d+ex)^n\right)}\,dx=\int \frac{1}{a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n)), x)
```

3.22 $\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.22.1 Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

output
```
(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e/exp(a/b/n)/n^2/((c*(e*x+d)^n)^
(1/n))+(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

3.22.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(be^{\frac{a}{bn}}n(c(d+ex)^n)^{\frac{1}{n}} - \text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+\right.}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

output
```
-(((d + e*x)*(b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1) - ExpIntegralEi[(a +
b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])))/(b^2*e*E^(a/(b*n
))*n^2*(c*(d + e*x)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])))
```

3.22.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2836, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{bn}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{bn^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{e}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

output
```
(((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*E^(a/(b*
n))*n^2*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(b*n*(a + b*Log[c*(d + e*x)^n]
)))/e
```

3.22.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.22.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{2(ex+d)}{\left(-ib\pi\ \mathrm{csgn}(ic(ex+d)^n)\ \mathrm{csgn}(ic)\ \mathrm{csgn}(i(ex+d)^n)+i\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}(ic(ex+d)^n)^2b+i\pi\ \mathrm{csgn}(i(ex+d)^n)\ \mathrm{csgn}(ic(ex+d)^n)^2b-i\pi\ \mathrm{csgn}$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)
*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*
Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)/b/n/e*(e*x+d)-
1/b^2/n^2/e*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^
n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c
)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b
*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{\left((benx+bdn)e^{\left(\frac{b\log(c)+a}{bn}\right)} - (bn\log(ex+d)+b\log(c)+a)\,\text{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)\right)e^{\left(-\frac{b\log(c)}{}\right.}}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
-((b*e*n*x + b*d*n)*e^((b*log(c) + a)/(b*n)) - (b*n*log(e*x + d) + b*log(c
) + a)*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))))*e^(-(b*log(c) + a
)/(b*n))/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)
```

3.22.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-2), x)
```

3.22.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*x + d)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n) + integra
te(1/(b^2*n*log((e*x + d)^n) + b^2*n*log(c) + a*b*n), x)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(95) = 190$.

Time $= 0.39$ (sec) , antiderivative size $= 286$, normalized size of antiderivative $= 2.98$

$$\begin{aligned}\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx &= \frac{bn\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}\log(ex+d)}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} \\ &\quad - \frac{(ex+d)bn}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2} \\ &\quad + \frac{b\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}\log(c)}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} \\ &\quad + \frac{a\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}}\end{aligned}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^3*
e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) - (e*x + d)*
b*n/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2) + b*Ei(log(c
)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e*n^3*log(e*x + d)
 + b^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) + a*Ei(log(c)/n + a/(b*n) + lo
g(e*x + d))*e^(-a/(b*n))/((b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b
^2*e*n^2)*c^(1/n))
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(a + b\,\ln(c\,(d + e\,x)^n))^2}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.23 $\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx$

3.23.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

output
```
1/2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e/exp(a/b/n)/n^3/((c*(e*x+d)
^n)^(1/n))+1/2*(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^2+1/2*(-e*x-d)/b^2/e/n
^2/(a+b*ln(c*(e*x+d)^n))
```

3.23.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(-\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2 + be^{\frac{a}{bn}}n(c(\right.}{2b^3en^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

output
```
-1/2*((d + e*x)*(-(ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*
Log[c*(d + e*x)^n])^2) + b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*(a + b*n +
 b*Log[c*(d + e*x)^n])))/(b^3*e*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1)*(a
+ b*Log[c*(d + e*x)^n])^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2836, 2734, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^2}d(d+ex)}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{bn}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{\frac{(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{bn^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

output
```
(-1/2*(d + e*x)/(b*n*(a + b*Log[c*(d + e*x)^n])^2) + (((d + e*x)*ExpIntegr
alEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*E^(a/(b*n))*n^2*(c*(d + e*x)^
n)^n^(-1)) - (d + e*x)/(b*n*(a + b*Log[c*(d + e*x)^n])))/(2*b*n))/e
```

3.23.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.23.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 734, normalized size of antiderivative = 5.44

method	result
risch	$-\frac{2benx+2bdn+i\pi bd\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2+i\pi bd\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2-i\pi bd\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex}{\left(-ib\pi\,\mathrm{csgn}(ic(ex+d)^n)\,\mathrm{csgn}\right.}$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-(2*b*e*n*x+2*b*d*n+I*Pi*b*d*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2+I*Pi*
b*d*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*d*csgn(I*c)*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)-I*Pi*b*d*csgn(I*c*(e*x+d)^n)^3-I*Pi*b*e*x*csgn(I*c*(e*x
+d)^n)^3+I*Pi*b*e*x*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e*x*csg
n(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*e*x*csgn(I*c)*csgn(I*c
*(e*x+d)^n)^2+2*ln(c)*b*e*x+2*b*e*x*ln((e*x+d)^n)+2*d*b*ln(c)+2*a*e*x+2*b*
d*ln((e*x+d)^n)+2*a*d)/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+
d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)
+2*a)^2/b^2/n^2/e-1/2/b^3/n^3/e*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1
/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)
*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*
Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^
n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(128) = 256.

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = -\frac{\left((b^2dn^2+abdn+(b^2en^2+aben)x+(b^2en^2x+b^2dn^2)\log\left(ex+d\right)+(b^2enx+b^2dn)\log\left(c\right))e^{\left(\frac{b\log(c)+a}{bn}\right)}\right.}{2\left(b^5en^5\log\left(ex+d\right)^2+b^5en^3\log\left(c\right.\right.}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*d*n^2 + a*b*d*n + (b^2*e*n^2 + a*b*e*n)*x + (b^2*e*n^2*x + b^2*
d*n^2)*log(e*x + d) + (b^2*e*n*x + b^2*d*n)*log(c))*e^((b*log(c) + a)/(b*n
)) - (b^2*n^2*log(e*x + d)^2 + b^2*log(c)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*
n*log(c) + a*b*n)*log(e*x + d))*log_integral((e*x + d)*e^((b*log(c) + a)/(
b*n))))*e^(-(b*log(c) + a)/(b*n))/(b^5*e*n^5*log(e*x + d)^2 + b^5*e*n^3*lo
g(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3 + 2*(b^5*e*n^4*log(c) + a*b^
4*e*n^4)*log(e*x + d))
```

3.23.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-3), x)
```

3.23.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
-1/2*((d*n + d*log(c))*b + a*d + ((e*n + e*log(c))*b + a*e)*x + (b*e*x + b
*d)*log((e*x + d)^n))/(b^4*e*n^2*log((e*x + d)^n)^2 + b^4*e*n^2*log(c)^2 +
 2*a*b^3*e*n^2*log(c) + a^2*b^2*e*n^2 + 2*(b^4*e*n^2*log(c) + a*b^3*e*n^2)
*log((e*x + d)^n)) + integrate(1/2/(b^3*n^2*log((e*x + d)^n) + b^3*n^2*log
(c) + a*b^2*n^2), x)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. 2(128) = 256.

Time = 0.41 (sec) , antiderivative size = 1218, normalized size of antiderivative = 9.02

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
1/2*b^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d
)^2/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4
*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*
e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n^2*log(e*x + d)/(b^5*e*n^5*log(e*x +
d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*
e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + b^2*n*Ei(log(c)/n
 + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e*n^5*lo
g(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d
) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) -
1/2*(e*x + d)*b^2*n^2/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)
*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*
log(c) + a^2*b^3*e*n^3) + a*b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-
a/(b*n))*log(e*x + d)/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d
)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3
*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n*log(c)/(b^5*e*n^5*
log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x +
 d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + 1/2*b^2
*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e*n^5*l
og(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x +
d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)...
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d+e\,x)^n))^3}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^3, x)
```

3.24 $\int (a+b\log(c(d+ex)^n))^{5/2}\,dx$

3.24.1 Optimal result

Integrand size = 18, antiderivative size = 179

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = -\frac{15b^{5/2}e^{-\frac{a}{bn}}n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{2e} + \frac{(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{e}$$

output
```
-5/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^(5/2)/e-15/8*b^(5/2)*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b
^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*n^2*(
e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

3.24.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx=\frac{(d+ex)\left(8(a+b\log (c(d+ex)^n))^{5/2}-5bn\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
((d + e*x)*(8*(a + b*Log[c*(d + e*x)^n])^(5/2) - 5*b*n*((3*b^(3/2)*n^(3/2)
*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b
*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a - 3*b
*n + 2*b*Log[c*(d + e*x)^n]))))/(8*e)
```

3.24.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2836, 2733, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx$$

$$\Big\downarrow \; 2836$$

$$\frac{\int (a+b\log (c(d+ex)^n))^{5/2}d(d+ex)}{e}$$

$$\Big\downarrow \; 2733$$

$$\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^{5/2}-\frac{5}{2}bn\int (a+b\log (c(d+ex)^n))^{3/2}d(d+ex)}{e}$$

$$\Big\downarrow \; 2733$$

$$\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^{5/2}-\frac{5}{2}bn\left((d+ex)\,(a+b\log (c(d+ex)^n))^{3/2}-\frac{3}{2}bn\int \sqrt{a+b\log (c(d+ex)^n)}d(d+ex)\right.}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+e\right.}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Bigg((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Bigg((d+ex)\sqrt{a+b\log\left(c(d+\right.}}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Bigg((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Bigg((d+ex)\sqrt{a+b\log\left(c(d+\right.}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+e\right.}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2) - (5*b*n*((d + e*x)*(a + b*Log
[c*(d + e*x)^n])^(3/2) - (3*b*n*(-1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*(d + e*x)*
Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d
 + e*x)^n)^n^(-1)) + (d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]))/2))/2)/e
```

3.24.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.24.4 Maple [F]

$$\int \left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{\frac{5}{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.24.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\log \left(c(d+ex)^{n}\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.24.6 Sympy [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \int (a+b\log(c(d+ex)^n))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(5/2), x)
```

3.24.7 Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \int (b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.24.8 Giac [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \int (b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^{5/2} \, dx = \int (a+b\,\ln (c\,(d+e\,x)^n))^{5/2} \,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.25 $\int (a+b\log (c(d+ex)^n))^{3/2}\,dx$

3.25.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\frac{3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e}-\frac{3bn(d+ex)\sqrt{a+b\log (c(d+ex)^n)}}{2e}+\frac{(d+ex)(a+b\log (c(d+ex)^n))^{3/2}}{e}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+3/4*b^(3/2)*n^(3/2)*(e*x+d)*erfi((a+
b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d
)^n)^(1/n))-3/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

3.25.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\frac{(d+ex)\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+2\sqrt{a+b\log \cdots}\right.}{4e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
((d + e*x)*((3*b^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*
Log[c*(d + e*x)^n]]*(2*a - 3*b*n + 2*b*Log[c*(d + e*x)^n])))/(4*e)
```

3.25.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2836, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^{3/2}d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\int\sqrt{a+b\log(c(d+ex)^n)}d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\left((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-\frac{1}{2}bn\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\left((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-\frac{1}{2}b(d+ex)(c(d+ex)^n)^{-1/n}\int\frac{(c}{\sqrt{a+}}\right.}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\left((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(}{}}\right.}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^{n}$$
$$e$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2) - (3*b*n*(-1/2*(Sqrt[b]*Sqrt[n
]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (d + e*x)*Sqrt[a + b*Log[c*(d +
e*x)^n]]))/2)/e
```

3.25.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.25.4 Maple [F]

$$\int (a+b\ln (c(ex+d)^n))^{\frac{3}{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.25.5 Fricas [F(-2)]

Exception generated.

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.25.6 Sympy [F]

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx = \int (a+b\log (c(d+ex)^n))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(3/2), x)
```

3.25.7 Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (b\log((ex+d)^n c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.25.8 Giac [F]

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (b\log((ex+d)^n c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (a+b\,\ln(c\,(d+e\,x)^n))^{3/2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.26 $\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$

3.26.1 Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e} + \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

output
```
-1/2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(
1/2)*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(e*x+d)*(a+b*ln(c*(e*x+d)
^n))^(1/2)/e
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{(d+ex)\left(-\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+2\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e}$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
((d + e*x)*(-((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))) + 2*Sqrt[a + b
*Log[c*(d + e*x)^n]]))/(2*e)
```

3.26.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2836, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \sqrt{a+b\log\left(c(d+ex)^n\right)}d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}bn\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}b(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log\left(c(d+ex)^n\right)}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{e}$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(-1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^
n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (d + e*x)*S
qrt[a + b*Log[c*(d + e*x)^n]])/e
```

3.26.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.26.4 Maple [F]

$$\int \sqrt{a + b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.26.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.26.6 Sympy [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.26.7 Maxima [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.26.8 Giac [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.27.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

output
```
(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a
/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

input
```
Integrate[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]
)/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

3.27.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2836, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \frac{\left(c(d+ex)^n\right)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log\left(c(d+ex)^n\right)}}{ben}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

input
```
Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]
)/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

3.27.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.27.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.27.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.27.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.27.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.27.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.28 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.28.1 Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

output
```
2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(3/
2)/e/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))-2*(e*x+d)/b/e/n/(a+b*ln(c*(e
*x+d)^n))^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\left(e^{\frac{a}{bn}}(c(d+ex)^n)^{\frac{1}{n}} - \Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\sqrt{-\frac{a+b\log(c(d+ex)^n)}{bn}}\right)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
```

output
```
(-2*(d + e*x)*(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1) - Gamma[1/2, -((a + b*Lo
g[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))]))/(b*e
*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

3.28.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2836, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{bn}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{2(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{bn^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{4(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2n^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
```

output
```
((2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n
])])/(b^(3/2)*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/
(b*n*Sqrt[a + b*Log[c*(d + e*x)^n]]))/e
```

3.28.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.28.4 Maple [F]

$$\int \frac{1}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.28.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.28.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-3/2), x)
```

3.28.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

3.28.8 Giac [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\ln(c(d+e\,x)^n))^{3/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.29 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.29.1 Optimal result

Integrand size = 18, antiderivative size = 156

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

output
```
-2/3*(e*x+d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(e*x+d)*erfi((a+b*ln(c*
(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e/exp(a/b/n)/n^(5/2)/(
(c*(e*x+d)^n)^(1/n))-4/3*(e*x+d)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(2bn\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{3/2}+e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}}(2a\ldots}{3b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

output
```
(-2*(d + e*x)*(2*b*n*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(-((a
 + b*Log[c*(d + e*x)^n])/(b*n)))^(3/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1
)*(2*a + b*n + 2*b*Log[c*(d + e*x)^n])))/(3*b^2*e*E^(a/(b*n))*n^2*(c*(d +
e*x)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])^(3/2))
```

3.29.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2836, 2734, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}d(d+ex)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{bn}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{2\left(\frac{2(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{bn^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{2\left(\frac{\frac{4(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2n^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

output
```
((-2*(d + e*x))/(3*b*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) + (2*((2*Sqrt[Pi]
*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(b^(3/2
)*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/(b*n*Sqrt[a
+ b*Log[c*(d + e*x)^n]])))/(3*b*n))/e
```

3.29.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.29.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.29.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log\left(c(d+ex)^{n}\right))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.29.6 Sympy [F]

$$\int \frac{1}{(a+b\log\left(c(d+ex)^{n}\right))^{5/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-5/2), x)
```

3.29.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

3.29.8 Giac [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.30 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx$

3.30.1 Optimal result

Integrand size = 18, antiderivative size = 192

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \frac{8e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{15b^{7/2}en^{7/2}} - \frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

output
```
-2/5*(e*x+d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(5/2)-4/15*(e*x+d)/b^2/e/n^2/(a+b
*ln(c*(e*x+d)^n))^(3/2)+8/15*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1
/2)/n^(1/2))*Pi^(1/2)/b^(7/2)/e/exp(a/b/n)/n^(7/2)/((c*(e*x+d)^n)^(1/n))-8
/15*(e*x+d)/b^3/e/n^3/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(-4b^2n^2\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{5/2}+e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}}}{15b^3en^3\left(a+b\log\left(c(d+ex)^n\right)\right.}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-7/2),x]
```

output
```
(-2*(d + e*x)*(-4*b^2*n^2*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*
(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^(5/2) + E^(a/(b*n))*(c*(d + e*x)^n)^
n^(-1)*(4*a^2 + 2*a*b*n + 3*b^2*n^2 + 2*b*(4*a + b*n)*Log[c*(d + e*x)^n] +
 4*b^2*Log[c*(d + e*x)^n]^2)))/(15*b^3*e*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n
^(-1)*(a + b*Log[c*(d + e*x)^n])^(5/2))
```

3.30.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2836, 2734, 2734, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}d(d+ex)}{5bn}-\frac{2(d+ex)}{5bn(a+b\log(c(d+ex)^n))^{5/2}}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}d(d+ex)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}\right)}{5bn}-\frac{2(d+ex)}{5bn(a+b\log(c(d+ex)^n))^{5/2}}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{bn}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}\right)}{5bn}-\frac{2(d+ex)}{5bn(a+b\log(c(d+ex)^n))^{5/2}}}{e}$$

$\Big\downarrow$ 2737

$$\frac{\frac{2\left(\frac{2\left(\frac{2(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{bn^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}\right)}{5bn}-\frac{2(d+ex)}{5bn(a+b\log(c(d+ex)}}{e}$$

$\Big\downarrow$ 2611

$$\frac{\frac{2\left(\frac{2\left(\frac{4(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2n^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}\right)}{5bn}-\frac{2(}{5bn(a+b\log}}{e}$$

$\Big\downarrow$ 2633

$$\frac{\frac{2\left(\frac{2\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}\right)}{5bn}-\frac{2(d+ex)}{5bn(a+b\log(c(d+ex)^r}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-7/2),x]
```

output
```
((-2*(d + e*x))/(5*b*n*(a + b*Log[c*(d + e*x)^n])^(5/2)) + (2*((-2*(d + e*
x))/(3*b*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) + (2*((2*Sqrt[Pi]*(d + e*x)*E
rfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*E^(a/(b*n)
)*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/(b*n*Sqrt[a + b*Log[c*(d
 + e*x)^n]])))/(3*b*n)))/(5*b*n))/e
```

3.30. $\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx$

3.30.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.30.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(7/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(7/2),x)
```

3.30.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.30.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(7/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-7/2), x)
```

3.30.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-7/2), x)
```

3.30.8 Giac [F]

$$\int \frac{1}{(a+b\log (c(d+ex)^n))^{7/2}}\,dx = \int \frac{1}{(b\log ((ex+d)^n c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-7/2), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log (c(d+ex)^n))^{7/2}}\,dx = \int \frac{1}{(a+b\,\ln (c\,(d+e\,x)^n))^{7/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(7/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(7/2), x)
```

3.31 $\int (a+b\log(c(d+ex)^n))^p\,dx$

3.31.1 Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (a+b\log(c(d+ex)^n))^p\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-p}}{e}$$

output
```
(e*x+d)*GAMMA(p+1,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^p/e/ex
p(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^p)
```

3.31.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d+ex)^n))^p\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-p}}{e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^p,x]
```

output
```
((d + e*x)*Gamma[1 + p, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*
(d + e*x)^n])^p)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d
+ e*x)^n])/(b*n)))^p)
```

3.31.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2836, 2737, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^p\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^p\,d(d+ex)}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \left(c(d+ex)^n\right)^{\frac{1}{n}}(a+b\log(c(d+ex)^n))^p\,d\log(c(d+ex)^n)}{en}$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^p,x]
```

output
```
((d + e*x)*Gamma[1 + p, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*
(d + e*x)^n])^p)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d
+ e*x)^n])/(b*n)))^p)
```

3.31.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.31.4 Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^p dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^p,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^p,x)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int (a+b\log(c(d+ex)^n))^p \, dx = \frac{e^{\left(-\frac{bnp\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(p+1,-\frac{bn\log(ex+d)+b\log(c)+a}{bn}\right)}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^p,x, algorithm="fricas")
```

output
```
e^(-(b*n*p*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(p + 1, -(b*n*log(e*x
 + d) + b*log(c) + a)/(b*n))/e
```

3.31.6 Sympy [F]

$$\int (a+b\log (c(d+ex)^n))^p \,dx=\int (a+b\log (c(d+ex)^n))^p \,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**p,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**p, x)
```

3.31.7 Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (c(d+ex)^n))^p \,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.31.8 Giac [F]

$$\int (a+b\log (c(d+ex)^n))^p \,dx=\int (b\log ((ex+d)^nc)+a)^p \,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^p, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^p\,dx = \int (a+b\ln(c\,(d+e\,x)^n))^p\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^p,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^p, x)
```

3.32 $\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p \, dx$

3.32.1 Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p \, dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\sqrt{d+ex}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d+ex}\right)}{b}\right)^{-p}}{c^2e}$$

output
```
GAMMA(p+1,-2*(a+b*ln(c*(e*x+d)^(1/2)))/b)*(a+b*ln(c*(e*x+d)^(1/2)))^p/(2^p
)/c^2/e/exp(2*a/b)/(((-a-b*ln(c*(e*x+d)^(1/2)))/b)^p)
```

3.32.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p \, dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\sqrt{d+ex}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d+ex}\right)}{b}\right)^{-p}}{c^2e}$$

input
```
Integrate[(a + b*Log[c*Sqrt[d + e*x]])^p,x]
```

output
```
(Gamma[1 + p, (-2*(a + b*Log[c*Sqrt[d + e*x]]))/b]*(a + b*Log[c*Sqrt[d + e
*x]])^p)/(2^p*c^2*e*E^((2*a)/b)*(-((a + b*Log[c*Sqrt[d + e*x]])/b))^p)
```

3.32.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2836, 2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\sqrt{d+ex}\right)\right)^p\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \left(a+b\log \left(c\sqrt{d+ex}\right)\right)^p d(d+ex)}{e}$$

$$\Big\downarrow \text{2736}$$

$$\frac{2\int c^2(d+ex)\left(a+b\log \left(c\sqrt{d+ex}\right)\right)^p d\log \left(c\sqrt{d+ex}\right)}{c^2e}$$

$$\Big\downarrow \text{2612}$$

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log \left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log \left(c\sqrt{d+ex}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log \left(c\sqrt{d+ex}\right)\right)}{b}\right)}{c^2e}$$

input
```
Int[(a + b*Log[c*Sqrt[d + e*x]])^p,x]
```

output
```
(Gamma[1 + p, (-2*(a + b*Log[c*Sqrt[d + e*x]]))/b]*(a + b*Log[c*Sqrt[d + e
*x]])^p)/(2^p*c^2*e*E^((2*a)/b)*(-((a + b*Log[c*Sqrt[d + e*x]])/b))^p)
```

3.32.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.32.4 Maple [F]

$$\int \left(a+b\ln\left(c\sqrt{ex+d}\right)\right)^p dx$$

input
```
int((a+b*ln(c*(e*x+d)^(1/2)))^p,x)
```

output
```
int((a+b*ln(c*(e*x+d)^(1/2)))^p,x)
```

3.32.5 Fricas [F]

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx=\int \left(b\log\left(\sqrt{ex+d}c\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(e*x+d)^(1/2)))^p,x, algorithm="fricas")
```

output
```
integral((b*log(sqrt(e*x + d)*c) + a)^p, x)
```

3.32.6 Sympy [F]

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx=\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**(1/2)))**p,x)
```

output
```
Integral((a + b*log(c*sqrt(d + e*x)))**p, x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx = -\frac{2\left(b\log\left(\sqrt{ex+d}c\right)+a\right)^{p+1}e^{\left(-\frac{2a}{b}\right)}E_{-p}\left(-\frac{2\left(b\log\left(\sqrt{ex+d}c\right)+a\right)}{b}\right)}{bc^2e}$$

input
```
integrate((a+b*log(c*(e*x+d)^(1/2)))^p,x, algorithm="maxima")
```

output
```
-2*(b*log(sqrt(e*x + d)*c) + a)^(p + 1)*e^(-2*a/b)*exp_integral_e(-p, -2*(
b*log(sqrt(e*x + d)*c) + a)/b)/(b*c^2*e)
```

3.32.8 Giac [F]

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx = \int \left(b\log\left(\sqrt{ex+d}c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^(1/2)))^p,x, algorithm="giac")
```

output
```
integrate((b*log(sqrt(e*x + d)*c) + a)^p, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx = \int \left(a+b\,\ln\left(c\,\sqrt{d+e\,x}\right)\right)^p\,dx$$

input
```
int((a + b*log(c*(d + e*x)^(1/2)))^p,x)
```

output
```
int((a + b*log(c*(d + e*x)^(1/2)))^p, x)
```

3.33 $\int \frac{(e+fx)^{-1+p}}{\log(d(e+fx)^p)}\,dx$

3.33.1 Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{(e+fx)^{-1+p}}{\log(d(e+fx)^p)}\,dx = \frac{\text{LogIntegral}(d(e+fx)^p)}{dfp}$$

output
```
Li(d*(f*x+e)^p)/d/f/p
```

3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)^{-1+p}}{\log(d(e+fx)^p)}\,dx = \frac{\text{LogIntegral}(d(e+fx)^p)}{dfp}$$

input
```
Integrate[(e + f*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
```

output
```
LogIntegral[d*(e + f*x)^p]/(d*f*p)
```

3.33.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2837, 2744, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{p-1}}{\log (d(e+fx)^p)} \, dx$$

$$\Big\downarrow \text{2837}$$

$$\frac{\int \frac{(e+fx)^{p-1}}{\log(d(e+fx)^p)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2744}$$

$$\frac{\int \frac{1}{\log(d(e+fx)^p)}d(e+fx)^p}{fp}$$

$$\Big\downarrow \text{2735}$$

$$\frac{\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

input
```
Int[(e + f*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
```

output
```
LogIntegral[d*(e + f*x)^p]/(d*f*p)
```

3.33.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2744
```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[1/n   Subst[Int[1/Lo
g[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.33.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result
default	$-\frac{\mathrm{Ei}_1(-\ln(d(fx+e)^p))}{pfd}$
risch	$-\frac{\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(id(fx+e)^p)(-\mathrm{csgn}(id(fx+e)^p)+\mathrm{csgn}(id))(-\mathrm{csgn}(id(fx+e)^p)+\mathrm{csgn}(i(fx+e)^p))}{2}}\ \mathrm{Ei}_1\left(-\ln(d)-\ln((fx+e)^p)-\frac{i\pi\ \mathrm{csgn}(i(fx+e)^p)\ \mathrm{c}}{2}\right.}{pfd}$

input
```
int((f*x+e)^(-1+p)/ln(d*(f*x+e)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/p/f/d*Ei(1,-ln(d*(f*x+e)^p))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^{-1+p}}{\log(d(e+fx)^p)}\,dx = \frac{\mathrm{Ei}(p\log(fx+e)+\log(d))}{dfp}$$

input
```
integrate((f*x+e)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="fricas")
```

output
```
Ei(p*log(f*x + e) + log(d))/(d*f*p)
```

3.33.6 Sympy [F]

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \int \frac{(e+fx)^{p-1}}{\log \left(d\left(e+fx\right)^p\right)}\,dx$$

input
```
integrate((f*x+e)**(-1+p)/ln(d*(f*x+e)**p),x)
```

output
```
Integral((e + f*x)**(p - 1)/log(d*(e + f*x)**p), x)
```

3.33.7 Maxima [F]

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \int \frac{(fx+e)^{p-1}}{\log \left((fx+e)^p d\right)}\,dx$$

input
```
integrate((f*x+e)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^(p - 1)/log((f*x + e)^p*d), x)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \frac{\mathrm{Ei}(p\log (fx+e)+\log (d))}{dfp}$$

input
```
integrate((f*x+e)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="giac")
```

output
```
Ei(p*log(f*x + e) + log(d))/(d*f*p)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \frac{\mathrm{logint}(d\,(e+f\,x)^p)}{d\,f\,p}$$

input
```
int((e + f*x)^(p - 1)/log(d*(e + f*x)^p),x)
```

output
```
logint(d*(e + f*x)^p)/(d*f*p)
```

3.34 $\int \frac{(eg+fgx)^{-1+p}}{\log(d(e+fx)^p)}\,dx$

3.34.1 Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx=\frac{(e+fx)^{1-p}(g(e+fx))^{-1+p}\,\mathrm{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

output
```
(f*x+e)^(1-p)*(g*(f*x+e))^(-1+p)*Li(d*(f*x+e)^p)/d/f/p
```

3.34.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx=\frac{(e+fx)^{1-p}(g(e+fx))^{-1+p}\,\mathrm{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

input
```
Integrate[(e*g + f*g*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
```

output
```
((e + f*x)^(1 - p)*(g*(e + f*x))^(-1 + p)*LogIntegral[d*(e + f*x)^p])/(d*f
*p)
```

3.34.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2837, 2745, 2744, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(eg+fgx)^{p-1}}{\log\left(d(e+fx)^p\right)} \, dx$$

$$\Big\downarrow \text{2837}$$

$$\frac{\int \frac{(g(e+fx))^{p-1}}{\log(d(e+fx)^p)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2745}$$

$$\frac{(e+fx)^{1-p}(g(e+fx))^{p-1}\int \frac{(e+fx)^{p-1}}{\log(d(e+fx)^p)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2744}$$

$$\frac{(e+fx)^{1-p}(g(e+fx))^{p-1}\int \frac{1}{\log(d(e+fx)^p)}d(e+fx)^p}{fp}$$

$$\Big\downarrow \text{2735}$$

$$\frac{(e+fx)^{1-p}(g(e+fx))^{p-1}\,\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

input
```
Int[(e*g + f*g*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
```

output
```
((e + f*x)^(1 - p)*(g*(e + f*x))^(-1 + p)*LogIntegral[d*(e + f*x)^p])/(d*f
*p)
```

3.34.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2744
```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[1/n   Subst[Int[1/Lo
g[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

rule 2745
```
Int[((d_)*(x_))^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[(d*x)^m/x^m
 Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.34.4 Maple [F]

$$\int \frac{(fgx+eg)^{-1+p}}{\ln(d(fx+e)^p)}dx$$

input
```
int((f*g*x+e*g)^(-1+p)/ln(d*(f*x+e)^p),x)
```

output
```
int((f*g*x+e*g)^(-1+p)/ln(d*(f*x+e)^p),x)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{(eg+fgx)^{-1+p}}{\log(d(e+fx)^p)}\,dx = \frac{g^{p-1}\mathrm{Ei}(p\log(fx+e)+\log(d))}{dfp}$$

input
```
integrate((f*g*x+e*g)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="fricas")
```

output
```
g^(p - 1)*Ei(p*log(f*x + e) + log(d))/(d*f*p)
```

3.34.6 Sympy [F]

$$\int \frac{(eg + fgx)^{-1+p}}{\log\left(d(e + fx)^p\right)}\,dx = \int \frac{(g(e + fx))^{p-1}}{\log\left(d\left(e + fx\right)^p\right)}\,dx$$

input
```
integrate((f*g*x+e*g)**(-1+p)/ln(d*(f*x+e)**p),x)
```

output
```
Integral((g*(e + f*x))**(p - 1)/log(d*(e + f*x)**p), x)
```

3.34.7 Maxima [F]

$$\int \frac{(eg + fgx)^{-1+p}}{\log\left(d(e + fx)^p\right)}\,dx = \int \frac{(fgx + eg)^{p-1}}{\log\left((fx + e)^p d\right)}\,dx$$

input
```
integrate((f*g*x+e*g)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="maxima")
```

output
```
integrate((f*g*x + e*g)^(p - 1)/log((f*x + e)^p*d), x)
```

3.34.8 Giac [F]

$$\int \frac{(eg + fgx)^{-1+p}}{\log\left(d(e + fx)^p\right)}\,dx = \int \frac{(fgx + eg)^{p-1}}{\log\left((fx + e)^p d\right)}\,dx$$

input
```
integrate((f*g*x+e*g)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="giac")
```

output
```
integrate((f*g*x + e*g)^(p - 1)/log((f*x + e)^p*d), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \int \frac{(e\,g+f\,g\,x)^{p-1}}{\ln\left(d\,(e+f\,x)^p\right)}\,dx$$

input
```
int((e*g + f*g*x)^(p - 1)/log(d*(e + f*x)^p),x)
```

output
```
int((e*g + f*g*x)^(p - 1)/log(d*(e + f*x)^p), x)
```

3.35 $\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx$

3.35.1 Optimal result

Integrand size = 22, antiderivative size = 178

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx = -\frac{b(ef-dg)^4nx}{5e^4} - \frac{b(ef-dg)^3n(f+gx)^2}{10e^3g} - \frac{b(ef-dg)^2n(f+gx)^3}{15e^2g} - \frac{b(ef-dg)n(f+gx)^4}{20eg} - \frac{bn(f+gx)^5}{25g} - \frac{b(ef-dg)^5n\log(d+ex)}{5e^5g} + \frac{(f+gx)^5(a+b\log(c(d+ex)^n))}{5g}$$

output
```
-1/5*b*(-d*g+e*f)^4*n*x/e^4-1/10*b*(-d*g+e*f)^3*n*(g*x+f)^2/e^3/g-1/15*b*(
-d*g+e*f)^2*n*(g*x+f)^3/e^2/g-1/20*b*(-d*g+e*f)*n*(g*x+f)^4/e/g-1/25*b*n*(
g*x+f)^5/g-1/5*b*(-d*g+e*f)^5*n*ln(e*x+d)/e^5/g+1/5*(g*x+f)^5*(a+b*ln(c*(e
*x+d)^n))/g
```

3.35.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.77

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx = \frac{ex(60ae^4(5f^4+10f^3gx+10f^2g^2x^2+5fg^3x^3+g^4x^4)-bn(60d^4g^4-30d^3eg^3(10f+gx)+10d^2e^2g^2(60f}{}$$

input
```
Integrate[(f + g*x)^4*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(e*x*(60*a*e^4*(5*f^4 + 10*f^3*g*x + 10*f^2*g^2*x^2 + 5*f*g^3*x^3 + g^4*x^
4) - b*n*(60*d^4*g^4 - 30*d^3*e*g^3*(10*f + g*x) + 10*d^2*e^2*g^2*(60*f^2
+ 15*f*g*x + 2*g^2*x^2) - 5*d*e^3*g*(120*f^3 + 60*f^2*g*x + 20*f*g^2*x^2 +
 3*g^3*x^3) + e^4*(300*f^4 + 300*f^3*g*x + 200*f^2*g^2*x^2 + 75*f*g^3*x^3
+ 12*g^4*x^4))) + 60*b*d^2*g*(-10*e^3*f^3 + 10*d*e^2*f^2*g - 5*d^2*e*f*g^2
 + d^3*g^3)*n*Log[d + e*x] + 60*b*e^4*(5*d*f^4 + e*x*(5*f^4 + 10*f^3*g*x +
 10*f^2*g^2*x^2 + 5*f*g^3*x^3 + g^4*x^4))*Log[c*(d + e*x)^n])/(300*e^5)
```

3.35.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(f+gx)^5 (a+b\log(c(d+ex)^n))}{5g}-\frac{ben\int \frac{(f+gx)^5}{d+ex}dx}{5g}$$

$$\Big\downarrow \text{49}$$

$$\frac{(f+gx)^5 (a+b\log(c(d+ex)^n))}{5g}-\frac{ben\int \left(\frac{(ef-dg)^5}{e^5(d+ex)}+\frac{g(ef-dg)^4}{e^5}+\frac{g(f+gx)(ef-dg)^3}{e^4}+\frac{g(f+gx)^2(ef-dg)^2}{e^3}+\frac{g(f+gx)^3(ef-dg)}{e^2}+\frac{g(f+gx)^4}{e}\right)dx}{5g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^5 (a+b\log(c(d+ex)^n))}{5g}-\frac{ben\left(\frac{(ef-dg)^5\log(d+ex)}{e^6}+\frac{gx(ef-dg)^4}{e^5}+\frac{(f+gx)^2(ef-dg)^3}{2e^4}+\frac{(f+gx)^3(ef-dg)^2}{3e^3}+\frac{(f+gx)^4(ef-dg)}{4e^2}+\frac{(f+gx)^5}{5e}\right)}{5g}$$

input
```
Int[(f + g*x)^4*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/5*(b*e*n*((g*(e*f - d*g)^4*x)/e^5 + ((e*f - d*g)^3*(f + g*x)^2)/(2*e^4)
 + ((e*f - d*g)^2*(f + g*x)^3)/(3*e^3) + ((e*f - d*g)*(f + g*x)^4)/(4*e^2)
 + (f + g*x)^5/(5*e) + ((e*f - d*g)^5*Log[d + e*x])/e^6))/g + ((f + g*x)^5
*(a + b*Log[c*(d + e*x)^n]))/(5*g)
```

3.35.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(164) = 328$.

Time = 1.62 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.31

method	result
parallelrisch	$\frac{300bd\,e^4f^4n+300x\ln(c(ex+d)^n)b\,e^5f^4-300xb\,e^5f^4n-300\ln(c(ex+d)^n)bd\,e^4f^4+60\ln(ex+d)b\,d^5g^4n+60x^5\ln(c(ex+d)^n)b\,e^5}{}$
risch	Expression too large to display

input
```
int((g*x+f)^4*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
1/300*(300*b*d*e^4*f^4*n+300*x*ln(c*(e*x+d)^n)*b*e^5*f^4-300*x*b*e^5*f^4*n
-300*ln(c*(e*x+d)^n)*b*d*e^4*f^4+60*ln(e*x+d)*b*d^5*g^4*n+60*x^5*ln(c*(e*x
+d)^n)*b*e^5*g^4-12*x^5*b*e^5*g^4*n+300*x^4*a*e^5*f*g^3+600*x^3*a*e^5*f^2*
g^2+600*x^2*a*e^5*f^3*g+60*x^5*a*e^5*g^4+600*ln(e*x+d)*b*d*e^4*f^4*n+15*x^
4*b*d*e^4*g^4*n-75*x^4*b*e^5*f*g^3*n+600*x^3*ln(c*(e*x+d)^n)*b*e^5*f^2*g^2
-20*x^3*b*d^2*e^3*g^4*n-200*x^3*b*e^5*f^2*g^2*n+600*x^2*ln(c*(e*x+d)^n)*b*
e^5*f^3*g+30*x^2*b*d^3*e^2*g^4*n-300*x^2*b*e^5*f^3*g*n-60*x*b*d^4*e*g^4*n+
300*x^4*ln(c*(e*x+d)^n)*b*e^5*f*g^3-300*b*d^4*e*f*g^3*n+600*b*d^3*e^2*f^2*
g^2*n-300*a*d*e^4*f^4-600*b*d^2*e^3*f^3*g*n+300*x*a*e^5*f^4-600*x*b*d^2*e^
3*f^2*g^2*n+600*x*b*d*e^4*f^3*g*n-300*ln(e*x+d)*b*d^4*e*f*g^3*n+600*ln(e*x
+d)*b*d^3*e^2*f^2*g^2*n+100*x^3*b*d*e^4*f*g^3*n-150*x^2*b*d^2*e^3*f*g^3*n+
300*x^2*b*d*e^4*f^2*g^2*n+300*x*b*d^3*e^2*f*g^3*n-600*ln(e*x+d)*b*d^2*e^3*
f^3*g*n+60*b*d^5*g^4*n)/e^5
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(164) = 328$.

Time = 0.32 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.65

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx = -\frac{12\,(be^5g^4n-5\,ae^5g^4)x^5-15\,(20\,ae^5fg^3-(5\,be^5fg^3-bde^4g^4)n)x^4-20\,(30\,ae^5f^2g^2-(10\,be^5f^2g^2-5}{}$$

input
```
integrate((g*x+f)^4*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/300*(12*(b*e^5*g^4*n - 5*a*e^5*g^4)*x^5 - 15*(20*a*e^5*f*g^3 - (5*b*e^5
*f*g^3 - b*d*e^4*g^4)*n)*x^4 - 20*(30*a*e^5*f^2*g^2 - (10*b*e^5*f^2*g^2 -
5*b*d*e^4*f*g^3 + b*d^2*e^3*g^4)*n)*x^3 - 30*(20*a*e^5*f^3*g - (10*b*e^5*f
^3*g - 10*b*d*e^4*f^2*g^2 + 5*b*d^2*e^3*f*g^3 - b*d^3*e^2*g^4)*n)*x^2 - 60
*(5*a*e^5*f^4 - (5*b*e^5*f^4 - 10*b*d*e^4*f^3*g + 10*b*d^2*e^3*f^2*g^2 - 5
*b*d^3*e^2*f*g^3 + b*d^4*e*g^4)*n)*x - 60*(b*e^5*g^4*n*x^5 + 5*b*e^5*f*g^3
*n*x^4 + 10*b*e^5*f^2*g^2*n*x^3 + 10*b*e^5*f^3*g*n*x^2 + 5*b*e^5*f^4*n*x +
 (5*b*d*e^4*f^4 - 10*b*d^2*e^3*f^3*g + 10*b*d^3*e^2*f^2*g^2 - 5*b*d^4*e*f*
g^3 + b*d^5*g^4)*n)*log(e*x + d) - 60*(b*e^5*g^4*x^5 + 5*b*e^5*f*g^3*x^4 +
 10*b*e^5*f^2*g^2*x^3 + 10*b*e^5*f^3*g*x^2 + 5*b*e^5*f^4*x)*log(c))/e^5
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(153) = 306$.

Time = 2.17 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.19

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx$$
$$= \begin{cases} af^4x + 2af^3gx^2 + 2af^2g^2x^3 + afg^3x^4 + \frac{ag^4x^5}{5} + \frac{bd^5g^4\log(c(d+ex)^n)}{5e^5} - \frac{bd^4fg^3\log(c(d+ex)^n)}{e^4} - \frac{bd^4g^4nx}{5e^4} + \frac{2bd^3f^2g \\ (a+b\log(cd^n))\left(f^4x + 2f^3gx^2 + 2f^2g^2x^3 + fg^3x^4 + \frac{g^4x^5}{5}\right) \end{cases}$$

input
```
integrate((g*x+f)**4*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f**4*x + 2*a*f**3*g*x**2 + 2*a*f**2*g**2*x**3 + a*f*g**3*x**4
 + a*g**4*x**5/5 + b*d**5*g**4*log(c*(d + e*x)**n)/(5*e**5) - b*d**4*f*g**
3*log(c*(d + e*x)**n)/e**4 - b*d**4*g**4*n*x/(5*e**4) + 2*b*d**3*f**2*g**2
*log(c*(d + e*x)**n)/e**3 + b*d**3*f*g**3*n*x/e**3 + b*d**3*g**4*n*x**2/(1
0*e**3) - 2*b*d**2*f**3*g*log(c*(d + e*x)**n)/e**2 - 2*b*d**2*f**2*g**2*n*
x/e**2 - b*d**2*f*g**3*n*x**2/(2*e**2) - b*d**2*g**4*n*x**3/(15*e**2) + b*
d*f**4*log(c*(d + e*x)**n)/e + 2*b*d*f**3*g*n*x/e + b*d*f**2*g**2*n*x**2/e
 + b*d*f*g**3*n*x**3/(3*e) + b*d*g**4*n*x**4/(20*e) - b*f**4*n*x + b*f**4*
x*log(c*(d + e*x)**n) - b*f**3*g*n*x**2 + 2*b*f**3*g*x**2*log(c*(d + e*x)*
*n) - 2*b*f**2*g**2*n*x**3/3 + 2*b*f**2*g**2*x**3*log(c*(d + e*x)**n) - b*
f*g**3*n*x**4/4 + b*f*g**3*x**4*log(c*(d + e*x)**n) - b*g**4*n*x**5/25 + b
*g**4*x**5*log(c*(d + e*x)**n)/5, Ne(e, 0)), ((a + b*log(c*d**n))*(f**4*x
+ 2*f**3*g*x**2 + 2*f**2*g**2*x**3 + f*g**3*x**4 + g**4*x**5/5), True))
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(164) = 328$.

Time = 0.21 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.21

$$\begin{aligned}
&\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx \\
&= \frac{1}{5}bg^4x^5\log((ex+d)^n c) + \frac{1}{5}ag^4x^5 + bfg^3x^4\log((ex+d)^n c) + afg^3x^4 \\
&\quad + 2bf^2g^2x^3\log((ex+d)^n c) + 2af^2g^2x^3 - bef^4n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) \\
&\quad + \frac{1}{300}beg^4n\left(\frac{60d^5\log(ex+d)}{e^6} - \frac{12e^4x^5 - 15de^3x^4 + 20d^2e^2x^3 - 30d^3ex^2 + 60d^4x}{e^5}\right) \\
&\quad - \frac{1}{12}befg^3n\left(\frac{12d^4\log(ex+d)}{e^5} + \frac{3e^3x^4 - 4de^2x^3 + 6d^2ex^2 - 12d^3x}{e^4}\right) \\
&\quad + \frac{1}{3}bef^2g^2n\left(\frac{6d^3\log(ex+d)}{e^4} - \frac{2e^2x^3 - 3dex^2 + 6d^2x}{e^3}\right) \\
&\quad - bef^3gn\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2dx}{e^2}\right) \\
&\quad + 2bf^3gx^2\log((ex+d)^n c) + 2af^3gx^2 + bf^4x\log((ex+d)^n c) + af^4x
\end{aligned}$$

input
```
integrate((g*x+f)^4*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/5*b*g^4*x^5*log((e*x + d)^n*c) + 1/5*a*g^4*x^5 + b*f*g^3*x^4*log((e*x +
d)^n*c) + a*f*g^3*x^4 + 2*b*f^2*g^2*x^3*log((e*x + d)^n*c) + 2*a*f^2*g^2*x
^3 - b*e*f^4*n*(x/e - d*log(e*x + d)/e^2) + 1/300*b*e*g^4*n*(60*d^5*log(e*
x + d)/e^6 - (12*e^4*x^5 - 15*d*e^3*x^4 + 20*d^2*e^2*x^3 - 30*d^3*e*x^2 +
60*d^4*x)/e^5) - 1/12*b*e*f*g^3*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 -
4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4) + 1/3*b*e*f^2*g^2*n*(6*d^3*log(
e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) - b*e*f^3*g*n*(2*d^2
*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*b*f^3*g*x^2*log((e*x + d)^n*c
) + 2*a*f^3*g*x^2 + b*f^4*x*log((e*x + d)^n*c) + a*f^4*x
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. 2(164) = 328.

Time = 0.32 (sec) , antiderivative size = 1209, normalized size of antiderivative = 6.79

$$\int (f+gx)^4 (a+b\log (c(d+ex)^n))\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
(e*x + d)*b*f^4*n*log(e*x + d)/e + 2*(e*x + d)^2*b*f^3*g*n*log(e*x + d)/e^
2 - 4*(e*x + d)*b*d*f^3*g*n*log(e*x + d)/e^2 + 2*(e*x + d)^3*b*f^2*g^2*n*l
og(e*x + d)/e^3 - 6*(e*x + d)^2*b*d*f^2*g^2*n*log(e*x + d)/e^3 + 6*(e*x +
d)*b*d^2*f^2*g^2*n*log(e*x + d)/e^3 + (e*x + d)^4*b*f*g^3*n*log(e*x + d)/e
^4 - 4*(e*x + d)^3*b*d*f*g^3*n*log(e*x + d)/e^4 + 6*(e*x + d)^2*b*d^2*f*g^
3*n*log(e*x + d)/e^4 - 4*(e*x + d)*b*d^3*f*g^3*n*log(e*x + d)/e^4 + 1/5*(e
*x + d)^5*b*g^4*n*log(e*x + d)/e^5 - (e*x + d)^4*b*d*g^4*n*log(e*x + d)/e^
5 + 2*(e*x + d)^3*b*d^2*g^4*n*log(e*x + d)/e^5 - 2*(e*x + d)^2*b*d^3*g^4*n
*log(e*x + d)/e^5 + (e*x + d)*b*d^4*g^4*n*log(e*x + d)/e^5 - (e*x + d)*b*f
^4*n/e - (e*x + d)^2*b*f^3*g*n/e^2 + 4*(e*x + d)*b*d*f^3*g*n/e^2 - 2/3*(e*
x + d)^3*b*f^2*g^2*n/e^3 + 3*(e*x + d)^2*b*d*f^2*g^2*n/e^3 - 6*(e*x + d)*b
*d^2*f^2*g^2*n/e^3 - 1/4*(e*x + d)^4*b*f*g^3*n/e^4 + 4/3*(e*x + d)^3*b*d*f
*g^3*n/e^4 - 3*(e*x + d)^2*b*d^2*f*g^3*n/e^4 + 4*(e*x + d)*b*d^3*f*g^3*n/e
^4 - 1/25*(e*x + d)^5*b*g^4*n/e^5 + 1/4*(e*x + d)^4*b*d*g^4*n/e^5 - 2/3*(e
*x + d)^3*b*d^2*g^4*n/e^5 + (e*x + d)^2*b*d^3*g^4*n/e^5 - (e*x + d)*b*d^4*
g^4*n/e^5 + (e*x + d)*b*f^4*log(c)/e + 2*(e*x + d)^2*b*f^3*g*log(c)/e^2 -
4*(e*x + d)*b*d*f^3*g*log(c)/e^2 + 2*(e*x + d)^3*b*f^2*g^2*log(c)/e^3 - 6*
(e*x + d)^2*b*d*f^2*g^2*log(c)/e^3 + 6*(e*x + d)*b*d^2*f^2*g^2*log(c)/e^3
+ (e*x + d)^4*b*f*g^3*log(c)/e^4 - 4*(e*x + d)^3*b*d*f*g^3*log(c)/e^4 + 6*
(e*x + d)^2*b*d^2*f*g^3*log(c)/e^4 - 4*(e*x + d)*b*d^3*f*g^3*log(c)/e^4...
```

3.35.9 Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.96

$$
\begin{aligned}
\int (f+gx)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = {} & x\left(\frac{5\,a\,e\,f^4+20\,a\,d\,f^3\,g-5\,b\,e\,f^4\,n}{5\,e} - \frac{d\left(\frac{d\left(\frac{d\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^4\,(5\,a-b\,n)}{5\,e}\right)}{e}-\frac{2\,f\,g^2\,(2\,a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{e}+\frac{2\,f^2\,g\,(3\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{e}\right) \\
& - x^3\left(\frac{d\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^4\,(5\,a-b\,n)}{5\,e}\right)}{3\,e}-\frac{2\,f\,g^2\,(2\,a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{3\,e}\right) \\
& + x^4\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{4\,e}-\frac{d\,g^4\,(5\,a-b\,n)}{20\,e}\right) \\
& + \ln\left(c\,(d+e\,x)^n\right)\,\left(b\,f^4\,x+2\,b\,f^3\,g\,x^2+2\,b\,f^2\,g^2\,x^3+b\,f\,g^3\,x^4+\frac{b\,g^4\,x^5}{5}\right) \\
& + x^2\left(\frac{d\left(\frac{d\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^4\,(5\,a-b\,n)}{5\,e}\right)}{e}-\frac{2\,f\,g^2\,(2\,a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{2\,e} + \frac{f^2\,g\,(3\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}\right) + \frac{g^4\,x^5\,(5\,a-b\,n)}{25} \\
& + \frac{\ln(d+e\,x)\,\left(b\,n\,d^5\,g^4-5\,b\,n\,d^4\,e\,f\,g^3+10\,b\,n\,d^3\,e^2\,f^2\,g^2-10\,b\,n\,d^2\,e^3\,f^3\,g+5\,b\,n\,d\,e^4\,f^4\right)}{5\,e^5}
\end{aligned}
$$

input
```
int((f + g*x)^4*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x*((5*a*e*f^4 + 20*a*d*f^3*g - 5*b*e*f^4*n)/(5*e) - (d*((d*((d*((g^3*(a*d*
g + 4*a*e*f - b*e*f*n))/e - (d*g^4*(5*a - b*n))/(5*e)))/e - (2*f*g^2*(2*a*
d*g + 3*a*e*f - b*e*f*n))/e))/e + (2*f^2*g*(3*a*d*g + 2*a*e*f - b*e*f*n))/
e))/e) - x^3*((d*((g^3*(a*d*g + 4*a*e*f - b*e*f*n))/e - (d*g^4*(5*a - b*n)
)/(5*e)))/(3*e) - (2*f*g^2*(2*a*d*g + 3*a*e*f - b*e*f*n))/(3*e)) + x^4*((g
^3*(a*d*g + 4*a*e*f - b*e*f*n))/(4*e) - (d*g^4*(5*a - b*n))/(20*e)) + log(
c*(d + e*x)^n)*((b*g^4*x^5)/5 + b*f^4*x + 2*b*f^2*g^2*x^3 + 2*b*f^3*g*x^2
+ b*f*g^3*x^4) + x^2*((d*((d*((g^3*(a*d*g + 4*a*e*f - b*e*f*n))/e - (d*g^4
*(5*a - b*n))/(5*e)))/e - (2*f*g^2*(2*a*d*g + 3*a*e*f - b*e*f*n))/e))/(2*e
) + (f^2*g*(3*a*d*g + 2*a*e*f - b*e*f*n))/e) + (g^4*x^5*(5*a - b*n))/25 +
(log(d + e*x)*(b*d^5*g^4*n + 5*b*d*e^4*f^4*n + 10*b*d^3*e^2*f^2*g^2*n - 5*
b*d^4*e*f*g^3*n - 10*b*d^2*e^3*f^3*g*n))/(5*e^5)
```

3.35. $\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx$

3.36 $\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx$

3.36.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx = -\frac{b(ef-dg)^3nx}{4e^3} - \frac{b(ef-dg)^2n(f+gx)^2}{8e^2g} - \frac{b(ef-dg)n(f+gx)^3}{12eg} - \frac{bn(f+gx)^4}{16g} - \frac{b(ef-dg)^4n\log(d+ex)}{4e^4g} + \frac{(f+gx)^4(a+b\log(c(d+ex)^n))}{4g}$$

output
```
-1/4*b*(-d*g+e*f)^3*n*x/e^3-1/8*b*(-d*g+e*f)^2*n*(g*x+f)^2/e^2/g-1/12*b*(-
d*g+e*f)*n*(g*x+f)^3/e/g-1/16*b*n*(g*x+f)^4/g-1/4*b*(-d*g+e*f)^4*n*ln(e*x+
d)/e^4/g+1/4*(g*x+f)^4*(a+b*ln(c*(e*x+d)^n))/g
```

3.36.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.52

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx$$
$$= \frac{ex(12ae^3(4f^3+6f^2gx+4fg^2x^2+g^3x^3) - bn(-12d^3g^3+6d^2eg^2(8f+gx) - 4de^2g(18f^2+6fgx+g^2x^2)}{}$$

input
```
Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(e*x*(12*a*e^3*(4*f^3 + 6*f^2*g*x + 4*f*g^2*x^2 + g^3*x^3) - b*n*(-12*d^3*
g^3 + 6*d^2*e*g^2*(8*f + g*x) - 4*d*e^2*g*(18*f^2 + 6*f*g*x + g^2*x^2) + e
^3*(48*f^3 + 36*f^2*g*x + 16*f*g^2*x^2 + 3*g^3*x^3))) - 12*b*d^2*g*(6*e^2*
f^2 - 4*d*e*f*g + d^2*g^2)*n*Log[d + e*x] + 12*b*e^3*(4*d*f^3 + e*x*(4*f^3
 + 6*f^2*g*x + 4*f*g^2*x^2 + g^3*x^3))*Log[c*(d + e*x)^n])/(48*e^4)
```

3.36.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))\, dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(f+gx)^4 (a+b\log (c(d+ex)^n))}{4g}-\frac{ben\int \frac{(f+gx)^4}{d+ex}dx}{4g}$$

$$\Big\downarrow \text{49}$$

$$\frac{(f+gx)^4 (a+b\log (c(d+ex)^n))}{4g}-\frac{ben\int \left(\frac{(ef-dg)^4}{e^4(d+ex)}+\frac{g(ef-dg)^3}{e^4}+\frac{g(f+gx)(ef-dg)^2}{e^3}+\frac{g(f+gx)^2(ef-dg)}{e^2}+\frac{g(f+gx)^3}{e}\right)dx}{4g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^4 (a+b\log (c(d+ex)^n))}{4g}-\frac{ben\left(\frac{(ef-dg)^4\log(d+ex)}{e^5}+\frac{gx(ef-dg)^3}{e^4}+\frac{(f+gx)^2(ef-dg)^2}{2e^3}+\frac{(f+gx)^3(ef-dg)}{3e^2}+\frac{(f+gx)^4}{4e}\right)}{4g}$$

input
```
Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/4*(b*e*n*((g*(e*f - d*g)^3*x)/e^4 + ((e*f - d*g)^2*(f + g*x)^2)/(2*e^3)
 + ((e*f - d*g)*(f + g*x)^3)/(3*e^2) + (f + g*x)^4/(4*e) + ((e*f - d*g)^4*
Log[d + e*x])/e^5))/g + ((f + g*x)^4*(a + b*Log[c*(d + e*x)^n]))/(4*g)
```

3.36.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(137) = 274$.

Time = 1.07 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.87

method	result
parallelrisch	$-\frac{-48b\,d^3 ef\,g^2 n+72b\,d^2 e^2 f^2 gn-72x^2 a\,e^4 f^2 g-48x\ln(c(ex+d)^n)b\,e^4 f^3+48xb\,e^4 f^3 n+48\ln(c(ex+d)^n)bd\,e^3 f^3+12\ln(ex+d)b}{}$
risch	Expression too large to display

input
```
int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-48*b*d^3*e*f*g^2*n+72*b*d^2*e^2*f^2*g*n-72*x^2*a*e^4*f^2*g-48*x*ln
(c*(e*x+d)^n)*b*e^4*f^3+48*x*b*e^4*f^3*n+48*ln(c*(e*x+d)^n)*b*d*e^3*f^3+12
*ln(e*x+d)*b*d^4*g^3*n-12*x^4*ln(c*(e*x+d)^n)*b*e^4*g^3+3*x^4*b*e^4*g^3*n-
48*x^3*a*e^4*f*g^2+48*a*d*f^3*e^3-48*x^3*ln(c*(e*x+d)^n)*b*e^4*f*g^2-4*x^3
*b*d*e^3*g^3*n+16*x^3*b*e^4*f*g^2*n-72*x^2*ln(c*(e*x+d)^n)*b*e^4*f^2*g+6*x
^2*b*d^2*e^2*g^3*n+36*x^2*b*e^4*f^2*g*n-12*x*b*d^3*e*g^3*n-96*ln(e*x+d)*b*
d*e^3*f^3*n-48*b*d*e^3*f^3*n+12*b*d^4*g^3*n-48*ln(e*x+d)*b*d^3*e*f*g^2*n+7
2*ln(e*x+d)*b*d^2*e^2*f^2*g*n+48*x*b*d^2*e^2*f*g^2*n-72*x*b*d*e^3*f^2*g*n-
24*x^2*b*d*e^3*f*g^2*n-12*x^4*a*e^4*g^3-48*x*a*e^4*f^3)/e^4
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(137) = 274$.

Time = 0.29 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.28

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx =$$
$$-\frac{3\,(be^4g^3n-4\,ae^4g^3)x^4-4\,(12\,ae^4fg^2-(4\,be^4fg^2-bde^3g^3)n)x^3-6\,(12\,ae^4f^2g-(6\,be^4f^2g-4\,bde^3fg\cdots}{}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/48*(3*(b*e^4*g^3*n - 4*a*e^4*g^3)*x^4 - 4*(12*a*e^4*f*g^2 - (4*b*e^4*f*
g^2 - b*d*e^3*g^3)*n)*x^3 - 6*(12*a*e^4*f^2*g - (6*b*e^4*f^2*g - 4*b*d*e^3
*f*g^2 + b*d^2*e^2*g^3)*n)*x^2 - 12*(4*a*e^4*f^3 - (4*b*e^4*f^3 - 6*b*d*e^
3*f^2*g + 4*b*d^2*e^2*f*g^2 - b*d^3*e*g^3)*n)*x - 12*(b*e^4*g^3*n*x^4 + 4*
b*e^4*f*g^2*n*x^3 + 6*b*e^4*f^2*g*n*x^2 + 4*b*e^4*f^3*n*x + (4*b*d*e^3*f^3
 - 6*b*d^2*e^2*f^2*g + 4*b*d^3*e*f*g^2 - b*d^4*g^3)*n)*log(e*x + d) - 12*(
b*e^4*g^3*x^4 + 4*b*e^4*f*g^2*x^3 + 6*b*e^4*f^2*g*x^2 + 4*b*e^4*f^3*x)*log
(c))/e^4
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(128) = 256$.

Time = 1.16 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.75

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx$$
$$= \begin{cases} af^3x + \frac{3af^2gx^2}{2} + afg^2x^3 + \frac{ag^3x^4}{4} - \frac{bd^4g^3\log(c(d+ex)^n)}{4e^4} + \frac{bd^3fg^2\log(c(d+ex)^n)}{e^3} + \frac{bd^3g^3nx}{4e^3} - \frac{3bd^2f^2g\log(c(d+ex)^n)}{2e^2} - \\ (a+b\log(cd^n))\left(f^3x + \frac{3f^2gx^2}{2} + fg^2x^3 + \frac{g^3x^4}{4}\right) \end{cases}$$

input
```
integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f**3*x + 3*a*f**2*g*x**2/2 + a*f*g**2*x**3 + a*g**3*x**4/4 -
b*d**4*g**3*log(c*(d + e*x)**n)/(4*e**4) + b*d**3*f*g**2*log(c*(d + e*x)**
n)/e**3 + b*d**3*g**3*n*x/(4*e**3) - 3*b*d**2*f**2*g*log(c*(d + e*x)**n)/(
2*e**2) - b*d**2*f*g**2*n*x/e**2 - b*d**2*g**3*n*x**2/(8*e**2) + b*d*f**3*
log(c*(d + e*x)**n)/e + 3*b*d*f**2*g*n*x/(2*e) + b*d*f*g**2*n*x**2/(2*e) +
 b*d*g**3*n*x**3/(12*e) - b*f**3*n*x + b*f**3*x*log(c*(d + e*x)**n) - 3*b*
f**2*g*n*x**2/4 + 3*b*f**2*g*x**2*log(c*(d + e*x)**n)/2 - b*f*g**2*n*x**3/
3 + b*f*g**2*x**3*log(c*(d + e*x)**n) - b*g**3*n*x**4/16 + b*g**3*x**4*log
(c*(d + e*x)**n)/4, Ne(e, 0)), ((a + b*log(c*d**n))*(f**3*x + 3*f**2*g*x**
2/2 + f*g**2*x**3 + g**3*x**4/4), True))
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(137) = 274$.

Time = 0.20 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.91

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx = \frac{1}{4}bg^3x^4\log((ex+d)^nc) + \frac{1}{4}ag^3x^4 + bfg^2x^3\log((ex+d)^nc) + afg^2x^3 - bef^3n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) - \frac{1}{48}beg^3n\left(\frac{12\,d^4\log(ex+d)}{e^5} + \frac{3\,e^3x^4 - 4\,de^2x^3 + 6\,d^2ex^2 - 12\,d^3x}{e^4}\right) + \frac{1}{6}befg^2n\left(\frac{6\,d^3\log(ex+d)}{e^4} - \frac{2\,e^2x^3 - 3\,dex^2 + 6\,d^2x}{e^3}\right) - \frac{3}{4}bef^2gn\left(\frac{2\,d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2\,dx}{e^2}\right) + \frac{3}{2}bf^2gx^2\log((ex+d)^nc) + \frac{3}{2}af^2gx^2 + bf^3x\log((ex+d)^nc) + af^3x$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/4*b*g^3*x^4*log((e*x + d)^n*c) + 1/4*a*g^3*x^4 + b*f*g^2*x^3*log((e*x +
d)^n*c) + a*f*g^2*x^3 - b*e*f^3*n*(x/e - d*log(e*x + d)/e^2) - 1/48*b*e*g^
3*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12
*d^3*x)/e^4) + 1/6*b*e*f*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*
e*x^2 + 6*d^2*x)/e^3) - 3/4*b*e*f^2*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 -
 2*d*x)/e^2) + 3/2*b*f^2*g*x^2*log((e*x + d)^n*c) + 3/2*a*f^2*g*x^2 + b*f^
3*x*log((e*x + d)^n*c) + a*f^3*x
```

3.36. $\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx$

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(137) = 274$.

Time = 0.32 (sec) , antiderivative size = 770, normalized size of antiderivative = 5.17

$$
\begin{aligned}
\int (f+gx)^3\,(a+b\log(c(d+ex)^n))\,dx ={}& \frac{(ex+d)bf^3n\log(ex+d)}{e} \\
&+ \frac{3\,(ex+d)^2bf^2gn\log(ex+d)}{2\,e^2} \\
&- \frac{3\,(ex+d)bdf^2gn\log(ex+d)}{e^2} \\
&+ \frac{(ex+d)^3bfg^2n\log(ex+d)}{e^3} \\
&- \frac{3\,(ex+d)^2bdfg^2n\log(ex+d)}{e^3} \\
&+ \frac{3\,(ex+d)bd^2fg^2n\log(ex+d)}{e^3} \\
&+ \frac{(ex+d)^4bg^3n\log(ex+d)}{4\,e^4} \\
&- \frac{(ex+d)^3bdg^3n\log(ex+d)}{e^4} \\
&+ \frac{3\,(ex+d)^2bd^2g^3n\log(ex+d)}{2\,e^4} \\
&- \frac{(ex+d)bd^3g^3n\log(ex+d)}{e^4} - \frac{(ex+d)bf^3n}{e} \\
&- \frac{3\,(ex+d)^2bf^2gn}{4\,e^2} + \frac{3\,(ex+d)bdf^2gn}{e^2} \\
&- \frac{(ex+d)^3bfg^2n}{3\,e^3} + \frac{3\,(ex+d)^2bdfg^2n}{2\,e^3} \\
&- \frac{3\,(ex+d)bd^2fg^2n}{e^3} - \frac{(ex+d)^4bg^3n}{16\,e^4} \\
&+ \frac{(ex+d)^3bdg^3n}{3\,e^4} - \frac{3\,(ex+d)^2bd^2g^3n}{4\,e^4} \\
&+ \frac{(ex+d)bd^3g^3n}{e^4} + \frac{(ex+d)bf^3\log(c)}{e} \\
&+ \frac{3\,(ex+d)^2bf^2g\log(c)}{2\,e^2} - \frac{3\,(ex+d)bdf^2g\log(c)}{e^2} \\
&+ \frac{(ex+d)^3bfg^2\log(c)}{e^3} - \frac{3\,(ex+d)^2bdfg^2\log(c)}{e^3} \\
&+ \frac{3\,(ex+d)bd^2fg^2\log(c)}{e^3} \\
&+ \frac{(ex+d)^4bg^3\log(c)}{4\,e^4} - \frac{(ex+d)^3bdg^3\log(c)}{e^4} \\
&+ \frac{3\,(ex+d)^2bd^2g^3\log(c)}{2\,e^4} - \frac{(ex+d)bd^3g^3\log(c)}{e^4} \\
&+ \frac{(ex+d)af^3}{e} + \frac{3\,(ex+d)^2af^2g}{2\,e^2} \\
&- \frac{3\,(ex+d)adf^2g}{e^2} + \frac{(ex+d)^3afg^2}{e^3} \\
&- \frac{3\,(ex+d)^2adfg^2}{e^3} + \frac{3\,(ex+d)ad^2fg^2}{e^3}
\end{aligned}
$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
(e*x + d)*b*f^3*n*log(e*x + d)/e + 3/2*(e*x + d)^2*b*f^2*g*n*log(e*x + d)/
e^2 - 3*(e*x + d)*b*d*f^2*g*n*log(e*x + d)/e^2 + (e*x + d)^3*b*f*g^2*n*log
(e*x + d)/e^3 - 3*(e*x + d)^2*b*d*f*g^2*n*log(e*x + d)/e^3 + 3*(e*x + d)*b
*d^2*f*g^2*n*log(e*x + d)/e^3 + 1/4*(e*x + d)^4*b*g^3*n*log(e*x + d)/e^4 -
 (e*x + d)^3*b*d*g^3*n*log(e*x + d)/e^4 + 3/2*(e*x + d)^2*b*d^2*g^3*n*log(
e*x + d)/e^4 - (e*x + d)*b*d^3*g^3*n*log(e*x + d)/e^4 - (e*x + d)*b*f^3*n/
e - 3/4*(e*x + d)^2*b*f^2*g*n/e^2 + 3*(e*x + d)*b*d*f^2*g*n/e^2 - 1/3*(e*x
 + d)^3*b*f*g^2*n/e^3 + 3/2*(e*x + d)^2*b*d*f*g^2*n/e^3 - 3*(e*x + d)*b*d^
2*f*g^2*n/e^3 - 1/16*(e*x + d)^4*b*g^3*n/e^4 + 1/3*(e*x + d)^3*b*d*g^3*n/e
^4 - 3/4*(e*x + d)^2*b*d^2*g^3*n/e^4 + (e*x + d)*b*d^3*g^3*n/e^4 + (e*x +
d)*b*f^3*log(c)/e + 3/2*(e*x + d)^2*b*f^2*g*log(c)/e^2 - 3*(e*x + d)*b*d*f
^2*g*log(c)/e^2 + (e*x + d)^3*b*f*g^2*log(c)/e^3 - 3*(e*x + d)^2*b*d*f*g^2
*log(c)/e^3 + 3*(e*x + d)*b*d^2*f*g^2*log(c)/e^3 + 1/4*(e*x + d)^4*b*g^3*l
og(c)/e^4 - (e*x + d)^3*b*d*g^3*log(c)/e^4 + 3/2*(e*x + d)^2*b*d^2*g^3*log
(c)/e^4 - (e*x + d)*b*d^3*g^3*log(c)/e^4 + (e*x + d)*a*f^3/e + 3/2*(e*x +
d)^2*a*f^2*g/e^2 - 3*(e*x + d)*a*d*f^2*g/e^2 + (e*x + d)^3*a*f*g^2/e^3 - 3
*(e*x + d)^2*a*d*f*g^2/e^3 + 3*(e*x + d)*a*d^2*f*g^2/e^3 + 1/4*(e*x + d)^4
*a*g^3/e^4 - (e*x + d)^3*a*d*g^3/e^4 + 3/2*(e*x + d)^2*a*d^2*g^3/e^4 - (e*
x + d)*a*d^3*g^3/e^4
```

3.36. $\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx$

3.36.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.36

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx$$

$$= x\left(\frac{4\,a\,e\,f^3+12\,a\,d\,f^2\,g-4\,b\,e\,f^3\,n}{4\,e} + \frac{d\left(\frac{d\left(\frac{g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^3\,(4\,a-b\,n)}{4\,e}\right)}{e}-\frac{3\,f\,g\,(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{2\,e}\right)}{e}\right)$$
$$+ x^3\left(\frac{g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{3\,e}-\frac{d\,g^3\,(4\,a-b\,n)}{12\,e}\right)$$
$$+ \ln\left(c\,(d+e\,x)^n\right)\left(b\,f^3\,x+\frac{3\,b\,f^2\,g\,x^2}{2}+b\,f\,g^2\,x^3+\frac{b\,g^3\,x^4}{4}\right)$$
$$- x^2\left(\frac{d\left(\frac{g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^3\,(4\,a-b\,n)}{4\,e}\right)}{2\,e}-\frac{3\,f\,g\,(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{4\,e}\right)$$
$$- \frac{\ln(d+e\,x)\,\left(b\,n\,d^4\,g^3-4\,b\,n\,d^3\,e\,f\,g^2+6\,b\,n\,d^2\,e^2\,f^2\,g-4\,b\,n\,d\,e^3\,f^3\right)}{4\,e^4}$$
$$+ \frac{g^3\,x^4\,(4\,a-b\,n)}{16}$$

input
```
int((f + g*x)^3*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x*((4*a*e*f^3 + 12*a*d*f^2*g - 4*b*e*f^3*n)/(4*e) + (d*((d*((g^2*(a*d*g +
3*a*e*f - b*e*f*n))/e - (d*g^3*(4*a - b*n))/(4*e)))/e - (3*f*g*(2*a*d*g +
2*a*e*f - b*e*f*n))/(2*e)))/e) + x^3*((g^2*(a*d*g + 3*a*e*f - b*e*f*n))/(3
*e) - (d*g^3*(4*a - b*n))/(12*e)) + log(c*(d + e*x)^n)*((b*g^3*x^4)/4 + b*
f^3*x + (3*b*f^2*g*x^2)/2 + b*f*g^2*x^3) - x^2*((d*((g^2*(a*d*g + 3*a*e*f
- b*e*f*n))/e - (d*g^3*(4*a - b*n))/(4*e)))/(2*e) - (3*f*g*(2*a*d*g + 2*a*
e*f - b*e*f*n))/(4*e)) - (log(d + e*x)*(b*d^4*g^3*n - 4*b*d*e^3*f^3*n - 4*
b*d^3*e*f*g^2*n + 6*b*d^2*e^2*f^2*g*n))/(4*e^4) + (g^3*x^4*(4*a - b*n))/16
```

3.37 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx$

3.37.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx = -\frac{b(ef-dg)^2nx}{3e^2} - \frac{b(ef-dg)n(f+gx)^2}{6eg} - \frac{bn(f+gx)^3}{9g} - \frac{b(ef-dg)^3n\log(d+ex)}{3e^3g} + \frac{(f+gx)^3(a+b\log(c(d+ex)^n))}{3g}$$

output
```
-1/3*b*(-d*g+e*f)^2*n*x/e^2-1/6*b*(-d*g+e*f)*n*(g*x+f)^2/e/g-1/9*b*n*(g*x+
f)^3/g-1/3*b*(-d*g+e*f)^3*n*ln(e*x+d)/e^3/g+1/3*(g*x+f)^3*(a+b*ln(c*(e*x+d
)^n))/g
```

3.37.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx$$
$$= \frac{6bd^2g(-3ef+dg)n\log(d+ex)+e(x(6ae^2(3f^2+3fgx+g^2x^2)-bn(6d^2g^2-3deg(6f+gx)+e^2(18f^2}{18e^3}$$

input
```
Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(6*b*d^2*g*(-3*e*f + d*g)*n*Log[d + e*x] + e*(x*(6*a*e^2*(3*f^2 + 3*f*g*x
+ g^2*x^2) - b*n*(6*d^2*g^2 - 3*d*e*g*(6*f + g*x) + e^2*(18*f^2 + 9*f*g*x
+ 2*g^2*x^2))) + 6*b*e*(3*d*f^2 + e*x*(3*f^2 + 3*f*g*x + g^2*x^2))*Log[c*(
d + e*x)^n]))/(18*e^3)
```

3.37.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(f+gx)^3 (a+b\log (c(d+ex)^n))}{3g}-\frac{ben\int \frac{(f+gx)^3}{d+ex}dx}{3g}$$

$$\Big\downarrow \text{49}$$

$$\frac{(f+gx)^3 (a+b\log (c(d+ex)^n))}{3g}-\frac{ben\int \left(\frac{(ef-dg)^3}{e^3(d+ex)}+\frac{g(ef-dg)^2}{e^3}+\frac{g(f+gx)(ef-dg)}{e^2}+\frac{g(f+gx)^2}{e}\right)dx}{3g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^3 (a+b\log (c(d+ex)^n))}{3g}-\frac{ben\left(\frac{(ef-dg)^3\log(d+ex)}{e^4}+\frac{gx(ef-dg)^2}{e^3}+\frac{(f+gx)^2(ef-dg)}{2e^2}+\frac{(f+gx)^3}{3e}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/3*(b*e*n*((g*(e*f - d*g)^2*x)/e^3 + ((e*f - d*g)*(f + g*x)^2)/(2*e^2) +
 (f + g*x)^3/(3*e) + ((e*f - d*g)^3*Log[d + e*x])/e^4))/g + ((f + g*x)^3*(
a + b*Log[c*(d + e*x)^n]))/(3*g)
```

3.37.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.37.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(110) = 220$.

Time = 0.85 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.21

method	result
parallelrisch	$\frac{6x^3 \ln(c(ex+d)^n)bd\,e^3 g^2 - 2x^3 bd\,e^3 g^2 n + 6x^3 ad\,e^3 g^2 + 18x^2 \ln(c(ex+d)^n)bd\,e^3 fg + 3x^2 b\,d^2 e^2 g^2 n - 9x^2 bd\,e^3 fgn + 6\ln(ex+d)b\,d^4}{}$
risch	$\frac{a\,g^2 x^3}{3} + a\,f^2 x - \frac{g^2 bn\,x^3}{9} + ga f\,x^2 + g\ln(c)\,bf\,x^2 - \frac{\ln(ex+d)b\,f^3 n}{3g} - \frac{ig^2 \pi b\,x^3 \operatorname{csgn}(ic(ex+d)^n)^3}{6} + \frac{(gx+}{}$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*x^3*ln(c*(e*x+d)^n)*b*d*e^3*g^2-2*x^3*b*d*e^3*g^2*n+6*x^3*a*d*e^3*
g^2+18*x^2*ln(c*(e*x+d)^n)*b*d*e^3*f*g+3*x^2*b*d^2*e^2*g^2*n-9*x^2*b*d*e^3
*f*g*n+6*ln(e*x+d)*b*d^4*g^2*n-18*ln(e*x+d)*b*d^3*e*f*g*n+36*ln(e*x+d)*b*d
^2*e^2*f^2*n+18*x^2*a*d*e^3*f*g+18*x*ln(c*(e*x+d)^n)*b*d*e^3*f^2-6*x*b*d^3
*e*g^2*n+18*x*b*d^2*e^2*f*g*n-18*x*b*d*e^3*f^2*n+18*x*a*d*e^3*f^2-18*ln(c*
(e*x+d)^n)*b*d^2*e^2*f^2)/d/e^3
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(110) = 220$.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.84

$$\int (f+gx)^2(a+b\log(c(d+ex)^n))\,dx = -\frac{2\,(be^3g^2n-3\,ae^3g^2)x^3-3\,(6\,ae^3fg-(3\,be^3fg-bde^2g^2)n)x^2-6\,(3\,ae^3f^2-(3\,be^3f^2-3\,bde^2fg+bd^2}{}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/18*(2*(b*e^3*g^2*n - 3*a*e^3*g^2)*x^3 - 3*(6*a*e^3*f*g - (3*b*e^3*f*g -
 b*d*e^2*g^2)*n)*x^2 - 6*(3*a*e^3*f^2 - (3*b*e^3*f^2 - 3*b*d*e^2*f*g + b*d
^2*e*g^2)*n)*x - 6*(b*e^3*g^2*n*x^3 + 3*b*e^3*f*g*n*x^2 + 3*b*e^3*f^2*n*x
+ (3*b*d*e^2*f^2 - 3*b*d^2*e*f*g + b*d^3*g^2)*n)*log(e*x + d) - 6*(b*e^3*g
^2*x^3 + 3*b*e^3*f*g*x^2 + 3*b*e^3*f^2*x)*log(c))/e^3
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(102) = 204$.

Time = 0.66 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.10

$$\int (f+gx)^2(a+b\log(c(d+ex)^n))\,dx = \begin{cases} af^2x+afgx^2+\frac{ag^2x^3}{3}+\frac{bd^3g^2\log(c(d+ex)^n)}{3e^3}-\frac{bd^2fg\log(c(d+ex)^n)}{e^2}-\frac{bd^2g^2nx}{3e^2}+\frac{bdf^2\log(c(d+ex)^n)}{e}+\frac{bdfgnx}{e}+\frac{bdg^2n}{6e} \\ (a+b\log(cd^n))\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right) \end{cases}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*d**3*g**2*log(c*(d +
e*x)**n)/(3*e**3) - b*d**2*f*g*log(c*(d + e*x)**n)/e**2 - b*d**2*g**2*n*x/
(3*e**2) + b*d*f**2*log(c*(d + e*x)**n)/e + b*d*f*g*n*x/e + b*d*g**2*n*x**
2/(6*e) - b*f**2*n*x + b*f**2*x*log(c*(d + e*x)**n) - b*f*g*n*x**2/2 + b*f
*g*x**2*log(c*(d + e*x)**n) - b*g**2*n*x**3/9 + b*g**2*x**3*log(c*(d + e*x
)**n)/3, Ne(e, 0)), ((a + b*log(c*d**n))*(f**2*x + f*g*x**2 + g**2*x**3/3)
, True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.56

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx$$
$$= \frac{1}{3}bg^2x^3\log((ex+d)^nc) + \frac{1}{3}ag^2x^3 - bef^2n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + \frac{1}{18}beg^2n\left(\frac{6d^3\log(ex+d)}{e^4} - \frac{2e^2x^3 - 3dex^2 + 6d^2x}{e^3}\right) - \frac{1}{2}befgn\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2dx}{e^2}\right) + bfgx^2\log((ex+d)^nc) + afgx^2 + bf^2x\log((ex+d)^nc) + af^2x$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/3*b*g^2*x^3*log((e*x + d)^n*c) + 1/3*a*g^2*x^3 - b*e*f^2*n*(x/e - d*log(
e*x + d)/e^2) + 1/18*b*e*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*
e*x^2 + 6*d^2*x)/e^3) - 1/2*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2
*d*x)/e^2) + b*f*g*x^2*log((e*x + d)^n*c) + a*f*g*x^2 + b*f^2*x*log((e*x +
 d)^n*c) + a*f^2*x
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(110) = 220$.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.53

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx = \frac{(ex+d)bf^2n\log(ex+d)}{e} + \frac{(ex+d)^2bfgn\log(ex+d)}{e^2} - \frac{2(ex+d)bdfgn\log(ex+d)}{e^2} + \frac{(ex+d)^3bg^2n\log(ex+d)}{3e^3} - \frac{(ex+d)^2bdg^2n\log(ex+d)}{e^3} + \frac{(ex+d)bd^2g^2n\log(ex+d)}{e^3} - \frac{(ex+d)bf^2n}{e} - \frac{(ex+d)^2bfgn}{2e^2} + \frac{2(ex+d)bdfgn}{e^2} - \frac{(ex+d)^3bg^2n}{9e^3} + \frac{(ex+d)^2bdg^2n}{2e^3} - \frac{(ex+d)bd^2g^2n}{e^3} + \frac{(ex+d)bf^2\log(c)}{e} + \frac{(ex+d)^2bfg\log(c)}{e^2} - \frac{2(ex+d)bdfg\log(c)}{e^2} + \frac{(ex+d)^3bg^2\log(c)}{3e^3} - \frac{(ex+d)^2bdg^2\log(c)}{e^3} + \frac{(ex+d)bd^2g^2\log(c)}{e^3} + \frac{(ex+d)af^2}{e} + \frac{(ex+d)^2afg}{e^2} - \frac{2(ex+d)adfg}{e^2} + \frac{(ex+d)^3ag^2}{3e^3} - \frac{(ex+d)^2adg^2}{e^3} + \frac{(ex+d)ad^2g^2}{e^3}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

3.37. $\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx$

output
```
(e*x + d)*b*f^2*n*log(e*x + d)/e + (e*x + d)^2*b*f*g*n*log(e*x + d)/e^2 -
2*(e*x + d)*b*d*f*g*n*log(e*x + d)/e^2 + 1/3*(e*x + d)^3*b*g^2*n*log(e*x +
 d)/e^3 - (e*x + d)^2*b*d*g^2*n*log(e*x + d)/e^3 + (e*x + d)*b*d^2*g^2*n*l
og(e*x + d)/e^3 - (e*x + d)*b*f^2*n/e - 1/2*(e*x + d)^2*b*f*g*n/e^2 + 2*(e
*x + d)*b*d*f*g*n/e^2 - 1/9*(e*x + d)^3*b*g^2*n/e^3 + 1/2*(e*x + d)^2*b*d*
g^2*n/e^3 - (e*x + d)*b*d^2*g^2*n/e^3 + (e*x + d)*b*f^2*log(c)/e + (e*x +
d)^2*b*f*g*log(c)/e^2 - 2*(e*x + d)*b*d*f*g*log(c)/e^2 + 1/3*(e*x + d)^3*b
*g^2*log(c)/e^3 - (e*x + d)^2*b*d*g^2*log(c)/e^3 + (e*x + d)*b*d^2*g^2*log
(c)/e^3 + (e*x + d)*a*f^2/e + (e*x + d)^2*a*f*g/e^2 - 2*(e*x + d)*a*d*f*g/
e^2 + 1/3*(e*x + d)^3*a*g^2/e^3 - (e*x + d)^2*a*d*g^2/e^3 + (e*x + d)*a*d^
2*g^2/e^3
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx$$
$$= x^2\left(\frac{g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{2\,e}-\frac{d\,g^2\,(3\,a-b\,n)}{6\,e}\right)$$
$$+x\left(\frac{3\,a\,e\,f^2-3\,b\,e\,f^2\,n+6\,a\,d\,f\,g}{3\,e}-\frac{d\left(\frac{g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^2\,(3\,a-b\,n)}{3\,e}\right)}{e}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)\left(b\,f^2\,x+b\,f\,g\,x^2+\frac{b\,g^2\,x^3}{3}\right)+\frac{g^2\,x^3\,(3\,a-b\,n)}{9}$$
$$+\frac{\ln(d+e\,x)\,\left(b\,n\,d^3\,g^2-3\,b\,n\,d^2\,e\,f\,g+3\,b\,n\,d\,e^2\,f^2\right)}{3\,e^3}$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x^2*((g*(a*d*g + 2*a*e*f - b*e*f*n))/(2*e) - (d*g^2*(3*a - b*n))/(6*e)) +
x*((3*a*e*f^2 - 3*b*e*f^2*n + 6*a*d*f*g)/(3*e) - (d*((g*(a*d*g + 2*a*e*f -
 b*e*f*n))/e - (d*g^2*(3*a - b*n))/(3*e)))/e) + log(c*(d + e*x)^n)*((b*g^2
*x^3)/3 + b*f^2*x + b*f*g*x^2) + (g^2*x^3*(3*a - b*n))/9 + (log(d + e*x)*(
b*d^3*g^2*n + 3*b*d*e^2*f^2*n - 3*b*d^2*e*f*g*n))/(3*e^3)
```

3.38 $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = -\frac{b(ef-dg)nx}{2e}-\frac{bn(f+gx)^2}{4g}-\frac{b(ef-dg)^2n\log(d+ex)}{2e^2g}+\frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}$$

output
```
-1/2*b*(-d*g+e*f)*n*x/e-1/4*b*n*(g*x+f)^2/g-1/2*b*(-d*g+e*f)^2*n*ln(e*x+d)
/e^2/g+1/2*(g*x+f)^2*(a+b*ln(c*(e*x+d)^n))/g
```

3.38.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = afx-bfnx+\frac{1}{2}agx^2-\frac{1}{2}bgn\left(-\frac{dx}{e}+\frac{x^2}{2}+\frac{d^2\log(d+ex)}{e^2}\right)+\frac{1}{2}bgx^2\log\left(c(d+ex)^n\right)+\frac{bf(d+ex)\log\left(c(d+ex)^n\right)}{e}$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
a*f*x - b*f*n*x + (a*g*x^2)/2 - (b*g*n*(-((d*x)/e) + x^2/2 + (d^2*Log[d +
e*x])/e^2))/2 + (b*g*x^2*Log[c*(d + e*x)^n])/2 + (b*f*(d + e*x)*Log[c*(d +
 e*x)^n])/e
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)dx$$

$$\Big\downarrow \ 2842$$

$$\frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}-\frac{ben\int\frac{(f+gx)^2}{d+ex}dx}{2g}$$

$$\Big\downarrow \ 49$$

$$\frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}-\frac{ben\int\left(\frac{(ef-dg)^2}{e^2(d+ex)}+\frac{g(ef-dg)}{e^2}+\frac{g(f+gx)}{e}\right)dx}{2g}$$

$$\Big\downarrow \ 2009$$

$$\frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}-\frac{ben\left(\frac{(ef-dg)^2\log(d+ex)}{e^3}+\frac{gx(ef-dg)}{e^2}+\frac{(f+gx)^2}{2e}\right)}{2g}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/2*(b*e*n*((g*(e*f - d*g)*x)/e^2 + (f + g*x)^2/(2*e) + ((e*f - d*g)^2*Lo
g[d + e*x])/e^3))/g + ((f + g*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*g)
```

3.38.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.38.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

method	result
parts	$a\left(\frac{1}{2}g\,x^{2}+fx\right)+b\Big(f\ln\left(c(ex+d)^{n}\right)x-fnx+\frac{fnd\ln(ex+d)}{e}+\frac{x^{2}g\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{gn\,x^{2}}{4}-$ $n\,d^{2}g$
norman	$\left(-\frac{1}{4}bgn+\frac{1}{2}ag\right)x^{2}+bfx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{(dgbn-2befn+2aef)x}{2e}+\frac{bg\,x^{2}\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-$ $n\left(b\,d^{2}g-2b\right.$
default	$afx+\frac{ag\,x^{2}}{2}+bf\ln\left(c(ex+d)^{n}\right)x-bfnx+\frac{bfnd\ln(ex+d)}{e}+\frac{bg\,x^{2}\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{bgn\,x^{2}}{4}-$ $nb\,d^{2}g$
parallelrisch	$-\frac{-2x^{2}\ln(c(ex+d)^{n})b\,e^{2}g+b\,e^{2}gn\,x^{2}+2\ln(ex+d)b\,d^{2}gn-8\ln(ex+d)bdefn-2a\,e^{2}g\,x^{2}-4x\ln(c(ex+d)^{n})b\,e^{2}f-2bdegnx+4b\,e}{4e^{2}}$
risch	$\frac{bx(gx+2f)\ln((ex+d)^{n})}{2}-\frac{i\pi bfx\ \mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^{n})\,\mathrm{csgn}(ic(ex+d)^{n})}{2}-\frac{i\pi bfx\,\mathrm{csgn}(ic(ex+d)^{n})^{3}}{2}-$ $i\pi bg\,x^{2}\,\mathrm{csgn}(ic$

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*g*x^2+f*x)+b*(f*ln(c*(e*x+d)^n)*x-f*n*x+f/e*n*d*ln(e*x+d)+1/2*x^2*g
*ln(c*exp(n*ln(e*x+d)))-1/4*g*n*x^2-1/2*n*d^2*g/e^2*ln(e*x+d)+1/2*d*g*n/e*
x)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = -\frac{(be^2gn-2\,ae^2g)x^2-2\,(2\,ae^2f-(2\,be^2f-bdeg)n)x-2\,(be^2gnx^2+2\,be^2fnx+(2\,bdef-bd^2g)n)\log}{4\,e^2}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/4*((b*e^2*g*n - 2*a*e^2*g)*x^2 - 2*(2*a*e^2*f - (2*b*e^2*f - b*d*e*g)*n
)*x - 2*(b*e^2*g*n*x^2 + 2*b*e^2*f*n*x + (2*b*d*e*f - b*d^2*g)*n)*log(e*x
+ d) - 2*(b*e^2*g*x^2 + 2*b*e^2*f*x)*log(c))/e^2
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx$$
$$= \begin{cases} afx+\frac{agx^2}{2}-\frac{bd^2g\log\left(c(d+ex)^n\right)}{2e^2}+\frac{bdf\log\left(c(d+ex)^n\right)}{e}+\frac{bdgnx}{2e}-bfnx+bfx\log\left(c(d+ex)^n\right)-\frac{bgnx^2}{4}+\frac{bgx^2\log(c}{2} \\ \left(a+b\log\left(cd^n\right)\right)\left(fx+\frac{gx^2}{2}\right) \end{cases}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f*x + a*g*x**2/2 - b*d**2*g*log(c*(d + e*x)**n)/(2*e**2) + b*
d*f*log(c*(d + e*x)**n)/e + b*d*g*n*x/(2*e) - b*f*n*x + b*f*x*log(c*(d + e
*x)**n) - b*g*n*x**2/4 + b*g*x**2*log(c*(d + e*x)**n)/2, Ne(e, 0)), ((a +
b*log(c*d**n))*(f*x + g*x**2/2), True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int (f+gx)(a+b\log(c(d+ex)^n))\,dx = -befn\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) - \frac{1}{4}begn\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right) + \frac{1}{2}bgx^2\log\left((ex+d)^nc\right) + \frac{1}{2}agx^2 + bfx\log\left((ex+d)^nc\right) + afx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
-b*e*f*n*(x/e - d*log(e*x + d)/e^2) - 1/4*b*e*g*n*(2*d^2*log(e*x + d)/e^3
+ (e*x^2 - 2*d*x)/e^2) + 1/2*b*g*x^2*log((e*x + d)^n*c) + 1/2*a*g*x^2 + b*
f*x*log((e*x + d)^n*c) + a*f*x
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(83) = 166.

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.01

$$\int (f+gx)(a+b\log(c(d+ex)^n))\,dx = \frac{(ex+d)bfn\log(ex+d)}{e} + \frac{(ex+d)^2bgn\log(ex+d)}{2e^2} - \frac{(ex+d)bdgn\log(ex+d)}{e^2} - \frac{(ex+d)bfn}{e} - \frac{(ex+d)^2bgn}{4e^2} + \frac{(ex+d)bdgn}{e^2} + \frac{(ex+d)bf\log(c)}{e} + \frac{(ex+d)^2bg\log(c)}{2e^2} - \frac{(ex+d)bdg\log(c)}{e^2} + \frac{(ex+d)af}{e} + \frac{(ex+d)^2ag}{2e^2} - \frac{(ex+d)adg}{e^2}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
(e*x + d)*b*f*n*log(e*x + d)/e + 1/2*(e*x + d)^2*b*g*n*log(e*x + d)/e^2 -
(e*x + d)*b*d*g*n*log(e*x + d)/e^2 - (e*x + d)*b*f*n/e - 1/4*(e*x + d)^2*b
*g*n/e^2 + (e*x + d)*b*d*g*n/e^2 + (e*x + d)*b*f*log(c)/e + 1/2*(e*x + d)^
2*b*g*log(c)/e^2 - (e*x + d)*b*d*g*log(c)/e^2 + (e*x + d)*a*f/e + 1/2*(e*x
 + d)^2*a*g/e^2 - (e*x + d)*a*d*g/e^2
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int (f+gx)\left(a+b\log \left(c(d+ex)^n\right)\right)\,dx = x\left(\frac{2\,a\,d\,g+2\,a\,e\,f-2\,b\,e\,f\,n}{2\,e}-\frac{d\,g\,\left(2\,a-b\,n\right)}{2\,e}\right) + \ln\left(c\,{\left(d+e\,x\right)}^n\right)\,\left(\frac{b\,g\,x^2}{2}+b\,f\,x\right) - \frac{\ln\left(d+e\,x\right)\,\left(b\,d^2\,g\,n-2\,b\,d\,e\,f\,n\right)}{2\,e^2} + \frac{g\,x^2\,\left(2\,a-b\,n\right)}{4}$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x*((2*a*d*g + 2*a*e*f - 2*b*e*f*n)/(2*e) - (d*g*(2*a - b*n))/(2*e)) + log(
c*(d + e*x)^n)*(b*f*x + (b*g*x^2)/2) - (log(d + e*x)*(b*d^2*g*n - 2*b*d*e*
f*n))/(2*e^2) + (g*x^2*(2*a - b*n))/4
```

3.39 $\int (a+b\log(c(d+ex)^n))\,dx$

3.39.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (a+b\log(c(d+ex)^n))\,dx = ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}$$

output
```
a*x-b*n*x+b*(e*x+d)*ln(c*(e*x+d)^n)/e
```

3.39.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d+ex)^n))\,dx = ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}$$

input
```
Integrate[a + b*Log[c*(d + e*x)^n],x]
```

output
```
a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

3.39.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b(d+ex)\log(c(d+ex)^n)}{e}-bnx$$

input
```
Int[a + b*Log[c*(d + e*x)^n],x]
```

output
```
a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.39.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

method	result
default	$ax+b\ln\left(c(ex+d)^n\right)x-bnx+\frac{bnd\ln(ex+d)}{e}$
parts	$ax+b\ln\left(c(ex+d)^n\right)x-bnx+\frac{bnd\ln(ex+d)}{e}$
norman	$(-bn+a)\,x+bx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{bnd\ln(ex+d)}{e}$
parallelrisch	$\frac{b(x\ln(c(ex+d)^n)den-xden^2+\ln(c(ex+d)^n)d^2n)}{den}+ax$
risch	$ax+bx\ln\left((ex+d)^n\right)-\frac{ib\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)}{2}+\frac{ib\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2}{2}+ib\pi$

input
```
int(a+b*ln(c*(e*x+d)^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*ln(c*(e*x+d)^n)*x-b*n*x+b/e*n*d*ln(e*x+d)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a+b\log (c(d+ex)^n))\,dx = \frac{bex\log (c) - (ben - ae)x + (benx + bdn)\log (ex+d)}{e}$$

input
```
integrate(a+b*log(c*(e*x+d)^n),x, algorithm="fricas")
```

output
```
(b*e*x*log(c) - (b*e*n - a*e)*x + (b*e*n*x + b*d*n)*log(e*x + d))/e
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a+b\log (c(d+ex)^n))\,dx = ax + b\left(\begin{cases} \frac{d\log (c(d+ex)^n)}{e} - nx + x\log (c(d+ex)^n) & \text{for } e \neq 0 \\ x\log (cd^n) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*ln(c*(e*x+d)**n),x)
```

output
```
a*x + b*Piecewise((d*log(c*(d + e*x)**n)/e - n*x + x*log(c*(d + e*x)**n),
Ne(e, 0)), (x*log(c*d**n), True))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a+b\log (c(d+ex)^n))\,dx = -ben\left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right) + bx\log ((ex+d)^n c) + ax$$

input
```
integrate(a+b*log(c*(e*x+d)^n),x, algorithm="maxima")
```

output
```
-b*e*n*(x/e - d*log(e*x + d)/e^2) + b*x*log((e*x + d)^n*c) + a*x
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int (a+b\log(c(d+ex)^n))\,dx = \left(\frac{(ex+d)n\log(ex+d)}{e} - \frac{(ex+d)n}{e} + \frac{(ex+d)\log(c)}{e}\right)b + ax$$

input
```
integrate(a+b*log(c*(e*x+d)^n),x, algorithm="giac")
```

output
```
((e*x + d)*n*log(e*x + d)/e - (e*x + d)*n/e + (e*x + d)*log(c)/e)*b + a*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int (a+b\log(c(d+ex)^n))\,dx = x\,(a-b\,n) + b\,x\,\ln\left(c\,(d+e\,x)^n\right) + \frac{b\,d\,n\,\ln(d+e\,x)}{e}$$

input
```
int(a + b*log(c*(d + e*x)^n),x)
```

output
```
x*(a - b*n) + b*x*log(c*(d + e*x)^n) + (b*d*n*log(d + e*x))/e
```

3.40 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx$

3.40.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output
```
(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g+b*n*polylog(2,-g*(e*x+d)/
(-d*g+e*f))/g
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyL
og[2, (g*(d + e*x))/(-(e*f) + d*g)])/g
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx$$

$$\Big\downarrow \ 2841$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \ 2840$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{bn\int\frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \ 2838$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyL
og[2, -((g*(d + e*x))/(e*f - d*g))])/g
```

3.40.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.40.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(gx+f)}{g} - \frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g} - \frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g} + \frac{\left(-\frac{ib\pi\,\mathrm{csgn}(ic(ex+d)^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}}{2}\right.}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)*ln(g*x+f)/g-b/g*n*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b/g
*n*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*
(e*x+d)^n)^3+b*ln(c)+a)*ln(g*x+f)/g
```

3.40.5 Fricas [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{f + gx}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{gx + f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.40.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.40.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(g*x + f), x) + a*log(g*x + f)/g
```

3.40.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x), x)
```

3.41 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx$

3.41.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx=\frac{ben\log(d+ex)}{g(ef-dg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}-\frac{ben\log(f+gx)}{g(ef-dg)}$$

output
```
b*e*n*ln(e*x+d)/g/(-d*g+e*f)+(-a-b*ln(c*(e*x+d)^n))/g/(g*x+f)-b*e*n*ln(g*x
+f)/g/(-d*g+e*f)
```

3.41.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx=\frac{-\frac{a+b\log(c(d+ex)^n)}{f+gx}+\frac{ben(\log(d+ex)-\log(f+gx))}{ef-dg}}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

output
```
(-((a + b*Log[c*(d + e*x)^n])/(f + g*x)) + (b*e*n*(Log[d + e*x] - Log[f +
g*x]))/(e*f - d*g))/g
```

3.41.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2842, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^2} \, dx$$

$$\Big\downarrow \ \textcolor{blue}{2842}$$

$$\frac{ben\int \frac{1}{(d+ex)(f+gx)}dx}{g}-\frac{a+b\log \left(c(d+ex)^n\right)}{g(f+gx)}$$

$$\Big\downarrow \ \textcolor{blue}{47}$$

$$\frac{ben\left(\frac{e\int \frac{1}{d+ex}dx}{ef-dg}-\frac{g\int \frac{1}{f+gx}dx}{ef-dg}\right)}{g}-\frac{a+b\log \left(c(d+ex)^n\right)}{g(f+gx)}$$

$$\Big\downarrow \ \textcolor{blue}{16}$$

$$\frac{ben\left(\frac{\log(d+ex)}{ef-dg}-\frac{\log(f+gx)}{ef-dg}\right)}{g}-\frac{a+b\log \left(c(d+ex)^n\right)}{g(f+gx)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

output
```
-((a + b*Log[c*(d + e*x)^n])/(g*(f + g*x))) + (b*e*n*(Log[d + e*x]/(e*f -
d*g) - Log[f + g*x]/(e*f - d*g)))/g
```

3.41.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.41.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

method	result
parallelrisch	$-\frac{\ln(ex+d)xb\,e^2gn-\ln(gx+f)xb\,e^2gn+\ln(ex+d)b\,e^2fn-\ln(gx+f)b\,e^2fn+\ln(c(ex+d)^n)bdeg-\ln(c(ex+d)^n)b\,e^2f+adeg-a\,e^2}{(dg-ef)(gx+f)eg}$
risch	$-\frac{b\ln((ex+d)^n)}{g(gx+f)}-\frac{i\pi\ \mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2bdg+i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2bdg+i\pi bef\,\mathrm{csgn}(ic(ex+d)^n)^3-i}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-(ln(e*x+d)*x*b*e^2*g*n-ln(g*x+f)*x*b*e^2*g*n+ln(e*x+d)*b*e^2*f*n-ln(g*x+f
)*b*e^2*f*n+ln(c*(e*x+d)^n)*b*d*e*g-ln(c*(e*x+d)^n)*b*e^2*f+a*d*e*g-a*e^2*
f)/(d*g-e*f)/(g*x+f)/e/g
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{aef-adg-(begnx+bdgn)\log\left(ex+d\right)+(begnx+befn)\log\left(gx+f\right)+(bef-bdg)\log\left(c\right)}{ef^2g-dfg^2+(efg^2-dg^3)x}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
-(a*e*f - a*d*g - (b*e*g*n*x + b*d*g*n)*log(e*x + d) + (b*e*g*n*x + b*e*f*
n)*log(g*x + f) + (b*e*f - b*d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 -
d*g^3)*x)
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(61) = 122.

Time = 3.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \begin{cases} \frac{ax+\frac{bd\log\left(c(d+ex)^n\right)}{e}-bnx+bx\log\left(c(d+ex)^n\right)}{f^2} \\ -\frac{a}{fg+g^2x}-\frac{bn}{fg+g^2x}-\frac{b\log\left(c\left(\frac{ef}{g}+ex\right)^n\right)}{fg+g^2x} \\ -\frac{adg}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{aef}{dfg^2+dg^3x-ef^2g-efg^2x}-\frac{bdg\log\left(c(d+ex)^n\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{befn\log\left(\frac{f}{g}+x\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{begnx\log\left(\frac{f}{g}\right.}{dfg^2+dg^3x-ef^2} \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

output
```
Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)*
*n))/f**2, Eq(g, 0)), (-a/(f*g + g**2*x) - b*n/(f*g + g**2*x) - b*log(c*(e
*f/g + e*x)**n)/(f*g + g**2*x), Eq(d, e*f/g)), (-a*d*g/(d*f*g**2 + d*g**3*
x - e*f**2*g - e*f*g**2*x) + a*e*f/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g
**2*x) - b*d*g*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g
**2*x) + b*e*f*n*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x
) + b*e*g*n*x*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x) -
 b*e*g*x*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x)
, True))
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = ben\left(\frac{\log(ex+d)}{efg-dg^2}-\frac{\log(gx+f)}{efg-dg^2}\right) - \frac{b\log\left((ex+d)^nc\right)}{g^2x+fg}-\frac{a}{g^2x+fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) - b*lo
g((e*x + d)^n*c)/(g^2*x + f*g) - a/(g^2*x + f*g)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log\left(ex+d\right)}{efg-dg^2} - \frac{ben\log\left(gx+f\right)}{efg-dg^2} - \frac{bn\log\left(ex+d\right)}{g^2x+fg} - \frac{b\log\left(c\right)+a}{g^2x+fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
b*e*n*log(e*x + d)/(e*f*g - d*g^2) - b*e*n*log(g*x + f)/(e*f*g - d*g^2) -
b*n*log(e*x + d)/(g^2*x + f*g) - (b*log(c) + a)/(g^2*x + f*g)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{a}{x\,g^2+f\,g} - \frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{g\,\left(f+g\,x\right)} + \frac{b\,e\,n\,\mathrm{atan}\left(\frac{e\,f\,2{}\mathrm{i}+e\,g\,x\,2{}\mathrm{i}}{d\,g-e\,f}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{g\,\left(d\,g-e\,f\right)}$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^2,x)
```

output
```
(b*e*n*atan((e*f*2i + e*g*x*2i)/(d*g - e*f) + 1i)*2i)/(g*(d*g - e*f)) - (b
*log(c*(d + e*x)^n))/(g*(f + g*x)) - a/(f*g + g^2*x)
```

3.42 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^3}\,dx$

3.42.1 Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = \frac{ben}{2g(ef-dg)(f+gx)}+\frac{be^2n\log(d+ex)}{2g(ef-dg)^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}-\frac{be^2n\log(f+gx)}{2g(ef-dg)^2}$$

output
```
1/2*b*e*n/g/(-d*g+e*f)/(g*x+f)+1/2*b*e^2*n*ln(e*x+d)/g/(-d*g+e*f)^2+1/2*(-
a-b*ln(c*(e*x+d)^n))/g/(g*x+f)^2-1/2*b*e^2*n*ln(g*x+f)/g/(-d*g+e*f)^2
```

3.42.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = -\frac{a+b\log\left(c(d+ex)^n\right)-\frac{ben(f+gx)(ef-dg+e(f+gx)\log(d+ex)-e(f+gx)\log(f+gx))}{(ef-dg)^2}}{2g(f+gx)^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n] - (b*e*n*(f + g*x)*(e*f - d*g + e*(f + g*x)
*Log[d + e*x] - e*(f + g*x)*Log[f + g*x]))/(e*f - d*g)^2)/(g*(f + g*x)^2)
```

3.42.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx$$

$$\Big\downarrow \ 2842$$

$$\frac{ben\int \frac{1}{(d+ex)(f+gx)^2}dx}{2g}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}$$

$$\Big\downarrow \ 54$$

$$\frac{ben\int \left(\frac{e^2}{(ef-dg)^2(d+ex)}-\frac{ge}{(ef-dg)^2(f+gx)}-\frac{g}{(ef-dg)(f+gx)^2}\right)dx}{2g}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{ben\left(\frac{1}{(f+gx)(ef-dg)}+\frac{e\log(d+ex)}{(ef-dg)^2}-\frac{e\log(f+gx)}{(ef-dg)^2}\right)}{2g}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n])/(g*(f + g*x)^2) + (b*e*n*(1/((e*f - d*g)*(
f + g*x)) + (e*Log[d + e*x])/(e*f - d*g)^2 - (e*Log[f + g*x])/(e*f - d*g)^
2))/(2*g)
```

3.42.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(107) = 214$.

Time = 1.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
parallelrisch	$\frac{-xbd\,e^2g^3n+xb\,e^3f\,g^2n+b\,e^3f^2gn+2ad\,e^2f\,g^2-\ln(c(ex+d)^n)b\,d^2e\,g^3-\ln(c(ex+d)^n)b\,e^3f^2g-bd\,e^2f\,g^2n+\ln(ex+d)b\,e^3f^2gn-}{2(g}$
risch	$-\frac{b\ln((ex+d)^n)}{2g(gx+f)^2}-\frac{2\ln(gx+f)b\,e^2f^2n-2\ln(-ex-d)b\,e^2f^2n+2a\,e^2f^2+2\ln(gx+f)b\,e^2g^2n\,x^2-2\ln(-ex-d)b\,e^2g^2n\,x^2-4\ln(}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x*b*d*e^2*g^3*n+x*b*e^3*f*g^2*n+b*e^3*f^2*g*n+2*a*d*e^2*f*g^2-ln(c*(
e*x+d)^n)*b*d^2*e*g^3-ln(c*(e*x+d)^n)*b*e^3*f^2*g-b*d*e^2*f*g^2*n+ln(e*x+d
)*b*e^3*f^2*g*n-ln(g*x+f)*b*e^3*f^2*g*n+2*ln(c*(e*x+d)^n)*b*d*e^2*f*g^2+ln
(e*x+d)*x^2*b*e^3*g^3*n-ln(g*x+f)*x^2*b*e^3*g^3*n+2*ln(e*x+d)*x*b*e^3*f*g^
2*n-2*ln(g*x+f)*x*b*e^3*f*g^2*n-a*d^2*e*g^3-a*e^3*f^2*g)/(d^2*g^2-2*d*e*f*
g+e^2*f^2)/(g*x+f)^2/e/g^2
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.45

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = -\frac{ae^2f^2-2\,adefg+ad^2g^2-(be^2fg-bdeg^2)nx-(be^2f^2-bdefg)n-(be^2g^2nx^2+2\,be^2fgnx+(2\,bdef}{2\,(e^2f^4g-2\,def^3g^2+d^2f^2g^3+(e^2f^2g^3-2}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*e^2*f^2 - 2*a*d*e*f*g + a*d^2*g^2 - (b*e^2*f*g - b*d*e*g^2)*n*x -
(b*e^2*f^2 - b*d*e*f*g)*n - (b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + (2*b*d*e*
f*g - b*d^2*g^2)*n)*log(e*x + d) + (b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + b*
e^2*f^2*n)*log(g*x + f) + (b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*log(c))/(e
^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^3 + (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*
g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2*g^3 + d^2*f*g^4)*x)
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. $2(97) = 194$.

Time = 13.16 (sec) , antiderivative size = 1945, normalized size of antiderivative = 17.37

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**3,x)
```

output
```
Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)*
*n))/f**3, Eq(g, 0)), (-2*a/(4*f**2*g + 8*f*g**2*x + 4*g**3*x**2) - b*n/(4
*f**2*g + 8*f*g**2*x + 4*g**3*x**2) - 2*b*log(c*(e*f/g + e*x)**n)/(4*f**2*
g + 8*f*g**2*x + 4*g**3*x**2), Eq(d, e*f/g)), (-a*d**2*g**2/(2*d**2*f**2*g
**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g*
*3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**
2*g**3*x**2) + 2*a*d*e*f*g/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g*
*5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2
*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) - a*e**2*f**2/(2*d**
2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e
*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*
e**2*f**2*g**3*x**2) - b*d**2*g**2*log(c*(d + e*x)**n)/(2*d**2*f**2*g**3 +
 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x
- 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**
3*x**2) - b*d*e*f*g*n/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x*
*2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4
*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) + 2*b*d*e*f*g*log(c*(d +
e*x)**n)/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f*
*3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f
**3*g**2*x + 2*e**2*f**2*g**3*x**2) - b*d*e*g**2*n*x/(2*d**2*f**2*g**3 ...
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.49

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^3}\,dx$$
$$=\frac{1}{2}\,ben\left(\frac{e\log(ex+d)}{e^2f^2g-2\,defg^2+d^2g^3}-\frac{e\log(gx+f)}{e^2f^2g-2\,defg^2+d^2g^3}+\frac{1}{ef^2g-dfg^2+(efg^2-dg^3)x}\right)$$
$$-\frac{b\log((ex+d)^nc)}{2\,(g^3x^2+2\,fg^2x+f^2g)}-\frac{a}{2\,(g^3x^2+2\,fg^2x+f^2g)}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^3,x, algorithm="maxima")
```

output
```
1/2*b*e*n*(e*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - e*log(g*x
+ f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) + 1/(e*f^2*g - d*f*g^2 + (e*f*g^2
 - d*g^3)*x)) - 1/2*b*log((e*x + d)^n*c)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1
/2*a/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.79

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^3}\,dx=\frac{be^2n\log(ex+d)}{2\,(e^2f^2g-2\,defg^2+d^2g^3)}$$
$$-\frac{be^2n\log(gx+f)}{2\,(e^2f^2g-2\,defg^2+d^2g^3)}-\frac{bn\log(ex+d)}{2\,(g^3x^2+2\,fg^2x+f^2g)}$$
$$+\frac{begnx+befn-bef\log(c)+bdg\log(c)-aef+adg}{2\,(efg^3x^2-dg^4x^2+2\,ef^2g^2x-2\,dfg^3x+ef^3g-df^2g^2)}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^3,x, algorithm="giac")
```

output
```
1/2*b*e^2*n*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - 1/2*b*e^2*n
*log(g*x + f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - 1/2*b*n*log(e*x + d)/(
g^3*x^2 + 2*f*g^2*x + f^2*g) + 1/2*(b*e*g*n*x + b*e*f*n - b*e*f*log(c) + b
*d*g*log(c) - a*e*f + a*d*g)/(e*f*g^3*x^2 - d*g^4*x^2 + 2*e*f^2*g^2*x - 2*
d*f*g^3*x + e*f^3*g - d*f^2*g^2)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = \frac{b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,d^2\,g^3-2\,e^2\,f^2\,g}{2\,g\,{\left(d\,g-e\,f\right)}^2}+\frac{2\,e\,g\,x}{d\,g-e\,f}\right)}{g\,{\left(d\,g-e\,f\right)}^2} - \frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)} - \frac{\frac{a\,d\,g-a\,e\,f+b\,e\,f\,n}{d\,g-e\,f}+\frac{b\,e\,g\,n\,x}{d\,g-e\,f}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^3,x)
```

output
```
(b*e^2*n*atanh((2*d^2*g^3 - 2*e^2*f^2*g)/(2*g*(d*g - e*f)^2) + (2*e*g*x)/(
d*g - e*f)))/(g*(d*g - e*f)^2) - (b*log(c*(d + e*x)^n))/(2*g*(f^2 + g^2*x^
2 + 2*f*g*x)) - ((a*d*g - a*e*f + b*e*f*n)/(d*g - e*f) + (b*e*g*n*x)/(d*g
- e*f))/(2*f^2*g + 2*g^3*x^2 + 4*f*g^2*x)
```

3.43 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^4}\,dx$

3.43.1 Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \frac{ben}{6g(ef-dg)(f+gx)^2} + \frac{be^2n}{3g(ef-dg)^2(f+gx)} + \frac{be^3n\log(d+ex)}{3g(ef-dg)^3} - \frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3} - \frac{be^3n\log(f+gx)}{3g(ef-dg)^3}$$

output
```
1/6*b*e*n/g/(-d*g+e*f)/(g*x+f)^2+1/3*b*e^2*n/g/(-d*g+e*f)^2/(g*x+f)+1/3*b*
e^3*n*ln(e*x+d)/g/(-d*g+e*f)^3+1/3*(-a-b*ln(c*(e*x+d)^n))/g/(g*x+f)^3-1/3*
b*e^3*n*ln(g*x+f)/g/(-d*g+e*f)^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.78

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \frac{-2\left(a+b\log\left(c(d+ex)^n\right)\right) + \frac{ben(f+gx)\left((ef-dg)(3ef-dg+2egx)+2e^2(f+gx)^2\log(d+ex)-2e^2(f+gx)^2\log(f+gx)\right)}{(ef-dg)^3}}{6g(f+gx)^3}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^4,x]
```

output
```
(-2*(a + b*Log[c*(d + e*x)^n]) + (b*e*n*(f + g*x)*((e*f - d*g)*(3*e*f - d*
g + 2*e*g*x) + 2*e^2*(f + g*x)^2*Log[d + e*x] - 2*e^2*(f + g*x)^2*Log[f +
g*x]))/(e*f - d*g)^3)/(6*g*(f + g*x)^3)
```

3.43.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx$$

$$\Big\downarrow \ 2842$$

$$\frac{ben\int \frac{1}{(d+ex)(f+gx)^3}dx}{3g}-\frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3}$$

$$\Big\downarrow \ 54$$

$$\frac{ben\int \left(\frac{e^3}{(ef-dg)^3(d+ex)}-\frac{ge^2}{(ef-dg)^3(f+gx)}-\frac{ge}{(ef-dg)^2(f+gx)^2}-\frac{g}{(ef-dg)(f+gx)^3}\right)dx}{3g}-\frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{ben\left(\frac{e^2\log(d+ex)}{(ef-dg)^3}-\frac{e^2\log(f+gx)}{(ef-dg)^3}+\frac{e}{(f+gx)(ef-dg)^2}+\frac{1}{2(f+gx)^2(ef-dg)}\right)}{3g}-\frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^4,x]
```

output
```
-1/3*(a + b*Log[c*(d + e*x)^n])/(g*(f + g*x)^3) + (b*e*n*(1/(2*(e*f - d*g)
*(f + g*x)^2) + e/((e*f - d*g)^2*(f + g*x)) + (e^2*Log[d + e*x])/(e*f - d*
g)^3 - (e^2*Log[f + g*x])/(e*f - d*g)^3))/(3*g)
```

3.43.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(134) = 268$.

Time = 1.61 (sec) , antiderivative size = 455, normalized size of antiderivative = 3.23

method	result
parallelrisch	$-\frac{2\ln(c(ex+d)^n)b\,d^3e\,g^5-2\ln(c(ex+d)^n)b\,e^4f^3g^2+3b\,e^4f^3g^2n-6a\,d^2e^2f\,g^4+6ad\,e^3f^2g^3+6\ln(ex+d)x^2b\,e^4f\,g^4n-6\ln(gx+\ldots}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*ln(c*(e*x+d)^n)*b*d^3*e*g^5-2*ln(c*(e*x+d)^n)*b*e^4*f^3*g^2+3*b*e^
4*f^3*g^2*n-6*a*d^2*e^2*f*g^4+6*a*d*e^3*f^2*g^3+6*ln(e*x+d)*x^2*b*e^4*f*g^
4*n-6*ln(g*x+f)*x^2*b*e^4*f*g^4*n+6*ln(e*x+d)*x*b*e^4*f^2*g^3*n+2*x^2*b*e^
4*f*g^4*n+x*b*d^2*e^2*g^5*n+5*x*b*e^4*f^2*g^3*n-6*ln(c*(e*x+d)^n)*b*d^2*e^
2*f*g^4+6*ln(c*(e*x+d)^n)*b*d*e^3*f^2*g^3+2*ln(e*x+d)*x^3*b*e^4*g^5*n-2*ln
(g*x+f)*x^3*b*e^4*g^5*n+2*ln(e*x+d)*b*e^4*f^3*g^2*n-2*ln(g*x+f)*b*e^4*f^3*
g^2*n+b*d^2*e^2*f*g^4*n-4*b*d*e^3*f^2*g^3*n+2*a*d^3*e*g^5-2*a*e^4*f^3*g^2-
2*x^2*b*d*e^3*g^5*n-6*ln(g*x+f)*x*b*e^4*f^2*g^3*n-6*x*b*d*e^3*f*g^4*n)/(d^
3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/(g*x+f)^3/g^3/e
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(131) = 262.

Time = 0.32 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.60

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = -\frac{2\,ae^3f^3 - 6\,ade^2f^2g + 6\,ad^2efg^2 - 2\,ad^3g^3 - 2\,(be^3fg^2 - bde^2g^3)nx^2 - (5\,be^3f^2g - 6\,bde^2fg^2 + bd^2eg^3}{6\,(e^3f^6g - 3\,de^2f^5g}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a*e^3*f^3 - 6*a*d*e^2*f^2*g + 6*a*d^2*e*f*g^2 - 2*a*d^3*g^3 - 2*(b
*e^3*f*g^2 - b*d*e^2*g^3)*n*x^2 - (5*b*e^3*f^2*g - 6*b*d*e^2*f*g^2 + b*d^2
*e*g^3)*n*x - (3*b*e^3*f^3 - 4*b*d*e^2*f^2*g + b*d^2*e*f*g^2)*n - 2*(b*e^3
*g^3*n*x^3 + 3*b*e^3*f*g^2*n*x^2 + 3*b*e^3*f^2*g*n*x + (3*b*d*e^2*f^2*g -
3*b*d^2*e*f*g^2 + b*d^3*g^3)*n)*log(e*x + d) + 2*(b*e^3*g^3*n*x^3 + 3*b*e^
3*f*g^2*n*x^2 + 3*b*e^3*f^2*g*n*x + b*e^3*f^3*n)*log(g*x + f) + 2*(b*e^3*f
^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*log(c))/(e^3*f^6*g - 3
*d*e^2*f^5*g^2 + 3*d^2*e*f^4*g^3 - d^3*f^3*g^4 + (e^3*f^3*g^4 - 3*d*e^2*f^
2*g^5 + 3*d^2*e*f*g^6 - d^3*g^7)*x^3 + 3*(e^3*f^4*g^3 - 3*d*e^2*f^3*g^4 +
3*d^2*e*f^2*g^5 - d^3*f*g^6)*x^2 + 3*(e^3*f^5*g^2 - 3*d*e^2*f^4*g^3 + 3*d^
2*e*f^3*g^4 - d^3*f^2*g^5)*x)
```

3.43.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**4,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(131) = 262$.

Time = 0.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.13

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^4}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,e^2\log\left(ex + d\right)}{e^3f^3g - 3\,de^2f^2g^2 + 3\,d^2efg^3 - d^3g^4} - \frac{2\,e^2\log\left(gx + f\right)}{e^3f^3g - 3\,de^2f^2g^2 + 3\,d^2efg^3 - d^3g^4} + \frac{}{e^2f^4g - 2\,def^3g^2 + d}\right.$$
$$- \frac{b\log\left((ex + d)^n c\right)}{3\left(g^4x^3 + 3\,fg^3x^2 + 3\,f^2g^2x + f^3g\right)} - \frac{a}{3\left(g^4x^3 + 3\,fg^3x^2 + 3\,f^2g^2x + f^3g\right)}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/6*(2*e^2*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3
*g^4) - 2*e^2*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 -
d^3*g^4) + (2*e*g*x + 3*e*f - d*g)/(e^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^
3 + (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2
*g^3 + d^2*f*g^4)*x))*b*e*n - 1/3*b*log((e*x + d)^n*c)/(g^4*x^3 + 3*f*g^3*
x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*a/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x +
f^3*g)
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(131) = 262$.

Time = 0.31 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.84

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^4}\,dx = \frac{be^3n\log\left(ex + d\right)}{3\left(e^3f^3g - 3\,de^2f^2g^2 + 3\,d^2efg^3 - d^3g^4\right)}$$
$$- \frac{be^3n\log\left(gx + f\right)}{3\left(e^3f^3g - 3\,de^2f^2g^2 + 3\,d^2efg^3 - d^3g^4\right)} - \frac{bn\log\left(ex + d\right)}{3\left(g^4x^3 + 3\,fg^3x^2 + 3\,f^2g^2x + f^3g\right)}$$
$$+ \frac{2\,be^2g^2nx^2 + 5\,be^2fgnx - bdeg^2nx + 3\,be^2f^2n - bdefgn - 2\,be^2f^2\log\left(c\right) + 4\,bdefg\log\left(c\right) - 2\,bd^2g^2\,\mathrm{l}}{6\left(e^2f^2g^4x^3 - 2\,defg^5x^3 + d^2g^6x^3 + 3\,e^2f^3g^3x^2 - 6\,def^2g^4x^2 + 3\,d^2fg^5x^2 + 3\,e^2f^4g^2x - 6\,def^3g^3x + 3\right.}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^4,x, algorithm="giac")
```

output
```
1/3*b*e^3*n*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^
3*g^4) - 1/3*b*e^3*n*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f
*g^3 - d^3*g^4) - 1/3*b*n*log(e*x + d)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*
x + f^3*g) + 1/6*(2*b*e^2*g^2*n*x^2 + 5*b*e^2*f*g*n*x - b*d*e*g^2*n*x + 3*
b*e^2*f^2*n - b*d*e*f*g*n - 2*b*e^2*f^2*log(c) + 4*b*d*e*f*g*log(c) - 2*b*
d^2*g^2*log(c) - 2*a*e^2*f^2 + 4*a*d*e*f*g - 2*a*d^2*g^2)/(e^2*f^2*g^4*x^3
 - 2*d*e*f*g^5*x^3 + d^2*g^6*x^3 + 3*e^2*f^3*g^3*x^2 - 6*d*e*f^2*g^4*x^2 +
 3*d^2*f*g^5*x^2 + 3*e^2*f^4*g^2*x - 6*d*e*f^3*g^3*x + 3*d^2*f^2*g^4*x + e
^2*f^5*g - 2*d*e*f^4*g^2 + d^2*f^3*g^3)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.01

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \frac{2\,a\,d\,e\,f}{3\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{a\,d^2\,g}{3\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{3\,g\,{\left(f+g\,x\right)}^3}-\frac{a\,e^2\,f^2}{3\,g\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{5\,b\,e^2\,f\,n\,x}{6\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{b\,e^2\,g\,n\,x^2}{3\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{b\,d\,e\,f\,n}{6\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{b\,e^2\,f^2\,n}{2\,g\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{b\,d\,e\,g\,n\,x}{6\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{b\,e^3\,n\,\mathrm{atan}\left(\frac{d\,g\,1{}\mathrm{i}+e\,f\,1{}\mathrm{i}+e\,g\,x\,2{}\mathrm{i}}{d\,g-e\,f}\right)\,2{}\mathrm{i}}{3\,g\,{\left(d\,g-e\,f\right)}^3}$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^4,x)
```

output
```
(2*a*d*e*f)/(3*(f + g*x)^3*(d*g - e*f)^2) - (a*d^2*g)/(3*(f + g*x)^3*(d*g
- e*f)^2) - (b*log(c*(d + e*x)^n))/(3*g*(f + g*x)^3) - (a*e^2*f^2)/(3*g*(f
 + g*x)^3*(d*g - e*f)^2) + (b*e^3*n*atan((d*g*1i + e*f*1i + e*g*x*2i)/(d*g
 - e*f))*2i)/(3*g*(d*g - e*f)^3) + (5*b*e^2*f*n*x)/(6*(f + g*x)^3*(d*g - e
*f)^2) + (b*e^2*g*n*x^2)/(3*(f + g*x)^3*(d*g - e*f)^2) - (b*d*e*f*n)/(6*(f
 + g*x)^3*(d*g - e*f)^2) + (b*e^2*f^2*n)/(2*g*(f + g*x)^3*(d*g - e*f)^2) -
 (b*d*e*g*n*x)/(6*(f + g*x)^3*(d*g - e*f)^2)
```

3.44 $\int (f+gx)^3 (a+b\log(c(d+ex)^n))^2\,dx$

3.44.1 Optimal result

Integrand size = 24, antiderivative size = 365

$$\begin{aligned}
&\int (f+gx)^3 (a+b\log(c(d+ex)^n))^2\,dx \\
&= \frac{2b^2(ef-dg)^3n^2x}{e^3} + \frac{3b^2g(ef-dg)^2n^2(d+ex)^2}{4e^4} + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3}{9e^4} \\
&+ \frac{b^2g^3n^2(d+ex)^4}{32e^4} + \frac{b^2(ef-dg)^4n^2\log^2(d+ex)}{4e^4g} \\
&- \frac{2b(ef-dg)^3n(d+ex)(a+b\log(c(d+ex)^n))}{e^4} \\
&- \frac{3bg(ef-dg)^2n(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^4} \\
&- \frac{2bg^2(ef-dg)n(d+ex)^3(a+b\log(c(d+ex)^n))}{3e^4} \\
&- \frac{bg^3n(d+ex)^4(a+b\log(c(d+ex)^n))}{8e^4} \\
&- \frac{b(ef-dg)^4n\log(d+ex)(a+b\log(c(d+ex)^n))}{2e^4g} + \frac{(f+gx)^4(a+b\log(c(d+ex)^n))^2}{4g}
\end{aligned}$$

output
```
2*b^2*(-d*g+e*f)^3*n^2*x/e^3+3/4*b^2*g*(-d*g+e*f)^2*n^2*(e*x+d)^2/e^4+2/9*
b^2*g^2*(-d*g+e*f)*n^2*(e*x+d)^3/e^4+1/32*b^2*g^3*n^2*(e*x+d)^4/e^4+1/4*b^
2*(-d*g+e*f)^4*n^2*ln(e*x+d)^2/e^4/g-2*b*(-d*g+e*f)^3*n*(e*x+d)*(a+b*ln(c*
(e*x+d)^n))/e^4-3/2*b*g*(-d*g+e*f)^2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^4
-2/3*b*g^2*(-d*g+e*f)*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^4-1/8*b*g^3*n*(e
*x+d)^4*(a+b*ln(c*(e*x+d)^n))/e^4-1/2*b*(-d*g+e*f)^4*n*ln(e*x+d)*(a+b*ln(c
*(e*x+d)^n))/e^4/g+1/4*(g*x+f)^4*(a+b*ln(c*(e*x+d)^n))^2/g
```

3.44.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.99

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx$$
$$= \frac{288(ef-dg)^3(d+ex)\left(a+b\log \left(c(d+ex)^n\right)\right)^2+432g(ef-dg)^2(d+ex)^2\left(a+b\log \left(c(d+ex)^n\right)\right)^2+28\ldots}{}$$

input
```
Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(288*(e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 + 432*g*(e*f - d
*g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + 288*g^2*(e*f - d*g)*(d +
e*x)^3*(a + b*Log[c*(d + e*x)^n])^2 + 72*g^3*(d + e*x)^4*(a + b*Log[c*(d +
 e*x)^n])^2 - 576*b*(e*f - d*g)^3*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d
+ e*x)^n]) + 216*b*g*(e*f - d*g)^2*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*
(a + b*Log[c*(d + e*x)^n])) + 64*b*g^2*(e*f - d*g)*n*(b*e*n*x*(3*d^2 + 3*d
*e*x + e^2*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])) + 9*b*g^3*n*(b
*e*n*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 4*(d + e*x)^4*(a + b*
Log[c*(d + e*x)^n])))/(288*e^4)
```

3.44.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx$$
$$\Big\downarrow \text{2845}$$
$$\frac{(f+gx)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{4g}-\frac{ben\int \frac{(f+gx)^4(a+b\log(c(d+ex)^n))}{d+ex}dx}{2g}$$
$$\Big\downarrow \text{2858}$$
$$\frac{(f+gx)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{4g}-\frac{bn\int \frac{\left(e\left(f-\frac{dg}{e}\right)+g(d+ex)\right)^4(a+b\log(c(d+ex)^n))}{e^4(d+ex)}d(d+ex)}{2g}$$

$$\downarrow \ 27$$

$$\frac{(f+gx)^4 (a+b\log (c(d+ex)^n))^2}{4g}-\frac{bn\int \frac{(ef-dg+g(d+ex))^4(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{2e^4g}$$

$$\downarrow \ 2772$$

$$\frac{(f+gx)^4 (a+b\log (c(d+ex)^n))^2}{4g}-$$
$$\frac{bn\left(-bn\int \left(\frac{1}{4}(d+ex)^3g^4+\frac{4}{3}(ef-dg)(d+ex)^2g^3+3(ef-dg)^2(d+ex)g^2+4(ef-dg)^3g+\frac{(ef-dg)^4\log(d+ex)}{d+ex}\right) c\right.}{}$$

$$\downarrow \ 2009$$

$$\frac{(f+gx)^4 (a+b\log (c(d+ex)^n))^2}{4g}-$$
$$\frac{bn\left(\frac{4}{3}g^3(d+ex)^3(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)+3g^2(d+ex)^2(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)+(ef-dg)^4\right.}{}$$

input
```
Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
((f + g*x)^4*(a + b*Log[c*(d + e*x)^n])^2)/(4*g) - (b*n*(-(b*n*(4*g*(e*f -
 d*g)^3*(d + e*x) + (3*g^2*(e*f - d*g)^2*(d + e*x)^2)/2 + (4*g^3*(e*f - d*
g)*(d + e*x)^3)/9 + (g^4*(d + e*x)^4)/16 + ((e*f - d*g)^4*Log[d + e*x]^2)/
2)) + 4*g*(e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^n]) + 3*g^2*(e*f
- d*g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]) + (4*g^3*(e*f - d*g)*(d +
e*x)^3*(a + b*Log[c*(d + e*x)^n]))/3 + (g^4*(d + e*x)^4*(a + b*Log[c*(d +
e*x)^n]))/4 + (e*f - d*g)^4*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n])))/(2*e
^4*g)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1342 vs. $2(347) = 694$.

Time = 2.20 (sec) , antiderivative size = 1343, normalized size of antiderivative = 3.68

method	result	size
parallelrisch	Expression too large to display	1343
risch	Expression too large to display	6770

input
```
int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/288*(192*a*b*e^4*f*g^2*n*x^3+300*b^2*d^3*e*g^3*n^2*x-288*a^2*e^4*f^3*x+
288*a^2*d*e^3*f^3-288*a*b*d*e^3*f*g^2*n*x^2-288*a^2*e^4*f*g^2*x^3+576*b*n*
a*e^4*f^3*x+36*a*b*e^4*g^3*n*x^4+28*b^2*d*e^3*g^3*n^2*x^3-64*b^2*e^4*f*g^2
*n^2*x^3-9*b^2*e^4*g^3*n^2*x^4+72*ln(c*(e*x+d)^n)^2*b^2*d^4*g^3-864*a*b*d*
e^3*f^2*g*n*x-576*b^2*n^2*e^4*f^3*x-300*b^2*d^4*g^3*n^2-288*ln(c*(e*x+d)^n
)^2*b^2*d*e^3*f^3+144*ln(c*(e*x+d)^n)*b^2*d^4*g^3*n-444*ln(e*x+d)*b^2*d^4*
g^3*n^2-72*a^2*e^4*g^3*x^4-78*b^2*d^2*e^2*g^3*n^2*x^2-216*b^2*e^4*f^2*g*n^
2*x^2-432*a^2*e^4*f^2*g*x^2+240*b^2*d*e^3*f*g^2*n^2*x^2+432*a*b*e^4*f^2*g*
n*x^2-1056*b^2*d^2*e^2*f*g^2*n^2*x+1296*b^2*d*e^3*f^2*g*n^2*x+576*b^2*d*e^
3*f^3*n^2+144*a*b*d^4*g^3*n-72*x^4*ln(c*(e*x+d)^n)^2*b^2*e^4*g^3-288*x*ln(
c*(e*x+d)^n)^2*b^2*e^4*f^3+576*x*ln(c*(e*x+d)^n)*b^2*d^2*e^2*f*g^2*n-864*x
*ln(c*(e*x+d)^n)*b^2*d*e^3*f^2*g*n+1056*b^2*d^3*e*f*g^2*n^2-1296*b^2*d^2*e
^2*f^2*g*n^2-576*a*b*d*e^3*f^3*n-576*ln(e*x+d)*a*b*d^3*e*f*g^2*n+864*ln(e*
x+d)*a*b*d^2*e^2*f^2*g*n+72*a*b*d^2*e^2*g^3*n*x^2-48*a*b*d*e^3*g^3*n*x^3-1
44*a*b*d^3*e*g^3*n*x-576*ln(c*(e*x+d)^n)*b^2*d^3*e*f*g^2*n+864*ln(c*(e*x+d
)^n)*b^2*d^2*e^2*f^2*g*n+36*x^4*ln(c*(e*x+d)^n)*b^2*e^4*g^3*n-144*x^4*ln(c
*(e*x+d)^n)*a*b*e^4*g^3-288*x^3*ln(c*(e*x+d)^n)^2*b^2*e^4*f*g^2-432*x^2*ln
(c*(e*x+d)^n)^2*b^2*e^4*f^2*g+576*x*ln(c*(e*x+d)^n)*b^2*e^4*f^3*n-576*x*ln
(c*(e*x+d)^n)*a*b*e^4*f^3-288*ln(c*(e*x+d)^n)^2*b^2*d^3*e*f*g^2+432*ln(c*(
e*x+d)^n)^2*b^2*d^2*e^2*f^2*g-576*ln(c*(e*x+d)^n)*b^2*d*e^3*f^3*n+576*l...
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(347) = 694$.

Time = 0.30 (sec) , antiderivative size = 1190, normalized size of antiderivative = 3.26

$$\int (f + gx)^3 (a + b\log(c(d + ex)^n))^2 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
1/288*(9*(b^2*e^4*g^3*n^2 - 4*a*b*e^4*g^3*n + 8*a^2*e^4*g^3)*x^4 + 4*(72*a
^2*e^4*f*g^2 + (16*b^2*e^4*f*g^2 - 7*b^2*d*e^3*g^3)*n^2 - 12*(4*a*b*e^4*f*
g^2 - a*b*d*e^3*g^3)*n)*x^3 + 6*(72*a^2*e^4*f^2*g + (36*b^2*e^4*f^2*g - 40
*b^2*d*e^3*f*g^2 + 13*b^2*d^2*e^2*g^3)*n^2 - 12*(6*a*b*e^4*f^2*g - 4*a*b*d
*e^3*f*g^2 + a*b*d^2*e^2*g^3)*n)*x^2 + 72*(b^2*e^4*g^3*n^2*x^4 + 4*b^2*e^4
*f*g^2*n^2*x^3 + 6*b^2*e^4*f^2*g*n^2*x^2 + 4*b^2*e^4*f^3*n^2*x + (4*b^2*d*
e^3*f^3 - 6*b^2*d^2*e^2*f^2*g + 4*b^2*d^3*e*f*g^2 - b^2*d^4*g^3)*n^2)*log(
e*x + d)^2 + 72*(b^2*e^4*g^3*x^4 + 4*b^2*e^4*f*g^2*x^3 + 6*b^2*e^4*f^2*g*x
^2 + 4*b^2*e^4*f^3*x)*log(c)^2 + 12*(24*a^2*e^4*f^3 + (48*b^2*e^4*f^3 - 10
8*b^2*d*e^3*f^2*g + 88*b^2*d^2*e^2*f*g^2 - 25*b^2*d^3*e*g^3)*n^2 - 12*(4*a
*b*e^4*f^3 - 6*a*b*d*e^3*f^2*g + 4*a*b*d^2*e^2*f*g^2 - a*b*d^3*e*g^3)*n)*x
 - 12*(3*(b^2*e^4*g^3*n^2 - 4*a*b*e^4*g^3*n)*x^4 - 4*(12*a*b*e^4*f*g^2*n -
 (4*b^2*e^4*f*g^2 - b^2*d*e^3*g^3)*n^2)*x^3 + (48*b^2*d*e^3*f^3 - 108*b^2*
d^2*e^2*f^2*g + 88*b^2*d^3*e*f*g^2 - 25*b^2*d^4*g^3)*n^2 - 6*(12*a*b*e^4*f
^2*g*n - (6*b^2*e^4*f^2*g - 4*b^2*d*e^3*f*g^2 + b^2*d^2*e^2*g^3)*n^2)*x^2
- 12*(4*a*b*d*e^3*f^3 - 6*a*b*d^2*e^2*f^2*g + 4*a*b*d^3*e*f*g^2 - a*b*d^4*
g^3)*n - 12*(4*a*b*e^4*f^3*n - (4*b^2*e^4*f^3 - 6*b^2*d*e^3*f^2*g + 4*b^2*
d^2*e^2*f*g^2 - b^2*d^3*e*g^3)*n^2)*x - 12*(b^2*e^4*g^3*n*x^4 + 4*b^2*e^4*
f*g^2*n*x^3 + 6*b^2*e^4*f^2*g*n*x^2 + 4*b^2*e^4*f^3*n*x + (4*b^2*d*e^3*f^3
 - 6*b^2*d^2*e^2*f^2*g + 4*b^2*d^3*e*f*g^2 - b^2*d^4*g^3)*n)*log(c))*lo...
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(348) = 696$.

Time = 2.35 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.40

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^2 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Piecewise((a**2*f**3*x + 3*a**2*f**2*g*x**2/2 + a**2*f*g**2*x**3 + a**2*g*
*3*x**4/4 - a*b*d**4*g**3*log(c*(d + e*x)**n)/(2*e**4) + 2*a*b*d**3*f*g**2
*log(c*(d + e*x)**n)/e**3 + a*b*d**3*g**3*n*x/(2*e**3) - 3*a*b*d**2*f**2*g
*log(c*(d + e*x)**n)/e**2 - 2*a*b*d**2*f*g**2*n*x/e**2 - a*b*d**2*g**3*n*x
**2/(4*e**2) + 2*a*b*d*f**3*log(c*(d + e*x)**n)/e + 3*a*b*d*f**2*g*n*x/e +
 a*b*d*f*g**2*n*x**2/e + a*b*d*g**3*n*x**3/(6*e) - 2*a*b*f**3*n*x + 2*a*b*
f**3*x*log(c*(d + e*x)**n) - 3*a*b*f**2*g*n*x**2/2 + 3*a*b*f**2*g*x**2*log
(c*(d + e*x)**n) - 2*a*b*f*g**2*n*x**3/3 + 2*a*b*f*g**2*x**3*log(c*(d + e*
x)**n) - a*b*g**3*n*x**4/8 + a*b*g**3*x**4*log(c*(d + e*x)**n)/2 + 25*b**2
*d**4*g**3*n*log(c*(d + e*x)**n)/(24*e**4) - b**2*d**4*g**3*log(c*(d + e*x
)**n)**2/(4*e**4) - 11*b**2*d**3*f*g**2*n*log(c*(d + e*x)**n)/(3*e**3) + b
**2*d**3*f*g**2*log(c*(d + e*x)**n)**2/e**3 - 25*b**2*d**3*g**3*n**2*x/(24
*e**3) + b**2*d**3*g**3*n*x*log(c*(d + e*x)**n)/(2*e**3) + 9*b**2*d**2*f**
2*g*n*log(c*(d + e*x)**n)/(2*e**2) - 3*b**2*d**2*f**2*g*log(c*(d + e*x)**n
)**2/(2*e**2) + 11*b**2*d**2*f*g**2*n**2*x/(3*e**2) - 2*b**2*d**2*f*g**2*n
*x*log(c*(d + e*x)**n)/e**2 + 13*b**2*d**2*g**3*n**2*x**2/(48*e**2) - b**2
*d**2*g**3*n*x**2*log(c*(d + e*x)**n)/(4*e**2) - 2*b**2*d*f**3*n*log(c*(d
+ e*x)**n)/e + b**2*d*f**3*log(c*(d + e*x)**n)**2/e - 9*b**2*d*f**2*g*n**2
*x/(2*e) + 3*b**2*d*f**2*g*n*x*log(c*(d + e*x)**n)/e - 5*b**2*d*f*g**2*n**
2*x**2/(6*e) + b**2*d*f*g**2*n*x**2*log(c*(d + e*x)**n)/e - 7*b**2*d*g*...
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(347) = 694$.

Time = 0.21 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.27

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^2 \, dx$$
$$= \frac{1}{4} b^2 g^3 x^4 \log \left((ex+d)^n c\right)^2 + \frac{1}{2} abg^3 x^4 \log \left((ex+d)^n c\right)$$
$$+ b^2 f g^2 x^3 \log \left((ex+d)^n c\right)^2 + \frac{1}{4} a^2 g^3 x^4 + 2\, abfg^2 x^3 \log \left((ex+d)^n c\right)$$
$$+ \frac{3}{2} b^2 f^2 g x^2 \log \left((ex+d)^n c\right)^2 + a^2 f g^2 x^3 - 2\, abef^3 n \left(\frac{x}{e} - \frac{d \log (ex+d)}{e^2}\right)$$
$$- \frac{1}{24} abeg^3 n \left(\frac{12\, d^4 \log (ex+d)}{e^5} + \frac{3\, e^3 x^4 - 4\, de^2 x^3 + 6\, d^2 e x^2 - 12\, d^3 x}{e^4}\right)$$
$$+ \frac{1}{3} abefg^2 n \left(\frac{6\, d^3 \log (ex+d)}{e^4} - \frac{2\, e^2 x^3 - 3\, dex^2 + 6\, d^2 x}{e^3}\right)$$
$$- \frac{3}{2} abef^2 gn \left(\frac{2\, d^2 \log (ex+d)}{e^3} + \frac{ex^2 - 2\, dx}{e^2}\right) + 3\, abf^2 g x^2 \log \left((ex+d)^n c\right)$$
$$+ b^2 f^3 x \log \left((ex+d)^n c\right)^2 + \frac{3}{2} a^2 f^2 g x^2 + 2\, abf^3 x \log \left((ex+d)^n c\right)$$
$$- \left(2\, en \left(\frac{x}{e} - \frac{d \log (ex+d)}{e^2}\right) \log \left((ex+d)^n c\right) + \frac{\left(d \log (ex+d)^2 - 2\, ex + 2\, d \log (ex+d)\right) n^2}{e}\right) b^2 f^3$$
$$- \frac{3}{4} \left(2\, en \left(\frac{2\, d^2 \log (ex+d)}{e^3} + \frac{ex^2 - 2\, dx}{e^2}\right) \log \left((ex+d)^n c\right) - \frac{\left(e^2 x^2 + 2\, d^2 \log (ex+d)^2 - 6\, dex + 6\, d^2 \mathrm{l}\right.}{e^2}\right.$$
$$+ \frac{1}{18} \left(6\, en \left(\frac{6\, d^3 \log (ex+d)}{e^4} - \frac{2\, e^2 x^3 - 3\, dex^2 + 6\, d^2 x}{e^3}\right) \log \left((ex+d)^n c\right) + \frac{\left(4\, e^3 x^3 - 15\, de^2 x^2 - 18\, d^3 \mathrm{l}\right.}{}\right.$$
$$- \frac{1}{288} \left(12\, en \left(\frac{12\, d^4 \log (ex+d)}{e^5} + \frac{3\, e^3 x^4 - 4\, de^2 x^3 + 6\, d^2 e x^2 - 12\, d^3 x}{e^4}\right) \log \left((ex+d)^n c\right) - \frac{\left(9\, e^4 x^4 - 28\right.}{}\right.$$
$$+ a^2 f^3 x$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

3.44. $\int (f+gx)^3 (a+b\log (c(d+ex)^n))^2 \, dx$

output
```
1/4*b^2*g^3*x^4*log((e*x + d)^n*c)^2 + 1/2*a*b*g^3*x^4*log((e*x + d)^n*c)
+ b^2*f*g^2*x^3*log((e*x + d)^n*c)^2 + 1/4*a^2*g^3*x^4 + 2*a*b*f*g^2*x^3*l
og((e*x + d)^n*c) + 3/2*b^2*f^2*g*x^2*log((e*x + d)^n*c)^2 + a^2*f*g^2*x^3
 - 2*a*b*e*f^3*n*(x/e - d*log(e*x + d)/e^2) - 1/24*a*b*e*g^3*n*(12*d^4*log
(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4) +
1/3*a*b*e*f*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2
*x)/e^3) - 3/2*a*b*e*f^2*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2
) + 3*a*b*f^2*g*x^2*log((e*x + d)^n*c) + b^2*f^3*x*log((e*x + d)^n*c)^2 +
3/2*a^2*f^2*g*x^2 + 2*a*b*f^3*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e
*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x
+ d))*n^2/e)*b^2*f^3 - 3/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x
)/e^2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*
d^2*log(e*x + d))*n^2/e^2)*b^2*f^2*g + 1/18*(6*e*n*(6*d^3*log(e*x + d)/e^4
 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c) + (4*e^3*x^3
- 15*d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e*x + d))
*n^2/e^3)*b^2*f*g^2 - 1/288*(12*e*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4
- 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4)*log((e*x + d)^n*c) - (9*e^4*x
^4 - 28*d*e^3*x^3 + 78*d^2*e^2*x^2 + 72*d^4*log(e*x + d)^2 - 300*d^3*e*x +
 300*d^4*log(e*x + d))*n^2/e^4)*b^2*g^3 + a^2*f^3*x
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. 2(347) = 694.

Time = 0.34 (sec) , antiderivative size = 2345, normalized size of antiderivative = 6.42

$$\int (f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
(e*x + d)*b^2*f^3*n^2*log(e*x + d)^2/e + 3/2*(e*x + d)^2*b^2*f^2*g*n^2*log
(e*x + d)^2/e^2 - 3*(e*x + d)*b^2*d*f^2*g*n^2*log(e*x + d)^2/e^2 + (e*x +
d)^3*b^2*f*g^2*n^2*log(e*x + d)^2/e^3 - 3*(e*x + d)^2*b^2*d*f*g^2*n^2*log(
e*x + d)^2/e^3 + 3*(e*x + d)*b^2*d^2*f*g^2*n^2*log(e*x + d)^2/e^3 + 1/4*(e
*x + d)^4*b^2*g^3*n^2*log(e*x + d)^2/e^4 - (e*x + d)^3*b^2*d*g^3*n^2*log(e
*x + d)^2/e^4 + 3/2*(e*x + d)^2*b^2*d^2*g^3*n^2*log(e*x + d)^2/e^4 - (e*x
+ d)*b^2*d^3*g^3*n^2*log(e*x + d)^2/e^4 - 2*(e*x + d)*b^2*f^3*n^2*log(e*x
+ d)/e - 3/2*(e*x + d)^2*b^2*f^2*g*n^2*log(e*x + d)/e^2 + 6*(e*x + d)*b^2*
d*f^2*g*n^2*log(e*x + d)/e^2 - 2/3*(e*x + d)^3*b^2*f*g^2*n^2*log(e*x + d)/
e^3 + 3*(e*x + d)^2*b^2*d*f*g^2*n^2*log(e*x + d)/e^3 - 6*(e*x + d)*b^2*d^2
*f*g^2*n^2*log(e*x + d)/e^3 - 1/8*(e*x + d)^4*b^2*g^3*n^2*log(e*x + d)/e^4
 + 2/3*(e*x + d)^3*b^2*d*g^3*n^2*log(e*x + d)/e^4 - 3/2*(e*x + d)^2*b^2*d^
2*g^3*n^2*log(e*x + d)/e^4 + 2*(e*x + d)*b^2*d^3*g^3*n^2*log(e*x + d)/e^4
+ 2*(e*x + d)*b^2*f^3*n*log(e*x + d)*log(c)/e + 3*(e*x + d)^2*b^2*f^2*g*n*
log(e*x + d)*log(c)/e^2 - 6*(e*x + d)*b^2*d*f^2*g*n*log(e*x + d)*log(c)/e^
2 + 2*(e*x + d)^3*b^2*f*g^2*n*log(e*x + d)*log(c)/e^3 - 6*(e*x + d)^2*b^2*
d*f*g^2*n*log(e*x + d)*log(c)/e^3 + 6*(e*x + d)*b^2*d^2*f*g^2*n*log(e*x +
d)*log(c)/e^3 + 1/2*(e*x + d)^4*b^2*g^3*n*log(e*x + d)*log(c)/e^4 - 2*(e*x
 + d)^3*b^2*d*g^3*n*log(e*x + d)*log(c)/e^4 + 3*(e*x + d)^2*b^2*d^2*g^3*n*
log(e*x + d)*log(c)/e^4 - 2*(e*x + d)*b^2*d^3*g^3*n*log(e*x + d)*log(c)...
```

3.44. $\int (f+gx)^3 (a+b\log(c(d+ex)^n))^2 \, dx$

3.44.9 Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.88

$$\int (f+gx)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^2 \, dx$$

$$= x\left(\frac{72\,a^2\,d\,e^2\,f^2\,g+24\,a^2\,e^3\,f^3-48\,a\,b\,e^3\,f^3\,n-12\,b^2\,d^3\,g^3\,n^2+48\,b^2\,d^2\,e\,f\,g^2\,n^2-72\,b^2\,d\,e^2\,f^2\,g\,n^2+48}{24\,e^3}\right.$$

$$+\frac{d\left(\frac{d\left(\frac{g^2\,\left(6\,a^2\,d\,g+18\,a^2\,e\,f-b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-12\,a\,b\,e\,f\,n\right)}{6\,e}-\frac{d\,g^3\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{8\,e}\right)}{e}-\frac{g\,\left(12\,a^2\,d\,e\,f\,g+12\,a^2\,e^2\,f^2-12\,a\,b\,e^2\,f^2\,n+b^2\,d^2}{4\,e^2}\right.}{e}$$

$$-x^2\left(\frac{d\left(\frac{g^2\,\left(6\,a^2\,d\,g+18\,a^2\,e\,f-b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-12\,a\,b\,e\,f\,n\right)}{6\,e}-\frac{d\,g^3\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{8\,e}\right)}{2\,e}\right.$$

$$\left.-\frac{g\,\left(12\,a^2\,d\,e\,f\,g+12\,a^2\,e^2\,f^2-12\,a\,b\,e^2\,f^2\,n+b^2\,d^2\,g^2\,n^2-4\,b^2\,d\,e\,f\,g\,n^2+6\,b^2\,e^2\,f^2\,n^2\right)}{8\,e^2}\right)$$

$$+\ln\left(c\,{\left(d+e\,x\right)}^n\right)^2\,\left(b^2\,f^3\,x-\frac{d\,\left(b^2\,d^3\,g^3-4\,b^2\,d^2\,e\,f\,g^2+6\,b^2\,d\,e^2\,f^2\,g-4\,b^2\,e^3\,f^3\right)}{4\,e^4}\right.$$

$$\left.+\frac{b^2\,g^3\,x^4}{4}+\frac{3\,b^2\,f^2\,g\,x^2}{2}+b^2\,f\,g^2\,x^3\right)$$

$$+x^3\left(\frac{g^2\,\left(6\,a^2\,d\,g+18\,a^2\,e\,f-b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-12\,a\,b\,e\,f\,n\right)}{18\,e}\right.$$

$$\left.-\frac{d\,g^3\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{24\,e}\right)$$

$$+\ln\left(c\,{\left(d+e\,x\right)}^n\right)\left(\frac{x\left(\frac{d\left(\frac{d\left(\frac{8\,b\,g^2\,\left(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n\right)}{e}-\frac{2\,b\,d\,g^3\,\left(4\,a-b\,n\right)}{e}\right)}{e}-\frac{12\,b\,f\,g\,\left(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n\right)}{e}\right)}{2\,e}+\frac{4\,b\,f^2\,\left(3\,a\,d\,g+a\,e\,f-b\right.}{e}\right)}{2}\right.$$

$$+\frac{x^3\,\left(\frac{4\,b\,g^2\,\left(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n\right)}{3\,e}-\frac{b\,d\,g^3\,\left(4\,a-b\,n\right)}{3\,e}\right)}{2}$$

$$\left(\frac{d\left(\frac{8\,b\,g^2\,\left(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n\right)}{\ldots}-\frac{2\,b\,d\,g^3\,\left(4\,a-b\,n\right)}{\ldots}\right)}{\ldots}\right)$$

input
```
int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
x*((24*a^2*e^3*f^3 - 12*b^2*d^3*g^3*n^2 + 48*b^2*e^3*f^3*n^2 - 48*a*b*e^3*
f^3*n + 72*a^2*d*e^2*f^2*g - 72*b^2*d*e^2*f^2*g*n^2 + 48*b^2*d^2*e*f*g^2*n
^2)/(24*e^3) + (d*((d*((g^2*(6*a^2*d*g + 18*a^2*e*f - b^2*d*g*n^2 + 4*b^2*
e*f*n^2 - 12*a*b*e*f*n))/(6*e) - (d*g^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(8*e)
))/e - (g*(12*a^2*e^2*f^2 + b^2*d^2*g^2*n^2 + 6*b^2*e^2*f^2*n^2 - 12*a*b*e
^2*f^2*n + 12*a^2*d*e*f*g - 4*b^2*d*e*f*g*n^2))/(4*e^2)))/e) - x^2*((d*((g
^2*(6*a^2*d*g + 18*a^2*e*f - b^2*d*g*n^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n))/
(6*e) - (d*g^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(8*e)))/(2*e) - (g*(12*a^2*e^2
*f^2 + b^2*d^2*g^2*n^2 + 6*b^2*e^2*f^2*n^2 - 12*a*b*e^2*f^2*n + 12*a^2*d*e
*f*g - 4*b^2*d*e*f*g*n^2))/(8*e^2)) + log(c*(d + e*x)^n)^2*(b^2*f^3*x - (d
*(b^2*d^3*g^3 - 4*b^2*e^3*f^3 + 6*b^2*d*e^2*f^2*g - 4*b^2*d^2*e*f*g^2))/(4
*e^4) + (b^2*g^3*x^4)/4 + (3*b^2*f^2*g*x^2)/2 + b^2*f*g^2*x^3) + x^3*((g^2
*(6*a^2*d*g + 18*a^2*e*f - b^2*d*g*n^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n))/(1
8*e) - (d*g^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(24*e)) + log(c*(d + e*x)^n)*((
x*((d*((d*((8*b*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (2*b*d*g^3*(4*a - b*n
))/e))/e - (12*b*f*g*(2*a*d*g + 2*a*e*f - b*e*f*n))/e))/(2*e) + (4*b*f^2*(
3*a*d*g + a*e*f - b*e*f*n))/e))/2 + (x^3*((4*b*g^2*(a*d*g + 3*a*e*f - b*e*
f*n))/(3*e) - (b*d*g^3*(4*a - b*n))/(3*e)))/2 - (x^2*((d*((8*b*g^2*(a*d*g
+ 3*a*e*f - b*e*f*n))/e - (2*b*d*g^3*(4*a - b*n))/e))/(4*e) - (3*b*f*g*(2*
a*d*g + 2*a*e*f - b*e*f*n))/e))/2 + (b*g^3*x^4*(4*a - b*n))/8) + (log(d...
```

3.44. $\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx$

3.45 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2\,dx$

3.45.1 Optimal result

Integrand size = 24, antiderivative size = 287

$$\begin{aligned}
&\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2\,dx \\
&= \frac{2b^2(ef-dg)^2n^2x}{e^2} + \frac{b^2g(ef-dg)n^2(d+ex)^2}{2e^3} + \frac{2b^2g^2n^2(d+ex)^3}{27e^3} \\
&\quad + \frac{b^2(ef-dg)^3n^2\log^2(d+ex)}{3e^3g} - \frac{2b(ef-dg)^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3} \\
&\quad - \frac{bg(ef-dg)n(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3} \\
&\quad - \frac{2bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3} \\
&\quad - \frac{2b(ef-dg)^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g} \\
&\quad + \frac{(f+gx)^3(a+b\log(c(d+ex)^n))^2}{3g}
\end{aligned}$$

output
```
2*b^2*(-d*g+e*f)^2*n^2*x/e^2+1/2*b^2*g*(-d*g+e*f)*n^2*(e*x+d)^2/e^3+2/27*b
^2*g^2*n^2*(e*x+d)^3/e^3+1/3*b^2*(-d*g+e*f)^3*n^2*ln(e*x+d)^2/e^3/g-2*b*(-
d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^3-b*g*(-d*g+e*f)*n*(e*x+d)^2*
(a+b*ln(c*(e*x+d)^n))/e^3-2/9*b*g^2*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^3-
2/3*b*(-d*g+e*f)^3*n*ln(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^3/g+1/3*(g*x+f)^3*(
a+b*ln(c*(e*x+d)^n))^2/g
```

3.45.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.86

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^2 \, dx$$
$$= \frac{54(ef-dg)^2(d+ex)(a+b\log (c(d+ex)^n))^2+54g(ef-dg)(d+ex)^2(a+b\log (c(d+ex)^n))^2+18g^2(\ldots}{}$$

input
```
Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(54*(e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 + 54*g*(e*f - d*g
)*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + 18*g^2*(d + e*x)^3*(a + b*Log
[c*(d + e*x)^n])^2 - 108*b*(e*f - d*g)^2*n*(e*(a - b*n)*x + b*(d + e*x)*Lo
g[c*(d + e*x)^n]) + 27*b*g*(e*f - d*g)*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x
)^2*(a + b*Log[c*(d + e*x)^n])) + 4*b*g^2*n*(b*e*n*x*(3*d^2 + 3*d*e*x + e^
2*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])))/(54*e^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^2 \, dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{(f+gx)^3 (a+b\log (c(d+ex)^n))^2}{3g}-\frac{2ben\int \frac{(f+gx)^3(a+b\log(c(d+ex)^n))}{d+ex}dx}{3g}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{(f+gx)^3 (a+b\log (c(d+ex)^n))^2}{3g}-\frac{2bn\int \frac{\left(e\left(f-\frac{dg}{e}\right)+g(d+ex)\right)^3(a+b\log(c(d+ex)^n))}{e^3(d+ex)}d(d+ex)}{3g}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-\frac{2bn\int\frac{(ef-dg+g(d+ex))^3(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{3e^3g}$$

$\downarrow$ 2772

$$\frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$
$$2bn\left(-bn\int\left(\frac{1}{3}(d+ex)^2g^3+\frac{3}{2}(ef-dg)(d+ex)g^2+3(ef-dg)^2g+\frac{(ef-dg)^3\log(d+ex)}{d+ex}\right)d(d+ex)+\frac{3}{2}g^2(d+ex)^2($$

$\downarrow$ 2009

$$\frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$
$$2bn\left(\frac{3}{2}g^2(d+ex)^2(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)+(ef-dg)^3\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)+3g(d+ex)\right.$$

input
```
Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
((f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^2)/(3*g) - (2*b*n*(-(b*n*(3*g*(e*f
 - d*g)^2*(d + e*x) + (3*g^2*(e*f - d*g)*(d + e*x)^2)/4 + (g^3*(d + e*x)^3
)/9 + ((e*f - d*g)^3*Log[d + e*x]^2)/2)) + 3*g*(e*f - d*g)^2*(d + e*x)*(a
+ b*Log[c*(d + e*x)^n]) + (3*g^2*(e*f - d*g)*(d + e*x)^2*(a + b*Log[c*(d +
 e*x)^n]))/2 + (g^3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/3 + (e*f - d*g
)^3*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n])))/(3*e^3*g)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(275) = 550$.

Time = 1.48 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.96

method	result
parallelrisch	$\frac{108x^2\ln(c(ex+d)^n)ab\,e^3fgn-108b^2d\,e^2f^2n^3+36ab\,d^3g^2n^2-54a^2d\,e^2f^2n+4x^3b^2e^3g^2n^3+18x^3a^2e^3g^2n+108x\,b^2e^3f^2n^3+18\ldots}{}$
risch	Expression too large to display

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/54*(108*x^2*ln(c*(e*x+d)^n)*a*b*e^3*f*g*n-108*b^2*d*e^2*f^2*n^3+36*a*b*d
^3*g^2*n^2-54*a^2*d*e^2*f^2*n+4*x^3*b^2*e^3*g^2*n^3+18*x^3*a^2*e^3*g^2*n+1
08*x*b^2*e^3*f^2*n^3+18*ln(c*(e*x+d)^n)^2*b^2*d^3*g^2*n-66*ln(c*(e*x+d)^n)
*b^2*d^3*g^2*n^2+54*x*a^2*e^3*f^2*n+162*b^2*d^2*e*f*g*n^3+108*a*b*d*e^2*f^
2*n^2+18*x^3*ln(c*(e*x+d)^n)^2*b^2*e^3*g^2*n-12*x^3*ln(c*(e*x+d)^n)*b^2*e^
3*g^2*n^2-12*x^3*a*b*e^3*g^2*n^2-15*x^2*b^2*d*e^2*g^2*n^3+27*x^2*b^2*e^3*f
*g*n^3+54*x*ln(c*(e*x+d)^n)^2*b^2*e^3*f^2*n-108*x*ln(c*(e*x+d)^n)*b^2*e^3*
f^2*n^2+66*x*b^2*d^2*e*g^2*n^3+108*x*ln(c*(e*x+d)^n)*b^2*d*e^2*f*g*n^2+108
*x*a*b*d*e^2*f*g*n^2-108*ln(c*(e*x+d)^n)*a*b*d^2*e*f*g*n+54*x^2*a^2*e^3*f*
g*n-108*x*a*b*e^3*f^2*n^2+54*ln(c*(e*x+d)^n)^2*b^2*d*e^2*f^2*n-108*ln(c*(e
*x+d)^n)*b^2*d*e^2*f^2*n^2+36*ln(c*(e*x+d)^n)*a*b*d^3*g^2*n-66*b^2*d^3*g^2
*n^3-108*a*b*d^2*e*f*g*n^2+36*x^3*ln(c*(e*x+d)^n)*a*b*e^3*g^2*n+54*x^2*ln(
c*(e*x+d)^n)^2*b^2*e^3*f*g*n+18*x^2*ln(c*(e*x+d)^n)*b^2*d*e^2*g^2*n^2-54*x
^2*ln(c*(e*x+d)^n)*b^2*e^3*f*g*n^2+18*x^2*a*b*d*e^2*g^2*n^2-54*x^2*a*b*e^3
*f*g*n^2-36*x*ln(c*(e*x+d)^n)*b^2*d^2*e*g^2*n^2-162*x*b^2*d*e^2*f*g*n^3+10
8*x*ln(c*(e*x+d)^n)*a*b*e^3*f^2*n-36*x*a*b*d^2*e*g^2*n^2-54*ln(c*(e*x+d)^n
)^2*b^2*d^2*e*f*g*n+162*ln(c*(e*x+d)^n)*b^2*d^2*e*f*g*n^2+108*ln(c*(e*x+d)
^n)*a*b*d*e^2*f^2*n)/n/e^3
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(275) = 550$.

Time = 0.30 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.65

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{2\,(2\,b^2e^3g^2n^2 - 6\,abe^3g^2n + 9\,a^2e^3g^2)x^3 + 3\,(18\,a^2e^3fg + (9\,b^2e^3fg - 5\,b^2de^2g^2)n^2 - 6\,(3\,abe^3fg - abde^2g}{}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
1/54*(2*(2*b^2*e^3*g^2*n^2 - 6*a*b*e^3*g^2*n + 9*a^2*e^3*g^2)*x^3 + 3*(18*
a^2*e^3*f*g + (9*b^2*e^3*f*g - 5*b^2*d*e^2*g^2)*n^2 - 6*(3*a*b*e^3*f*g - a
*b*d*e^2*g^2)*n)*x^2 + 18*(b^2*e^3*g^2*n^2*x^3 + 3*b^2*e^3*f*g*n^2*x^2 + 3
*b^2*e^3*f^2*n^2*x + (3*b^2*d*e^2*f^2 - 3*b^2*d^2*e*f*g + b^2*d^3*g^2)*n^2
)*log(e*x + d)^2 + 18*(b^2*e^3*g^2*x^3 + 3*b^2*e^3*f*g*x^2 + 3*b^2*e^3*f^2
*x)*log(c)^2 + 6*(9*a^2*e^3*f^2 + (18*b^2*e^3*f^2 - 27*b^2*d*e^2*f*g + 11*
b^2*d^2*e*g^2)*n^2 - 6*(3*a*b*e^3*f^2 - 3*a*b*d*e^2*f*g + a*b*d^2*e*g^2)*n
)*x - 6*(2*(b^2*e^3*g^2*n^2 - 3*a*b*e^3*g^2*n)*x^3 + (18*b^2*d*e^2*f^2 - 2
7*b^2*d^2*e*f*g + 11*b^2*d^3*g^2)*n^2 - 3*(6*a*b*e^3*f*g*n - (3*b^2*e^3*f*
g - b^2*d*e^2*g^2)*n^2)*x^2 - 6*(3*a*b*d*e^2*f^2 - 3*a*b*d^2*e*f*g + a*b*d
^3*g^2)*n - 6*(3*a*b*e^3*f^2*n - (3*b^2*e^3*f^2 - 3*b^2*d*e^2*f*g + b^2*d^
2*e*g^2)*n^2)*x - 6*(b^2*e^3*g^2*n*x^3 + 3*b^2*e^3*f*g*n*x^2 + 3*b^2*e^3*f
^2*n*x + (3*b^2*d*e^2*f^2 - 3*b^2*d^2*e*f*g + b^2*d^3*g^2)*n)*log(c))*log(
e*x + d) - 6*(2*(b^2*e^3*g^2*n - 3*a*b*e^3*g^2)*x^3 - 3*(6*a*b*e^3*f*g - (
3*b^2*e^3*f*g - b^2*d*e^2*g^2)*n)*x^2 - 6*(3*a*b*e^3*f^2 - (3*b^2*e^3*f^2
- 3*b^2*d*e^2*f*g + b^2*d^2*e*g^2)*n)*x)*log(c))/e^3
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(274) = 548$.

Time = 1.23 (sec) , antiderivative size = 774, normalized size of antiderivative = 2.70

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2\,dx$$

$$= \begin{cases} a^2f^2x + a^2fgx^2 + \frac{a^2g^2x^3}{3} + \frac{2abd^3g^2\log(c(d+ex)^n)}{3e^3} - \frac{2abd^2fg\log(c(d+ex)^n)}{e^2} - \frac{2abd^2g^2nx}{3e^2} + \frac{2abdf^2\log(c(d+ex)^n)}{e} + \frac{2abd \\ (a+b\log(cd^n))^2\left(f^2x + fgx^2 + \frac{g^2x^3}{3}\right) \end{cases}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output

```
Piecewise((a**2*f**2*x + a**2*f*g*x**2 + a**2*g**2*x**3/3 + 2*a*b*d**3*g**
2*log(c*(d + e*x)**n)/(3*e**3) - 2*a*b*d**2*f*g*log(c*(d + e*x)**n)/e**2 -
 2*a*b*d**2*g**2*n*x/(3*e**2) + 2*a*b*d*f**2*log(c*(d + e*x)**n)/e + 2*a*b
*d*f*g*n*x/e + a*b*d*g**2*n*x**2/(3*e) - 2*a*b*f**2*n*x + 2*a*b*f**2*x*log
(c*(d + e*x)**n) - a*b*f*g*n*x**2 + 2*a*b*f*g*x**2*log(c*(d + e*x)**n) - 2
*a*b*g**2*n*x**3/9 + 2*a*b*g**2*x**3*log(c*(d + e*x)**n)/3 - 11*b**2*d**3*
g**2*n*log(c*(d + e*x)**n)/(9*e**3) + b**2*d**3*g**2*log(c*(d + e*x)**n)**
2/(3*e**3) + 3*b**2*d**2*f*g*n*log(c*(d + e*x)**n)/e**2 - b**2*d**2*f*g*lo
g(c*(d + e*x)**n)**2/e**2 + 11*b**2*d**2*g**2*n**2*x/(9*e**2) - 2*b**2*d**
2*g**2*n*x*log(c*(d + e*x)**n)/(3*e**2) - 2*b**2*d*f**2*n*log(c*(d + e*x)*
*n)/e + b**2*d*f**2*log(c*(d + e*x)**n)**2/e - 3*b**2*d*f*g*n**2*x/e + 2*b
**2*d*f*g*n*x*log(c*(d + e*x)**n)/e - 5*b**2*d*g**2*n**2*x**2/(18*e) + b**
2*d*g**2*n*x**2*log(c*(d + e*x)**n)/(3*e) + 2*b**2*f**2*n**2*x - 2*b**2*f*
*2*n*x*log(c*(d + e*x)**n) + b**2*f**2*x*log(c*(d + e*x)**n)**2 + b**2*f*g
*n**2*x**2/2 - b**2*f*g*n*x**2*log(c*(d + e*x)**n) + b**2*f*g*x**2*log(c*(
d + e*x)**n)**2 + 2*b**2*g**2*n**2*x**3/27 - 2*b**2*g**2*n*x**3*log(c*(d +
 e*x)**n)/9 + b**2*g**2*x**3*log(c*(d + e*x)**n)**2/3, Ne(e, 0)), ((a + b*
log(c*d**n))**2*(f**2*x + f*g*x**2 + g**2*x**3/3), True))
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(275) = 550$.

Time = 0.21 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.93

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx$$
$$= \frac{1}{3}\, b^2 g^2 x^3 \log \left((ex+d)^n c\right)^2 + \frac{2}{3}\, abg^2 x^3 \log \left((ex+d)^n c\right) + b^2 fgx^2 \log \left((ex+d)^n c\right)^2 + \frac{1}{3}\, a^2 g^2 x^3 - 2\, abef^2 n \left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right) + \frac{1}{9}\, abeg^2 n \left(\frac{6\, d^3 \log (ex+d)}{e^4} - \frac{2\, e^2 x^3 - 3\, dex^2 + 6\, d^2 x}{e^3}\right) - abefgn \left(\frac{2\, d^2 \log (ex+d)}{e^3} + \frac{ex^2 - 2\, dx}{e^2}\right) + 2\, abfgx^2 \log \left((ex+d)^n c\right) + b^2 f^2 x \log \left((ex+d)^n c\right)^2 + a^2 fgx^2 + 2\, abf^2 x \log \left((ex+d)^n c\right) - \left(2\, en \left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right) \log \left((ex+d)^n c\right) + \frac{\left(d\log (ex+d)^2 - 2\, ex + 2\, d\log (ex+d)\right) n^2}{e}\right) b^2 f^2 - \frac{1}{2} \left(2\, en \left(\frac{2\, d^2 \log (ex+d)}{e^3} + \frac{ex^2 - 2\, dx}{e^2}\right) \log \left((ex+d)^n c\right) - \frac{\left(e^2 x^2 + 2\, d^2 \log (ex+d)^2 - 6\, dex + 6\, d^2 \mathrm{l}\right.}{e^2} + \frac{1}{54} \left(6\, en \left(\frac{6\, d^3 \log (ex+d)}{e^4} - \frac{2\, e^2 x^3 - 3\, dex^2 + 6\, d^2 x}{e^3}\right) \log \left((ex+d)^n c\right) + \frac{\left(4\, e^3 x^3 - 15\, de^2 x^2 - 18\, d^3 \mathrm{l}\right.}{} + a^2 f^2 x$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*g^2*x^3*log((e*x + d)^n*c)^2 + 2/3*a*b*g^2*x^3*log((e*x + d)^n*c)
+ b^2*f*g*x^2*log((e*x + d)^n*c)^2 + 1/3*a^2*g^2*x^3 - 2*a*b*e*f^2*n*(x/e
- d*log(e*x + d)/e^2) + 1/9*a*b*e*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x
^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) - a*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*
x^2 - 2*d*x)/e^2) + 2*a*b*f*g*x^2*log((e*x + d)^n*c) + b^2*f^2*x*log((e*x
+ d)^n*c)^2 + a^2*f*g*x^2 + 2*a*b*f^2*x*log((e*x + d)^n*c) - (2*e*n*(x/e -
 d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*
log(e*x + d))*n^2/e)*b^2*f^2 - 1/2*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2
 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*
e*x + 6*d^2*log(e*x + d))*n^2/e^2)*b^2*f*g + 1/54*(6*e*n*(6*d^3*log(e*x +
d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c) + (4*e^
3*x^3 - 15*d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e*x
 + d))*n^2/e^3)*b^2*g^2 + a^2*f^2*x
```

3.45. $\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx$

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(275) = 550$.

Time = 0.41 (sec) , antiderivative size = 1315, normalized size of antiderivative = 4.58

$$\int (f+gx)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
(e*x + d)*b^2*f^2*n^2*log(e*x + d)^2/e + (e*x + d)^2*b^2*f*g*n^2*log(e*x +
 d)^2/e^2 - 2*(e*x + d)*b^2*d*f*g*n^2*log(e*x + d)^2/e^2 + 1/3*(e*x + d)^3
*b^2*g^2*n^2*log(e*x + d)^2/e^3 - (e*x + d)^2*b^2*d*g^2*n^2*log(e*x + d)^2
/e^3 + (e*x + d)*b^2*d^2*g^2*n^2*log(e*x + d)^2/e^3 - 2*(e*x + d)*b^2*f^2*
n^2*log(e*x + d)/e - (e*x + d)^2*b^2*f*g*n^2*log(e*x + d)/e^2 + 4*(e*x + d
)*b^2*d*f*g*n^2*log(e*x + d)/e^2 - 2/9*(e*x + d)^3*b^2*g^2*n^2*log(e*x + d
)/e^3 + (e*x + d)^2*b^2*d*g^2*n^2*log(e*x + d)/e^3 - 2*(e*x + d)*b^2*d^2*g
^2*n^2*log(e*x + d)/e^3 + 2*(e*x + d)*b^2*f^2*n*log(e*x + d)*log(c)/e + 2*
(e*x + d)^2*b^2*f*g*n*log(e*x + d)*log(c)/e^2 - 4*(e*x + d)*b^2*d*f*g*n*lo
g(e*x + d)*log(c)/e^2 + 2/3*(e*x + d)^3*b^2*g^2*n*log(e*x + d)*log(c)/e^3
- 2*(e*x + d)^2*b^2*d*g^2*n*log(e*x + d)*log(c)/e^3 + 2*(e*x + d)*b^2*d^2*
g^2*n*log(e*x + d)*log(c)/e^3 + 2*(e*x + d)*b^2*f^2*n^2/e + 1/2*(e*x + d)^
2*b^2*f*g*n^2/e^2 - 4*(e*x + d)*b^2*d*f*g*n^2/e^2 + 2/27*(e*x + d)^3*b^2*g
^2*n^2/e^3 - 1/2*(e*x + d)^2*b^2*d*g^2*n^2/e^3 + 2*(e*x + d)*b^2*d^2*g^2*n
^2/e^3 + 2*(e*x + d)*a*b*f^2*n*log(e*x + d)/e + 2*(e*x + d)^2*a*b*f*g*n*lo
g(e*x + d)/e^2 - 4*(e*x + d)*a*b*d*f*g*n*log(e*x + d)/e^2 + 2/3*(e*x + d)^
3*a*b*g^2*n*log(e*x + d)/e^3 - 2*(e*x + d)^2*a*b*d*g^2*n*log(e*x + d)/e^3
+ 2*(e*x + d)*a*b*d^2*g^2*n*log(e*x + d)/e^3 - 2*(e*x + d)*b^2*f^2*n*log(c
)/e - (e*x + d)^2*b^2*f*g*n*log(c)/e^2 + 4*(e*x + d)*b^2*d*f*g*n*log(c)/e^
2 - 2/9*(e*x + d)^3*b^2*g^2*n*log(c)/e^3 + (e*x + d)^2*b^2*d*g^2*n*log(...
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.06

$$\int (f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$

$$= \ln\left(c\,(d+e\,x)^n\right)\left(\frac{x^2\left(\frac{3\,b\,g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{b\,d\,g^2\,(3\,a-b\,n)}{e}\right)}{3} - \frac{x\left(\frac{d\left(\frac{18\,b\,g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{6\,b\,d\,g^2\,(3\,a-b\,n)}{e}\right)}{3\,e}-\frac{6\,b\,f\,(2\,a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}\right)}{3} + \frac{2\,b\,g^2\,x^3\,(3\,a-b\,n)}{9}\right)$$
$$+x\left(\frac{18\,a^2\,d\,e\,f\,g+9\,a^2\,e^2\,f^2-18\,a\,b\,e^2\,f^2\,n+6\,b^2\,d^2\,g^2\,n^2-18\,b^2\,d\,e\,f\,g\,n^2+18\,b^2\,e^2\,f^2\,n^2}{9\,e^2} - \frac{d\left(\frac{g\,(3\,a^2\,d\,g+6\,a^2\,e\,f-b^2\,d\,g\,n^2+3\,b^2\,e\,f\,n^2-6\,a\,b\,e\,f\,n)}{3\,e}-\frac{d\,g^2\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{9\,e}\right)}{e}\right)$$
$$+x^2\left(\frac{g\,(3\,a^2\,d\,g+6\,a^2\,e\,f-b^2\,d\,g\,n^2+3\,b^2\,e\,f\,n^2-6\,a\,b\,e\,f\,n)}{6\,e} - \frac{d\,g^2\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{18\,e}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)^2\left(b^2\,f^2\,x+\frac{b^2\,g^2\,x^3}{3}+\frac{d\,(b^2\,d^2\,g^2-3\,b^2\,d\,e\,f\,g+3\,b^2\,e^2\,f^2)}{3\,e^3}+b^2\,f\,g\,x^2\right)$$
$$-\frac{\ln\left(d+e\,x\right)\,(11\,b^2\,d^3\,g^2\,n^2-27\,b^2\,d^2\,e\,f\,g\,n^2+18\,b^2\,d\,e^2\,f^2\,n^2-6\,a\,b\,d^3\,g^2\,n+18\,a\,b\,d^2\,e\,f\,g\,n-18\,a\,b\,d}{9\,e^3}$$
$$+\frac{g^2\,x^3\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{27}$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
log(c*(d + e*x)^n)*((x^2*((3*b*g*(a*d*g + 2*a*e*f - b*e*f*n))/e - (b*d*g^2
*(3*a - b*n))/e))/3 - (x*((d*((18*b*g*(a*d*g + 2*a*e*f - b*e*f*n))/e - (6*
b*d*g^2*(3*a - b*n))/e))/(3*e) - (6*b*f*(2*a*d*g + a*e*f - b*e*f*n))/e))/3
 + (2*b*g^2*x^3*(3*a - b*n))/9) + x*((9*a^2*e^2*f^2 + 6*b^2*d^2*g^2*n^2 +
18*b^2*e^2*f^2*n^2 - 18*a*b*e^2*f^2*n + 18*a^2*d*e*f*g - 18*b^2*d*e*f*g*n^
2)/(9*e^2) - (d*((g*(3*a^2*d*g + 6*a^2*e*f - b^2*d*g*n^2 + 3*b^2*e*f*n^2 -
 6*a*b*e*f*n))/(3*e) - (d*g^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(9*e)))/e) +
x^2*((g*(3*a^2*d*g + 6*a^2*e*f - b^2*d*g*n^2 + 3*b^2*e*f*n^2 - 6*a*b*e*f*n
))/(6*e) - (d*g^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(18*e)) + log(c*(d + e*x)
^n)^2*(b^2*f^2*x + (b^2*g^2*x^3)/3 + (d*(b^2*d^2*g^2 + 3*b^2*e^2*f^2 - 3*b
^2*d*e*f*g))/(3*e^3) + b^2*f*g*x^2) - (log(d + e*x)*(11*b^2*d^3*g^2*n^2 -
6*a*b*d^3*g^2*n + 18*b^2*d*e^2*f^2*n^2 - 18*a*b*d*e^2*f^2*n - 27*b^2*d^2*e
*f*g*n^2 + 18*a*b*d^2*e*f*g*n))/(9*e^3) + (g^2*x^3*(9*a^2 + 2*b^2*n^2 - 6*
a*b*n))/27
```

3.45. $\int (f+gx)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

3.46 $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

3.46.1 Optimal result

Integrand size = 22, antiderivative size = 186

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = -\frac{2ab(ef-dg)nx}{e} + \frac{2b^2(ef-dg)n^2x}{e} + \frac{b^2gn^2(d+ex)^2}{4e^2} - \frac{2b^2(ef-dg)n(d+ex)\log\left(c(d+ex)^n\right)}{e^2} - \frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}$$

output
```
-2*a*b*(-d*g+e*f)*n*x/e+2*b^2*(-d*g+e*f)*n^2*x/e+1/4*b^2*g*n^2*(e*x+d)^2/e
^2-2*b^2*(-d*g+e*f)*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2-1/2*b*g*n*(e*x+d)^2*(a+b
*ln(c*(e*x+d)^n))/e^2+(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2+1/2*g
*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2
```

3.46.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$
$$=\frac{4(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2+2g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2-8b(ef-dg)n(e(a-}{4e^2}$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(4*(e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 + 2*g*(d + e*x)^2*(a
 + b*Log[c*(d + e*x)^n])^2 - 8*b*(e*f - d*g)*n*(e*(a - b*n)*x + b*(d + e*x
)*Log[c*(d + e*x)^n]) + b*g*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*(a + b*
Log[c*(d + e*x)^n])))/(4*e^2)
```

3.46.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}-\frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}-\frac{2abnx(ef-dg)}{e}-\frac{2b^2n(d+ex)(ef-dg)\log\left(c(d+ex)^n\right)}{e^2}+\frac{b^2gn^2(d+ex)^2}{4e^2}+\frac{2b^2n^2x(ef-dg)}{e}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(-2*a*b*(e*f - d*g)*n*x)/e + (2*b^2*(e*f - d*g)*n^2*x)/e + (b^2*g*n^2*(d +
 e*x)^2)/(4*e^2) - (2*b^2*(e*f - d*g)*n*(d + e*x)*Log[c*(d + e*x)^n])/e^2
- (b*g*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2) + ((e*f - d*g)*(d
 + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 + (g*(d + e*x)^2*(a + b*Log[c*(d
 + e*x)^n])^2)/(2*e^2)
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(180) = 360$.

Time = 0.61 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.47

method	result
parallelrisch	$-\frac{2x^{2}\ln\left(c\left(ex+d\right)^{n}\right)b^{2}e^{2}gn-4x^{2}\ln\left(c\left(ex+d\right)^{n}\right)ab\,e^{2}g+8x\ln\left(c\left(ex+d\right)^{n}\right)b^{2}e^{2}fn-8x\ln\left(c\left(ex+d\right)^{n}\right)ab\,e^{2}f-8\ln\left(c\left(ex+d\right)^{n}\right)b^{2}def}{}$
risch	Expression too large to display

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*x^2*ln(c*(e*x+d)^n)*b^2*e^2*g*n-4*x^2*ln(c*(e*x+d)^n)*a*b*e^2*g+8*
x*ln(c*(e*x+d)^n)*b^2*e^2*f*n-8*x*ln(c*(e*x+d)^n)*a*b*e^2*f-8*ln(c*(e*x+d)
^n)*b^2*d*e*f*n+8*ln(c*(e*x+d)^n)*a*b*d*e*f-10*ln(e*x+d)*b^2*d^2*g*n^2-6*b
^2*d^2*g*n^2-b^2*e^2*g*n^2*x^2+8*b^2*d*e*f*n^2+16*ln(e*x+d)*b^2*d*e*f*n^2+
4*ln(e*x+d)*a*b*d^2*g*n-4*ln(c*(e*x+d)^n)^2*b^2*d*e*f+4*ln(c*(e*x+d)^n)*b^
2*d^2*g*n-2*x^2*ln(c*(e*x+d)^n)^2*b^2*e^2*g-4*x*ln(c*(e*x+d)^n)^2*b^2*e^2*
f+4*a*b*d^2*g*n+2*a*b*e^2*g*n*x^2+6*b^2*d*e*g*n^2*x+8*a*b*e^2*f*n*x-8*b^2*
e^2*f*n^2*x-8*a*b*d*e*f*n-4*a*b*d*e*g*n*x-16*ln(e*x+d)*a*b*d*e*f*n-2*a^2*e
^2*g*x^2-4*a^2*e^2*f*x+4*a^2*d*e*f-4*x*ln(c*(e*x+d)^n)*b^2*d*e*g*n+2*ln(c*
(e*x+d)^n)^2*b^2*d^2*g)/e^2
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(180) = 360$.

Time = 0.28 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.16

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$
$$=\frac{(b^2e^2gn^2-2\,abe^2gn+2\,a^2e^2g)x^2+2\,(b^2e^2gn^2x^2+2\,b^2e^2fn^2x+(2\,b^2def-b^2d^2g)n^2)\log\left(ex+d\right)^2+2\,(b}{}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
1/4*((b^2*e^2*g*n^2 - 2*a*b*e^2*g*n + 2*a^2*e^2*g)*x^2 + 2*(b^2*e^2*g*n^2*
x^2 + 2*b^2*e^2*f*n^2*x + (2*b^2*d*e*f - b^2*d^2*g)*n^2)*log(e*x + d)^2 +
2*(b^2*e^2*g*x^2 + 2*b^2*e^2*f*x)*log(c)^2 + 2*(2*a^2*e^2*f + (4*b^2*e^2*f
 - 3*b^2*d*e*g)*n^2 - 2*(2*a*b*e^2*f - a*b*d*e*g)*n)*x - 2*((4*b^2*d*e*f -
 3*b^2*d^2*g)*n^2 + (b^2*e^2*g*n^2 - 2*a*b*e^2*g*n)*x^2 - 2*(2*a*b*d*e*f -
 a*b*d^2*g)*n - 2*(2*a*b*e^2*f*n - (2*b^2*e^2*f - b^2*d*e*g)*n^2)*x - 2*(b
^2*e^2*g*n*x^2 + 2*b^2*e^2*f*n*x + (2*b^2*d*e*f - b^2*d^2*g)*n)*log(c))*lo
g(e*x + d) - 2*((b^2*e^2*g*n - 2*a*b*e^2*g)*x^2 - 2*(2*a*b*e^2*f - (2*b^2*
e^2*f - b^2*d*e*g)*n)*x)*log(c))/e^2
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(177) = 354$.

Time = 0.69 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.12

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$
$$=\begin{cases} a^2fx+\frac{a^2gx^2}{2}-\frac{abd^2g\log\left(c(d+ex)^n\right)}{e^2}+\frac{2abdf\log\left(c(d+ex)^n\right)}{e}+\frac{abdgnx}{e}-2abfnx+2abfx\log\left(c(d+ex)^n\right)-\frac{abgnx^2}{2} \\ \left(a+b\log\left(cd^n\right)\right)^2\left(fx+\frac{gx^2}{2}\right) \end{cases}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Piecewise((a**2*f*x + a**2*g*x**2/2 - a*b*d**2*g*log(c*(d + e*x)**n)/e**2
+ 2*a*b*d*f*log(c*(d + e*x)**n)/e + a*b*d*g*n*x/e - 2*a*b*f*n*x + 2*a*b*f*
x*log(c*(d + e*x)**n) - a*b*g*n*x**2/2 + a*b*g*x**2*log(c*(d + e*x)**n) +
3*b**2*d**2*g*n*log(c*(d + e*x)**n)/(2*e**2) - b**2*d**2*g*log(c*(d + e*x)
**n)**2/(2*e**2) - 2*b**2*d*f*n*log(c*(d + e*x)**n)/e + b**2*d*f*log(c*(d
+ e*x)**n)**2/e - 3*b**2*d*g*n**2*x/(2*e) + b**2*d*g*n*x*log(c*(d + e*x)**
n)/e + 2*b**2*f*n**2*x - 2*b**2*f*n*x*log(c*(d + e*x)**n) + b**2*f*x*log(c
*(d + e*x)**n)**2 + b**2*g*n**2*x**2/4 - b**2*g*n*x**2*log(c*(d + e*x)**n)
/2 + b**2*g*x**2*log(c*(d + e*x)**n)**2/2, Ne(e, 0)), ((a + b*log(c*d**n))
**2*(f*x + g*x**2/2), True))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.69

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{1}{2}\,b^2gx^2\log\left((ex+d)^nc\right)^2$$
$$-2\,abefn\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right)-\frac{1}{2}\,abegn\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)$$
$$+abgx^2\log\left((ex+d)^nc\right)+b^2fx\log\left((ex+d)^nc\right)^2+\frac{1}{2}\,a^2gx^2+2\,abfx\log\left((ex+d)^nc\right)$$
$$-\left(2\,en\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right)\log\left((ex+d)^nc\right)+\frac{\left(d\log\left(ex+d\right)^2-2\,ex+2\,d\log\left(ex+d\right)\right)n^2}{e}\right)b^2f$$
$$-\frac{1}{4}\left(2\,en\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)\log\left((ex+d)^nc\right)-\frac{\left(e^2x^2+2\,d^2\log\left(ex+d\right)^2-6\,dex+6\,d^2\,\mathrm{l}\right.}{e^2}\right.$$
$$+a^2fx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*g*x^2*log((e*x + d)^n*c)^2 - 2*a*b*e*f*n*(x/e - d*log(e*x + d)/e^2
) - 1/2*a*b*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + a*b*g*x
^2*log((e*x + d)^n*c) + b^2*f*x*log((e*x + d)^n*c)^2 + 1/2*a^2*g*x^2 + 2*a
*b*f*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d
)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2*f - 1/4*
(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) -
 (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*
b^2*g + a^2*f*x
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(180) = 360$.

Time = 0.38 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.13

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{(ex+d)b^2fn^2\log(ex+d)^2}{e} + \frac{(ex+d)^2b^2gn^2\log(ex+d)^2}{2\,e^2} - \frac{(ex+d)b^2dgn^2\log(ex+d)^2}{e^2} - \frac{2\,(ex+d)b^2fn^2\log(ex+d)}{e} - \frac{(ex+d)^2b^2gn^2\log(ex+d)}{2\,e^2} + \frac{2\,(ex+d)b^2dgn^2\log(ex+d)}{e^2} + \frac{2\,(ex+d)b^2fn\log(ex+d)\log(c)}{e} + \frac{(ex+d)^2b^2gn\log(ex+d)\log(c)}{e^2} - \frac{2\,(ex+d)b^2dgn\log(ex+d)\log(c)}{e^2} + \frac{2\,(ex+d)b^2fn^2}{e} + \frac{(ex+d)^2b^2gn^2}{4\,e^2} - \frac{2\,(ex+d)b^2dgn^2}{e^2} + \frac{2\,(ex+d)abfn\log(ex+d)}{e} + \frac{(ex+d)^2abgn\log(ex+d)}{e^2} - \frac{2\,(ex+d)abdgn\log(ex+d)}{e^2} - \frac{2\,(ex+d)b^2fn\log(c)}{e} - \frac{(ex+d)^2b^2gn\log(c)}{2\,e^2} + \frac{2\,(ex+d)b^2dgn\log(c)}{e^2} + \frac{(ex+d)b^2f\log(c)^2}{e} + \frac{(ex+d)^2b^2g\log(c)^2}{2\,e^2} - \frac{(ex+d)b^2dg\log(c)^2}{e^2} - \frac{2\,(ex+d)abfn}{e} - \frac{(ex+d)^2abgn}{2\,e^2} + \frac{2\,(ex+d)abdgn}{e^2} + \frac{2\,(ex+d)abf\log(c)}{e} + \frac{(ex+d)^2abg\log(c)}{e^2} - \frac{2\,(ex+d)abdg\log(c)}{e^2} + \frac{(ex+d)a^2f}{e} + \frac{(ex+d)^2a^2g}{2\,e^2} - \frac{(ex+d)a^2dg}{e^2}$$

3.46. $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
(e*x + d)*b^2*f*n^2*log(e*x + d)^2/e + 1/2*(e*x + d)^2*b^2*g*n^2*log(e*x +
 d)^2/e^2 - (e*x + d)*b^2*d*g*n^2*log(e*x + d)^2/e^2 - 2*(e*x + d)*b^2*f*n
^2*log(e*x + d)/e - 1/2*(e*x + d)^2*b^2*g*n^2*log(e*x + d)/e^2 + 2*(e*x +
d)*b^2*d*g*n^2*log(e*x + d)/e^2 + 2*(e*x + d)*b^2*f*n*log(e*x + d)*log(c)/
e + (e*x + d)^2*b^2*g*n*log(e*x + d)*log(c)/e^2 - 2*(e*x + d)*b^2*d*g*n*lo
g(e*x + d)*log(c)/e^2 + 2*(e*x + d)*b^2*f*n^2/e + 1/4*(e*x + d)^2*b^2*g*n^
2/e^2 - 2*(e*x + d)*b^2*d*g*n^2/e^2 + 2*(e*x + d)*a*b*f*n*log(e*x + d)/e +
 (e*x + d)^2*a*b*g*n*log(e*x + d)/e^2 - 2*(e*x + d)*a*b*d*g*n*log(e*x + d)
/e^2 - 2*(e*x + d)*b^2*f*n*log(c)/e - 1/2*(e*x + d)^2*b^2*g*n*log(c)/e^2 +
 2*(e*x + d)*b^2*d*g*n*log(c)/e^2 + (e*x + d)*b^2*f*log(c)^2/e + 1/2*(e*x
+ d)^2*b^2*g*log(c)^2/e^2 - (e*x + d)*b^2*d*g*log(c)^2/e^2 - 2*(e*x + d)*a
*b*f*n/e - 1/2*(e*x + d)^2*a*b*g*n/e^2 + 2*(e*x + d)*a*b*d*g*n/e^2 + 2*(e*
x + d)*a*b*f*log(c)/e + (e*x + d)^2*a*b*g*log(c)/e^2 - 2*(e*x + d)*a*b*d*g
*log(c)/e^2 + (e*x + d)*a^2*f/e + 1/2*(e*x + d)^2*a^2*g/e^2 - (e*x + d)*a^
2*d*g/e^2
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.44

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$
$$= \ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{b^2\,g\,x^2}{2}-\frac{d\,(b^2\,d\,g-2\,b^2\,e\,f)}{2\,e^2}+b^2\,f\,x\right)+x\left(\frac{2\,a^2\,d\,g+2\,a^2\,e\,f-2\,b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-4\,a\,b\,e\,f\,n}{2\,e}-\frac{d\,g\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{2\,e}\right)+\ln\left(c\,(d+e\,x)^n\right)\left(\frac{b\,g\,(2\,a-b\,n)\,x^2}{2}+\left(\frac{2\,b\,(a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}-\frac{b\,d\,g\,(2\,a-b\,n)}{e}\right)x\right)+\frac{g\,x^2\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{4}+\frac{\ln(d+e\,x)\,(3\,g\,b^2\,d^2\,n^2-4\,e\,f\,b^2\,d\,n^2-2\,a\,g\,b\,d^2\,n+4\,a\,e\,f\,b\,d\,n)}{2\,e^2}$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
log(c*(d + e*x)^n)^2*((b^2*g*x^2)/2 - (d*(b^2*d*g - 2*b^2*e*f))/(2*e^2) +
b^2*f*x) + x*((2*a^2*d*g + 2*a^2*e*f - 2*b^2*d*g*n^2 + 4*b^2*e*f*n^2 - 4*a
*b*e*f*n)/(2*e) - (d*g*(2*a^2 + b^2*n^2 - 2*a*b*n))/(2*e)) + log(c*(d + e*
x)^n)*(x*((2*b*(a*d*g + a*e*f - b*e*f*n))/e - (b*d*g*(2*a - b*n))/e) + (b*
g*x^2*(2*a - b*n))/2) + (g*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/4 + (log(d + e
*x)*(3*b^2*d^2*g*n^2 - 4*b^2*d*e*f*n^2 - 2*a*b*d^2*g*n + 4*a*b*d*e*f*n))/(
2*e^2)
```

3.46. $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

3.47 $\int (a+b\log(c(d+ex)^n))^2\,dx$

3.47.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int (a+b\log(c(d+ex)^n))^2\,dx = -2abnx+2b^2n^2x-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}$$

output
```
-2*a*b*n*x+2*b^2*n^2*x-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e+(e*x+d)*(a+b*ln(c
*(e*x+d)^n))^2/e
```

3.47.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-2bn\left(ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - 2*b*n*(a*x - b*n*x + (b*(d +
e*x)*Log[c*(d + e*x)^n])/e)
```

3.47.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2836, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^2\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^2\,d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2-2bn\int (a+b\log(c(d+ex)^n))\,d(d+ex)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2-2bn(a(d+ex)+b(d+ex)\log(c(d+ex)^n)-bn(d+ex))}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(a*(d + e*x) - b*n*(d + e*
x) + b*(d + e*x)*Log[c*(d + e*x)^n]))/e
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.47.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

method	result
norman	$\left(2b^2n^2-2abn+a^2\right)x+b^2x\ln\left(c\,e^{n\ln(ex+d)}\right)^2+\left(-2b^2n+2ab\right)x\ln\left(c\,e^{n\ln(ex+d)}\right)+\frac{b^2d\ln\left(c\,e^{n\ln\dots}\right)}{e}$
default	$a^2x+b^2x\ln\left(c\,e^{n\ln(ex+d)}\right)^2+\frac{b^2d\ln\left(c\,e^{n\ln(ex+d)}\right)^2}{e}+2b^2n^2x-2b^2nx\ln\left(c\,e^{n\ln(ex+d)}\right)-\frac{2n^2b^2d\ln(e\dots}{e}$
parts	$a^2x+b^2x\ln\left(c\,e^{n\ln(ex+d)}\right)^2+\frac{b^2d\ln\left(c\,e^{n\ln(ex+d)}\right)^2}{e}+2b^2n^2x-2b^2nx\ln\left(c\,e^{n\ln(ex+d)}\right)-\frac{2n^2b^2d\ln(e\dots}{e}$
parallelrisch	$\frac{x\ln(c(ex+d)^n)^2b^2den-2x\ln(c(ex+d)^n)b^2de\,n^2+2x\,b^2de\,n^3+2x\ln(c(ex+d)^n)abden-2xabde\,n^2+\ln(c(ex+d)^n)^2b^2d^2n-2\ln(\dots}{end}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b^2*n^2-2*a*b*n+a^2)*x+b^2*x*ln(c*exp(n*ln(e*x+d)))^2+(-2*b^2*n+2*a*b)*
x*ln(c*exp(n*ln(e*x+d)))+b^2*d/e*ln(c*exp(n*ln(e*x+d)))^2+n*(-2*b^2*d*n+2*
a*b*d)/e*ln(e*x+d)
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.15

$$\int (a+b\log(c(d+ex)^n))^2\,dx$$
$$=\frac{b^2ex\log(c)^2+(b^2en^2x+b^2dn^2)\log(ex+d)^2-2(b^2en-abe)x\log(c)+(2b^2en^2-2aben+a^2e)x-2(b\dots}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
(b^2*e*x*log(c)^2 + (b^2*e*n^2*x + b^2*d*n^2)*log(e*x + d)^2 - 2*(b^2*e*n
- a*b*e)*x*log(c) + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*e)*x - 2*(b^2*d*n^2 - a
*b*d*n + (b^2*e*n^2 - a*b*e*n)*x - (b^2*e*n*x + b^2*d*n)*log(c))*log(e*x +
 d))/e
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (a+b\log (c(d+ex)^n))^2\,dx$$
$$= \begin{cases} a^2x + \frac{2abd\log (c(d+ex)^n)}{e} - 2abnx + 2abx\log (c(d+ex)^n) - \frac{2b^2dn\log (c(d+ex)^n)}{e} + \frac{b^2d\log (c(d+ex)^n)^2}{e} + 2b^2n^2x - \\ x(a+b\log (cd^n))^2 \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*d*log(c*(d + e*x)**n)/e - 2*a*b*n*x + 2*a*b*x*lo
g(c*(d + e*x)**n) - 2*b**2*d*n*log(c*(d + e*x)**n)/e + b**2*d*log(c*(d + e
*x)**n)**2/e + 2*b**2*n**2*x - 2*b**2*n*x*log(c*(d + e*x)**n) + b**2*x*log
(c*(d + e*x)**n)**2, Ne(e, 0)), (x*(a + b*log(c*d**n))**2, True))
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int (a+b\log (c(d+ex)^n))^2\,dx$$
$$= -2\,aben\left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right) + b^2x\log ((ex+d)^nc)^2 + 2\,abx\log ((ex+d)^nc) - \left(2\,en\left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right)\log ((ex+d)^nc) + \frac{(d\log (ex+d)^2 - 2\,ex + 2\,d\log (ex+d))n^2}{e}\right)b^2 + a^2x$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-2*a*b*e*n*(x/e - d*log(e*x + d)/e^2) + b^2*x*log((e*x + d)^n*c)^2 + 2*a*b
*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*
c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2 + a^2*x
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(65) = 130$.

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.68

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(ex+d)b^2n^2\log(ex+d)^2}{e} - \frac{2(ex+d)b^2n^2\log(ex+d)}{e} + \frac{2(ex+d)b^2n\log(ex+d)\log(c)}{e} + \frac{2(ex+d)b^2n^2}{e} + \frac{2(ex+d)abn\log(ex+d)}{e} - \frac{2(ex+d)b^2n\log(c)}{e} + \frac{(ex+d)b^2\log(c)^2}{e} - \frac{2(ex+d)abn}{e} + \frac{2(ex+d)ab\log(c)}{e} + \frac{(ex+d)a^2}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
(e*x + d)*b^2*n^2*log(e*x + d)^2/e - 2*(e*x + d)*b^2*n^2*log(e*x + d)/e +
2*(e*x + d)*b^2*n*log(e*x + d)*log(c)/e + 2*(e*x + d)*b^2*n^2/e + 2*(e*x +
 d)*a*b*n*log(e*x + d)/e - 2*(e*x + d)*b^2*n*log(c)/e + (e*x + d)*b^2*log(
c)^2/e - 2*(e*x + d)*a*b*n/e + 2*(e*x + d)*a*b*log(c)/e + (e*x + d)*a^2/e
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int (a+b\log(c(d+ex)^n))^2\,dx = x\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right) + \ln\left(c\,(d+e\,x)^n\right)^2\left(b^2\,x+\frac{b^2\,d}{e}\right) - \frac{\ln(d+e\,x)\,\left(2\,b^2\,d\,n^2-2\,a\,b\,d\,n\right)}{e} + 2\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,(a-b\,n)$$

input
```
int((a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + log(c*(d + e*x)^n)^2*(b^2*x + (b^2*d)/e) -
 (log(d + e*x)*(2*b^2*d*n^2 - 2*a*b*d*n))/e + 2*b*x*log(c*(d + e*x)^n)*(a
- b*n)
```

3.48 $\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$

3.48.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output
```
(a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g+2*b*n*(a+b*ln(c*(e*x+d)
^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-2*b^2*n^2*polylog(3,-g*(e*x+d)/(-d
*g+e*f))/g
```

3.48.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx)+2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\log\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 2*b*n*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[(e*(f + g*x))/
(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^2*n^2*(Log[d
+ e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d
+ e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

3.48.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}dx$$

$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2ben\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\left(bn\int\frac{\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right)}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g} - \frac{2bn\left(bn\,\mathrm{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)-\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right)}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g - (2*b*n*(
-((a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]) + b
*n*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))]))/g
```

3.48.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.48.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 737, normalized size of antiderivative = 6.64

method	result
risch	$\frac{b^2 \ln(g(ex+d)-dg+ef)\ln(ex+d)^2 n^2}{g} - \frac{2b^2 \ln(g(ex+d)-dg+ef)\ln((ex+d)^n)\ln(ex+d)n}{g} + \frac{b^2 \ln(g(ex+d)-dg+ef)\ln((ex+d)^n)^2}{g}$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^2*n^2-2*b^2*ln(g*(e*x+d)-d*g+e*f)/g*
ln((e*x+d)^n)*ln(e*x+d)*n+b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2+b^2*
n^2/g*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*b^2*n^2/g*ln(e*x+d)*polylog(
2,g*(e*x+d)/(d*g-e*f))-2*b^2*n^2/g*polylog(3,g*(e*x+d)/(d*g-e*f))-2*b^2*n^
2*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)+2*b^2*n*dilog((g*(e*x+
d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)-2*b^2*n^2*ln(e*x+d)^2*ln((g*(e*x+d
)-d*g+e*f)/(-d*g+e*f))/g+2*b^2*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*
f))/g*ln((e*x+d)^n)+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^
n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*
(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(ln((e*x+d)^n
)*ln(g*x+f)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln
(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e))+1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(
I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*
(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+
2*a)^2*ln(g*x+f)/g
```

3.48.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x
+ f), x)
```

3.48.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x), x)
```

3.48.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")
```

output
```
a^2*log(g*x + f)/g + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*
a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x + f), x)
```

3.48.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x), x)
```

3.49 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx$

3.49.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(f+gx)} - \frac{2ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{2b^2en^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/(-d*g+e*f)/(g*x+f)-2*b*e*n*(a+b*ln(c*(e*x+
d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)-2*b^2*e*n^2*polylog(2,-g*(e*x
+d)/(-d*g+e*f))/g/(-d*g+e*f)
```

3.49.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \frac{-\left((a+b\log(c(d+ex)^n))\left(ag(d+ex)+bg(d+ex)\log(c(d+ex)^n)-2ben(f+gx)\log\left(\frac{e(f+gx)}{ef-dg}\right)\right)\right)+}{g(-ef+dg)(f+gx)}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^2,x]
```

output
```
(-((a + b*Log[c*(d + e*x)^n])*(a*g*(d + e*x) + b*g*(d + e*x)*Log[c*(d + e*
x)^n] - 2*b*e*n*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)])) + 2*b^2*e*n^2*(
f + g*x)*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/(g*(-(e*f) + d*g)*(f +
g*x))
```

3.49.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2844, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2844}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(f+gx)(ef-dg)}-\frac{2ben\int\frac{a+b\log(c(d+ex)^n)}{f+gx}dx}{ef-dg}$$

$$\Big\downarrow \text{2841}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(f+gx)(ef-dg)}-\frac{2ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{ben\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}\right)}{ef-dg}$$

$$\Big\downarrow \text{2840}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(f+gx)(ef-dg)}-\frac{2ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{bn\int\frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)}{d+ex}d(d+ex)}{g}\right)}{ef-dg}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(f+gx)(ef-dg)}-\frac{2ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}\right)}{ef-dg}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/((e*f - d*g)*(f + g*x)) - (2*b*e*
n*(((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*Po
lyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g))/(e*f - d*g)
```

3.49.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

3.49.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.07

method	result
risch	$-\frac{b^2 \ln((ex+d)^n)^2}{(gx+f)g} - \frac{2b^2 ne \ln((ex+d)^n) \ln(ex+d)}{g(dg-ef)} + \frac{2b^2 ne \ln((ex+d)^n) \ln(gx+f)}{g(dg-ef)} - \frac{2b^2 n^2 e \,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g(dg-ef)} - \frac{2b^2 n^2}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2*ln((e*x+d)^n)^2/(g*x+f)/g-2*b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)*ln(e*x+
d)+2*b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)*ln(g*x+f)-2*b^2/g*n^2*e/(d*g-e*f)*d
ilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-2*b^2/g*n^2*e/(d*g-e*f)*ln(g*x+f)*ln((
(g*x+f)*e+d*g-e*f)/(d*g-e*f))+b^2/g*n^2*e/(d*g-e*f)*ln(e*x+d)^2+(-I*b*Pi*c
sgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*
x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(
e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(-ln((e*x+d)^n)/(g*x+f)/g+1/g*n*e*(-1/(d*g-
e*f)*ln(e*x+d)+1/(d*g-e*f)*ln(g*x+f)))-1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*cs
gn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(
c)+2*a)^2/(g*x+f)/g
```

3.49.5 Fricas [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(b\log ((ex+d)^n c)+a)^2}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*
x^2 + 2*f*g*x + f^2), x)
```

3.49.6 Sympy [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(a+b\log (c(d+ex)^n))^2}{(f+gx)^2}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**2, x)
```

3.49.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
2*a*b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) -
b^2*(log((e*x + d)^n)^2/(g^2*x + f*g) - integrate((e*g*x*log(c)^2 + d*g*lo
g(c)^2 + 2*(e*f*n + d*g*log(c) + (e*g*n + e*g*log(c))*x)*log((e*x + d)^n))
/(e*g^3*x^3 + d*f^2*g + (2*e*f*g^2 + d*g^3)*x^2 + (e*f^2*g + 2*d*f*g^2)*x)
, x)) - 2*a*b*log((e*x + d)^n*c)/(g^2*x + f*g) - a^2/(g^2*x + f*g)
```

3.49.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^2, x)
```

3.50 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx$

3.50.1 Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = -\frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^2(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g(f+gx)^2} + \frac{b^2e^2n^2\log(f+gx)}{g(ef-dg)^2} - \frac{be^2n(a+b\log\left(c(d+ex)^n\right))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} + \frac{b^2e^2n^2\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}$$

output
```
-b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)^2/(g*x+f)-1/2*(a+b*ln(c*(e
*x+d)^n))^2/g/(g*x+f)^2+b^2*e^2*n^2*ln(g*x+f)/g/(-d*g+e*f)^2-b*e^2*n*(a+b*
ln(c*(e*x+d)^n))*ln(1+(-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^2+b^2*e^2*n^2*pol
ylog(2,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^2
```

3.50.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx$$
$$= \frac{-(a+b\log(c(d+ex)^n))^2 + \frac{e(f+gx)\left(2b(ef-dg)n(a+b\log(c(d+ex)^n))+e(f+gx)(a+b\log(c(d+ex)^n))^2-2b^2en^2(f+gx)(\log(d+ex)-\right.}{(ef-d}}{2g(f+gx)^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^3,x]
```

output
```
(-(a + b*Log[c*(d + e*x)^n])^2 + (e*(f + g*x)*(2*b*(e*f - d*g)*n*(a + b*Lo
g[c*(d + e*x)^n]) + e*(f + g*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b^2*e*n^2
*(f + g*x)*(Log[d + e*x] - Log[f + g*x]) - 2*b*e*n*(f + g*x)*(a + b*Log[c*
(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - 2*b^2*e*n^2*(f + g*x)*PolyL
og[2, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*f - d*g)^2)/(2*g*(f + g*x)^2)
```

3.50.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2845, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^2}dx}{g} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2858}$$

$$\frac{bn\int \frac{e^2(a+b\log(c(d+ex)^n))}{(d+ex)\left(e\left(f-\frac{dg}{e}\right)+g(d+ex)\right)^2}d(d+ex)}{g} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{be^2n\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2789$$

$$\frac{be^2n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2751$$

$$\frac{be^2n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\int \frac{1}{ef-dg+g(d+ex)}d(d+ex)}{ef-dg}\right)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{be^2n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\log(g(d+ex)-dg+ef)}{g(ef-dg)}\right)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2779$$

$$\frac{be^2n\left(\frac{\frac{bn\int \frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)}{d+ex}d(d+ex)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\log(g(d+ex)-dg+ef)}{g(ef-dg)}\right)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{be^2n\left(\frac{\frac{bn\,\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\log(g(d+ex)-dg+ef)}{g(ef-dg)}\right)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}$$

3.50. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n])^2/(g*(f + g*x)^2) + (b*e^2*n*(-((g*(((d +
e*x)*(a + b*Log[c*(d + e*x)^n]))/((e*f - d*g)*(e*f - d*g + g*(d + e*x))) -
 (b*n*Log[e*f - d*g + g*(d + e*x)])/(g*(e*f - d*g))))/(e*f - d*g)) + (-(((
a + b*Log[c*(d + e*x)^n])*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(e*f - d*g))
 + (b*n*PolyLog[2, -((e*f - d*g)/(g*(d + e*x)))])/(e*f - d*g))/(e*f - d*g)
))/g
```

3.50.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.50.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.27

method	result
risch	$-\frac{b^2 \ln((ex+d)^n)^2}{2(gx+f)^2 g}+\frac{b^2 n e^2 \ln((ex+d)^n) \ln(ex+d)}{g(dg-ef)^2}-\frac{b^2 n e \ln((ex+d)^n)}{g(dg-ef)(gx+f)}-\frac{b^2 n e^2 \ln((ex+d)^n) \ln(gx+f)}{g(dg-ef)^2}-\frac{b^2 n^2 e^2 \ln(ex+d)^2}{2g(dg-ef)^2}$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*ln((e*x+d)^n)^2/(g*x+f)^2/g+b^2/g*n*e^2*ln((e*x+d)^n)/(d*g-e*f)^2
*ln(e*x+d)-b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)/(g*x+f)-b^2/g*n*e^2*ln((e*x+d
)^n)/(d*g-e*f)^2*ln(g*x+f)-1/2*b^2/g*n^2*e^2/(d*g-e*f)^2*ln(e*x+d)^2-b^2/g
*n^2*e^2/(d*g-e*f)^2*ln(e*x+d)+b^2/g*n^2*e^2/(d*g-e*f)^2*ln(g*x+f)+b^2/g*n
^2*e^2/(d*g-e*f)^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b^2/g*n^2*e^2/(d*g
-e*f)^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-I*b*Pi*csgn(I*c*(e*x
+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*
b+2*b*ln(c)+2*a)*b*(-1/2*ln((e*x+d)^n)/(g*x+f)^2/g+1/2/g*n*e*(e/(d*g-e*f)^
2*ln(e*x+d)-1/(d*g-e*f)/(g*x+f)-e/(d*g-e*f)^2*ln(g*x+f)))-1/8*(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*
x+d)^n)^3*b+2*b*ln(c)+2*a)^2/(g*x+f)^2/g
```

3.50.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^3*
x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

3.50.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**3, x)
```

3.50.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^3,x, algorithm="maxima")
```

output
```
a*b*e*n*(e*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - e*log(g*x +
f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) + 1/(e*f^2*g - d*f*g^2 + (e*f*g^2 -
 d*g^3)*x)) - 1/2*b^2*(log((e*x + d)^n)^2/(g^3*x^2 + 2*f*g^2*x + f^2*g) -
2*integrate((e*g*x*log(c)^2 + d*g*log(c)^2 + (e*f*n + 2*d*g*log(c) + (e*g*
n + 2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^4*x^4 + d*f^3*g + (3*e*f*g^3 +
 d*g^4)*x^3 + 3*(e*f^2*g^2 + d*f*g^3)*x^2 + (e*f^3*g + 3*d*f^2*g^2)*x), x)
) - a*b*log((e*x + d)^n*c)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*a^2/(g^3*x^
2 + 2*f*g^2*x + f^2*g)
```

3.50.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^3, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = \int \frac{(a+b\ln(c(d+e\,x)^n))^2}{(f+g\,x)^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^3,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^3, x)
```

3.51 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$

3.51.1 Optimal result

Integrand size = 24, antiderivative size = 317

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx &= -\frac{b^2e^2n^2}{3g(ef-dg)^2(f+gx)}-\frac{b^2e^3n^2\log(d+ex)}{3g(ef-dg)^3}\\ &\quad+\frac{ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)(f+gx)^2}\\ &\quad-\frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3(ef-dg)^3(f+gx)}\\ &\quad-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}+\frac{b^2e^3n^2\log(f+gx)}{g(ef-dg)^3}\\ &\quad-\frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3}\\ &\quad+\frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3}\end{aligned}$$

output

```
-1/3*b^2*e^2*n^2/g/(-d*g+e*f)^2/(g*x+f)-1/3*b^2*e^3*n^2*ln(e*x+d)/g/(-d*g+
e*f)^3+1/3*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)/(g*x+f)^2-2/3*b*e^2*n*
(e*x+d)*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)^3/(g*x+f)-1/3*(a+b*ln(c*(e*x+d)^n
))^2/g/(g*x+f)^3+b^2*e^3*n^2*ln(g*x+f)/g/(-d*g+e*f)^3-2/3*b*e^3*n*(a+b*ln(
c*(e*x+d)^n))*ln(1+(-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^3+2/3*b^2*e^3*n^2*po
lylog(2,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^3
```

3.51.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$$
$$= \frac{-(a+b\log(c(d+ex)^n))^2 + \frac{e(f+gx)\left(b(ef-dg)^2n(a+b\log(c(d+ex)^n))+2be(ef-dg)n(f+gx)(a+b\log(c(d+ex)^n))+e^2(f+gx)^2(a+b}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^4,x]
```

output
```
(-(a + b*Log[c*(d + e*x)^n])^2 + (e*(f + g*x)*(b*(e*f - d*g)^2*n*(a + b*Lo
g[c*(d + e*x)^n]) + 2*b*e*(e*f - d*g)*n*(f + g*x)*(a + b*Log[c*(d + e*x)^n
]) + e^2*(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2 - 2*b^2*e^2*n^2*(f + g*x
)^2*(Log[d + e*x] - Log[f + g*x]) - b^2*e*n^2*(f + g*x)*(e*f - d*g + e*(f
+ g*x)*Log[d + e*x] - e*(f + g*x)*Log[f + g*x]) - 2*b*e^2*n*(f + g*x)^2*(a
 + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - 2*b^2*e^2*n^2*(f
 + g*x)^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*f - d*g)^3)/(3*g*(
f + g*x)^3)
```

3.51.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^3}dx}{3g}-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2bn\int \frac{e^3(a+b\log(c(d+ex)^n))}{(d+ex)\left(e\left(f-\frac{dg}{e}\right)+g(d+ex)\right)^3}d(d+ex)}{3g}-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2be^3n\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^3}d(d+ex)}{3g}-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \; 2789$$

$$\frac{2be^3n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{(ef-dg+g(d+ex))^3}d(d+ex)}{ef-dg}\right)}{3g}-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \; 2756$$

$$\frac{2be^3n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}-\frac{g\left(\frac{bn\int \frac{1}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{2g}-\frac{a+b\log(c(d+ex)^n)}{2g(g(d+ex)-dg+ef)^2}\right)}{ef-dg}\right)}{3g}-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \; 54$$

$$\frac{2be^3n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}-\frac{g\left(\frac{bn\int \left(-\frac{g}{(ef-dg)^2(ef-dg+g(d+ex))}-\frac{g}{(ef-dg)(ef-dg+g(d+ex))^2}+\frac{1}{(ef-dg)^2(d+ex)}\right)d(d+ex)}{2g}-\frac{a+}{2g(g}\right.}{ef-dg}\right.}{3g}$$

$$\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{2be^3n\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{1}{(ef-dg)(g(d+ex)-dg+ef)}+\frac{\log(d+ex)}{(ef-dg)^2}-\frac{\log(g(d+ex)-dg+ef)}{(ef-dg)^2}\right)}{2g}-\frac{a+b\log(c(d+ex)^n)}{2g(g(d+ex)-dg+ef)^2}\right)}{ef-dg}\right)}{3g}-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

$$\Big\downarrow \; 2789$$

3.51. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$

$$\frac{2be^3n\left(\frac{\frac{\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log\left(c(d+ex)^n\right)}{(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{1}{(ef-dg)(g(d+ex)-dg+ef)}+\frac{\log(d+ex)}{(ef-dg)^2}-\frac{\log(g(d+ex)-dg+ef)}{(ef-dg)^2}\right)}{2g}-\cdots\right.}{ef-dg}\cdots\right.}{3g}\cdots$$

$$\frac{(a+b\log\left(c(d+ex)^n\right))^2}{3g(f+gx)^3}$$

$\downarrow$ 2751

$$\frac{2be^3n\left(\frac{\frac{\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\int \frac{1}{ef-dg+g(d+ex)}d(d+ex)}{ef-dg}\right)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{1}{(ef-dg)(g(d+ex)-dg+ef)}+\cdots\right.}{2g}\cdots\right.}{\cdots}\cdots\right.}{3g}\cdots$$

$$\frac{(a+b\log\left(c(d+ex)^n\right))^2}{3g(f+gx)^3}$$

$\downarrow$ 16

$$\frac{2be^3n\left(\frac{\frac{\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\log(g(d+ex)-dg+ef)}{g(ef-dg)}\right)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{1}{(ef-dg)(g(d+ex)-dg+ef)}+\frac{\log(d+ex)}{(ef-dg)^2}\cdots\right.}{2g}\cdots\right.}{\cdots}\cdots\right.}{3g}\cdots$$

$$\frac{(a+b\log\left(c(d+ex)^n\right))^2}{3g(f+gx)^3}$$

$\downarrow$ 2779

$$\frac{2be^3n\left(\frac{\frac{\frac{bn\int \frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)}{d+ex}d(d+ex)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\log(g(d+ex)-dg+ef)}{g(ef-dg)}\right)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{3g}\cdots$$

$$\frac{(a+b\log\left(c(d+ex)^n\right))^2}{3g(f+gx)^3}$$

$\downarrow$ 2838

3.51. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$

$$\frac{2be^3n\left(\frac{\frac{\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{bn\log(g(d+ex)-dg+ef)}{g(ef-dg)}\right)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{\cdots}{(ef-\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right)}{3g}$$

$$-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^4,x]
```

output
```
-1/3*(a + b*Log[c*(d + e*x)^n])^2/(g*(f + g*x)^3) + (2*b*e^3*n*(-((g*(-1/2
*(a + b*Log[c*(d + e*x)^n])/(g*(e*f - d*g + g*(d + e*x))^2) + (b*n*(1/((e*
f - d*g)*(e*f - d*g + g*(d + e*x))) + Log[d + e*x]/(e*f - d*g)^2 - Log[e*f
 - d*g + g*(d + e*x)]/(e*f - d*g)^2))/(2*g)))/(e*f - d*g)) + (-((g*(((d +
e*x)*(a + b*Log[c*(d + e*x)^n]))/((e*f - d*g)*(e*f - d*g + g*(d + e*x))) -
 (b*n*Log[e*f - d*g + g*(d + e*x)])/(g*(e*f - d*g))))/(e*f - d*g)) + (-(((
a + b*Log[c*(d + e*x)^n])*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(e*f - d*g))
 + (b*n*PolyLog[2, -((e*f - d*g)/(g*(d + e*x)))])/(e*f - d*g))/(e*f - d*g)
)/(e*f - d*g)))/(3*g)
```

3.51.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

3.51. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.51.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.38

method	result
risch	$-\frac{b^2 \ln((ex+d)^n)^2}{3(gx+f)^3 g} - \frac{2b^2 n e^3 \ln((ex+d)^n) \ln(ex+d)}{3g(dg-ef)^3} - \frac{b^2 n e \ln((ex+d)^n)}{3g(dg-ef)(gx+f)^2} + \frac{2b^2 n e^3 \ln((ex+d)^n) \ln(gx+f)}{3g(dg-ef)^3} + \frac{2b^2 n e^2 \ln((ex\cdots}{3g(dg-ef)^2(g\cdots}$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*ln((e*x+d)^n)^2/(g*x+f)^3/g-2/3*b^2/g*n*e^3*ln((e*x+d)^n)/(d*g-e*
f)^3*ln(e*x+d)-1/3*b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)/(g*x+f)^2+2/3*b^2/g*n
*e^3*ln((e*x+d)^n)/(d*g-e*f)^3*ln(g*x+f)+2/3*b^2/g*n*e^2*ln((e*x+d)^n)/(d*
g-e*f)^2/(g*x+f)+b^2/g*n^2*e^3/(d*g-e*f)^3*ln(e*x+d)-1/3*b^2/g*n^2*e^2/(d*
g-e*f)^2/(g*x+f)-b^2/g*n^2*e^3/(d*g-e*f)^3*ln(g*x+f)+1/3*b^2/g*n^2*e^3/(d*
g-e*f)^3*ln(e*x+d)^2-2/3*b^2/g*n^2*e^3/(d*g-e*f)^3*dilog(((g*x+f)*e+d*g-e*
f)/(d*g-e*f))-2/3*b^2/g*n^2*e^3/(d*g-e*f)^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*
f)/(d*g-e*f))+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*P
i*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(-1/3*ln((e*x+d)^n)
/(g*x+f)^3/g+1/3/g*n*e*(-e^2/(d*g-e*f)^3*ln(e*x+d)-1/2/(d*g-e*f)/(g*x+f)^2
+e^2/(d*g-e*f)^3*ln(g*x+f)+e/(d*g-e*f)^2/(g*x+f)))-1/12*(-I*b*Pi*csgn(I*c*
(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^
2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n
)^3*b+2*b*ln(c)+2*a)^2/(g*x+f)^3/g
```

3.51.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^4,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^4*
x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

3.51.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**4,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**4, x)
```

3.51.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/3*(2*e^2*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3
*g^4) - 2*e^2*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 -
d^3*g^4) + (2*e*g*x + 3*e*f - d*g)/(e^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^
3 + (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2
*g^3 + d^2*f*g^4)*x))*a*b*e*n - 1/3*b^2*(log((e*x + d)^n)^2/(g^4*x^3 + 3*f
*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 3*integrate(1/3*(3*e*g*x*log(c)^2 + 3*d*
g*log(c)^2 + 2*(e*f*n + 3*d*g*log(c) + (e*g*n + 3*e*g*log(c))*x)*log((e*x
+ d)^n))/(e*g^5*x^5 + d*f^4*g + (4*e*f*g^4 + d*g^5)*x^4 + 2*(3*e*f^2*g^3 +
 2*d*f*g^4)*x^3 + 2*(2*e*f^3*g^2 + 3*d*f^2*g^3)*x^2 + (e*f^4*g + 4*d*f^3*g
^2)*x), x)) - 2/3*a*b*log((e*x + d)^n*c)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^
2*x + f^3*g) - 1/3*a^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

3.51.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^4,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^4, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^4}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^4,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^4, x)
```

3.52 $\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx$

3.52.1 Optimal result

Integrand size = 24, antiderivative size = 598

$$
\begin{aligned}
&\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx \\
&= \frac{6ab^2(ef-dg)^3n^2x}{e^3} - \frac{6b^3(ef-dg)^3n^3x}{e^3} - \frac{9b^3g(ef-dg)^2n^3(d+ex)^2}{8e^4} \\
&\quad - \frac{2b^3g^2(ef-dg)n^3(d+ex)^3}{9e^4} - \frac{3b^3g^3n^3(d+ex)^4}{128e^4} \\
&\quad + \frac{6b^3(ef-dg)^3n^2(d+ex)\log \left(c(d+ex)^n\right)}{e^4} \\
&\quad + \frac{9b^2g(ef-dg)^2n^2(d+ex)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)}{4e^4} \\
&\quad + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)}{3e^4} \\
&\quad + \frac{3b^2g^3n^2(d+ex)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)}{32e^4} \\
&\quad - \frac{3b(ef-dg)^3n(d+ex) \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e^4} \\
&\quad - \frac{9bg(ef-dg)^2n(d+ex)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{4e^4} \\
&\quad - \frac{bg^2(ef-dg)n(d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e^4} \\
&\quad - \frac{3bg^3n(d+ex)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{16e^4} + \frac{(ef-dg)^3(d+ex) \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^4} \\
&\quad + \frac{3g(ef-dg)^2(d+ex)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{2e^4} \\
&\quad + \frac{g^2(ef-dg)(d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^4} + \frac{g^3(d+ex)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{4e^4}
\end{aligned}
$$

output
```
6*a*b^2*(-d*g+e*f)^3*n^2*x/e^3-6*b^3*(-d*g+e*f)^3*n^3*x/e^3-9/8*b^3*g*(-d*
g+e*f)^2*n^3*(e*x+d)^2/e^4-2/9*b^3*g^2*(-d*g+e*f)*n^3*(e*x+d)^3/e^4-3/128*
b^3*g^3*n^3*(e*x+d)^4/e^4+6*b^3*(-d*g+e*f)^3*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e
^4+9/4*b^2*g*(-d*g+e*f)^2*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^4+2/3*b^2*
g^2*(-d*g+e*f)*n^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^4+3/32*b^2*g^3*n^2*(e
*x+d)^4*(a+b*ln(c*(e*x+d)^n))/e^4-3*b*(-d*g+e*f)^3*n*(e*x+d)*(a+b*ln(c*(e*
x+d)^n))^2/e^4-9/4*b*g*(-d*g+e*f)^2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^
4-b*g^2*(-d*g+e*f)*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^2/e^4-3/16*b*g^3*n*(e
*x+d)^4*(a+b*ln(c*(e*x+d)^n))^2/e^4+(-d*g+e*f)^3*(e*x+d)*(a+b*ln(c*(e*x+d)
^n))^3/e^4+3/2*g*(-d*g+e*f)^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^4+g^2*(-
d*g+e*f)*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^3/e^4+1/4*g^3*(e*x+d)^4*(a+b*ln(c
*(e*x+d)^n))^3/e^4
```

3.52.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.79

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^3 \, dx$$
$$= \frac{1152(ef-dg)^3(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3+1728g(ef-dg)^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3+}{}$$

input
```
Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(1152*(e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 + 1728*g*(e*f -
 d*g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3 + 1152*g^2*(e*f - d*g)*(d
 + e*x)^3*(a + b*Log[c*(d + e*x)^n])^3 + 288*g^3*(d + e*x)^4*(a + b*Log[c*
(d + e*x)^n])^3 - 3456*b*(e*f - d*g)^3*n*((d + e*x)*(a + b*Log[c*(d + e*x)
^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])) - 1296*b*
g*(e*f - d*g)^2*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n
*x*(2*d + e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))) - 128*b*g^2*(e
*f - d*g)*n*(9*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^2 + 2*b*n*(b*e*n*x*(
3*d^2 + 3*d*e*x + e^2*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))) -
27*b*g^3*n*(8*(d + e*x)^4*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n*x*(4*d
^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 4*(d + e*x)^4*(a + b*Log[c*(d +
e*x)^n]))))/(1152*e^4)
```

3.52.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^3 \, dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{3g^2(d+ex)^2(ef-dg) (a+b\log (c(d+ex)^n))^3}{e^3}+\frac{(ef-dg)^3 (a+b\log (c(d+ex)^n))^3}{e^3}+\frac{3g(d+ex)(ef-dg\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^2g^2n^2(d+ex)^3(ef-dg) (a+b\log (c(d+ex)^n))}{3e^4}+\frac{9b^2gn^2(d+ex)^2(ef-dg)^2 (a+b\log (c(d+ex)^n))}{4e^4}+\frac{3b^2g^3n^2(d+ex)^4 (a+b\log (c(d+ex)^n))}{32e^4}+\frac{6ab^2n^2x(ef-dg)^3}{e^3}+\frac{g^2(d+ex)^3(ef-dg) (a+b\log (c(d+ex)^n))^3}{e^4}-\frac{bg^2n(d+ex)^3(ef-dg) (a+b\log (c(d+ex)^n))^2}{e^4}+\frac{3g(d+ex)^2(ef-dg)^2 (a+b\log (c(d+ex)^n))^3}{2e^4}-\frac{9bgn(d+ex)^2(ef-dg)^2 (a+b\log (c(d+ex)^n))^2}{4e^4}+\frac{(d+ex)(ef-dg)^3 (a+b\log (c(d+ex)^n))^3}{e^4}-\frac{3bn(d+ex)(ef-dg)^3 (a+b\log (c(d+ex)^n))^2}{e^4}+\frac{g^3(d+ex)^4 (a+b\log (c(d+ex)^n))^3}{4e^4}-\frac{3bg^3n(d+ex)^4 (a+b\log (c(d+ex)^n))^2}{16e^4}+\frac{6b^3n^2(d+ex)(ef-dg)^3\log (c(d+ex)^n)}{e^4}-\frac{2b^3g^2n^3(d+ex)^3(ef-dg)}{9e^4}-\frac{9b^3gn^3(d+ex)^2(ef-dg)^2}{8e^4}-\frac{3b^3g^3n^3(d+ex)^4}{128e^4}-\frac{6b^3n^3x(ef-dg)^3}{e^3}$$

input
```
Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(6*a*b^2*(e*f - d*g)^3*n^2*x)/e^3 - (6*b^3*(e*f - d*g)^3*n^3*x)/e^3 - (9*b
^3*g*(e*f - d*g)^2*n^3*(d + e*x)^2)/(8*e^4) - (2*b^3*g^2*(e*f - d*g)*n^3*(
d + e*x)^3)/(9*e^4) - (3*b^3*g^3*n^3*(d + e*x)^4)/(128*e^4) + (6*b^3*(e*f
- d*g)^3*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e^4 + (9*b^2*g*(e*f - d*g)^2*n^
2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^4) + (2*b^2*g^2*(e*f - d*g)
*n^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*e^4) + (3*b^2*g^3*n^2*(d +
 e*x)^4*(a + b*Log[c*(d + e*x)^n]))/(32*e^4) - (3*b*(e*f - d*g)^3*n*(d + e
*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^4 - (9*b*g*(e*f - d*g)^2*n*(d + e*x)^2
*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^4) - (b*g^2*(e*f - d*g)*n*(d + e*x)^3*
(a + b*Log[c*(d + e*x)^n])^2)/e^4 - (3*b*g^3*n*(d + e*x)^4*(a + b*Log[c*(d
 + e*x)^n])^2)/(16*e^4) + ((e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^
n])^3)/e^4 + (3*g*(e*f - d*g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/
(2*e^4) + (g^2*(e*f - d*g)*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^3)/e^4 +
 (g^3*(d + e*x)^4*(a + b*Log[c*(d + e*x)^n])^3)/(4*e^4)
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2718 vs. $2(578) = 1156$.

Time = 5.42 (sec) , antiderivative size = 2719, normalized size of antiderivative = 4.55

method	result	size
parallelrisch	Expression too large to display	2719
risch	Expression too large to display	30495

input
```
int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/1152*(-108*a*b^2*e^4*g^3*n^2*x^4-1152*a^3*e^4*f^3*x+1152*a^3*d*e^3*f^3-
148*b^3*d*e^3*g^3*n^3*x^3+256*b^3*e^4*f*g^2*n^3*x^3+216*a^2*b*e^4*g^3*n*x^
4+690*b^3*d^2*e^2*g^3*n^3*x^2+1296*b^3*e^4*f^2*g*n^3*x^2-4980*b^3*d^3*e*g^
3*n^3*x-16320*b^3*d^3*e*f*g^2*n^3+18144*b^3*d^2*e^2*f^2*g*n^3+6912*a*b^2*d
*e^3*f^3*n^2-288*a^3*e^4*g^3*x^4+6912*b^3*n^3*e^4*f^3*x+3456*b*n*a^2*e^4*f
^3*x-6912*b^2*n^2*a*e^4*f^3*x-3456*a^2*b*d^3*e*f*g^2*n+5184*a^2*b*d^2*e^2*
f^2*g*n+2880*a*b^2*d*e^3*f*g^2*n^2*x^2-12672*a*b^2*d^2*e^2*f*g^2*n^2*x+155
52*a*b^2*d*e^3*f^2*g*n^2*x+8580*ln(e*x+d)*b^3*d^4*g^3*n^3+4980*b^3*d^4*g^3
*n^3+432*a^2*b*d^2*e^2*g^3*n*x^2+864*a^2*b*d^4*g^3*n+27*b^3*e^4*g^3*n^3*x^
4-1152*a^3*e^4*f*g^2*x^3-1728*a^3*e^4*f^2*g*x^2-6912*b^3*d*e^3*f^3*n^3-360
0*a*b^2*d^4*g^3*n^2-1728*a^2*b*d*e^3*f*g^2*n*x^2+2592*a^2*b*e^4*f^2*g*n*x^
2+3600*a*b^2*d^3*e*g^3*n^2*x+336*a*b^2*d*e^3*g^3*n^2*x^3-768*a*b^2*e^4*f*g
^2*n^2*x^3-1824*b^3*d*e^3*f*g^2*n^3*x^2+1152*a^2*b*e^4*f*g^2*n*x^3-936*a*b
^2*d^2*e^2*g^3*n^2*x^2-2592*a*b^2*e^4*f^2*g*n^2*x^2+16320*b^3*d^2*e^2*f*g^
2*n^3*x-18144*b^3*d*e^3*f^2*g*n^3*x+12672*a*b^2*d^3*e*f*g^2*n^2-15552*a*b^
2*d^2*e^2*f^2*g*n^2-288*x^4*ln(c*(e*x+d)^n)^3*b^3*e^4*g^3-1152*x*ln(c*(e*x
+d)^n)^3*b^3*e^4*f^3-1152*ln(c*(e*x+d)^n)^3*b^3*d*e^3*f^3-1800*ln(c*(e*x+d
)^n)^2*b^3*d^4*g^3*n-3600*ln(c*(e*x+d)^n)*b^3*d^4*g^3*n^2+864*ln(c*(e*x+d)
^n)^2*a*b^2*d^4*g^3-13824*ln(e*x+d)*b^3*d*e^3*f^3*n^3-5328*ln(e*x+d)*a*b^2
*d^4*g^3*n^2+864*ln(e*x+d)*a^2*b*d^4*g^3*n+288*ln(c*(e*x+d)^n)^3*b^3*d^...
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2802 vs. $2(578) = 1156$.

Time = 0.38 (sec) , antiderivative size = 2802, normalized size of antiderivative = 4.69

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
-1/1152*(9*(3*b^3*e^4*g^3*n^3 - 12*a*b^2*e^4*g^3*n^2 + 24*a^2*b*e^4*g^3*n
- 32*a^3*e^4*g^3)*x^4 - 4*(288*a^3*e^4*f*g^2 - (64*b^3*e^4*f*g^2 - 37*b^3*
d*e^3*g^3)*n^3 + 12*(16*a*b^2*e^4*f*g^2 - 7*a*b^2*d*e^3*g^3)*n^2 - 72*(4*a
^2*b*e^4*f*g^2 - a^2*b*d*e^3*g^3)*n)*x^3 - 288*(b^3*e^4*g^3*n^3*x^4 + 4*b^
3*e^4*f*g^2*n^3*x^3 + 6*b^3*e^4*f^2*g*n^3*x^2 + 4*b^3*e^4*f^3*n^3*x + (4*b
^3*d*e^3*f^3 - 6*b^3*d^2*e^2*f^2*g + 4*b^3*d^3*e*f*g^2 - b^3*d^4*g^3)*n^3)
*log(e*x + d)^3 - 288*(b^3*e^4*g^3*x^4 + 4*b^3*e^4*f*g^2*x^3 + 6*b^3*e^4*f
^2*g*x^2 + 4*b^3*e^4*f^3*x)*log(c)^3 - 6*(288*a^3*e^4*f^2*g - (216*b^3*e^4
*f^2*g - 304*b^3*d*e^3*f*g^2 + 115*b^3*d^2*e^2*g^3)*n^3 + 12*(36*a*b^2*e^4
*f^2*g - 40*a*b^2*d*e^3*f*g^2 + 13*a*b^2*d^2*e^2*g^3)*n^2 - 72*(6*a^2*b*e^
4*f^2*g - 4*a^2*b*d*e^3*f*g^2 + a^2*b*d^2*e^2*g^3)*n)*x^2 + 72*(3*(b^3*e^4
*g^3*n^3 - 4*a*b^2*e^4*g^3*n^2)*x^4 + (48*b^3*d*e^3*f^3 - 108*b^3*d^2*e^2*
f^2*g + 88*b^3*d^3*e*f*g^2 - 25*b^3*d^4*g^3)*n^3 - 4*(12*a*b^2*e^4*f*g^2*n
^2 - (4*b^3*e^4*f*g^2 - b^3*d*e^3*g^3)*n^3)*x^3 - 12*(4*a*b^2*d*e^3*f^3 -
6*a*b^2*d^2*e^2*f^2*g + 4*a*b^2*d^3*e*f*g^2 - a*b^2*d^4*g^3)*n^2 - 6*(12*a
*b^2*e^4*f^2*g*n^2 - (6*b^3*e^4*f^2*g - 4*b^3*d*e^3*f*g^2 + b^3*d^2*e^2*g^
3)*n^3)*x^2 - 12*(4*a*b^2*e^4*f^3*n^2 - (4*b^3*e^4*f^3 - 6*b^3*d*e^3*f^2*g
 + 4*b^3*d^2*e^2*f*g^2 - b^3*d^3*e*g^3)*n^3)*x - 12*(b^3*e^4*g^3*n^2*x^4 +
 4*b^3*e^4*f*g^2*n^2*x^3 + 6*b^3*e^4*f^2*g*n^2*x^2 + 4*b^3*e^4*f^3*n^2*x +
 (4*b^3*d*e^3*f^3 - 6*b^3*d^2*e^2*f^2*g + 4*b^3*d^3*e*f*g^2 - b^3*d^4*g...
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2594 vs. $2(583) = 1166$.

Time = 4.44 (sec) , antiderivative size = 2594, normalized size of antiderivative = 4.34

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^3\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Piecewise((a**3*f**3*x + 3*a**3*f**2*g*x**2/2 + a**3*f*g**2*x**3 + a**3*g*
*3*x**4/4 - 3*a**2*b*d**4*g**3*log(c*(d + e*x)**n)/(4*e**4) + 3*a**2*b*d**
3*f*g**2*log(c*(d + e*x)**n)/e**3 + 3*a**2*b*d**3*g**3*n*x/(4*e**3) - 9*a*
*2*b*d**2*f**2*g*log(c*(d + e*x)**n)/(2*e**2) - 3*a**2*b*d**2*f*g**2*n*x/e
**2 - 3*a**2*b*d**2*g**3*n*x**2/(8*e**2) + 3*a**2*b*d*f**3*log(c*(d + e*x)
**n)/e + 9*a**2*b*d*f**2*g*n*x/(2*e) + 3*a**2*b*d*f*g**2*n*x**2/(2*e) + a*
*2*b*d*g**3*n*x**3/(4*e) - 3*a**2*b*f**3*n*x + 3*a**2*b*f**3*x*log(c*(d +
e*x)**n) - 9*a**2*b*f**2*g*n*x**2/4 + 9*a**2*b*f**2*g*x**2*log(c*(d + e*x)
**n)/2 - a**2*b*f*g**2*n*x**3 + 3*a**2*b*f*g**2*x**3*log(c*(d + e*x)**n) -
 3*a**2*b*g**3*n*x**4/16 + 3*a**2*b*g**3*x**4*log(c*(d + e*x)**n)/4 + 25*a
*b**2*d**4*g**3*n*log(c*(d + e*x)**n)/(8*e**4) - 3*a*b**2*d**4*g**3*log(c*
(d + e*x)**n)**2/(4*e**4) - 11*a*b**2*d**3*f*g**2*n*log(c*(d + e*x)**n)/e*
*3 + 3*a*b**2*d**3*f*g**2*log(c*(d + e*x)**n)**2/e**3 - 25*a*b**2*d**3*g**
3*n**2*x/(8*e**3) + 3*a*b**2*d**3*g**3*n*x*log(c*(d + e*x)**n)/(2*e**3) +
27*a*b**2*d**2*f**2*g*n*log(c*(d + e*x)**n)/(2*e**2) - 9*a*b**2*d**2*f**2*
g*log(c*(d + e*x)**n)**2/(2*e**2) + 11*a*b**2*d**2*f*g**2*n**2*x/e**2 - 6*
a*b**2*d**2*f*g**2*n*x*log(c*(d + e*x)**n)/e**2 + 13*a*b**2*d**2*g**3*n**2
*x**2/(16*e**2) - 3*a*b**2*d**2*g**3*n*x**2*log(c*(d + e*x)**n)/(4*e**2) -
 6*a*b**2*d*f**3*n*log(c*(d + e*x)**n)/e + 3*a*b**2*d*f**3*log(c*(d + e*x)
**n)**2/e - 27*a*b**2*d*f**2*g*n**2*x/(2*e) + 9*a*b**2*d*f**2*g*n*x*log...
```

3.52.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. 2(578) = 1156.

Time = 0.26 (sec) , antiderivative size = 1687, normalized size of antiderivative = 2.82

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
1/4*b^3*g^3*x^4*log((e*x + d)^n*c)^3 + 3/4*a*b^2*g^3*x^4*log((e*x + d)^n*c
)^2 + b^3*f*g^2*x^3*log((e*x + d)^n*c)^3 + 3/4*a^2*b*g^3*x^4*log((e*x + d)
^n*c) + 3*a*b^2*f*g^2*x^3*log((e*x + d)^n*c)^2 + 3/2*b^3*f^2*g*x^2*log((e*
x + d)^n*c)^3 + 1/4*a^3*g^3*x^4 + 3*a^2*b*f*g^2*x^3*log((e*x + d)^n*c) + 9
/2*a*b^2*f^2*g*x^2*log((e*x + d)^n*c)^2 + b^3*f^3*x*log((e*x + d)^n*c)^3 +
 a^3*f*g^2*x^3 - 3*a^2*b*e*f^3*n*(x/e - d*log(e*x + d)/e^2) - 1/16*a^2*b*e
*g^3*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 -
 12*d^3*x)/e^4) + 1/2*a^2*b*e*f*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3
 - 3*d*e*x^2 + 6*d^2*x)/e^3) - 9/4*a^2*b*e*f^2*g*n*(2*d^2*log(e*x + d)/e^3
 + (e*x^2 - 2*d*x)/e^2) + 9/2*a^2*b*f^2*g*x^2*log((e*x + d)^n*c) + 3*a*b^2
*f^3*x*log((e*x + d)^n*c)^2 + 3/2*a^3*f^2*g*x^2 + 3*a^2*b*f^3*x*log((e*x +
 d)^n*c) - 3*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log
(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a*b^2*f^3 - (3*e*n*(x/e - d
*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log
(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*
e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*b^3*f^3 - 9/4*(2*e*n*(2
*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^2*x^2
 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*a*b^2*f^2
*g - 3/8*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x +
d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*x + d...
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5182 vs. $2(578) = 1156$.

Time = 0.41 (sec) , antiderivative size = 5182, normalized size of antiderivative = 8.67

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
(e*x + d)*b^3*f^3*n^3*log(e*x + d)^3/e + 3/2*(e*x + d)^2*b^3*f^2*g*n^3*log
(e*x + d)^3/e^2 - 3*(e*x + d)*b^3*d*f^2*g*n^3*log(e*x + d)^3/e^2 + (e*x +
d)^3*b^3*f*g^2*n^3*log(e*x + d)^3/e^3 - 3*(e*x + d)^2*b^3*d*f*g^2*n^3*log(
e*x + d)^3/e^3 + 3*(e*x + d)*b^3*d^2*f*g^2*n^3*log(e*x + d)^3/e^3 + 1/4*(e
*x + d)^4*b^3*g^3*n^3*log(e*x + d)^3/e^4 - (e*x + d)^3*b^3*d*g^3*n^3*log(e
*x + d)^3/e^4 + 3/2*(e*x + d)^2*b^3*d^2*g^3*n^3*log(e*x + d)^3/e^4 - (e*x
+ d)*b^3*d^3*g^3*n^3*log(e*x + d)^3/e^4 - 3*(e*x + d)*b^3*f^3*n^3*log(e*x
+ d)^2/e - 9/4*(e*x + d)^2*b^3*f^2*g*n^3*log(e*x + d)^2/e^2 + 9*(e*x + d)*
b^3*d*f^2*g*n^3*log(e*x + d)^2/e^2 - (e*x + d)^3*b^3*f*g^2*n^3*log(e*x + d
)^2/e^3 + 9/2*(e*x + d)^2*b^3*d*f*g^2*n^3*log(e*x + d)^2/e^3 - 9*(e*x + d)
*b^3*d^2*f*g^2*n^3*log(e*x + d)^2/e^3 - 3/16*(e*x + d)^4*b^3*g^3*n^3*log(e
*x + d)^2/e^4 + (e*x + d)^3*b^3*d*g^3*n^3*log(e*x + d)^2/e^4 - 9/4*(e*x +
d)^2*b^3*d^2*g^3*n^3*log(e*x + d)^2/e^4 + 3*(e*x + d)*b^3*d^3*g^3*n^3*log(
e*x + d)^2/e^4 + 3*(e*x + d)*b^3*f^3*n^2*log(e*x + d)^2*log(c)/e + 9/2*(e*
x + d)^2*b^3*f^2*g*n^2*log(e*x + d)^2*log(c)/e^2 - 9*(e*x + d)*b^3*d*f^2*g
*n^2*log(e*x + d)^2*log(c)/e^2 + 3*(e*x + d)^3*b^3*f*g^2*n^2*log(e*x + d)^
2*log(c)/e^3 - 9*(e*x + d)^2*b^3*d*f*g^2*n^2*log(e*x + d)^2*log(c)/e^3 + 9
*(e*x + d)*b^3*d^2*f*g^2*n^2*log(e*x + d)^2*log(c)/e^3 + 3/4*(e*x + d)^4*b
^3*g^3*n^2*log(e*x + d)^2*log(c)/e^4 - 3*(e*x + d)^3*b^3*d*g^3*n^2*log(e*x
 + d)^2*log(c)/e^4 + 9/2*(e*x + d)^2*b^3*d^2*g^3*n^2*log(e*x + d)^2*log...
```

3.52.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 2133, normalized size of antiderivative = 3.57

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
x^3*((g^2*(24*a^3*d*g + 72*a^3*e*f + 7*b^3*d*g*n^3 - 16*b^3*e*f*n^3 - 12*a
*b^2*d*g*n^2 + 48*a*b^2*e*f*n^2 - 72*a^2*b*e*f*n))/(72*e) - (d*g^3*(32*a^3
 - 3*b^3*n^3 + 12*a*b^2*n^2 - 24*a^2*b*n))/(96*e)) + log(c*(d + e*x)^n)^3*
(b^3*f^3*x - (d*(b^3*d^3*g^3 - 4*b^3*e^3*f^3 + 6*b^3*d*e^2*f^2*g - 4*b^3*d
^2*e*f*g^2))/(4*e^4) + (b^3*g^3*x^4)/4 + (3*b^3*f^2*g*x^2)/2 + b^3*f*g^2*x
^3) - x^2*((d*((g^2*(24*a^3*d*g + 72*a^3*e*f + 7*b^3*d*g*n^3 - 16*b^3*e*f*
n^3 - 12*a*b^2*d*g*n^2 + 48*a*b^2*e*f*n^2 - 72*a^2*b*e*f*n))/(24*e) - (d*g
^3*(32*a^3 - 3*b^3*n^3 + 12*a*b^2*n^2 - 24*a^2*b*n))/(32*e)))/(2*e) - (g*(
48*a^3*e^2*f^2 - 13*b^3*d^2*g^2*n^3 - 36*b^3*e^2*f^2*n^3 - 72*a^2*b*e^2*f^
2*n + 48*a^3*d*e*f*g + 12*a*b^2*d^2*g^2*n^2 + 72*a*b^2*e^2*f^2*n^2 + 40*b^
3*d*e*f*g*n^3 - 48*a*b^2*d*e*f*g*n^2))/(32*e^2)) + log(c*(d + e*x)^n)^2*((
x^3*((4*b^2*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (b^2*d*g^3*(4*a - b*n))/e
))/4 - (x^2*((d*((48*b^2*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (12*b^2*d*g^
3*(4*a - b*n))/e))/(8*e) - (9*b^2*f*g*(2*a*d*g + 2*a*e*f - b*e*f*n))/e))/4
 + (x*((d*((d*((48*b^2*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (12*b^2*d*g^3*
(4*a - b*n))/e))/e - (72*b^2*f*g*(2*a*d*g + 2*a*e*f - b*e*f*n))/e))/(4*e)
+ (12*b^2*f^2*(3*a*d*g + a*e*f - b*e*f*n))/e))/4 - (12*a*b^2*d^4*g^3 - 25*
b^3*d^4*g^3*n - 48*a*b^2*d*e^3*f^3 + 48*b^3*d*e^3*f^3*n + 72*a*b^2*d^2*e^2
*f^2*g - 108*b^3*d^2*e^2*f^2*g*n - 48*a*b^2*d^3*e*f*g^2 + 88*b^3*d^3*e*f*g
^2*n)/(16*e^4) + (3*b^2*g^3*x^4*(4*a - b*n))/16) + x*((96*a^3*e^3*f^3 +...
```

3.52. $\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx$

3.53 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3\,dx$

3.53.1 Optimal result

Integrand size = 24, antiderivative size = 432

$$\begin{aligned}
&\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3\,dx \\
&= \frac{6ab^2(ef-dg)^2n^2x}{e^2} - \frac{6b^3(ef-dg)^2n^3x}{e^2} - \frac{3b^3g(ef-dg)n^3(d+ex)^2}{4e^3} \\
&\quad - \frac{2b^3g^2n^3(d+ex)^3}{27e^3} + \frac{6b^3(ef-dg)^2n^2(d+ex)\log(c(d+ex)^n)}{e^3} \\
&\quad + \frac{3b^2g(ef-dg)n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^3} \\
&\quad + \frac{2b^2g^2n^2(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3} \\
&\quad - \frac{3b(ef-dg)^2n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^3} \\
&\quad - \frac{3bg(ef-dg)n(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^3} \\
&\quad - \frac{bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))^2}{3e^3} + \frac{(ef-dg)^2(d+ex)(a+b\log(c(d+ex)^n))^3}{e^3} \\
&\quad + \frac{g(ef-dg)(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^3} + \frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^3}{3e^3}
\end{aligned}$$

output
```
6*a*b^2*(-d*g+e*f)^2*n^2*x/e^2-6*b^3*(-d*g+e*f)^2*n^3*x/e^2-3/4*b^3*g*(-d*
g+e*f)*n^3*(e*x+d)^2/e^3-2/27*b^3*g^2*n^3*(e*x+d)^3/e^3+6*b^3*(-d*g+e*f)^2
*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^3+3/2*b^2*g*(-d*g+e*f)*n^2*(e*x+d)^2*(a+b*l
n(c*(e*x+d)^n))/e^3+2/9*b^2*g^2*n^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^3-3*
b*(-d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^3-3/2*b*g*(-d*g+e*f)*n*
(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^3-1/3*b*g^2*n*(e*x+d)^3*(a+b*ln(c*(e*x
+d)^n))^2/e^3+(-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^3+g*(-d*g+e*f
)*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^3+1/3*g^2*(e*x+d)^3*(a+b*ln(c*(e*x+d
)^n))^3/e^3
```

3.53.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.77

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \frac{108(ef-dg)^2(d+ex)\left(a+b\log \left(c(d+ex)^n\right)\right)^3+108g(ef-dg)(d+ex)^2\left(a+b\log \left(c(d+ex)^n\right)\right)^3+36g}{}$$

input
```
Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(108*(e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 + 108*g*(e*f - d
*g)*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3 + 36*g^2*(d + e*x)^3*(a + b*L
og[c*(d + e*x)^n])^3 - 324*b*(e*f - d*g)^2*n*((d + e*x)*(a + b*Log[c*(d +
e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])) - 81*
b*g*(e*f - d*g)*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n
*x*(2*d + e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))) - 4*b*g^2*n*(9
*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^2 + 2*b*n*(b*e*n*x*(3*d^2 + 3*d*e*
x + e^2*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))))/(108*e^3)
```

3.53.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(ef-dg)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^2}+\frac{2g(d+ex)(ef-dg) \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^2}+\frac{g^2(d+ex)^2 \left(a+b\log\right.}{e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^2gn^2(d+ex)^2(ef-dg) \left(a+b\log \left(c(d+ex)^n\right)\right)}{2e^3}+\frac{2b^2g^2n^2(d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)}{9e^3}+\frac{6ab^2n^2x(ef-dg)^2}{e^2}-\frac{3bgn(d+ex)^2(ef-dg) \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{2e^3}-\frac{3bn(d+ex)(ef-dg)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e^3}+\frac{g(d+ex)^2(ef-dg) \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^3}+\frac{(d+ex)(ef-dg)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^3}-\frac{bg^2n(d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{3e^3}+\frac{g^2(d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{3e^3}+\frac{6b^3n^2(d+ex)(ef-dg)^2 \log \left(c(d+ex)^n\right)}{e^3}-\frac{3b^3gn^3(d+ex)^2(ef-dg)}{4e^3}-\frac{2b^3g^2n^3(d+ex)^3}{27e^3}-\frac{6b^3n^3x(ef-dg)^2}{e^2}$$

input
```
Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(6*a*b^2*(e*f - d*g)^2*n^2*x)/e^2 - (6*b^3*(e*f - d*g)^2*n^3*x)/e^2 - (3*b
^3*g*(e*f - d*g)*n^3*(d + e*x)^2)/(4*e^3) - (2*b^3*g^2*n^3*(d + e*x)^3)/(2
7*e^3) + (6*b^3*(e*f - d*g)^2*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e^3 + (3*b
^2*g*(e*f - d*g)*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^3) + (2*
b^2*g^2*n^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(9*e^3) - (3*b*(e*f -
d*g)^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^3 - (3*b*g*(e*f - d*g)*
n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^3) - (b*g^2*n*(d + e*x)^3
*(a + b*Log[c*(d + e*x)^n])^2)/(3*e^3) + ((e*f - d*g)^2*(d + e*x)*(a + b*L
og[c*(d + e*x)^n])^3)/e^3 + (g*(e*f - d*g)*(d + e*x)^2*(a + b*Log[c*(d + e
*x)^n])^3)/e^3 + (g^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^3)/(3*e^3)
```

3.53. $\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx$

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. $2(418) = 836$.

Time = 3.55 (sec) , antiderivative size = 1733, normalized size of antiderivative = 4.01

method	result	size
parallelrisch	Expression too large to display	1733
risch	Expression too large to display	20417

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*(-108*a^2*b*d^2*e*g^2*n*x-81*b^3*e^3*f*g*n^3*x^2-36*a^2*b*e^3*g^2*n*
x^3-510*b^3*d^2*e*g^2*n^3*x+36*x^3*ln(c*(e*x+d)^n)^3*b^3*e^3*g^2+108*x*ln(
c*(e*x+d)^n)^3*b^3*e^3*f^2+108*ln(c*(e*x+d)^n)^3*b^3*d*e^2*f^2-198*ln(c*(e
*x+d)^n)^2*b^3*d^3*g^2*n-396*ln(c*(e*x+d)^n)*b^3*d^3*g^2*n^2+108*ln(c*(e*x
+d)^n)^2*a*b^2*d^3*g^2-90*a*b^2*d*e^2*g^2*n^2*x^2+162*a*b^2*e^3*f*g*n^2*x^
2+1134*b^3*d*e^2*f*g*n^3*x-162*a^2*b*e^3*f*g*n*x^2+396*a*b^2*d^2*e*g^2*n^2
*x-648*b^3*e^3*f^2*n^3*x+648*b^3*d*e^2*f^2*n^3-396*a*b^2*d^3*g^2*n^2+648*a
*b^2*e^3*f^2*n^2*x-324*a^2*b*e^3*f^2*n*x+108*a^2*b*d^3*g^2*n-1134*b^3*d^2*
e*f*g*n^3-648*a*b^2*d*e^2*f^2*n^2+36*a^3*e^3*g^2*x^3+324*a^2*b*d*e^2*f*g*n
*x-8*b^3*e^3*g^2*n^3*x^3+108*a^3*e^3*f*g*x^2+24*a*b^2*e^3*g^2*n^2*x^3+57*b
^3*d*e^2*g^2*n^3*x^2-108*a^3*d*e^2*f^2+108*a^3*e^3*f^2*x+510*b^3*d^3*g^2*n
^3+906*ln(e*x+d)*b^3*d^3*g^2*n^3-972*a*b^2*d*e^2*f*g*n^2*x+324*a^2*b*d*e^2
*f^2*n+36*ln(c*(e*x+d)^n)^3*b^3*d^3*g^2+972*a*b^2*d^2*e*f*g*n^2+108*x^2*ln
(c*(e*x+d)^n)*a*b^2*d*e^2*g^2*n+1296*ln(e*x+d)*b^3*d*e^2*f^2*n^3-612*ln(e*
x+d)*a*b^2*d^3*g^2*n^2+108*ln(e*x+d)*a^2*b*d^3*g^2*n+324*x*ln(c*(e*x+d)^n)
^2*a*b^2*e^3*f^2-108*ln(c*(e*x+d)^n)^3*b^3*d^2*e*f*g-324*ln(c*(e*x+d)^n)^2
*b^3*d*e^2*f^2*n-648*ln(c*(e*x+d)^n)*b^3*d*e^2*f^2*n^2+324*x*ln(c*(e*x+d)^
n)*a^2*b*e^3*f^2+324*ln(c*(e*x+d)^n)^2*a*b^2*d*e^2*f^2+216*ln(c*(e*x+d)^n)
*a*b^2*d^3*g^2*n-324*ln(c*(e*x+d)^n)*a^2*b*d*e^2*f^2-324*a^2*b*d^2*e*f*g*n
-324*x^2*ln(c*(e*x+d)^n)*a*b^2*e^3*f*g*n+324*x*ln(c*(e*x+d)^n)^2*b^3*d*...
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. 2(418) = 836.

Time = 0.34 (sec) , antiderivative size = 1771, normalized size of antiderivative = 4.10

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
-1/108*(4*(2*b^3*e^3*g^2*n^3 - 6*a*b^2*e^3*g^2*n^2 + 9*a^2*b*e^3*g^2*n - 9
*a^3*e^3*g^2)*x^3 - 36*(b^3*e^3*g^2*n^3*x^3 + 3*b^3*e^3*f*g*n^3*x^2 + 3*b^
3*e^3*f^2*n^3*x + (3*b^3*d*e^2*f^2 - 3*b^3*d^2*e*f*g + b^3*d^3*g^2)*n^3)*l
og(e*x + d)^3 - 36*(b^3*e^3*g^2*x^3 + 3*b^3*e^3*f*g*x^2 + 3*b^3*e^3*f^2*x)
*log(c)^3 - 3*(36*a^3*e^3*f*g - (27*b^3*e^3*f*g - 19*b^3*d*e^2*g^2)*n^3 +
6*(9*a*b^2*e^3*f*g - 5*a*b^2*d*e^2*g^2)*n^2 - 18*(3*a^2*b*e^3*f*g - a^2*b*
d*e^2*g^2)*n)*x^2 + 18*((18*b^3*d*e^2*f^2 - 27*b^3*d^2*e*f*g + 11*b^3*d^3*
g^2)*n^3 + 2*(b^3*e^3*g^2*n^3 - 3*a*b^2*e^3*g^2*n^2)*x^3 - 6*(3*a*b^2*d*e^
2*f^2 - 3*a*b^2*d^2*e*f*g + a*b^2*d^3*g^2)*n^2 - 3*(6*a*b^2*e^3*f*g*n^2 -
(3*b^3*e^3*f*g - b^3*d*e^2*g^2)*n^3)*x^2 - 6*(3*a*b^2*e^3*f^2*n^2 - (3*b^3
*e^3*f^2 - 3*b^3*d*e^2*f*g + b^3*d^2*e*g^2)*n^3)*x - 6*(b^3*e^3*g^2*n^2*x^
3 + 3*b^3*e^3*f*g*n^2*x^2 + 3*b^3*e^3*f^2*n^2*x + (3*b^3*d*e^2*f^2 - 3*b^3
*d^2*e*f*g + b^3*d^3*g^2)*n^2)*log(c))*log(e*x + d)^2 + 18*(2*(b^3*e^3*g^2
*n - 3*a*b^2*e^3*g^2)*x^3 - 3*(6*a*b^2*e^3*f*g - (3*b^3*e^3*f*g - b^3*d*e^
2*g^2)*n)*x^2 - 6*(3*a*b^2*e^3*f^2 - (3*b^3*e^3*f^2 - 3*b^3*d*e^2*f*g + b^
3*d^2*e*g^2)*n)*x)*log(c)^2 - 6*(18*a^3*e^3*f^2 - (108*b^3*e^3*f^2 - 189*b
^3*d*e^2*f*g + 85*b^3*d^2*e*g^2)*n^3 + 6*(18*a*b^2*e^3*f^2 - 27*a*b^2*d*e^
2*f*g + 11*a*b^2*d^2*e*g^2)*n^2 - 18*(3*a^2*b*e^3*f^2 - 3*a^2*b*d*e^2*f*g
+ a^2*b*d^2*e*g^2)*n)*x - 6*((108*b^3*d*e^2*f^2 - 189*b^3*d^2*e*f*g + 85*b
^3*d^3*g^2)*n^3 + 2*(2*b^3*e^3*g^2*n^3 - 6*a*b^2*e^3*g^2*n^2 + 9*a^2*b*...
```

3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1578 vs. $2(422) = 844$.

Time = 2.41 (sec) , antiderivative size = 1578, normalized size of antiderivative = 3.65

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Piecewise((a**3*f**2*x + a**3*f*g*x**2 + a**3*g**2*x**3/3 + a**2*b*d**3*g*
*2*log(c*(d + e*x)**n)/e**3 - 3*a**2*b*d**2*f*g*log(c*(d + e*x)**n)/e**2 -
 a**2*b*d**2*g**2*n*x/e**2 + 3*a**2*b*d*f**2*log(c*(d + e*x)**n)/e + 3*a**
2*b*d*f*g*n*x/e + a**2*b*d*g**2*n*x**2/(2*e) - 3*a**2*b*f**2*n*x + 3*a**2*
b*f**2*x*log(c*(d + e*x)**n) - 3*a**2*b*f*g*n*x**2/2 + 3*a**2*b*f*g*x**2*l
og(c*(d + e*x)**n) - a**2*b*g**2*n*x**3/3 + a**2*b*g**2*x**3*log(c*(d + e*
x)**n) - 11*a*b**2*d**3*g**2*n*log(c*(d + e*x)**n)/(3*e**3) + a*b**2*d**3*
g**2*log(c*(d + e*x)**n)**2/e**3 + 9*a*b**2*d**2*f*g*n*log(c*(d + e*x)**n)
/e**2 - 3*a*b**2*d**2*f*g*log(c*(d + e*x)**n)**2/e**2 + 11*a*b**2*d**2*g**
2*n**2*x/(3*e**2) - 2*a*b**2*d**2*g**2*n*x*log(c*(d + e*x)**n)/e**2 - 6*a*
b**2*d*f**2*n*log(c*(d + e*x)**n)/e + 3*a*b**2*d*f**2*log(c*(d + e*x)**n)*
*2/e - 9*a*b**2*d*f*g*n**2*x/e + 6*a*b**2*d*f*g*n*x*log(c*(d + e*x)**n)/e
- 5*a*b**2*d*g**2*n**2*x**2/(6*e) + a*b**2*d*g**2*n*x**2*log(c*(d + e*x)**
n)/e + 6*a*b**2*f**2*n**2*x - 6*a*b**2*f**2*n*x*log(c*(d + e*x)**n) + 3*a*
b**2*f**2*x*log(c*(d + e*x)**n)**2 + 3*a*b**2*f*g*n**2*x**2/2 - 3*a*b**2*f
*g*n*x**2*log(c*(d + e*x)**n) + 3*a*b**2*f*g*x**2*log(c*(d + e*x)**n)**2 +
 2*a*b**2*g**2*n**2*x**3/9 - 2*a*b**2*g**2*n*x**3*log(c*(d + e*x)**n)/3 +
a*b**2*g**2*x**3*log(c*(d + e*x)**n)**2 + 85*b**3*d**3*g**2*n**2*log(c*(d
+ e*x)**n)/(18*e**3) - 11*b**3*d**3*g**2*n*log(c*(d + e*x)**n)**2/(6*e**3)
 + b**3*d**3*g**2*log(c*(d + e*x)**n)**3/(3*e**3) - 21*b**3*d**2*f*g*n*...
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(418) = 836$.

Time = 0.24 (sec) , antiderivative size = 1140, normalized size of antiderivative = 2.64

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*g^2*x^3*log((e*x + d)^n*c)^3 + a*b^2*g^2*x^3*log((e*x + d)^n*c)^2
+ b^3*f*g*x^2*log((e*x + d)^n*c)^3 + a^2*b*g^2*x^3*log((e*x + d)^n*c) + 3*
a*b^2*f*g*x^2*log((e*x + d)^n*c)^2 + b^3*f^2*x*log((e*x + d)^n*c)^3 + 1/3*
a^3*g^2*x^3 - 3*a^2*b*e*f^2*n*(x/e - d*log(e*x + d)/e^2) + 1/6*a^2*b*e*g^2
*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) - 3/2*
a^2*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 3*a^2*b*f*g
*x^2*log((e*x + d)^n*c) + 3*a*b^2*f^2*x*log((e*x + d)^n*c)^2 + a^3*f*g*x^2
 + 3*a^2*b*f^2*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e - d*log(e*x + d)/e^2)*
log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*
a*b^2*f^2 - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*(
(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2
 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^
2))*b^3*f^2 - 3/2*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*lo
g((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e
*x + d))*n^2/e^2)*a*b^2*f*g - 1/4*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2
- 2*d*x)/e^2)*log((e*x + d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^
2 + 18*d^2*log(e*x + d)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e
^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n*log((e*x +
 d)^n*c)/e^3))*b^3*f*g + 1/18*(6*e*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3
- 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c) + (4*e^3*x^3 - 15*d*e^2*...
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2932 vs. $2(418) = 836$.

Time = 0.38 (sec) , antiderivative size = 2932, normalized size of antiderivative = 6.79

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
(e*x + d)*b^3*f^2*n^3*log(e*x + d)^3/e + (e*x + d)^2*b^3*f*g*n^3*log(e*x +
 d)^3/e^2 - 2*(e*x + d)*b^3*d*f*g*n^3*log(e*x + d)^3/e^2 + 1/3*(e*x + d)^3
*b^3*g^2*n^3*log(e*x + d)^3/e^3 - (e*x + d)^2*b^3*d*g^2*n^3*log(e*x + d)^3
/e^3 + (e*x + d)*b^3*d^2*g^2*n^3*log(e*x + d)^3/e^3 - 3*(e*x + d)*b^3*f^2*
n^3*log(e*x + d)^2/e - 3/2*(e*x + d)^2*b^3*f*g*n^3*log(e*x + d)^2/e^2 + 6*
(e*x + d)*b^3*d*f*g*n^3*log(e*x + d)^2/e^2 - 1/3*(e*x + d)^3*b^3*g^2*n^3*l
og(e*x + d)^2/e^3 + 3/2*(e*x + d)^2*b^3*d*g^2*n^3*log(e*x + d)^2/e^3 - 3*(
e*x + d)*b^3*d^2*g^2*n^3*log(e*x + d)^2/e^3 + 3*(e*x + d)*b^3*f^2*n^2*log(
e*x + d)^2*log(c)/e + 3*(e*x + d)^2*b^3*f*g*n^2*log(e*x + d)^2*log(c)/e^2
- 6*(e*x + d)*b^3*d*f*g*n^2*log(e*x + d)^2*log(c)/e^2 + (e*x + d)^3*b^3*g^
2*n^2*log(e*x + d)^2*log(c)/e^3 - 3*(e*x + d)^2*b^3*d*g^2*n^2*log(e*x + d)
^2*log(c)/e^3 + 3*(e*x + d)*b^3*d^2*g^2*n^2*log(e*x + d)^2*log(c)/e^3 + 6*
(e*x + d)*b^3*f^2*n^3*log(e*x + d)/e + 3/2*(e*x + d)^2*b^3*f*g*n^3*log(e*x
 + d)/e^2 - 12*(e*x + d)*b^3*d*f*g*n^3*log(e*x + d)/e^2 + 2/9*(e*x + d)^3*
b^3*g^2*n^3*log(e*x + d)/e^3 - 3/2*(e*x + d)^2*b^3*d*g^2*n^3*log(e*x + d)/
e^3 + 6*(e*x + d)*b^3*d^2*g^2*n^3*log(e*x + d)/e^3 + 3*(e*x + d)*a*b^2*f^2
*n^2*log(e*x + d)^2/e + 3*(e*x + d)^2*a*b^2*f*g*n^2*log(e*x + d)^2/e^2 - 6
*(e*x + d)*a*b^2*d*f*g*n^2*log(e*x + d)^2/e^2 + (e*x + d)^3*a*b^2*g^2*n^2*
log(e*x + d)^2/e^3 - 3*(e*x + d)^2*a*b^2*d*g^2*n^2*log(e*x + d)^2/e^3 + 3*
(e*x + d)*a*b^2*d^2*g^2*n^2*log(e*x + d)^2/e^3 - 6*(e*x + d)*b^3*f^2*n^...
```

3.53. $\int (f+gx)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx$

3.53.9 Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 1157, normalized size of antiderivative = 2.68

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx$$

$$= \ln\left(c\,(d+e\,x)^n\right)^2 \left(x^2\left(\frac{3\,b^2\,g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{2\,e}-\frac{b^2\,d\,g^2\,(3\,a-b\,n)}{2\,e}\right)\right.$$

$$-x\left(\frac{d\left(\frac{3\,b^2\,g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{b^2\,d\,g^2\,(3\,a-b\,n)}{e}\right)}{e}-\frac{3\,b^2\,f\,(2\,a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}\right)$$

$$+\frac{d\,(-11\,n\,b^3\,d^2\,g^2+27\,n\,b^3\,d\,e\,f\,g-18\,n\,b^3\,e^2\,f^2+6\,a\,b^2\,d^2\,g^2-18\,a\,b^2\,d\,e\,f\,g+18\,a\,b^2\,e^2\,f^2)}{6\,e^3}$$

$$\left.+\frac{b^2\,g^2\,x^3\,(3\,a-b\,n)}{3}\right)$$

$$+x\left(\frac{36\,a^3\,d\,e\,f\,g+18\,a^3\,e^2\,f^2-54\,a^2\,b\,e^2\,f^2\,n+36\,a\,b^2\,d^2\,g^2\,n^2-108\,a\,b^2\,d\,e\,f\,g\,n^2+108\,a\,b^2\,e^2\,f^2\,n^2-\cdots}{18\,e^2}\right.$$

$$\left.-\frac{d\left(\frac{g\,(6\,a^3\,d\,g+12\,a^3\,e\,f+5\,b^3\,d\,g\,n^3-9\,b^3\,e\,f\,n^3-6\,a\,b^2\,d\,g\,n^2+18\,a\,b^2\,e\,f\,n^2-18\,a^2\,b\,e\,f\,n)}{6\,e}-\frac{d\,g^2\,(9\,a^3-9\,a^2\,b\,n+6\,a\,b^2\,n^2-2\,b^3\,n^3)}{9\,e}\right)}{e}\right)$$

$$+x^2\left(\frac{g\,(6\,a^3\,d\,g+12\,a^3\,e\,f+5\,b^3\,d\,g\,n^3-9\,b^3\,e\,f\,n^3-6\,a\,b^2\,d\,g\,n^2+18\,a\,b^2\,e\,f\,n^2-18\,a^2\,b\,e\,f\,n)}{12\,e}\right.$$

$$\left.-\frac{d\,g^2\,(9\,a^3-9\,a^2\,b\,n+6\,a\,b^2\,n^2-2\,b^3\,n^3)}{18\,e}\right)$$

$$+\ln\left(c\,(d+e\,x)^n\right)^3\left(b^3\,f^2\,x+\frac{b^3\,g^2\,x^3}{3}+\frac{d\,(b^3\,d^2\,g^2-3\,b^3\,d\,e\,f\,g+3\,b^3\,e^2\,f^2)}{3\,e^3}+b^3\,f\,g\,x^2\right)$$

$$+\frac{g^2\,x^3\,(9\,a^3-9\,a^2\,b\,n+6\,a\,b^2\,n^2-2\,b^3\,n^3)}{27}$$

$$+\frac{\ln(d+e\,x)\,\left(18\,a^2\,b\,d^3\,g^2\,n-54\,a^2\,b\,d^2\,e\,f\,g\,n+54\,a^2\,b\,d\,e^2\,f^2\,n-66\,a\,b^2\,d^3\,g^2\,n^2+162\,a\,b^2\,d^2\,e\,f\,g\,n^2-\cdots\right.}{18\,e^3}$$

$$+\frac{\ln\left(c\,(d+e\,x)^n\right)\left(\frac{x^2\,\left(9\,b\,e\,g\,(3\,a^2\,d\,g+6\,a^2\,e\,f-b^2\,d\,g\,n^2+3\,b^2\,e\,f\,n^2-6\,a\,b\,e\,f\,n)-3\,b\,d\,e\,g^2\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)\right)}{6\,e}+\frac{x\left(\frac{54\,a^2\,b\,d\,e^2\,f\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
log(c*(d + e*x)^n)^2*(x^2*((3*b^2*g*(a*d*g + 2*a*e*f - b*e*f*n))/(2*e) - (
b^2*d*g^2*(3*a - b*n))/(2*e)) - x*((d*((3*b^2*g*(a*d*g + 2*a*e*f - b*e*f*n
))/e - (b^2*d*g^2*(3*a - b*n))/e))/e - (3*b^2*f*(2*a*d*g + a*e*f - b*e*f*n
))/e) + (d*(6*a*b^2*d^2*g^2 + 18*a*b^2*e^2*f^2 - 11*b^3*d^2*g^2*n - 18*b^3
*e^2*f^2*n - 18*a*b^2*d*e*f*g + 27*b^3*d*e*f*g*n))/(6*e^3) + (b^2*g^2*x^3*
(3*a - b*n))/3) + x*((18*a^3*e^2*f^2 - 66*b^3*d^2*g^2*n^3 - 108*b^3*e^2*f^
2*n^3 - 54*a^2*b*e^2*f^2*n + 36*a^3*d*e*f*g + 36*a*b^2*d^2*g^2*n^2 + 108*a
*b^2*e^2*f^2*n^2 + 162*b^3*d*e*f*g*n^3 - 108*a*b^2*d*e*f*g*n^2)/(18*e^2) -
 (d*((g*(6*a^3*d*g + 12*a^3*e*f + 5*b^3*d*g*n^3 - 9*b^3*e*f*n^3 - 6*a*b^2*
d*g*n^2 + 18*a*b^2*e*f*n^2 - 18*a^2*b*e*f*n))/(6*e) - (d*g^2*(9*a^3 - 2*b^
3*n^3 + 6*a*b^2*n^2 - 9*a^2*b*n))/(9*e)))/e) + x^2*((g*(6*a^3*d*g + 12*a^3
*e*f + 5*b^3*d*g*n^3 - 9*b^3*e*f*n^3 - 6*a*b^2*d*g*n^2 + 18*a*b^2*e*f*n^2
- 18*a^2*b*e*f*n))/(12*e) - (d*g^2*(9*a^3 - 2*b^3*n^3 + 6*a*b^2*n^2 - 9*a^
2*b*n))/(18*e)) + log(c*(d + e*x)^n)^3*(b^3*f^2*x + (b^3*g^2*x^3)/3 + (d*(
b^3*d^2*g^2 + 3*b^3*e^2*f^2 - 3*b^3*d*e*f*g))/(3*e^3) + b^3*f*g*x^2) + (g^
2*x^3*(9*a^3 - 2*b^3*n^3 + 6*a*b^2*n^2 - 9*a^2*b*n))/27 + (log(d + e*x)*(8
5*b^3*d^3*g^2*n^3 + 18*a^2*b*d^3*g^2*n - 66*a*b^2*d^3*g^2*n^2 + 108*b^3*d*
e^2*f^2*n^3 - 108*a*b^2*d*e^2*f^2*n^2 + 54*a^2*b*d*e^2*f^2*n - 189*b^3*d^2
*e*f*g*n^3 + 162*a*b^2*d^2*e*f*g*n^2 - 54*a^2*b*d^2*e*f*g*n))/(18*e^3) + (
log(c*(d + e*x)^n)*((x^2*(9*b*e*g*(3*a^2*d*g + 6*a^2*e*f - b^2*d*g*n^2 ...
```

3.53. $\int (f + gx)^2 (a + b\log(c(d + ex)^n))^3 \, dx$

3.54 $\int (f+gx)(a+b\log(c(d+ex)^n))^3\,dx$

3.54.1 Optimal result

Integrand size = 22, antiderivative size = 265

$$\int (f+gx)(a+b\log(c(d+ex)^n))^3\,dx = \frac{6ab^2(ef-dg)n^2x}{e}-\frac{6b^3(ef-dg)n^3x}{e} - \frac{3b^3gn^3(d+ex)^2}{8e^2} + \frac{6b^3(ef-dg)n^2(d+ex)\log(c(d+ex)^n)}{e^2} + \frac{3b^2gn^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2} - \frac{3b(ef-dg)n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2} - \frac{3bgn(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} + \frac{(ef-dg)(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2} + \frac{g(d+ex)^2(a+b\log(c(d+ex)^n))^3}{2e^2}$$

output

```
6*a*b^2*(-d*g+e*f)*n^2*x/e-6*b^3*(-d*g+e*f)*n^3*x/e-3/8*b^3*g*n^3*(e*x+d)^
2/e^2+6*b^3*(-d*g+e*f)*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2+3/4*b^2*g*n^2*(e*x+
d)^2*(a+b*ln(c*(e*x+d)^n))/e^2-3*b*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^
n))^2/e^2-3/4*b*g*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+(-d*g+e*f)*(e*x+
d)*(a+b*ln(c*(e*x+d)^n))^3/e^2+1/2*g*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.76

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$=\frac{8(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3+4g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3-24b(ef-dg)n((d\cdot$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(8*(e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 + 4*g*(d + e*x)^2*(a
 + b*Log[c*(d + e*x)^n])^3 - 24*b*(e*f - d*g)*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])) -
3*b*g*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n*x*(2*d +
e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))))/(8*e^2)
```

3.54.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^2}+\frac{6ab^2n^2x(ef-dg)}{e}-\frac{3bn(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}-\frac{3bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2}+\frac{6b^3n^2(d+ex)(ef-dg)\log\left(c(d+ex)^n\right)}{e^2}-\frac{3b^3gn^3(d+ex)^2}{8e^2}-\frac{6b^3n^3x(ef-dg)}{e}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(6*a*b^2*(e*f - d*g)*n^2*x)/e - (6*b^3*(e*f - d*g)*n^3*x)/e - (3*b^3*g*n^3
*(d + e*x)^2)/(8*e^2) + (6*b^3*(e*f - d*g)*n^2*(d + e*x)*Log[c*(d + e*x)^n
])/e^2 + (3*b^2*g*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^2) - (3
*b*(e*f - d*g)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 - (3*b*g*n*(d
 + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2) + ((e*f - d*g)*(d + e*x)*(
a + b*Log[c*(d + e*x)^n])^3)/e^2 + (g*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n
])^3)/(2*e^2)
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.54.4 Maple [B] (verified)

Leave count of result is larger than twice the leaf count of optimal. 905 vs. $2(257) = 514$.

Time = 1.74 (sec) , antiderivative size = 906, normalized size of antiderivative = 3.42

method	result
parallelrisch	$-\frac{-6a\,b^2e^2g\,n^2x^2-42b^3deg\,n^3x+6a^2b\,e^2gn\,x^2-48a\,b^2e^2f\,n^2x+24a^2b\,e^2fnx-48b^3def\,n^3+3b^3e^2g\,n^3x^2+48b^3e^2f\,n^3x-36a\,b}{}$
risch	Expression too large to display

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-6*a*b^2*e^2*g*n^2*x^2-42*b^3*d*e*g*n^3*x+6*a^2*b*e^2*g*n*x^2-48*a*b
^2*e^2*f*n^2*x+24*a^2*b*e^2*f*n*x-48*b^3*d*e*f*n^3+3*b^3*e^2*g*n^3*x^2+48*
b^3*e^2*f*n^3*x-36*a*b^2*d^2*g*n^2+36*a*b^2*d*e*g*n^2*x+12*a^2*b*d^2*g*n-4
*a^3*e^2*g*x^2-8*a^3*e^2*f*x+8*a^3*d*e*f+42*b^3*d^2*g*n^3-96*ln(e*x+d)*b^3
*d*e*f*n^3-60*ln(e*x+d)*a*b^2*d^2*g*n^2+12*ln(e*x+d)*a^2*b*d^2*g*n-4*x^2*l
n(c*(e*x+d)^n)^3*b^3*e^2*g-8*x*ln(c*(e*x+d)^n)^3*b^3*e^2*f-8*ln(c*(e*x+d)^
n)^3*b^3*d*e*f-18*ln(c*(e*x+d)^n)^2*b^3*d^2*g*n-36*ln(c*(e*x+d)^n)*b^3*d^2
*g*n^2+12*ln(c*(e*x+d)^n)^2*a*b^2*d^2*g+6*x^2*ln(c*(e*x+d)^n)^2*b^3*e^2*g*
n-6*x^2*ln(c*(e*x+d)^n)*b^3*e^2*g*n^2-12*x^2*ln(c*(e*x+d)^n)^2*a*b^2*e^2*g
+24*x*ln(c*(e*x+d)^n)^2*b^3*e^2*f*n-48*x*ln(c*(e*x+d)^n)*b^3*e^2*f*n^2-12*
x^2*ln(c*(e*x+d)^n)*a^2*b*e^2*g-24*x*ln(c*(e*x+d)^n)^2*a*b^2*e^2*f+24*ln(c
*(e*x+d)^n)^2*b^3*d*e*f*n+48*ln(c*(e*x+d)^n)*b^3*d*e*f*n^2-24*x*ln(c*(e*x+
d)^n)*a^2*b*e^2*f-24*ln(c*(e*x+d)^n)^2*a*b^2*d*e*f+24*ln(c*(e*x+d)^n)*a*b^
2*d^2*g*n+24*ln(c*(e*x+d)^n)*a^2*b*d*e*f+48*a*b^2*d*e*f*n^2-24*a^2*b*d*e*f
*n+78*ln(e*x+d)*b^3*d^2*g*n^3-12*a^2*b*d*e*g*n*x+4*ln(c*(e*x+d)^n)^3*b^3*d
^2*g+12*x^2*ln(c*(e*x+d)^n)*a*b^2*e^2*g*n-12*x*ln(c*(e*x+d)^n)^2*b^3*d*e*g
*n+36*x*ln(c*(e*x+d)^n)*b^3*d*e*g*n^2+96*ln(e*x+d)*a*b^2*d*e*f*n^2-48*ln(e
*x+d)*a^2*b*d*e*f*n+48*x*ln(c*(e*x+d)^n)*a*b^2*e^2*f*n-48*ln(c*(e*x+d)^n)*
a*b^2*d*e*f*n-24*x*ln(c*(e*x+d)^n)*a*b^2*d*e*g*n)/e^2
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(257) = 514$.

Time = 0.31 (sec) , antiderivative size = 923, normalized size of antiderivative = 3.48

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$= \frac{4\left(b^3e^2gn^3x^2+2\,b^3e^2fn^3x+\left(2\,b^3def-b^3d^2g\right)n^3\right)\log\left(ex+d\right)^3+4\left(b^3e^2gx^2+2\,b^3e^2fx\right)\log\left(c\right)^3-\left(3\,b^3e^2\right.}{}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
1/8*(4*(b^3*e^2*g*n^3*x^2 + 2*b^3*e^2*f*n^3*x + (2*b^3*d*e*f - b^3*d^2*g)*
n^3)*log(e*x + d)^3 + 4*(b^3*e^2*g*x^2 + 2*b^3*e^2*f*x)*log(c)^3 - (3*b^3*
e^2*g*n^3 - 6*a*b^2*e^2*g*n^2 + 6*a^2*b*e^2*g*n - 4*a^3*e^2*g)*x^2 - 6*((4
*b^3*d*e*f - 3*b^3*d^2*g)*n^3 - 2*(2*a*b^2*d*e*f - a*b^2*d^2*g)*n^2 + (b^3
*e^2*g*n^3 - 2*a*b^2*e^2*g*n^2)*x^2 - 2*(2*a*b^2*e^2*f*n^2 - (2*b^3*e^2*f
- b^3*d*e*g)*n^3)*x - 2*(b^3*e^2*g*n^2*x^2 + 2*b^3*e^2*f*n^2*x + (2*b^3*d*
e*f - b^3*d^2*g)*n^2)*log(c))*log(e*x + d)^2 - 6*((b^3*e^2*g*n - 2*a*b^2*e
^2*g)*x^2 - 2*(2*a*b^2*e^2*f - (2*b^3*e^2*f - b^3*d*e*g)*n)*x)*log(c)^2 +
2*(4*a^3*e^2*f - 3*(8*b^3*e^2*f - 7*b^3*d*e*g)*n^3 + 6*(4*a*b^2*e^2*f - 3*
a*b^2*d*e*g)*n^2 - 6*(2*a^2*b*e^2*f - a^2*b*d*e*g)*n)*x + 6*((8*b^3*d*e*f
- 7*b^3*d^2*g)*n^3 - 2*(4*a*b^2*d*e*f - 3*a*b^2*d^2*g)*n^2 + (b^3*e^2*g*n^
3 - 2*a*b^2*e^2*g*n^2 + 2*a^2*b*e^2*g*n)*x^2 + 2*(b^3*e^2*g*n*x^2 + 2*b^3*
e^2*f*n*x + (2*b^3*d*e*f - b^3*d^2*g)*n)*log(c)^2 + 2*(2*a^2*b*d*e*f - a^2
*b*d^2*g)*n + 2*(2*a^2*b*e^2*f*n + (4*b^3*e^2*f - 3*b^3*d*e*g)*n^3 - 2*(2*
a*b^2*e^2*f - a*b^2*d*e*g)*n^2)*x - 2*((4*b^3*d*e*f - 3*b^3*d^2*g)*n^2 + (
b^3*e^2*g*n^2 - 2*a*b^2*e^2*g*n)*x^2 - 2*(2*a*b^2*d*e*f - a*b^2*d^2*g)*n -
 2*(2*a*b^2*e^2*f*n - (2*b^3*e^2*f - b^3*d*e*g)*n^2)*x)*log(c))*log(e*x +
d) + 6*((b^3*e^2*g*n^2 - 2*a*b^2*e^2*g*n + 2*a^2*b*e^2*g)*x^2 + 2*(2*a^2*b
*e^2*f + (4*b^3*e^2*f - 3*b^3*d*e*g)*n^2 - 2*(2*a*b^2*e^2*f - a*b^2*d*e*g)
*n)*x)*log(c))/e^2
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(258) = 516$.

Time = 1.27 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.15

$$\int (f + gx) (a + b\log (c(d + ex)^n))^3 \, dx$$
$$= \begin{cases} a^3fx + \frac{a^3gx^2}{2} - \frac{3a^2bd^2g\log (c(d+ex)^n)}{2e^2} + \frac{3a^2bdf\log (c(d+ex)^n)}{e} + \frac{3a^2bdgnx}{2e} - 3a^2bfnx + 3a^2bfx\log (c(d + ex)^n) - \\ (a + b\log (cd^n))^3 \left(fx + \frac{gx^2}{2}\right) \end{cases}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Piecewise((a**3*f*x + a**3*g*x**2/2 - 3*a**2*b*d**2*g*log(c*(d + e*x)**n)/
(2*e**2) + 3*a**2*b*d*f*log(c*(d + e*x)**n)/e + 3*a**2*b*d*g*n*x/(2*e) - 3
*a**2*b*f*n*x + 3*a**2*b*f*x*log(c*(d + e*x)**n) - 3*a**2*b*g*n*x**2/4 + 3
*a**2*b*g*x**2*log(c*(d + e*x)**n)/2 + 9*a*b**2*d**2*g*n*log(c*(d + e*x)**
n)/(2*e**2) - 3*a*b**2*d**2*g*log(c*(d + e*x)**n)**2/(2*e**2) - 6*a*b**2*d
*f*n*log(c*(d + e*x)**n)/e + 3*a*b**2*d*f*log(c*(d + e*x)**n)**2/e - 9*a*b
**2*d*g*n**2*x/(2*e) + 3*a*b**2*d*g*n*x*log(c*(d + e*x)**n)/e + 6*a*b**2*f
*n**2*x - 6*a*b**2*f*n*x*log(c*(d + e*x)**n) + 3*a*b**2*f*x*log(c*(d + e*x
)**n)**2 + 3*a*b**2*g*n**2*x**2/4 - 3*a*b**2*g*n*x**2*log(c*(d + e*x)**n)/
2 + 3*a*b**2*g*x**2*log(c*(d + e*x)**n)**2/2 - 21*b**3*d**2*g*n**2*log(c*(
d + e*x)**n)/(4*e**2) + 9*b**3*d**2*g*n*log(c*(d + e*x)**n)**2/(4*e**2) -
b**3*d**2*g*log(c*(d + e*x)**n)**3/(2*e**2) + 6*b**3*d*f*n**2*log(c*(d + e
*x)**n)/e - 3*b**3*d*f*n*log(c*(d + e*x)**n)**2/e + b**3*d*f*log(c*(d + e*
x)**n)**3/e + 21*b**3*d*g*n**3*x/(4*e) - 9*b**3*d*g*n**2*x*log(c*(d + e*x)
**n)/(2*e) + 3*b**3*d*g*n*x*log(c*(d + e*x)**n)**2/(2*e) - 6*b**3*f*n**3*x
 + 6*b**3*f*n**2*x*log(c*(d + e*x)**n) - 3*b**3*f*n*x*log(c*(d + e*x)**n)*
*2 + b**3*f*x*log(c*(d + e*x)**n)**3 - 3*b**3*g*n**3*x**2/8 + 3*b**3*g*n**
2*x**2*log(c*(d + e*x)**n)/4 - 3*b**3*g*n*x**2*log(c*(d + e*x)**n)**2/4 +
b**3*g*x**2*log(c*(d + e*x)**n)**3/2, Ne(e, 0)), ((a + b*log(c*d**n))**3*(
f*x + g*x**2/2), True))
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(257) = 514$.

Time = 0.22 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.50

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$= \frac{1}{2}b^3gx^2\log\left((ex+d)^nc\right)^3 + \frac{3}{2}ab^2gx^2\log\left((ex+d)^nc\right)^2 + b^3fx\log\left((ex+d)^nc\right)^3 - 3a^2befn\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) - \frac{3}{4}a^2begn\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right) + \frac{3}{2}a^2bgx^2\log\left((ex+d)^nc\right) + 3ab^2fx\log\left((ex+d)^nc\right)^2 + \frac{1}{2}a^3gx^2 + 3a^2bfx\log\left((ex+d)^nc\right) - 3\left(2en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right) + \frac{\left(d\log(ex+d)^2-2ex+2d\log(ex+d)\right)n^2}{e}\right)ab^2f - \left(3en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right)^2 - en\left(\frac{\left(d\log(ex+d)^3+3d\log(ex+d)^2-6ex+6d\log\right.}{e^2}\right.\right. - \frac{3}{4}\left(2en\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right)\log\left((ex+d)^nc\right) - \frac{\left(e^2x^2+2d^2\log(ex+d)^2-6dex+6d^2\log\right.}{e^2}\right. - \frac{1}{8}\left(6en\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right)\log\left((ex+d)^nc\right)^2 + en\left(\frac{\left(4d^2\log(ex+d)^3+3e^2x^2+18d^2\log\right.}{}\right.\right. + a^3fx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*g*x^2*log((e*x + d)^n*c)^3 + 3/2*a*b^2*g*x^2*log((e*x + d)^n*c)^2
+ b^3*f*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*f*n*(x/e - d*log(e*x + d)/e^2)
- 3/4*a^2*b*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 3/2*a^2
*b*g*x^2*log((e*x + d)^n*c) + 3*a*b^2*f*x*log((e*x + d)^n*c)^2 + 1/2*a^3*g
*x^2 + 3*a^2*b*f*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e - d*log(e*x + d)/e^2
)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e
)*a*b^2*f - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*(
(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2
 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^
2))*b^3*f - 3/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log(
(e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x
 + d))*n^2/e^2)*a*b^2*g - 1/8*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*
d*x)/e^2)*log((e*x + d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 +
18*d^2*log(e*x + d)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x
^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n*log((e*x + d)^
n*c)/e^3))*b^3*g + a^3*f*x
```

3.54. $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. $2(257) = 514$.

Time = 0.33 (sec) , antiderivative size = 1321, normalized size of antiderivative = 4.98

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
(e*x + d)*b^3*f*n^3*log(e*x + d)^3/e + 1/2*(e*x + d)^2*b^3*g*n^3*log(e*x +
 d)^3/e^2 - (e*x + d)*b^3*d*g*n^3*log(e*x + d)^3/e^2 - 3*(e*x + d)*b^3*f*n
^3*log(e*x + d)^2/e - 3/4*(e*x + d)^2*b^3*g*n^3*log(e*x + d)^2/e^2 + 3*(e*
x + d)*b^3*d*g*n^3*log(e*x + d)^2/e^2 + 3*(e*x + d)*b^3*f*n^2*log(e*x + d)
^2*log(c)/e + 3/2*(e*x + d)^2*b^3*g*n^2*log(e*x + d)^2*log(c)/e^2 - 3*(e*x
 + d)*b^3*d*g*n^2*log(e*x + d)^2*log(c)/e^2 + 6*(e*x + d)*b^3*f*n^3*log(e*
x + d)/e + 3/4*(e*x + d)^2*b^3*g*n^3*log(e*x + d)/e^2 - 6*(e*x + d)*b^3*d*
g*n^3*log(e*x + d)/e^2 + 3*(e*x + d)*a*b^2*f*n^2*log(e*x + d)^2/e + 3/2*(e
*x + d)^2*a*b^2*g*n^2*log(e*x + d)^2/e^2 - 3*(e*x + d)*a*b^2*d*g*n^2*log(e
*x + d)^2/e^2 - 6*(e*x + d)*b^3*f*n^2*log(e*x + d)*log(c)/e - 3/2*(e*x + d
)^2*b^3*g*n^2*log(e*x + d)*log(c)/e^2 + 6*(e*x + d)*b^3*d*g*n^2*log(e*x +
d)*log(c)/e^2 + 3*(e*x + d)*b^3*f*n*log(e*x + d)*log(c)^2/e + 3/2*(e*x + d
)^2*b^3*g*n*log(e*x + d)*log(c)^2/e^2 - 3*(e*x + d)*b^3*d*g*n*log(e*x + d)
*log(c)^2/e^2 - 6*(e*x + d)*b^3*f*n^3/e - 3/8*(e*x + d)^2*b^3*g*n^3/e^2 +
6*(e*x + d)*b^3*d*g*n^3/e^2 - 6*(e*x + d)*a*b^2*f*n^2*log(e*x + d)/e - 3/2
*(e*x + d)^2*a*b^2*g*n^2*log(e*x + d)/e^2 + 6*(e*x + d)*a*b^2*d*g*n^2*log(
e*x + d)/e^2 + 6*(e*x + d)*b^3*f*n^2*log(c)/e + 3/4*(e*x + d)^2*b^3*g*n^2*
log(c)/e^2 - 6*(e*x + d)*b^3*d*g*n^2*log(c)/e^2 + 6*(e*x + d)*a*b^2*f*n*lo
g(e*x + d)*log(c)/e + 3*(e*x + d)^2*a*b^2*g*n*log(e*x + d)*log(c)/e^2 - 6*
(e*x + d)*a*b^2*d*g*n*log(e*x + d)*log(c)/e^2 - 3*(e*x + d)*b^3*f*n*log...
```

3.54.9 Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.93

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$= \ln\left(c\,(d+e\,x)^n\right)^3\left(\frac{b^3\,g\,x^2}{2}-\frac{d\,(b^3\,d\,g-2\,b^3\,e\,f)}{2\,e^2}+b^3\,f\,x\right)+\ln\left(c\,(d+e\,x)^n\right)\left(\frac{x\left(\frac{12\,a^2\,b\,d\,g+12\,a^2\,b\,e\,f-12\,b^3\,d\,g\,n^2+24\,b^3\,e\,f\,n^2-24\,a\,b^2\,e\,f\,n}{2\,e}-\frac{3\,b\,d\,g\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}\right)}{2}+\frac{3\,b\,g\,x^2\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{4}\right)+\ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{x\left(\frac{6\,b^2\,(a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}-\frac{3\,b^2\,d\,g\,(2\,a-b\,n)}{e}\right)}{2}-\frac{3\,d\,(2\,a\,b^2\,d\,g-4\,a\,b^2\,e\,f-3\,b^3\,d\,g\,n+4\,b^3\,e\,f\,n)}{4\,e^2}+\frac{3\,b^2\,g\,x^2\,(2\,a-b\,n)}{4}\right)+x\left(\frac{4\,a^3\,d\,g+4\,a^3\,e\,f+18\,b^3\,d\,g\,n^3-24\,b^3\,e\,f\,n^3-12\,a\,b^2\,d\,g\,n^2+24\,a\,b^2\,e\,f\,n^2-12\,a^2\,b\,e\,f\,n}{4\,e}-\frac{d\,g\,(4\,a^3-6\,a^2\,b\,n+6\,a\,b^2\,n^2-3\,b^3\,n^3)}{4\,e}\right)+\frac{g\,x^2\,(4\,a^3-6\,a^2\,b\,n+6\,a\,b^2\,n^2-3\,b^3\,n^3)}{8}-\frac{\ln(d+e\,x)\,(6\,g\,a^2\,b\,d^2\,n-12\,e\,f\,a^2\,b\,d\,n-18\,g\,a\,b^2\,d^2\,n^2+24\,e\,f\,a\,b^2\,d\,n^2+21\,g\,b^3\,d^2\,n^3-24\,e\,f\,b^3\,d\,n^3}{4\,e^2}$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
log(c*(d + e*x)^n)^3*((b^3*g*x^2)/2 - (d*(b^3*d*g - 2*b^3*e*f))/(2*e^2) +
b^3*f*x) + log(c*(d + e*x)^n)*((x*((12*a^2*b*d*g + 12*a^2*b*e*f - 12*b^3*d
*g*n^2 + 24*b^3*e*f*n^2 - 24*a*b^2*e*f*n)/(2*e) - (3*b*d*g*(2*a^2 + b^2*n^
2 - 2*a*b*n))/e))/2 + (3*b*g*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/4) + log(c*(
d + e*x)^n)^2*((x*((6*b^2*(a*d*g + a*e*f - b*e*f*n))/e - (3*b^2*d*g*(2*a -
 b*n))/e))/2 - (3*d*(2*a*b^2*d*g - 4*a*b^2*e*f - 3*b^3*d*g*n + 4*b^3*e*f*n
))/(4*e^2) + (3*b^2*g*x^2*(2*a - b*n))/4) + x*((4*a^3*d*g + 4*a^3*e*f + 18
*b^3*d*g*n^3 - 24*b^3*e*f*n^3 - 12*a*b^2*d*g*n^2 + 24*a*b^2*e*f*n^2 - 12*a
^2*b*e*f*n)/(4*e) - (d*g*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n))/(4
*e)) + (g*x^2*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n))/8 - (log(d +
e*x)*(21*b^3*d^2*g*n^3 + 6*a^2*b*d^2*g*n - 24*b^3*d*e*f*n^3 - 18*a*b^2*d^2
*g*n^2 - 12*a^2*b*d*e*f*n + 24*a*b^2*d*e*f*n^2))/(4*e^2)
```

3.55 $\int (a+b\log(c(d+ex)^n))^3\,dx$

3.55.1 Optimal result

Integrand size = 16, antiderivative size = 99

$$\int (a+b\log(c(d+ex)^n))^3\,dx = 6ab^2n^2x-6b^3n^3x+\frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{e}-\frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}$$

output
```
6*a*b^2*n^2*x-6*b^3*n^3*x+6*b^3*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-3*b*n*(e*x+d
)*(a+b*ln(c*(e*x+d)^n))^2/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e
```

3.55.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int (a+b\log(c(d+ex)^n))^3\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^3-3bn\left((d+ex)(a+b\log(c(d+ex)^n))^2-2bn(e(a-bn)x+b(d+ex)\right.}{e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])))/e
```

3.55.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2836, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^3\,dx$$

$$\Big\downarrow \ 2836$$

$$\frac{\int (a+b\log(c(d+ex)^n))^3\,d(d+ex)}{e}$$

$$\Big\downarrow \ 2733$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^3-3bn\int (a+b\log(c(d+ex)^n))^2\,d(d+ex)}{e}$$

$$\Big\downarrow \ 2733$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^3-3bn\Big((d+ex)(a+b\log(c(d+ex)^n))^2-2bn\int (a+b\log(c(d+ex)^n))\,d(d+e}{e}$$

$$\Big\downarrow \ 2009$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^3-3bn\Big((d+ex)(a+b\log(c(d+ex)^n))^2-2bn(a(d+ex)+b(d+ex)\log(c(d+}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^2 - 2*b*n*(a*(d + e*x) - b*n*(d + e*x) + b*(d + e*x)*Log[c*(d
+ e*x)^n])))/e
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(99) = 198$.

Time = 0.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.25

method	result
parallelrisch	$\frac{x\ln(c(ex+d)^n)^3 b^3 en-3x\ln(c(ex+d)^n)^2 b^3 e n^2+6x\ln(c(ex+d)^n) b^3 e n^3-6x b^3 e n^4+3x\ln(c(ex+d)^n)^2 a b^2 en-6x\ln(c(ex+d)^n)}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
(x*ln(c*(e*x+d)^n)^3*b^3*e*n-3*x*ln(c*(e*x+d)^n)^2*b^3*e*n^2+6*x*ln(c*(e*x
+d)^n)*b^3*e*n^3-6*x*b^3*e*n^4+3*x*ln(c*(e*x+d)^n)^2*a*b^2*e*n-6*x*ln(c*(e
*x+d)^n)*a*b^2*e*n^2+6*x*a*b^2*e*n^3+ln(c*(e*x+d)^n)^3*b^3*d*n-3*ln(c*(e*x
+d)^n)^2*b^3*d*n^2+6*ln(c*(e*x+d)^n)*b^3*d*n^3+6*b^3*d*n^4+3*x*ln(c*(e*x+d
)^n)*a^2*b*e*n-3*x*a^2*b*e*n^2+3*ln(c*(e*x+d)^n)^2*a*b^2*d*n-6*ln(c*(e*x+d
)^n)*a*b^2*d*n^2-6*a*b^2*d*n^3+x*a^3*e*n+3*ln(c*(e*x+d)^n)*a^2*b*d*n+3*a^2
*b*d*n^2-a^3*d*n)/e/n
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(99) = 198.

Time = 0.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.27

$$\int (a+b\log(c(d+ex)^n))^3\,dx$$
$$= \frac{b^3ex\log(c)^3 + (b^3en^3x + b^3dn^3)\log(ex+d)^3 - 3(b^3en - ab^2e)x\log(c)^2 - 3(b^3dn^3 - ab^2dn^2 + (b^3en^3 -$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
(b^3*e*x*log(c)^3 + (b^3*e*n^3*x + b^3*d*n^3)*log(e*x + d)^3 - 3*(b^3*e*n
- a*b^2*e)*x*log(c)^2 - 3*(b^3*d*n^3 - a*b^2*d*n^2 + (b^3*e*n^3 - a*b^2*e*
n^2)*x - (b^3*e*n^2*x + b^3*d*n^2)*log(c))*log(e*x + d)^2 + 3*(2*b^3*e*n^2
 - 2*a*b^2*e*n + a^2*b*e)*x*log(c) - (6*b^3*e*n^3 - 6*a*b^2*e*n^2 + 3*a^2*
b*e*n - a^3*e)*x + 3*(2*b^3*d*n^3 - 2*a*b^2*d*n^2 + a^2*b*d*n + (b^3*e*n*x
 + b^3*d*n)*log(c)^2 + (2*b^3*e*n^3 - 2*a*b^2*e*n^2 + a^2*b*e*n)*x - 2*(b^
3*d*n^2 - a*b^2*d*n + (b^3*e*n^2 - a*b^2*e*n)*x)*log(c))*log(e*x + d))/e
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(95) = 190.

Time = 0.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.97

$$\int (a+b\log(c(d+ex)^n))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2bd\log(c(d+ex)^n)}{e} - 3a^2bnx + 3a^2bx\log(c(d+ex)^n) - \frac{6ab^2dn\log(c(d+ex)^n)}{e} + \frac{3ab^2d\log(c(d+ex)^n)^2}{e} + 6ab \\ x(a+b\log(cd^n))^3 \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*d*log(c*(d + e*x)**n)/e - 3*a**2*b*n*x + 3*a*
*2*b*x*log(c*(d + e*x)**n) - 6*a*b**2*d*n*log(c*(d + e*x)**n)/e + 3*a*b**2
*d*log(c*(d + e*x)**n)**2/e + 6*a*b**2*n**2*x - 6*a*b**2*n*x*log(c*(d + e*
x)**n) + 3*a*b**2*x*log(c*(d + e*x)**n)**2 + 6*b**3*d*n**2*log(c*(d + e*x)
**n)/e - 3*b**3*d*n*log(c*(d + e*x)**n)**2/e + b**3*d*log(c*(d + e*x)**n)*
*3/e - 6*b**3*n**3*x + 6*b**3*n**2*x*log(c*(d + e*x)**n) - 3*b**3*n*x*log(
c*(d + e*x)**n)**2 + b**3*x*log(c*(d + e*x)**n)**3, Ne(e, 0)), (x*(a + b*l
og(c*d**n))**3, True))
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.85

$$\int (a+b\log(c(d+ex)^n))^3\,dx = b^3x\log\left((ex+d)^nc\right)^3 - 3\,a^2ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) + 3\,ab^2x\log\left((ex+d)^nc\right)^2 + 3\,a^2bx\log\left((ex+d)^nc\right) - 3\left(2\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right) + \frac{\left(d\log(ex+d)^2-2\,ex+2\,d\log(ex+d)\right)n^2}{e}\right)ab^2 - \left(3\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right)^2 - en\left(\frac{\left(d\log(ex+d)^3+3\,d\log(ex+d)^2-6\,ex+6\,d\log\right.}{e^2}\right.\right. + a^3x$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
b^3*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*n*(x/e - d*log(e*x + d)/e^2) + 3*a*
b^2*x*log((e*x + d)^n*c)^2 + 3*a^2*b*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e
- d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d
*log(e*x + d))*n^2/e)*a*b^2 - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x +
 d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log
(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log
((e*x + d)^n*c)/e^2))*b^3 + a^3*x
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(99) = 198.

Time = 0.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.03

$$\int (a+b\log(c(d+ex)^n))^3\,dx = \frac{(ex+d)b^3n^3\log(ex+d)^3}{e} - \frac{3(ex+d)b^3n^3\log(ex+d)^2}{e} + \frac{3(ex+d)b^3n^2\log(ex+d)^2\log(c)}{e} + \frac{6(ex+d)b^3n^3\log(ex+d)}{e} + \frac{3(ex+d)ab^2n^2\log(ex+d)^2}{e} - \frac{6(ex+d)b^3n^2\log(ex+d)\log(c)}{e} + \frac{3(ex+d)b^3n\log(ex+d)\log(c)^2}{e} - \frac{6(ex+d)b^3n^3}{e} - \frac{6(ex+d)ab^2n^2\log(ex+d)}{e} + \frac{6(ex+d)b^3n^2\log(c)}{e} + \frac{6(ex+d)ab^2n\log(ex+d)\log(c)}{e} - \frac{3(ex+d)b^3n\log(c)^2}{e} + \frac{(ex+d)b^3\log(c)^3}{e} + \frac{6(ex+d)ab^2n^2}{e} + \frac{3(ex+d)a^2bn\log(ex+d)}{e} - \frac{6(ex+d)ab^2n\log(c)}{e} + \frac{3(ex+d)ab^2\log(c)^2}{e} - \frac{3(ex+d)a^2bn}{e} + \frac{3(ex+d)a^2b\log(c)}{e} + \frac{(ex+d)a^3}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
(e*x + d)*b^3*n^3*log(e*x + d)^3/e - 3*(e*x + d)*b^3*n^3*log(e*x + d)^2/e
+ 3*(e*x + d)*b^3*n^2*log(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^3*n^3*log(e*
x + d)/e + 3*(e*x + d)*a*b^2*n^2*log(e*x + d)^2/e - 6*(e*x + d)*b^3*n^2*lo
g(e*x + d)*log(c)/e + 3*(e*x + d)*b^3*n*log(e*x + d)*log(c)^2/e - 6*(e*x +
 d)*b^3*n^3/e - 6*(e*x + d)*a*b^2*n^2*log(e*x + d)/e + 6*(e*x + d)*b^3*n^2
*log(c)/e + 6*(e*x + d)*a*b^2*n*log(e*x + d)*log(c)/e - 3*(e*x + d)*b^3*n*
log(c)^2/e + (e*x + d)*b^3*log(c)^3/e + 6*(e*x + d)*a*b^2*n^2/e + 3*(e*x +
 d)*a^2*b*n*log(e*x + d)/e - 6*(e*x + d)*a*b^2*n*log(c)/e + 3*(e*x + d)*a*
b^2*log(c)^2/e - 3*(e*x + d)*a^2*b*n/e + 3*(e*x + d)*a^2*b*log(c)/e + (e*x
 + d)*a^3/e
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int (a+b\log (c(d+ex)^n))^3 \, dx = x\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right) + \ln\left(c\left(d+e\,x\right)^n\right)^3\left(b^3\,x+\frac{b^3\,d}{e}\right) + \ln\left(c\left(d+e\,x\right)^n\right)^2\left(\frac{3\left(a\,b^2\,d-b^3\,d\,n\right)}{e}+3\,b^2\,x\left(a-b\,n\right)\right) + \frac{\ln\left(d+e\,x\right)\,\left(3\,d\,a^2\,b\,n-6\,d\,a\,b^2\,n^2+6\,d\,b^3\,n^3\right)}{e} + 3\,b\,x\,\ln\left(c\left(d+e\,x\right)^n\right)\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)$$

input
```
int((a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
x*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n) + log(c*(d + e*x)^n)^3*(b^3*
x + (b^3*d)/e) + log(c*(d + e*x)^n)^2*((3*(a*b^2*d - b^3*d*n))/e + 3*b^2*x
*(a - b*n)) + (log(d + e*x)*(6*b^3*d*n^3 + 3*a^2*b*d*n - 6*a*b^2*d*n^2))/e
 + 3*b*x*log(c*(d + e*x)^n)*(a^2 + 2*b^2*n^2 - 2*a*b*n)
```

3.56 $\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

3.56.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{6b^3n^3\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output

```
(a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g+3*b*n*(a+b*ln(c*(e*x+d)
^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-6*b^2*n^2*(a+b*ln(c*(e*x+d)^n))*
polylog(3,-g*(e*x+d)/(-d*g+e*f))/g+6*b^3*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*
f))/g
```

3.56.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(158) = 316$.

Time = 0.18 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(f+gx)+3bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\left(\mathrm{lo}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] + 3*b*n*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*Log[(e*(f + g*x)
)/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 6*b^2*n^2*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*
x))/(e*f - d*g)])/2 + Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(Log[d + e*x]^3*Lo
g[(e*(f + g*x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(
-(e*f) + d*g)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] +
 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

3.56.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$\Big\downarrow \text{2843}$$
$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3ben\int \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$
$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g}-\frac{3bn\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}$$

$\downarrow$ 2821

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g}-\frac{3bn\left(2bn\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\right)}{g}$$

$\downarrow$ 2830

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g}-\frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-bn\int\frac{\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)\right)-\text{PolyLog}\left(2,-\frac{g}{}\right.}{g}$$

$\downarrow$ 7143

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g}-\frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-bn\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)\right)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)\left(\right.\right.}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g - (3*b*n*(
-((a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]) +
 2*b*n*((a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))
] - b*n*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])))/g
```

3.56. $\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx$

3.56.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.56.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 1396, normalized size of antiderivative = 8.84

method	result	size
risch	Expression too large to display	1396

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
-b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^3*n^3+3*b^3*ln(g*(e*x+d)-d*g+e*f)/g
*ln((e*x+d)^n)*ln(e*x+d)^2*n^2-3*b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)
^2*ln(e*x+d)*n+b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^3-2*b^3*n^3/g*ln(
e*x+d)^3*ln(1-g*(e*x+d)/(d*g-e*f))-3*b^3*n^3/g*ln(e*x+d)^2*polylog(2,g*(e*
x+d)/(d*g-e*f))+6*b^3*n^3/g*polylog(4,g*(e*x+d)/(d*g-e*f))+3*b^3*n^3*dilog
((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)^2-6*b^3*n^2*dilog((g*(e*x+d)-
d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)*ln(e*x+d)+3*b^3*n*dilog((g*(e*x+d)-d*
g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)^2+3*b^3*n^3*ln(e*x+d)^3*ln((g*(e*x+d)-d
*g+e*f)/(-d*g+e*f))/g-6*b^3*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e
*f))/g*ln((e*x+d)^n)+3*b^3*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/
g*ln((e*x+d)^n)^2+3*b^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*
g-e*f))+6*b^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))
-6*b^3*n^2/g*ln((e*x+d)^n)*polylog(3,g*(e*x+d)/(d*g-e*f))+1/8*(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*
x+d)^n)^3*b+2*b*ln(c)+2*a)^3*ln(g*x+f)/g+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*
csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*cs
gn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*l
n(c)+2*a)*b^2*((ln((e*x+d)^n)-n*ln(e*x+d))^2*ln(g*(e*x+d)-d*g+e*f)/g+n^2/g
*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*n^2/g*ln(e*x+d)*polylog(2,g*(e...
```

3.56.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^3}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g*x + f), x)
```

3.56.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x), x)
```

3.56.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")
```

output
```
a^3*log(g*x + f)/g + integrate((b^3*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*
a*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^
2 + 3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log((e*x + d)^n))/(g*x + f),
 x)
```

3.56.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x), x)
```

3.57 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2} dx$

3.57.1 Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2} dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{(ef-dg)(f+gx)} - \frac{3ben(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{6b^2en^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{6b^3en^3\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/(-d*g+e*f)/(g*x+f)-3*b*e*n*(a+b*ln(c*(e*x+
d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)-6*b^2*e*n^2*(a+b*ln(c*(e*x+
d)^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)+6*b^3*e*n^3*polylog(3
,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)
```

3.57.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. 2(190) = 380.

Time = 0.25 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.16

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$$
$$= \frac{-3b(ef-dg)n\log(d+ex)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2+3ben(f+gx)\log(d+ex)\left(a-bn\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^2,x]
```

output
```
(-3*b*(e*f - d*g)*n*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)
^n])^2 + 3*b*e*n*(f + g*x)*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[c*(d
 + e*x)^n])^2 - (e*f - d*g)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^
3 - 3*b*e*n*(f + g*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[
f + g*x] + 3*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d
+ e*x]*(g*(d + e*x)*Log[d + e*x] - 2*e*(f + g*x)*Log[(e*(f + g*x))/(e*f -
d*g)]) - 2*e*(f + g*x)*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3
*(Log[d + e*x]^2*(g*(d + e*x)*Log[d + e*x] - 3*e*(f + g*x)*Log[(e*(f + g*x
))/(e*f - d*g)]) - 6*e*(f + g*x)*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(
e*f) + d*g)] + 6*e*(f + g*x)*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(g
*(e*f - d*g)*(f + g*x))
```

3.57.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2844, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$$
$$\Big\downarrow \ 2844$$
$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{(f+gx)(ef-dg)}-\frac{3ben\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}dx}{ef-dg}$$

$$\Big\downarrow \; 2843$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(f+gx)(ef-dg)}-\frac{3ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2ben\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}\right)}{ef-dg}$$

$$\Big\downarrow \; 2881$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(f+gx)(ef-dg)}-\frac{3ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}\right)}{ef-dg}$$

$$\Big\downarrow \; 2821$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(f+gx)(ef-dg)}-\frac{3ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\left(bn\int\frac{\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right)}{g}\right)}{ef-dg}$$

$$\Big\downarrow \; 7143$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(f+gx)(ef-dg)}-\frac{3ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\left(bn\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right)}{g}\right)}{ef-dg}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/((e*f - d*g)*(f + g*x)) - (3*b*e*
n*(((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g - (2*b*
n*(-((a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])
+ b*n*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))]))/g))/(e*f - d*g)
```

3.57. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$

3.57.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.57. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$

3.57.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 1268, normalized size of antiderivative = 6.67

method	result	size
risch	Expression too large to display	1268

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^3*ln((e*x+d)^n)^3/(g*x+f)/g+3*b^3/g*n^3*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*
f)*ln(e*x+d)^2-6*b^3/g*n^2*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)
*ln(e*x+d)+3*b^3/g*n*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^2+3*b
^3/g*n^2*e/(d*g-e*f)*ln(e*x+d)^2*ln((e*x+d)^n)-3*b^3/g*n*e/(d*g-e*f)*ln(e*
x+d)*ln((e*x+d)^n)^2+3*b^3/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln(1+g*(e*x+d)/(-
d*g+e*f))+6*b^3/g*n^3*e/(d*g-e*f)*ln(e*x+d)*polylog(2,-g*(e*x+d)/(-d*g+e*f
))-6*b^3/g*n^3*e/(d*g-e*f)*polylog(3,-g*(e*x+d)/(-d*g+e*f))+b^3/g*n^3*e/(-
d*g+e*f)*ln(e*x+d)^3-6*b^3/g*n^3*e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d
*g+e*f))*ln(e*x+d)+6*b^3/g*n^2*e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g
+e*f))*ln((e*x+d)^n)-6*b^3/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln((g*(e*x+d)-d*g
+e*f)/(-d*g+e*f))+6*b^3/g*n^2*e/(d*g-e*f)*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)
/(-d*g+e*f))*ln((e*x+d)^n)-1/8*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn
(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n
)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^3/(g
*x+f)/g+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*
csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^
n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^2*(-ln((e*x+d)^n)^2/(
g*x+f)/g+2/g*n*e*(-ln((e*x+d)^n)/(d*g-e*f)*ln(e*x+d)+ln((e*x+d)^n)/(d*g-e*
f)*ln(g*x+f)-e*n*(1/(d*g-e*f)*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g
*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e)-1/2/(d*g-e*f)/e*ln(e*x+d)^2)...
```

3.57.5 Fricas [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^3}{(f+gx)^2}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^3}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.57.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x)**2, x)
```

3.57.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^2,x, algorithm="maxima")
```

output
```
3*a^2*b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2))
- b^3*log((e*x + d)^n)^3/(g^2*x + f*g) - 3*a^2*b*log((e*x + d)^n*c)/(g^2*x
 + f*g) - a^3/(g^2*x + f*g) + integrate((b^3*d*g*log(c)^3 + 3*a*b^2*d*g*lo
g(c)^2 + 3*(a*b^2*d*g + (e*f*n + d*g*log(c))*b^3 + (a*b^2*e*g + (e*g*n + e
*g*log(c))*b^3)*x)*log((e*x + d)^n)^2 + (b^3*e*g*log(c)^3 + 3*a*b^2*e*g*lo
g(c)^2)*x + 3*(b^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c) + (b^3*e*g*log(c)^2 +
 2*a*b^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^3*x^3 + d*f^2*g + (2*e*f*g^
2 + d*g^3)*x^2 + (e*f^2*g + 2*d*f*g^2)*x), x)
```

3.57.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f)^2, x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^2, x)
```

3.58 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx$

3.58.1 Optimal result

Integrand size = 24, antiderivative size = 342

$$
\begin{aligned}
\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx = & -\frac{3ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2(ef-dg)^2(f+gx)} \\
& -\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2g(f+gx)^2} \\
& +\frac{3b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^2} \\
& -\frac{3be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{2g(ef-dg)^2} \\
& +\frac{3b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} \\
& +\frac{3b^3e^2n^3\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^2} \\
& +\frac{3b^3e^2n^3\operatorname{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}
\end{aligned}
$$

output
```
-3/2*b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/(-d*g+e*f)^2/(g*x+f)-1/2*(a+b*l
n(c*(e*x+d)^n))^3/g/(g*x+f)^2+3*b^2*e^2*n^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*
x+f)/(-d*g+e*f))/g/(-d*g+e*f)^2-3/2*b*e^2*n*(a+b*ln(c*(e*x+d)^n))^2*ln(1+(
-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^2+3*b^2*e^2*n^2*(a+b*ln(c*(e*x+d)^n))*po
lylog(2,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^2+3*b^3*e^2*n^3*polylog(2,-g*(e*
x+d)/(-d*g+e*f))/g/(-d*g+e*f)^2+3*b^3*e^2*n^3*polylog(3,(d*g-e*f)/g/(e*x+d
))/g/(-d*g+e*f)^2
```

3.58.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.81

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx =$$
$$-\frac{-3be(ef-dg)n(f+gx)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2+3b(ef-dg)^2n\log(d+ex)\left(a-b\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^3,x]
```

output
```
-1/2*(-3*b*e*(e*f - d*g)*n*(f + g*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d +
e*x)^n])^2 + 3*b*(e*f - d*g)^2*n*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Lo
g[c*(d + e*x)^n])^2 - 3*b*e^2*n*(f + g*x)^2*Log[d + e*x]*(a - b*n*Log[d +
e*x] + b*Log[c*(d + e*x)^n])^2 + (e*f - d*g)^2*(a - b*n*Log[d + e*x] + b*L
og[c*(d + e*x)^n])^3 + 3*b*e^2*n*(f + g*x)^2*(a - b*n*Log[d + e*x] + b*Log
[c*(d + e*x)^n])^2*Log[f + g*x] + 3*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log[
c*(d + e*x)^n])*(g*(d + e*x)*(d*g - e*(2*f + g*x))*Log[d + e*x]^2 - 2*e^2*
(f + g*x)^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*e*(f + g*x)*Log[d + e*x]*(g
*(d + e*x) + e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) + 2*e^2*(f + g*x)
^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(g*(d + e*x)*(d*g -
 e*(2*f + g*x))*Log[d + e*x]^3 + 3*e*(f + g*x)*Log[d + e*x]^2*(g*(d + e*x)
 + e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) - 6*e^2*(f + g*x)^2*Log[d +
 e*x]*(Log[(e*(f + g*x))/(e*f - d*g)] - PolyLog[2, (g*(d + e*x))/(-(e*f) +
 d*g)]) - 6*e^2*(f + g*x)^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*e
^2*(f + g*x)^2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(g*(e*f - d*g)^2
*(f + g*x)^2)
```

3.58.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2845, 2858, 27, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{3ben\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(f+gx)^2}dx}{2g}-\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2858}$$

$$\frac{3bn\int \frac{e^2(a+b\log(c(d+ex)^n))^2}{(d+ex)\left(e\left(f-\frac{dg}{e}\right)+g(d+ex)\right)^2}d(d+ex)}{2g}-\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3be^2n\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{2g}-\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2789}$$

$$\frac{3be^2n\left(\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}\right)}{2g}-\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2755}$$

$$\frac{3be^2n\left(\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{2bn\int \frac{a+b\log(c(d+ex)^n)}{ef-dg+g(d+ex)}d(d+ex)}{ef-dg}\right)}{ef-dg}\right)}{2g}-\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2754}$$

$$\frac{3be^2n\left(\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{2bn\left(\frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)(a+b\log(c(d+ex)^n))}{g}-\frac{bn\int \frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)}{d+ex}d(}{g}\right.}{ef-dg}\right.}{ef-dg}\right.}{2g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$\downarrow$ 2779

$$\frac{3be^2n\left(\frac{\frac{2bn\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{ef-dg}{g(d+ex)}+1\right)}{d+ex}d(d+ex)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{2bn\left(\frac{\log\left(\frac{g(}{e}\right.}{}\right.}{}\right.}{}\right.}{2g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$\downarrow$ 2821

3.58. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx$

$$\frac{3be^2n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))-bn\int\frac{\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{d+ex}d(d+ex)\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)}{(ef-d\cdots}\right.}{\cdots}\right.}{2g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$\downarrow$ 2838

$$\frac{3be^2n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))-bn\int\frac{\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{d+ex}d(d+ex)\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)}{(ef-d\cdots}\right.}{\cdots}\right.}{2g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

$\downarrow$ 7143

$$\frac{3be^2n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(\cdots}{(ef-dg)(g(d+ex)\cdots}\right.}{\cdots}\right.}{2g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2}$$

3.58. $\int\frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n])^3/(g*(f + g*x)^2) + (3*b*e^2*n*(-((g*(((d
+ e*x)*(a + b*Log[c*(d + e*x)^n])^2)/((e*f - d*g)*(e*f - d*g + g*(d + e*x)
)) - (2*b*n*(((a + b*Log[c*(d + e*x)^n])*Log[1 + (g*(d + e*x))/(e*f - d*g)
])/g + (b*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g))/(e*f - d*g)))/(e
*f - d*g)) + (-(((a + b*Log[c*(d + e*x)^n])^2*Log[1 + (e*f - d*g)/(g*(d +
e*x))])/(e*f - d*g)) + (2*b*n*((a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((e*
f - d*g)/(g*(d + e*x)))] + b*n*PolyLog[3, -((e*f - d*g)/(g*(d + e*x)))]))/
(e*f - d*g))/(e*f - d*g)))/(2*g)
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.58.4 Maple [F]

$$\int \frac{\left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{3}}{\left(gx+f\right)^{3}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^3,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^3,x)
```

3.58.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

3.58.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x)**3, x)
```

3.58.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^3,x, algorithm="maxima")
```

output
```
3/2*a^2*b*e*n*(e*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - e*log(
g*x + f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) + 1/(e*f^2*g - d*f*g^2 + (e*f
*g^2 - d*g^3)*x)) - 1/2*b^3*log((e*x + d)^n)^3/(g^3*x^2 + 2*f*g^2*x + f^2*
g) - 3/2*a^2*b*log((e*x + d)^n*c)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*a^3/
(g^3*x^2 + 2*f*g^2*x + f^2*g) + integrate(1/2*(2*b^3*d*g*log(c)^3 + 6*a*b^
2*d*g*log(c)^2 + 3*(2*a*b^2*d*g + (e*f*n + 2*d*g*log(c))*b^3 + (2*a*b^2*e*
g + (e*g*n + 2*e*g*log(c))*b^3)*x)*log((e*x + d)^n)^2 + 2*(b^3*e*g*log(c)^
3 + 3*a*b^2*e*g*log(c)^2)*x + 6*(b^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c) + (
b^3*e*g*log(c)^2 + 2*a*b^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^4*x^4 + d
*f^3*g + (3*e*f*g^3 + d*g^4)*x^3 + 3*(e*f^2*g^2 + d*f*g^3)*x^2 + (e*f^3*g
+ 3*d*f^2*g^2)*x), x)
```

3.58.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^3}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f)^3, x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^3,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^3, x)
```

3.59 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

3.59.1 Optimal result

Integrand size = 24, antiderivative size = 564

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \frac{b^2e^2n^2(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)^3(f+gx)} + \frac{ben(a+b\log(c(d+ex)^n))^2}{2g(ef-dg)(f+gx)^2} - \frac{be^2n(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)^3(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} - \frac{b^3e^3n^3\log(f+gx)}{g(ef-dg)^3} + \frac{2b^2e^3n^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{b^2e^3n^2(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{be^3n(a+b\log(c(d+ex)^n))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{b^3e^3n^3\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^2e^3n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\operatorname{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3}$$

output
```
b^2*e^2*n^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)^3/(g*x+f)+1/2*b*e*n*(
a+b*ln(c*(e*x+d)^n))^2/g/(-d*g+e*f)/(g*x+f)^2-b*e^2*n*(e*x+d)*(a+b*ln(c*(e
*x+d)^n))^2/(-d*g+e*f)^3/(g*x+f)-1/3*(a+b*ln(c*(e*x+d)^n))^3/g/(g*x+f)^3-b
^3*e^3*n^3*ln(g*x+f)/g/(-d*g+e*f)^3+2*b^2*e^3*n^2*(a+b*ln(c*(e*x+d)^n))*ln
(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)^3+b^2*e^3*n^2*(a+b*ln(c*(e*x+d)^n))*ln
(1+(-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^3-b*e^3*n*(a+b*ln(c*(e*x+d)^n))^2*ln
(1+(-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^3-b^3*e^3*n^3*polylog(2,(d*g-e*f)/g/
(e*x+d))/g/(-d*g+e*f)^3+2*b^2*e^3*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,(d*g
-e*f)/g/(e*x+d))/g/(-d*g+e*f)^3+2*b^3*e^3*n^3*polylog(2,-g*(e*x+d)/(-d*g+e
*f))/g/(-d*g+e*f)^3+2*b^3*e^3*n^3*polylog(3,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e
*f)^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$$
$$= \frac{3be(ef-dg)^2n(f+gx)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2+6be^2(ef-dg)n(f+gx)^2\left(a-bn\log\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^4,x]
```

output
```
(3*b*e*(e*f - d*g)^2*n*(f + g*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)
^n])^2 + 6*b*e^2*(e*f - d*g)*n*(f + g*x)^2*(a - b*n*Log[d + e*x] + b*Log[c
*(d + e*x)^n])^2 - 6*b*(e*f - d*g)^3*n*Log[d + e*x]*(a - b*n*Log[d + e*x]
+ b*Log[c*(d + e*x)^n])^2 + 6*b*e^3*n*(f + g*x)^3*Log[d + e*x]*(a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*(e*f - d*g)^3*(a - b*n*Log[d + e*
x] + b*Log[c*(d + e*x)^n])^3 - 6*b*e^3*n*(f + g*x)^3*(a - b*n*Log[d + e*x]
 + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 6*b^2*n^2*(a - b*n*Log[d + e*x]
+ b*Log[c*(d + e*x)^n])*(e^2*g*(d + e*x)*(f + g*x)^2 + g*(3*d*e^2*f^2 - 3*
d^2*e*f*g + d^3*g^2 + e^3*x*(3*f^2 + 3*f*g*x + g^2*x^2))*Log[d + e*x]^2 +
3*e^3*(f + g*x)^3*Log[(e*(f + g*x))/(e*f - d*g)] + e*(f + g*x)*Log[d + e*x
]*(g^2*(d + e*x)^2 - 4*e*g*(d + e*x)*(f + g*x) - 2*e^2*(f + g*x)^2*Log[(e*
(f + g*x))/(e*f - d*g)]) - 2*e^3*(f + g*x)^3*PolyLog[2, (g*(d + e*x))/(-(e
*f) + d*g)]) + b^3*n^3*(2*g*(3*d*e^2*f^2 - 3*d^2*e*f*g + d^3*g^2 + e^3*x*(
3*f^2 + 3*f*g*x + g^2*x^2))*Log[d + e*x]^3 - 6*e^3*(f + g*x)^3*Log[(e*(f +
 g*x))/(e*f - d*g)] + 3*e*(f + g*x)*Log[d + e*x]^2*(g^2*(d + e*x)^2 - 4*e*
g*(d + e*x)*(f + g*x) - 2*e^2*(f + g*x)^2*Log[(e*(f + g*x))/(e*f - d*g)])
+ 18*e^3*(f + g*x)^3*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] + 6*e^2*(f +
 g*x)^2*Log[d + e*x]*(g*(d + e*x) + 3*e*(f + g*x)*Log[(e*(f + g*x))/(e*f -
 d*g)] - 2*e*(f + g*x)*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 12*e^3*
(f + g*x)^3*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(6*g*(e*f - d*g)...
```

3.59.3 Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2845, 2858, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4} \, dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{ben\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(f+gx)^3}dx}{g}-\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{bn\int \frac{e^3(a+b\log(c(d+ex)^n))^2}{(d+ex)\left(e\left(f-\frac{dg}{e}\right)+g(d+ex)\right)^3}d(d+ex)}{g}-\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{be^3n\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))^3}d(d+ex)}{g}-\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \ 2789$$

$$\frac{be^3n\left(\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{(ef-dg+g(d+ex))^3}d(d+ex)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \ 2756$$

$$\frac{be^3n\left(\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}-\frac{g\left(\frac{bn\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))^2}d(d+ex)}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(g(d+ex)-dg+ef)^2}\right)}{ef-dg}\right)}{g}-\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \ 2789$$

$$\frac{be^3n\left(\frac{\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{(ef-dg+g(d+ex))^2}d(\cdots}{ef-dg}\cdots\right.}{g}\cdots\right.}{ef-dg}\cdots\right.}{g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \ 2751$$

3.59. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

$$\frac{be^3n\left(\frac{\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+e\cdots}{(ef-dg)(g(d+ex)-dg\cdots}\right.}{\cdots}\right.}{g}\right.}{ef-\cdots}\right.}{g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$\Big\downarrow$ 16

$$\frac{be^3n\left(\frac{\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{(ef-dg+g(d+ex))^2}d(d+ex)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex\cdots}{(ef-dg)(g(d+ex)-dg\cdots}\right.}{\cdots}\right.}{g}\right.}{ef-dg}\right.}{g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$\Big\downarrow$ 2755

$$\frac{be^3n\left(\frac{\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{2bn\int \frac{a+b\log(c(d+ex)^n)}{ef-dg+g(d+ex)}d(d+ex)}{ef-dg}\right)}{ef-dg}}{ef-dg}-\frac{g\left(\frac{bn\left(\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(ef-dg+g(d+ex))}\cdots}{ef-dg}\right.}{\cdots}\right.}{\cdots}\right.}{g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

3.59. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

$$\Big\downarrow \text{2754}$$

$$\frac{be^3n\left(\frac{\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(ef-dg+g(d+ex))}d(d+ex)}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{2bn\left(\frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)(a+b\log(c(d+ex)^n))}{g}-\frac{bn\int\frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)}{d+ex}d(d+ex)}{g}\right)}{ef-dg}\right)}{ef-dg}}{ef-dg}\right)}{g}$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \text{2779}$$

$$be^3n\left(\frac{\frac{\frac{2bn\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{ef-dg}{g(d+ex)}+1\right)}{d+ex}d(d+ex)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(g(d+ex)-dg+ef)}-\frac{2bn\left(\frac{\log\left(\frac{g(d+ex)}{ef-dg}\right.}{}\right.}{}\right.}{}}{ef-dg}\right)$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$$\Big\downarrow \text{2821}$$

3.59. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

$$be^3n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))-bn\int\frac{\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{d+ex}d(d+ex)\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b}{(ef-dg)(g}\right.}{}\right.}{ef-dg}\right.$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$\downarrow$ 2838

$$be^3n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))-bn\int\frac{\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{d+ex}d(d+ex)\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b}{(ef-dg)(g}\right.}{}\right.}{ef-dg}\right.$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

$\downarrow$ 7143

3.59. $\int\frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

$$be^3n\left(\frac{\frac{\frac{2bn\left(\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)\right)}{ef-dg}-\frac{\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{ef-dg}}{ef-dg}}{ef-dg}-\frac{g\left(\frac{(d+ex)(a+b\log(c(d+ex}{(ef-dg)(g(d+ex)-dg}\right.}{}\right.$$

$$\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^4,x]
```

output
```
-1/3*(a + b*Log[c*(d + e*x)^n])^3/(g*(f + g*x)^3) + (b*e^3*n*(-((g*(-1/2*(
a + b*Log[c*(d + e*x)^n])^2/(g*(e*f - d*g + g*(d + e*x))^2) + (b*n*(-((g*(
((d + e*x)*(a + b*Log[c*(d + e*x)^n]))/((e*f - d*g)*(e*f - d*g + g*(d + e*
x))) - (b*n*Log[e*f - d*g + g*(d + e*x)])/(g*(e*f - d*g))))/(e*f - d*g)) +
 (-(((a + b*Log[c*(d + e*x)^n])*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(e*f -
 d*g)) + (b*n*PolyLog[2, -((e*f - d*g)/(g*(d + e*x)))])/(e*f - d*g))/(e*f
- d*g)))/g))/(e*f - d*g)) + (-((g*(((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2
)/((e*f - d*g)*(e*f - d*g + g*(d + e*x))) - (2*b*n*(((a + b*Log[c*(d + e*x
)^n])*Log[1 + (g*(d + e*x))/(e*f - d*g)])/g + (b*n*PolyLog[2, -((g*(d + e*
x))/(e*f - d*g))])/g))/(e*f - d*g)))/(e*f - d*g)) + (-(((a + b*Log[c*(d +
e*x)^n])^2*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(e*f - d*g)) + (2*b*n*((a +
 b*Log[c*(d + e*x)^n])*PolyLog[2, -((e*f - d*g)/(g*(d + e*x)))] + b*n*Poly
Log[3, -((e*f - d*g)/(g*(d + e*x)))]))/(e*f - d*g))/(e*f - d*g))/(e*f - d*
g)))/g
```

3.59. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

3.59.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

3.59. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.59. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx$

3.59.4 Maple [F]

$$\int \frac{(a+b\ln (c(ex+d)^n))^3}{(gx+f)^4}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^4,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^4,x)
```

3.59.5 Fricas [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^4}\, dx = \int \frac{(b\log ((ex+d)^n c)+a)^3}{(gx+f)^4}\, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^4,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3
*g*x + f^4), x)
```

3.59.6 Sympy [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^4}\, dx = \int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^4}\, dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)**4,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x)**4, x)
```

3.59.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{(gx+f)^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/2*(2*e^2*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3
*g^4) - 2*e^2*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 -
d^3*g^4) + (2*e*g*x + 3*e*f - d*g)/(e^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^
3 + (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2
*g^3 + d^2*f*g^4)*x))*a^2*b*e*n - 1/3*b^3*log((e*x + d)^n)^3/(g^4*x^3 + 3*
f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - a^2*b*log((e*x + d)^n*c)/(g^4*x^3 + 3*f
*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*a^3/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g
^2*x + f^3*g) + integrate((b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c)^2 + (3*a*
b^2*d*g + (e*f*n + 3*d*g*log(c))*b^3 + (3*a*b^2*e*g + (e*g*n + 3*e*g*log(c
))*b^3)*x)*log((e*x + d)^n)^2 + (b^3*e*g*log(c)^3 + 3*a*b^2*e*g*log(c)^2)*
x + 3*(b^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c) + (b^3*e*g*log(c)^2 + 2*a*b^2
*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^5*x^5 + d*f^4*g + (4*e*f*g^4 + d*g^
5)*x^4 + 2*(3*e*f^2*g^3 + 2*d*f*g^4)*x^3 + 2*(2*e*f^3*g^2 + 3*d*f^2*g^3)*x
^2 + (e*f^4*g + 4*d*f^3*g^2)*x), x)
```

3.59.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{(gx+f)^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^4,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f)^4, x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)^4}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^4,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^4, x)
```

3.60 $\int (f+gx) (a+b \log (c(d+ex)^n))^4 \, dx$

3.60.1 Optimal result

Integrand size = 22, antiderivative size = 340

$$\begin{aligned}
&\int (f+gx) (a+b \log (c(d+ex)^n))^4 \, dx \\
&= -\frac{24ab^3(ef-dg)n^3x}{e}+\frac{24b^4(ef-dg)n^4x}{e}+\frac{3b^4gn^4(d+ex)^2}{4e^2} \\
&-\frac{24b^4(ef-dg)n^3(d+ex)\log (c(d+ex)^n)}{e^2}-\frac{3b^3gn^3(d+ex)^2 (a+b\log (c(d+ex)^n))}{2e^2} \\
&+\frac{12b^2(ef-dg)n^2(d+ex) (a+b\log (c(d+ex)^n))^2}{e^2} \\
&+\frac{3b^2gn^2(d+ex)^2 (a+b\log (c(d+ex)^n))^2}{2e^2} \\
&-\frac{4b(ef-dg)n(d+ex) (a+b\log (c(d+ex)^n))^3}{e^2} \\
&-\frac{bgn(d+ex)^2 (a+b\log (c(d+ex)^n))^3}{e^2} \\
&+\frac{(ef-dg)(d+ex) (a+b\log (c(d+ex)^n))^4}{e^2}+\frac{g(d+ex)^2 (a+b\log (c(d+ex)^n))^4}{2e^2}
\end{aligned}$$

output
```
-24*a*b^3*(-d*g+e*f)*n^3*x/e+24*b^4*(-d*g+e*f)*n^4*x/e+3/4*b^4*g*n^4*(e*x+
d)^2/e^2-24*b^4*(-d*g+e*f)*n^3*(e*x+d)*ln(c*(e*x+d)^n)/e^2-3/2*b^3*g*n^3*(
e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+12*b^2*(-d*g+e*f)*n^2*(e*x+d)*(a+b*ln(c
*(e*x+d)^n))^2/e^2+3/2*b^2*g*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2-4*b
*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^2-b*g*n*(e*x+d)^2*(a+b*ln(
c*(e*x+d)^n))^3/e^2+(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/e^2+1/2*g*(
e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^4/e^2
```

3.60.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.76

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$$
$$=\frac{4(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4+2g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4-16b(ef-dg)n((d\cdots}{\cdots}$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^4,x]
```

output
```
(4*(e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 + 2*g*(d + e*x)^2*(a
 + b*Log[c*(d + e*x)^n])^4 - 16*b*(e*f - d*g)*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(
a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n]))) - b*g*n*(4*(d + e*x)^2*(a +
 b*Log[c*(d + e*x)^n])^3 - 3*b*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])
^2 + b*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])))
))/(4*e^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3b^3gn^3(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}-\frac{24ab^3n^3x(ef-dg)}{e}+\frac{12b^2n^2(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}+\frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}-\frac{4bn(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}-\frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}-\frac{24b^4n^3(d+ex)(ef-dg)\log\left(c(d+ex)^n\right)}{e^2}+\frac{3b^4gn^4(d+ex)^2}{4e^2}+\frac{24b^4n^4x(ef-dg)}{e}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^4,x]
```

output
```
(-24*a*b^3*(e*f - d*g)*n^3*x)/e + (24*b^4*(e*f - d*g)*n^4*x)/e + (3*b^4*g*
n^4*(d + e*x)^2)/(4*e^2) - (24*b^4*(e*f - d*g)*n^3*(d + e*x)*Log[c*(d + e*
x)^n])/e^2 - (3*b^3*g*n^3*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2)
+ (12*b^2*(e*f - d*g)*n^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 + (3
*b^2*g*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2) - (4*b*(e*f -
 d*g)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/e^2 - (b*g*n*(d + e*x)^2*(
a + b*Log[c*(d + e*x)^n])^3)/e^2 + ((e*f - d*g)*(d + e*x)*(a + b*Log[c*(d
+ e*x)^n])^4)/e^2 + (g*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^4)/(2*e^2)
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1492 vs. $2(332) = 664$.

Time = 3.52 (sec) , antiderivative size = 1493, normalized size of antiderivative = 4.39

method	result	size
parallelrisch	Expression too large to display	1493
risch	Expression too large to display	37938

input

```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^4,x,method=_RETURNVERBOSE)
```

output

```
-1/4*(-16*a^3*b*d*e*f*n-90*b^4*n^4*d^2*g+4*a^4*d*e*f-2*a^4*e^2*g*x^2+8*a^3
*b*d^2*g*n-4*a^4*e^2*f*x-36*b^2*n^2*a^2*d^2*g+84*b^3*n^3*a*d^2*g-3*b^4*n^4
*e^2*g*x^2-96*b^4*n^4*e^2*f*x+6*b^3*n^3*a*e^2*g*x^2+90*b^4*n^4*d*e*g*x-6*b
^2*n^2*a^2*e^2*g*x^2+96*b^3*n^3*a*e^2*f*x-48*b^2*n^2*a^2*e^2*f*x+4*b*n*a^3
*e^2*g*x^2+16*b*n*a^3*e^2*f*x-174*ln(e*x+d)*b^4*d^2*g*n^4-96*b^3*n^3*a*d*e
*f+48*b^2*n^2*a^2*d*e*f+96*b^4*n^4*d*e*f+192*ln(e*x+d)*b^4*d*e*f*n^4+156*l
n(e*x+d)*a*b^3*d^2*g*n^3-60*ln(e*x+d)*a^2*b^2*d^2*g*n^2+8*ln(e*x+d)*a^3*b*
d^2*g*n-2*x^2*ln(c*(e*x+d)^n)^4*b^4*e^2*g-4*x*ln(c*(e*x+d)^n)^4*b^4*e^2*f-
4*ln(c*(e*x+d)^n)^4*b^4*d*e*f-12*ln(c*(e*x+d)^n)^3*b^4*d^2*g*n+42*ln(c*(e*
x+d)^n)^2*b^4*d^2*g*n^2+84*ln(c*(e*x+d)^n)*b^4*d^2*g*n^3+8*ln(c*(e*x+d)^n)
^3*a*b^3*d^2*g+12*ln(c*(e*x+d)^n)^2*a^2*b^2*d^2*g+2*ln(c*(e*x+d)^n)^4*b^4*
d^2*g+4*x^2*ln(c*(e*x+d)^n)^3*b^4*e^2*g*n-6*x^2*ln(c*(e*x+d)^n)^2*b^4*e^2*
g*n^2+6*x^2*ln(c*(e*x+d)^n)*b^4*e^2*g*n^3-8*x^2*ln(c*(e*x+d)^n)^3*a*b^3*e^
2*g+16*x*ln(c*(e*x+d)^n)^3*b^4*e^2*f*n-48*x*ln(c*(e*x+d)^n)^2*b^4*e^2*f*n^
2+96*x*ln(c*(e*x+d)^n)*b^4*e^2*f*n^3-12*x^2*ln(c*(e*x+d)^n)^2*a^2*b^2*e^2*
g-16*x*ln(c*(e*x+d)^n)^3*a*b^3*e^2*f+16*ln(c*(e*x+d)^n)^3*b^4*d*e*f*n-84*b
^3*n^3*a*d*e*g*x+36*b^2*n^2*a^2*d*e*g*x-192*ln(e*x+d)*a*b^3*d*e*f*n^3+96*l
n(e*x+d)*a^2*b^2*d*e*f*n^2-32*ln(e*x+d)*a^3*b*d*e*f*n-24*x*ln(c*(e*x+d)^n)
^2*a*b^3*d*e*g*n-48*ln(c*(e*x+d)^n)^2*b^4*d*e*f*n^2-96*ln(c*(e*x+d)^n)*b^4
*d*e*f*n^3-8*x^2*ln(c*(e*x+d)^n)*a^3*b*e^2*g-24*x*ln(c*(e*x+d)^n)^2*a^2...
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1756 vs. $2(332) = 664$.

Time = 0.31 (sec) , antiderivative size = 1756, normalized size of antiderivative = 5.16

$$\int (f + gx) (a + b\log(c(d + ex)^n))^4 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^4,x, algorithm="fricas")
```

output
```
1/4*(2*(b^4*e^2*g*n^4*x^2 + 2*b^4*e^2*f*n^4*x + (2*b^4*d*e*f - b^4*d^2*g)*
n^4)*log(e*x + d)^4 + 2*(b^4*e^2*g*x^2 + 2*b^4*e^2*f*x)*log(c)^4 - 4*((4*b
^4*d*e*f - 3*b^4*d^2*g)*n^4 - 2*(2*a*b^3*d*e*f - a*b^3*d^2*g)*n^3 + (b^4*e
^2*g*n^4 - 2*a*b^3*e^2*g*n^3)*x^2 - 2*(2*a*b^3*e^2*f*n^3 - (2*b^4*e^2*f -
b^4*d*e*g)*n^4)*x - 2*(b^4*e^2*g*n^3*x^2 + 2*b^4*e^2*f*n^3*x + (2*b^4*d*e*
f - b^4*d^2*g)*n^3)*log(c))*log(e*x + d)^3 - 4*((b^4*e^2*g*n - 2*a*b^3*e^2
*g)*x^2 - 2*(2*a*b^3*e^2*f - (2*b^4*e^2*f - b^4*d*e*g)*n)*x)*log(c)^3 + (3
*b^4*e^2*g*n^4 - 6*a*b^3*e^2*g*n^3 + 6*a^2*b^2*e^2*g*n^2 - 4*a^3*b*e^2*g*n
 + 2*a^4*e^2*g)*x^2 + 6*((8*b^4*d*e*f - 7*b^4*d^2*g)*n^4 - 2*(4*a*b^3*d*e*
f - 3*a*b^3*d^2*g)*n^3 + 2*(2*a^2*b^2*d*e*f - a^2*b^2*d^2*g)*n^2 + (b^4*e^
2*g*n^4 - 2*a*b^3*e^2*g*n^3 + 2*a^2*b^2*e^2*g*n^2)*x^2 + 2*(b^4*e^2*g*n^2*
x^2 + 2*b^4*e^2*f*n^2*x + (2*b^4*d*e*f - b^4*d^2*g)*n^2)*log(c)^2 + 2*(2*a
^2*b^2*e^2*f*n^2 + (4*b^4*e^2*f - 3*b^4*d*e*g)*n^4 - 2*(2*a*b^3*e^2*f - a*
b^3*d*e*g)*n^3)*x - 2*((4*b^4*d*e*f - 3*b^4*d^2*g)*n^3 - 2*(2*a*b^3*d*e*f
- a*b^3*d^2*g)*n^2 + (b^4*e^2*g*n^3 - 2*a*b^3*e^2*g*n^2)*x^2 - 2*(2*a*b^3*
e^2*f*n^2 - (2*b^4*e^2*f - b^4*d*e*g)*n^3)*x)*log(c))*log(e*x + d)^2 + 6*(
(b^4*e^2*g*n^2 - 2*a*b^3*e^2*g*n + 2*a^2*b^2*e^2*g)*x^2 + 2*(2*a^2*b^2*e^2
*f + (4*b^4*e^2*f - 3*b^4*d*e*g)*n^2 - 2*(2*a*b^3*e^2*f - a*b^3*d*e*g)*n)*
x)*log(c)^2 + 2*(2*a^4*e^2*f + 3*(16*b^4*e^2*f - 15*b^4*d*e*g)*n^4 - 6*(8*
a*b^3*e^2*f - 7*a*b^3*d*e*g)*n^3 + 6*(4*a^2*b^2*e^2*f - 3*a^2*b^2*d*e*g...
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. $2(332) = 664$.

Time = 2.31 (sec) , antiderivative size = 1372, normalized size of antiderivative = 4.04

$$\int (f + gx) (a + b\log(c(d + ex)^n))^4 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**4,x)
```

output
```
Piecewise((a**4*f*x + a**4*g*x**2/2 - 2*a**3*b*d**2*g*log(c*(d + e*x)**n)/
e**2 + 4*a**3*b*d*f*log(c*(d + e*x)**n)/e + 2*a**3*b*d*g*n*x/e - 4*a**3*b*
f*n*x + 4*a**3*b*f*x*log(c*(d + e*x)**n) - a**3*b*g*n*x**2 + 2*a**3*b*g*x*
*2*log(c*(d + e*x)**n) + 9*a**2*b**2*d**2*g*n*log(c*(d + e*x)**n)/e**2 - 3
*a**2*b**2*d**2*g*log(c*(d + e*x)**n)**2/e**2 - 12*a**2*b**2*d*f*n*log(c*(
d + e*x)**n)/e + 6*a**2*b**2*d*f*log(c*(d + e*x)**n)**2/e - 9*a**2*b**2*d*
g*n**2*x/e + 6*a**2*b**2*d*g*n*x*log(c*(d + e*x)**n)/e + 12*a**2*b**2*f*n*
*2*x - 12*a**2*b**2*f*n*x*log(c*(d + e*x)**n) + 6*a**2*b**2*f*x*log(c*(d +
 e*x)**n)**2 + 3*a**2*b**2*g*n**2*x**2/2 - 3*a**2*b**2*g*n*x**2*log(c*(d +
 e*x)**n) + 3*a**2*b**2*g*x**2*log(c*(d + e*x)**n)**2 - 21*a*b**3*d**2*g*n
**2*log(c*(d + e*x)**n)/e**2 + 9*a*b**3*d**2*g*n*log(c*(d + e*x)**n)**2/e*
*2 - 2*a*b**3*d**2*g*log(c*(d + e*x)**n)**3/e**2 + 24*a*b**3*d*f*n**2*log(
c*(d + e*x)**n)/e - 12*a*b**3*d*f*n*log(c*(d + e*x)**n)**2/e + 4*a*b**3*d*
f*log(c*(d + e*x)**n)**3/e + 21*a*b**3*d*g*n**3*x/e - 18*a*b**3*d*g*n**2*x
*log(c*(d + e*x)**n)/e + 6*a*b**3*d*g*n*x*log(c*(d + e*x)**n)**2/e - 24*a*
b**3*f*n**3*x + 24*a*b**3*f*n**2*x*log(c*(d + e*x)**n) - 12*a*b**3*f*n*x*l
og(c*(d + e*x)**n)**2 + 4*a*b**3*f*x*log(c*(d + e*x)**n)**3 - 3*a*b**3*g*n
**3*x**2/2 + 3*a*b**3*g*n**2*x**2*log(c*(d + e*x)**n) - 3*a*b**3*g*n*x**2*
log(c*(d + e*x)**n)**2 + 2*a*b**3*g*x**2*log(c*(d + e*x)**n)**3 + 45*b**4*
d**2*g*n**3*log(c*(d + e*x)**n)/(2*e**2) - 21*b**4*d**2*g*n**2*log(c*(d...
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(332) = 664$.

Time = 0.24 (sec) , antiderivative size = 1163, normalized size of antiderivative = 3.42

$$\int (f + gx) (a + b \log (c(d + ex)^n))^4 \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^4,x, algorithm="maxima")
```

output
```
1/2*b^4*g*x^2*log((e*x + d)^n*c)^4 + 2*a*b^3*g*x^2*log((e*x + d)^n*c)^3 +
b^4*f*x*log((e*x + d)^n*c)^4 + 3*a^2*b^2*g*x^2*log((e*x + d)^n*c)^2 + 4*a*
b^3*f*x*log((e*x + d)^n*c)^3 - 4*a^3*b*e*f*n*(x/e - d*log(e*x + d)/e^2) -
a^3*b*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*a^3*b*g*x^2
*log((e*x + d)^n*c) + 6*a^2*b^2*f*x*log((e*x + d)^n*c)^2 + 1/2*a^4*g*x^2 +
 4*a^3*b*f*x*log((e*x + d)^n*c) - 6*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log(
(e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a^2*
b^2*f - 4*(3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d
*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 -
 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2)
)*a*b^3*f - (4*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^3 + (e*n*
((d*log(e*x + d)^4 + 4*d*log(e*x + d)^3 + 12*d*log(e*x + d)^2 - 24*e*x + 2
4*d*log(e*x + d))*n^2/e^3 - 4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e
*x + 6*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^3) + 6*(d*log(e*x + d)^2 - 2
*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)^2/e^2)*e*n)*b^4*f - 3/2*(2*e
*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^
2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*a^2*
b^2*g - 1/2*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x
 + d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*x + d
)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*log(...
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2488 vs. $2(332) = 664$.

Time = 0.35 (sec) , antiderivative size = 2488, normalized size of antiderivative = 7.32

$$\int (f + gx)\left(a + b\log\left(c(d + ex)^n\right)\right)^4\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^4,x, algorithm="giac")
```

output
```
(e*x + d)*b^4*f*n^4*log(e*x + d)^4/e + 1/2*(e*x + d)^2*b^4*g*n^4*log(e*x +
 d)^4/e^2 - (e*x + d)*b^4*d*g*n^4*log(e*x + d)^4/e^2 - 4*(e*x + d)*b^4*f*n
^4*log(e*x + d)^3/e - (e*x + d)^2*b^4*g*n^4*log(e*x + d)^3/e^2 + 4*(e*x +
d)*b^4*d*g*n^4*log(e*x + d)^3/e^2 + 4*(e*x + d)*b^4*f*n^3*log(e*x + d)^3*l
og(c)/e + 2*(e*x + d)^2*b^4*g*n^3*log(e*x + d)^3*log(c)/e^2 - 4*(e*x + d)*
b^4*d*g*n^3*log(e*x + d)^3*log(c)/e^2 + 12*(e*x + d)*b^4*f*n^4*log(e*x + d
)^2/e + 3/2*(e*x + d)^2*b^4*g*n^4*log(e*x + d)^2/e^2 - 12*(e*x + d)*b^4*d*
g*n^4*log(e*x + d)^2/e^2 + 4*(e*x + d)*a*b^3*f*n^3*log(e*x + d)^3/e + 2*(e
*x + d)^2*a*b^3*g*n^3*log(e*x + d)^3/e^2 - 4*(e*x + d)*a*b^3*d*g*n^3*log(e
*x + d)^3/e^2 - 12*(e*x + d)*b^4*f*n^3*log(e*x + d)^2*log(c)/e - 3*(e*x +
d)^2*b^4*g*n^3*log(e*x + d)^2*log(c)/e^2 + 12*(e*x + d)*b^4*d*g*n^3*log(e*
x + d)^2*log(c)/e^2 + 6*(e*x + d)*b^4*f*n^2*log(e*x + d)^2*log(c)^2/e + 3*
(e*x + d)^2*b^4*g*n^2*log(e*x + d)^2*log(c)^2/e^2 - 6*(e*x + d)*b^4*d*g*n^
2*log(e*x + d)^2*log(c)^2/e^2 - 24*(e*x + d)*b^4*f*n^4*log(e*x + d)/e - 3/
2*(e*x + d)^2*b^4*g*n^4*log(e*x + d)/e^2 + 24*(e*x + d)*b^4*d*g*n^4*log(e*
x + d)/e^2 - 12*(e*x + d)*a*b^3*f*n^3*log(e*x + d)^2/e - 3*(e*x + d)^2*a*b
^3*g*n^3*log(e*x + d)^2/e^2 + 12*(e*x + d)*a*b^3*d*g*n^3*log(e*x + d)^2/e^
2 + 24*(e*x + d)*b^4*f*n^3*log(e*x + d)*log(c)/e + 3*(e*x + d)^2*b^4*g*n^3
*log(e*x + d)*log(c)/e^2 - 24*(e*x + d)*b^4*d*g*n^3*log(e*x + d)*log(c)/e^
2 + 12*(e*x + d)*a*b^3*f*n^2*log(e*x + d)^2*log(c)/e + 6*(e*x + d)^2*a*...
```

3.60. $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$

3.60.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.42

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$$
$$=x\left(\frac{2a^4dg+2a^4ef-42b^4dgn^4+48b^4efn^4+36ab^3dgn^3-48ab^3efn^3-12a^2b^2dgn^2+24a^2b^2}{2e}\right.$$
$$\left.-\frac{dg\left(2a^4-4a^3bn+6a^2b^2n^2-6ab^3n^3+3b^4n^4\right)}{2e}\right)$$
$$+\ln\left(c\left(d+ex\right)^n\right)^4\left(\frac{b^4gx^2}{2}-\frac{d\left(b^4dg-2b^4ef\right)}{2e^2}+b^4fx\right)$$
$$+\ln\left(c\left(d+ex\right)^n\right)\left(\frac{bg\left(4a^3-6a^2bn+6ab^2n^2-3b^3n^3\right)x^2}{2}\right.$$
$$+\left(\frac{4a^3bdg+4a^3bef+18b^4dgn^3-24b^4efn^3-12a^2b^2efn-12ab^3dgn^2+24ab^3efn^2}{e}\right.$$
$$\left.\left.-\frac{bdg\left(4a^3-6a^2bn+6ab^2n^2-3b^3n^3\right)}{e}\right)x\right)$$
$$+\ln\left(c\left(d+ex\right)^n\right)^3\left(x\left(\frac{4b^3\left(adg+aef-befn\right)}{e}-\frac{2b^3dg\left(2a-bn\right)}{e}\right)\right.$$
$$\left.-\frac{d\left(2ab^3dg-4ab^3ef-3b^4dgn+4b^4efn\right)}{e^2}+b^3gx^2\left(2a-bn\right)\right)$$
$$+\ln\left(c\left(d+ex\right)^n\right)^2\left(x\left(\frac{6a^2b^2dg+6a^2b^2ef-6b^4dgn^2+12b^4efn^2-12ab^3efn}{e}\right.\right.$$
$$\left.-\frac{3b^2dg\left(2a^2-2abn+b^2n^2\right)}{e}\right)$$
$$-\frac{3d\left(2a^2b^2dg-4a^2b^2ef+7b^4dgn^2-8b^4efn^2-6ab^3dgn+8ab^3efn\right)}{2e^2}$$
$$\left.+\frac{3b^2gx^2\left(2a^2-2abn+b^2n^2\right)}{2}\right)$$
$$+\frac{\ln\left(d+ex\right)\left(-4ga^3bd^2n+8efa^3bdn+18ga^2b^2d^2n^2-24efa^2b^2dn^2-42gab^3d^2n^3+48efa}{2e^2}$$
$$+\frac{gx^2\left(2a^4-4a^3bn+6a^2b^2n^2-6ab^3n^3+3b^4n^4\right)}{4}$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^4,x)
```

output
```
x*((2*a^4*d*g + 2*a^4*e*f - 42*b^4*d*g*n^4 + 48*b^4*e*f*n^4 + 36*a*b^3*d*g
*n^3 - 48*a*b^3*e*f*n^3 - 12*a^2*b^2*d*g*n^2 + 24*a^2*b^2*e*f*n^2 - 8*a^3*
b*e*f*n)/(2*e) - (d*g*(2*a^4 + 3*b^4*n^4 - 6*a*b^3*n^3 + 6*a^2*b^2*n^2 - 4
*a^3*b*n))/(2*e)) + log(c*(d + e*x)^n)^4*((b^4*g*x^2)/2 - (d*(b^4*d*g - 2*
b^4*e*f))/(2*e^2) + b^4*f*x) + log(c*(d + e*x)^n)*(x*((4*a^3*b*d*g + 4*a^3
*b*e*f + 18*b^4*d*g*n^3 - 24*b^4*e*f*n^3 - 12*a^2*b^2*e*f*n - 12*a*b^3*d*g
*n^2 + 24*a*b^3*e*f*n^2)/e - (b*d*g*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a
^2*b*n))/e) + (b*g*x^2*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n))/2) +
 log(c*(d + e*x)^n)^3*(x*((4*b^3*(a*d*g + a*e*f - b*e*f*n))/e - (2*b^3*d*g
*(2*a - b*n))/e) - (d*(2*a*b^3*d*g - 4*a*b^3*e*f - 3*b^4*d*g*n + 4*b^4*e*f
*n))/e^2 + b^3*g*x^2*(2*a - b*n)) + log(c*(d + e*x)^n)^2*(x*((6*a^2*b^2*d*
g + 6*a^2*b^2*e*f - 6*b^4*d*g*n^2 + 12*b^4*e*f*n^2 - 12*a*b^3*e*f*n)/e - (
3*b^2*d*g*(2*a^2 + b^2*n^2 - 2*a*b*n))/e) - (3*d*(2*a^2*b^2*d*g - 4*a^2*b^
2*e*f + 7*b^4*d*g*n^2 - 8*b^4*e*f*n^2 - 6*a*b^3*d*g*n + 8*a*b^3*e*f*n))/(2
*e^2) + (3*b^2*g*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/2) + (log(d + e*x)*(45*b
^4*d^2*g*n^4 - 4*a^3*b*d^2*g*n - 48*b^4*d*e*f*n^4 - 42*a*b^3*d^2*g*n^3 + 1
8*a^2*b^2*d^2*g*n^2 + 8*a^3*b*d*e*f*n + 48*a*b^3*d*e*f*n^3 - 24*a^2*b^2*d*
e*f*n^2))/(2*e^2) + (g*x^2*(2*a^4 + 3*b^4*n^4 - 6*a*b^3*n^3 + 6*a^2*b^2*n^
2 - 4*a^3*b*n))/4
```

3.60. $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$

3.61 $\int (a+b\log(c(d+ex)^n))^4\,dx$

3.61.1 Optimal result

Integrand size = 16, antiderivative size = 131

$$\int (a+b\log(c(d+ex)^n))^4\,dx = -24ab^3n^3x+24b^4n^4x-\frac{24b^4n^3(d+ex)\log(c(d+ex)^n)}{e} + \frac{12b^2n^2(d+ex)(a+b\log(c(d+ex)^n))^2}{e} - \frac{4bn(d+ex)(a+b\log(c(d+ex)^n))^3}{e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{e}$$

output
```
-24*a*b^3*n^3*x+24*b^4*n^4*x-24*b^4*n^3*(e*x+d)*ln(c*(e*x+d)^n)/e+12*b^2*n
^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e-4*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3
/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/e
```

3.61.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int (a+b\log(c(d+ex)^n))^4\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^4-4bn\big((d+ex)(a+b\log(c(d+ex)^n))^3-3bn\big((d+ex)(a+b\log(c(d$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^4,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 - 4*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(
a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n]))))/e
```

3.61.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2836, 2733, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^4\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^4\,d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^4-4bn\int (a+b\log(c(d+ex)^n))^3\,d(d+ex)}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^4-4bn\Big((d+ex)(a+b\log(c(d+ex)^n))^3-3bn\int (a+b\log(c(d+ex)^n))^2\,d(d+}{e}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^4-4bn\Big((d+ex)(a+b\log(c(d+ex)^n))^3-3bn\Big((d+ex)(a+b\log(c(d+ex)^n))}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^4-4bn\Big((d+ex)(a+b\log(c(d+ex)^n))^3-3bn\Big((d+ex)(a+b\log(c(d+ex)^n))}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^4,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 - 4*b*n*((d + e*x)*(a + b*Log[c*(d
 + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(a*(
d + e*x) - b*n*(d + e*x) + b*(d + e*x)*Log[c*(d + e*x)^n]))))/e
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(131) = 262$.

Time = 0.59 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.01

method	result
parallelrisch	$\frac{-12x\ln(c(ex+d)^n)a^2b^2en^2+4x\ln(c(ex+d)^n)a^3ben+4x\ln(c(ex+d)^n)^3ab^3en-12x\ln(c(ex+d)^n)^2ab^3en^2+24x\ln(c(ex+d)^n)}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))^4,x,method=_RETURNVERBOSE)
```

output
```
(-12*x*ln(c*(e*x+d)^n)*a^2*b^2*e*n^2+4*x*ln(c*(e*x+d)^n)*a^3*b*e*n+4*x*ln(
c*(e*x+d)^n)^3*a*b^3*e*n-12*x*ln(c*(e*x+d)^n)^2*a*b^3*e*n^2+24*x*ln(c*(e*x
+d)^n)*a*b^3*e*n^3+6*x*ln(c*(e*x+d)^n)^2*a^2*b^2*e*n+24*a*b^3*d*n^4-12*a^2
*b^2*d*n^3+4*a^3*b*d*n^2+ln(c*(e*x+d)^n)^4*b^4*d*n-4*ln(c*(e*x+d)^n)^3*b^4
*d*n^2+12*ln(c*(e*x+d)^n)^2*b^4*d*n^3-24*ln(c*(e*x+d)^n)*b^4*d*n^4+x*a^4*e
*n+24*x*b^4*e*n^5+12*x*a^2*b^2*e*n^3+4*ln(c*(e*x+d)^n)^3*a*b^3*d*n-12*ln(c
*(e*x+d)^n)^2*a*b^3*d*n^2+24*ln(c*(e*x+d)^n)*a*b^3*d*n^3-4*x*a^3*b*e*n^2+6
*ln(c*(e*x+d)^n)^2*a^2*b^2*d*n-12*ln(c*(e*x+d)^n)*a^2*b^2*d*n^2+4*ln(c*(e*
x+d)^n)*a^3*b*d*n+x*ln(c*(e*x+d)^n)^4*b^4*e*n-4*x*ln(c*(e*x+d)^n)^3*b^4*e*
n^2+12*x*ln(c*(e*x+d)^n)^2*b^4*e*n^3-24*x*ln(c*(e*x+d)^n)*b^4*e*n^4-24*x*a
*b^3*e*n^4-24*b^4*d*n^5-a^4*d*n)/e/n
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(131) = 262$.

Time = 0.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.69

$$\int (a + b\log(c(d+ex)^n))^4\,dx$$
$$= \frac{b^4ex\log(c)^4 + (b^4en^4x + b^4dn^4)\log(ex+d)^4 - 4(b^4en - ab^3e)x\log(c)^3 - 4(b^4dn^4 - ab^3dn^3 + (b^4en^4 - \cdots}{}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="fricas")
```

output
```
(b^4*e*x*log(c)^4 + (b^4*e*n^4*x + b^4*d*n^4)*log(e*x + d)^4 - 4*(b^4*e*n
- a*b^3*e)*x*log(c)^3 - 4*(b^4*d*n^4 - a*b^3*d*n^3 + (b^4*e*n^4 - a*b^3*e*
n^3)*x - (b^4*e*n^3*x + b^4*d*n^3)*log(c))*log(e*x + d)^3 + 6*(2*b^4*e*n^2
 - 2*a*b^3*e*n + a^2*b^2*e)*x*log(c)^2 + 6*(2*b^4*d*n^4 - 2*a*b^3*d*n^3 +
a^2*b^2*d*n^2 + (b^4*e*n^2*x + b^4*d*n^2)*log(c)^2 + (2*b^4*e*n^4 - 2*a*b^
3*e*n^3 + a^2*b^2*e*n^2)*x - 2*(b^4*d*n^3 - a*b^3*d*n^2 + (b^4*e*n^3 - a*b
^3*e*n^2)*x)*log(c))*log(e*x + d)^2 - 4*(6*b^4*e*n^3 - 6*a*b^3*e*n^2 + 3*a
^2*b^2*e*n - a^3*b*e)*x*log(c) + (24*b^4*e*n^4 - 24*a*b^3*e*n^3 + 12*a^2*b
^2*e*n^2 - 4*a^3*b*e*n + a^4*e)*x - 4*(6*b^4*d*n^4 - 6*a*b^3*d*n^3 + 3*a^2
*b^2*d*n^2 - a^3*b*d*n - (b^4*e*n*x + b^4*d*n)*log(c)^3 + 3*(b^4*d*n^2 - a
*b^3*d*n + (b^4*e*n^2 - a*b^3*e*n)*x)*log(c)^2 + (6*b^4*e*n^4 - 6*a*b^3*e*
n^3 + 3*a^2*b^2*e*n^2 - a^3*b*e*n)*x - 3*(2*b^4*d*n^3 - 2*a*b^3*d*n^2 + a^
2*b^2*d*n + (2*b^4*e*n^3 - 2*a*b^3*e*n^2 + a^2*b^2*e*n)*x)*log(c))*log(e*x
 + d))/e
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(126) = 252$.

Time = 0.90 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.78

$$\int (a+b\log (c(d+ex)^n))^4\,dx$$
$$=\begin{cases} a^4x+\frac{4a^3bd\log (c(d+ex)^n)}{e}-4a^3bnx+4a^3bx\log (c(d+ex)^n)-\frac{12a^2b^2dn\log (c(d+ex)^n)}{e}+\frac{6a^2b^2d\log (c(d+ex)^n)^2}{e}+1 \\ x(a+b\log (cd^n))^4 \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*d*log(c*(d + e*x)**n)/e - 4*a**3*b*n*x + 4*a*
*3*b*x*log(c*(d + e*x)**n) - 12*a**2*b**2*d*n*log(c*(d + e*x)**n)/e + 6*a*
*2*b**2*d*log(c*(d + e*x)**n)**2/e + 12*a**2*b**2*n**2*x - 12*a**2*b**2*n*
x*log(c*(d + e*x)**n) + 6*a**2*b**2*x*log(c*(d + e*x)**n)**2 + 24*a*b**3*d
*n**2*log(c*(d + e*x)**n)/e - 12*a*b**3*d*n*log(c*(d + e*x)**n)**2/e + 4*a
*b**3*d*log(c*(d + e*x)**n)**3/e - 24*a*b**3*n**3*x + 24*a*b**3*n**2*x*log
(c*(d + e*x)**n) - 12*a*b**3*n*x*log(c*(d + e*x)**n)**2 + 4*a*b**3*x*log(c
*(d + e*x)**n)**3 - 24*b**4*d*n**3*log(c*(d + e*x)**n)/e + 12*b**4*d*n**2*
log(c*(d + e*x)**n)**2/e - 4*b**4*d*n*log(c*(d + e*x)**n)**3/e + b**4*d*lo
g(c*(d + e*x)**n)**4/e + 24*b**4*n**4*x - 24*b**4*n**3*x*log(c*(d + e*x)**
n) + 12*b**4*n**2*x*log(c*(d + e*x)**n)**2 - 4*b**4*n*x*log(c*(d + e*x)**n
)**3 + b**4*x*log(c*(d + e*x)**n)**4, Ne(e, 0)), (x*(a + b*log(c*d**n))**4
, True))
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(131) = 262$.

Time = 0.22 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.82

$$\int (a+b\log(c(d+ex)^n))^4\,dx = b^4x\log((ex+d)^nc)^4 + 4\,ab^3x\log((ex+d)^nc)^3 - 4\,a^3ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) + 6\,a^2b^2x\log((ex+d)^nc)^2 + 4\,a^3bx\log((ex+d)^nc) - 6\left(2\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2-2\,ex+2\,d\log(ex+d))n^2}{e}\right)a^2b^2 - 4\left(3\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^2 - en\left(\frac{(d\log(ex+d)^3+3\,d\log(ex+d)^2-6\,ex+6\,d\mathrm{l}}{e^2}\right.\right. - \left(4\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^3 + \left(en\left(\frac{(d\log(ex+d)^4+4\,d\log(ex+d)^3+12\,d\log(e\right.}{e^3}\right.\right.\right. + a^4x$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="maxima")
```

output
```
b^4*x*log((e*x + d)^n*c)^4 + 4*a*b^3*x*log((e*x + d)^n*c)^3 - 4*a^3*b*e*n*
(x/e - d*log(e*x + d)/e^2) + 6*a^2*b^2*x*log((e*x + d)^n*c)^2 + 4*a^3*b*x*
log((e*x + d)^n*c) - 6*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c
) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a^2*b^2 - 4*(3*e*
n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3
 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x +
 d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*a*b^3 - (4*e*
n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^3 + (e*n*((d*log(e*x + d)^
4 + 4*d*log(e*x + d)^3 + 12*d*log(e*x + d)^2 - 24*e*x + 24*d*log(e*x + d))
*n^2/e^3 - 4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x
+ d))*n*log((e*x + d)^n*c)/e^3) + 6*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*
x + d))*n*log((e*x + d)^n*c)^2/e^2)*e*n)*b^4 + a^4*x
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(131) = 262$.

Time = 0.33 (sec) , antiderivative size = 758, normalized size of antiderivative = 5.79

$$
\begin{aligned}
\int (a+b\log (c(d+ex)^n))^4\,dx = {} & \frac{(ex+d)b^4n^4\log (ex+d)^4}{e} - \frac{4\,(ex+d)b^4n^4\log (ex+d)^3}{e} \\
& + \frac{4\,(ex+d)b^4n^3\log (ex+d)^3\log (c)}{e} \\
& + \frac{12\,(ex+d)b^4n^4\log (ex+d)^2}{e} \\
& + \frac{4\,(ex+d)ab^3n^3\log (ex+d)^3}{e} \\
& - \frac{12\,(ex+d)b^4n^3\log (ex+d)^2\log (c)}{e} \\
& + \frac{6\,(ex+d)b^4n^2\log (ex+d)^2\log (c)^2}{e} \\
& - \frac{24\,(ex+d)b^4n^4\log (ex+d)}{e} \\
& - \frac{12\,(ex+d)ab^3n^3\log (ex+d)^2}{e} \\
& + \frac{24\,(ex+d)b^4n^3\log (ex+d)\log (c)}{e} \\
& + \frac{12\,(ex+d)ab^3n^2\log (ex+d)^2\log (c)}{e} \\
& - \frac{12\,(ex+d)b^4n^2\log (ex+d)\log (c)^2}{e} \\
& + \frac{4\,(ex+d)b^4n\log (ex+d)\log (c)^3}{e} \\
& + \frac{24\,(ex+d)b^4n^4}{e} + \frac{24\,(ex+d)ab^3n^3\log (ex+d)}{e} \\
& + \frac{6\,(ex+d)a^2b^2n^2\log (ex+d)^2}{e} - \frac{24\,(ex+d)b^4n^3\log (c)}{e} \\
& - \frac{24\,(ex+d)ab^3n^2\log (ex+d)\log (c)}{e} \\
& + \frac{12\,(ex+d)b^4n^2\log (c)^2}{e} \\
& + \frac{12\,(ex+d)ab^3n\log (ex+d)\log (c)^2}{e} \\
& - \frac{4\,(ex+d)b^4n\log (c)^3}{e} + \frac{(ex+d)b^4\log (c)^4}{e} \\
& - \frac{24\,(ex+d)ab^3n^3}{e} - \frac{12\,(ex+d)a^2b^2n^2\log (ex+d)}{e} \\
& + \frac{24\,(ex+d)ab^3n^2\log (c)}{e} \\
& + \frac{12\,(ex+d)a^2b^2n\log (ex+d)\log (c)}{e} \\
& - \frac{12\,(ex+d)ab^3n\log (c)^2}{e} + \frac{4\,(ex+d)ab^3\log (c)^3}{e} \\
& + \frac{12\,(ex+d)a^2b^2n^2}{e} + \frac{4\,(ex+d)a^3bn\log (ex+d)}{e}
\end{aligned}
$$

3.61. $\int (a+b\log (c(d+ex)^n))^4\,dx$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="giac")
```

output
```
(e*x + d)*b^4*n^4*log(e*x + d)^4/e - 4*(e*x + d)*b^4*n^4*log(e*x + d)^3/e
+ 4*(e*x + d)*b^4*n^3*log(e*x + d)^3*log(c)/e + 12*(e*x + d)*b^4*n^4*log(e
*x + d)^2/e + 4*(e*x + d)*a*b^3*n^3*log(e*x + d)^3/e - 12*(e*x + d)*b^4*n^
3*log(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^4*n^2*log(e*x + d)^2*log(c)^2/e
- 24*(e*x + d)*b^4*n^4*log(e*x + d)/e - 12*(e*x + d)*a*b^3*n^3*log(e*x + d
)^2/e + 24*(e*x + d)*b^4*n^3*log(e*x + d)*log(c)/e + 12*(e*x + d)*a*b^3*n^
2*log(e*x + d)^2*log(c)/e - 12*(e*x + d)*b^4*n^2*log(e*x + d)*log(c)^2/e +
 4*(e*x + d)*b^4*n*log(e*x + d)*log(c)^3/e + 24*(e*x + d)*b^4*n^4/e + 24*(
e*x + d)*a*b^3*n^3*log(e*x + d)/e + 6*(e*x + d)*a^2*b^2*n^2*log(e*x + d)^2
/e - 24*(e*x + d)*b^4*n^3*log(c)/e - 24*(e*x + d)*a*b^3*n^2*log(e*x + d)*l
og(c)/e + 12*(e*x + d)*b^4*n^2*log(c)^2/e + 12*(e*x + d)*a*b^3*n*log(e*x +
 d)*log(c)^2/e - 4*(e*x + d)*b^4*n*log(c)^3/e + (e*x + d)*b^4*log(c)^4/e -
 24*(e*x + d)*a*b^3*n^3/e - 12*(e*x + d)*a^2*b^2*n^2*log(e*x + d)/e + 24*(
e*x + d)*a*b^3*n^2*log(c)/e + 12*(e*x + d)*a^2*b^2*n*log(e*x + d)*log(c)/e
 - 12*(e*x + d)*a*b^3*n*log(c)^2/e + 4*(e*x + d)*a*b^3*log(c)^3/e + 12*(e*
x + d)*a^2*b^2*n^2/e + 4*(e*x + d)*a^3*b*n*log(e*x + d)/e - 12*(e*x + d)*a
^2*b^2*n*log(c)/e + 6*(e*x + d)*a^2*b^2*log(c)^2/e - 4*(e*x + d)*a^3*b*n/e
 + 4*(e*x + d)*a^3*b*log(c)/e + (e*x + d)*a^4/e
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.10

$$\int (a + b\log (c(d+ex)^n))^4 \, dx$$
$$= \ln \left(c\,{\left(d+e\,x\right)}^n\right)^2\,\left(\frac{6\,\left(d\,a^2\,b^2-2\,d\,a\,b^3\,n+2\,d\,b^4\,n^2\right)}{e}+6\,b^2\,x\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)\right) + x\,\left(a^4-4\,a^3\,b\,n+12\,a^2\,b^2\,n^2-24\,a\,b^3\,n^3+24\,b^4\,n^4\right)+\ln \left(c\,{\left(d+e\,x\right)}^n\right)^4\,\left(b^4\,x+\frac{b^4\,d}{e}\right) + \ln \left(c\,{\left(d+e\,x\right)}^n\right)^3\,\left(\frac{4\,\left(a\,b^3\,d-b^4\,d\,n\right)}{e}+4\,b^3\,x\,\left(a-b\,n\right)\right) - \frac{\ln \left(d+e\,x\right)\,\left(-4\,d\,a^3\,b\,n+12\,d\,a^2\,b^2\,n^2-24\,d\,a\,b^3\,n^3+24\,d\,b^4\,n^4\right)}{e} + 4\,b\,x\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\,\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right)$$

input
```
int((a + b*log(c*(d + e*x)^n))^4,x)
```

output
```
log(c*(d + e*x)^n)^2*((6*(a^2*b^2*d + 2*b^4*d*n^2 - 2*a*b^3*d*n))/e + 6*b^
2*x*(a^2 + 2*b^2*n^2 - 2*a*b*n)) + x*(a^4 + 24*b^4*n^4 - 24*a*b^3*n^3 + 12
*a^2*b^2*n^2 - 4*a^3*b*n) + log(c*(d + e*x)^n)^4*(b^4*x + (b^4*d)/e) + log
(c*(d + e*x)^n)^3*((4*(a*b^3*d - b^4*d*n))/e + 4*b^3*x*(a - b*n)) - (log(d
 + e*x)*(24*b^4*d*n^4 + 12*a^2*b^2*d*n^2 - 4*a^3*b*d*n - 24*a*b^3*d*n^3))/
e + 4*b*x*log(c*(d + e*x)^n)*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n)
```

3.61. $\int (a+b\log(c(d+ex)^n))^4\, dx$

3.62 $\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx$

3.62.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx &= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}\\ &\quad+\frac{4bn(a+b\log(c(d+ex)^n))^3\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}\\ &\quad-\frac{12b^2n^2(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}\\ &\quad+\frac{24b^3n^3(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}\\ &\quad-\frac{24b^4n^4\operatorname{PolyLog}\left(5,-\frac{g(d+ex)}{ef-dg}\right)}{g}\end{aligned}$$

output
```
(a+b*ln(c*(e*x+d)^n))^4*ln(e*(g*x+f)/(-d*g+e*f))/g+4*b*n*(a+b*ln(c*(e*x+d)
^n))^3*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-12*b^2*n^2*(a+b*ln(c*(e*x+d)^n))
^2*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g+24*b^3*n^3*(a+b*ln(c*(e*x+d)^n))*pol
ylog(4,-g*(e*x+d)/(-d*g+e*f))/g-24*b^4*n^4*polylog(5,-g*(e*x+d)/(-d*g+e*f)
)/g
```

3.62.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 503 vs. $2(205) = 410$.

Time = 0.18 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^4\log(f+gx)+4bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\left(\mathrm{lo}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^4*Log[f + g*x] + 4*b*n*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*(Log[d + e*x]*Log[(e*(f + g*x)
)/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 6*b^2*n^2*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]^2*Log[(e*(f + g
*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) - 4*b^3*n^3*(-a + b*n*Log[
d + e*x] - b*Log[c*(d + e*x)^n])*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f -
d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Log[
d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d + e
*x))/(-(e*f) + d*g)]) + b^4*n^4*(Log[d + e*x]^4*Log[(e*(f + g*x))/(e*f - d
*g)] + 4*Log[d + e*x]^3*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 12*Log[
d + e*x]^2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 24*Log[d + e*x]*Poly
Log[4, (g*(d + e*x))/(-(e*f) + d*g)] - 24*PolyLog[5, (g*(d + e*x))/(-(e*f)
 + d*g)]))/g
```

3.62.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2843, 2881, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx$$
$$\Big\downarrow \text{ 2843}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{4ben\int\frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{4bn\int\frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{4bn\left(3bn\int\frac{(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3\right)}{g}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{4bn\left(3bn\left(\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2-2bn\int\frac{(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)\right)\right.}{g}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{4bn\left(3bn\left(\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2-2bn\left(\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right.\right.\right.}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{4bn\left(3bn\left(\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2-2bn\left(\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))-\right.\right.\right.}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x),x]
```

3.62. $\int\frac{(a+b\log(c(d+ex)^n))^4}{f+gx}dx$

output
```
((a + b*Log[c*(d + e*x)^n])^4*Log[(e*(f + g*x))/(e*f - d*g)])/g - (4*b*n*(
-((a + b*Log[c*(d + e*x)^n])^3*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]) +
 3*b*n*((a + b*Log[c*(d + e*x)^n])^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g
))] - 2*b*n*((a + b*Log[c*(d + e*x)^n])*PolyLog[4, -((g*(d + e*x))/(e*f -
d*g))] - b*n*PolyLog[5, -((g*(d + e*x))/(e*f - d*g))]))))/g
```

3.62.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.62.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.02 (sec) , antiderivative size = 2172, normalized size of antiderivative = 10.60

method	result	size
risch	Expression too large to display	2172

input
```
int((a+b*ln(c*(e*x+d)^n))^4/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
-12*b^4*n^2/g*ln((e*x+d)^n)^2*polylog(3,g*(e*x+d)/(d*g-e*f))+24*b^4*n^3/g*
ln((e*x+d)^n)*polylog(4,g*(e*x+d)/(d*g-e*f))+3*b^4*n^4/g*ln(e*x+d)^4*ln(1-
g*(e*x+d)/(d*g-e*f))+4*b^4*n^4/g*ln(e*x+d)^3*polylog(2,g*(e*x+d)/(d*g-e*f)
)+b^4*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^4*n^4-4*b^4*n^4*dilog((g*(e*x+d)-d
*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)^3+4*b^4*n*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+
e*f))/g*ln((e*x+d)^n)^3-4*b^4*n^4*ln(e*x+d)^4*ln((g*(e*x+d)-d*g+e*f)/(-d*g
+e*f))/g+1/16*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*P
i*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^4*ln(g*x+f)/g+3/2*(-I
*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(
I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csg
n(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2*b^2*((ln((e*x+d)^n)-n*ln(e*x+d))^2*l
n(g*(e*x+d)-d*g+e*f)/g+n^2/g*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*n^2/g
*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))-2*n^2/g*polylog(3,g*(e*x+d)/(d*g
-e*f))+2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f
))/g+2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*
g+e*f))/g)+1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*
Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+
d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^3*b*(ln((e*x+d)^n)*l
n(g*x+f)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln...
```

3.62.5 Fricas [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^4}{f + gx} \, dx = \int \frac{(b\log((ex + d)^n c) + a)^4}{gx + f} \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b^4*log((e*x + d)^n*c)^4 + 4*a*b^3*log((e*x + d)^n*c)^3 + 6*a^2*
b^2*log((e*x + d)^n*c)^2 + 4*a^3*b*log((e*x + d)^n*c) + a^4)/(g*x + f), x)
```

3.62.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**4/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**4/(f + g*x), x)
```

3.62.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^4}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f),x, algorithm="maxima")
```

output
```
a^4*log(g*x + f)/g + integrate((b^4*log((e*x + d)^n)^4 + b^4*log(c)^4 + 4*
a*b^3*log(c)^3 + 6*a^2*b^2*log(c)^2 + 4*a^3*b*log(c) + 4*(b^4*log(c) + a*b
^3)*log((e*x + d)^n)^3 + 6*(b^4*log(c)^2 + 2*a*b^3*log(c) + a^2*b^2)*log((
e*x + d)^n)^2 + 4*(b^4*log(c)^3 + 3*a*b^3*log(c)^2 + 3*a^2*b^2*log(c) + a^
3*b)*log((e*x + d)^n))/(g*x + f), x)
```

3.62.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^4}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^4/(g*x + f), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log{(c(d + ex)^n)})^4}{f + gx}\,dx = \int \frac{(a + b\,\ln{(c\,(d + e\,x)^n)})^4}{f + g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^4/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^4/(f + g*x), x)
```

3.63 $\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx$

3.63.1 Optimal result

Integrand size = 24, antiderivative size = 248

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx &= \frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{(ef-dg)(f+gx)} \\ &\quad - \frac{4ben(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} \\ &\quad - \frac{12b^2en^2(a+b\log(c(d+ex)^n))^2\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} \\ &\quad + \frac{24b^3en^3(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} \\ &\quad - \frac{24b^4en^4\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}\end{aligned}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/(-d*g+e*f)/(g*x+f)-4*b*e*n*(a+b*ln(c*(e*x+
d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)-12*b^2*e*n^2*(a+b*ln(c*(e*x
+d)^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)+24*b^3*e*n^3*(a+b*
ln(c*(e*x+d)^n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)-24*b^4*e*n^
4*polylog(4,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)
```

3.63.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 531 vs. $2(248) = 496$.

Time = 0.46 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.14

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx$$
$$= \frac{-\left((ef-dg)\left(a-bn\log(d+ex)+b\log(c(d+ex)^n)\right)^4\right)+4bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\,(}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x)^2,x]
```

output
```
(-((e*f - d*g)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^4) + 4*b*n*(a
 - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*(g*(d + e*x)*Log[d + e*x] -
e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) + 6*b^2*n^2*(a - b*n*Log[d + e
*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*(g*(d + e*x)*Log[d + e*x] - 2*
e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) - 2*e*(f + g*x)*PolyLog[2, (g*
(d + e*x))/(-(e*f) + d*g)]) + 4*b^3*n^3*(a - b*n*Log[d + e*x] + b*Log[c*(d
 + e*x)^n])*(Log[d + e*x]^2*(g*(d + e*x)*Log[d + e*x] - 3*e*(f + g*x)*Log[
(e*(f + g*x))/(e*f - d*g)]) - 6*e*(f + g*x)*Log[d + e*x]*PolyLog[2, (g*(d
+ e*x))/(-(e*f) + d*g)] + 6*e*(f + g*x)*PolyLog[3, (g*(d + e*x))/(-(e*f) +
 d*g)]) + b^4*n^4*(g*(d + e*x)*Log[d + e*x]^4 - 4*e*(f + g*x)*Log[d + e*x]
^3*Log[(e*(f + g*x))/(e*f - d*g)] - 12*e*(f + g*x)*Log[d + e*x]^2*PolyLog[
2, (g*(d + e*x))/(-(e*f) + d*g)] + 24*e*(f + g*x)*Log[d + e*x]*PolyLog[3,
(g*(d + e*x))/(-(e*f) + d*g)] - 24*e*(f + g*x)*PolyLog[4, (g*(d + e*x))/(-
(e*f) + d*g)]))/(g*(e*f - d*g)*(f + g*x))
```

3.63.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2844, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx$$
$$\Big\downarrow \; 2844$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)(ef-dg)}-\frac{4ben\int\frac{(a+b\log(c(d+ex)^n))^3}{f+gx}dx}{ef-dg}$$

$\Big\downarrow$ 2843

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)(ef-dg)}-\frac{4ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3ben\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}\right)}{ef-dg}$$

$\Big\downarrow$ 2881

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)(ef-dg)}-\frac{4ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}\right)}{ef-dg}$$

$\Big\downarrow$ 2821

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)(ef-dg)}-\frac{4ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\left(2bn\int\frac{(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c\right.}{g}}{ef-dg}$$

$\Big\downarrow$ 2830

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)(ef-dg)}-\frac{4ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\left(2bn\left(\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))-bn\int\frac{\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)\right)-\right.}{g}}{ef-dg}$$

$\Big\downarrow$ 7143

3.63. $\int\frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)(ef-dg)} - \frac{4ben\left(\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g} - \frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))-bn\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)\right)-\text{PolyLog}\left(2,\ldots\right.\right.}{g}\right.}{ef-dg}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x)^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4)/((e*f - d*g)*(f + g*x)) - (4*b*e*
n*(((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g - (3*b*
n*(-((a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]
) + 2*b*n*((a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*
g))] - b*n*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])))/g))/(e*f - d*g)
```

3.63.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.63.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.22 (sec) , antiderivative size = 2156, normalized size of antiderivative = 8.69

method	result	size
risch	Expression too large to display	2156

input
```
int((a+b*ln(c*(e*x+d)^n))^4/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
24*b^4/g*n^4*e/(d*g-e*f)*polylog(4,-g*(e*x+d)/(-d*g+e*f))-3*b^4/g*n^4*e/(-
d*g+e*f)*ln(e*x+d)^4-2*b^4/g*n^4*e/(d*g-e*f)*ln(e*x+d)^4-12*b^4/g*n^4*e/(d
*g-e*f)*ln(e*x+d)^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))-4*b^4/g*n^4*e/(d*g-e*
f)*ln(g*(e*x+d)-d*g+e*f)*ln(e*x+d)^3+4*b^4/g*n*e/(d*g-e*f)*ln(g*(e*x+d)-d*
g+e*f)*ln((e*x+d)^n)^3+4*b^4/g*n^3*e/(-d*g+e*f)*ln(e*x+d)^3*ln((e*x+d)^n)-
8*b^4/g*n^4*e/(d*g-e*f)*ln(e*x+d)^3*ln(1+g*(e*x+d)/(-d*g+e*f))+12*b^4/g*n^
3*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)*ln(e*x+d)^2-12*b^4/g*n^2
*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^2*ln(e*x+d)-24*b^4/g*n^3*
e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)*ln(e*x+d)-
24*b^4/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln
((e*x+d)^n)+12*b^4/g*n^2*e/(d*g-e*f)*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*
g+e*f))*ln((e*x+d)^n)^2+12*b^4/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln(1+g*(e*x+d
)/(-d*g+e*f))*ln((e*x+d)^n)+24*b^4/g*n^3*e/(d*g-e*f)*ln(e*x+d)*polylog(2,-
g*(e*x+d)/(-d*g+e*f))*ln((e*x+d)^n)+6*b^4/g*n^2*e/(d*g-e*f)*ln(e*x+d)^2*ln
((e*x+d)^n)^2-4*b^4/g*n*e/(d*g-e*f)*ln(e*x+d)*ln((e*x+d)^n)^3+12*b^4/g*n^2
*e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)^2+12*b^4/
g*n^4*e/(d*g-e*f)*ln(e*x+d)^3*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))-24*b^4/g*
n^3*e/(d*g-e*f)*polylog(3,-g*(e*x+d)/(-d*g+e*f))*ln((e*x+d)^n)+12*b^4/g*n^
4*e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln(e*x+d)^2+1/2*(-I*b*
Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(...
```

3.63.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^4}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b^4*log((e*x + d)^n*c)^4 + 4*a*b^3*log((e*x + d)^n*c)^3 + 6*a^2*
b^2*log((e*x + d)^n*c)^2 + 4*a^3*b*log((e*x + d)^n*c) + a^4)/(g^2*x^2 + 2*
f*g*x + f^2), x)
```

3.63.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**4/(g*x+f)**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**4/(f + g*x)**2, x)
```

3.63.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^4}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f)^2,x, algorithm="maxima")
```

output
```
4*a^3*b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2))
- b^4*log((e*x + d)^n)^4/(g^2*x + f*g) - 4*a^3*b*log((e*x + d)^n*c)/(g^2*x
 + f*g) - a^4/(g^2*x + f*g) + integrate((b^4*d*g*log(c)^4 + 4*a*b^3*d*g*lo
g(c)^3 + 6*a^2*b^2*d*g*log(c)^2 + 4*(a*b^3*d*g + (e*f*n + d*g*log(c))*b^4
+ (a*b^3*e*g + (e*g*n + e*g*log(c))*b^4)*x)*log((e*x + d)^n)^3 + 6*(b^4*d*
g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d*g + (b^4*e*g*log(c)^2 + 2*a*b^
3*e*g*log(c) + a^2*b^2*e*g)*x)*log((e*x + d)^n)^2 + (b^4*e*g*log(c)^4 + 4*
a*b^3*e*g*log(c)^3 + 6*a^2*b^2*e*g*log(c)^2)*x + 4*(b^4*d*g*log(c)^3 + 3*a
*b^3*d*g*log(c)^2 + 3*a^2*b^2*d*g*log(c) + (b^4*e*g*log(c)^3 + 3*a*b^3*e*g
*log(c)^2 + 3*a^2*b^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^3*x^3 + d*f^2*
g + (2*e*f*g^2 + d*g^3)*x^2 + (e*f^2*g + 2*d*f*g^2)*x), x)
```

3.63.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^4}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^4/(g*x + f)^2, x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^4}{(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^4/(f + g*x)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^4/(f + g*x)^2, x)
```

3.64 $\int \log(a+bx)\,dx$

3.64.1 Optimal result

Integrand size = 6, antiderivative size = 19

$$\int \log(a+bx)\,dx=-x+\frac{(a+bx)\log(a+bx)}{b}$$

output
```
-x+(b*x+a)*ln(b*x+a)/b
```

3.64.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \log(a+bx)\,dx=-x+\frac{(a+bx)\log(a+bx)}{b}$$

input
```
Integrate[Log[a + b*x],x]
```

output
```
-x + ((a + b*x)*Log[a + b*x])/b
```

3.64.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a+bx)\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+bx)\log(a+bx)-a-bx}{b}$$

input
```
Int[Log[a + b*x],x]
```

output
```
(-a - b*x + (a + b*x)*Log[a + b*x])/b
```

3.64.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.64.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
norman	$x \ln(bx+a)+\frac{a\ln(bx+a)}{b}-x$	24
risch	$x \ln(bx+a)+\frac{a\ln(bx+a)}{b}-x$	24
derivativedivides	$\frac{(bx+a)\ln(bx+a)-bx-a}{b}$	25
default	$\frac{(bx+a)\ln(bx+a)-bx-a}{b}$	25
parallelrisch	$\frac{\ln(bx+a)xb-bx+a\ln(bx+a)+a}{b}$	28
parts	$x\ln(bx+a)-b\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}\right)$	31

input
```
int(ln(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*ln(b*x+a)+a/b*ln(b*x+a)-x
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \log(a+bx)\,dx=-\frac{bx-(bx+a)\log(bx+a)}{b}$$

input
```
integrate(log(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x - (b*x + a)*log(b*x + a))/b
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \log(a+bx)\,dx=-b\left(-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}\right)+x\log(a+bx)$$

input
```
integrate(ln(b*x+a),x)
```

output
```
-b*(-a*log(a + b*x)/b**2 + x/b) + x*log(a + b*x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \log(a+bx)\,dx = -\frac{bx-(bx+a)\log{(bx+a)}+a}{b}$$

input
```
integrate(log(b*x+a),x, algorithm="maxima")
```

output
```
-(b*x - (b*x + a)*log(b*x + a) + a)/b
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \log(a+bx)\,dx = -\frac{bx-(bx+a)\log{(bx+a)}+a}{b}$$

input
```
integrate(log(b*x+a),x, algorithm="giac")
```

output
```
-(b*x - (b*x + a)*log(b*x + a) + a)/b
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \log(a+bx)\,dx = x\,\ln{(a+b\,x)}-x+\frac{a\,\ln{(a+b\,x)}}{b}$$

input
```
int(log(a + b*x),x)
```

output
```
x*log(a + b*x) - x + (a*log(a + b*x))/b
```

3.65 $\int \log^2(a+bx)\,dx$

3.65.1 Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \log^2(a+bx)\,dx = 2x - \frac{2(a+bx)\log(a+bx)}{b} + \frac{(a+bx)\log^2(a+bx)}{b}$$

output
```
2*x-2*(b*x+a)*ln(b*x+a)/b+(b*x+a)*ln(b*x+a)^2/b
```

3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \log^2(a+bx)\,dx = \frac{2bx - 2(a+bx)\log(a+bx) + (a+bx)\log^2(a+bx)}{b}$$

input
```
Integrate[Log[a + b*x]^2,x]
```

output
```
(2*b*x - 2*(a + b*x)*Log[a + b*x] + (a + b*x)*Log[a + b*x]^2)/b
```

3.65.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2836, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2(a+bx)\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^2(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(a+bx)\log^2(a+bx)-2\int \log(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+bx)\log^2(a+bx)-2((a+bx)\log(a+bx)-a-bx)}{b}$$

input
```
Int[Log[a + b*x]^2,x]
```

output
```
((a + b*x)*Log[a + b*x]^2 - 2*(-a - b*x + (a + b*x)*Log[a + b*x]))/b
```

3.65.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.65.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(bx+a)^2(bx+a)-2(bx+a)\ln(bx+a)+2bx+2a}{b}$	40
default	$\frac{\ln(bx+a)^2(bx+a)-2(bx+a)\ln(bx+a)+2bx+2a}{b}$	40
risch	$\frac{(bx+a)\ln(bx+a)^2}{b}-2x\ln(bx+a)+2x-\frac{2a\ln(bx+a)}{b}$	43
norman	$x\ln(bx+a)^2+\frac{a\ln(bx+a)^2}{b}+2x-2x\ln(bx+a)-\frac{2a\ln(bx+a)}{b}$	49
parallelrisch	$\frac{x\ln(bx+a)^2b-2\ln(bx+a)xb+\ln(bx+a)^2a+2bx-2a\ln(bx+a)-2a}{b}$	53

input
```
int(ln(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(b*x+a)^2*(b*x+a)-2*(b*x+a)*ln(b*x+a)+2*b*x+2*a)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \log^2(a+bx)\,dx=\frac{(bx+a)\log(bx+a)^2+2\,bx-2\,(bx+a)\log(bx+a)}{b}$$

input
```
integrate(log(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a)^2 + 2*b*x - 2*(b*x + a)*log(b*x + a))/b
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \log^2(a+bx)\,dx=2b\left(-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}\right)-2x\log(a+bx)+\frac{(a+bx)\log(a+bx)^2}{b}$$

input
```
integrate(ln(b*x+a)**2,x)
```

output
```
2*b*(-a*log(a + b*x)/b**2 + x/b) - 2*x*log(a + b*x) + (a + b*x)*log(a + b*
x)**2/b
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \log^2(a+bx)\,dx = \frac{(bx+a)\left(\log(bx+a)^2 - 2\,\log(bx+a) + 2\right)}{b}$$

input
```
integrate(log(b*x+a)^2,x, algorithm="maxima")
```

output
```
(b*x + a)*(log(b*x + a)^2 - 2*log(b*x + a) + 2)/b
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \log^2(a+bx)\,dx = \frac{(bx+a)\log(bx+a)^2}{b} - \frac{2\,(bx+a)\log(bx+a)}{b} + \frac{2\,(bx+a)}{b}$$

input
```
integrate(log(b*x+a)^2,x, algorithm="giac")
```

output
```
(b*x + a)*log(b*x + a)^2/b - 2*(b*x + a)*log(b*x + a)/b + 2*(b*x + a)/b
```

3.65.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \log^2(a+bx)\,dx = 2\,x - 2\,x\,\ln(a+b\,x) + x\ln(a+b\,x)^2 + \frac{a\ln(a+b\,x)^2}{b} - \frac{2\,a\,\ln(a+b\,x)}{b}$$

input
```
int(log(a + b*x)^2,x)
```

output
```
2*x - 2*x*log(a + b*x) + x*log(a + b*x)^2 + (a*log(a + b*x)^2)/b - (2*a*lo
g(a + b*x))/b
```

3.66 $\int \log^3(a+bx)\,dx$

3.66.1 Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \log^3(a+bx)\,dx = -6x+\frac{6(a+bx)\log(a+bx)}{b}-\frac{3(a+bx)\log^2(a+bx)}{b}+\frac{(a+bx)\log^3(a+bx)}{b}$$

output
```
-6*x+6*(b*x+a)*ln(b*x+a)/b-3*(b*x+a)*ln(b*x+a)^2/b+(b*x+a)*ln(b*x+a)^3/b
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \log^3(a+bx)\,dx = \frac{-6bx+6(a+bx)\log(a+bx)-3(a+bx)\log^2(a+bx)+(a+bx)\log^3(a+bx)}{b}$$

input
```
Integrate[Log[a + b*x]^3,x]
```

output
```
(-6*b*x + 6*(a + b*x)*Log[a + b*x] - 3*(a + b*x)*Log[a + b*x]^2 + (a + b*x
)*Log[a + b*x]^3)/b
```

3.66.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2836, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3(a+bx)\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^3(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(a+bx)\log^3(a+bx)-3\int \log^2(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(a+bx)\log^3(a+bx)-3\left((a+bx)\log^2(a+bx)-2\int \log(a+bx)d(a+bx)\right)}{b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+bx)\log^3(a+bx)-3\left((a+bx)\log^2(a+bx)-2((a+bx)\log(a+bx)-a-bx)\right)}{b}$$

input
```
Int[Log[a + b*x]^3,x]
```

output
```
((a + b*x)*Log[a + b*x]^3 - 3*((a + b*x)*Log[a + b*x]^2 - 2*(-a - b*x + (a
 + b*x)*Log[a + b*x])))/b
```

3.66.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.66.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\ln(bx+a)^3(bx+a)-3\ln(bx+a)^2(bx+a)+6(bx+a)\ln(bx+a)-6bx-6a}{b}$
default	$\frac{\ln(bx+a)^3(bx+a)-3\ln(bx+a)^2(bx+a)+6(bx+a)\ln(bx+a)-6bx-6a}{b}$
risch	$\frac{(bx+a)\ln(bx+a)^3}{b}-\frac{3(bx+a)\ln(bx+a)^2}{b}+6x\ln(bx+a)-6x+\frac{6a\ln(bx+a)}{b}$
norman	$x\ln(bx+a)^3+\frac{a\ln(bx+a)^3}{b}-6x+6x\ln(bx+a)-3x\ln(bx+a)^2+\frac{6a\ln(bx+a)}{b}-\frac{3a\ln(bx-}{b}$
parallelrisch	$\frac{x\ln(bx+a)^3b-3x\ln(bx+a)^2b+\ln(bx+a)^3a+6\ln(bx+a)xb-3\ln(bx+a)^2a-6bx+6a\ln(bx+a)+6a}{b}$

input
```
int(ln(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(b*x+a)^3*(b*x+a)-3*ln(b*x+a)^2*(b*x+a)+6*(b*x+a)*ln(b*x+a)-6*b*x-6
*a)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \log^3(a+bx)\,dx$$
$$=\frac{(bx+a)\log(bx+a)^3-3(bx+a)\log(bx+a)^2-6bx+6(bx+a)\log(bx+a)}{b}$$

input
```
integrate(log(b*x+a)^3,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a)^3 - 3*(b*x + a)*log(b*x + a)^2 - 6*b*x + 6*(b*x +
a)*log(b*x + a))/b
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \log^3(a+bx)\,dx = -6b\left(-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}\right)+6x\log(a+bx)+\frac{(-3a-3bx)\log(a+bx)^2}{b}+\frac{(a+bx)\log(a+bx)^3}{b}$$

input
```
integrate(ln(b*x+a)**3,x)
```

output
```
-6*b*(-a*log(a + b*x)/b**2 + x/b) + 6*x*log(a + b*x) + (-3*a - 3*b*x)*log(
a + b*x)**2/b + (a + b*x)*log(a + b*x)**3/b
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \log^3(a+bx)\,dx = \frac{\left(\log(bx+a)^3-3\log(bx+a)^2+6\log(bx+a)-6\right)(bx+a)}{b}$$

input
```
integrate(log(b*x+a)^3,x, algorithm="maxima")
```

output
```
(log(b*x + a)^3 - 3*log(b*x + a)^2 + 6*log(b*x + a) - 6)*(b*x + a)/b
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \log^3(a+bx)\,dx = \frac{(bx+a)\log(bx+a)^3}{b}-\frac{3(bx+a)\log(bx+a)^2}{b}+\frac{6(bx+a)\log(bx+a)}{b}-\frac{6(bx+a)}{b}$$

input
```
integrate(log(b*x+a)^3,x, algorithm="giac")
```

output
```
(b*x + a)*log(b*x + a)^3/b - 3*(b*x + a)*log(b*x + a)^2/b + 6*(b*x + a)*lo
g(b*x + a)/b - 6*(b*x + a)/b
```

3.66.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \log^3(a+bx)\,dx = 6\,x\,\ln(a+b\,x) - 6\,x - 3\,x\ln(a+b\,x)^2 + x\ln(a+b\,x)^3 - \frac{3\,a\ln(a+b\,x)^2}{b} + \frac{a\ln(a+b\,x)^3}{b} + \frac{6\,a\,\ln(a+b\,x)}{b}$$

input
```
int(log(a + b*x)^3,x)
```

output
```
6*x*log(a + b*x) - 6*x - 3*x*log(a + b*x)^2 + x*log(a + b*x)^3 - (3*a*log(
a + b*x)^2)/b + (a*log(a + b*x)^3)/b + (6*a*log(a + b*x))/b
```

3.67 $\int \log(a+bx+cx)\,dx$

3.67.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \log(a+bx+cx)\,dx=-x+\frac{(a+(b+c)x)\log(a+(b+c)x)}{b+c}$$

output
```
-x+(a+(b+c)*x)*ln(a+(b+c)*x)/(b+c)
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log(a+bx+cx)\,dx=-x+\frac{(a+(b+c)x)\log(a+(b+c)x)}{b+c}$$

input
```
Integrate[Log[a + b*x + c*x],x]
```

output
```
-x + ((a + (b + c)*x)*Log[a + (b + c)*x])/(b + c)
```

3.67.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2894, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a+bx+cx)\,dx$$

$$\Big\downarrow \text{2894}$$

$$\int \log(a+x(b+c))dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log(a+(b+c)x)d(a+(b+c)x)}{b+c}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+x(b+c))\log(a+x(b+c))-a-x(b+c)}{b+c}$$

input
```
Int[Log[a + b*x + c*x],x]
```

output
```
(-a - (b + c)*x + (a + (b + c)*x)*Log[a + (b + c)*x])/(b + c)
```

3.67.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

3.67.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
norman	$x \ln \left(bx+xc+a\right)+\frac{a \ln \left(bx+xc+a\right)}{b+c}-x$	32
derivativedivides	$\frac{\left(a+\left(b+c\right)x\right) \ln \left(a+\left(b+c\right)x\right)-a-\left(b+c\right)x}{b+c}$	33
default	$\frac{\left(a+\left(b+c\right)x\right) \ln \left(a+\left(b+c\right)x\right)-a-\left(b+c\right)x}{b+c}$	33
parts	$x \ln \left(bx+xc+a\right)-\left(b+c\right) \left(\frac{x}{b+c}-\frac{a \ln \left(bx+xc+a\right)}{\left(b+c\right)^{2}}\right)$	43
risch	$x \ln \left(bx+xc+a\right)+\frac{a \ln \left(a+\left(b+c\right)x\right)}{b+c}-\frac{bx}{b+c}-\frac{xc}{b+c}$	46
parallelrisch	$\frac{x \ln \left(bx+xc+a\right) a b+x \ln \left(bx+xc+a\right) a c-a b x-x c a+\ln \left(bx+xc+a\right) a^{2}}{a \left(b+c\right)}$	60

input
```
int(ln(b*x+c*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*ln(b*x+c*x+a)+a/(b+c)*ln(b*x+c*x+a)-x
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \log(a+bx+cx)\,dx=-\frac{(b+c)x-((b+c)x+a)\log\left((b+c)x+a\right)}{b+c}$$

input
```
integrate(log(b*x+c*x+a),x, algorithm="fricas")
```

output
```
-((b + c)*x - ((b + c)*x + a)*log((b + c)*x + a))/(b + c)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \log(a+bx+cx)\,dx = x\log(a+bx+cx) + (-b-c)\left(-\frac{a\log(a+x(b+c))}{(b+c)^2} + \frac{x}{b+c}\right)$$

input
```
integrate(ln(b*x+c*x+a),x)
```

output
```
x*log(a + b*x + c*x) + (-b - c)*(-a*log(a + x*(b + c))/(b + c)**2 + x/(b +
 c))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \log(a+bx+cx)\,dx = -\frac{bx+cx-(bx+cx+a)\log(bx+cx+a)+a}{b+c}$$

input
```
integrate(log(b*x+c*x+a),x, algorithm="maxima")
```

output
```
-(b*x + c*x - (b*x + c*x + a)*log(b*x + c*x + a) + a)/(b + c)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \log(a+bx+cx)\,dx = -\frac{bx+cx-(bx+cx+a)\log(bx+cx+a)+a}{b+c}$$

input
```
integrate(log(b*x+c*x+a),x, algorithm="giac")
```

output
```
-(b*x + c*x - (b*x + c*x + a)*log(b*x + c*x + a) + a)/(b + c)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \log(a+bx+cx)\,dx = x\,\ln\left(a+b\,x+c\,x\right)-x+\frac{a\,\ln\left(a+b\,x+c\,x\right)}{b+c}$$

input
```
int(log(a + b*x + c*x),x)
```

output
```
x*log(a + b*x + c*x) - x + (a*log(a + b*x + c*x))/(b + c)
```

3.68 $\int \log^2(a+bx+cx)\,dx$

3.68.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \log^2(a+bx+cx)\,dx = 2x-\frac{2(a+(b+c)x)\log(a+(b+c)x)}{b+c}+\frac{(a+(b+c)x)\log^2(a+(b+c)x)}{b+c}$$

output
```
2*x-2*(a+(b+c)*x)*ln(a+(b+c)*x)/(b+c)+(a+(b+c)*x)*ln(a+(b+c)*x)^2/(b+c)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \log^2(a+bx+cx)\,dx = \frac{2(b+c)x-2(a+(b+c)x)\log(a+(b+c)x)+(a+(b+c)x)\log^2(a+(b+c)x)}{b+c}$$

input
```
Integrate[Log[a + b*x + c*x]^2,x]
```

output
```
(2*(b + c)*x - 2*(a + (b + c)*x)*Log[a + (b + c)*x] + (a + (b + c)*x)*Log[
a + (b + c)*x]^2)/(b + c)
```

3.68.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2894, 2836, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2(a+bx+cx)\,dx$$

$$\Big\downarrow \text{2894}$$

$$\int \log^2(a+x(b+c))dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^2(a+(b+c)x)d(a+(b+c)x)}{b+c}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(a+x(b+c))\log^2(a+x(b+c))-2\int \log(a+(b+c)x)d(a+(b+c)x)}{b+c}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+x(b+c))\log^2(a+x(b+c))-2((a+x(b+c))\log(a+x(b+c))-a-x(b+c))}{b+c}$$

input
```
Int[Log[a + b*x + c*x]^2,x]
```

output
```
((a + (b + c)*x)*Log[a + (b + c)*x]^2 - 2*(-a - (b + c)*x + (a + (b + c)*x
)*Log[a + (b + c)*x]))/(b + c)
```

3.68.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

3.68.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\ln(a+(b+c)x)^2(a+(b+c)x)-2(a+(b+c)x)\ln(a+(b+c)x)+2a+2(b+c)x}{b+c}$
default	$\frac{\ln(a+(b+c)x)^2(a+(b+c)x)-2(a+(b+c)x)\ln(a+(b+c)x)+2a+2(b+c)x}{b+c}$
norman	$x\ln(bx+xc+a)^2+\frac{a\ln(bx+xc+a)^2}{b+c}+2x-2x\ln(bx+xc+a)-\frac{2a\ln(bx+xc+a)}{b+c}$
risch	$\frac{\ln(bx+xc+a)^2(bx+xc+a)}{b+c}-2x\ln(bx+xc+a)-\frac{2a\ln(a+(b+c)x)}{b+c}+\frac{2bx}{b+c}+\frac{2xc}{b+c}$
parallelrisch	$\frac{x\ln(bx+xc+a)^2ab+x\ln(bx+xc+a)^2ac-2x\ln(bx+xc+a)ab-2x\ln(bx+xc+a)ac+\ln(bx+xc+a)^2a^2+2abx+2xca-2\ln(b}{a(b+c)}$

input
```
int(ln(b*x+c*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(b+c)*(ln(a+(b+c)*x)^2*(a+(b+c)*x)-2*(a+(b+c)*x)*ln(a+(b+c)*x)+2*a+2*(b+
c)*x)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \log^2(a+bx+cx)\,dx = \frac{((b+c)x+a)\log\left((b+c)x+a\right)^2+2\,(b+c)x-2\,((b+c)x+a)\log\left((b+c)x+a\right)}{b+c}$$

input
```
integrate(log(b*x+c*x+a)^2,x, algorithm="fricas")
```

output
```
(((b + c)*x + a)*log((b + c)*x + a)^2 + 2*(b + c)*x - 2*((b + c)*x + a)*lo
g((b + c)*x + a))/(b + c)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \log^2(a+bx+cx)\,dx = -2x\log(a+bx+cx)+(2b+2c)\left(-\frac{a\log(a+x(b+c))}{(b+c)^2}+\frac{x}{b+c}\right) + \frac{(a+bx+cx)\log(a+bx+cx)^2}{b+c}$$

input
```
integrate(ln(b*x+c*x+a)**2,x)
```

output
```
-2*x*log(a + b*x + c*x) + (2*b + 2*c)*(-a*log(a + x*(b + c))/(b + c)**2 +
x/(b + c)) + (a + b*x + c*x)*log(a + b*x + c*x)**2/(b + c)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \log^2(a+bx+cx)\,dx = \frac{(bx+cx+a)\left(\log(bx+cx+a)^2-2\log(bx+cx+a)+2\right)}{b+c}$$

input
```
integrate(log(b*x+c*x+a)^2,x, algorithm="maxima")
```

output
```
(b*x + c*x + a)*(log(b*x + c*x + a)^2 - 2*log(b*x + c*x + a) + 2)/(b + c)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \log^2(a+bx+cx)\,dx = \frac{(bx+cx+a)\log(bx+cx+a)^2}{b+c} - \frac{2(bx+cx+a)\log(bx+cx+a)}{b+c} + \frac{2(bx+cx+a)}{b+c}$$

input
```
integrate(log(b*x+c*x+a)^2,x, algorithm="giac")
```

output
```
(b*x + c*x + a)*log(b*x + c*x + a)^2/(b + c) - 2*(b*x + c*x + a)*log(b*x +
 c*x + a)/(b + c) + 2*(b*x + c*x + a)/(b + c)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int \log^2(a+bx+cx)\,dx$$
$$= \frac{2\,b\,x+2\,c\,x-2\,a\,\ln\left(a+b\,x+c\,x\right)+a\ln\left(a+b\,x+c\,x\right)^2+b\,x\ln\left(a+b\,x+c\,x\right)^2+c\,x\ln\left(a+b\,x+c\,x\right)^2}{b+c}$$

input
```
int(log(a + b*x + c*x)^2,x)
```

output
```
(2*b*x + 2*c*x - 2*a*log(a + b*x + c*x) + a*log(a + b*x + c*x)^2 + b*x*log
(a + b*x + c*x)^2 + c*x*log(a + b*x + c*x)^2 - 2*b*x*log(a + b*x + c*x) -
2*c*x*log(a + b*x + c*x))/(b + c)
```

3.69 $\int \log^3(a+bx+cx)\,dx$

3.69.1 Optimal result

Integrand size = 11, antiderivative size = 73

$$\int \log^3(a+bx+cx)\,dx = -6x+\frac{6(a+(b+c)x)\log(a+(b+c)x)}{b+c} - \frac{3(a+(b+c)x)\log^2(a+(b+c)x)}{b+c} + \frac{(a+(b+c)x)\log^3(a+(b+c)x)}{b+c}$$

output
```
-6*x+6*(a+(b+c)*x)*ln(a+(b+c)*x)/(b+c)-3*(a+(b+c)*x)*ln(a+(b+c)*x)^2/(b+c)
+(a+(b+c)*x)*ln(a+(b+c)*x)^3/(b+c)
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \log^3(a+bx+cx)\,dx = \frac{-6(b+c)x+6(a+(b+c)x)\log(a+(b+c)x)-3(a+(b+c)x)\log^2(a+(b+c)x)+(a+(b+c)x)\log^3}{b+c}$$

input
```
Integrate[Log[a + b*x + c*x]^3,x]
```

output
```
(-6*(b + c)*x + 6*(a + (b + c)*x)*Log[a + (b + c)*x] - 3*(a + (b + c)*x)*L
og[a + (b + c)*x]^2 + (a + (b + c)*x)*Log[a + (b + c)*x]^3)/(b + c)
```

3.69.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2894, 2836, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3(a+bx+cx)\,dx$$

$$\Big\downarrow \text{2894}$$

$$\int \log^3(a+x(b+c))dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^3(a+(b+c)x)d(a+(b+c)x)}{b+c}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(a+x(b+c))\log^3(a+x(b+c))-3\int \log^2(a+(b+c)x)d(a+(b+c)x)}{b+c}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(a+x(b+c))\log^3(a+x(b+c))-3\big((a+x(b+c))\log^2(a+x(b+c))-2\int \log(a+(b+c)x)d(a+(b+c)x)\big)}{b+c}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+x(b+c))\log^3(a+x(b+c))-3\big((a+x(b+c))\log^2(a+x(b+c))-2((a+x(b+c))\log(a+x(b+c))-a-}{b+c}$$

input
```
Int[Log[a + b*x + c*x]^3,x]
```

output
```
((a + (b + c)*x)*Log[a + (b + c)*x]^3 - 3*((a + (b + c)*x)*Log[a + (b + c)
*x]^2 - 2*(-a - (b + c)*x + (a + (b + c)*x)*Log[a + (b + c)*x])))/(b + c)
```

3.69.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

3.69.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\ln(a+(b+c)x)^3(a+(b+c)x)-3\ln(a+(b+c)x)^2(a+(b+c)x)+6(a+(b+c)x)\ln(a+(b+c)x)-6a-6(b+c)x}{b+c}$
default	$\frac{\ln(a+(b+c)x)^3(a+(b+c)x)-3\ln(a+(b+c)x)^2(a+(b+c)x)+6(a+(b+c)x)\ln(a+(b+c)x)-6a-6(b+c)x}{b+c}$
norman	$x\ln(bx+xc+a)^3+\frac{a\ln(bx+xc+a)^3}{b+c}-6x+6x\ln(bx+xc+a)-3x\ln(bx+xc+a)^2+$
risch	$\frac{\ln(bx+xc+a)^3(bx+xc+a)}{b+c}-\frac{3\ln(bx+xc+a)^2(bx+xc+a)}{b+c}+6x\ln(bx+xc+a)+\frac{6a\ln(a+(b+c)x)}{b+c}-\frac{6bx}{b+c}$
parallelrisch	$\frac{x\ln(bx+xc+a)^3ab+x\ln(bx+xc+a)^3ac-3x\ln(bx+xc+a)^2ab-3x\ln(bx+xc+a)^2ac+\ln(bx+xc+a)^3a^2+6x\ln(bx+xc+a)}{(b+c)a}$

input
```
int(ln(b*x+c*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(b+c)*(ln(a+(b+c)*x)^3*(a+(b+c)*x)-3*ln(a+(b+c)*x)^2*(a+(b+c)*x)+6*(a+(b
+c)*x)*ln(a+(b+c)*x)-6*a-6*(b+c)*x)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \log^3(a+bx+cx)\,dx$$
$$=\frac{((b+c)x+a)\log\left((b+c)x+a\right)^3-3\left((b+c)x+a\right)\log\left((b+c)x+a\right)^2-6\,(b+c)x+6\left((b+c)x+a\right)\log}{b+c}$$

input
```
integrate(log(b*x+c*x+a)^3,x, algorithm="fricas")
```

output
```
(((b + c)*x + a)*log((b + c)*x + a)^3 - 3*((b + c)*x + a)*log((b + c)*x +
a)^2 - 6*(b + c)*x + 6*((b + c)*x + a)*log((b + c)*x + a))/(b + c)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \log^3(a+bx+cx)\,dx=6x\log(a+bx+cx)+(-6b-6c)\left(-\frac{a\log(a+x(b+c))}{(b+c)^2}+\frac{x}{b+c}\right)+\frac{(-3a-3bx-3cx)\log(a+bx+cx)^2}{b+c}+\frac{(a+bx+cx)\log(a+bx+cx)^3}{b+c}$$

input
```
integrate(ln(b*x+c*x+a)**3,x)
```

output
```
6*x*log(a + b*x + c*x) + (-6*b - 6*c)*(-a*log(a + x*(b + c))/(b + c)**2 +
x/(b + c)) + (-3*a - 3*b*x - 3*c*x)*log(a + b*x + c*x)**2/(b + c) + (a + b
*x + c*x)*log(a + b*x + c*x)**3/(b + c)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \log^3(a+bx+cx)\,dx = \frac{\left(\log\left(bx+cx+a\right)^3-3\,\log\left(bx+cx+a\right)^2+6\,\log\left(bx+cx+a\right)-6\right)\left(bx+cx+a\right)}{b+c}$$

input
```
integrate(log(b*x+c*x+a)^3,x, algorithm="maxima")
```

output
```
(log(b*x + c*x + a)^3 - 3*log(b*x + c*x + a)^2 + 6*log(b*x + c*x + a) - 6)
*(b*x + c*x + a)/(b + c)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \log^3(a+bx+cx)\,dx = \frac{(bx+cx+a)\log\left(bx+cx+a\right)^3}{b+c} - \frac{3\,(bx+cx+a)\log\left(bx+cx+a\right)^2}{b+c} + \frac{6\,(bx+cx+a)\log\left(bx+cx+a\right)}{b+c} - \frac{6\,(bx+cx+a)}{b+c}$$

input
```
integrate(log(b*x+c*x+a)^3,x, algorithm="giac")
```

output
```
(b*x + c*x + a)*log(b*x + c*x + a)^3/(b + c) - 3*(b*x + c*x + a)*log(b*x +
 c*x + a)^2/(b + c) + 6*(b*x + c*x + a)*log(b*x + c*x + a)/(b + c) - 6*(b*
x + c*x + a)/(b + c)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.89

$$\int \log^3(a+bx+cx)\,dx$$
$$= \frac{6\,a\,\ln\left(a+b\,x+c\,x\right)-6\,c\,x-6\,b\,x-3\,a\ln\left(a+b\,x+c\,x\right)^2+a\ln\left(a+b\,x+c\,x\right)^3-3\,b\,x\ln\left(a+b\,x+c\,x\right.}{}$$

input
```
int(log(a + b*x + c*x)^3,x)
```

output
```
(6*a*log(a + b*x + c*x) - 6*c*x - 6*b*x - 3*a*log(a + b*x + c*x)^2 + a*log
(a + b*x + c*x)^3 - 3*b*x*log(a + b*x + c*x)^2 + b*x*log(a + b*x + c*x)^3
- 3*c*x*log(a + b*x + c*x)^2 + c*x*log(a + b*x + c*x)^3 + 6*b*x*log(a + b*
x + c*x) + 6*c*x*log(a + b*x + c*x))/(b + c)
```

3.70 $\int \log(c(d+ex)^n)\,dx$

3.70.1 Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \log(c(d+ex)^n)\,dx = -nx + \frac{(d+ex)\log(c(d+ex)^n)}{e}$$

output
```
-n*x+(e*x+d)*ln(c*(e*x+d)^n)/e
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \log(c(d+ex)^n)\,dx = -nx + \frac{(d+ex)\log(c(d+ex)^n)}{e}$$

input
```
Integrate[Log[c*(d + e*x)^n],x]
```

output
```
-(n*x) + ((d + e*x)*Log[c*(d + e*x)^n])/e
```

3.70.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(d+ex)^n\right) \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log \left(c(d+ex)^n\right) d(d+ex)}{e}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(d+ex) \log \left(c(d+ex)^n\right) - n(d+ex)}{e}$$

input
```
Int[Log[c*(d + e*x)^n],x]
```

output
```
(-(n*(d + e*x)) + (d + e*x)*Log[c*(d + e*x)^n])/e
```

3.70.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.70.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result
norman	$x \ln\left(c\,e^{n \ln(ex+d)}\right)+\frac{nd\ln(ex+d)}{e}-nx$
default	$\ln\left(c(ex+d)^{n}\right)x-en\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^{2}}\right)$
parts	$\ln\left(c(ex+d)^{n}\right)x-en\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^{2}}\right)$
parallelrisch	$\frac{x\ln(c(ex+d)^{n})den-xden^{2}+\ln(c(ex+d)^{n})d^{2}n}{den}$
risch	$x\ln\left((ex+d)^{n}\right)-\frac{i\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^{n})\,\mathrm{csgn}(ic(ex+d)^{n})}{2}+\frac{i\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^{n})^{2}}{2}+\frac{i\pi x\ \mathrm{csgn}(i(ex$

input
```
int(ln(c*(e*x+d)^n),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*exp(n*ln(e*x+d)))+n*d/e*ln(e*x+d)-n*x
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \log\left(c(d+ex)^{n}\right)\,dx=-\frac{enx-ex\log(c)-(enx+dn)\log(ex+d)}{e}$$

input
```
integrate(log(c*(e*x+d)^n),x, algorithm="fricas")
```

output
```
-(e*n*x - e*x*log(c) - (e*n*x + d*n)*log(e*x + d))/e
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \log\left(c(d+ex)^{n}\right)\,dx=\begin{cases}\frac{d\log(c(d+ex)^{n})}{e}-nx+x\log\left(c(d+ex)^{n}\right) & \text{for } e\neq 0\\ x\log\left(cd^{n}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(ln(c*(e*x+d)**n),x)
```

output
```
Piecewise((d*log(c*(d + e*x)**n)/e - n*x + x*log(c*(d + e*x)**n), Ne(e, 0)
), (x*log(c*d**n), True))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \log \left(c(d+ex)^n\right) \, dx = -en\left(\frac{x}{e} - \frac{d\log \left(ex+d\right)}{e^2}\right) + x\log \left((ex+d)^n c\right)$$

input
```
integrate(log(c*(e*x+d)^n),x, algorithm="maxima")
```

output
```
-e*n*(x/e - d*log(e*x + d)/e^2) + x*log((e*x + d)^n*c)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \log \left(c(d+ex)^n\right) \, dx = \frac{(ex+d)n\log \left(ex+d\right)}{e} - \frac{(ex+d)n}{e} + \frac{(ex+d)\log \left(c\right)}{e}$$

input
```
integrate(log(c*(e*x+d)^n),x, algorithm="giac")
```

output
```
(e*x + d)*n*log(e*x + d)/e - (e*x + d)*n/e + (e*x + d)*log(c)/e
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \log \left(c(d+ex)^n\right) \, dx = x\,\ln \left(c\,{\left(d+e\,x\right)}^n\right) - n\,x + \frac{d\,n\,\ln \left(d+e\,x\right)}{e}$$

input
```
int(log(c*(d + e*x)^n),x)
```

output
```
x*log(c*(d + e*x)^n) - n*x + (d*n*log(d + e*x))/e
```

3.71 $\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx$

3.71.1 Optimal result

Integrand size = 27, antiderivative size = 24

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx=-\frac{\text{PolyLog}\left(2,\frac{e(f+gx)}{ef-dg}\right)}{g}$$

output
```
-polylog(2,e*(g*x+f)/(-d*g+e*f))/g
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx=-\frac{\text{PolyLog}\left(2,\frac{e(f+gx)}{ef-dg}\right)}{g}$$

input
```
Integrate[Log[-((g*(d + e*x))/(e*f - d*g))]/(f + g*x),x]
```

output
```
-(PolyLog[2, (e*(f + g*x))/(e*f - d*g)]/g)
```

3.71.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx$$

$$\Big\downarrow \text{ 2840}$$

$$\frac{\int \frac{\log\left(1-\frac{e(f+gx)}{ef-dg}\right)}{f+gx}d(f+gx)}{g}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{\text{PolyLog}\left(2,\frac{e(f+gx)}{ef-dg}\right)}{g}$$

input
```
Int[Log[-((g*(d + e*x))/(e*f - d*g))]/(f + g*x),x]
```

output
```
-(PolyLog[2, (e*(f + g*x))/(e*f - d*g)]/g)
```

3.71.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 1.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

method	result	size
derivativedivides	$\frac{(-dg+ef)\,\mathrm{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)}{g(dg-ef)}$	54
default	$\frac{(-dg+ef)\,\mathrm{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)}{g(dg-ef)}$	54
risch	$-\frac{\mathrm{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)d}{dg-ef}+\frac{\mathrm{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)ef}{g(dg-ef)}$	93
parts	$\frac{\ln\left(-\frac{g(ex+d)}{-dg+ef}\right)\ln(gx+f)}{g}-\frac{e\left(\frac{\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{e}+\frac{\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{e}\right)}{g}$	106

input
```
int(ln(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/g*(-d*g+e*f)/(d*g-e*f)*dilog(-g*e/(-d*g+e*f)*x-d*g/(-d*g+e*f))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\mathrm{Li}_2\left(\frac{egx+dg}{ef-dg}+1\right)}{g}$$

input
```
integrate(log(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x, algorithm="fricas")
```

output
```
-dilog((e*g*x + d*g)/(e*f - d*g) + 1)/g
```

3.71.6 Sympy [F]

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = \int \frac{\log\left(-\frac{dg}{-dg+ef}-\frac{egx}{-dg+ef}\right)}{f+gx}\,dx$$

input
```
integrate(ln(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x)
```

output
```
Integral(log(-d*g/(-d*g + e*f) - e*g*x/(-d*g + e*f))/(f + g*x), x)
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\log(ex+d)\log(gx+f)}{g} + \frac{\log(gx+f)\log\left(-\frac{(ex+d)g}{ef-dg}\right)}{g} + \frac{\log(ex+d)\log\left(\frac{egx+dg}{ef-dg}+1\right)+\mathrm{Li}_2\left(-\frac{egx+dg}{ef-dg}\right)}{g}$$

input
```
integrate(log(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x, algorithm="maxima")
```

output
```
-log(e*x + d)*log(g*x + f)/g + log(g*x + f)*log(-(e*x + d)*g/(e*f - d*g))/
g + (log(e*x + d)*log((e*g*x + d*g)/(e*f - d*g) + 1) + dilog(-(e*g*x + d*g
)/(e*f - d*g)))/g
```

3.71.8 Giac [F]

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = \int \frac{\log\left(-\frac{(ex+d)g}{ef-dg}\right)}{gx+f}\,dx$$

input
```
integrate(log(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x, algorithm="giac")
```

output
```
integrate(log(-(e*x + d)*g/(e*f - d*g))/(g*x + f), x)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx=-\frac{\mathrm{Li}_2\left(\frac{g\,(d+e\,x)}{d\,g-e\,f}\right)}{g}$$

input
```
int(log((g*(d + e*x))/(d*g - e*f))/(f + g*x),x)
```

output
```
-dilog((g*(d + e*x))/(d*g - e*f))/g
```

3.72 $\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx$

3.72.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log(x) - b\,\mathrm{PolyLog}(2,-cex)$$

output
```
a*ln(x)-b*polylog(2,-c*e*x)
```

3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log(x) - b\,\mathrm{PolyLog}(2,-cex)$$

input
```
Integrate[(a + b*Log[c*(c^(-1) + e*x)])/x,x]
```

output
```
a*Log[x] - b*PolyLog[2, -(c*e*x)]
```

3.72.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{ 2839}$$

$$b\int \frac{\log(cex+1)}{x}dx + a\log(x)$$

$$\Big\downarrow \text{ 2838}$$

$$a\log(x) - b\,\text{PolyLog}(2,-cex)$$

input
```
Int[(a + b*Log[c*(c^(-1) + e*x)])/x,x]
```

output
```
a*Log[x] - b*PolyLog[2, -(c*e*x)]
```

3.72.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$\ln(x)\,a - b\,\text{dilog}(cex+1)$	16
parts	$\ln(x)\,a - b\,\text{dilog}(cex+1)$	16
derivativedivides	$a\ln(cex) - b\,\text{dilog}(cex+1)$	19
default	$a\ln(cex) - b\,\text{dilog}(cex+1)$	19

input
```
int((a+b*ln(c*(1/c+e*x)))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a-b*dilog(c*e*x+1)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = -b\text{Li}_2(-cex) + a\log(x)$$

input
```
integrate((a+b*log(c*(1/c+e*x)))/x,x, algorithm="fricas")
```

output
```
-b*dilog(-c*e*x) + a*log(x)
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log(x) - b\,\text{Li}_2\left(cexe^{i\pi}\right)$$

input
```
integrate((a+b*ln(c*(1/c+e*x)))/x,x)
```

output
```
a*log(x) - b*polylog(2, c*e*x*exp_polar(I*pi))
```

3.72.7 Maxima [F]

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = \int \frac{b\log\left(\left(ex+\frac{1}{c}\right)c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(1/c+e*x)))/x,x, algorithm="maxima")
```

output
```
b*integrate(log(c*e*x + 1)/x, x) + a*log(x)
```

3.72.8 Giac [F]

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = \int \frac{b\log\left(\left(ex+\frac{1}{c}\right)c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(1/c+e*x)))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + 1/c)*c) + a)/x, x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\ \ln(x) - b\,\mathrm{polylog}(2,-c\,e\,x)$$

input
```
int((a + b*log(c*(e*x + 1/c)))/x,x)
```

output
```
a*log(x) - b*polylog(2, -c*e*x)
```

3.73 $\int \frac{\log(3+ex)}{x} \, dx$

3.73.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{\log(3+ex)}{x} \, dx = \log(3)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{3}\right)$$

output
```
ln(3)*ln(x)-polylog(2,-1/3*e*x)
```

3.73.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log(3+ex)}{x} \, dx = \log(3)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{3}\right)$$

input
```
Integrate[Log[3 + e*x]/x,x]
```

output
```
Log[3]*Log[x] - PolyLog[2, -1/3*(e*x)]
```

3.73.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+3)}{x}\,dx$$
$$\Big\downarrow \text{ 2839}$$
$$\int \frac{\log\left(\frac{ex}{3}+1\right)}{x}dx+\log(3)\log(x)$$
$$\Big\downarrow \text{ 2838}$$
$$\log(3)\log(x)-\text{PolyLog}\left(2,-\frac{ex}{3}\right)$$

input
```
Int[Log[3 + e*x]/x,x]
```

output
```
Log[3]*Log[x] - PolyLog[2, -1/3*(e*x)]
```

3.73.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

method	result	size
derivativedivides	$\left(\ln \left(ex+3\right)-\ln \left(\frac{ex}{3}+1\right)\right) \ln \left(-\frac{ex}{3}\right)-\operatorname{dilog}\left(\frac{ex}{3}+1\right)$	33
default	$\left(\ln \left(ex+3\right)-\ln \left(\frac{ex}{3}+1\right)\right) \ln \left(-\frac{ex}{3}\right)-\operatorname{dilog}\left(\frac{ex}{3}+1\right)$	33
risch	$\left(\ln \left(ex+3\right)-\ln \left(\frac{ex}{3}+1\right)\right) \ln \left(-\frac{ex}{3}\right)-\operatorname{dilog}\left(\frac{ex}{3}+1\right)$	33
parts	$\ln \left(ex+3\right) \ln \left(x\right)-e\left(\frac{\operatorname{dilog}\left(\frac{ex}{3}+1\right)}{e}+\frac{\ln \left(x\right) \ln \left(\frac{ex}{3}+1\right)}{e}\right)$	39

input
```
int(ln(e*x+3)/x,x,method=_RETURNVERBOSE)
```

output
```
(ln(e*x+3)-ln(1/3*e*x+1))*ln(-1/3*e*x)-dilog(1/3*e*x+1)
```

3.73.5 Fricas [F]

$$\int \frac{\log(3+ex)}{x}\,dx = \int \frac{\log\left(ex+3\right)}{x}\,dx$$

input
```
integrate(log(e*x+3)/x,x, algorithm="fricas")
```

output
```
integral(log(e*x + 3)/x, x)
```

3.73.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{\log(3+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(3)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } |x|<1 \\ -\log(3)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(3)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(3)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(e*x+3)/x,x)
```

output
```
Piecewise((-polylog(2, e*x*exp_polar(I*pi)/3), (Abs(x) < 1) & (1/Abs(x) <
1)), (log(3)*log(x) - polylog(2, e*x*exp_polar(I*pi)/3), Abs(x) < 1), (-lo
g(3)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/3), 1/Abs(x) < 1), (-meijer
g(((), (1, 1)), ((0, 0), ()), x)*log(3) + meijerg(((1, 1), ()), ((), (0, 0
)), x)*log(3) - polylog(2, e*x*exp_polar(I*pi)/3), True))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\log(3+ex)}{x}\,dx = \log(ex+3)\log\left(-\frac{1}{3}\,ex\right)+\operatorname{Li}_2\left(\frac{1}{3}\,ex+1\right)$$

input
```
integrate(log(e*x+3)/x,x, algorithm="maxima")
```

output
```
log(e*x + 3)*log(-1/3*e*x) + dilog(1/3*e*x + 1)
```

3.73.8 Giac [F]

$$\int \frac{\log(3+ex)}{x}\,dx = \int \frac{\log(ex+3)}{x}\,dx$$

input
```
integrate(log(e*x+3)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x + 3)/x, x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\log(3+ex)}{x}\,dx = \mathrm{Li}_2\left(-\frac{e\,x}{3}\right) + \ln(e\,x+3)\,\ln\left(-\frac{e\,x}{3}\right)$$

input
```
int(log(e*x + 3)/x,x)
```

output
```
dilog(-(e*x)/3) + log(e*x + 3)*log(-(e*x)/3)
```

3.74 $\int \frac{\log(2+ex)}{x} \, dx$

3.74.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{\log(2+ex)}{x} \, dx = \log(2)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{2}\right)$$

output
```
ln(2)*ln(x)-polylog(2,-1/2*e*x)
```

3.74.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log(2+ex)}{x} \, dx = \log(2)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{2}\right)$$

input
```
Integrate[Log[2 + e*x]/x,x]
```

output
```
Log[2]*Log[x] - PolyLog[2, -1/2*(e*x)]
```

3.74.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+2)}{x} \, dx$$

$$\Big\downarrow \text{2839}$$

$$\int \frac{\log\left(\frac{ex}{2}+1\right)}{x}dx+\log(2)\log(x)$$

$$\Big\downarrow \text{2838}$$

$$\log(2)\log(x)-\text{PolyLog}\left(2,-\frac{ex}{2}\right)$$

input
```
Int[Log[2 + e*x]/x,x]
```

output
```
Log[2]*Log[x] - PolyLog[2, -1/2*(e*x)]
```

3.74.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

method	result	size
derivativedivides	$\left(\ln \left(e x+2\right)-\ln \left(\frac{e x}{2}+1\right)\right) \ln \left(-\frac{e x}{2}\right)-\operatorname{dilog}\left(\frac{e x}{2}+1\right)$	33
default	$\left(\ln \left(e x+2\right)-\ln \left(\frac{e x}{2}+1\right)\right) \ln \left(-\frac{e x}{2}\right)-\operatorname{dilog}\left(\frac{e x}{2}+1\right)$	33
risch	$\left(\ln \left(e x+2\right)-\ln \left(\frac{e x}{2}+1\right)\right) \ln \left(-\frac{e x}{2}\right)-\operatorname{dilog}\left(\frac{e x}{2}+1\right)$	33
parts	$\ln \left(e x+2\right) \ln \left(x\right)-e \left(\frac{\operatorname{dilog}\left(\frac{e x}{2}+1\right)}{e}+\frac{\ln \left(x\right) \ln \left(\frac{e x}{2}+1\right)}{e}\right)$	39

input
```
int(ln(e*x+2)/x,x,method=_RETURNVERBOSE)
```

output
```
(ln(e*x+2)-ln(1/2*e*x+1))*ln(-1/2*e*x)-dilog(1/2*e*x+1)
```

3.74.5 Fricas [F]

$$\int \frac{\log(2+ex)}{x}\,dx = \int \frac{\log\left(ex+2\right)}{x}\,dx$$

input
```
integrate(log(e*x+2)/x,x, algorithm="fricas")
```

output
```
integral(log(e*x + 2)/x, x)
```

3.74.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{\log(2+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(2)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } |x| < 1 \\ -\log(2)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(2) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(2) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(e*x+2)/x,x)
```

output
```
Piecewise((-polylog(2, e*x*exp_polar(I*pi)/2), (Abs(x) < 1) & (1/Abs(x) <
1)), (log(2)*log(x) - polylog(2, e*x*exp_polar(I*pi)/2), Abs(x) < 1), (-lo
g(2)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/2), 1/Abs(x) < 1), (-meijer
g(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ()), ((), (0, 0
)), x)*log(2) - polylog(2, e*x*exp_polar(I*pi)/2), True))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\log(2+ex)}{x}\,dx = \log(ex+2)\log\left(-\frac{1}{2}\,ex\right) + \operatorname{Li}_2\left(\frac{1}{2}\,ex+1\right)$$

input
```
integrate(log(e*x+2)/x,x, algorithm="maxima")
```

output
```
log(e*x + 2)*log(-1/2*e*x) + dilog(1/2*e*x + 1)
```

3.74.8 Giac [F]

$$\int \frac{\log(2+ex)}{x}\,dx = \int \frac{\log(ex+2)}{x}\,dx$$

input
```
integrate(log(e*x+2)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x + 2)/x, x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\log(2+ex)}{x}\,dx = \mathrm{Li}_2\left(-\frac{e\,x}{2}\right) + \ln(e\,x+2)\,\ln\left(-\frac{e\,x}{2}\right)$$

input
```
int(log(e*x + 2)/x,x)
```

output
```
dilog(-(e*x)/2) + log(e*x + 2)*log(-(e*x)/2)
```

3.75 $\int \frac{\log(1+ex)}{x} \, dx$

3.75.1 Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{\log(1+ex)}{x} \, dx = -\operatorname{PolyLog}(2,-ex)$$

output
```
-polylog(2,-e*x)
```

3.75.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+ex)}{x} \, dx = -\operatorname{PolyLog}(2,-ex)$$

input
```
Integrate[Log[1 + e*x]/x,x]
```

output
```
-PolyLog[2, -(e*x)]
```

3.75.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)}{x} \, dx$$
$$\Big\downarrow \; 2838$$
$$-\operatorname{PolyLog}(2,-ex)$$

input
```
Int[Log[1 + e*x]/x,x]
```

output
```
-PolyLog[2, -(e*x)]
```

3.75.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.75.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\operatorname{dilog}(ex+1)$	9
default	$-\operatorname{dilog}(ex+1)$	9
meijerg	$-\operatorname{Li}_2(-ex)$	9
risch	$-\operatorname{dilog}(ex+1)$	9
parts	$\ln(ex+1)\ln(x) - e\left(\frac{\operatorname{dilog}(ex+1)}{e} + \frac{\ln(x)\ln(ex+1)}{e}\right)$	37

input
```
int(1/x*ln(e*x+1),x,method=_RETURNVERBOSE)
```

output
```
-dilog(e*x+1)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\log(1+ex)}{x}\,dx = -\mathrm{Li}_2(-ex)$$

input
```
integrate(log(e*x+1)/x,x, algorithm="fricas")
```

output
```
-dilog(-e*x)
```

3.75.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\log(1+ex)}{x}\,dx = -\operatorname{Li}_2\left(exe^{i\pi}\right)$$

input
```
integrate(ln(e*x+1)/x,x)
```

output
```
-polylog(2, e*x*exp_polar(I*pi))
```

3.75.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{\log(1+ex)}{x}\,dx = \log\left(ex+1\right)\log\left(-ex\right) + \mathrm{Li}_2(ex+1)$$

input
```
integrate(log(e*x+1)/x,x, algorithm="maxima")
```

output
```
log(e*x + 1)*log(-e*x) + dilog(e*x + 1)
```

3.75.8 Giac [F]

$$\int \frac{\log(1+ex)}{x}\,dx = \int \frac{\log(ex+1)}{x}\,dx$$

input
```
integrate(log(e*x+1)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x + 1)/x, x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{\log(1+ex)}{x}\,dx = \mathrm{Li}_2(-e\,x) + \ln(e\,x+1)\,\ln(-e\,x)$$

input
```
int(log(e*x + 1)/x,x)
```

output
```
dilog(-e*x) + log(e*x + 1)*log(-e*x)
```

3.76 $\int \frac{\log(ex)}{x} \, dx$

3.76.1 Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \frac{\log(ex)}{x} \, dx = \frac{1}{2} \log^2(ex)$$

output
```
1/2*ln(e*x)^2
```

3.76.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log(ex)}{x} \, dx = \frac{1}{2} \log^2(ex)$$

input
```
Integrate[Log[e*x]/x,x]
```

output
```
Log[e*x]^2/2
```

3.76.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex)}{x} \, dx$$
$$\Big\downarrow \; 2738$$
$$\frac{1}{2}\log^2(ex)$$

input
```
Int[Log[e*x]/x,x]
```

output
```
Log[e*x]^2/2
```

3.76.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

3.76.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\ln(ex)^2}{2}$	9
default	$\frac{\ln(ex)^2}{2}$	9
norman	$\frac{\ln(ex)^2}{2}$	9
risch	$\frac{\ln(ex)^2}{2}$	9
parts	$\ln(ex)\ln(x) - \frac{\ln(x)^2}{2}$	15

input
```
int(ln(e*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(e*x)^2
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log{(ex)}^2$$

input
```
integrate(log(e*x)/x,x, algorithm="fricas")
```

output
```
1/2*log(e*x)^2
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\log(ex)}{x}\,dx = \frac{\log{(ex)}^2}{2}$$

input
```
integrate(ln(e*x)/x,x)
```

output
```
log(e*x)**2/2
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log{(ex)}^2$$

input
```
integrate(log(e*x)/x,x, algorithm="maxima")
```

output
```
1/2*log(e*x)^2
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log{(ex)}^2$$

input
```
integrate(log(e*x)/x,x, algorithm="giac")
```

output
```
1/2*log(e*x)^2
```

3.76.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{\ln{(e\,x)}^2}{2}$$

input
```
int(log(e*x)/x,x)
```

output
```
log(e*x)^2/2
```

3.77 $\int \frac{\log(-1+ex)}{x}\,dx$

3.77.1 Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{\log(-1+ex)}{x}\,dx = \log(ex)\log(-1+ex) + \text{PolyLog}(2,1-ex)$$

output
```
ln(e*x)*ln(e*x-1)+polylog(2,-e*x+1)
```

3.77.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log(-1+ex)}{x}\,dx = \log(ex)\log(-1+ex) + \text{PolyLog}(2,1-ex)$$

input
```
Integrate[Log[-1 + e*x]/x,x]
```

output
```
Log[e*x]*Log[-1 + e*x] + PolyLog[2, 1 - e*x]
```

3.77.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2841, 25, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex-1)}{x}\,dx$$

$$\Big\downarrow \text{ 2841}$$

$$\log(ex)\log(ex-1)-e\int -\frac{\log(ex)}{1-ex}dx$$

$$\Big\downarrow \text{ 25}$$

$$e\int \frac{\log(ex)}{1-ex}dx+\log(ex)\log(ex-1)$$

$$\Big\downarrow \text{ 2752}$$

$$\text{PolyLog}(2,1-ex)+\log(ex)\log(ex-1)$$

input
```
Int[Log[-1 + e*x]/x,x]
```

output
```
Log[e*x]*Log[-1 + e*x] + PolyLog[2, 1 - e*x]
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.77.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\operatorname{dilog}(ex)+\ln(ex)\ln(ex-1)$	17
default	$\operatorname{dilog}(ex)+\ln(ex)\ln(ex-1)$	17
risch	$\operatorname{dilog}(ex)+\ln(ex)\ln(ex-1)$	17
parts	$\ln(ex-1)\ln(x)-e\left(\frac{(\ln(x)-\ln(ex))\ln(-ex+1)}{e}-\frac{\operatorname{dilog}(ex)}{e}\right)$	44

input
```
int(ln(e*x-1)/x,x,method=_RETURNVERBOSE)
```

output
```
dilog(e*x)+ln(e*x)*ln(e*x-1)
```

3.77.5 Fricas [F]

$$\int \frac{\log(-1+ex)}{x}\,dx = \int \frac{\log(ex-1)}{x}\,dx$$

input
```
integrate(log(e*x-1)/x,x, algorithm="fricas")
```

output
```
integral(log(e*x - 1)/x, x)
```

3.77.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.00

$$\int \frac{\log(-1+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2(ex) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ i\pi\log(x)-\operatorname{Li}_2(ex) & \text{for } |x|<1 \\ -i\pi\log\left(\frac{1}{x}\right)-\operatorname{Li}_2(ex) & \text{for } \frac{1}{|x|}<1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)-\operatorname{Li}_2(ex) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(e*x-1)/x,x)
```

output
```
Piecewise((-polylog(2, e*x), (Abs(x) < 1) & (1/Abs(x) < 1)), (I*pi*log(x)
- polylog(2, e*x), Abs(x) < 1), (-I*pi*log(1/x) - polylog(2, e*x), 1/Abs(x
) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*meijerg(((1,
1), ()), ((), (0, 0)), x) - polylog(2, e*x), True))
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log(-1+ex)}{x}\,dx = \log(ex-1)\log(ex) + \mathrm{Li}_2(-ex+1)$$

input
```
integrate(log(e*x-1)/x,x, algorithm="maxima")
```

output
```
log(e*x - 1)*log(e*x) + dilog(-e*x + 1)
```

3.77.8 Giac [F]

$$\int \frac{\log(-1+ex)}{x}\,dx = \int \frac{\log(ex-1)}{x}\,dx$$

input
```
integrate(log(e*x-1)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x - 1)/x, x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\log(-1+ex)}{x}\,dx = \mathrm{Li}_2(e\,x) + \ln(e\,x-1)\,\ln(e\,x)$$

input
```
int(log(e*x - 1)/x,x)
```

output
```
dilog(e*x) + log(e*x - 1)*log(e*x)
```

3.78 $\int \frac{\log(-2+ex)}{x}\,dx$

3.78.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{\log(-2+ex)}{x}\,dx = \log\left(\frac{ex}{2}\right)\log(-2+ex) + \text{PolyLog}\left(2, 1-\frac{ex}{2}\right)$$

output
```
ln(1/2*e*x)*ln(e*x-2)+polylog(2,1-1/2*e*x)
```

3.78.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\log(-2+ex)}{x}\,dx = \log\left(\frac{ex}{2}\right)\log(-2+ex) + \text{PolyLog}\left(2, \frac{1}{2}(2-ex)\right)$$

input
```
Integrate[Log[-2 + e*x]/x,x]
```

output
```
Log[(e*x)/2]*Log[-2 + e*x] + PolyLog[2, (2 - e*x)/2]
```

3.78.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2841, 25, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex-2)}{x} dx$$

$$\Big\downarrow \text{2841}$$

$$\log\left(\frac{ex}{2}\right)\log(ex-2) - e\int -\frac{\log\left(\frac{ex}{2}\right)}{2-ex}dx$$

$$\Big\downarrow \text{25}$$

$$e\int \frac{\log\left(\frac{ex}{2}\right)}{2-ex}dx + \log\left(\frac{ex}{2}\right)\log(ex-2)$$

$$\Big\downarrow \text{2752}$$

$$\text{PolyLog}\left(2, 1-\frac{ex}{2}\right) + \log\left(\frac{ex}{2}\right)\log(ex-2)$$

input
```
Int[Log[-2 + e*x]/x,x]
```

output
```
Log[(e*x)/2]*Log[-2 + e*x] + PolyLog[2, 1 - (e*x)/2]
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.78.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\operatorname{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln\left(ex-2\right)$	19
default	$\operatorname{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln\left(ex-2\right)$	19
risch	$\operatorname{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln\left(ex-2\right)$	19
parts	$\ln\left(ex-2\right)\ln\left(x\right)-e\left(\frac{\left(\ln(x)-\ln\left(\frac{ex}{2}\right)\right)\ln\left(1-\frac{ex}{2}\right)}{e}-\frac{\operatorname{dilog}\left(\frac{ex}{2}\right)}{e}\right)$	46

input
```
int(ln(e*x-2)/x,x,method=_RETURNVERBOSE)
```

output
```
dilog(1/2*e*x)+ln(1/2*e*x)*ln(e*x-2)
```

3.78.5 Fricas [F]

$$\int \frac{\log(-2+ex)}{x}\,dx=\int \frac{\log\left(ex-2\right)}{x}\,dx$$

input
```
integrate(log(e*x-2)/x,x, algorithm="fricas")
```

output
```
integral(log(e*x - 2)/x, x)
```

3.78.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.08

$$\int \frac{\log(-2+ex)}{x}\,dx$$
$$=\begin{cases}-\operatorname{Li}_2\left(\frac{ex}{2}\right)\\ \log\left(2\right)\log\left(x\right)+3i\pi\log\left(x\right)-\operatorname{Li}_2\left(\frac{ex}{2}\right)\\ -\log\left(2\right)\log\left(\frac{1}{x}\right)-3i\pi\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{ex}{2}\right)\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log\left(2\right)-3i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log\left(2\right)+3i\pi G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\right.\end{cases}$$

input
```
integrate(ln(e*x-2)/x,x)
```

output
```
Piecewise((-polylog(2, e*x/2), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(2)*log
(x) + 3*I*pi*log(x) - polylog(2, e*x/2), Abs(x) < 1), (-log(2)*log(1/x) -
3*I*pi*log(1/x) - polylog(2, e*x/2), 1/Abs(x) < 1), (-meijerg(((), (1, 1))
, ((0, 0), ()), x)*log(2) - 3*I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x)
+ meijerg(((1, 1), ()), ((), (0, 0)), x)*log(2) + 3*I*pi*meijerg(((1, 1),
()), ((), (0, 0)), x) - polylog(2, e*x/2), True))
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\log(-2+ex)}{x}\,dx = \log\left(ex-2\right)\log\left(\frac{1}{2}\,ex\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ex+1\right)$$

input
```
integrate(log(e*x-2)/x,x, algorithm="maxima")
```

output
```
log(e*x - 2)*log(1/2*e*x) + dilog(-1/2*e*x + 1)
```

3.78.8 Giac [F]

$$\int \frac{\log(-2+ex)}{x}\,dx = \int \frac{\log\left(ex-2\right)}{x}\,dx$$

input
```
integrate(log(e*x-2)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x - 2)/x, x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{\log(-2+ex)}{x}\,dx = \mathrm{Li}_2\left(\frac{e\,x}{2}\right) + \ln\left(e\,x-2\right)\,\ln\left(\frac{e\,x}{2}\right)$$

input
```
int(log(e*x - 2)/x,x)
```

output
```
dilog((e*x)/2) + log(e*x - 2)*log((e*x)/2)
```

3.79 $\int \frac{a+b\log(3+ex)}{x}\,dx$

3.79.1 Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{a+b\log(3+ex)}{x}\,dx = (a+b\log(3))\log(x)-b\,\text{PolyLog}\left(2,-\frac{ex}{3}\right)$$

output
```
(a+b*ln(3))*ln(x)-b*polylog(2,-1/3*e*x)
```

3.79.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(3+ex)}{x}\,dx = a\log(x)+b\log(3)\log(x)-b\,\text{PolyLog}\left(2,-\frac{ex}{3}\right)$$

input
```
Integrate[(a + b*Log[3 + e*x])/x,x]
```

output
```
a*Log[x] + b*Log[3]*Log[x] - b*PolyLog[2, -1/3*(e*x)]
```

3.79.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(ex+3)}{x}\,dx$$

$$\Big\downarrow \text{ 2839}$$

$$b\int \frac{\log\left(\frac{ex}{3}+1\right)}{x}dx+\log(x)(a+b\log(3))$$

$$\Big\downarrow \text{ 2838}$$

$$\log(x)(a+b\log(3))-b\,\text{PolyLog}\left(2,-\frac{ex}{3}\right)$$

input
```
Int[(a + b*Log[3 + e*x])/x,x]
```

output
```
(a + b*Log[3])*Log[x] - b*PolyLog[2, -1/3*(e*x)]
```

3.79.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result	size
parts	$\ln(x) a+b\left(\left(\ln(ex+3)-\ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right)-\operatorname{dilog}\left(\frac{ex}{3}+1\right)\right)$	40
derivativedivides	$a\ln(ex)+b\left(\left(\ln(ex+3)-\ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right)-\operatorname{dilog}\left(\frac{ex}{3}+1\right)\right)$	42
default	$a\ln(ex)+b\left(\left(\ln(ex+3)-\ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right)-\operatorname{dilog}\left(\frac{ex}{3}+1\right)\right)$	42
risch	$\ln(x) a+\ln(ex+3)\ln\left(-\frac{ex}{3}\right) b-\ln\left(\frac{ex}{3}+1\right)\ln\left(-\frac{ex}{3}\right) b-\operatorname{dilog}\left(\frac{ex}{3}+1\right) b$	44

input
```
int((a+b*ln(e*x+3))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a+b*((ln(e*x+3)-ln(1/3*e*x+1))*ln(-1/3*e*x)-dilog(1/3*e*x+1))
```

3.79.5 Fricas [F]

$$\int \frac{a+b\log(3+ex)}{x}\,dx=\int \frac{b\log(ex+3)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x+3))/x,x, algorithm="fricas")
```

output
```
integral((b*log(e*x + 3) + a)/x, x)
```

3.79.6 Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.48

$$\int \frac{a+b\log(3+ex)}{x}\,dx = a\log(x) + b\left(\begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(3)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } |x|<1 \\ -\log(3)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(3) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(3) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*ln(e*x+3))/x,x)
```

output
```
a*log(x) + b*Piecewise((-polylog(2, e*x*exp_polar(I*pi)/3), (Abs(x) < 1) &
 (1/Abs(x) < 1)), (log(3)*log(x) - polylog(2, e*x*exp_polar(I*pi)/3), Abs(
x) < 1), (-log(3)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/3), 1/Abs(x) <
 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(3) + meijerg(((1, 1), ()
), ((), (0, 0)), x)*log(3) - polylog(2, e*x*exp_polar(I*pi)/3), True))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{a+b\log(3+ex)}{x}\,dx = \left(\log(ex+3)\log\left(-\frac{1}{3}ex\right) + \operatorname{Li}_2\left(\frac{1}{3}ex+1\right)\right)b + a\log(x)$$

input
```
integrate((a+b*log(e*x+3))/x,x, algorithm="maxima")
```

output
```
(log(e*x + 3)*log(-1/3*e*x) + dilog(1/3*e*x + 1))*b + a*log(x)
```

3.79.8 Giac [F]

$$\int \frac{a+b\log(3+ex)}{x}\,dx=\int \frac{b\log\left(ex+3\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x+3))/x,x, algorithm="giac")
```

output
```
integrate((b*log(e*x + 3) + a)/x, x)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(3+ex)}{x}\,dx=b\,\mathrm{Li}_2\left(-\frac{e\,x}{3}\right)+a\,\ln\left(x\right)+b\,\ln\left(e\,x+3\right)\,\ln\left(-\frac{e\,x}{3}\right)$$

input
```
int((a + b*log(e*x + 3))/x,x)
```

output
```
b*dilog(-(e*x)/3) + a*log(x) + b*log(e*x + 3)*log(-(e*x)/3)
```

3.80 $\int \frac{a+b\log(2+ex)}{x}\,dx$

3.80.1 Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{a+b\log(2+ex)}{x}\,dx=(a+b\log(2))\log(x)-b\,\text{PolyLog}\left(2,-\frac{ex}{2}\right)$$

output
```
(a+b*ln(2))*ln(x)-b*polylog(2,-1/2*e*x)
```

3.80.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(2+ex)}{x}\,dx=a\log(x)+b\log(2)\log(x)-b\,\text{PolyLog}\left(2,-\frac{ex}{2}\right)$$

input
```
Integrate[(a + b*Log[2 + e*x])/x,x]
```

output
```
a*Log[x] + b*Log[2]*Log[x] - b*PolyLog[2, -1/2*(e*x)]
```

3.80.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(ex+2)}{x}\,dx$$

$$\Big\downarrow \ 2839$$

$$b\int \frac{\log\left(\frac{ex}{2}+1\right)}{x}dx+\log(x)(a+b\log(2))$$

$$\Big\downarrow \ 2838$$

$$\log(x)(a+b\log(2))-b\,\text{PolyLog}\left(2,-\frac{ex}{2}\right)$$

input
```
Int[(a + b*Log[2 + e*x])/x,x]
```

output
```
(a + b*Log[2])*Log[x] - b*PolyLog[2, -1/2*(e*x)]
```

3.80.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result	size
parts	$\ln(x) a + b\left(\left(\ln(ex+2) - \ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right) - \operatorname{dilog}\left(\frac{ex}{2}+1\right)\right)$	40
derivativedivides	$a\ln(ex) + b\left(\left(\ln(ex+2) - \ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right) - \operatorname{dilog}\left(\frac{ex}{2}+1\right)\right)$	42
default	$a\ln(ex) + b\left(\left(\ln(ex+2) - \ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right) - \operatorname{dilog}\left(\frac{ex}{2}+1\right)\right)$	42
risch	$\ln(x) a + \ln(ex+2)\ln\left(-\frac{ex}{2}\right) b - \ln\left(\frac{ex}{2}+1\right)\ln\left(-\frac{ex}{2}\right) b - \operatorname{dilog}\left(\frac{ex}{2}+1\right) b$	44

input
```
int((a+b*ln(e*x+2))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a+b*((ln(e*x+2)-ln(1/2*e*x+1))*ln(-1/2*e*x)-dilog(1/2*e*x+1))
```

3.80.5 Fricas [F]

$$\int \frac{a + b\log(2+ex)}{x}\,dx = \int \frac{b\log(ex+2) + a}{x}\,dx$$

input
```
integrate((a+b*log(e*x+2))/x,x, algorithm="fricas")
```

output
```
integral((b*log(e*x + 2) + a)/x, x)
```

3.80.6 Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.48

$$\int \frac{a + b\log(2 + ex)}{x}\,dx = a\log(x) + b\left(\begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(2)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } |x| < 1 \\ -\log(2)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(2) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(2) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*ln(e*x+2))/x,x)
```

output
```
a*log(x) + b*Piecewise((-polylog(2, e*x*exp_polar(I*pi)/2), (Abs(x) < 1) &
 (1/Abs(x) < 1)), (log(2)*log(x) - polylog(2, e*x*exp_polar(I*pi)/2), Abs(
x) < 1), (-log(2)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/2), 1/Abs(x) <
 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ()
), ((), (0, 0)), x)*log(2) - polylog(2, e*x*exp_polar(I*pi)/2), True))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{a + b\log(2 + ex)}{x}\,dx = \left(\log(ex + 2)\log\left(-\frac{1}{2}ex\right) + \operatorname{Li}_2\left(\frac{1}{2}ex + 1\right)\right)b + a\log(x)$$

input
```
integrate((a+b*log(e*x+2))/x,x, algorithm="maxima")
```

output
```
(log(e*x + 2)*log(-1/2*e*x) + dilog(1/2*e*x + 1))*b + a*log(x)
```

3.80.8 Giac [F]

$$\int \frac{a+b\log(2+ex)}{x}\,dx = \int \frac{b\log(ex+2)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x+2))/x,x, algorithm="giac")
```

output
```
integrate((b*log(e*x + 2) + a)/x, x)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(2+ex)}{x}\,dx = b\,\mathrm{Li}_2\left(-\frac{e\,x}{2}\right) + a\,\ln(x) + b\,\ln(e\,x+2)\,\ln\left(-\frac{e\,x}{2}\right)$$

input
```
int((a + b*log(e*x + 2))/x,x)
```

output
```
b*dilog(-(e*x)/2) + a*log(x) + b*log(e*x + 2)*log(-(e*x)/2)
```

3.81 $\int \frac{a+b\log(1+ex)}{x}\,dx$

3.81.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\mathrm{PolyLog}(2,-ex)$$

output
```
a*ln(x)-b*polylog(2,-e*x)
```

3.81.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\mathrm{PolyLog}(2,-ex)$$

input
```
Integrate[(a + b*Log[1 + e*x])/x,x]
```

output
```
a*Log[x] - b*PolyLog[2, -(e*x)]
```

3.81.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(ex+1)}{x}\,dx$$

$$\Big\downarrow \text{2839}$$

$$b\int \frac{\log(ex+1)}{x}dx + a\log(x)$$

$$\Big\downarrow \text{2838}$$

$$a\log(x) - b\,\text{PolyLog}(2,-ex)$$

input
```
Int[(a + b*Log[1 + e*x])/x,x]
```

output
```
a*Log[x] - b*PolyLog[2, -(e*x)]
```

3.81.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

3.81.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
risch	$\ln(x)a - b\,\mathrm{dilog}(ex+1)$	15
parts	$\ln(x)a - b\,\mathrm{dilog}(ex+1)$	15
derivativedivides	$a\ln(ex) - b\,\mathrm{dilog}(ex+1)$	17
default	$a\ln(ex) - b\,\mathrm{dilog}(ex+1)$	17

input
```
int((a+b*ln(e*x+1))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a-b*dilog(e*x+1)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log(1+ex)}{x}\,dx = -b\mathrm{Li}_2(-ex) + a\log(x)$$

input
```
integrate((a+b*log(e*x+1))/x,x, algorithm="fricas")
```

output
```
-b*dilog(-e*x) + a*log(x)
```

3.81.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\mathrm{Li}_2\left(exe^{i\pi}\right)$$

input
```
integrate((a+b*ln(e*x+1))/x,x)
```

output
```
a*log(x) - b*polylog(2, e*x*exp_polar(I*pi))
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{a+b\log(1+ex)}{x}\,dx = (\log(ex+1)\log(-ex) + \mathrm{Li}_2(ex+1))b + a\log(x)$$

input
```
integrate((a+b*log(e*x+1))/x,x, algorithm="maxima")
```

output
```
(log(e*x + 1)*log(-e*x) + dilog(e*x + 1))*b + a*log(x)
```

3.81.8 Giac [F]

$$\int \frac{a+b\log(1+ex)}{x}\,dx = \int \frac{b\log(ex+1)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x+1))/x,x, algorithm="giac")
```

output
```
integrate((b*log(e*x + 1) + a)/x, x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\,\ln(x) - b\,\mathrm{polylog}(2,-e\,x)$$

input
```
int((a + b*log(e*x + 1))/x,x)
```

output
```
a*log(x) - b*polylog(2, -e*x)
```

3.82 $\int \frac{a+b\log(ex)}{x}\,dx$

3.82.1 Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{(a+b\log(ex))^2}{2b}$$

output
```
1/2*(a+b*ln(e*x))^2/b
```

3.82.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log(ex)}{x}\,dx = a\log(x) + \frac{1}{2}b\log^2(ex)$$

input
```
Integrate[(a + b*Log[e*x])/x,x]
```

output
```
a*Log[x] + (b*Log[e*x]^2)/2
```

3.82.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(ex)}{x}\,dx$$

$$\Big\downarrow \text{ 2738}$$

$$\frac{(a+b\log(ex))^2}{2b}$$

input
```
Int[(a + b*Log[e*x])/x,x]
```

output
```
(a + b*Log[e*x])^2/(2*b)
```

3.82.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

3.82.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{b\ln(ex)^2}{2}+\ln\left(x\right)a$	15
parts	$\frac{b\ln(ex)^2}{2}+\ln\left(x\right)a$	15
derivativedivides	$\frac{b\ln(ex)^2}{2}+a\ln\left(ex\right)$	17
default	$\frac{b\ln(ex)^2}{2}+a\ln\left(ex\right)$	17
norman	$\frac{b\ln(ex)^2}{2}+a\ln\left(ex\right)$	17
parallelrisch	$\frac{b\ln(ex)^2}{2}+a\ln\left(ex\right)$	17

input
```
int((a+b*ln(e*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln(e*x)^2+ln(x)*a
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{1}{2}\,b\log\left(ex\right)^2 + a\log\left(ex\right)$$

input
```
integrate((a+b*log(e*x))/x,x, algorithm="fricas")
```

output
```
1/2*b*log(e*x)^2 + a*log(e*x)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(ex)}{x}\,dx = a\log\left(x\right) + \frac{b\log\left(ex\right)^2}{2}$$

input
```
integrate((a+b*ln(e*x))/x,x)
```

output
```
a*log(x) + b*log(e*x)**2/2
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{\left(b\log\left(ex\right)+a\right)^2}{2\,b}$$

input
```
integrate((a+b*log(e*x))/x,x, algorithm="maxima")
```

output
```
1/2*(b*log(e*x) + a)^2/b
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{1}{2}\,b\log{(ex)}^2 + a\log{(x)}$$

input
```
integrate((a+b*log(e*x))/x,x, algorithm="giac")
```

output
```
1/2*b*log(e*x)^2 + a*log(x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{b\ln{(e\,x)}^2}{2} + a\,\ln{(x)}$$

input
```
int((a + b*log(e*x))/x,x)
```

output
```
a*log(x) + (b*log(e*x)^2)/2
```

3.83 $\int \frac{a+b\log(-1+ex)}{x}\,dx$

3.83.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = \log(ex)(a+b\log(-1+ex))+b\,\mathrm{PolyLog}(2,1-ex)$$

output
```
ln(e*x)*(a+b*ln(e*x-1))+b*polylog(2,-e*x+1)
```

3.83.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = a\log(x)+b\log(ex)\log(-1+ex)+b\,\mathrm{PolyLog}(2,1-ex)$$

input
```
Integrate[(a + b*Log[-1 + e*x])/x,x]
```

output
```
a*Log[x] + b*Log[e*x]*Log[-1 + e*x] + b*PolyLog[2, 1 - e*x]
```

3.83.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2841, 25, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(ex-1)}{x}dx$$

$$\Big\downarrow \; 2841$$

$$\log(ex)(a+b\log(ex-1))-be\int -\frac{\log(ex)}{1-ex}dx$$

$$\Big\downarrow \; 25$$

$$be\int \frac{\log(ex)}{1-ex}dx+\log(ex)(a+b\log(ex-1))$$

$$\Big\downarrow \; 2752$$

$$\log(ex)(a+b\log(ex-1))+b\,\text{PolyLog}(2,1-ex)$$

input
```
Int[(a + b*Log[-1 + e*x])/x,x]
```

output
```
Log[e*x]*(a + b*Log[-1 + e*x]) + b*PolyLog[2, 1 - e*x]
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.83.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
risch	$\ln(x) a+\ln(ex-1) \ln(ex) b+\operatorname{dilog}(ex) b$	24
parts	$\ln(x) a+b(\operatorname{dilog}(ex)+\ln(ex) \ln(ex-1))$	24
derivativedivides	$a \ln(ex)+b(\operatorname{dilog}(ex)+\ln(ex) \ln(ex-1))$	26
default	$a \ln(ex)+b(\operatorname{dilog}(ex)+\ln(ex) \ln(ex-1))$	26

input
```
int((a+b*ln(e*x-1))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a+ln(e*x-1)*ln(e*x)*b+dilog(e*x)*b
```

3.83.5 Fricas [F]

$$\int \frac{a+b\log(-1+ex)}{x}\,dx=\int \frac{b\log(ex-1)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x-1))/x,x, algorithm="fricas")
```

output
```
integral((b*log(e*x - 1) + a)/x, x)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = a\log(x) + b\left(\begin{cases} -\operatorname{Li}_2(ex) & \text{for } \frac{1}{|x|}<1\wedge|x|<1 \\ i\pi\log(x)-\operatorname{Li}_2(ex) & \text{for } |x|<1 \\ -i\pi\log\left(\frac{1}{x}\right)-\operatorname{Li}_2(ex) & \text{for } \frac{1}{|x|}<1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)-\operatorname{Li}_2(ex) & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*ln(e*x-1))/x,x)
```

output
```
a*log(x) + b*Piecewise((-polylog(2, e*x), (Abs(x) < 1) & (1/Abs(x) < 1)),
(I*pi*log(x) - polylog(2, e*x), Abs(x) < 1), (-I*pi*log(1/x) - polylog(2,
e*x), 1/Abs(x) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*
meijerg(((1, 1), ()), ((), (0, 0)), x) - polylog(2, e*x), True))
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = (\log(ex-1)\log(ex)+\mathrm{Li}_2(-ex+1))b + a\log(x)$$

input
```
integrate((a+b*log(e*x-1))/x,x, algorithm="maxima")
```

output
```
(log(e*x - 1)*log(e*x) + dilog(-e*x + 1))*b + a*log(x)
```

3.83.8 Giac [F]

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = \int \frac{b\log(ex-1)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x-1))/x,x, algorithm="giac")
```

output
```
integrate((b*log(e*x - 1) + a)/x, x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = b\,\mathrm{Li}_2(e\,x)+a\,\ln(x)+b\,\ln(e\,x-1)\,\ln(e\,x)$$

input
```
int((a + b*log(e*x - 1))/x,x)
```

output
```
b*dilog(e*x) + a*log(x) + b*log(e*x - 1)*log(e*x)
```

3.84 $\int \frac{a+b\log(-2+ex)}{x}\,dx$

3.84.1 Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = \log\left(\frac{ex}{2}\right)(a+b\log(-2+ex))+b\,\text{PolyLog}\left(2,1-\frac{ex}{2}\right)$$

output
```
ln(1/2*e*x)*(a+b*ln(e*x-2))+b*polylog(2,1-1/2*e*x)
```

3.84.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = a\log(x)+b\log\left(\frac{ex}{2}\right)\log(-2+ex)+b\,\text{PolyLog}\left(2,\frac{1}{2}(2-ex)\right)$$

input
```
Integrate[(a + b*Log[-2 + e*x])/x,x]
```

output
```
a*Log[x] + b*Log[(e*x)/2]*Log[-2 + e*x] + b*PolyLog[2, (2 - e*x)/2]
```

3.84.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2841, 25, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(ex-2)}{x}dx$$

$$\Big\downarrow \text{2841}$$

$$\log\left(\frac{ex}{2}\right)(a+b\log(ex-2))-be\int -\frac{\log\left(\frac{ex}{2}\right)}{2-ex}dx$$

$$\Big\downarrow \text{25}$$

$$be\int \frac{\log\left(\frac{ex}{2}\right)}{2-ex}dx+\log\left(\frac{ex}{2}\right)(a+b\log(ex-2))$$

$$\Big\downarrow \text{2752}$$

$$\log\left(\frac{ex}{2}\right)(a+b\log(ex-2))+b\,\text{PolyLog}\left(2,1-\frac{ex}{2}\right)$$

input
```
Int[(a + b*Log[-2 + e*x])/x,x]
```

output
```
Log[(e*x)/2]*(a + b*Log[-2 + e*x]) + b*PolyLog[2, 1 - (e*x)/2]
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
risch	$\ln(x) a+\ln(ex-2) \ln\left(\frac{ex}{2}\right) b+\operatorname{dilog}\left(\frac{ex}{2}\right) b$	26
parts	$\ln(x) a+b\left(\operatorname{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right) \ln(ex-2)\right)$	26
derivativedivides	$a \ln(ex)+b\left(\operatorname{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right) \ln(ex-2)\right)$	28
default	$a \ln(ex)+b\left(\operatorname{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right) \ln(ex-2)\right)$	28

input
```
int((a+b*ln(e*x-2))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a+ln(e*x-2)*ln(1/2*e*x)*b+dilog(1/2*e*x)*b
```

3.84.5 Fricas [F]

$$\int \frac{a+b\log(-2+ex)}{x}\,dx=\int \frac{b\log(ex-2)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x-2))/x,x, algorithm="fricas")
```

output
```
integral((b*log(e*x - 2) + a)/x, x)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{a+b\log(-2+ex)}{x}\,dx=a\log(x)$$
$$+b\left(\begin{cases}-\operatorname{Li}_2\left(\frac{ex}{2}\right)\\ \log(2)\log(x)+3i\pi\log(x)-\operatorname{Li}_2\left(\frac{ex}{2}\right)\\ -\log(2)\log\left(\frac{1}{x}\right)-3i\pi\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{ex}{2}\right)\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(2)-3i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(2)+3i\pi G_{2,2}^{0,2}\left(\begin{matrix}1,1\end{matrix}\right.\end{cases}\right.$$

input
```
integrate((a+b*ln(e*x-2))/x,x)
```

output
```
a*log(x) + b*Piecewise((-polylog(2, e*x/2), (Abs(x) < 1) & (1/Abs(x) < 1))
, (log(2)*log(x) + 3*I*pi*log(x) - polylog(2, e*x/2), Abs(x) < 1), (-log(2
)*log(1/x) - 3*I*pi*log(1/x) - polylog(2, e*x/2), 1/Abs(x) < 1), (-meijerg
(((), (1, 1)), ((0, 0), ()), x)*log(2) - 3*I*pi*meijerg(((), (1, 1)), ((0,
 0), ()), x) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(2) + 3*I*pi*meij
erg(((1, 1), ()), ((), (0, 0)), x) - polylog(2, e*x/2), True))
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = \left(\log\left(ex-2\right)\log\left(\frac{1}{2}\,ex\right) + \mathrm{Li}_2\left(-\frac{1}{2}\,ex+1\right)\right)b + a\log\left(x\right)$$

input
```
integrate((a+b*log(e*x-2))/x,x, algorithm="maxima")
```

output
```
(log(e*x - 2)*log(1/2*e*x) + dilog(-1/2*e*x + 1))*b + a*log(x)
```

3.84.8 Giac [F]

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = \int \frac{b\log\left(ex-2\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(e*x-2))/x,x, algorithm="giac")
```

output
```
integrate((b*log(e*x - 2) + a)/x, x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = b\,\mathrm{Li}_2\left(\frac{e\,x}{2}\right)+a\,\ln\left(x\right)+b\,\ln\left(e\,x-2\right)\,\ln\left(\frac{e\,x}{2}\right)$$

input
```
int((a + b*log(e*x - 2))/x,x)
```

output
```
b*dilog((e*x)/2) + a*log(x) + b*log(e*x - 2)*log((e*x)/2)
```

3.85 $\int x^2 \log^2(c(a+bx)^n)\,dx$

3.85.1 Optimal result

Integrand size = 16, antiderivative size = 187

$$\begin{aligned}\int x^2 \log^2(c(a+bx)^n)\,dx = {} & \frac{2a^2n^2x}{b^2} - \frac{an^2(a+bx)^2}{2b^3} + \frac{2n^2(a+bx)^3}{27b^3} \\ & - \frac{a^3n^2\log^2(a+bx)}{3b^3} - \frac{2a^2n(a+bx)\log(c(a+bx)^n)}{b^3} \\ & + \frac{an(a+bx)^2\log(c(a+bx)^n)}{b^3} - \frac{2n(a+bx)^3\log(c(a+bx)^n)}{9b^3} \\ & + \frac{2a^3n\log(a+bx)\log(c(a+bx)^n)}{3b^3} + \frac{1}{3}x^3\log^2(c(a+bx)^n)\end{aligned}$$

output
```
2*a^2*n^2*x/b^2-1/2*a*n^2*(b*x+a)^2/b^3+2/27*n^2*(b*x+a)^3/b^3-1/3*a^3*n^2
*ln(b*x+a)^2/b^3-2*a^2*n*(b*x+a)*ln(c*(b*x+a)^n)/b^3+a*n*(b*x+a)^2*ln(c*(b
*x+a)^n)/b^3-2/9*n*(b*x+a)^3*ln(c*(b*x+a)^n)/b^3+2/3*a^3*n*ln(b*x+a)*ln(c*
(b*x+a)^n)/b^3+1/3*x^3*ln(c*(b*x+a)^n)^2
```

3.85.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.87

$$\int x^2 \log^2\left(c(a+bx)^n\right) dx = \frac{11a^2n^2x}{9b^2} - \frac{5an^2x^2}{18b} + \frac{2n^2x^3}{27} - \frac{11a^3n\log\left(c(a+bx)^n\right)}{9b^3} - \frac{2a^2nx\log\left(c(a+bx)^n\right)}{3b^2} + \frac{anx^2\log\left(c(a+bx)^n\right)}{3b} - \frac{2}{9}nx^3\log\left(c(a+bx)^n\right) + \frac{a^3\log^2\left(c(a+bx)^n\right)}{3b^3} + \frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

input
```
Integrate[x^2*Log[c*(a + b*x)^n]^2,x]
```

output
```
(11*a^2*n^2*x)/(9*b^2) - (5*a*n^2*x^2)/(18*b) + (2*n^2*x^3)/27 - (11*a^3*n
*Log[c*(a + b*x)^n])/(9*b^3) - (2*a^2*n*x*Log[c*(a + b*x)^n])/(3*b^2) + (a
*n*x^2*Log[c*(a + b*x)^n])/(3*b) - (2*n*x^3*Log[c*(a + b*x)^n])/9 + (a^3*L
og[c*(a + b*x)^n]^2)/(3*b^3) + (x^3*Log[c*(a + b*x)^n]^2)/3
```

3.85.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^2\left(c(a+bx)^n\right) dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right) - \frac{2}{3}bn\int \frac{x^3\log\left(c(a+bx)^n\right)}{a+bx}dx$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right) - \frac{2}{3}n\int \frac{x^3\log\left(c(a+bx)^n\right)}{a+bx}d(a+bx)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{3}n\int -\frac{x^3\log\left(c(a+bx)^n\right)}{a+bx}d(a+bx)+\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2n\int -\frac{b^3x^3\log(c(a+bx)^n)}{a+bx}d(a+bx)}{3b^3}+\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

$$\Big\downarrow \text{2772}$$

$$\frac{2n\left(-n\int\left(\frac{\log(a+bx)a^3}{a+bx}-3a^2+\frac{3}{2}(a+bx)a-\frac{1}{3}(a+bx)^2\right)d(a+bx)+a^3\log(a+bx)\log\left(c(a+bx)^n\right)-3a^2(a+bx}{3b^3}$$

$$\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2n\left(a^3\log(a+bx)\log\left(c(a+bx)^n\right)-3a^2(a+bx)\log\left(c(a+bx)^n\right)-n\left(\frac{1}{2}a^3\log^2(a+bx)-3a^2(a+bx)+\frac{3}{4}a(a+bx\right.\right.}{3b^3}$$

$$\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

input
```
Int[x^2*Log[c*(a + b*x)^n]^2,x]
```

output
```
(x^3*Log[c*(a + b*x)^n]^2)/3 + (2*n*(-(n*(-3*a^2*(a + b*x) + (3*a*(a + b*x
)^2)/4 - (a + b*x)^3/9 + (a^3*Log[a + b*x]^2)/2)) - 3*a^2*(a + b*x)*Log[c*
(a + b*x)^n] + (3*a*(a + b*x)^2*Log[c*(a + b*x)^n])/2 - ((a + b*x)^3*Log[c
*(a + b*x)^n])/3 + a^3*Log[a + b*x]*Log[c*(a + b*x)^n]))/(3*b^3)
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.85.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{-18x^3\ln(c(bx+a)^n)^2b^3+12x^3\ln(c(bx+a)^n)b^3n-4b^3n^2x^3-18x^2\ln(c(bx+a)^n)a\,b^2n+15a\,b^2n^2x^2+102\ln(bx+a)a^3n^2+36x}{54b^3}$
risch	Expression too large to display

input
```
int(x^2*ln(c*(b*x+a)^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-18*x^3*ln(c*(b*x+a)^n)^2*b^3+12*x^3*ln(c*(b*x+a)^n)*b^3*n-4*b^3*n^
2*x^3-18*x^2*ln(c*(b*x+a)^n)*a*b^2*n+15*a*b^2*n^2*x^2+102*ln(b*x+a)*a^3*n^
2+36*x*ln(c*(b*x+a)^n)*a^2*b*n-66*a^2*b*n^2*x-18*ln(c*(b*x+a)^n)^2*a^3-36*
ln(c*(b*x+a)^n)*a^3*n+66*a^3*n^2)/b^3
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.96

$$\int x^2 \log^2 (c(a + bx)^n) \, dx$$
$$= \frac{4 b^3 n^2 x^3 + 18 b^3 x^3 \log (c)^2 - 15 a b^2 n^2 x^2 + 66 a^2 b n^2 x + 18 (b^3 n^2 x^3 + a^3 n^2) \log (bx + a)^2 - 6 (2 b^3 n^2 x^3 - 3 a}{}$$

input
```
integrate(x^2*log(c*(b*x+a)^n)^2,x, algorithm="fricas")
```

output
```
1/54*(4*b^3*n^2*x^3 + 18*b^3*x^3*log(c)^2 - 15*a*b^2*n^2*x^2 + 66*a^2*b*n^
2*x + 18*(b^3*n^2*x^3 + a^3*n^2)*log(b*x + a)^2 - 6*(2*b^3*n^2*x^3 - 3*a*b
^2*n^2*x^2 + 6*a^2*b*n^2*x + 11*a^3*n^2 - 6*(b^3*n*x^3 + a^3*n)*log(c))*lo
g(b*x + a) - 6*(2*b^3*n*x^3 - 3*a*b^2*n*x^2 + 6*a^2*b*n*x)*log(c))/b^3
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int x^2 \log^2 (c(a + bx)^n) \, dx$$
$$= \begin{cases} -\frac{11a^3 n \log (c(a+bx)^n)}{9b^3} + \frac{a^3 \log (c(a+bx)^n)^2}{3b^3} + \frac{11a^2 n^2 x}{9b^2} - \frac{2a^2 n x \log (c(a+bx)^n)}{3b^2} - \frac{5an^2 x^2}{18b} + \frac{anx^2 \log (c(a+bx)^n)}{3b} + \frac{2n^2 x^3}{27} - \frac{2}{} \\ \frac{x^3 \log (a^n c)^2}{3} \end{cases}$$

input
```
integrate(x**2*ln(c*(b*x+a)**n)**2,x)
```

output
```
Piecewise((-11*a**3*n*log(c*(a + b*x)**n)/(9*b**3) + a**3*log(c*(a + b*x)*
*n)**2/(3*b**3) + 11*a**2*n**2*x/(9*b**2) - 2*a**2*n*x*log(c*(a + b*x)**n)
/(3*b**2) - 5*a*n**2*x**2/(18*b) + a*n*x**2*log(c*(a + b*x)**n)/(3*b) + 2*
n**2*x**3/27 - 2*n*x**3*log(c*(a + b*x)**n)/9 + x**3*log(c*(a + b*x)**n)**
2/3, Ne(b, 0)), (x**3*log(a**n*c)**2/3, True))
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int x^2 \log^2 \left(c(a+bx)^n\right)\, dx = \frac{1}{3}\, x^3 \log\left((bx+a)^n c\right)^2 + \frac{1}{9}\, bn\left(\frac{6\, a^3 \log\left(bx+a\right)}{b^4} - \frac{2\, b^2 x^3 - 3\, abx^2 + 6\, a^2 x}{b^3}\right) \log\left((bx+a)^n c\right) + \frac{\left(4\, b^3 x^3 - 15\, ab^2 x^2 - 18\, a^3 \log\left(bx+a\right)^2 + 66\, a^2 bx - 66\, a^3 \log\left(bx+a\right)\right) n^2}{54\, b^3}$$

input
```
integrate(x^2*log(c*(b*x+a)^n)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*log((b*x + a)^n*c)^2 + 1/9*b*n*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^
3 - 3*a*b*x^2 + 6*a^2*x)/b^3)*log((b*x + a)^n*c) + 1/54*(4*b^3*x^3 - 15*a*
b^2*x^2 - 18*a^3*log(b*x + a)^2 + 66*a^2*b*x - 66*a^3*log(b*x + a))*n^2/b^
3
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.83

$$\int x^2 \log^2 \left(c(a+bx)^n\right)\, dx = \frac{(bx+a)^3 n^2 \log\left(bx+a\right)^2}{3\, b^3} - \frac{(bx+a)^2 an^2 \log\left(bx+a\right)^2}{b^3} + \frac{(bx+a)a^2 n^2 \log\left(bx+a\right)^2}{b^3} - \frac{2\,(bx+a)^3 n^2 \log\left(bx+a\right)}{9\, b^3} + \frac{(bx+a)^2 an^2 \log\left(bx+a\right)}{b^3} - \frac{2\,(bx+a)a^2 n^2 \log\left(bx+a\right)}{b^3} + \frac{2\,(bx+a)^3 n \log\left(bx+a\right) \log\left(c\right)}{3\, b^3} - \frac{2\,(bx+a)^2 an \log\left(bx+a\right) \log\left(c\right)}{b^3} + \frac{2\,(bx+a)a^2 n \log\left(bx+a\right) \log\left(c\right)}{b^3} + \frac{2\,(bx+a)^3 n^2}{27\, b^3} - \frac{(bx+a)^2 an^2}{2\, b^3} + \frac{2\,(bx+a)a^2 n^2}{b^3} - \frac{2\,(bx+a)^3 n \log\left(c\right)}{9\, b^3} + \frac{(bx+a)^2 an \log\left(c\right)}{b^3} - \frac{2\,(bx+a)a^2 n \log\left(c\right)}{b^3} + \frac{(bx+a)^3 \log\left(c\right)^2}{3\, b^3} - \frac{(bx+a)^2 a \log\left(c\right)^2}{b^3} + \frac{(bx+a)a^2 \log\left(c\right)^2}{b^3}$$

input
```
integrate(x^2*log(c*(b*x+a)^n)^2,x, algorithm="giac")
```

output
```
1/3*(b*x + a)^3*n^2*log(b*x + a)^2/b^3 - (b*x + a)^2*a*n^2*log(b*x + a)^2/
b^3 + (b*x + a)*a^2*n^2*log(b*x + a)^2/b^3 - 2/9*(b*x + a)^3*n^2*log(b*x +
 a)/b^3 + (b*x + a)^2*a*n^2*log(b*x + a)/b^3 - 2*(b*x + a)*a^2*n^2*log(b*x
 + a)/b^3 + 2/3*(b*x + a)^3*n*log(b*x + a)*log(c)/b^3 - 2*(b*x + a)^2*a*n*
log(b*x + a)*log(c)/b^3 + 2*(b*x + a)*a^2*n*log(b*x + a)*log(c)/b^3 + 2/27
*(b*x + a)^3*n^2/b^3 - 1/2*(b*x + a)^2*a*n^2/b^3 + 2*(b*x + a)*a^2*n^2/b^3
 - 2/9*(b*x + a)^3*n*log(c)/b^3 + (b*x + a)^2*a*n*log(c)/b^3 - 2*(b*x + a)
*a^2*n*log(c)/b^3 + 1/3*(b*x + a)^3*log(c)^2/b^3 - (b*x + a)^2*a*log(c)^2/
b^3 + (b*x + a)*a^2*log(c)^2/b^3
```

3.85.9 Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

$$\int x^2 \log^2\left(c(a+bx)^n\right)\,dx = \frac{2\,n^2\,x^3}{27} + \ln\left(c\,(a+b\,x)^n\right)^2\left(\frac{x^3}{3}+\frac{a^3}{3\,b^3}\right) - \ln\left(c\,(a+b\,x)^n\right)\left(\frac{2\,n\,x^3}{9}-\frac{a\,n\,x^2}{3\,b}+\frac{2\,a^2\,n\,x}{3\,b^2}\right) - \frac{11\,a^3\,n^2\,\ln\left(a+b\,x\right)}{9\,b^3} - \frac{5\,a\,n^2\,x^2}{18\,b} + \frac{11\,a^2\,n^2\,x}{9\,b^2}$$

input
```
int(x^2*log(c*(a + b*x)^n)^2,x)
```

output
```
(2*n^2*x^3)/27 + log(c*(a + b*x)^n)^2*(x^3/3 + a^3/(3*b^3)) - log(c*(a + b
*x)^n)*((2*n*x^3)/9 - (a*n*x^2)/(3*b) + (2*a^2*n*x)/(3*b^2)) - (11*a^3*n^2
*log(a + b*x))/(9*b^3) - (5*a*n^2*x^2)/(18*b) + (11*a^2*n^2*x)/(9*b^2)
```

3.86 $\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx$

3.86.1 Optimal result

Integrand size = 16, antiderivative size = 177

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx = -\frac{b^2n^2}{3a^2x}-\frac{b^3n^2\log(x)}{a^3}+\frac{b^3n^2\log(a+bx)}{3a^3}-\frac{bn\log(c(a+bx)^n)}{3ax^2}+\frac{2b^2n(a+bx)\log(c(a+bx)^n)}{3a^3x}-\frac{\log^2(c(a+bx)^n)}{3x^3}+\frac{2b^3n\log(c(a+bx)^n)\log\left(1-\frac{a}{a+bx}\right)}{3a^3}-\frac{2b^3n^2\,\mathrm{PolyLog}\left(2,\frac{a}{a+bx}\right)}{3a^3}$$

output
```
-1/3*b^2*n^2/a^2/x-b^3*n^2*ln(x)/a^3+1/3*b^3*n^2*ln(b*x+a)/a^3-1/3*b*n*ln(
c*(b*x+a)^n)/a/x^2+2/3*b^2*n*(b*x+a)*ln(c*(b*x+a)^n)/a^3/x-1/3*ln(c*(b*x+a
)^n)^2/x^3+2/3*b^3*n*ln(c*(b*x+a)^n)*ln(1-a/(b*x+a))/a^3-2/3*b^3*n^2*polyl
og(2,a/(b*x+a))/a^3
```

3.86.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx = -\frac{ab^2n^2x^2+3b^3n^2x^3\log(x)-3b^3n^2x^3\log(a+bx)+a^2bnx\log(c(a+bx)^n)-2ab^2nx^2\log(c(a+bx)^n)-}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^2/x^4,x]
```

output
```
-1/3*(a*b^2*n^2*x^2 + 3*b^3*n^2*x^3*Log[x] - 3*b^3*n^2*x^3*Log[a + b*x] +
a^2*b*n*x*Log[c*(a + b*x)^n] - 2*a*b^2*n*x^2*Log[c*(a + b*x)^n] - 2*b^3*n*
x^3*Log[-((b*x)/a)]*Log[c*(a + b*x)^n] + a^3*Log[c*(a + b*x)^n]^2 + b^3*x^
3*Log[c*(a + b*x)^n]^2 - 2*b^3*n^2*x^3*PolyLog[2, 1 + (b*x)/a])/(a^3*x^3)
```

3.86.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {2845, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2}{3}bn\int \frac{\log\left(c(a+bx)^n\right)}{x^3(a+bx)}dx-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2}{3}n\int \frac{\log\left(c(a+bx)^n\right)}{x^3(a+bx)}d(a+bx)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{3}n\int -\frac{\log\left(c(a+bx)^n\right)}{x^3(a+bx)}d(a+bx)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}b^3n\int -\frac{\log\left(c(a+bx)^n\right)}{b^3x^3(a+bx)}d(a+bx)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \text{2789}$$

$$-\frac{2}{3}b^3n\left(\frac{\int -\frac{\log(c(a+bx)^n)}{b^3x^3}d(a+bx)}{a}+\frac{\int \frac{\log(c(a+bx)^n)}{b^2x^2(a+bx)}d(a+bx)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \text{2756}$$

$$-\frac{2}{3}b^3n\left(\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\int\frac{1}{b^2x^2(a+bx)}d(a+bx)}{a}+\frac{\int\frac{\log(c(a+bx)^n)}{b^2x^2(a+bx)}d(a+bx)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \ 54$$

$$-\frac{2}{3}b^3n\left(\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\int\left(\frac{1}{b^2x^2a}-\frac{1}{bxa^2}+\frac{1}{(a+bx)a^2}\right)d(a+bx)}{a}+\frac{\int\frac{\log(c(a+bx)^n)}{b^2x^2(a+bx)}d(a+bx)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{2}{3}b^3n\left(\frac{\int\frac{\log(c(a+bx)^n)}{b^2x^2(a+bx)}d(a+bx)}{a}+\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\left(-\frac{\log(-bx)}{a^2}+\frac{\log(a+bx)}{a^2}-\frac{1}{abx}\right)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \ 2789$$

$$-\frac{2}{3}b^3n\left(\frac{\frac{\int\frac{\log(c(a+bx)^n)}{b^2x^2}d(a+bx)}{a}+\frac{\int-\frac{\log(c(a+bx)^n)}{bx(a+bx)}d(a+bx)}{a}}{a}+\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\left(-\frac{\log(-bx)}{a^2}+\frac{\log(a+bx)}{a^2}-\frac{1}{abx}\right)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \ 2751$$

$$-\frac{2}{3}b^3n\left(\frac{\frac{-\frac{n\int-\frac{1}{bx}d(a+bx)}{a}-\frac{(a+bx)\log\left(c(a+bx)^n\right)}{abx}}{a}+\frac{\int-\frac{\log\left(c(a+bx)^n\right)}{bx(a+bx)}d(a+bx)}{a}}{a}+\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\left(-\frac{\log(-bx)}{a^2}+\frac{\log(a+bx)}{a^2}-\frac{1}{abx}\right)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \ 16$$

$$-\frac{2}{3}b^3n\left(\frac{\frac{\int-\frac{\log\left(c(a+bx)^n\right)}{bx(a+bx)}d(a+bx)}{a}+\frac{\frac{n\log(-bx)}{a}-\frac{(a+bx)\log\left(c(a+bx)^n\right)}{abx}}{a}}{a}+\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\left(-\frac{\log(-bx)}{a^2}+\frac{\log(a+bx)}{a^2}-\frac{1}{abx}\right)}{a}\right)-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \ 2779$$

3.86. $\int\frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx$

$$-\frac{2}{3}b^3n\left(\frac{\frac{\frac{n\int\frac{\log\left(1-\frac{a}{a+bx}\right)}{a+bx}d(a+bx)}{a}-\frac{\log\left(1-\frac{a}{a+bx}\right)\log(c(a+bx)^n)}{a}}{a}+\frac{\frac{n\log(-bx)}{a}-\frac{(a+bx)\log(c(a+bx)^n)}{abx}}{a}}{a}+\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\left(-\frac{\log(-bx)}{a^2}+\right.}{a}\right.$$

$$\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2}{3}b^3n\left(\frac{\frac{\log(c(a+bx)^n)}{2b^2x^2}-\frac{1}{2}n\left(-\frac{\log(-bx)}{a^2}+\frac{\log(a+bx)}{a^2}-\frac{1}{abx}\right)}{a}+\frac{\frac{\frac{n\operatorname{PolyLog}\left(2,\frac{a}{a+bx}\right)}{a}-\frac{\log\left(1-\frac{a}{a+bx}\right)\log(c(a+bx)^n)}{a}}{a}+\frac{\frac{n\log(-bx)}{a}-\frac{(a+b}{a}}{a}\right.$$

$$\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}$$

input
```
Int[Log[c*(a + b*x)^n]^2/x^4,x]
```

output
```
-1/3*Log[c*(a + b*x)^n]^2/x^3 - (2*b^3*n*((-1/2*(n*(-(1/(a*b*x)) - Log[-(b
*x)]/a^2 + Log[a + b*x]/a^2)) + Log[c*(a + b*x)^n]/(2*b^2*x^2))/a + (((n*L
og[-(b*x)])/a - ((a + b*x)*Log[c*(a + b*x)^n])/(a*b*x))/a + (-((Log[c*(a +
 b*x)^n]*Log[1 - a/(a + b*x)])/a) + (n*PolyLog[2, a/(a + b*x)])/a)/a)/a))/
3
```

3.86.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

3.86. $\int \frac{\log^2(c(a+bx)^n)}{x^4} \, dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.86.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.73

method	result
risch	$-\frac{\ln((bx+a)^n)^2}{3x^3}-\frac{2b^3n\ln((bx+a)^n)\ln(bx+a)}{3a^3}-\frac{bn\ln((bx+a)^n)}{3a\,x^2}+\frac{2b^3n\ln((bx+a)^n)\ln(x)}{3a^3}+\frac{2b^2n\ln((bx+a)^n)}{3a^2x}+\frac{b^3n^2\ln(b}{a^3}$

input
```
int(ln(c*(b*x+a)^n)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln((b*x+a)^n)^2/x^3-2/3*b^3*n*ln((b*x+a)^n)/a^3*ln(b*x+a)-1/3*b*n*ln(
(b*x+a)^n)/a/x^2+2/3*b^3*n*ln((b*x+a)^n)/a^3*ln(x)+2/3*b^2*n*ln((b*x+a)^n)
/a^2/x+b^3*n^2*ln(b*x+a)/a^3-1/3*b^2*n^2/a^2/x-b^3*n^2*ln(x)/a^3-2/3*b^3*n
^2/a^3*dilog((b*x+a)/a)-2/3*b^3*n^2/a^3*ln(x)*ln((b*x+a)/a)+1/3*b^3*n^2/a^
3*ln(b*x+a)^2+(-I*Pi*csgn(I*c*(b*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn
(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n
)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*ln(c))*(-1/3*ln((b*x+a)^n)/x^3+1/3*b*n*(-b
^2/a^3*ln(b*x+a)-1/2/a/x^2+b^2/a^3*ln(x)+b/a^2/x))-1/12*(-I*Pi*csgn(I*c*(b
*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x
+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*
ln(c))^2/x^3
```

3.86.5 Fricas [F]

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx=\int \frac{\log\left((bx+a)^n c\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/x^4,x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^2/x^4, x)
```

3.86.6 Sympy [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx=\int \frac{\log\left(c(a+bx)^n\right)^2}{x^4}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)**2/x**4,x)
```

output
```
Integral(log(c*(a + b*x)**n)**2/x**4, x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx=-\frac{1}{3}\,b^2n^2\left(\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)b}{a^3}-\frac{3\,b\log\left(bx+a\right)}{a^3}-\frac{bx\log\left(bx+a\right)^2-3\,bx\log\left(x\right)-a}{a^3x}\right)-\frac{1}{3}\,bn\left(\frac{2\,b^2\log\left(bx+a\right)}{a^3}-\frac{2\,b^2\log\left(x\right)}{a^3}-\frac{2\,bx-a}{a^2x^2}\right)\log\left((bx+a)^nc\right)-\frac{\log\left((bx+a)^nc\right)^2}{3\,x^3}$$

input
```
integrate(log(c*(b*x+a)^n)^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^2*n^2*(2*(log(b*x/a + 1)*log(x) + dilog(-b*x/a))*b/a^3 - 3*b*log(b*
x + a)/a^3 - (b*x*log(b*x + a)^2 - 3*b*x*log(x) - a)/(a^3*x)) - 1/3*b*n*(2
*b^2*log(b*x + a)/a^3 - 2*b^2*log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*log((b*x
 + a)^n*c) - 1/3*log((b*x + a)^n*c)^2/x^3
```

3.86.8 Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/x^4,x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^2/x^4, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{x^4}\,dx$$

input
```
int(log(c*(a + b*x)^n)^2/x^4,x)
```

output
```
int(log(c*(a + b*x)^n)^2/x^4, x)
```

3.87 $\int x^2 \log^3 (c(a+bx)^n) \, dx$

3.87.1 Optimal result

Integrand size = 16, antiderivative size = 285

$$\begin{aligned}\int x^2 \log^3 (c(a+bx)^n) \, dx = & -\frac{6a^2n^3x}{b^2} + \frac{3an^3(a+bx)^2}{4b^3} - \frac{2n^3(a+bx)^3}{27b^3} \\ & + \frac{6a^2n^2(a+bx)\log(c(a+bx)^n)}{b^3} \\ & - \frac{3an^2(a+bx)^2\log(c(a+bx)^n)}{2b^3} \\ & + \frac{2n^2(a+bx)^3\log(c(a+bx)^n)}{9b^3} \\ & - \frac{3a^2n(a+bx)\log^2(c(a+bx)^n)}{b^3} \\ & + \frac{3an(a+bx)^2\log^2(c(a+bx)^n)}{2b^3} \\ & - \frac{n(a+bx)^3\log^2(c(a+bx)^n)}{3b^3} + \frac{a^2(a+bx)\log^3(c(a+bx)^n)}{b^3} \\ & - \frac{a(a+bx)^2\log^3(c(a+bx)^n)}{b^3} + \frac{(a+bx)^3\log^3(c(a+bx)^n)}{3b^3}\end{aligned}$$

output
```
-6*a^2*n^3*x/b^2+3/4*a*n^3*(b*x+a)^2/b^3-2/27*n^3*(b*x+a)^3/b^3+6*a^2*n^2*
(b*x+a)*ln(c*(b*x+a)^n)/b^3-3/2*a*n^2*(b*x+a)^2*ln(c*(b*x+a)^n)/b^3+2/9*n^
2*(b*x+a)^3*ln(c*(b*x+a)^n)/b^3-3*a^2*n*(b*x+a)*ln(c*(b*x+a)^n)^2/b^3+3/2*
a*n*(b*x+a)^2*ln(c*(b*x+a)^n)^2/b^3-1/3*n*(b*x+a)^3*ln(c*(b*x+a)^n)^2/b^3+
a^2*(b*x+a)*ln(c*(b*x+a)^n)^3/b^3-a*(b*x+a)^2*ln(c*(b*x+a)^n)^3/b^3+1/3*(b
*x+a)^3*ln(c*(b*x+a)^n)^3/b^3
```

3.87.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.91

$$\int x^2 \log^3\left(c(a+bx)^n\right) dx = -\frac{85a^2n^3x}{18b^2} + \frac{19an^3x^2}{36b} - \frac{2n^3x^3}{27} + \frac{85a^3n^2\log\left(c(a+bx)^n\right)}{18b^3} + \frac{11a^2n^2x\log\left(c(a+bx)^n\right)}{3b^2} - \frac{5an^2x^2\log\left(c(a+bx)^n\right)}{6b} + \frac{2}{9}n^2x^3\log\left(c(a+bx)^n\right) - \frac{11a^3n\log^2\left(c(a+bx)^n\right)}{6b^3} - \frac{a^2nx\log^2\left(c(a+bx)^n\right)}{b^2} + \frac{anx^2\log^2\left(c(a+bx)^n\right)}{2b} - \frac{1}{3}nx^3\log^2\left(c(a+bx)^n\right) + \frac{a^3\log^3\left(c(a+bx)^n\right)}{3b^3} + \frac{1}{3}x^3\log^3\left(c(a+bx)^n\right)$$

input
```
Integrate[x^2*Log[c*(a + b*x)^n]^3,x]
```

output
```
(-85*a^2*n^3*x)/(18*b^2) + (19*a*n^3*x^2)/(36*b) - (2*n^3*x^3)/27 + (85*a^
3*n^2*Log[c*(a + b*x)^n])/(18*b^3) + (11*a^2*n^2*x*Log[c*(a + b*x)^n])/(3*
b^2) - (5*a*n^2*x^2*Log[c*(a + b*x)^n])/(6*b) + (2*n^2*x^3*Log[c*(a + b*x)
^n])/9 - (11*a^3*n*Log[c*(a + b*x)^n]^2)/(6*b^3) - (a^2*n*x*Log[c*(a + b*x
)^n]^2)/b^2 + (a*n*x^2*Log[c*(a + b*x)^n]^2)/(2*b) - (n*x^3*Log[c*(a + b*x
)^n]^2)/3 + (a^3*Log[c*(a + b*x)^n]^3)/(3*b^3) + (x^3*Log[c*(a + b*x)^n]^3
)/3
```

3.87.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^3\left(c(a+bx)^n\right) dx$$

$$\Big\downarrow \text{ 2848}$$

$$\int \left(\frac{a^2 \log^3 (c(a+bx)^n)}{b^2} + \frac{(a+bx)^2 \log^3 (c(a+bx)^n)}{b^2} - \frac{2a(a+bx) \log^3 (c(a+bx)^n)}{b^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6a^2n^2(a+bx)\log (c(a+bx)^n)}{b^3} + \frac{a^2(a+bx)\log^3 (c(a+bx)^n)}{b^3} - \frac{3a^2n(a+bx)\log^2 (c(a+bx)^n)}{b^3} - \frac{6a^2n^3x}{b^2} + \frac{2n^2(a+bx)^3\log (c(a+bx)^n)}{9b^3} - \frac{3an^2(a+bx)^2\log (c(a+bx)^n)}{2b^3} + \frac{(a+bx)^3\log^3 (c(a+bx)^n)}{3b^3} - \frac{a(a+bx)^2\log^3 (c(a+bx)^n)}{b^3} - \frac{n(a+bx)^3\log^2 (c(a+bx)^n)}{3b^3} + \frac{3an(a+bx)^2\log^2 (c(a+bx)^n)}{2b^3} - \frac{2n^3(a+bx)^3}{27b^3} + \frac{3an^3(a+bx)^2}{4b^3}$$

input
```
Int[x^2*Log[c*(a + b*x)^n]^3,x]
```

output
```
(-6*a^2*n^3*x)/b^2 + (3*a*n^3*(a + b*x)^2)/(4*b^3) - (2*n^3*(a + b*x)^3)/(
27*b^3) + (6*a^2*n^2*(a + b*x)*Log[c*(a + b*x)^n])/b^3 - (3*a*n^2*(a + b*x
)^2*Log[c*(a + b*x)^n])/(2*b^3) + (2*n^2*(a + b*x)^3*Log[c*(a + b*x)^n])/(
9*b^3) - (3*a^2*n*(a + b*x)*Log[c*(a + b*x)^n]^2)/b^3 + (3*a*n*(a + b*x)^2
*Log[c*(a + b*x)^n]^2)/(2*b^3) - (n*(a + b*x)^3*Log[c*(a + b*x)^n]^2)/(3*b
^3) + (a^2*(a + b*x)*Log[c*(a + b*x)^n]^3)/b^3 - (a*(a + b*x)^2*Log[c*(a +
 b*x)^n]^3)/b^3 + ((a + b*x)^3*Log[c*(a + b*x)^n]^3)/(3*b^3)
```

3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.87.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{36x^3 \ln(c(bx+a)^n)^3 b^3 - 36x^3 \ln(c(bx+a)^n)^2 b^3 n + 24x^3 \ln(c(bx+a)^n) b^3 n^2 - 8b^3 n^3 x^3 + 54x^2 \ln(c(bx+a)^n)^2 a\, b^2 n - 90x^2 \ln(c(bx+}{}$
risch	Expression too large to display

input
```
int(x^2*ln(c*(b*x+a)^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*(36*x^3*ln(c*(b*x+a)^n)^3*b^3-36*x^3*ln(c*(b*x+a)^n)^2*b^3*n+24*x^3*
ln(c*(b*x+a)^n)*b^3*n^2-8*b^3*n^3*x^3+54*x^2*ln(c*(b*x+a)^n)^2*a*b^2*n-90*
x^2*ln(c*(b*x+a)^n)*a*b^2*n^2+57*a*b^2*n^3*x^2+906*ln(b*x+a)*a^3*n^3-108*x
*ln(c*(b*x+a)^n)^2*a^2*b*n+396*x*ln(c*(b*x+a)^n)*a^2*b*n^2-510*a^2*b*n^3*x
+36*ln(c*(b*x+a)^n)^3*a^3-198*ln(c*(b*x+a)^n)^2*a^3*n-396*ln(c*(b*x+a)^n)*
a^3*n^2+510*a^3*n^3)/b^3
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.20

$$\int x^2 \log^3 (c(a+bx)^n)\, dx =$$
$$-\frac{8\,b^3n^3x^3 - 36\,b^3x^3\log(c)^3 - 57\,ab^2n^3x^2 + 510\,a^2bn^3x - 36\,(b^3n^3x^3 + a^3n^3)\log(bx+a)^3 + 18\,(2\,b^3n^3x^3}{}$$

input
```
integrate(x^2*log(c*(b*x+a)^n)^3,x, algorithm="fricas")
```

output
```
-1/108*(8*b^3*n^3*x^3 - 36*b^3*x^3*log(c)^3 - 57*a*b^2*n^3*x^2 + 510*a^2*b
*n^3*x - 36*(b^3*n^3*x^3 + a^3*n^3)*log(b*x + a)^3 + 18*(2*b^3*n^3*x^3 - 3
*a*b^2*n^3*x^2 + 6*a^2*b*n^3*x + 11*a^3*n^3 - 6*(b^3*n^2*x^3 + a^3*n^2)*lo
g(c))*log(b*x + a)^2 + 18*(2*b^3*n*x^3 - 3*a*b^2*n*x^2 + 6*a^2*b*n*x)*log(
c)^2 - 6*(4*b^3*n^3*x^3 - 15*a*b^2*n^3*x^2 + 66*a^2*b*n^3*x + 85*a^3*n^3 +
 18*(b^3*n*x^3 + a^3*n)*log(c)^2 - 6*(2*b^3*n^2*x^3 - 3*a*b^2*n^2*x^2 + 6*
a^2*b*n^2*x + 11*a^3*n^2)*log(c))*log(b*x + a) - 6*(4*b^3*n^2*x^3 - 15*a*b
^2*n^2*x^2 + 66*a^2*b*n^2*x)*log(c))/b^3
```

3.87.6 Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx$$
$$= \begin{cases} \frac{85a^3n^2\log\left(c(a+bx)^n\right)}{18b^3} - \frac{11a^3n\log\left(c(a+bx)^n\right)^2}{6b^3} + \frac{a^3\log\left(c(a+bx)^n\right)^3}{3b^3} - \frac{85a^2n^3x}{18b^2} + \frac{11a^2n^2x\log\left(c(a+bx)^n\right)}{3b^2} - \frac{a^2nx\log\left(c(a+bx)^n\right)^2}{b^2} \\ \frac{x^3\log\left(a^nc\right)^3}{3} \end{cases}$$

input
```
integrate(x**2*ln(c*(b*x+a)**n)**3,x)
```

output
```
Piecewise((85*a**3*n**2*log(c*(a + b*x)**n)/(18*b**3) - 11*a**3*n*log(c*(a
 + b*x)**n)**2/(6*b**3) + a**3*log(c*(a + b*x)**n)**3/(3*b**3) - 85*a**2*n
**3*x/(18*b**2) + 11*a**2*n**2*x*log(c*(a + b*x)**n)/(3*b**2) - a**2*n*x*l
og(c*(a + b*x)**n)**2/b**2 + 19*a*n**3*x**2/(36*b) - 5*a*n**2*x**2*log(c*(
a + b*x)**n)/(6*b) + a*n*x**2*log(c*(a + b*x)**n)**2/(2*b) - 2*n**3*x**3/2
7 + 2*n**2*x**3*log(c*(a + b*x)**n)/9 - n*x**3*log(c*(a + b*x)**n)**2/3 +
x**3*log(c*(a + b*x)**n)**3/3, Ne(b, 0)), (x**3*log(a**n*c)**3/3, True))
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.75

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx = \frac{1}{3}\,x^3\log\left((bx+a)^nc\right)^3$$
$$+\frac{1}{6}\,bn\left(\frac{6\,a^3\log\left(bx+a\right)}{b^4} - \frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{b^3}\right)\log\left((bx+a)^nc\right)^2$$
$$-\frac{1}{108}\,bn\left(\frac{\left(8\,b^3x^3-36\,a^3\log\left(bx+a\right)^3-57\,ab^2x^2-198\,a^3\log\left(bx+a\right)^2+510\,a^2bx-510\,a^3\log\left(bx+a\right)\right.}{b^4}\right.$$

input
```
integrate(x^2*log(c*(b*x+a)^n)^3,x, algorithm="maxima")
```

output
```
1/3*x^3*log((b*x + a)^n*c)^3 + 1/6*b*n*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^
3 - 3*a*b*x^2 + 6*a^2*x)/b^3)*log((b*x + a)^n*c)^2 - 1/108*b*n*((8*b^3*x^3
 - 36*a^3*log(b*x + a)^3 - 57*a*b^2*x^2 - 198*a^3*log(b*x + a)^2 + 510*a^2
*b*x - 510*a^3*log(b*x + a))*n^2/b^4 - 6*(4*b^3*x^3 - 15*a*b^2*x^2 - 18*a^
3*log(b*x + a)^2 + 66*a^2*b*x - 66*a^3*log(b*x + a))*n*log((b*x + a)^n*c)/
b^4)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(271) = 542$.

Time = 0.32 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.20

$$\begin{aligned}\int x^2\log^3\left(c(a+bx)^n\right)\,dx &= \frac{(bx+a)^3n^3\log(bx+a)^3}{3\,b^3} - \frac{(bx+a)^2an^3\log(bx+a)^3}{b^3} \\ &+ \frac{(bx+a)a^2n^3\log(bx+a)^3}{b^3} - \frac{(bx+a)^3n^3\log(bx+a)^2}{3\,b^3} \\ &+ \frac{3\,(bx+a)^2an^3\log(bx+a)^2}{2\,b^3} - \frac{3\,(bx+a)a^2n^3\log(bx+a)^2}{b^3} \\ &+ \frac{(bx+a)^3n^2\log(bx+a)^2\log(c)}{b^3} \\ &- \frac{3\,(bx+a)^2an^2\log(bx+a)^2\log(c)}{b^3} \\ &+ \frac{3\,(bx+a)a^2n^2\log(bx+a)^2\log(c)}{b^3} \\ &+ \frac{2\,(bx+a)^3n^3\log(bx+a)}{9\,b^3} - \frac{3\,(bx+a)^2an^3\log(bx+a)}{2\,b^3} \\ &+ \frac{6\,(bx+a)a^2n^3\log(bx+a)}{b^3} - \frac{2\,(bx+a)^3n^2\log(bx+a)\log(c)}{3\,b^3} \\ &+ \frac{3\,(bx+a)^2an^2\log(bx+a)\log(c)}{b^3} \\ &- \frac{6\,(bx+a)a^2n^2\log(bx+a)\log(c)}{b^3} \\ &+ \frac{(bx+a)^3n\log(bx+a)\log(c)^2}{b^3} \\ &- \frac{3\,(bx+a)^2an\log(bx+a)\log(c)^2}{b^3} \\ &+ \frac{3\,(bx+a)a^2n\log(bx+a)\log(c)^2}{b^3} - \frac{2\,(bx+a)^3n^3}{27\,b^3} \\ &+ \frac{3\,(bx+a)^2an^3}{4\,b^3} - \frac{6\,(bx+a)a^2n^3}{b^3} + \frac{2\,(bx+a)^3n^2\log(c)}{9\,b^3} \\ &- \frac{3\,(bx+a)^2an^2\log(c)}{2\,b^3} + \frac{6\,(bx+a)a^2n^2\log(c)}{b^3} \\ &- \frac{(bx+a)^3n\log(c)^2}{3\,b^3} + \frac{3\,(bx+a)^2an\log(c)^2}{2\,b^3} \\ &- \frac{3\,(bx+a)a^2n\log(c)^2}{b^3} + \frac{(bx+a)^3\log(c)^3}{3\,b^3} \\ &- \frac{(bx+a)^2a\log(c)^3}{b^3} + \frac{(bx+a)a^2\log(c)^3}{b^3}\end{aligned}$$

input
```
integrate(x^2*log(c*(b*x+a)^n)^3,x, algorithm="giac")
```

output
```
1/3*(b*x + a)^3*n^3*log(b*x + a)^3/b^3 - (b*x + a)^2*a*n^3*log(b*x + a)^3/
b^3 + (b*x + a)*a^2*n^3*log(b*x + a)^3/b^3 - 1/3*(b*x + a)^3*n^3*log(b*x +
 a)^2/b^3 + 3/2*(b*x + a)^2*a*n^3*log(b*x + a)^2/b^3 - 3*(b*x + a)*a^2*n^3
*log(b*x + a)^2/b^3 + (b*x + a)^3*n^2*log(b*x + a)^2*log(c)/b^3 - 3*(b*x +
 a)^2*a*n^2*log(b*x + a)^2*log(c)/b^3 + 3*(b*x + a)*a^2*n^2*log(b*x + a)^2
*log(c)/b^3 + 2/9*(b*x + a)^3*n^3*log(b*x + a)/b^3 - 3/2*(b*x + a)^2*a*n^3
*log(b*x + a)/b^3 + 6*(b*x + a)*a^2*n^3*log(b*x + a)/b^3 - 2/3*(b*x + a)^3
*n^2*log(b*x + a)*log(c)/b^3 + 3*(b*x + a)^2*a*n^2*log(b*x + a)*log(c)/b^3
 - 6*(b*x + a)*a^2*n^2*log(b*x + a)*log(c)/b^3 + (b*x + a)^3*n*log(b*x + a
)*log(c)^2/b^3 - 3*(b*x + a)^2*a*n*log(b*x + a)*log(c)^2/b^3 + 3*(b*x + a)
*a^2*n*log(b*x + a)*log(c)^2/b^3 - 2/27*(b*x + a)^3*n^3/b^3 + 3/4*(b*x + a
)^2*a*n^3/b^3 - 6*(b*x + a)*a^2*n^3/b^3 + 2/9*(b*x + a)^3*n^2*log(c)/b^3 -
 3/2*(b*x + a)^2*a*n^2*log(c)/b^3 + 6*(b*x + a)*a^2*n^2*log(c)/b^3 - 1/3*(
b*x + a)^3*n*log(c)^2/b^3 + 3/2*(b*x + a)^2*a*n*log(c)^2/b^3 - 3*(b*x + a)
*a^2*n*log(c)^2/b^3 + 1/3*(b*x + a)^3*log(c)^3/b^3 - (b*x + a)^2*a*log(c)^
3/b^3 + (b*x + a)*a^2*log(c)^3/b^3
```

3.87.9 Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.60

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx = \ln\left(c\,(a+b\,x)^n\right)^3\left(\frac{x^3}{3}+\frac{a^3}{3\,b^3}\right)-\frac{2\,n^3\,x^3}{27} - \ln\left(c\,(a+b\,x)^n\right)^2\left(\frac{n\,x^3}{3}+\frac{11\,a^3\,n}{6\,b^3}-\frac{a\,n\,x^2}{2\,b}+\frac{a^2\,n\,x}{b^2}\right) + \frac{\ln\left(c\,(a+b\,x)^n\right)\left(\frac{2\,b\,n^2\,x^3}{3}-\frac{5\,a\,n^2\,x^2}{2}+\frac{11\,a^2\,n^2\,x}{b}\right)}{3\,b} + \frac{85\,a^3\,n^3\,\ln\left(a+b\,x\right)}{18\,b^3}+\frac{19\,a\,n^3\,x^2}{36\,b}-\frac{85\,a^2\,n^3\,x}{18\,b^2}$$

input
```
int(x^2*log(c*(a + b*x)^n)^3,x)
```

output
```
log(c*(a + b*x)^n)^3*(x^3/3 + a^3/(3*b^3)) - (2*n^3*x^3)/27 - log(c*(a + b
*x)^n)^2*((n*x^3)/3 + (11*a^3*n)/(6*b^3) - (a*n*x^2)/(2*b) + (a^2*n*x)/b^2
) + (log(c*(a + b*x)^n)*((2*b*n^2*x^3)/3 - (5*a*n^2*x^2)/2 + (11*a^2*n^2*x
)/b))/(3*b) + (85*a^3*n^3*log(a + b*x))/(18*b^3) + (19*a*n^3*x^2)/(36*b) -
 (85*a^2*n^3*x)/(18*b^2)
```

3.88 $\int \frac{(f+gx)^3}{a+b\log(c(d+ex)^n)}\,dx$

3.88.1 Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \frac{(f+gx)^3}{a+b\log(c(d+ex)^n)}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^4n} + \frac{3e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n} + \frac{3e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n} + \frac{e^{-\frac{4a}{bn}}g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}$$

output
```
(-d*g+e*f)^3*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e^4/exp(a/b/n)/n/((c*
(e*x+d)^n)^(1/n))+3*g*(-d*g+e*f)^2*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/
n)/b/e^4/exp(2*a/b/n)/n/((c*(e*x+d)^n)^(2/n))+3*g^2*(-d*g+e*f)*(e*x+d)^3*E
i(3*(a+b*ln(c*(e*x+d)^n))/b/n)/b/e^4/exp(3*a/b/n)/n/((c*(e*x+d)^n)^(3/n))+
g^3*(e*x+d)^4*Ei(4*(a+b*ln(c*(e*x+d)^n))/b/n)/b/e^4/exp(4*a/b/n)/n/((c*(e*
x+d)^n)^(4/n))
```

3.88.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$= \frac{e^{-\frac{4a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-4/n}\left(e^{\frac{3a}{bn}}(ef-dg)^3\left(c(d+ex)^n\right)^{3/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)+g(d+ex\right.}{}$$

input
```
Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((d + e*x)*(E^((3*a)/(b*n))*(e*f - d*g)^3*(c*(d + e*x)^n)^(3/n)*ExpIntegra
lEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] + g*(d + e*x)*(3*E^((2*a)/(b*n))*(e*
f - d*g)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n
]))/(b*n)] - g*(d + e*x)*(-3*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1
)*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)] - g*(d + e*x)*ExpInt
egralEi[(4*(a + b*Log[c*(d + e*x)^n]))/(b*n)]))))/(b*e^4*E^((4*a)/(b*n))*n
*(c*(d + e*x)^n)^(4/n))
```

3.88.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$\Big\downarrow \text{2846}$$
$$\int \left(\frac{3g^2(d+ex)^2(ef-dg)}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{(ef-dg)^3}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{3g(d+ex)(ef-dg)^2}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{g^3(d+ex)^3}{e^3\left(a+b\log\left(c(d+ex\right.\right.}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{3g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\,(c(d+ex)^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}+\frac{3ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\,(c(d+ex)^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\,(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^4n}+\frac{g^3e^{-\frac{4a}{bn}}(d+ex)^4\,(c(d+ex)^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}$$

input
```
Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((e*f - d*g)^3*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/
(b*e^4*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (3*g*(e*f - d*g)^2*(d + e*x
)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^4*E^((2*a)/(
b*n))*n*(c*(d + e*x)^n)^(2/n)) + (3*g^2*(e*f - d*g)*(d + e*x)^3*ExpIntegra
lEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^4*E^((3*a)/(b*n))*n*(c*(d
+ e*x)^n)^(3/n)) + (g^3*(d + e*x)^4*ExpIntegralEi[(4*(a + b*Log[c*(d + e*x
)^n]))/(b*n)])/(b*e^4*E^((4*a)/(b*n))*n*(c*(d + e*x)^n)^(4/n))
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

3.88.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 9346, normalized size of antiderivative = 31.26

method	result	size
risch	Expression too large to display	9346

input
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\left(g^3\,\mathrm{log\_integral}\left((e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4)e^{\left(\frac{4\,(b\log(c)+a)}{bn}\right)}\right)+3\,(efg^2-dg^3)e^{\left(\frac{b\log(c)+a}{bn}\right)}\,\mathrm{lo}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
(g^3*log_integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4
)*e^(4*(b*log(c) + a)/(b*n))) + 3*(e*f*g^2 - d*g^3)*e^((b*log(c) + a)/(b*n
))*log_integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c) +
 a)/(b*n))) + 3*(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3)*e^(2*(b*log(c) + a)/(b
*n))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))) +
(e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*e^(3*(b*log(c) + a)/(b
*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))))*e^(-4*(b*log(c) + a
)/(b*n))/(b*e^4*n)
```

3.88.6 Sympy [F]

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(f+gx)^3}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n)), x)
```

3.88.7 Maxima [F]

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(gx+f)^3}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a), x)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.95

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{f^3\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en} - \frac{3\,df^2g\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^2n} + \frac{3\,d^2fg^2\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^3n} - \frac{d^3g^3\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^4n} + \frac{3\,f^2g\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(ex+d)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^2n} - \frac{6\,dfg^2\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(ex+d)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^3n} + \frac{3\,d^2g^3\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(ex+d)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^4n} + \frac{3\,fg^2\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(ex+d)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}e^3n} - \frac{3\,dg^3\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(ex+d)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}e^4n} + \frac{g^3\mathrm{Ei}\left(\frac{4\,\log(c)}{n}+\frac{4\,a}{bn}+4\,\log(ex+d)\right)e^{\left(-\frac{4\,a}{bn}\right)}}{bc^{\frac{4}{n}}e^4n}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
f^3*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n) - 3
*d*f^2*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^2
*n) + 3*d^2*f*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^
(1/n)*e^3*n) - d^3*g^3*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/
(b*c^(1/n)*e^4*n) + 3*f^2*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^
(-2*a/(b*n))/(b*c^(2/n)*e^2*n) - 6*d*f*g^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*l
og(e*x + d))*e^(-2*a/(b*n))/(b*c^(2/n)*e^3*n) + 3*d^2*g^3*Ei(2*log(c)/n +
2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/(b*c^(2/n)*e^4*n) + 3*f*g^2*Ei(
3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/(b*c^(3/n)*e^3*n)
- 3*d*g^3*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/(b*c^
(3/n)*e^4*n) + g^3*Ei(4*log(c)/n + 4*a/(b*n) + 4*log(e*x + d))*e^(-4*a/(b*
n))/(b*c^(4/n)*e^4*n)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{{\left(f+g\,x\right)}^3}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n)), x)
```

3.89 $\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx$

3.89.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^3n} + \frac{2e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n} + \frac{e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}$$

output
```
(-d*g+e*f)^2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e^3/exp(a/b/n)/n/((c*
(e*x+d)^n)^(1/n))+2*g*(-d*g+e*f)*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)
/b/e^3/exp(2*a/b/n)/n/((c*(e*x+d)^n)^(2/n))+g^2*(e*x+d)^3*Ei(3*(a+b*ln(c*(
e*x+d)^n))/b/n)/b/e^3/exp(3*a/b/n)/n/((c*(e*x+d)^n)^(3/n))
```

3.89.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(e^{\frac{2a}{bn}}(ef-dg)^2\left(c(d+ex)^n\right)^{2/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)-g(d+ex\right.}{}$$

input
```
Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((d + e*x)*(E^((2*a)/(b*n))*(e*f - d*g)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegra
lEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] - g*(d + e*x)*(-2*E^(a/(b*n))*(e*f -
 d*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/
(b*n)] - g*(d + e*x)*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]))
)/(b*e^3*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n))
```

3.89.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$\Big\downarrow \text{2846}$$
$$\int \left(\frac{(ef-dg)^2}{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{2g(d+ex)(ef-dg)}{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{g^2(d+ex)^2}{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^3n}+\frac{g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}$$

input
```
Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((e*f - d*g)^2*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/
(b*e^3*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (2*g*(e*f - d*g)*(d + e*x)^
2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^3*E^((2*a)/(b*
n))*n*(c*(d + e*x)^n)^(2/n)) + (g^2*(d + e*x)^3*ExpIntegralEi[(3*(a + b*Lo
g[c*(d + e*x)^n]))/(b*n)])/(b*e^3*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n))
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

3.89.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 1889, normalized size of antiderivative = 8.63

method	result	size
risch	Expression too large to display	1889

input
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/e^3*g^2/b/n*(e*x+d)^3*((e*x+d)^n)^(-3/n)*c^(-3/n)*exp(-3/2*(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*
x+d)^n)^3*b+2*a)/b/n)*Ei(1,-3*ln(e*x+d)-3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*c
sgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csg
n(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln
(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)-1/e*f^2/b/n*(e*x+d)*((e*x+d)
^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I
*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*
x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csg
n(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^
2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))
+2*a)/b/n)-1/e^3*g^2*d^2/b/n*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*
(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*cs
gn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*
csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e
*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*
b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^
3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)+2/e^3*d*g^2/b/n...
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx$$
$$= \frac{\left(g^2 \log\_integral\left((e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)e^{\left(\frac{3\,(b\log(c)+a)}{bn}\right)}\right)+2\,(efg-dg^2)e^{\left(\frac{b\log(c)+a}{bn}\right)}\log\_integral\left(\right.\right.}{}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
(g^2*log_integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c)
 + a)/(b*n))) + 2*(e*f*g - d*g^2)*e^((b*log(c) + a)/(b*n))*log_integral((e
^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))) + (e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)*e^(2*(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c)
 + a)/(b*n))))*e^(-3*(b*log(c) + a)/(b*n))/(b*e^3*n)
```

3.89.6 Sympy [F]

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+gx)^2}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n)), x)
```

3.89.7 Maxima [F]

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(gx+f)^2}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a), x)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx &= \frac{f^2\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en} \\ &\quad-\frac{2\,dfg\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^2n} \\ &\quad+\frac{d^2g^2\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^3n} \\ &\quad+\frac{2\,fg\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log\left(ex+d\right)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^2n} \\ &\quad-\frac{2\,dg^2\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log\left(ex+d\right)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^3n} \\ &\quad+\frac{g^2\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log\left(ex+d\right)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}e^3n}\end{aligned}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
f^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n) - 2
*d*f*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^2*n
) + d^2*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*
e^3*n) + 2*f*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/
(b*c^(2/n)*e^2*n) - 2*d*g^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^
(-2*a/(b*n))/(b*c^(2/n)*e^3*n) + g^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x
 + d))*e^(-3*a/(b*n))/(b*c^(3/n)*e^3*n)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(f+g\,x)^2}{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n)), x)
```

3.90 $\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx$

3.90.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n} + \frac{e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^2n}$$

output
```
(-d*g+e*f)*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e^2/exp(a/b/n)/n/((c*(e
*x+d)^n)^(1/n))+g*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b/e^2/exp(2*a/
b/n)/n/((c*(e*x+d)^n)^(2/n))
```

3.90.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(e^{\frac{a}{bn}}(ef-dg)\left(c(d+ex)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)+g(d+ex)\right.}{be^2n}$$

input
```
Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((d + e*x)*(E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(
a + b*Log[c*(d + e*x)^n])/(b*n)] + g*(d + e*x)*ExpIntegralEi[(2*(a + b*Log
[c*(d + e*x)^n]))/(b*n)]))/(b*e^2*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n))
```

3.90.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}dx$$

$$\Big\downarrow \text{2846}$$

$$\int \left(\frac{ef-dg}{e\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{g(d+ex)}{e\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n}+\frac{ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^2n}$$

input
```
Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b
*e^2*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (g*(d + e*x)^2*ExpIntegralEi[
(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^2*E^((2*a)/(b*n))*n*(c*(d + e*
x)^n)^(2/n))
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

3.90.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 937, normalized size of antiderivative = 6.74

method	result	size
risch	Expression too large to display	937

input
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2*g/b/n*(e*x+d)^2*c^(-2/n)*((e*x+d)^n)^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*
(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^
2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n
)^3*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*c
sgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d
)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(l
n((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)-1/e*f/b/n*(e*x+d)*c^(-1/n)*((e*x+d)^n)
^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+
I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*
x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I
*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(
I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csg
n(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)+1
/e^2*d*g/b/n*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*
c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n
)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)
^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*
c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e
*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*
b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\left((ef-dg)e^{\left(\frac{b\log(c)+a}{bn}\right)}\,\mathrm{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)+g\,\mathrm{log\_integral}\left((e^2x^2+2\,dex+d^2)e^{\left(\frac{2\,(b\log(c)}{bn}\right.}\right.\right.}{be^2n}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
((e*f - d*g)*e^((b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c)
+ a)/(b*n))) + g*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a
)/(b*n))))*e^(-2*(b*log(c) + a)/(b*n))/(b*e^2*n)
```

3.90.6 Sympy [F]

$$\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{f+gx}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((f + g*x)/(a + b*log(c*(d + e*x)**n)), x)
```

3.90.7 Maxima [F]

$$\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{gx+f}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(b*log((e*x + d)^n*c) + a), x)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx = \frac{f\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en} - \frac{dg\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^2 n} + \frac{g\mathrm{Ei}\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+2\log\left(ex+d\right)\right)e^{\left(-\frac{2a}{bn}\right)}}{bc^{\frac{2}{n}}e^2 n}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
f*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n) - d*g
*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^2*n) + g*
Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/(b*c^(2/n)*e^2*
n)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{f+g\,x}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n)), x)
```

3.91 $\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx$

3.91.1 Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

output
```
(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e/exp(a/b/n)/n/((c*(e*x+d)^n)^(1/n
))
```

3.91.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-1),x]
```

output
```
((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n
))*n*(c*(d + e*x)^n)^n^(-1))
```

3.91.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2836, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2609}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-1),x]
```

output
```
((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n
))*n*(c*(d + e*x)^n)^n^(-1))
```

3.91.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.91.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.90

method	result
risch	$-\frac{(ex+d)c^{-\frac{1}{n}}((ex+d)^n)^{-\frac{1}{n}}\mathrm{e}^{-\frac{-ib\pi\ \mathrm{csgn}(ic(ex+d)^n)\ \mathrm{csgn}(ic)\ \mathrm{csgn}(i(ex+d)^n)+i\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}(ic(ex+d)^n)^2 b+i\pi\ \mathrm{csgn}(i(ex+d)^n)\ \mathrm{csgn}(ic(ex-}{2bn}}}{}$

input
```
int(1/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/e/b/n*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e
*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*
b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^
3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)+1/2*I*(b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)-b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-b*Pi*csgn(I*(e*x+d)^n
)*csgn(I*c*(e*x+d)^n)^2+b*Pi*csgn(I*c*(e*x+d)^n)^3+2*I*b*ln(c)+2*I*b*(ln((
e*x+d)^n)-n*ln(e*x+d))+2*I*a)/b/n)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{\left(-\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)}{ben}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
e^(-(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)))
/(b*e*n)
```

3.91.6 Sympy [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(1/(a + b*log(c*(d + e*x)**n)), x)
```

3.91.7 Maxima [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{1}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(1/(b*log((e*x + d)^n*c) + a), x)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{1}{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n)), x)
```

3.92 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx$

3.92.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.92.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

3.92.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}dx$$
$$\Big\downarrow \text{ 2867}$$
$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.92.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.92.4 Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)(a+b\ln(c(ex+d)^n))}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.92.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(1/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

3.92.6 Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)\left(f+gx\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

3.92.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.92.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.92.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))), x)
```

3.93 $\int \frac{1}{(f+gx)^2(a+b\log(c(d+ex)^n))}\,dx$

3.93.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

3.93.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])), x]
```

3.93.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))} \, dx$$
$$\Big\downarrow \; 2867$$
$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))}dx$$

input
```
Int[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.93.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.93.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 (a+b\ln (c (ex+d)^n))}dx$$

input
```
int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

3.93.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))} \, dx = \int \frac{1}{(gx+f)^2(b\log \left((ex+d)^n c\right)+a)} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(1/(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2
)*log((e*x + d)^n*c)), x)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))} \, dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Timed out
```

3.93.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))} \, dx = \int \frac{1}{(gx+f)^2(b\log \left((ex+d)^n c\right)+a)} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)), x)
```

3.93.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{(gx+f)^2(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)), x)
```

3.93.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))), x)
```

3.94 $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.94.1 Optimal result

Integrand size = 24, antiderivative size = 339

$$\begin{aligned}\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^4n^2}\\ &+\frac{6e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\ &+\frac{9e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\ &+\frac{4e^{-\frac{4a}{bn}}g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\ &-\frac{(d+ex)(f+gx)^3}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}\end{aligned}$$

output
```
(-d*g+e*f)^3*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^4/exp(a/b/n)/n^2/
((c*(e*x+d)^n)^(1/n))+6*g*(-d*g+e*f)^2*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n)
)/b/n)/b^2/e^4/exp(2*a/b/n)/n^2/((c*(e*x+d)^n)^(2/n))+9*g^2*(-d*g+e*f)*(e*
x+d)^3*Ei(3*(a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^4/exp(3*a/b/n)/n^2/((c*(e*x+d
)^n)^(3/n))+4*g^3*(e*x+d)^4*Ei(4*(a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^4/exp(4*
a/b/n)/n^2/((c*(e*x+d)^n)^(4/n))-(e*x+d)*(g*x+f)^3/b/e/n/(a+b*ln(c*(e*x+d)
^n))
```

3.94.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1674 vs. $2(339) = 678$.

Time = 0.46 (sec) , antiderivative size = 1674, normalized size of antiderivative = 4.94

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(-(b*d*e^3*E^((4*a)/(b*n))*f^3*n*(c*(d + e*x)^n)^(4/n)) - b*e^4*E^((4*a)/(
b*n))*f^3*n*x*(c*(d + e*x)^n)^(4/n) - 3*b*d*e^3*E^((4*a)/(b*n))*f^2*g*n*x*
(c*(d + e*x)^n)^(4/n) - 3*b*e^4*E^((4*a)/(b*n))*f^2*g*n*x^2*(c*(d + e*x)^n
)^(4/n) - 3*b*d*e^3*E^((4*a)/(b*n))*f*g^2*n*x^2*(c*(d + e*x)^n)^(4/n) - 3*
b*e^4*E^((4*a)/(b*n))*f*g^2*n*x^3*(c*(d + e*x)^n)^(4/n) - b*d*e^3*E^((4*a)
/(b*n))*g^3*n*x^3*(c*(d + e*x)^n)^(4/n) - b*e^4*E^((4*a)/(b*n))*g^3*n*x^4*
(c*(d + e*x)^n)^(4/n) + a*e^3*E^((3*a)/(b*n))*f^3*(d + e*x)*(c*(d + e*x)^n
)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] - 3*a*d*e^2*E^((3*
a)/(b*n))*f^2*g*(d + e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c
*(d + e*x)^n])/(b*n)] + 3*a*d^2*e*E^((3*a)/(b*n))*f*g^2*(d + e*x)*(c*(d +
e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] - a*d^3*E^((
3*a)/(b*n))*g^3*(d + e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c
*(d + e*x)^n])/(b*n)] + 6*a*e^2*E^((2*a)/(b*n))*f^2*g*(d + e*x)^2*(c*(d +
e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] - 12*a*d
*e*E^((2*a)/(b*n))*f*g^2*(d + e*x)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(
2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 6*a*d^2*E^((2*a)/(b*n))*g^3*(d + e*
x)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b
*n)] + 9*a*e*E^(a/(b*n))*f*g^2*(d + e*x)^3*(c*(d + e*x)^n)^n^(-1)*ExpInteg
ralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)] - 9*a*d*E^(a/(b*n))*g^3*(d + e
*x)^3*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]...
```

3.94.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2847, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx$$

$\big\downarrow$ 2847

$$-\frac{3(ef-dg)\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{4\int \frac{(f+gx)^3}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)^3}{ben(a+b\log(c(d+ex)^n))}$$

$\big\downarrow$ 2846

$$\frac{4\int \left(\frac{(ef-dg)^3}{e^3(a+b\log(c(d+ex)^n))}+\frac{3g(d+ex)(ef-dg)^2}{e^3(a+b\log(c(d+ex)^n))}+\frac{3g^2(d+ex)^2(ef-dg)}{e^3(a+b\log(c(d+ex)^n))}+\frac{g^3(d+ex)^3}{e^3(a+b\log(c(d+ex)^n))}\right)dx}{bn}-\frac{3(ef-dg)\int \left(\frac{(ef-dg)^2}{e^2(a+b\log(c(d+ex)^n))}+\frac{2g(d+ex)(ef-dg)}{e^2(a+b\log(c(d+ex)^n))}+\frac{g^2(d+ex)^2}{e^2(a+b\log(c(d+ex)^n))}\right)dx}{ben}-\frac{(d+ex)(f+gx)^3}{ben(a+b\log(c(d+ex)^n))}$$

$\big\downarrow$ 2009

$$\frac{4\left(\frac{3g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)(c(d+ex)^n)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}+\frac{3ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2(c(d+ex)^n)^{-2/n}\text{ExpIntegralE}}{be^4n}\right.}{}$$

$$\frac{3(ef-dg)\left(\frac{2ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\text{ExpI}}{be^3n}\right.}{ben}$$

$$\frac{(d+ex)(f+gx)^3}{ben(a+b\log(c(d+ex)^n))}$$

input
```
Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^2,x]
```

3.94. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx$

output
```
(-3*(e*f - d*g)*(((e*f - d*g)^2*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d +
e*x)^n])/(b*n)])/(b*e^3*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (2*g*(e*f
- d*g)*(d + e*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b
*e^3*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n)) + (g^2*(d + e*x)^3*ExpIntegr
alEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^3*E^((3*a)/(b*n))*n*(c*(d
 + e*x)^n)^(3/n))))/(b*e*n) + (4*(((e*f - d*g)^3*(d + e*x)*ExpIntegralEi[(
a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e^4*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(
-1)) + (3*g*(e*f - d*g)^2*(d + e*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e
*x)^n]))/(b*n)])/(b*e^4*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n)) + (3*g^2*
(e*f - d*g)*(d + e*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)
])/(b*e^4*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n)) + (g^3*(d + e*x)^4*ExpI
ntegralEi[(4*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^4*E^((4*a)/(b*n))*n*
(c*(d + e*x)^n)^(4/n))))/(b*n) - ((d + e*x)*(f + g*x)^3)/(b*e*n*(a + b*Log
[c*(d + e*x)^n]))
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.94. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.94.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 9517, normalized size of antiderivative = 28.07

method	result	size
risch	Expression too large to display	9517

input
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.01

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx$$
$$= \frac{\left(9\left(aefg^2-adg^3+(befg^2-bdg^3)n\log(ex+d)+(befg^2-bdg^3)\log(c)\right)e^{\left(\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left((e^3x^3\right.\right.}{}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
(9*(a*e*f*g^2 - a*d*g^3 + (b*e*f*g^2 - b*d*g^3)*n*log(e*x + d) + (b*e*f*g^
2 - b*d*g^3)*log(c))*e^((b*log(c) + a)/(b*n))*log_integral((e^3*x^3 + 3*d*
e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c) + a)/(b*n))) + 6*(a*e^2*f^2*g -
2*a*d*e*f*g^2 + a*d^2*g^3 + (b*e^2*f^2*g - 2*b*d*e*f*g^2 + b*d^2*g^3)*n*lo
g(e*x + d) + (b*e^2*f^2*g - 2*b*d*e*f*g^2 + b*d^2*g^3)*log(c))*e^(2*(b*log
(c) + a)/(b*n))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)
/(b*n))) + (a*e^3*f^3 - 3*a*d*e^2*f^2*g + 3*a*d^2*e*f*g^2 - a*d^3*g^3 + (b
*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*n*log(e*x + d) +
 (b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*log(c))*e^(3*
(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) - (
b*e^4*g^3*n*x^4 + b*d*e^3*f^3*n + (3*b*e^4*f*g^2 + b*d*e^3*g^3)*n*x^3 + 3*
(b*e^4*f^2*g + b*d*e^3*f*g^2)*n*x^2 + (b*e^4*f^3 + 3*b*d*e^3*f^2*g)*n*x)*e
^(4*(b*log(c) + a)/(b*n)) + 4*(b*g^3*n*log(e*x + d) + b*g^3*log(c) + a*g^3
)*log_integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*e
^(4*(b*log(c) + a)/(b*n))))*e^(-4*(b*log(c) + a)/(b*n))/(b^3*e^4*n^3*log(e
*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)
```

3.94.6 Sympy [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx$$

input
```
integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n))**2, x)
```

3.94.7 Maxima [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*g^3*x^4 + d*f^3 + (3*e*f*g^2 + d*g^3)*x^3 + 3*(e*f^2*g + d*f*g^2)*x^2
+ (e*f^3 + 3*d*f^2*g)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*
e*n) + integrate((4*e*g^3*x^3 + e*f^3 + 3*d*f^2*g + 3*(3*e*f*g^2 + d*g^3)*
x^2 + 6*(e*f^2*g + d*f*g^2)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c)
+ a*b*e*n), x)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3473 vs. $2(341) = 682$.

Time = 0.42 (sec) , antiderivative size = 3473, normalized size of antiderivative = 10.24

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
b*e^3*f^3*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d
)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)
) - 3*b*d*e^2*f^2*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*l
og(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^
2)*c^(1/n)) + 3*b*d^2*e*f*g^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-
a/(b*n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*
b^2*e^4*n^2)*c^(1/n)) - b*d^3*g^3*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*
e^(-a/(b*n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c)
+ a*b^2*e^4*n^2)*c^(1/n)) - (e*x + d)*b*e^3*f^3*n/(b^3*e^4*n^3*log(e*x + d
) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) - 3*(e*x + d)^2*b*e^2*f^2*g*n/(b^3
*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) + 3*(e*x + d)*
b*d*e^2*f^2*g*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4
*n^2) - 3*(e*x + d)^3*b*e*f*g^2*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*
log(c) + a*b^2*e^4*n^2) + 6*(e*x + d)^2*b*d*e*f*g^2*n/(b^3*e^4*n^3*log(e*x
 + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) - 3*(e*x + d)*b*d^2*e*f*g^2*n/
(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) - (e*x + d
)^4*b*g^3*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2
) + 3*(e*x + d)^3*b*d*g^3*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c)
 + a*b^2*e^4*n^2) - 3*(e*x + d)^2*b*d^2*g^3*n/(b^3*e^4*n^3*log(e*x + d) +
b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) + (e*x + d)*b*d^3*g^3*n/(b^3*e^4*n^...
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+g\,x)^3}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

input

```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^2,x)
```

output

```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.95 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.95.1 Optimal result

Integrand size = 24, antiderivative size = 259

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^3n^2} + \frac{4e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2} + \frac{3e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2} - \frac{(d+ex)(f+gx)^2}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

output
```
(-d*g+e*f)^2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^3/exp(a/b/n)/n^2/
((c*(e*x+d)^n)^(1/n))+4*g*(-d*g+e*f)*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/
b/n)/b^2/e^3/exp(2*a/b/n)/n^2/((c*(e*x+d)^n)^(2/n))+3*g^2*(e*x+d)^3*Ei(3*(
a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^3/exp(3*a/b/n)/n^2/((c*(e*x+d)^n)^(3/n))-(
e*x+d)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

3.95.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1015 vs. $2(259) = 518$.

Time = 0.25 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.92

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}(c(d+ex)^n)^{-3/n}\left(-bde^2e^{\frac{3a}{bn}}f^2n(c(d+ex)^n)^{3/n} - be^3e^{\frac{3a}{bn}}f^2nx(c(d+ex)^n)^{3/n} - 2bde^2e^{\frac{3a}{bn}}fgnx(c(d+\right.}{}$$

input
```
Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(-(b*d*e^2*E^((3*a)/(b*n))*f^2*n*(c*(d + e*x)^n)^(3/n)) - b*e^3*E^((3*a)/(
b*n))*f^2*n*x*(c*(d + e*x)^n)^(3/n) - 2*b*d*e^2*E^((3*a)/(b*n))*f*g*n*x*(c
*(d + e*x)^n)^(3/n) - 2*b*e^3*E^((3*a)/(b*n))*f*g*n*x^2*(c*(d + e*x)^n)^(3
/n) - b*d*e^2*E^((3*a)/(b*n))*g^2*n*x^2*(c*(d + e*x)^n)^(3/n) - b*e^3*E^((
3*a)/(b*n))*g^2*n*x^3*(c*(d + e*x)^n)^(3/n) + a*e^2*E^((2*a)/(b*n))*f^2*(d
 + e*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*
n)] - 2*a*d*e*E^((2*a)/(b*n))*f*g*(d + e*x)*(c*(d + e*x)^n)^(2/n)*ExpInteg
ralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] + a*d^2*E^((2*a)/(b*n))*g^2*(d + e
*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]
+ 4*a*e*E^(a/(b*n))*f*g*(d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(
2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] - 4*a*d*E^(a/(b*n))*g^2*(d + e*x)^2*(
c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]
+ 3*a*g^2*(d + e*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]
+ b*e^2*E^((2*a)/(b*n))*f^2*(d + e*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[
(a + b*Log[c*(d + e*x)^n])/(b*n)]*Log[c*(d + e*x)^n] - 2*b*d*e*E^((2*a)/(b
*n))*f*g*(d + e*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e
*x)^n])/(b*n)]*Log[c*(d + e*x)^n] + b*d^2*E^((2*a)/(b*n))*g^2*(d + e*x)*(c
*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*Log[c*
(d + e*x)^n] + 4*b*e*E^(a/(b*n))*f*g*(d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*Ex
pIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n] - ...
```

3.95.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2847, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}dx$$

$$\Big\downarrow \ 2847$$

$$-\frac{2(ef-dg)\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{3\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)^2}{ben\,(a+b\log(c(d+ex)^n))}$$

$$\Big\downarrow \ 2846$$

$$\frac{3\int \left(\frac{(ef-dg)^2}{e^2(a+b\log(c(d+ex)^n))}+\frac{2g(d+ex)(ef-dg)}{e^2(a+b\log(c(d+ex)^n))}+\frac{g^2(d+ex)^2}{e^2(a+b\log(c(d+ex)^n))}\right)dx}{bn}-\frac{2(ef-dg)\int \left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{ben}-\frac{(d+ex)(f+gx)^2}{ben\,(a+b\log(c(d+ex)^n))}$$

$$\Big\downarrow \ 2009$$

$$\frac{3\left(\frac{2ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+\dots}{}\right)}{be^3n}\dots\right)}{bn}$$
$$\frac{2(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n}+\frac{ge^{-\frac{2a}{bn}}(d+ex)^2(c(d+ex)^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(\dots}{}\right)}{be^2n}\dots\right)}{ben}$$
$$\frac{(d+ex)(f+gx)^2}{ben\,(a+b\log(c(d+ex)^n))}$$

input
```
Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(-2*(e*f - d*g)*(((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*
x)^n])/(b*n)])/(b*e^2*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (g*(d + e*x)
^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^2*E^((2*a)/(b
*n))*n*(c*(d + e*x)^n)^(2/n))))/(b*e*n) + (3*(((e*f - d*g)^2*(d + e*x)*Exp
IntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e^3*E^(a/(b*n))*n*(c*(d +
 e*x)^n)^n^(-1)) + (2*g*(e*f - d*g)*(d + e*x)^2*ExpIntegralEi[(2*(a + b*Lo
g[c*(d + e*x)^n]))/(b*n)])/(b*e^3*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n))
 + (g^2*(d + e*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(
b*e^3*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n))))/(b*n) - ((d + e*x)*(f + g
*x)^2)/(b*e*n*(a + b*Log[c*(d + e*x)^n]))
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.95.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 5089, normalized size of antiderivative = 19.65

method	result	size
risch	Expression too large to display	5089

input
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.67

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx$$
$$= \frac{\left(4\left(aefg-adg^2+(befg-bdg^2)n\log(ex+d)+(befg-bdg^2)\log(c)\right)e^{\left(\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left(\left(e^2x^2+\right.\right.\right.}{}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
(4*(a*e*f*g - a*d*g^2 + (b*e*f*g - b*d*g^2)*n*log(e*x + d) + (b*e*f*g - b*
d*g^2)*log(c))*e^((b*log(c) + a)/(b*n))*log_integral((e^2*x^2 + 2*d*e*x +
d^2)*e^(2*(b*log(c) + a)/(b*n))) + (a*e^2*f^2 - 2*a*d*e*f*g + a*d^2*g^2 +
(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*log(e*x + d) + (b*e^2*f^2 - 2*b*d*
e*f*g + b*d^2*g^2)*log(c))*e^(2*(b*log(c) + a)/(b*n))*log_integral((e*x +
d)*e^((b*log(c) + a)/(b*n))) - (b*e^3*g^2*n*x^3 + b*d*e^2*f^2*n + (2*b*e^3
*f*g + b*d*e^2*g^2)*n*x^2 + (b*e^3*f^2 + 2*b*d*e^2*f*g)*n*x)*e^(3*(b*log(c
) + a)/(b*n)) + 3*(b*g^2*n*log(e*x + d) + b*g^2*log(c) + a*g^2)*log_integr
al((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c) + a)/(b*n))))*
e^(-3*(b*log(c) + a)/(b*n))/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c)
 + a*b^2*e^3*n^2)
```

3.95.6 Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx$$

input
```
integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**2, x)
```

3.95.7 Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*g^2*x^3 + d*f^2 + (2*e*f*g + d*g^2)*x^2 + (e*f^2 + 2*d*f*g)*x)/(b^2*e*
n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n) + integrate((3*e*g^2*x^2 +
e*f^2 + 2*d*f*g + 2*(2*e*f*g + d*g^2)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e
*n*log(c) + a*b*e*n), x)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2031 vs. 2(260) = 520.

Time = 0.39 (sec) , antiderivative size = 2031, normalized size of antiderivative = 7.84

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
b*e^2*f^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d
)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/n)
) - 2*b*d*e*f*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e
*x + d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c
^(1/n)) + b*d^2*g^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*l
og(e*x + d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^
2)*c^(1/n)) - (e*x + d)*b*e^2*f^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^
2*log(c) + a*b^2*e^3*n^2) - 2*(e*x + d)^2*b*e*f*g*n/(b^3*e^3*n^3*log(e*x +
 d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2) + 2*(e*x + d)*b*d*e*f*g*n/(b^3*e
^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2) - (e*x + d)^3*b*
g^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2) + 2*
(e*x + d)^2*b*d*g^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b
^2*e^3*n^2) - (e*x + d)*b*d^2*g^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^
2*log(c) + a*b^2*e^3*n^2) + 4*b*e*f*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(
e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3
*n^2*log(c) + a*b^2*e^3*n^2)*c^(2/n)) - 4*b*d*g^2*n*Ei(2*log(c)/n + 2*a/(b
*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^3*n^3*log(e*x +
d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(2/n)) + b*e^2*f^2*Ei(log(c)/n
+ a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e^3*n^3*log(e*x + d) +
 b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/n)) - 2*b*d*e*f*g*Ei(log(c)/n...
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

input
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.96 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.96.1 Optimal result

Integrand size = 22, antiderivative size = 177

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2} + \frac{2e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^2n^2} - \frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

output
```
(-d*g+e*f)*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^2/exp(a/b/n)/n^2/((
c*(e*x+d)^n)^(1/n))+2*g*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^2/
exp(2*a/b/n)/n^2/((c*(e*x+d)^n)^(2/n))-(e*x+d)*(g*x+f)/b/e/n/(a+b*ln(c*(e*
x+d)^n))
```

3.96.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\int \frac{f+gx}{(a+b\log (c(d+ex)^n))^2}\,dx =$$
$$-\frac{e^{-\frac{2a}{bn}}(d+ex)\,(c(d+ex)^n)^{-2/n}\left(bee^{\frac{2a}{bn}}n(c(d+ex)^n)^{2/n}\,(f+gx)-e^{\frac{a}{bn}}(ef-dg)\,(c(d+ex)^n)^{\frac{1}{n}}\,\text{ExpInteg}\right.}{b^2e^2n^2}$$

input
```
Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
-(((d + e*x)*(b*e*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n)*(f + g*x) - E^(a
/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(a + b*Log[c*(d +
 e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n]) - 2*g*(d + e*x)*ExpIntegralEi[
(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n])))/(b^2*e^
2*E^((2*a)/(b*n))*n^2*(c*(d + e*x)^n)^(2/n)*(a + b*Log[c*(d + e*x)^n])))
```

3.96.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(a+b\log (c(d+ex)^n))^2}\,dx$$
$$\Big\downarrow \; 2847$$
$$-\frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben\,(a+b\log (c(d+ex)^n))}$$
$$\Big\downarrow \; 2836$$
$$-\frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{be^2n}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben\,(a+b\log (c(d+ex)^n))}$$
$$\Big\downarrow \; 2737$$

$$-\frac{(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\int \frac{\left(c(d+ex)^n\right)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log\left(c(d+ex)^n\right)}{be^2n^2}+$$
$$\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

$\Big\downarrow$ 2609

$$\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-$$
$$\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}-$$
$$\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

$\Big\downarrow$ 2846

$$\frac{2\int \left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{bn}-$$
$$\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}-$$
$$\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

$\Big\downarrow$ 2009

$$-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}+$$
$$\frac{2\left(\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n}+\frac{ge^{-\frac{2a}{bn}}(d+ex)^2(c(d+ex)^n)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^2n}\right)}{bn}$$
$$\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

input
```
Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
-(((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/
(b^2*e^2*E^(a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1))) + (2*(((e*f - d*g)*(d +
e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e^2*E^(a/(b*n))*n
*(c*(d + e*x)^n)^n^(-1)) + (g*(d + e*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d
 + e*x)^n]))/(b*n)])/(b*e^2*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n))))/(b*
n) - ((d + e*x)*(f + g*x))/(b*e*n*(a + b*Log[c*(d + e*x)^n]))
```

3.96. $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.96.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.01 (sec) , antiderivative size = 2300, normalized size of antiderivative = 12.99

method	result	size
risch	Expression too large to display	2300

input
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(e*x+d)*(g*x+f)/b/e/n/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*
x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(
c)+2*a)-1/b^2/n^2*f*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^
n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c
)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b
*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*x-1/b^2/e/n^2*f*((e*x+d)^n)^(-1/n)*
c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)
+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e
*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-
I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn
(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*cs
gn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*
d-2/b^2/n^2*g*((e*x+d)^n)^(-2/n)*c^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+d)^n
)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a
)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*...
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.35

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx$$

$$= \frac{\left(\left(aef-adg+(bef-bdg)n\log(ex+d)+(bef-bdg)\log(c)\right)e^{\left(\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)\cdots}{bn}\right.}\right.\right.}{}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
((a*e*f - a*d*g + (b*e*f - b*d*g)*n*log(e*x + d) + (b*e*f - b*d*g)*log(c))
*e^((b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)))
 - (b*e^2*g*n*x^2 + b*d*e*f*n + (b*e^2*f + b*d*e*g)*n*x)*e^(2*(b*log(c) +
a)/(b*n)) + 2*(b*g*n*log(e*x + d) + b*g*log(c) + a*g)*log_integral((e^2*x^
2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))))*e^(-2*(b*log(c) + a)/(b*n)
)/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)
```

3.96.6 Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{f+gx}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2}\,dx$$

input
```
integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**2, x)
```

3.96.7 Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{gx+f}{\left(b\log\left((ex+d)^n c\right)+a\right)^2}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*g*x^2 + d*f + (e*f + d*g)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c
) + a*b*e*n) + integrate((2*e*g*x + e*f + d*g)/(b^2*e*n*log((e*x + d)^n) +
 b^2*e*n*log(c) + a*b*e*n), x)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 973, normalized size of antiderivative = 5.50

$$\int \frac{f + gx}{(a + b\log(c(d + ex)^n))^2} \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
b*e*f*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((
b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) -
b*d*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((
b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) -
(e*x + d)*b*e*f*n/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e
^2*n^2) - (e*x + d)^2*b*g*n/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c)
 + a*b^2*e^2*n^2) + (e*x + d)*b*d*g*n/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*
n^2*log(c) + a*b^2*e^2*n^2) + 2*b*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*
x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n
^2*log(c) + a*b^2*e^2*n^2)*c^(2/n)) + b*e*f*Ei(log(c)/n + a/(b*n) + log(e*
x + d))*e^(-a/(b*n))*log(c)/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c
) + a*b^2*e^2*n^2)*c^(1/n)) - b*d*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*
e^(-a/(b*n))*log(c)/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^
2*e^2*n^2)*c^(1/n)) + a*e*f*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b
*n))/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1
/n)) - a*d*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^2*
n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) + 2*b*g*Ei
(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e^2*
n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(2/n)) + 2*a*g*Ei
(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^2*n^3*...
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f + gx}{(a + b\log(c(d + ex)^n))^2} \, dx = \int \frac{f + g\,x}{(a + b\,\ln(c\,(d + e\,x)^n))^2} \, dx$$

input
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.97 $\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.97.1 Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

output
```
(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e/exp(a/b/n)/n^2/((c*(e*x+d)^n)^
(1/n))+(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

3.97.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Bigl(be^{\frac{a}{bn}}n(c(d+ex)^n)^{\frac{1}{n}} - \text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

output
```
-(((d + e*x)*(b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1) - ExpIntegralEi[(a +
b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])))/(b^2*e*E^(a/(b*n
))*n^2*(c*(d + e*x)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])))
```

3.97.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2836, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{bn}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{bn^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{e}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

output
```
(((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*E^(a/(b*
n))*n^2*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(b*n*(a + b*Log[c*(d + e*x)^n]
)))/e
```

3.97.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.97.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{2(ex+d)}{\left(-ib\pi\,\mathrm{csgn}(ic(ex+d)^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)+i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2b+i\pi\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2b-i\pi\,\mathrm{csgn}\right.}$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)
*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*
Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)/b/n/e*(e*x+d)-
1/b^2/n^2/e*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^
n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c
)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b
*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{\left((benx+bdn)e^{\left(\frac{b\log(c)+a}{bn}\right)} - (bn\log(ex+d)+b\log(c)+a)\,\mathrm{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)\right)e^{\left(-\frac{b\log(c)+a}{bn}\right)}}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
-((b*e*n*x + b*d*n)*e^((b*log(c) + a)/(b*n)) - (b*n*log(e*x + d) + b*log(c
) + a)*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))))*e^(-(b*log(c) + a
)/(b*n))/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)
```

3.97.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-2), x)
```

3.97.7 Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\log\left((ex+d)^nc\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*x + d)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n) + integra
te(1/(b^2*n*log((e*x + d)^n) + b^2*n*log(c) + a*b*n), x)
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(95) = 190$.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.98

$$\begin{aligned}\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx &= \frac{bn\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}\log\left(ex+d\right)}{(b^3en^3\log\left(ex+d\right)+b^3en^2\log\left(c\right)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} \\ &\quad-\frac{(ex+d)bn}{b^3en^3\log\left(ex+d\right)+b^3en^2\log\left(c\right)+ab^2en^2} \\ &\quad+\frac{b\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}\log\left(c\right)}{(b^3en^3\log\left(ex+d\right)+b^3en^2\log\left(c\right)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} \\ &\quad+\frac{a\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{(b^3en^3\log\left(ex+d\right)+b^3en^2\log\left(c\right)+ab^2en^2)c^{\left(\frac{1}{n}\right)}}\end{aligned}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^3*
e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) - (e*x + d)*
b*n/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2) + b*Ei(log(c
)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e*n^3*log(e*x + d)
 + b^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) + a*Ei(log(c)/n + a/(b*n) + lo
g(e*x + d))*e^(-a/(b*n))/((b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b
^2*e*n^2)*c^(1/n))
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.98 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx$

3.98.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.98.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

3.98.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

$$\Big\downarrow \text{ 2867}$$

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
$Aborted
```

3.98.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.98.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.98.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log((e*x + d)^n*c)^2 + 2*(
a*b*g*x + a*b*f)*log((e*x + d)^n*c)), x)
```

3.98.6 Sympy [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)), x)
```

3.98.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 7.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
(e*f - d*g)*integrate(1/(b^2*e*f^2*n*log(c) + a*b*e*f^2*n + (b^2*e*g^2*n*l
og(c) + a*b*e*g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(c) + a*b*e*f*g*n)*x + (b^2*e
*g^2*n*x^2 + 2*b^2*e*f*g*n*x + b^2*e*f^2*n)*log((e*x + d)^n)), x) - (e*x +
 d)/(b^2*e*f*n*log(c) + a*b*e*f*n + (b^2*e*g*n*log(c) + a*b*e*g*n)*x + (b^
2*e*g*n*x + b^2*e*f*n)*log((e*x + d)^n))
```

3.98.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^2), x)
```

3.98.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

3.99 $\int \frac{1}{(f+gx)^2(a+b\log(c(d+ex)^n))^2}\,dx$

3.99.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.99.2 Mathematica [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2), x]
```

3.99.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2} \, dx$$
$$\Big\downarrow \ 2867$$
$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2}dx$$

input
```
Int[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
$Aborted
```

3.99.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.99.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 (a+b\ln (c (ex+d)^n))^2}dx$$

input
```
int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.99.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.33

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2} \, dx = \int \frac{1}{(gx+f)^2(b\log ((ex+d)^n c)+a)^2} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*g^2*x^2 + 2*a^2*f*g*x + a^2*f^2 + (b^2*g^2*x^2 + 2*b^2*f*g
*x + b^2*f^2)*log((e*x + d)^n*c)^2 + 2*(a*b*g^2*x^2 + 2*a*b*f*g*x + a*b*f^
2)*log((e*x + d)^n*c)), x)
```

3.99.6 Sympy [N/A]

Not integrable

Time = 11.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2} \, dx = \int \frac{1}{(a+b\log (c(d+ex)^n))^2 (f+gx)^2} \, dx$$

input
```
integrate(1/(g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)**2), x)
```

3.99.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 12.04

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2} \, dx = \int \frac{1}{(gx+f)^2(b\log ((ex+d)^n c)+a)^2} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*x + d)/(b^2*e*f^2*n*log(c) + a*b*e*f^2*n + (b^2*e*g^2*n*log(c) + a*b*e
*g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(c) + a*b*e*f*g*n)*x + (b^2*e*g^2*n*x^2 +
2*b^2*e*f*g*n*x + b^2*e*f^2*n)*log((e*x + d)^n)) - integrate((e*g*x - e*f
+ 2*d*g)/(b^2*e*f^3*n*log(c) + a*b*e*f^3*n + (b^2*e*g^3*n*log(c) + a*b*e*g
^3*n)*x^3 + 3*(b^2*e*f*g^2*n*log(c) + a*b*e*f*g^2*n)*x^2 + 3*(b^2*e*f^2*g*
n*log(c) + a*b*e*f^2*g*n)*x + (b^2*e*g^3*n*x^3 + 3*b^2*e*f*g^2*n*x^2 + 3*b
^2*e*f^2*g*n*x + b^2*e*f^3*n)*log((e*x + d)^n)), x)
```

3.99.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2} \, dx = \int \frac{1}{(gx+f)^2(b\log \left((ex+d)^n c\right)+a)^2} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^2), x)
```

3.99.9 Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^2} \, dx = \int \frac{1}{(f+g\,x)^2\,(a+b\,\ln\left(c\,(d+e\,x)^n\right))^2} \, dx$$

input
```
int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^2), x)
```

3.100 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3} dx$

3.100.1 Optimal result

Integrand size = 24, antiderivative size = 351

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3} dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^3n^3} + \frac{4e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^3n^3} + \frac{9e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{2b^3e^3n^3} - \frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

output
```
1/2*(-d*g+e*f)^2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^3/exp(a/b/n)/
n^3/((c*(e*x+d)^n)^(1/n))+4*g*(-d*g+e*f)*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^
n))/b/n)/b^3/e^3/exp(2*a/b/n)/n^3/((c*(e*x+d)^n)^(2/n))+9/2*g^2*(e*x+d)^3*
Ei(3*(a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^3/exp(3*a/b/n)/n^3/((c*(e*x+d)^n)^(3
/n))-1/2*(e*x+d)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))^2+(-d*g+e*f)*(e*x+d
)*(g*x+f)/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))-3/2*(e*x+d)*(g*x+f)^2/b^2/e/n^
2/(a+b*ln(c*(e*x+d)^n))
```

3.100.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(e^{\frac{2a}{bn}}(ef-dg)^2\left(c(d+ex)^n\right)^{2/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(\right.}{}$$

input
```
Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
((d + e*x)*(E^((2*a)/(b*n))*(e*f - d*g)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegra
lEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])^2 - 8*E^(
a/(b*n))*g*(-(e*f) + d*g)*(d + e*x)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(
2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n])^2 + 9*g^2*
(d + e*x)^2*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log
[c*(d + e*x)^n])^2 - b*e*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n)*(f + g*x)
*(b*e*n*(f + g*x) + a*(e*f + 2*d*g + 3*e*g*x) + b*(2*d*g + e*(f + 3*g*x))*
Log[c*(d + e*x)^n])))/(2*b^3*e^3*E^((3*a)/(b*n))*n^3*(c*(d + e*x)^n)^(3/n)
*(a + b*Log[c*(d + e*x)^n])^2)
```

3.100.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 753 vs. 2(351) = 702.

Time = 1.75 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2847, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$$

$$\Big\downarrow \text{ 2847}$$

$$-\frac{(ef-dg)\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}dx}{ben}+\frac{3\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}dx}{2bn}-\frac{(d+ex)(f+gx)^2}{2ben\,(a+b\log(c(d+ex)^n))^2}$$

$$\Big\downarrow \text{ 2847}$$

$$-\frac{(ef-dg)\left(-\frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}\right)}{ben}+\frac{3\left(-\frac{2(ef-dg)\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{3\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)^2}{ben(a+b\log(c(d+ex)^n))}\right)}{2bn}-\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \ 2836$$

$$-\frac{(ef-dg)\left(-\frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{be^2n}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}\right)}{ben}+\frac{3\left(-\frac{2(ef-dg)\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{3\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)^2}{ben(a+b\log(c(d+ex)^n))}\right)}{2bn}-\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \ 2737$$

$$-\frac{(ef-dg)\left(-\frac{(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{be^2n^2}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}\right)}{ben}+\frac{3\left(-\frac{2(ef-dg)\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{3\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)^2}{ben(a+b\log(c(d+ex)^n))}\right)}{2bn}-\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \ 2609$$

$$-\frac{(ef-dg)\left(\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}\right)}{ben}+\frac{3\left(-\frac{2(ef-dg)\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{3\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)^2}{ben(a+b\log(c(d+ex)^n))}\right)}{2bn}-\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \ 2846$$

3.100. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$

$$-\frac{(ef-dg)\left(\frac{2\int\left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{bn}-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}\right.}{ben}$$

$$\frac{3\left(\frac{3\int\left(\frac{(ef-dg)^2}{e^2(a+b\log(c(d+ex)^n))}+\frac{2g(d+ex)(ef-dg)}{e^2(a+b\log(c(d+ex)^n))}+\frac{g^2(d+ex)^2}{e^2(a+b\log(c(d+ex)^n))}\right)dx}{bn}-\frac{2(ef-dg)\int\left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{ben}\right.}{2bn}$$

$$\frac{(d+ex)(f+gx)^2}{2ben\,(a+b\log(c(d+ex)^n))^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(ef-dg)\left(-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}+\frac{2\left(\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegr}}{be^2n}\right.}{}\right.}{ben}$$

$$\frac{3\left(\frac{3\left(\frac{2ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d}{bn}\right.}{be^3n}\right.}{bn}\right.}{}$$

$$\frac{(d+ex)(f+gx)^2}{2ben\,(a+b\log(c(d+ex)^n))^2}$$

input
```
Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^3,x]
```

3.100. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$

output
```
-1/2*((d + e*x)*(f + g*x)^2)/(b*e*n*(a + b*Log[c*(d + e*x)^n])^2) - ((e*f
- d*g)*(-(((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/
(b*n)])/(b^2*e^2*E^(a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1))) + (2*(((e*f - d*
g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e^2*E^(a/
(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (g*(d + e*x)^2*ExpIntegralEi[(2*(a + b*
Log[c*(d + e*x)^n]))/(b*n)])/(b*e^2*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n
))))/(b*n) - ((d + e*x)*(f + g*x))/(b*e*n*(a + b*Log[c*(d + e*x)^n]))))/(b
*e*n) + (3*((-2*(e*f - d*g)*(((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*L
og[c*(d + e*x)^n])/(b*n)])/(b*e^2*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) +
(g*(d + e*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^2
*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n))))/(b*e*n) + (3*(((e*f - d*g)^2*(
d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e^3*E^(a/(b*n
))*n*(c*(d + e*x)^n)^n^(-1)) + (2*g*(e*f - d*g)*(d + e*x)^2*ExpIntegralEi[
(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^3*E^((2*a)/(b*n))*n*(c*(d + e*
x)^n)^(2/n)) + (g^2*(d + e*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]
))/(b*n)])/(b*e^3*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n))))/(b*n) - ((d +
 e*x)*(f + g*x)^2)/(b*e*n*(a + b*Log[c*(d + e*x)^n]))))/(2*b*n)
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.100. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.100.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.73 (sec) , antiderivative size = 6545, normalized size of antiderivative = 18.65

method	result	size
risch	Expression too large to display	6545

input
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1090 vs. 2(344) = 688.

Time = 0.31 (sec) , antiderivative size = 1090, normalized size of antiderivative = 3.11

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
1/2*(8*(a^2*e*f*g - a^2*d*g^2 + (b^2*e*f*g - b^2*d*g^2)*n^2*log(e*x + d)^2
 + (b^2*e*f*g - b^2*d*g^2)*log(c)^2 + 2*((b^2*e*f*g - b^2*d*g^2)*n*log(c)
+ (a*b*e*f*g - a*b*d*g^2)*n)*log(e*x + d) + 2*(a*b*e*f*g - a*b*d*g^2)*log(
c))*e^((b*log(c) + a)/(b*n))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(
b*log(c) + a)/(b*n))) + (a^2*e^2*f^2 - 2*a^2*d*e*f*g + a^2*d^2*g^2 + (b^2*
e^2*f^2 - 2*b^2*d*e*f*g + b^2*d^2*g^2)*n^2*log(e*x + d)^2 + (b^2*e^2*f^2 -
 2*b^2*d*e*f*g + b^2*d^2*g^2)*log(c)^2 + 2*((b^2*e^2*f^2 - 2*b^2*d*e*f*g +
 b^2*d^2*g^2)*n*log(c) + (a*b*e^2*f^2 - 2*a*b*d*e*f*g + a*b*d^2*g^2)*n)*lo
g(e*x + d) + 2*(a*b*e^2*f^2 - 2*a*b*d*e*f*g + a*b*d^2*g^2)*log(c))*e^(2*(b
*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) - (b^
2*d*e^2*f^2*n^2 + (b^2*e^3*g^2*n^2 + 3*a*b*e^3*g^2*n)*x^3 + ((2*b^2*e^3*f*
g + b^2*d*e^2*g^2)*n^2 + (4*a*b*e^3*f*g + 5*a*b*d*e^2*g^2)*n)*x^2 + (a*b*d
*e^2*f^2 + 2*a*b*d^2*e*f*g)*n + ((b^2*e^3*f^2 + 2*b^2*d*e^2*f*g)*n^2 + (a*
b*e^3*f^2 + 6*a*b*d*e^2*f*g + 2*a*b*d^2*e*g^2)*n)*x + (3*b^2*e^3*g^2*n^2*x
^3 + (4*b^2*e^3*f*g + 5*b^2*d*e^2*g^2)*n^2*x^2 + (b^2*e^3*f^2 + 6*b^2*d*e^
2*f*g + 2*b^2*d^2*e*g^2)*n^2*x + (b^2*d*e^2*f^2 + 2*b^2*d^2*e*f*g)*n^2)*lo
g(e*x + d) + (3*b^2*e^3*g^2*n*x^3 + (4*b^2*e^3*f*g + 5*b^2*d*e^2*g^2)*n*x^
2 + (b^2*e^3*f^2 + 6*b^2*d*e^2*f*g + 2*b^2*d^2*e*g^2)*n*x + (b^2*d*e^2*f^2
 + 2*b^2*d^2*e*f*g)*n)*log(c))*e^(3*(b*log(c) + a)/(b*n)) + 9*(b^2*g^2*n^2
*log(e*x + d)^2 + b^2*g^2*log(c)^2 + 2*a*b*g^2*log(c) + a^2*g^2 + 2*(b^...
```

3.100.6 Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log (c(d+ex)^n))^3}\,dx=\int \frac{(f+gx)^2}{(a+b\log (c\,(d+ex)^n))^3}\,dx$$

input
```
integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**3, x)
```

3.100.7 Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^3}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
-1/2*((3*a*e^2*g^2 + (e^2*g^2*n + 3*e^2*g^2*log(c))*b)*x^3 + ((4*e^2*f*g +
 5*d*e*g^2)*a + (2*e^2*f*g*n + d*e*g^2*n + (4*e^2*f*g + 5*d*e*g^2)*log(c))
*b)*x^2 + (d*e*f^2 + 2*d^2*f*g)*a + (d*e*f^2*n + (d*e*f^2 + 2*d^2*f*g)*log
(c))*b + ((e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*a + (e^2*f^2*n + 2*d*e*f*g*n +
 (e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*log(c))*b)*x + (3*b*e^2*g^2*x^3 + (4*e^
2*f*g + 5*d*e*g^2)*b*x^2 + (e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*b*x + (d*e*f^
2 + 2*d^2*f*g)*b)*log((e*x + d)^n))/(b^4*e^2*n^2*log((e*x + d)^n)^2 + b^4*
e^2*n^2*log(c)^2 + 2*a*b^3*e^2*n^2*log(c) + a^2*b^2*e^2*n^2 + 2*(b^4*e^2*n
^2*log(c) + a*b^3*e^2*n^2)*log((e*x + d)^n)) + integrate(1/2*(9*e^2*g^2*x^
2 + e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2 + 2*(4*e^2*f*g + 5*d*e*g^2)*x)/(b^3*e^
2*n^2*log((e*x + d)^n) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2), x)
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8422 vs. 2(344) = 688.

Time = 0.49 (sec) , antiderivative size = 8422, normalized size of antiderivative = 23.99

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
1/2*b^2*e^2*f^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log
(e*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(
c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3
*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - b^2*d*e*f*g*n^2*Ei(log(c)/n + a/(b*n
) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2
 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*
e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 1/
2*b^2*d^2*g^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e
*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c)
 + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*l
og(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*e^2*f^2*n^2*log(e*x
+ d)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a
*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c)
+ a^2*b^3*e^3*n^3) - 2*(e*x + d)^2*b^2*e*f*g*n^2*log(e*x + d)/(b^5*e^3*n^5
*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(
e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3
) + (e*x + d)*b^2*d*e*f*g*n^2*log(e*x + d)/(b^5*e^3*n^5*log(e*x + d)^2 + 2
*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*
n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 3/2*(e*x + d)^3
*b^2*g^2*n^2*log(e*x + d)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*l...
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^3}\,dx$$

input
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^3, x)
```

3.101 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$

3.101.1 Optimal result

Integrand size = 22, antiderivative size = 261

$$\begin{aligned}\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^2n^3} \\ &\quad+\frac{2e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^2n^3} \\ &\quad-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} \\ &\quad+\frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)}-\frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\end{aligned}$$

output
```
1/2*(-d*g+e*f)*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^2/exp(a/b/n)/n^
3/((c*(e*x+d)^n)^(1/n))+2*g*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b^3/
e^2/exp(2*a/b/n)/n^3/((c*(e*x+d)^n)^(2/n))-1/2*(e*x+d)*(g*x+f)/b/e/n/(a+b*
ln(c*(e*x+d)^n))^2+1/2*(-d*g+e*f)*(e*x+d)/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n)
)-(e*x+d)*(g*x+f)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))
```

3.101.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx =$$
$$-\frac{e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(-e^{\frac{a}{bn}}(ef-dg)\left(c(d+ex)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log\right.}{}$$

input
```
Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
-1/2*((d + e*x)*(-(E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*ExpInteg
ralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])^2) - 4*
g*(d + e*x)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log
[c*(d + e*x)^n])^2 + b*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n)*(b*e*n*(f +
 g*x) + a*(e*f + d*g + 2*e*g*x) + b*(d*g + e*(f + 2*g*x))*Log[c*(d + e*x)^
n])))/(b^3*e^2*E^((2*a)/(b*n))*n^3*(c*(d + e*x)^n)^(2/n)*(a + b*Log[c*(d +
 e*x)^n])^2)
```

3.101.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2847, 2836, 2734, 2737, 2609, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$$
$$\Big\downarrow \text{2847}$$
$$-\frac{(ef-dg)\int \frac{1}{(a+b\log(c(d+ex)^n))^2}dx}{2ben}+\frac{\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}dx}{bn}-\frac{(d+ex)(f+gx)}{2ben(a+b\log(c(d+ex)^n))^2}$$
$$\Big\downarrow \text{2836}$$
$$-\frac{(ef-dg)\int \frac{1}{(a+b\log(c(d+ex)^n))^2}d(d+ex)}{2be^2n}+\frac{\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}dx}{bn}-\frac{(d+ex)(f+gx)}{2ben(a+b\log(c(d+ex)^n))^2}$$
$$\Big\downarrow \text{2734}$$

$$-\frac{(ef-dg)\left(\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{bn}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}+\frac{\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}dx}{bn}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \text{ 2737}$$

$$-\frac{(ef-dg)\left(\frac{(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{bn^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}+\frac{\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}dx}{bn}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}dx}{bn}-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \text{ 2847}$$

$$\frac{-\frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}dx}{ben}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}}{bn}-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{-\frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{be^2n}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}}{bn}-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$$\Big\downarrow \text{ 2737}$$

3.101. $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$

$$\frac{-\frac{(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{be^2n^2}+\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}}{bn}-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$\Big\downarrow$ 2609

$$\frac{\frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}dx}{bn}-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}-\frac{(d+ex)(f+gx)}{ben(a+b\log(c(d+ex)^n))}}{bn}-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$\Big\downarrow$ 2846

$$\frac{\frac{2\int \left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{bn}-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}-\frac{(d+ex)(\cdots}{ben(a+b\log(\cdots}}{bn}-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

$\Big\downarrow$ 2009

$$-\frac{(ef-dg)\left(\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}\right)}{2be^2n}+\frac{-\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}+\frac{2\left(\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+e\cdots}{bn}\right.}{be^2n}\cdots\right.}{\cdots}}{bn}-\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

input
```
Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^3,x]
```

3.101. $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$

output
```
-1/2*((d + e*x)*(f + g*x))/(b*e*n*(a + b*Log[c*(d + e*x)^n])^2) - ((e*f -
d*g)*(((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*E^(
a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(b*n*(a + b*Log[c*(d + e*
x)^n]))))/(2*b*e^2*n) + (-(((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log
[c*(d + e*x)^n])/(b*n)])/(b^2*e^2*E^(a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1)))
 + (2*(((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*
n)])/(b*e^2*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (g*(d + e*x)^2*ExpInte
gralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^2*E^((2*a)/(b*n))*n*(c*
(d + e*x)^n)^(2/n))))/(b*n) - ((d + e*x)*(f + g*x))/(b*e*n*(a + b*Log[c*(d
 + e*x)^n])))/(b*n)
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.101.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 3114, normalized size of antiderivative = 11.93

method	result	size
risch	Expression too large to display	3114

input
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-(2*ln(c)*b*d^2*g-I*Pi*b*d^2*g*csgn(I*c*(e*x+d)^n)^3+6*b*d*e*g*x*ln((e*x+d
)^n)-3*I*Pi*b*d*e*g*x*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+6*ln
(c)*b*d*e*g*x+2*b*e^2*g*n*x^2+2*b*e^2*f*n*x+2*b*e^2*f*x*ln((e*x+d)^n)+2*b*
d*e*f*ln((e*x+d)^n)+I*Pi*b*d^2*g*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*d^
2*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2+2*ln(c)*b*d*e*f-2*I*Pi*b*e^2*g
*x^2*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+3*I*Pi*b*d*e*g*x*csgn
(I*c)*csgn(I*c*(e*x+d)^n)^2+2*b*d*e*g*n*x+4*b*e^2*g*x^2*ln((e*x+d)^n)+4*ln
(c)*b*e^2*g*x^2+2*ln(c)*b*e^2*f*x+6*a*d*e*g*x+3*I*Pi*b*d*e*g*x*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e^2*f*x*csgn(I*c*(e*x+d)^n)^3-2*I*Pi*b
*e^2*g*x^2*csgn(I*c*(e*x+d)^n)^3+2*a*d^2*g+4*a*e^2*g*x^2+2*a*e^2*f*x+2*b*d
^2*g*ln((e*x+d)^n)-I*Pi*b*d*e*f*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+
d)^n)+2*I*Pi*b*e^2*g*x^2*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*d^2*g*csgn
(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+2*I*Pi*b*e^2*g*x^2*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)^2-3*I*Pi*b*d*e*g*x*csgn(I*c*(e*x+d)^n)^3-I*Pi*b
*d*e*f*csgn(I*c*(e*x+d)^n)^3+2*b*d*e*f*n-I*Pi*b*e^2*f*x*csgn(I*c)*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)+2*a*d*e*f+I*Pi*b*d*e*f*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)^2+I*Pi*b*e^2*f*x*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*d
*e*f*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*e^2*f*x*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)^2)/b^2/e^2/n^2/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn
(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d...
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(255) = 510$.

Time = 0.32 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.25

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{\left(((b^2ef-b^2dg)n^2\log(ex+d)^2+a^2ef-a^2dg+(b^2ef-b^2dg)\log(c)^2+2((b^2ef-b^2dg)n\log(c)+(ab\right.}{}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
1/2*(((b^2*e*f - b^2*d*g)*n^2*log(e*x + d)^2 + a^2*e*f - a^2*d*g + (b^2*e*
f - b^2*d*g)*log(c)^2 + 2*((b^2*e*f - b^2*d*g)*n*log(c) + (a*b*e*f - a*b*d
*g)*n)*log(e*x + d) + 2*(a*b*e*f - a*b*d*g)*log(c))*e^((b*log(c) + a)/(b*n
))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) - (b^2*d*e*f*n^2 + (b^
2*e^2*g*n^2 + 2*a*b*e^2*g*n)*x^2 + (a*b*d*e*f + a*b*d^2*g)*n + ((b^2*e^2*f
 + b^2*d*e*g)*n^2 + (a*b*e^2*f + 3*a*b*d*e*g)*n)*x + (2*b^2*e^2*g*n^2*x^2
+ (b^2*e^2*f + 3*b^2*d*e*g)*n^2*x + (b^2*d*e*f + b^2*d^2*g)*n^2)*log(e*x +
 d) + (2*b^2*e^2*g*n*x^2 + (b^2*e^2*f + 3*b^2*d*e*g)*n*x + (b^2*d*e*f + b^
2*d^2*g)*n)*log(c))*e^(2*(b*log(c) + a)/(b*n)) + 4*(b^2*g*n^2*log(e*x + d)
^2 + b^2*g*log(c)^2 + 2*a*b*g*log(c) + a^2*g + 2*(b^2*g*n*log(c) + a*b*g*n
)*log(e*x + d))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)
/(b*n))))*e^(-2*(b*log(c) + a)/(b*n))/(b^5*e^2*n^5*log(e*x + d)^2 + b^5*e^
2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3 + 2*(b^5*e^2*n^4
*log(c) + a*b^4*e^2*n^4)*log(e*x + d))
```

3.101.6 Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$$

input
```
integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**3, x)
```

3.101.7 Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{gx+f}{(b\log((ex+d)^n c)+a)^3}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a*e^2*g + (e^2*g*n + 2*e^2*g*log(c))*b)*x^2 + (d*e*f + d^2*g)*a +
 (d*e*f*n + (d*e*f + d^2*g)*log(c))*b + ((e^2*f + 3*d*e*g)*a + (e^2*f*n +
d*e*g*n + (e^2*f + 3*d*e*g)*log(c))*b)*x + (2*b*e^2*g*x^2 + (e^2*f + 3*d*e
*g)*b*x + (d*e*f + d^2*g)*b)*log((e*x + d)^n))/(b^4*e^2*n^2*log((e*x + d)^
n)^2 + b^4*e^2*n^2*log(c)^2 + 2*a*b^3*e^2*n^2*log(c) + a^2*b^2*e^2*n^2 + 2
*(b^4*e^2*n^2*log(c) + a*b^3*e^2*n^2)*log((e*x + d)^n)) + integrate(1/2*(4
*e*g*x + e*f + 3*d*g)/(b^3*e*n^2*log((e*x + d)^n) + b^3*e*n^2*log(c) + a*b
^2*e*n^2), x)
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4112 vs. $2(255) = 510$.

Time = 0.43 (sec) , antiderivative size = 4112, normalized size of antiderivative = 15.75

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
1/2*b^2*e*f*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x
 + d)^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) +
 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log
(c) + a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2*b^2*d*g*n^2*Ei(log(c)/n + a/(b*n) +
log(e*x + d))*e^(-a/(b*n))*log(e*x + d)^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2
*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*
n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2*(e
*x + d)*b^2*e*f*n^2*log(e*x + d)/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n
^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)
^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) - (e*x + d)^2*b^2*g*n^2*log
(e*x + d)/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c)
+ 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*lo
g(c) + a^2*b^3*e^2*n^3) + 1/2*(e*x + d)*b^2*d*g*n^2*log(e*x + d)/(b^5*e^2*
n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*l
og(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*
n^3) + 2*b^2*g*n^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*
n))*log(e*x + d)^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x +
d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*
e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(2/n)) + b^2*e*f*n*Ei(log(c)/n + a/(b*
n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e^2*n^5*log(e...
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{f+g\,x}{(a+b\,\ln(c\,(d+e\,x)^n))^3}\,dx$$

input
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^3, x)
```

3.102 $\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx$

3.102.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2ben(a+b\log(c(d+ex)^n))^2} - \frac{d+ex}{2b^2en^2(a+b\log(c(d+ex)^n))}$$

output
```
1/2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e/exp(a/b/n)/n^3/((c*(e*x+d)
^n)^(1/n))+1/2*(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^2+1/2*(-e*x-d)/b^2/e/n
^2/(a+b*ln(c*(e*x+d)^n))
```

3.102.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\left(-\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^2 + be^{\frac{a}{bn}}n(c(\ldots\right.}{2b^3en^3(a+b\log(c(d+ex)^n))^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

output
```
-1/2*((d + e*x)*(-(ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*
Log[c*(d + e*x)^n])^2) + b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*(a + b*n +
 b*Log[c*(d + e*x)^n])))/(b^3*e*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1)*(a
+ b*Log[c*(d + e*x)^n])^2)
```

3.102.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2836, 2734, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^2}d(d+ex)}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\frac{\int \frac{1}{a+b\log(c(d+ex)^n)}d(d+ex)}{bn}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{\frac{(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{a+b\log(c(d+ex)^n)}d\log(c(d+ex)^n)}{bn^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{\frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2n^2}-\frac{d+ex}{bn(a+b\log(c(d+ex)^n))}}{2bn}-\frac{d+ex}{2bn(a+b\log(c(d+ex)^n))^2}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

output
```
(-1/2*(d + e*x)/(b*n*(a + b*Log[c*(d + e*x)^n])^2) + (((d + e*x)*ExpIntegr
alEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*E^(a/(b*n))*n^2*(c*(d + e*x)^
n)^n^(-1)) - (d + e*x)/(b*n*(a + b*Log[c*(d + e*x)^n])))/(2*b*n))/e
```

3.102.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.102.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 734, normalized size of antiderivative = 5.44

method	result
risch	$-\frac{2benx+2bdn+i\pi bd\ \mathrm{csgn}(i(ex+d)^n)\ \mathrm{csgn}(ic(ex+d)^n)^2+i\pi bd\ \mathrm{csgn}(ic)\ \mathrm{csgn}(ic(ex+d)^n)^2-i\pi bd\ \mathrm{csgn}(ic)\ \mathrm{csgn}(i(ex+d)^n)\ \mathrm{csgn}(ic(ex}{\left(-ib\pi\ \mathrm{csgn}(ic(ex+d)^n)\ \mathrm{csgn}}$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-(2*b*e*n*x+2*b*d*n+I*Pi*b*d*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2+I*Pi*
b*d*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*d*csgn(I*c)*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)-I*Pi*b*d*csgn(I*c*(e*x+d)^n)^3-I*Pi*b*e*x*csgn(I*c*(e*x
+d)^n)^3+I*Pi*b*e*x*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e*x*csg
n(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*e*x*csgn(I*c)*csgn(I*c
*(e*x+d)^n)^2+2*ln(c)*b*e*x+2*b*e*x*ln((e*x+d)^n)+2*d*b*ln(c)+2*a*e*x+2*b*
d*ln((e*x+d)^n)+2*a*d)/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+
d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)
+2*a)^2/b^2/n^2/e-1/2/b^3/n^3/e*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1
/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)
*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*
Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^
n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(128) = 256$.

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx =$$

$$-\frac{\left((b^2dn^2+abdn+(b^2en^2+aben)x+(b^2en^2x+b^2dn^2)\log(ex+d)+(b^2enx+b^2dn)\log(c)\right)e^{\left(\frac{b\log(c)+a}{bn}\right.}}{2\left(b^5en^5\log(ex+d)^2+b^5en^3\log(c\right.}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*d*n^2 + a*b*d*n + (b^2*e*n^2 + a*b*e*n)*x + (b^2*e*n^2*x + b^2*
d*n^2)*log(e*x + d) + (b^2*e*n*x + b^2*d*n)*log(c))*e^((b*log(c) + a)/(b*n
)) - (b^2*n^2*log(e*x + d)^2 + b^2*log(c)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*
n*log(c) + a*b*n)*log(e*x + d))*log_integral((e*x + d)*e^((b*log(c) + a)/(
b*n))))*e^(-(b*log(c) + a)/(b*n))/(b^5*e*n^5*log(e*x + d)^2 + b^5*e*n^3*lo
g(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3 + 2*(b^5*e*n^4*log(c) + a*b^
4*e*n^4)*log(e*x + d))
```

3.102.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{3}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-3), x)
```

3.102.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{3}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
-1/2*((d*n + d*log(c))*b + a*d + ((e*n + e*log(c))*b + a*e)*x + (b*e*x + b
*d)*log((e*x + d)^n))/(b^4*e*n^2*log((e*x + d)^n)^2 + b^4*e*n^2*log(c)^2 +
 2*a*b^3*e*n^2*log(c) + a^2*b^2*e*n^2 + 2*(b^4*e*n^2*log(c) + a*b^3*e*n^2)
*log((e*x + d)^n)) + integrate(1/2/(b^3*n^2*log((e*x + d)^n) + b^3*n^2*log
(c) + a*b^2*n^2), x)
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. 2(128) = 256.

Time = 0.33 (sec) , antiderivative size = 1218, normalized size of antiderivative = 9.02

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
1/2*b^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d
)^2/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4
*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*
e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n^2*log(e*x + d)/(b^5*e*n^5*log(e*x +
d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*
e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + b^2*n*Ei(log(c)/n
 + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e*n^5*lo
g(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d
) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) -
1/2*(e*x + d)*b^2*n^2/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)
*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*
log(c) + a^2*b^3*e*n^3) + a*b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-
a/(b*n))*log(e*x + d)/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d
)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3
*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n*log(c)/(b^5*e*n^5*
log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x +
 d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + 1/2*b^2
*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e*n^5*l
og(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x +
d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)...
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d+e\,x)^n))^3}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^3, x)
```

3.103 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^3} dx$

3.103.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3} dx = \operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}, x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x)
```

3.103.2 Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3} dx = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3} dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3), x]
```

3.103.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^3} \, dx$$
$$\Big\downarrow \text{ 2867}$$
$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^3}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3),x]
```

output
```
$Aborted
```

3.103.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.103.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{3}}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x)
```

3.103.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.96

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^3}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*g*x + a^3*f + (b^3*g*x + b^3*f)*log((e*x + d)^n*c)^3 + 3*(
a*b^2*g*x + a*b^2*f)*log((e*x + d)^n*c)^2 + 3*(a^2*b*g*x + a^2*b*f)*log((e
*x + d)^n*c)), x)
```

3.103.6 Sympy [N/A]

Not integrable

Time = 6.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^3(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**3*(f + g*x)), x)
```

3.103.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 703, normalized size of antiderivative = 29.29

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^3}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
-1/2*(b*e^2*g*n*x^2 + (d*e*f - d^2*g)*a + (d*e*f*n + (d*e*f - d^2*g)*log(c
))*b + ((e^2*f - d*e*g)*a + (e^2*f*n + d*e*g*n + (e^2*f - d*e*g)*log(c))*b
)*x + ((e^2*f - d*e*g)*b*x + (d*e*f - d^2*g)*b)*log((e*x + d)^n))/(b^4*e^2
*f^2*n^2*log(c)^2 + 2*a*b^3*e^2*f^2*n^2*log(c) + a^2*b^2*e^2*f^2*n^2 + (b^
4*e^2*g^2*n^2*log(c)^2 + 2*a*b^3*e^2*g^2*n^2*log(c) + a^2*b^2*e^2*g^2*n^2)
*x^2 + (b^4*e^2*g^2*n^2*x^2 + 2*b^4*e^2*f*g*n^2*x + b^4*e^2*f^2*n^2)*log((
e*x + d)^n)^2 + 2*(b^4*e^2*f*g*n^2*log(c)^2 + 2*a*b^3*e^2*f*g*n^2*log(c) +
 a^2*b^2*e^2*f*g*n^2)*x + 2*(b^4*e^2*f^2*n^2*log(c) + a*b^3*e^2*f^2*n^2 +
(b^4*e^2*g^2*n^2*log(c) + a*b^3*e^2*g^2*n^2)*x^2 + 2*(b^4*e^2*f*g*n^2*log(
c) + a*b^3*e^2*f*g*n^2)*x)*log((e*x + d)^n)) + integrate(1/2*(e^2*f^2 - 3*
d*e*f*g + 2*d^2*g^2 - (e^2*f*g - d*e*g^2)*x)/(b^3*e^2*f^3*n^2*log(c) + a*b
^2*e^2*f^3*n^2 + (b^3*e^2*g^3*n^2*log(c) + a*b^2*e^2*g^3*n^2)*x^3 + 3*(b^3
*e^2*f*g^2*n^2*log(c) + a*b^2*e^2*f*g^2*n^2)*x^2 + 3*(b^3*e^2*f^2*g*n^2*lo
g(c) + a*b^2*e^2*f^2*g*n^2)*x + (b^3*e^2*g^3*n^2*x^3 + 3*b^3*e^2*f*g^2*n^2
*x^2 + 3*b^3*e^2*f^2*g*n^2*x + b^3*e^2*f^3*n^2)*log((e*x + d)^n)), x)
```

3.103.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^3}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^3), x)
```

3.103.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^3}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^3),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^3), x)
```

3.104 $\int \frac{1}{(f+gx)^2(a+b\log(c(d+ex)^n))^3}\,dx$

3.104.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x)
```

3.104.2 Mathematica [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3),x]
```

output
```
Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3), x]
```

3.104.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^3} \, dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^3}dx$$

input
```
Int[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3),x]
```

output
```
$Aborted
```

3.104.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.104.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{2} \left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{3}}dx$$

input
```
int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x)
```

output
```
int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x)
```

3.104.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.38

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^3} \, dx = \int \frac{1}{(gx+f)^2(b\log ((ex+d)^n c)+a)^3} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*g^2*x^2 + 2*a^3*f*g*x + a^3*f^2 + (b^3*g^2*x^2 + 2*b^3*f*g
*x + b^3*f^2)*log((e*x + d)^n*c)^3 + 3*(a*b^2*g^2*x^2 + 2*a*b^2*f*g*x + a*
b^2*f^2)*log((e*x + d)^n*c)^2 + 3*(a^2*b*g^2*x^2 + 2*a^2*b*f*g*x + a^2*b*f
^2)*log((e*x + d)^n*c)), x)
```

3.104.6 Sympy [N/A]

Not integrable

Time = 35.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^3} \, dx = \int \frac{1}{(a+b\log (c(d+ex)^n))^3 (f+gx)^2} \, dx$$

input
```
integrate(1/(g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**3*(f + g*x)**2), x)
```

3.104.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 934, normalized size of antiderivative = 38.92

$$\int \frac{1}{(f+gx)^2 (a+b\log (c(d+ex)^n))^3} \, dx = \int \frac{1}{(gx+f)^2(b\log ((ex+d)^n c)+a)^3} \, dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
1/2*((a*e^2*g - (e^2*g*n - e^2*g*log(c))*b)*x^2 - (d*e*f - 2*d^2*g)*a - (d
*e*f*n + (d*e*f - 2*d^2*g)*log(c))*b - ((e^2*f - 3*d*e*g)*a + (e^2*f*n + d
*e*g*n + (e^2*f - 3*d*e*g)*log(c))*b)*x + (b*e^2*g*x^2 - (e^2*f - 3*d*e*g)
*b*x - (d*e*f - 2*d^2*g)*b)*log((e*x + d)^n))/(b^4*e^2*f^3*n^2*log(c)^2 +
2*a*b^3*e^2*f^3*n^2*log(c) + a^2*b^2*e^2*f^3*n^2 + (b^4*e^2*g^3*n^2*log(c)
^2 + 2*a*b^3*e^2*g^3*n^2*log(c) + a^2*b^2*e^2*g^3*n^2)*x^3 + 3*(b^4*e^2*f*
g^2*n^2*log(c)^2 + 2*a*b^3*e^2*f*g^2*n^2*log(c) + a^2*b^2*e^2*f*g^2*n^2)*x
^2 + (b^4*e^2*g^3*n^2*x^3 + 3*b^4*e^2*f*g^2*n^2*x^2 + 3*b^4*e^2*f^2*g*n^2*
x + b^4*e^2*f^3*n^2)*log((e*x + d)^n)^2 + 3*(b^4*e^2*f^2*g*n^2*log(c)^2 +
2*a*b^3*e^2*f^2*g*n^2*log(c) + a^2*b^2*e^2*f^2*g*n^2)*x + 2*(b^4*e^2*f^3*n
^2*log(c) + a*b^3*e^2*f^3*n^2 + (b^4*e^2*g^3*n^2*log(c) + a*b^3*e^2*g^3*n^
2)*x^3 + 3*(b^4*e^2*f*g^2*n^2*log(c) + a*b^3*e^2*f*g^2*n^2)*x^2 + 3*(b^4*e
^2*f^2*g*n^2*log(c) + a*b^3*e^2*f^2*g*n^2)*x)*log((e*x + d)^n)) + integrat
e(1/2*(e^2*g^2*x^2 + e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2 - 2*(2*e^2*f*g - 3*d*
e*g^2)*x)/(b^3*e^2*f^4*n^2*log(c) + a*b^2*e^2*f^4*n^2 + (b^3*e^2*g^4*n^2*l
og(c) + a*b^2*e^2*g^4*n^2)*x^4 + 4*(b^3*e^2*f*g^3*n^2*log(c) + a*b^2*e^2*f
*g^3*n^2)*x^3 + 6*(b^3*e^2*f^2*g^2*n^2*log(c) + a*b^2*e^2*f^2*g^2*n^2)*x^2
 + 4*(b^3*e^2*f^3*g*n^2*log(c) + a*b^2*e^2*f^3*g*n^2)*x + (b^3*e^2*g^4*n^2
*x^4 + 4*b^3*e^2*f*g^3*n^2*x^3 + 6*b^3*e^2*f^2*g^2*n^2*x^2 + 4*b^3*e^2*f^3
*g*n^2*x + b^3*e^2*f^4*n^2)*log((e*x + d)^n)), x)
```

3.104.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(gx+f)^2(b\log\left((ex+d)^n c\right)+a)^3}\,dx$$

input
```
integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^3), x)
```

3.104.9 Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^3} \, dx = \int \frac{1}{(f+g\,x)^2 \left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^3} \, dx$$

input
```
int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^3),x)
```

output
```
int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^3), x)
```

3.105 $\int (f+gx)^2 \sqrt{a+b\log(c(d+ex)^n)}\,dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 404

$$\begin{aligned}
&\int (f+gx)^2\sqrt{a+b\log(c(d+ex)^n)}\,dx \\
&= -\frac{\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} \\
&- \frac{\sqrt{b}e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{n}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} \\
&- \frac{\sqrt{b}e^{-\frac{3a}{bn}}g^2\sqrt{n}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{6e^3} \\
&+ \frac{(ef-dg)^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{e^3} \\
&+ \frac{g(ef-dg)(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{e^3} + \frac{g^2(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{3e^3}
\end{aligned}$$

output
```
-1/18*g^2*(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/
2))*b^(1/2)*n^(1/2)*3^(1/2)*Pi^(1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n)
)-1/4*g*(-d*g+e*f)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1
/2)/n^(1/2))*b^(1/2)*n^(1/2)*2^(1/2)*Pi^(1/2)/e^3/exp(2*a/b/n)/((c*(e*x+d)
^n)^(2/n))-1/2*(-d*g+e*f)^2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/
2)/n^(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+
(-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3+g*(-d*g+e*f)*(e*x+d)^
2*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3+1/3*g^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^
(1/2)/e^3
```

3.105.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.93

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$= \frac{(d+ex)\left(-18\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{n}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+9\sqrt{b}e^{-\frac{2a}{bn}}g(-ef+dg)\sqrt{}\right.}{}$$

input
```
Integrate[(f + g*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
((d + e*x)*((-18*Sqrt[b]*(e*f - d*g)^2*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Lo
g[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))
 + (9*Sqrt[b]*g*(-(e*f) + d*g)*Sqrt[n]*Sqrt[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*
Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))*(c*(d
 + e*x)^n)^(2/n)) - (2*Sqrt[b]*g^2*Sqrt[n]*Sqrt[3*Pi]*(d + e*x)^2*Erfi[(Sq
rt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((3*a)/(b*n))
*(c*(d + e*x)^n)^(3/n)) + 36*(e*f - d*g)^2*Sqrt[a + b*Log[c*(d + e*x)^n]]
+ 36*g*(e*f - d*g)*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]] + 12*g^2*(d +
e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]]))/(36*e^3)
```

3.105.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(ef-dg)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2}+\frac{2g(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2}+\frac{g^2(d+ex)^2\sqrt{a+b\log(}}{e^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}g\sqrt{n}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3}-\frac{\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}g^2\sqrt{n}e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{6e^3}+\frac{g(d+ex)^2(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3}+\frac{(d+ex)(ef-dg)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3}+\frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3}$$

input
```
Int[(f + g*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
-1/2*(Sqrt[b]*(e*f - d*g)^2*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log
[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(e^3*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-
1)) - (Sqrt[b]*g*(e*f - d*g)*Sqrt[n]*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*
Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(2*e^3*E^((2*a)/(b*n))
*(c*(d + e*x)^n)^(2/n)) - (Sqrt[b]*g^2*Sqrt[n]*Sqrt[Pi/3]*(d + e*x)^3*Erfi
[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(6*e^3*E^((3
*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + ((e*f - d*g)^2*(d + e*x)*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/e^3 + (g*(e*f - d*g)*(d + e*x)^2*Sqrt[a + b*Log[c*(d +
e*x)^n]])/e^3 + (g^2*(d + e*x)^3*Sqrt[a + b*Log[c*(d + e*x)^n]])/(3*e^3)
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.105.4 Maple [F]

$$\int (gx+f)^2 \sqrt{a+b\ln \left(c \left(ex+d\right)^n\right)}dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.105.5 Fricas [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.105.6 Sympy [F]

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x)**2, x)
```

3.105.7 Maxima [F]

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (gx+f)^2\sqrt{b\log\left((ex+d)^nc\right)+a}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.105.8 Giac [F]

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (gx+f)^2\sqrt{b\log\left((ex+d)^nc\right)+a}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (f+g\,x)^2\,\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.106 $\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$

3.106.1 Optimal result

Integrand size = 24, antiderivative size = 255

$$\begin{aligned}\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx &= -\frac{\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^2}\\ &\quad-\frac{\sqrt{b}e^{-\frac{2a}{bn}}g\sqrt{n}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}\\ &\quad+\frac{(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2}+\frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}\end{aligned}$$

output
```
-1/8*g*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))
*b^(1/2)*n^(1/2)*2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))-1
/2*(-d*g+e*f)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*b^
(1/2)*n^(1/2)*Pi^(1/2)/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(-d*g+e*f)*(e*
x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2+1/2*g*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))
^(1/2)/e^2
```

3.106.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx =$$
$$-\frac{e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(4\sqrt{b}e^{\frac{a}{bn}}(ef-dg)\sqrt{n}\sqrt{\pi}(c(d+ex)^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+\sqrt{b}g\sqrt{n}\right.}{8}$$

input
```
Integrate[(f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
-1/8*((d + e*x)*(4*Sqrt[b]*E^(a/(b*n))*(e*f - d*g)*Sqrt[n]*Sqrt[Pi]*(c*(d
+ e*x)^n)^n^(-1)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] +
Sqrt[b]*g*Sqrt[n]*Sqrt[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d +
 e*x)^n]])/(Sqrt[b]*Sqrt[n])] - 4*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(2
*e*f - d*g + e*g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]))/(e^2*E^((2*a)/(b*n))*
(c*(d + e*x)^n)^(2/n))
```

3.106.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$\Big\downarrow \; 2848$$
$$\int \left(\frac{(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}+\frac{g(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}g\sqrt{n}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}+\frac{(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2}+\frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}$$

input
```
Int[(f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
-1/2*(Sqrt[b]*(e*f - d*g)*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c
*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)
) - (Sqrt[b]*g*Sqrt[n]*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log
[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(4*e^2*E^((2*a)/(b*n))*(c*(d + e*x)^
n)^(2/n)) + ((e*f - d*g)*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/e^2 + (
g*(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(2*e^2)
```

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.106.4 Maple [F]

$$\int (gx+f)\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.106.5 Fricas [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.106.6 Sympy [F]

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\,(d+ex)^n\right)}(f+gx)\,dx$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x), x)
```

3.106.7 Maxima [F]

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int (gx+f)\sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.106.8 Giac [F]

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (gx+f)\sqrt{b\log\left((ex+d)^nc\right)+a}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (f+g\,x)\,\sqrt{a+b\,\ln\left(c\,{(d+e\,x)}^n\right)}\,dx$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.107 $\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$

3.107.1 Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e} + \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

output
```
-1/2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(
1/2)*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(e*x+d)*(a+b*ln(c*(e*x+d)
^n))^(1/2)/e
```

3.107.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{(d+ex)\left(-\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+2\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e}$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
((d + e*x)*(-((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))) + 2*Sqrt[a + b
*Log[c*(d + e*x)^n]]))/(2*e)
```

3.107.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2836, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \sqrt{a+b\log\left(c(d+ex)^n\right)}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2733}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}bn\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}b(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log\left(c(d+ex)^n\right)}{e}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{e}$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(-1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^
n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (d + e*x)*S
qrt[a + b*Log[c*(d + e*x)^n]])/e
```

3.107.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.107.4 Maple [F]

$$\int \sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.107.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.107.6 Sympy [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\,(d+ex)^n\right)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.107.7 Maxima [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.107.8 Giac [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.108 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx$

3.108.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\text{Int}\left(\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)
```

3.108.2 Mathematica [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x),x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x), x]
```

3.108.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx$$
$$\Big\downarrow \; 2867$$
$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}dx$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x),x]
```

output
```
$Aborted
```

3.108.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.108.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}{gx+f}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)
```

3.108.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.108.6 Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.108.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.108.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.108.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x), x)
```

3.109 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^2}\,dx$

3.109.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(ef-dg)(f+gx)}-\frac{ben\text{Int}\left(\frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{2(ef-dg)}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/(-d*g+e*f)/(g*x+f)-1/2*b*e*n*Unintegra
ble(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/(-d*g+e*f)
```

3.109.2 Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^2,x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^2, x]
```

3.109.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2844, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2844}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)(ef-dg)}-\frac{ben\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}dx}{2(ef-dg)}$$

$$\Big\downarrow \text{2867}$$

$$\frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)(ef-dg)}-\frac{ben\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}dx}{2(ef-dg)}$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^2,x]
```

output
```
$Aborted
```

3.109.3.1 Defintions of rubi rules used

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.109.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ln \left(c \left(ex+d\right)^{n}\right)}}{\left(gx+f\right)^{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x)
```

3.109.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.109.6 Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}{\left(f+gx\right)^{2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**2,x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x)**2, x)
```

3.109.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^2, x)
```

3.109.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^2, x)
```

3.109.9 Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{{\left(f+g\,x\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^2, x)
```

3.110 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^3}\,dx$

3.110.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^3}\,dx = -\frac{\sqrt{a+b\log(c(d+ex)^n)}}{2g(f+gx)^2} + \frac{ben\text{Int}\left(\frac{1}{(d+ex)(f+gx)^2\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{4g}$$

output
```
-1/2*(a+b*ln(c*(e*x+d)^n))^(1/2)/g/(g*x+f)^2+1/4*b*e*n*Unintegrable(1/(e*x
+d)/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

3.110.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^3}\,dx = \int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^3}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^3,x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^3, x]
```

3.110.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2845, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{ben\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+b\log(c(d+ex)^n)}}dx}{4g}-\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2867}$$

$$\frac{ben\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+b\log(c(d+ex)^n)}}dx}{4g}-\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2g(f+gx)^2}$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^3,x]
```

output
```
$Aborted
```

3.110.3.1 Defintions of rubi rules used

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.110.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}{\left(gx+f\right)^{3}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x)
```

3.110.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}{(f+gx)^{3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.110.6 Sympy [N/A]

Not integrable

Time = 8.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}{(f+gx)^{3}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}{\left(f+gx\right)^{3}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**3,x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x)**3, x)
```

3.110.7 Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^3, x)
```

3.110.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^3, x)
```

3.110.9 Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{{\left(f+g\,x\right)}^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^3,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^3, x)
```

3.111 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx$

3.111.1 Optimal result

Integrand size = 26, antiderivative size = 526

$$\begin{aligned}\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx &= \frac{3b^{3/2}e^{-\frac{a}{bn}}(ef-dg)^2n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^3}\\
&+\frac{3b^{3/2}e^{-\frac{2a}{bn}}g(ef-dg)n^{3/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}\\
&+\frac{b^{3/2}e^{-\frac{3a}{bn}}g^2n^{3/2}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{12e^3}\\
&-\frac{3b(ef-dg)^2n(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e^3}\\
&-\frac{3bg(ef-dg)n(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{4e^3}\\
&-\frac{bg^2n(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{6e^3}+\frac{(ef-dg)^2(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{e^3}\\
&+\frac{g(ef-dg)(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{e^3}+\frac{g^2(d+ex)^3\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{3e^3}\end{aligned}$$

output
```
(-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3+g*(-d*g+e*f)*(e*x+d)^
2*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3+1/3*g^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^
(3/2)/e^3+1/36*b^(3/2)*g^2*n^(3/2)*(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d
)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)
^n)^(3/n))+3/16*b^(3/2)*g*(-d*g+e*f)*n^(3/2)*(e*x+d)^2*erfi(2^(1/2)*(a+b*l
n(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/e^3/exp(2*a/b/n)/(
(c*(e*x+d)^n)^(2/n))+3/4*b^(3/2)*(-d*g+e*f)^2*n^(3/2)*(e*x+d)*erfi((a+b*ln
(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e^3/exp(a/b/n)/((c*(e*x+d)^
n)^(1/n))-3/2*b*(-d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3-3/4
*b*g*(-d*g+e*f)*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3-1/6*b*g^2*n*(e
*x+d)^3*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3
```

3.111.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.85

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \frac{(d+ex)\left(144(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}+144g(ef-dg)(d+ex)\left(a\right.\right.}{}$$

input
```
Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
((d + e*x)*(144*(e*f - d*g)^2*(a + b*Log[c*(d + e*x)^n])^(3/2) + 144*g*(e*
f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2) + 48*g^2*(d + e*x)^2*(
a + b*Log[c*(d + e*x)^n])^(3/2) + 4*b*g^2*n*(d + e*x)^2*((Sqrt[b]*Sqrt[n]*
Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])
])/(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) - 6*Sqrt[a + b*Log[c*(d + e*x)^
n]]) + 27*b*g*(e*f - d*g)*n*(d + e*x)*((Sqrt[b]*Sqrt[n]*Sqrt[2*Pi]*Erfi[(S
qrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n)
)*(c*(d + e*x)^n)^(2/n)) - 4*Sqrt[a + b*Log[c*(d + e*x)^n]]) + 108*b*(e*f
- d*g)^2*n*((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/
(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - 2*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])))/(144*e^3)
```

3.111.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(ef-dg)^2 (a+b\log(c(d+ex)^n))^{3/2}}{e^2}+\frac{2g(d+ex)(ef-dg)(a+b\log(c(d+ex)^n))^{3/2}}{e^2}+\frac{g^2(d+ex)^2 (a+b$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}gn^{3/2}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}+\frac{3\sqrt{\pi}b^{3/2}n^{3/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}g^2n^{3/2}e^{-\frac{3a}{bn}}(d+ex)^3(c(d+ex)^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{12e^3}+\frac{g(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))^{3/2}}{e^3}+\frac{(d+ex)(ef-dg)^2(a+b\log(c(d+ex)^n))^{3/2}}{e^3}-\frac{3bgn(d+ex)^2(ef-dg)\sqrt{a+b\log(c(d+ex)^n)}}{4e^3}-\frac{3bn(d+ex)(ef-dg)^2\sqrt{a+b\log(c(d+ex)^n)}}{2e^3}+\frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^{3/2}}{3e^3}-\frac{bg^2n(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{6e^3}$$

input
```
Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
(3*b^(3/2)*(e*f - d*g)^2*n^(3/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*
(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(4*e^3*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1
)) + (3*b^(3/2)*g*(e*f - d*g)*n^(3/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]
*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(8*e^3*E^((2*a)/(b*n)
)*(c*(d + e*x)^n)^(2/n)) + (b^(3/2)*g^2*n^(3/2)*Sqrt[Pi/3]*(d + e*x)^3*Erf
i[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(12*e^3*E^(
(3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) - (3*b*(e*f - d*g)^2*n*(d + e*x)*Sqrt[
a + b*Log[c*(d + e*x)^n]])/(2*e^3) - (3*b*g*(e*f - d*g)*n*(d + e*x)^2*Sqrt
[a + b*Log[c*(d + e*x)^n]])/(4*e^3) - (b*g^2*n*(d + e*x)^3*Sqrt[a + b*Log[
c*(d + e*x)^n]])/(6*e^3) + ((e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d + e*x)
^n])^(3/2))/e^3 + (g*(e*f - d*g)*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^(3
/2))/e^3 + (g^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^(3/2))/(3*e^3)
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.111.4 Maple [F]

$$\int (gx+f)^2 (a+b\ln (c(ex+d)^n))^{\frac{3}{2}} dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.111.5 Fricas [F(-2)]

Exception generated.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.111.6 Sympy [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (a+b\log(c(d+ex)^n))^{\frac{3}{2}} (f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(3/2)*(f + g*x)**2, x)
```

3.111.7 Maxima [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (gx+f)^2 (b\log((ex+d)^n c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.111.8 Giac [F]

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{3/2}\, dx = \int (gx+f)^2 (b\log \left((ex+d)^n c\right)+a)^{\frac{3}{2}}\, dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{3/2}\, dx = \int (f+g\,x)^2 \left(a+b\,\ln \left(c\,(d+e\,x)^n\right)\right)^{3/2}\, dx$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.112 $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx$

3.112.1 Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx &= \frac{3b^{3/2}e^{-\frac{a}{bn}}(ef-dg)n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}\\ &+\frac{3b^{3/2}e^{-\frac{2a}{bn}}gn^{3/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{16e^2}\\ &-\frac{3b(ef-dg)n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}\\ &+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}\end{aligned}$$

output
```
(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^2+1/2*g*(e*x+d)^2*(a+b*ln
(c*(e*x+d)^n))^(3/2)/e^2+3/32*b^(3/2)*g*n^(3/2)*(e*x+d)^2*erfi(2^(1/2)*(a+
b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n
)/((c*(e*x+d)^n)^(2/n))+3/4*b^(3/2)*(-d*g+e*f)*n^(3/2)*(e*x+d)*erfi((a+b*l
n(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e^2/exp(a/b/n)/((c*(e*x+d)
^n)^(1/n))-3/2*b*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2-3/8*
b*g*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2
```

3.112.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.85

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\frac{(d+ex)\left(32(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}+16g(d+ex)\left(a+b\log\left(c(d+\right.\right.}{}$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
((d + e*x)*(32*(e*f - d*g)*(a + b*Log[c*(d + e*x)^n])^(3/2) + 16*g*(d + e*
x)*(a + b*Log[c*(d + e*x)^n])^(3/2) + 3*b*g*n*(d + e*x)*((Sqrt[b]*Sqrt[n]*
Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])
])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) - 4*Sqrt[a + b*Log[c*(d + e*x)^
n]]) + 24*b*(e*f - d*g)*n*((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c
*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) -
2*Sqrt[a + b*Log[c*(d + e*x)^n]])))/(32*e^2)
```

3.112.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}b^{3/2}n^{3/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}gn^{3/2}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{16e^2}+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}-\frac{3bn(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
(3*b^(3/2)*(e*f - d*g)*n^(3/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d
 + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(4*e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))
 + (3*b^(3/2)*g*n^(3/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(16*e^2*E^((2*a)/(b*n))*(c*(d + e*x
)^n)^(2/n)) - (3*b*(e*f - d*g)*n*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])
/(2*e^2) - (3*b*g*n*(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(8*e^2) +
((e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2))/e^2 + (g*(d + e*x
)^2*(a + b*Log[c*(d + e*x)^n])^(3/2))/(2*e^2)
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.112.4 Maple [F]

$$\int (gx+f)\left(a+b\ln\left(c(ex+d)^n\right)\right)^{\frac{3}{2}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.112.5 Fricas [F(-2)]

Exception generated.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.112.6 Sympy [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx = \int \left(a+b\log\left(c(d+ex)^n\right)\right)^{\frac{3}{2}}(f+gx)\,dx$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(3/2)*(f + g*x), x)
```

3.112.7 Maxima [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx = \int (gx+f)(b\log\left((ex+d)^nc\right)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.112.8 Giac [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\int (gx+f)(b\log\left((ex+d)^nc\right)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\int (f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.113 $\int (a+b\log (c(d+ex)^n))^{3/2}\, dx$

3.113.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int (a+b\log (c(d+ex)^n))^{3/2}\, dx = \frac{3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e} - \frac{3bn(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+3/4*b^(3/2)*n^(3/2)*(e*x+d)*erfi((a+
b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d
)^n)^(1/n))-3/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

3.113.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (a+b\log (c(d+ex)^n))^{3/2}\, dx = \frac{(d+ex)\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right) + 2\sqrt{a+b\log}\right.}{4e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
((d + e*x)*((3*b^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*
Log[c*(d + e*x)^n]]*(2*a - 3*b*n + 2*b*Log[c*(d + e*x)^n])))/(4*e)
```

3.113.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2836, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^{3/2}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\int \sqrt{a+b\log(c(d+ex)^n)}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2733}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\left((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-\frac{1}{2}bn\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\left((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-\frac{1}{2}b(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c\ldots}{\sqrt{a+\ldots}}\right.}{e}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}-\frac{3}{2}bn\left((d+ex)\sqrt{a+b\log(c(d+ex)^n)}-(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(\ldots}{\ldots}}\right.}{e}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^{n}\right.}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2) - (3*b*n*(-1/2*(Sqrt[b]*Sqrt[n
]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (d + e*x)*Sqrt[a + b*Log[c*(d +
e*x)^n]]))/2)/e
```

3.113.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.113.4 Maple [F]

$$\int (a+b\ln (c(ex+d)^n))^{\frac{3}{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.113.5 Fricas [F(-2)]

Exception generated.

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.113.6 Sympy [F]

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\int (a+b\log (c(d+ex)^n))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(3/2), x)
```

3.113.7 Maxima [F]

$$\int (a+b\log (c(d+ex)^n))^{3/2} \, dx = \int (b\log ((ex+d)^n c)+a)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.113.8 Giac [F]

$$\int (a+b\log (c(d+ex)^n))^{3/2} \, dx = \int (b\log ((ex+d)^n c)+a)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^{3/2} \, dx = \int \left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)^{3/2} \,d x$$

input
```
int((a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.114 $\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx} dx$

3.114.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx} dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}, x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)
```

3.114.2 Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx} dx = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx} dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x),x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x), x]
```

3.114.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}dx$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x),x]
```

output
```
$Aborted
```

3.114.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.114.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}{gx+f}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)
```

3.114.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(3/2)/(g*x+f),x)
```

output
```
Timed out
```

3.114.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^{\frac{3}{2}}}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f), x)
```

3.114.8 Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f), x)
```

3.114.9 Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x), x)
```

3.115 $\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx$

3.115.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{(ef-dg)(f+gx)} - \frac{3ben\text{Int}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx},x\right)}{2(ef-dg)}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/(-d*g+e*f)/(g*x+f)-3/2*b*e*n*Unintegra
ble((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)/(-d*g+e*f)
```

3.115.2 Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^2, x]
```

3.115.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2844, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2844}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{(f+gx)(ef-dg)}-\frac{3ben\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}dx}{2(ef-dg)}$$

$$\Big\downarrow \text{2867}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{(f+gx)(ef-dg)}-\frac{3ben\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}dx}{2(ef-dg)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^2,x]
```

output
```
$Aborted
```

3.115.3.1 Defintions of rubi rules used

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.115.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{\frac{3}{2}}}{\left(gx+f\right)^{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x)
```

3.115.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log\left(c(d+ex)^{n}\right))^{3/2}}{(f+gx)^{2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d+ex)^{n}\right))^{3/2}}{(f+gx)^{2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(3/2)/(g*x+f)**2,x)
```

output
```
Timed out
```

3.115.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^2, x)
```

3.115.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^2, x)
```

3.115.9 Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}{(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^2, x)
```

3.116 $\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx$

3.116.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx=-\frac{(a+b\log(c(d+ex)^n))^{3/2}}{2g(f+gx)^2}+\frac{3ben\text{Int}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{(d+ex)(f+gx)^2},x\right)}{4g}$$

output
```
-1/2*(a+b*ln(c*(e*x+d)^n))^(3/2)/g/(g*x+f)^2+3/4*b*e*n*Unintegrable((a+b*l
n(c*(e*x+d)^n))^(1/2)/(e*x+d)/(g*x+f)^2,x)/g
```

3.116.2 Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^3, x]
```

3.116.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2845, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3} \, dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{3ben\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(d+ex)(f+gx)^2}dx}{4g}-\frac{(a+b\log(c(d+ex)^n))^{3/2}}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2867}$$

$$\frac{3ben\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(d+ex)(f+gx)^2}dx}{4g}-\frac{(a+b\log(c(d+ex)^n))^{3/2}}{2g(f+gx)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^3,x]
```

output
```
$Aborted
```

3.116.3.1 Defintions of rubi rules used

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.116.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}{(gx+f)^3}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x)
```

3.116.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(3/2)/(g*x+f)**3,x)
```

output
```
Timed out
```

3.116.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^3, x)
```

3.116.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^3, x)
```

3.116.9 Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}{(f+g\,x)^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^3,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^3, x)
```

3.117 $\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{5/2}\,dx$

3.117.1 Optimal result

Integrand size = 26, antiderivative size = 660

$$\begin{aligned}
&\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{5/2}\,dx = \\
&-\frac{15b^{5/2}e^{-\frac{a}{bn}}(ef-dg)^2n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3} \\
&-\frac{15b^{5/2}e^{-\frac{2a}{bn}}g(ef-dg)n^{5/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{32e^3} \\
&-\frac{5b^{5/2}e^{-\frac{3a}{bn}}g^2n^{5/2}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{72e^3} \\
&+\frac{15b^2(ef-dg)^2n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3} \\
&+\frac{15b^2g(ef-dg)n^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{16e^3} \\
&+\frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{36e^3} \\
&-\frac{5b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^3} \\
&-\frac{5bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{4e^3} \\
&-\frac{5bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{18e^3}+\frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} \\
&+\frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3}+\frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{3e^3}
\end{aligned}$$

output
```
-5/2*b*(-d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3-5/4*b*g*(-d*
g+e*f)*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3-5/18*b*g^2*n*(e*x+d)^3*
(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3+(-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))
^(5/2)/e^3+g*(-d*g+e*f)*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(5/2)/e^3+1/3*g^2*
(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^(5/2)/e^3-5/216*b^(5/2)*g^2*n^(5/2)*(e*x+d
)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(
1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))-15/64*b^(5/2)*g*(-d*g+e*f)*n^(
5/2)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2
^(1/2)*Pi^(1/2)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))-15/8*b^(5/2)*(-d*g+
e*f)^2*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*P
i^(1/2)/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*(-d*g+e*f)^2*n^2*(e*
x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3+15/16*b^2*g*(-d*g+e*f)*n^2*(e*x+d)^2*
(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3+5/36*b^2*g^2*n^2*(e*x+d)^3*(a+b*ln(c*(e*x+
d)^n))^(1/2)/e^3
```

3.117.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.77

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx = \frac{(d+ex)\left(1728(ef-dg)^2(a+b\log(c(d+ex)^n))^{5/2}+1728g(ef-dg)(d+ex)\right.}{}$$

input
```
Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
((d + e*x)*(1728*(e*f - d*g)^2*(a + b*Log[c*(d + e*x)^n])^(5/2) + 1728*g*(
e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2) + 576*g^2*(d + e*x)^
2*(a + b*Log[c*(d + e*x)^n])^(5/2) - 1080*b*(e*f - d*g)^2*n*((3*b^(3/2)*n^
(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^
(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a
- 3*b*n + 2*b*Log[c*(d + e*x)^n])) - 40*b*g^2*n*(d + e*x)^2*((b^(3/2)*n^(3
/2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt
[n])])/(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + 6*Sqrt[a + b*Log[c*(d + e
*x)^n]]*(2*a - b*n + 2*b*Log[c*(d + e*x)^n])) - 135*b*g*(e*f - d*g)*n*(d +
 e*x)*((3*b^(3/2)*n^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e
*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + 4*S
qrt[a + b*Log[c*(d + e*x)^n]]*(4*a - 3*b*n + 4*b*Log[c*(d + e*x)^n]))))/(1
728*e^3)
```

3.117.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(ef-dg)^2 (a+b\log(c(d+ex)^n))^{5/2}}{e^2}+\frac{2g(d+ex)(ef-dg)(a+b\log(c(d+ex)^n))^{5/2}}{e^2}+\frac{g^2(d+ex)^2 (a+b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}gn^{5/2}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{32e^3}-\frac{15\sqrt{\pi}b^{5/2}n^{5/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}g^2n^{5/2}e^{-\frac{3a}{bn}}(d+ex)^3(c(d+ex)^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{72e^3}+\frac{15b^2gn^2(d+ex)^2(ef-dg)\sqrt{a+b\log(c(d+ex)^n)}}{16e^3}+\frac{15b^2n^2(d+ex)(ef-dg)^2\sqrt{a+b\log(c(d+ex)^n)}}{4e^3}+\frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{36e^3}+\frac{g(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))^{5/2}}{e^3}+\frac{(d+ex)(ef-dg)^2(a+b\log(c(d+ex)^n))^{5/2}}{e^3}-\frac{5bgn(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))^{3/2}}{4e^3}-\frac{5bn(d+ex)(ef-dg)^2(a+b\log(c(d+ex)^n))^{3/2}}{2e^3}+\frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^{5/2}}{3e^3}-\frac{5bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))^{3/2}}{18e^3}$$

input
```
Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
(-15*b^(5/2)*(e*f - d*g)^2*n^(5/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[
c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(8*e^3*E^(a/(b*n))*(c*(d + e*x)^n)^n^(
-1)) - (15*b^(5/2)*g*(e*f - d*g)*n^(5/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt
[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(32*e^3*E^((2*a)/(
b*n))*(c*(d + e*x)^n)^(2/n)) - (5*b^(5/2)*g^2*n^(5/2)*Sqrt[Pi/3]*(d + e*x)
^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(72*e
^3*E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + (15*b^2*(e*f - d*g)^2*n^2*(d +
 e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(4*e^3) + (15*b^2*g*(e*f - d*g)*n^2*
(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(16*e^3) + (5*b^2*g^2*n^2*(d +
 e*x)^3*Sqrt[a + b*Log[c*(d + e*x)^n]])/(36*e^3) - (5*b*(e*f - d*g)^2*n*(d
 + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2))/(2*e^3) - (5*b*g*(e*f - d*g)*n*(
d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2))/(4*e^3) - (5*b*g^2*n*(d + e*x
)^3*(a + b*Log[c*(d + e*x)^n])^(3/2))/(18*e^3) + ((e*f - d*g)^2*(d + e*x)*
(a + b*Log[c*(d + e*x)^n])^(5/2))/e^3 + (g*(e*f - d*g)*(d + e*x)^2*(a + b*
Log[c*(d + e*x)^n])^(5/2))/e^3 + (g^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n
])^(5/2))/(3*e^3)
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.117.4 Maple [F]

$$\int (gx+f)^2 (a+b\ln(c(ex+d)^n))^{\frac{5}{2}} dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.117.5 Fricas [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.117.6 Sympy [F]

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{5/2} \, dx = \int \left(a+b\log \left(c(d+ex)^n\right)\right)^{\frac{5}{2}} (f+gx)^2 \, dx$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(5/2)*(f + g*x)**2, x)
```

3.117.7 Maxima [F]

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^{5/2} \, dx = \int (gx+f)^2 (b\log \left((ex+d)^n c\right)+a)^{\frac{5}{2}} \, dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.117.8 Giac [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx = \int (gx+f)^2(b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx = \int (f+g\,x)^2\,(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}\,dx$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.118 $\int (f+gx) (a+b\log (c(d+ex)^n))^{5/2} \, dx$

3.118.1 Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}
\int (f+gx) (a+b\log (c(d+ex)^n))^{5/2} \, dx = & -\frac{15b^{5/2}e^{-\frac{a}{bn}}(ef-dg)n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^2} \\
& -\frac{15b^{5/2}e^{-\frac{2a}{bn}}gn^{5/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{64e^2} \\
& +\frac{15b^2(ef-dg)n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e^2} \\
& +\frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{32e^2} \\
& -\frac{5b(ef-dg)n(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{2e^2} \\
& -\frac{5bgn(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{8e^2} \\
& +\frac{(ef-dg)(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{e^2} \\
& +\frac{g(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{2e^2}
\end{aligned}$$

output
```
-5/2*b*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^2-5/8*b*g*n*(e*x
+d)^2*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^2+(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)
^n))^(5/2)/e^2+1/2*g*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(5/2)/e^2-15/128*b^(5
/2)*g*n^(5/2)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n
^(1/2))*2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))-15/8*b^(5/
2)*(-d*g+e*f)*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(
1/2))*Pi^(1/2)/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*(-d*g+e*f)*n^
2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2+15/32*b^2*g*n^2*(e*x+d)^2*(a+b*l
n(c*(e*x+d)^n))^(1/2)/e^2
```

3.118.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.79

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\frac{(d+ex)\left(128(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}+64g(d+ex)\left(a+b\log\left(c(d\right.\right.}{}$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
((d + e*x)*(128*(e*f - d*g)*(a + b*Log[c*(d + e*x)^n])^(5/2) + 64*g*(d + e
*x)*(a + b*Log[c*(d + e*x)^n])^(5/2) - 80*b*(e*f - d*g)*n*((3*b^(3/2)*n^(3
/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a
/(b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a -
3*b*n + 2*b*Log[c*(d + e*x)^n])) - 5*b*g*n*(d + e*x)*((3*b^(3/2)*n^(3/2)*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]
)/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + 4*Sqrt[a + b*Log[c*(d + e*x)^n
]]*(4*a - 3*b*n + 4*b*Log[c*(d + e*x)^n]))))/(128*e^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\pi}b^{5/2}n^{5/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}gn^{5/2}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{64e^2}+\frac{15b^2n^2(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^2}+\frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{32e^2}+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2}-\frac{5bn(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2}-\frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
(-15*b^(5/2)*(e*f - d*g)*n^(5/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*
(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]])/(8*e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1
)) - (15*b^(5/2)*g*n^(5/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b
*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]])/(64*e^2*E^((2*a)/(b*n))*(c*(d +
e*x)^n)^(2/n)) + (15*b^2*(e*f - d*g)*n^2*(d + e*x)*Sqrt[a + b*Log[c*(d + e
*x)^n]])/(4*e^2) + (15*b^2*g*n^2*(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]
])/(32*e^2) - (5*b*(e*f - d*g)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2
))/(2*e^2) - (5*b*g*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2))/(8*e^2
) + ((e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2))/e^2 + (g*(d +
 e*x)^2*(a + b*Log[c*(d + e*x)^n])^(5/2))/(2*e^2)
```

3.118. $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx$

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.118.4 Maple [F]

$$\int (gx+f)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{\frac{5}{2}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.118.5 Fricas [F(-2)]

Exception generated.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.118.6 Sympy [F]

$$\int (f+gx)\,(a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (a+b\log(c(d+ex)^n))^{\frac{5}{2}}\,(f+gx)\,dx$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(5/2)*(f + g*x), x)
```

3.118.7 Maxima [F]

$$\int (f+gx)\,(a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (gx+f)(b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.118.8 Giac [F]

$$\int (f+gx)\,(a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (gx+f)(b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\int (f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{5/2}\,dx$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.119 $\int (a+b\log(c(d+ex)^n))^{5/2}\,dx$

3.119.1 Optimal result

Integrand size = 18, antiderivative size = 179

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = -\frac{15b^{5/2}e^{-\frac{a}{bn}}n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{2e} + \frac{(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{e}$$

output
```
-5/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^(5/2)/e-15/8*b^(5/2)*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b
^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*n^2*(
e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

3.119.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int (a+b\log (c(d+ex)^n))^{5/2}\, dx = \frac{(d+ex)\left(8(a+b\log (c(d+ex)^n))^{5/2}-5bn\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\,\mathrm{e}\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
((d + e*x)*(8*(a + b*Log[c*(d + e*x)^n])^(5/2) - 5*b*n*((3*b^(3/2)*n^(3/2)
*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b
*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a - 3*b
*n + 2*b*Log[c*(d + e*x)^n]))))/(8*e)
```

3.119.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2836, 2733, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d+ex)^n))^{5/2}\, dx$$

$$\Big\downarrow \; 2836$$

$$\frac{\int (a+b\log (c(d+ex)^n))^{5/2}d(d+ex)}{e}$$

$$\Big\downarrow \; 2733$$

$$\frac{(d+ex)(a+b\log (c(d+ex)^n))^{5/2}-\frac{5}{2}bn\int (a+b\log (c(d+ex)^n))^{3/2}d(d+ex)}{e}$$

$$\Big\downarrow \; 2733$$

$$\frac{(d+ex)(a+b\log (c(d+ex)^n))^{5/2}-\frac{5}{2}bn\left((d+ex)(a+b\log (c(d+ex)^n))^{3/2}-\frac{3}{2}bn\int \sqrt{a+b\log (c(d+ex)^n)}d(d\right.}{e}$$

$\downarrow$ 2733

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+e\right.}}{e}$$

$\downarrow$ 2737

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+\right.}}{e}$$

$\downarrow$ 2611

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+\right.}}{e}$$

$\downarrow$ 2633

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}-\frac{5}{2}bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}-\frac{3}{2}bn\Big((d+ex)\sqrt{a+b\log\left(c(d+e\right.}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2) - (5*b*n*((d + e*x)*(a + b*Log
[c*(d + e*x)^n])^(3/2) - (3*b*n*(-1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*(d + e*x)*
Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d
 + e*x)^n)^n^(-1)) + (d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]))/2))/2)/e
```

3.119.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.119.4 Maple [F]

$$\int (a+b\ln (c(ex+d)^{n}))^{\frac{5}{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.119.5 Fricas [F(-2)]

Exception generated.

$$\int (a+b\log (c(d+ex)^{n}))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.119.6 Sympy [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (a+b\log(c(d+ex)^n))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(5/2), x)
```

3.119.7 Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (b\log((ex+d)^nc)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.119.8 Giac [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (b\log((ex+d)^nc)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx=\int (a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right))^{5/2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.120 $\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx$

3.120.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f),x)
```

3.120.2 Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x),x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x), x]
```

3.120.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}dx$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x),x]
```

output
```
$Aborted
```

3.120.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.120.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}{gx+f}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f),x)
```

3.120.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(5/2)/(g*x+f),x)
```

output
```
Timed out
```

3.120.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f), x)
```

3.120.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f), x)
```

3.120.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x), x)
```

3.121 $\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx$

3.121.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{5/2}}{(ef-dg)(f+gx)} - \frac{5ben\text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx},x\right)}{2(ef-dg)}$$

output
```
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(5/2)/(-d*g+e*f)/(g*x+f)-5/2*b*e*n*Unintegra
ble((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)/(-d*g+e*f)
```

3.121.2 Mathematica [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^2, x]
```

3.121.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2844, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2844}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{(f+gx)(ef-dg)}-\frac{5ben\int\frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}dx}{2(ef-dg)}$$

$$\Big\downarrow \text{2867}$$

$$\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{(f+gx)(ef-dg)}-\frac{5ben\int\frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}dx}{2(ef-dg)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^2,x]
```

output
```
$Aborted
```

3.121.3.1 Defintions of rubi rules used

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.121.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{\frac{5}{2}}}{\left(gx+f\right)^{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x)
```

3.121.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log\left(c(d+ex)^{n}\right))^{5/2}}{(f+gx)^{2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d+ex)^{n}\right))^{5/2}}{(f+gx)^{2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(5/2)/(g*x+f)**2,x)
```

output
```
Timed out
```

3.121.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^2, x)
```

3.121.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^2, x)
```

3.121.9 Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}{(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^2, x)
```

3.122 $\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx$

3.122.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = -\frac{(a+b\log(c(d+ex)^n))^{5/2}}{2g(f+gx)^2} + \frac{5ben\text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{3/2}}{(d+ex)(f+gx)^2},x\right)}{4g}$$

output
```
-1/2*(a+b*ln(c*(e*x+d)^n))^(5/2)/g/(g*x+f)^2+5/4*b*e*n*Unintegrable((a+b*l
n(c*(e*x+d)^n))^(3/2)/(e*x+d)/(g*x+f)^2,x)/g
```

3.122.2 Mathematica [N/A]

Not integrable

Time = 5.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^3, x]
```

3.122.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2845, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx$$
$$\Big\downarrow \text{2845}$$
$$\frac{5ben\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(d+ex)(f+gx)^2}dx}{4g}-\frac{(a+b\log(c(d+ex)^n))^{5/2}}{2g(f+gx)^2}$$
$$\Big\downarrow \text{2867}$$
$$\frac{5ben\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(d+ex)(f+gx)^2}dx}{4g}-\frac{(a+b\log(c(d+ex)^n))^{5/2}}{2g(f+gx)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^3,x]
```

output
```
$Aborted
```

3.122.3.1 Defintions of rubi rules used

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.122.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}{(gx+f)^3}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x)
```

3.122.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(5/2)/(g*x+f)**3,x)
```

output
```
Timed out
```

3.122.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^3, x)
```

3.122.8 Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}{(gx+f)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^3, x)
```

3.122.9 Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}{(f+g\,x)^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^3,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^3, x)
```

3.123 $\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.123.1 Optimal result

Integrand size = 26, antiderivative size = 383

$$\begin{aligned}\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} \\ &+ \frac{e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2\sqrt{b}e^4\sqrt{n}} \\ &+ \frac{3e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} \\ &+ \frac{e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}\end{aligned}$$

output
```
3/2*g*(-d*g+e*f)^2*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1
/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/e^4/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/b^(1/
2)/n^(1/2)+(-d*g+e*f)^3*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n
^(1/2))*Pi^(1/2)/e^4/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)+1/2*
g^3*(e*x+d)^4*erfi(2*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)
/e^4/exp(4*a/b/n)/((c*(e*x+d)^n)^(4/n))/b^(1/2)/n^(1/2)+g^2*(-d*g+e*f)*(e*
x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*P
i^(1/2)/e^4/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/b^(1/2)/n^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$= \frac{e^{-\frac{4a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-4/n}\left(2e^{\frac{3a}{bn}}(ef-dg)^3\left(c(d+ex)^n\right)^{3/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+g^3(d+ex)^3\text{er}\right.}{}$$

input
```
Integrate[(f + g*x)^3/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*(2*E^((3*a)/(b*n))*(e*f - d*g)^3*(c*(d + e*x)^n)^(3/n)
*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + g^3*(d + e*x)^3*
Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + 3*Sqrt[2]*E^(
(2*a)/(b*n))*g*(e*f - d*g)^2*(d + e*x)*(c*(d + e*x)^n)^(2/n)*Erfi[(Sqrt[2]
*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + 2*Sqrt[3]*E^(a/(b*n)
)*g^2*(e*f - d*g)*(d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[3]*Sqrt[a
+ b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(2*Sqrt[b]*e^4*E^((4*a)/(b*n
))*Sqrt[n]*(c*(d + e*x)^n)^(4/n))
```

3.123.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$\Big\downarrow \; 2848$$
$$\int \left(\frac{3g^2(d+ex)^2(ef-dg)}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{(ef-dg)^3}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{3g(d+ex)(ef-dg)^2}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{g^3(d+ex)^3}{e^3\sqrt{a+b\log\left(c(d+e\right.}}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}+\frac{3\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}+\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}+\frac{\sqrt{\pi}g^3e^{-\frac{4a}{bn}}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2\sqrt{b}e^4\sqrt{n}}$$

input
```
Int[(f + g*x)^3/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
((e*f - d*g)^3*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqr
t[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) +
 (g^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b
]*Sqrt[n])])/(2*Sqrt[b]*e^4*E^((4*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(4/n))
 + (3*g*(e*f - d*g)^2*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[
c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^((2*a)/(b*n))*Sqrt[n]*
(c*(d + e*x)^n)^(2/n)) + (g^2*(e*f - d*g)*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqr
t[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^((
3*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.123.4 Maple [F]

$$\int \frac{(gx+f)^3}{\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}}dx$$

input
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.123.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.123.6 Sympy [F]

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{\left(f+gx\right)^3}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

input
```
integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral((f + g*x)**3/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.123.7 Maxima [F]

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(gx+f)^3}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^3/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.123.8 Giac [F]

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(gx+f)^3}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^3/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(f+g\,x)^3}{\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

input
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.124 $\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.124.1 Optimal result

Integrand size = 26, antiderivative size = 283

$$\begin{aligned}\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}\\&\quad+\frac{e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}\\&\quad+\frac{e^{-\frac{3a}{bn}}g^2\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}\end{aligned}$$

output
```
1/3*g^2*(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2)
)*3^(1/2)*Pi^(1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/b^(1/2)/n^(1/2)+
(-d*g+e*f)^2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^
(1/2)/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)+g*(-d*g+e*f)*(e
*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*
Pi^(1/2)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/b^(1/2)/n^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(3e^{\frac{2a}{bn}}(ef-dg)^2\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+3\sqrt{2}e^{\frac{a}{bn}}g(ef\ \cdots}{3\sqrt{b}e^3\sqrt{n}\ \cdots}$$

input
```
Integrate[(f + g*x)^2/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*(3*E^((2*a)/(b*n))*(e*f - d*g)^2*(c*(d + e*x)^n)^(2/n)
*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + 3*Sqrt[2]*E^(a/(
b*n))*g*(e*f - d*g)*(d + e*x)*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a
+ b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + Sqrt[3]*g^2*(d + e*x)^2*Erfi
[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(3*Sqrt[b]*
e^3*E^((3*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))
```

3.124.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{2g(d+ex)(ef-dg)}{e^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}+\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}+\frac{\sqrt{\frac{\pi}{3}}g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}$$

input
```
Int[(f + g*x)^2/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
((e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqr
t[b]*Sqrt[n])]])/(Sqrt[b]*e^3*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) +
 (g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d +
 e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((2*a)/(b*n))*Sqrt[n]*(c*(d
+ e*x)^n)^(2/n)) + (g^2*Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((3*a)/(b*n))*Sqrt[n
]*(c*(d + e*x)^n)^(3/n))
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.124.4 Maple [F]

$$\int \frac{(gx+f)^2}{\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}}dx$$

input
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.124.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.124.6 Sympy [F]

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c\,(d+ex)^n\right)}}\,dx$$

input
```
integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral((f + g*x)**2/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.124.7 Maxima [F]

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(gx+f)^2}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.124.8 Giac [F]

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(gx+f)^2}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(f+g\,x)^2}{\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

input
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.125 $\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.125.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}} + \frac{e^{-\frac{2a}{bn}}g\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}$$

output
```
1/2*g*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*
2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/b^(1/2)/n^(1/2)+(-
d*g+e*f)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2
)/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{2a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(2e^{\frac{a}{bn}}(ef-dg)\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+\sqrt{2}g(d+ex)\operatorname{erfi}\right.}{2\sqrt{b}e^2\sqrt{n}}$$

input
```
Integrate[(f + g*x)/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*(2*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Erfi
[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + Sqrt[2]*g*(d + e*x)*E
rfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(2*Sqrt[
b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n))
```

3.125.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{ef-dg}{e\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{g(d+ex)}{e\sqrt{a+b\log\left(c(d+ex)^n\right)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}+\frac{\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}$$

input
```
Int[(f + g*x)/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[
b]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (
g*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sq
rt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n
))
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.125.4 Maple [F]

$$\int \frac{gx+f}{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

input
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.125.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.125.6 Sympy [F]

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{f+gx}{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}\,dx$$

input
```
integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral((f + g*x)/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.125.7 Maxima [F]

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{gx+f}{\sqrt{b\log\left(\left(ex+d\right)^{n}c\right)+a}}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.125.8 Giac [F]

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{gx+f}{\sqrt{b\log\left(\left(ex+d\right)^{n}c\right)+a}}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{f+g\,x}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}} \,d x$$

input
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.126 $\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.126.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

output
```
(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a
/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

input
```
Integrate[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]
)/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

3.126.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2836, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\log \left(c(d+ex)^n\right)}}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\sqrt{a+b\log (c(d+ex)^n)}}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log (c(d+ex)^n)}}d\log \left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int e^{\frac{a+b\log (c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log \left(c(d+ex)^n\right)}}{ben}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log (c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

input
```
Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]
)/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

3.126.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.126.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.126.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.126.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.126.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.126.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.127 $\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.127.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Int}\left(\frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.127.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

input
```
Integrate[1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

output
```
Integrate[1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

3.127.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx$$

$$\Big\downarrow \text{2867}$$

$$\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}dx$$

input
```
Int[1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

output
```
$Aborted
```

3.127.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.127.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.127.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.127.6 Sympy [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\,(d+ex)^n\right)}\,(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

3.127.7 Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(gx+f)\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

3.127.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx=\int \frac{1}{(gx+f)\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

3.127.9 Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx=\int \frac{1}{(f+g\,x)\,\sqrt{a+b\,\ln\left(c\,{(d+e\,x)}^n\right)}}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2)),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2)), x)
```

3.128 $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.128.1 Optimal result

Integrand size = 26, antiderivative size = 422

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{4e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{6e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{6e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$-\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

output
```
2*(-d*g+e*f)^3*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*P
i^(1/2)/b^(3/2)/e^4/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))+4*g^3*(e*x+d)
^4*erfi(2*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(3/2)/e^
4/exp(4*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(4/n))+6*g*(-d*g+e*f)^2*(e*x+d)^2*er
fi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b
^(3/2)/e^4/exp(2*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(2/n))+6*g^2*(-d*g+e*f)*(e*
x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*P
i^(1/2)/b^(3/2)/e^4/exp(3*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(3/n))-2*(e*x+d)*(
g*x+f)^3/b/e/n/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.128.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1281 vs. $2(422) = 844$.

Time = 1.70 (sec) , antiderivative size = 1281, normalized size of antiderivative = 3.04

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2\left(-\sqrt{b}de^3f^3\sqrt{n}-\sqrt{b}e^4f^3\sqrt{n}x-3\sqrt{b}de^3f^2g\sqrt{n}x-3\sqrt{b}e^4f^2g\sqrt{n}x^2-\right.}{}$$

input
```
Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
(2*(-(Sqrt[b]*d*e^3*f^3*Sqrt[n]) - Sqrt[b]*e^4*f^3*Sqrt[n]*x - 3*Sqrt[b]*d
*e^3*f^2*g*Sqrt[n]*x - 3*Sqrt[b]*e^4*f^2*g*Sqrt[n]*x^2 - 3*Sqrt[b]*d*e^3*f
*g^2*Sqrt[n]*x^2 - 3*Sqrt[b]*e^4*f*g^2*Sqrt[n]*x^3 - Sqrt[b]*d*e^3*g^3*Sqr
t[n]*x^3 - Sqrt[b]*e^4*g^3*Sqrt[n]*x^4 - (6*d*e^2*f^2*g*Sqrt[Pi]*(d + e*x)
*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(
d + e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (3*d^2*e*f*g^2*Sqrt[P
i]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a
 + b*Log[c*(d + e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - (d^3*g^3*
Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*
Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (2*
g^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*
Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((4*a)/(b*n))*(c*(d + e*x)^n)
^(4/n)) + (3*e^2*f^2*g*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log
[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((
2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) - (6*d*e*f*g^2*Sqrt[2*Pi]*(d + e*x)^2*E
rfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b
*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (3*d^2*g^3
*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqr
t[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c*(d + e*
x)^n)^(2/n)) + (3*e*f*g^2*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a +...
```

3.128.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.74, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2847, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.128. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}dx$$

$$\Big\downarrow \text{2847}$$

$$-\frac{6(ef-dg)\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{8\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

$$\Big\downarrow \text{2848}$$

$$\frac{8\int \left(\frac{(ef-dg)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}}+\frac{3g(d+ex)(ef-dg)^2}{e^3\sqrt{a+b\log(c(d+ex)^n)}}+\frac{3g^2(d+ex)^2(ef-dg)}{e^3\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g^3(d+ex)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-$$
$$\frac{6(ef-dg)\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{2g(d+ex)(ef-dg)}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{ben}-$$
$$\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)(c(d+ex)^n)^{-3/n}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}+\frac{3\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2(c(d+ex)^n)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b}\ldots\right.}{\sqrt{b}e^4\sqrt{n}}\ldots\right.}{\ldots}$$

$$\frac{6(ef-dg)\left(\frac{\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}+\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\text{erfi}\left(\ldots\right.}{\sqrt{b}e^3\sqrt{n}}\ldots\right.}{ben}$$

$$\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

input
```
Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

3.128. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}dx$

output
```
(-6*(e*f - d*g)*(((e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(
d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^(a/(b*n))*Sqrt[n]*(c*(d +
e*x)^n)^n^(-1)) + (g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt
[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((2*a)/(b*n
))*Sqrt[n]*(c*(d + e*x)^n)^(2/n)) + (g^2*Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sqrt
[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((3
*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))))/(b*e*n) + (8*(((e*f - d*g)^3*S
qrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/
(Sqrt[b]*e^4*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (g^3*Sqrt[Pi]*(
d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(2*
Sqrt[b]*e^4*E^((4*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(4/n)) + (3*g*(e*f - d
*g)^2*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])
/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^
(2/n)) + (g^2*(e*f - d*g)*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*
Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^((3*a)/(b*n))*Sqrt
[n]*(c*(d + e*x)^n)^(3/n))))/(b*n) - (2*(d + e*x)*(f + g*x)^3)/(b*e*n*Sqrt
[a + b*Log[c*(d + e*x)^n]])
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.128. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.128.4 Maple [F]

$$\int \frac{(gx+f)^3}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.128.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.128.6 Sympy [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n))**(3/2), x)
```

3.128.7 Maxima [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.128.8 Giac [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(f+g\,x)^3}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

input
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.129 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.129.1 Optimal result

Integrand size = 26, antiderivative size = 325

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}}$$
$$+\frac{4e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}}$$
$$+\frac{2e^{-\frac{3a}{bn}}g^2\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}}$$
$$-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

output
```
2*(-d*g+e*f)^2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*P
i^(1/2)/b^(3/2)/e^3/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))+4*g*(-d*g+e*f
)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1
/2)*Pi^(1/2)/b^(3/2)/e^3/exp(2*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(2/n))+2*g^2*
(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2
)*Pi^(1/2)/b^(3/2)/e^3/exp(3*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(3/n))-2*(e*x+d
)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.129.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 828 vs. $2(325) = 650$.

Time = 0.70 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.55

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2\left(-\sqrt{b}de^2f^2\sqrt{n}-\sqrt{b}e^3f^2\sqrt{n}x-2\sqrt{b}de^2fg\sqrt{n}x-2\sqrt{b}e^3fg\sqrt{n}x^2-\sqrt{\right.}{}$$

input
```
Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
(2*(-(Sqrt[b]*d*e^2*f^2*Sqrt[n]) - Sqrt[b]*e^3*f^2*Sqrt[n]*x - 2*Sqrt[b]*d
*e^2*f*g*Sqrt[n]*x - 2*Sqrt[b]*e^3*f*g*Sqrt[n]*x^2 - Sqrt[b]*d*e^2*g^2*Sqr
t[n]*x^2 - Sqrt[b]*e^3*g^2*Sqrt[n]*x^3 - (4*d*e*f*g*Sqrt[Pi]*(d + e*x)*Erf
i[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d +
e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (d^2*g^2*Sqrt[Pi]*(d + e*
x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c
*(d + e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (2*e*f*g*Sqrt[2*Pi]
*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n
])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)
) - (2*d*g^2*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*
x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n)
)*(c*(d + e*x)^n)^(2/n)) + (g^2*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[
a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n
]])/(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + (Sqrt[b]*e^2*f^2*Sqrt[n]*(d
+ e*x)*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c
*(d + e*x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (2*Sqrt[b]*
d*e*f*g*Sqrt[n]*(d + e*x)*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*
Sqrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^
(-1))))/(b^(3/2)*e^3*n^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

3.129.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2847, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

$\Big\downarrow$ 2847

$$-\frac{4(ef-dg)\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

$\Big\downarrow$ 2848

$$\frac{6\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{2g(d+ex)(ef-dg)}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{4(ef-dg)\int \left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{ben}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

$\Big\downarrow$ 2009

$$\frac{6\left(\frac{\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)(c(d+ex)^n)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}+\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+}{\sqrt{b}\sqrt{n}}\right.}{\sqrt{b}e^3\sqrt{n}}\right.}{bn}$$

$$\frac{4(ef-dg)\left(\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}+\frac{\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2(c(d+ex)^n)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c}{\sqrt{b}\sqrt{n}}\right.}{\sqrt{b}e^2\sqrt{n}}\right.}{ben}$$

$$\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

input
```
Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
(-4*(e*f - d*g)*(((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d
+ e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d + e*
x)^n)^n^(-1)) + (g*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(
d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*
(d + e*x)^n)^(2/n))))/(b*e*n) + (6*(((e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi
[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^(a/(b*n
))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2
*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b
]*e^3*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n)) + (g^2*Sqrt[Pi/3]*(d
+ e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])
/(Sqrt[b]*e^3*E^((3*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))))/(b*n) - (2*
(d + e*x)*(f + g*x)^2)/(b*e*n*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

3.129. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.129.4 Maple [F]

$$\int \frac{(gx+f)^2}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.129.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.129.6 Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(f+gx)^2}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**(3/2), x)
```

3.129.7 Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(gx+f)^2}{\left(b\log\left((ex+d)^n c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.129.8 Giac [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(gx+f)^2}{\left(b\log\left((ex+d)^n c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

input
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.130 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.130.1 Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} + \frac{2e^{-\frac{2a}{bn}}g\sqrt{2\pi}(d+ex)^2\,(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} - \frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

output
```
2*(-d*g+e*f)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^
(1/2)/b^(3/2)/e^2/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))+2*g*(e*x+d)^2*e
rfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/
b^(3/2)/e^2/exp(2*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(2/n))-2*(e*x+d)*(g*x+f)/b
/e/n/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.54

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(-2de^{\frac{a}{bn}}g\sqrt{\pi}\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(}}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
(2*(d + e*x)*(-2*d*E^(a/(b*n))*g*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)*Erfi[Sqrt
[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n
]] + g*Sqrt[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/
(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]] + Sqrt[b]*E^(a/(b*n))*Sq
rt[n]*(c*(d + e*x)^n)^n^(-1)*(-(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(f +
g*x)) + (e*f + d*g)*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-
((a + b*Log[c*(d + e*x)^n])/(b*n))])))/(b^(3/2)*e^2*E^((2*a)/(b*n))*n^(3/2
)*(c*(d + e*x)^n)^(2/n)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

3.130.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.45, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2847, 2836, 2737, 2611, 2633, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

$$\Big\downarrow \; 2847$$

$$-\frac{2(ef-dg)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

$$\Big\downarrow \; 2836$$

$$-\frac{2(ef-dg)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{be^2n}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

$$\Big\downarrow \; 2737$$

$$-\frac{2(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\int\frac{\left(c(d+ex)^n\right)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log\left(c(d+ex)^n\right)}{be^2n^2}+\frac{4\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{4(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log\left(c(d+ex)^n\right)}}{b^2e^2n^2}+\frac{4\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{4\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

$$\Big\downarrow \text{2848}$$

$$\frac{4\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}+\frac{4\left(\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}+\frac{\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}\right)}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

input
```
Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

3.130. $\int\frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

output
```
(-2*(e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sq
rt[b]*Sqrt[n])])/(b^(3/2)*e^2*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1))
+ (4*(((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/
(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1
)) + (g*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]
])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n
)^(2/n))))/(b*n) - (2*(d + e*x)*(f + g*x))/(b*e*n*Sqrt[a + b*Log[c*(d + e*
x)^n]])
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.130.4 Maple [F]

$$\int \frac{gx+f}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.130.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.130.6 Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**(3/2), x)
```

3.130.7 Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{gx+f}{(b\log((ex+d)^nc)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.130.8 Giac [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{gx+f}{(b\log((ex+d)^nc)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{f+g\,x}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

input
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.131 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.131.1 Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

output
```
2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(3/
2)/e/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))-2*(e*x+d)/b/e/n/(a+b*ln(c*(e
*x+d)^n))^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}} - \Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\sqrt{-\frac{a+b\log(c(d+ex)^n)}{bn}}\right)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
```

output
```
(-2*(d + e*x)*(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1) - Gamma[1/2, -((a + b*Lo
g[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))]))/(b*e
*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

3.131.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2836, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{bn}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{2(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{bn^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{4(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2n^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
```

output
```
((2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n
]])])/(b^(3/2)*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/
(b*n*Sqrt[a + b*Log[c*(d + e*x)^n]]))/e
```

3.131.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.131.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.131.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.131.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-3/2), x)
```

3.131.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

3.131.8 Giac [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.132 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.132.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)\,(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \mathrm{Int}\left(\frac{1}{(f+gx)\,(a+b\log(c(d+ex)^n))^{3/2}},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.132.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\,(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{1}{(f+gx)\,(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)), x]
```

3.132.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)),x]
```

output
```
$Aborted
```

3.132.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.132.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.132.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.132.6 Sympy [N/A]

Not integrable

Time = 5.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{\frac{3}{2}}\left(f+gx\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**(3/2)*(f + g*x)), x)
```

3.132.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^{n}c\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2)), x)
```

3.132.8 Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2)), x)
```

3.132.9 Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx=\int \frac{1}{\left(f+g\,x\right)\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^{3/2}}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2)),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2)), x)
```

3.133 $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.133.1 Optimal result

Integrand size = 26, antiderivative size = 520

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}}$$
$$+\frac{32e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}}$$
$$+\frac{8e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}}$$
$$+\frac{12e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}}$$
$$-\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}+\frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$-\frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

output
```
-2/3*(e*x+d)*(g*x+f)^3/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(-d*g+e*f)^3*
(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)
/e^4/exp(a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(1/n))+32/3*g^3*(e*x+d)^4*erfi(2*(a
+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e^4/exp(4*a/b/
n)/n^(5/2)/((c*(e*x+d)^n)^(4/n))+8*g*(-d*g+e*f)^2*(e*x+d)^2*erfi(2^(1/2)*(
a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/e^4/e
xp(2*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(2/n))+12*g^2*(-d*g+e*f)*(e*x+d)^3*erfi
(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/b^(
5/2)/e^4/exp(3*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(3/n))+4*(-d*g+e*f)*(e*x+d)*(
g*x+f)^2/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)-16/3*(e*x+d)*(g*x+f)^3/b^
2/e/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.133.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1523 vs. $2(520) = 1040$.

Time = 6.12 (sec) , antiderivative size = 1523, normalized size of antiderivative = 2.93

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
(2*(d + e*x)*((-24*d*e^2*f^2*g*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - (6*d^2*e*f*g^
2*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(
b*n))*(c*(d + e*x)^n)^n^(-1)) - (2*d^3*g^3*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*
(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (
16*g^3*Sqrt[Pi]*(d + e*x)^3*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[
b]*Sqrt[n])])/(E^((4*a)/(b*n))*(c*(d + e*x)^n)^(4/n)) + (12*e^2*f^2*g*Sqrt
[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sq
rt[n])])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (30*d*e*f*g^2*Sqrt[2*Pi
]*(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n]
)])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (6*d^2*g^3*Sqrt[2*Pi]*(d + e
*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^(
(2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (16*d*g^3*Sqrt[Pi]*(d + e*x)*(3*Sqrt
[2]*d*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d
 + e*x)^n]])/(Sqrt[b]*Sqrt[n])] - 2*Sqrt[3]*(d + e*x)*Erfi[(Sqrt[3]*Sqrt[a
 + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(E^((3*a)/(b*n))*(c*(d + e*
x)^n)^(3/n)) - (14*d*g^3*Sqrt[Pi]*(d + e*x)*(3*Sqrt[2]*d*E^(a/(b*n))*(c*(d
 + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*S
qrt[n])] - Sqrt[3]*(d + e*x)*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])
/(Sqrt[b]*Sqrt[n])]))/(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + (18*e*f...
```

3.133.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1341 vs. $2(520) = 1040$.

Time = 3.01 (sec) , antiderivative size = 1341, normalized size of antiderivative = 2.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2847, 2847, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$$

$$\Big\downarrow \text{2847}$$

$$-\frac{2(ef-dg)\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{ben}+\frac{8\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(d+ex)(f+gx)^3}{3ben\,(a+b\log(c(d+ex)^n))^{3/2}}$$

$$\Big\downarrow \text{2847}$$

$$-\frac{2(ef-dg)\left(-\frac{4(ef-dg)\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{ben}+\frac{8\left(-\frac{6(ef-dg)\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{8\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \textbf{2848}$$

$$-\frac{2(ef-dg)\left(\frac{6\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{2g(d+ex)(ef-dg)}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{4(ef-dg)\int \left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(}{e\sqrt{a+b\log}}\right.}{ben}\right.}{ben}$$

$$\frac{8\left(\frac{8\int \left(\frac{(ef-dg)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}}+\frac{3g(d+ex)(ef-dg)^2}{e^3\sqrt{a+b\log(c(d+ex)^n)}}+\frac{3g^2(d+ex)^2(ef-dg)}{e^3\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g^3(d+ex)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{6(ef-dg)\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\right.}{}\right.}{3bn}$$

$$\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}-$$

$$\frac{2(ef-dg)\left(-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}-\frac{4(ef-dg)\left(\frac{e^{-\frac{2a}{bn}}g\sqrt{\frac{\pi}{2}}(d+ex)^2\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)\left(c(d+ex)^n\right)^{-2/n}}{\sqrt{b}e^2\sqrt{n}}+\frac{e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+}{}\right.}{ben}\right.}{}$$

$$8\left(-\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}-\frac{6(ef-dg)\left(\frac{e^{-\frac{3a}{bn}}g^2\sqrt{\frac{\pi}{3}}(d+ex)^3\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)\left(c(d+ex)^n\right)^{-3/n}}{\sqrt{b}e^3\sqrt{n}}+\frac{e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\text{erfi}\left(\right.}{}\right.}{be}\right.$$

input
```
Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

3.133. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

output
```
(-2*(d + e*x)*(f + g*x)^3)/(3*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) - (2
*(e*f - d*g)*((-4*(e*f - d*g)*(((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a
 + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt
[n]*(c*(d + e*x)^n)^n^(-1)) + (g*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt
[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n
))*Sqrt[n]*(c*(d + e*x)^n)^(2/n))))/(b*e*n) + (6*(((e*f - d*g)^2*Sqrt[Pi]*
(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]
*e^3*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (g*(e*f - d*g)*Sqrt[2*P
i]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt
[n])])/(Sqrt[b]*e^3*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n)) + (g^2*
Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt
[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((3*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))
))/(b*n) - (2*(d + e*x)*(f + g*x)^2)/(b*e*n*Sqrt[a + b*Log[c*(d + e*x)^n]]
)))/(b*e*n) + (8*((-6*(e*f - d*g)*(((e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[
Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^(a/(b*n)
)*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*
Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]
*e^3*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n)) + (g^2*Sqrt[Pi/3]*(d +
 e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/
(Sqrt[b]*e^3*E^((3*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))))/(b*e*n) +...
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.133. $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.133.4 Maple [F]

$$\int \frac{(gx+f)^3}{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.133.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.133.6 Sympy [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n))**(5/2), x)
```

3.133.7 Maxima [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.133.8 Giac [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+g\,x)^3}{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}\,dx$$

input
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.134 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}} dx$

3.134.1 Optimal result

Integrand size = 26, antiderivative size = 421

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}} dx = \frac{4e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+\frac{16e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+\frac{4e^{-\frac{3a}{bn}}g^2\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^3n^{5/2}}$$
$$-\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}+\frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$-\frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

output
```
-2/3*(e*x+d)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(-d*g+e*f)^2*
(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)
/e^3/exp(a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(1/n))+16/3*g*(-d*g+e*f)*(e*x+d)^2*
erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)
/b^(5/2)/e^3/exp(2*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(2/n))+4*g^2*(e*x+d)^3*er
fi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/b
^(5/2)/e^3/exp(3*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(3/n))+8/3*(-d*g+e*f)*(e*x+
d)*(g*x+f)/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)-4*(e*x+d)*(g*x+f)^2/b^2
/e/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.25

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx =$$

$$-\frac{2e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(2de^{\frac{2a}{bn}}g(8ef+dg)\sqrt{\pi}\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)(a+b\log(c(\ldots}{}$$

input
```
Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
(-2*(d + e*x)*(2*d*E^((2*a)/(b*n))*g*(8*e*f + d*g)*Sqrt[Pi]*(c*(d + e*x)^n
)^(2/n)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*(a + b*Log[
c*(d + e*x)^n])^(3/2) + 8*E^(a/(b*n))*g*(-(e*f) + d*g)*Sqrt[2*Pi]*(d + e*x
)*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sq
rt[b]*Sqrt[n])]*(a + b*Log[c*(d + e*x)^n])^(3/2) - 6*g^2*Sqrt[3*Pi]*(d + e
*x)^2*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*(a
+ b*Log[c*(d + e*x)^n])^(3/2) + Sqrt[b]*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*
x)^n)^(2/n)*(2*b*(e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*n*Gamma[1/2, -((a + b*L
og[c*(d + e*x)^n])/(b*n))]*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^(3/2) + e
*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(f + g*x)*(b*e*n*(f + g*x) + 2*a*(e*f
+ 2*d*g + 3*e*g*x) + 2*b*(2*d*g + e*(f + 3*g*x))*Log[c*(d + e*x)^n]))))/(3
*b^(5/2)*e^3*E^((3*a)/(b*n))*n^(5/2)*(c*(d + e*x)^n)^(3/n)*(a + b*Log[c*(d
 + e*x)^n])^(3/2))
```

3.134.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 924 vs. 2(421) = 842.

Time = 2.60 (sec) , antiderivative size = 924, normalized size of antiderivative = 2.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2847, 2847, 2836, 2737, 2611, 2633, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}dx$$

$$\Big\downarrow \text{ 2847}$$

$$-\frac{4(ef-dg)\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3ben}+\frac{2\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\big\downarrow$ 2847

$$-\frac{4(ef-dg)\left(-\frac{2(ef-dg)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3ben}+\frac{2\left(-\frac{4(ef-dg)\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{bn}-\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\big\downarrow$ 2836

$$-\frac{4(ef-dg)\left(-\frac{2(ef-dg)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{be^2n}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3ben}+\frac{2\left(-\frac{4(ef-dg)\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{bn}-\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\big\downarrow$ 2737

$$-\frac{4(ef-dg)\left(-\frac{2(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{be^2n^2}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+g}{ben\sqrt{a+b\log(c(d-}}\right.}{3ben}+\frac{2\left(-\frac{4(ef-dg)\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{bn}-\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\big\downarrow$ 2611

3.134. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

$$-\frac{4(ef-dg)\left(-\frac{4(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2e^2n^2}+\frac{4\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d}{ben\sqrt{a-}}\right.}{3ben}$$

$$\frac{2\left(-\frac{4(ef-dg)\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int\frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{bn}-$$

$$\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\downarrow$ 2633

$$-\frac{4(ef-dg)\left(\frac{4\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)}}\right.}{3ben}$$

$$\frac{2\left(-\frac{4(ef-dg)\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{6\int\frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{bn}-$$

$$\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\downarrow$ 2848

$$-\frac{4(ef-dg)\left(\frac{4\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}\right.}{3ben}$$

$$\frac{2\left(\frac{6\int\left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{2g(d+ex)(ef-dg)}{e^2\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{4(ef-dg)\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{ben}\right.}{bn}$$

$$\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\downarrow$ 2009

3.134. $\int\frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

$$-\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}-$$

$$\frac{4(ef-dg)\left(-\frac{2e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)(c(d+ex)^n)^{-1/n}}{b^{3/2}e^2n^{3/2}}+\frac{4\left(\frac{e^{-\frac{2a}{bn}}g\sqrt{\frac{\pi}{2}}(d+ex)^2\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)(c}{\sqrt{b}e^2\sqrt{n}}\right.}{}\right.}{3ben}$$

$$\frac{2\left(-\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}-\frac{4(ef-dg)\left(\frac{e^{-\frac{2a}{bn}}g\sqrt{\frac{\pi}{2}}(d+ex)^2\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)(c(d+ex)^n)^{-2/n}}{\sqrt{b}e^2\sqrt{n}}+\frac{e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\operatorname{erfi}\left(\frac{\sqrt{a+}}{}\right.}{\sqrt{b}e^2}\right.}{ben}\right.}{}$$

input
```
Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output
```
(-2*(d + e*x)*(f + g*x)^2)/(3*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) - (4
*(e*f - d*g)*((-2*(e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d
+ e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^2*E^(a/(b*n))*n^(3/2)*(c*(d + e*
x)^n)^n^(-1)) + (4*(((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*
(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d +
 e*x)^n)^n^(-1)) + (g*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[
c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*
(c*(d + e*x)^n)^(2/n))))/(b*n) - (2*(d + e*x)*(f + g*x))/(b*e*n*Sqrt[a + b
*Log[c*(d + e*x)^n]])))/(3*b*e*n) + (2*((-4*(e*f - d*g)*(((e*f - d*g)*Sqrt
[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(Sq
rt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (g*Sqrt[Pi/2]*(d +
 e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/
(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n))))/(b*e*n) + (6
*(((e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(S
qrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
 + (g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d
 + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((2*a)/(b*n))*Sqrt[n]*(c*(
d + e*x)^n)^(2/n)) + (g^2*Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*
Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((3*a)/(b*n))*Sqrt
[n]*(c*(d + e*x)^n)^(3/n))))/(b*n) - (2*(d + e*x)*(f + g*x)^2)/(b*e*n*S...
```

3.134. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.134. $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.134.4 Maple [F]

$$\int \frac{(gx+f)^2}{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.134.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.134.6 Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**(5/2), x)
```

3.134.7 Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.134.8 Giac [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}\,dx$$

input
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.135 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.135.1 Optimal result

Integrand size = 24, antiderivative size = 311

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}} + \frac{8e^{-\frac{2a}{bn}}g\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}} - \frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

output

```
-2/3*(e*x+d)*(g*x+f)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(-d*g+e*f)*(e*x
+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e^2
/exp(a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(1/n))+8/3*g*(e*x+d)^2*erfi(2^(1/2)*(a+
b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/e^2/exp
(2*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(2/n))+4/3*(-d*g+e*f)*(e*x+d)/b^2/e^2/n^2
/(a+b*ln(c*(e*x+d)^n))^(1/2)-8/3*(e*x+d)*(g*x+f)/b^2/e/n^2/(a+b*ln(c*(e*x+
d)^n))^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.14

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{2e^{-\frac{2a}{bn}}(d+ex)\,(c(d+ex)^n)^{-2/n}\left(-8de^{\frac{a}{bn}}g\sqrt{\pi}(c(d+ex)^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log}}{\sqrt{b}}\right.\right.}{}$$

input

```
Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

output

```
(2*(d + e*x)*(-8*d*E^(a/(b*n))*g*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)*Erfi[Sqrt
[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + 4*g*Sqrt[2*Pi]*(d + e*x)*E
rfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] - (Sqrt[b]
*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)*(2*b*(e*f + 3*d*g)*n*Gamma[1/2
, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(-((a + b*Log[c*(d + e*x)^n])/(b*n)
))^(3/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(b*e*n*(f + g*x) + 2*a*(e*f
+ d*g + 2*e*g*x) + 2*b*(d*g + e*(f + 2*g*x))*Log[c*(d + e*x)^n])))/(a + b*
Log[c*(d + e*x)^n])^(3/2)))/(3*b^(5/2)*e^2*E^((2*a)/(b*n))*n^(5/2)*(c*(d +
 e*x)^n)^(2/n))
```

3.135.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.61, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2847, 2836, 2734, 2737, 2611, 2633, 2847, 2836, 2737, 2611, 2633, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$$

$$\Big\downarrow \text{2847}$$

$$-\frac{2(ef-dg)\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3ben}+\frac{4\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(d+ex)(f+gx)}{3ben\,(a+b\log(c(d+ex)^n))^{3/2}}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{2(ef-dg)\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}d(d+ex)}{3be^2n}+\frac{4\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \text{ 2734}$$

$$-\frac{2(ef-dg)\left(\frac{2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{bn}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}+\frac{4\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \text{ 2737}$$

$$-\frac{2(ef-dg)\left(\frac{2(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{bn^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}+\frac{4\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \text{ 2611}$$

$$-\frac{2(ef-dg)\left(\frac{4(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2n^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}+\frac{4\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{4\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}dx}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$$\Big\downarrow \text{ 2847}$$

3.135. $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

$$\frac{4\left(-\frac{2(ef-dg)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{ben}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\downarrow$ 2836

$$\frac{4\left(-\frac{2(ef-dg)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{be^2n}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\downarrow$ 2737

$$\frac{4\left(-\frac{2(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{be^2n^2}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\downarrow$ 2611

$$\frac{4\left(-\frac{4(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2e^2n^2}+\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

3.135. $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

$\Big\downarrow$ 2633

$$\frac{4\left(\frac{4\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{bn}-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}-\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\Big\downarrow$ 2848

$$\frac{4\left(\frac{4\int \left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}-\frac{2(d+ex)\cdots}{ben\sqrt{a+b\cdots}}\cdots\right)}{3bn}-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{2(ef-dg)\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3be^2n}+\frac{4\left(-\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}}+\frac{4\left(\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n}\cdots}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}\cdots\right)}{\cdots}\right)}{3bn}-\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

input
```
Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

3.135. $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

output
```
(-2*(d + e*x)*(f + g*x))/(3*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) - (2*(
e*f - d*g)*((2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqr
t[b]*Sqrt[n])])/(b^(3/2)*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*
(d + e*x))/(b*n*Sqrt[a + b*Log[c*(d + e*x)^n]])))/(3*b*e^2*n) + (4*((-2*(e
*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*
Sqrt[n])])/(b^(3/2)*e^2*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) + (4*(
((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[
b]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (
g*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sq
rt[b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n
))))/(b*n) - (2*(d + e*x)*(f + g*x))/(b*e*n*Sqrt[a + b*Log[c*(d + e*x)^n]]
)))/(3*b*n)
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.135.4 Maple [F]

$$\int \frac{gx + f}{\left(a + b\ln\left(c\left(ex + d\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.135.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{f + gx}{(a + b\log(c(d + ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.135.6 Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{f+gx}{(a+b\log\left(c\left(d+ex\right)^n\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**(5/2), x)
```

3.135.7 Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{gx+f}{(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.135.8 Giac [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{gx+f}{(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{f+g\,x}{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{5/2}}\,dx$$

input
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.136 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.136.1 Optimal result

Integrand size = 18, antiderivative size = 156

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx=\frac{4e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}}-\frac{2(d+ex)}{3ben\,(a+b\log(c(d+ex)^n))^{3/2}}-\frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}}$$

output
```
-2/3*(e*x+d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(e*x+d)*erfi((a+b*ln(c*
(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e/exp(a/b/n)/n^(5/2)/(
(c*(e*x+d)^n)^(1/n))-4/3*(e*x+d)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx=-\frac{2e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\left(2bn\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{3/2}+e^{\frac{a}{bn}}(c(d+ex)^n)^{\frac{1}{n}}(2c\right.}{3b^2en^2\,(a+b\log(c(d+ex)^n))^{3/2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

output
```
(-2*(d + e*x)*(2*b*n*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(-((a
 + b*Log[c*(d + e*x)^n])/(b*n)))^(3/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1
)*(2*a + b*n + 2*b*Log[c*(d + e*x)^n])))/(3*b^2*e*E^(a/(b*n))*n^2*(c*(d +
e*x)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])^(3/2))
```

3.136.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2836, 2734, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{2\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}d(d+ex)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}d(d+ex)}{bn}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{\frac{2\left(\frac{2(d+ex)(c(d+ex)^n)^{-1/n}\int \frac{(c(d+ex)^n)^{\frac{1}{n}}}{\sqrt{a+b\log(c(d+ex)^n)}}d\log(c(d+ex)^n)}{bn^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{\frac{2\left(\frac{4(d+ex)(c(d+ex)^n)^{-1/n}\int e^{\frac{a+b\log(c(d+ex)^n)}{bn}-\frac{a}{bn}}d\sqrt{a+b\log(c(d+ex)^n)}}{b^2n^2}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\left(\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}n^{3/2}}-\frac{2(d+ex)}{bn\sqrt{a+b\log(c(d+ex)^n)}}\right)}{3bn}-\frac{2(d+ex)}{3bn(a+b\log(c(d+ex)^n))^{3/2}}}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

output
```
((-2*(d + e*x))/(3*b*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) + (2*((2*Sqrt[Pi]
*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(b^(3/2
)*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/(b*n*Sqrt[a
+ b*Log[c*(d + e*x)^n]])))/(3*b*n))/e
```

3.136.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.136.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.136.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.136.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**(-5/2), x)
```

3.136.7 Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\log\left((ex+d)^n c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

3.136.8 Giac [F]

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\log\left((ex+d)^n c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d + e*x)^n))^(5/2),x)
```

output
```
int(1/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

3.137 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^{5/2}}\,dx$

3.137.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.137.2 Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)), x]
```

3.137.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^{5/2}}\,dx$$
$$\Big\downarrow \; 2867$$
$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^{5/2}}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)),x]
```

output
```
$Aborted
```

3.137.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.137.4 Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

3.137.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

output
```
Timed out
```

3.137.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2)), x)
```

3.137.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2)), x)
```

3.137.9 Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{5/2}}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2)),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2)), x)
```

3.138 $\int (f+gx)^{3/2}(a+b\log(c(d+ex)^n))\,dx$

3.138.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int (f+gx)^{3/2}(a+b\log(c(d+ex)^n))\,dx=-\frac{4b(ef-dg)^2n\sqrt{f+gx}}{5e^2g}-\frac{4b(ef-dg)n(f+gx)^{3/2}}{15eg}-\frac{4bn(f+gx)^{5/2}}{25g}+\frac{4b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5e^{5/2}g}+\frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5g}$$

output
```
-4/15*b*(-d*g+e*f)*n*(g*x+f)^(3/2)/e/g-4/25*b*n*(g*x+f)^(5/2)/g+4/5*b*(-d*
g+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/g+2
/5*(g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/g-4/5*b*(-d*g+e*f)^2*n*(g*x+f)^(1/2
)/e^2/g
```

3.138.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int (f+gx)^{3/2}(a+b\log(c(d+ex)^n))\,dx=\frac{2\left(-\frac{2}{5}bn(f+gx)^{5/2}-\frac{2b(ef-dg)n\left(\sqrt{e}\sqrt{f+gx}(4ef-3dg+egx)-3(ef-dg)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\right)}{3e^{5/2}}\right.}{5g}$$

input
```
Integrate[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(2*((-2*b*n*(f + g*x)^(5/2))/5 - (2*b*(e*f - d*g)*n*(Sqrt[e]*Sqrt[f + g*x]
*(4*e*f - 3*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g
*x])/Sqrt[e*f - d*g]]))/(3*e^(5/2)) + (f + g*x)^(5/2)*(a + b*Log[c*(d + e*
x)^n])))/(5*g)
```

3.138.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2842, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n)) \, dx$$

$$\Big\downarrow \; 2842$$

$$\frac{2(f+gx)^{5/2} (a+b\log (c(d+ex)^n))}{5g}-\frac{2ben\int \frac{(f+gx)^{5/2}}{d+ex}dx}{5g}$$

$$\Big\downarrow \; 60$$

$$\frac{2(f+gx)^{5/2} (a+b\log (c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{(ef-dg)\int \frac{(f+gx)^{3/2}}{d+ex}dx}{e}+\frac{2(f+gx)^{5/2}}{5e}\right)}{5g}$$

$$\Big\downarrow \; 60$$

$$\frac{2(f+gx)^{5/2} (a+b\log (c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{(ef-dg)\left(\frac{(ef-dg)\int \frac{\sqrt{f+gx}}{d+ex}dx}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{e}+\frac{2(f+gx)^{5/2}}{5e}\right)}{5g}$$

$$\Big\downarrow \; 60$$

$$\frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{(ef-dg)\left(\frac{(ef-dg)\left(\frac{(ef-dg)\int\frac{1}{(d+ex)\sqrt{f+gx}}dx}{e}+\frac{2\sqrt{f+gx}}{e}\right)}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{e}+\frac{2(f+gx)^{5/2}}{5e}\right)}{5g}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{(ef-dg)\left(\frac{(ef-dg)\left(\frac{2(ef-dg)\int\frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{eg}+\frac{2\sqrt{f+gx}}{e}\right)}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{e}+\frac{2(f+gx)^{5/2}}{5e}\right)}{5g}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{(ef-dg)\left(\frac{(ef-dg)\left(\frac{2\sqrt{f+gx}}{e}-\frac{2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}}\right)}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{e}+\frac{2(f+gx)^{5/2}}{5e}\right)}{5g}$$

input
```
Int[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]),x]
```

3.138. $\int(f+gx)^{3/2}(a+b\log(c(d+ex)^n))\,dx$

output
```
(-2*b*e*n*((2*(f + g*x)^(5/2))/(5*e) + ((e*f - d*g)*((2*(f + g*x)^(3/2))/(
3*e) + ((e*f - d*g)*((2*Sqrt[f + g*x])/e - (2*Sqrt[e*f - d*g]*ArcTanh[(Sqr
t[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/e^(3/2)))/e))/e))/(5*g) + (2*(f + g*
x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(5*g)
```

3.138.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.138.4 Maple [F]

$$\int (gx+f)^{\frac{3}{2}} \left(a+b\ln \left(c(ex+d)^{n}\right)\right) dx$$

input
```
int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n)),x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 538, normalized size of antiderivative = 3.30

$$\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))\,dx = \left[\frac{2\left(15\left(be^2f^2-2\,bdefg+bd^2g^2\right)n\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right)+(15\right.}{}\right.$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
[2/75*(15*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*sqrt((e*f - d*g)/e)*log(
(e*g*x + 2*e*f - d*g + 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) +
 (15*a*e^2*f^2 - 3*(2*b*e^2*g^2*n - 5*a*e^2*g^2)*x^2 - 2*(23*b*e^2*f^2 - 3
5*b*d*e*f*g + 15*b*d^2*g^2)*n + 2*(15*a*e^2*f*g - (11*b*e^2*f*g - 5*b*d*e*
g^2)*n)*x + 15*(b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + b*e^2*f^2*n)*log(e*x +
 d) + 15*(b*e^2*g^2*x^2 + 2*b*e^2*f*g*x + b*e^2*f^2)*log(c))*sqrt(g*x + f)
)/(e^2*g), 2/75*(30*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*sqrt(-(e*f - d
*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f - d*g)) + (15*a*e
^2*f^2 - 3*(2*b*e^2*g^2*n - 5*a*e^2*g^2)*x^2 - 2*(23*b*e^2*f^2 - 35*b*d*e*
f*g + 15*b*d^2*g^2)*n + 2*(15*a*e^2*f*g - (11*b*e^2*f*g - 5*b*d*e*g^2)*n)*
x + 15*(b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + b*e^2*f^2*n)*log(e*x + d) + 15
*(b*e^2*g^2*x^2 + 2*b*e^2*f*g*x + b*e^2*f^2)*log(c))*sqrt(g*x + f))/(e^2*g
)]
```

3.138.6 Sympy [F]

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int \left(a+b\log\left(c(d+ex)^n\right)\right)(f+gx)^{\frac{3}{2}}\,dx$$

input
```
integrate((g*x+f)**(3/2)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(f + g*x)**(3/2), x)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.138.8 Giac [F]

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int (gx+f)^{\frac{3}{2}}(b\log\left((ex+d)^nc\right)+a)\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int (f+g\,x)^{3/2}\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)\,dx$$

input
```
int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)), x)
```

3.139 $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx$

3.139.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = -\frac{4b(ef-dg)n\sqrt{f+gx}}{3eg}-\frac{4bn(f+gx)^{3/2}}{9g}+\frac{4b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{3/2}g}+\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}$$

output
```
-4/9*b*n*(g*x+f)^(3/2)/g+4/3*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^
(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/g+2/3*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/
g-4/3*b*(-d*g+e*f)*n*(g*x+f)^(1/2)/e/g
```

3.139.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx$$

$$=\frac{2\left(6b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+\sqrt{e}\sqrt{f+gx}(3ae(f+gx)-2bn(4ef-3dg+egx)+3be(f+gx\right.}{9e^{3/2}g}$$

input
```
Integrate[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(2*(6*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g
]] + Sqrt[e]*Sqrt[f + g*x]*(3*a*e*(f + g*x) - 2*b*n*(4*e*f - 3*d*g + e*g*x
) + 3*b*e*(f + g*x)*Log[c*(d + e*x)^n])))/(9*e^(3/2)*g)
```

3.139.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2842, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\int \frac{(f+gx)^{3/2}}{d+ex}dx}{3g}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\frac{(ef-dg)\int \frac{\sqrt{f+gx}}{d+ex}dx}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{3g}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\frac{(ef-dg)\left(\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}}dx}{e}+\frac{2\sqrt{f+gx}}{e}\right)}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{3g}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g} - \frac{2ben\left(\frac{(ef-dg)\left(\frac{2(ef-dg)\int \frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{eg}+\frac{2\sqrt{f+gx}}{e}\right)}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{3g}$$

$$\Big\downarrow \; 221$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g} - \frac{2ben\left(\frac{(ef-dg)\left(\frac{2\sqrt{f+gx}}{e}-\frac{2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}}\right)}{e}+\frac{2(f+gx)^{3/2}}{3e}\right)}{3g}$$

input
```
Int[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(-2*b*e*n*((2*(f + g*x)^(3/2))/(3*e) + ((e*f - d*g)*((2*Sqrt[f + g*x]])/e -
 (2*Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/e^(3
/2)))/e))/(3*g) + (2*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(3*g)
```

3.139.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.139.4 Maple [F]

$$\int \sqrt{gx+f}\,(a+b\ln(c(ex+d)^n))\,dx$$

input
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n)),x)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.36

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx$$
$$= \left[-\frac{2\left(3\,(bef-bdg)n\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right)-(3\,aef-2\,(4\,bef-3\,bdg)n-(2\,begn-\right.}{9\,eg}\right.$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
[-2/9*(3*(b*e*f - b*d*g)*n*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g -
2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) - (3*a*e*f - 2*(4*b*e*f
- 3*b*d*g)*n - (2*b*e*g*n - 3*a*e*g)*x + 3*(b*e*g*n*x + b*e*f*n)*log(e*x +
 d) + 3*(b*e*g*x + b*e*f)*log(c))*sqrt(g*x + f))/(e*g), 2/9*(6*(b*e*f - b*
d*g)*n*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(
e*f - d*g)) + (3*a*e*f - 2*(4*b*e*f - 3*b*d*g)*n - (2*b*e*g*n - 3*a*e*g)*x
 + 3*(b*e*g*n*x + b*e*f*n)*log(e*x + d) + 3*(b*e*g*x + b*e*f)*log(c))*sqrt
(g*x + f))/(e*g)]
```

3.139.6 Sympy [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \int (a+b\log(c(d+ex)^n))\sqrt{f+gx}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*sqrt(f + g*x), x)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.139.8 Giac [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \int \sqrt{gx+f}(b\log((ex+d)^n c)+a)\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \int \sqrt{f+g\,x}\,(a+b\,\ln(c\,(d+e\,x)^n))\,dx$$

input
```
int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)), x)
```

3.140 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx}}\,dx$

3.140.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx=-\frac{4bn\sqrt{f+gx}}{g}+\frac{4b\sqrt{ef-dg}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g}+\frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}$$

output
```
4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(-d*g+e*f)^(1/2)/g/e
^(1/2)-4*b*n*(g*x+f)^(1/2)/g+2*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/g
```

3.140.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx=\frac{2\left(\frac{2b\sqrt{ef-dg}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}}+\sqrt{f+gx}\left(a-2bn+b\log\left(c(d+ex)^n\right)\right)\right)}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x],x]
```

output
```
(2*((2*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/Sqrt[e] + Sqrt[f + g*x]*(a - 2*b*n + b*Log[c*(d + e*x)^n])))/g
```

3.140.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2842, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\int \frac{\sqrt{f+gx}}{d+ex}dx}{g}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}}dx}{e}+\frac{2\sqrt{f+gx}}{e}\right)}{g}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\frac{2(ef-dg)\int \frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{eg}+\frac{2\sqrt{f+gx}}{e}\right)}{g}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\frac{2\sqrt{f+gx}}{e}-\frac{2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}}\right)}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x],x]
```

output
```
(-2*b*e*n*((2*Sqrt[f + g*x])/e - (2*Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[
f + g*x])/Sqrt[e*f - d*g]])/e^(3/2)))/g + (2*Sqrt[f + g*x]*(a + b*Log[c*(d
 + e*x)^n]))/g
```

3.140.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.140.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{2\sqrt{gx+f}\,a+2b\left(\ln\left(c\left(\frac{(gx+f)e+dg-ef}{g}\right)^{n}\right)\sqrt{gx+f}-2en\left(\frac{\sqrt{gx+f}}{e}+\frac{(-dg+ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e\sqrt{(dg-ef)e}}\right)\right)}{g}$	113
default	$\frac{2\sqrt{gx+f}\,a+2b\left(\ln\left(c\left(\frac{(gx+f)e+dg-ef}{g}\right)^{n}\right)\sqrt{gx+f}-2en\left(\frac{\sqrt{gx+f}}{e}+\frac{(-dg+ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e\sqrt{(dg-ef)e}}\right)\right)}{g}$	113
parts	$\frac{2a\sqrt{gx+f}}{g}+\frac{2b\left(\ln\left(c\left(\frac{(gx+f)e+dg-ef}{g}\right)^{n}\right)\sqrt{gx+f}-2en\left(\frac{\sqrt{gx+f}}{e}+\frac{(-dg+ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e\sqrt{(dg-ef)e}}\right)\right)}{g}$	116

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/g*((g*x+f)^(1/2)*a+b*(ln(c*(((g*x+f)*e+d*g-e*f)/g)^n)*(g*x+f)^(1/2)-2*e*
n*((g*x+f)^(1/2)/e+(-d*g+e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)
/((d*g-e*f)*e)^(1/2)))))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.91

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx$$
$$= \left[\frac{2\left(bn\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right)+(bn\log\left(ex+d\right)-2\,bn+b\log\left(c\right)+a)\sqrt{gx+f}\right)}{g}, \frac{2\left(\right.}{}\right.$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[2*(b*n*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g + 2*sqrt(g*x + f)*e*s
qrt((e*f - d*g)/e))/(e*x + d)) + (b*n*log(e*x + d) - 2*b*n + b*log(c) + a)
*sqrt(g*x + f))/g, 2*(2*b*n*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*s
qrt(-(e*f - d*g)/e)/(e*f - d*g)) + (b*n*log(e*x + d) - 2*b*n + b*log(c) +
a)*sqrt(g*x + f))/g]
```

3.140.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/sqrt(f + g*x), x)
```

3.140.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx}}\,dx = -\frac{2\left(\left(2e\left(\frac{(ef-dg)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+dege}}+\frac{\sqrt{gx+f}}{e}\right)-\sqrt{gx+f}\log(ex+d)\right)bn-\sqrt{gx+f}b\log(c)-\sqrt{gx+\cdots}\right)}{g}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-2*((2*e*((e*f - d*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-
e^2*f + d*e*g)*e) + sqrt(g*x + f)/e) - sqrt(g*x + f)*log(e*x + d))*b*n - s
qrt(g*x + f)*b*log(c) - sqrt(g*x + f)*a)/g
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{f+g\,x}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(1/2), x)
```

3.141 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{3/2}}\,dx$

3.141.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=-\frac{4b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{g\sqrt{f+gx}}$$

output
```
-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*e^(1/2)/g/(-d*g+e*f
)^(1/2)-2*(a+b*ln(c*(e*x+d)^n))/g/(g*x+f)^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=\frac{2\left(-\frac{2b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx}}\right)}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(3/2),x]
```

output
```
(2*((-2*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt
[e*f - d*g] - (a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x]))/g
```

3.141.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2842, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \ 2842$$

$$\frac{2ben\int \frac{1}{(d+ex)\sqrt{f+gx}}dx}{g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{g\sqrt{f+gx}}$$

$$\Big\downarrow \ 73$$

$$\frac{4ben\int \frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{g^2}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{g\sqrt{f+gx}}$$

$$\Big\downarrow \ 221$$

$$-\frac{2(a+b\log\left(c(d+ex)^n\right))}{g\sqrt{f+gx}}-\frac{4b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(3/2),x]
```

output
```
(-4*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(g*Sqrt[
e*f - d*g]) - (2*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt[f + g*x])
```

3.141.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.141.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{(gx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(3/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(3/2),x)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.77

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(f+gx)^{3/2}}\,dx = \left[\frac{2\left((bgnx+bfn)\sqrt{\frac{e}{ef-dg}}\log\left(\frac{egx+2\,ef-dg-2\,(ef-dg)\sqrt{gx+f}\sqrt{\frac{e}{ef-dg}}}{ex+d}\right)-(bn\log\left(e\right.\right.}{g^{2}x+fg}\right.$$

$$-\frac{2\left(2\,(bgnx+bfn)\sqrt{-\frac{e}{ef-dg}}\arctan\left(-\frac{(ef-dg)\sqrt{gx+f}\sqrt{-\frac{e}{ef-dg}}}{egx+ef}\right)+(bn\log\left(ex+d\right)+b\log\left(c\right)+a)\sqrt{gx+f}\right)}{g^{2}x+fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[2*((b*g*n*x + b*f*n)*sqrt(e/(e*f - d*g))*log((e*g*x + 2*e*f - d*g - 2*(e*
f - d*g)*sqrt(g*x + f)*sqrt(e/(e*f - d*g)))/(e*x + d)) - (b*n*log(e*x + d)
 + b*log(c) + a)*sqrt(g*x + f))/(g^2*x + f*g), -2*(2*(b*g*n*x + b*f*n)*sqr
t(-e/(e*f - d*g))*arctan(-(e*f - d*g)*sqrt(g*x + f)*sqrt(-e/(e*f - d*g))/(
e*g*x + e*f)) + (b*n*log(e*x + d) + b*log(c) + a)*sqrt(g*x + f))/(g^2*x +
f*g)]
```

3.141.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(f + g*x)**(3/2), x)
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=\frac{4\,ben\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}g}-\frac{2\,bn\log\left((gx+f)e-ef+dg\right)}{\sqrt{gx+f}g}+\frac{2\left(bn\log\left(g\right)-b\log\left(c\right)-a\right)}{\sqrt{gx+f}g}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
4*b*e*n*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)
*g) - 2*b*n*log((g*x + f)*e - e*f + d*g)/(sqrt(g*x + f)*g) + 2*(b*n*log(g)
 - b*log(c) - a)/(sqrt(g*x + f)*g)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(3/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(3/2), x)
```

3.142 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{5/2}}\,dx$

3.142.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=\frac{4ben}{3g(ef-dg)\sqrt{f+gx}}-\frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g(f+gx)^{3/2}}$$

output
```
-4/3*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*g+e
*f)^(3/2)-2/3*(a+b*ln(c*(e*x+d)^n))/g/(g*x+f)^(3/2)+4/3*b*e*n/g/(-d*g+e*f)
/(g*x+f)^(1/2)
```

3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=-\frac{4ben\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{e(f+gx)}{ef-dg}\right)}{3g(-ef+dg)\sqrt{f+gx}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g(f+gx)^{3/2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(5/2),x]
```

output
```
(-4*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*g)])/(3*g
*(-(e*f) + d*g)*Sqrt[f + g*x]) - (2*(a + b*Log[c*(d + e*x)^n]))/(3*g*(f +
g*x)^(3/2))
```

3.142.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2842, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2ben\int \frac{1}{(d+ex)(f+gx)^{3/2}}dx}{3g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2ben\left(\frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}}dx}{ef-dg}+\frac{2}{\sqrt{f+gx}(ef-dg)}\right)}{3g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2ben\left(\frac{2e\int \frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{g(ef-dg)}+\frac{2}{\sqrt{f+gx}(ef-dg)}\right)}{3g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2ben\left(\frac{2}{\sqrt{f+gx}(ef-dg)}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{3g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(5/2),x]
```

output
```
(2*b*e*n*(2/((e*f - d*g)*Sqrt[f + g*x]) - (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]])/(e*f - d*g)^(3/2)))/(3*g) - (2*(a + b*Log[c*(
d + e*x)^n]))/(3*g*(f + g*x)^(3/2))
```

3.142.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.142.4 Maple [F]

$$\int \frac{a + b\ln\left(c(ex+d)^{n}\right)}{(gx+f)^{\frac{5}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(5/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(5/2),x)
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(94) = 188$.

Time = 0.36 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.73

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = \left[-\frac{2\left(\left(beg^2nx^2+2\,befgnx+bef^2n\right)\sqrt{\frac{e}{ef-dg}}\log\left(\frac{egx+2\,ef-dg+2\,(ef-dg)\sqrt{gx+f}\sqrt{}}{ex+d}\right.\right.}{3\left(ef^3g-df^2\right.}\right.$$

$$-\frac{2\left(2\left(beg^2nx^2+2\,befgnx+bef^2n\right)\sqrt{-\frac{e}{ef-dg}}\arctan\left(-\frac{(ef-dg)\sqrt{gx+f}\sqrt{-\frac{e}{ef-dg}}}{egx+ef}\right)-\left(2\,begnx+2\,befn-aef\right.\right.}{3\left(ef^3g-df^2g^2+(efg^3-dg^4)x^2+2\left(ef^2g^2-\right.\right.}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(5/2),x, algorithm="fricas")
```

output
```
[-2/3*((b*e*g^2*n*x^2 + 2*b*e*f*g*n*x + b*e*f^2*n)*sqrt(e/(e*f - d*g))*log
((e*g*x + 2*e*f - d*g + 2*(e*f - d*g)*sqrt(g*x + f)*sqrt(e/(e*f - d*g)))/(
e*x + d)) - (2*b*e*g*n*x + 2*b*e*f*n - a*e*f + a*d*g - (b*e*f - b*d*g)*n*l
og(e*x + d) - (b*e*f - b*d*g)*log(c))*sqrt(g*x + f))/(e*f^3*g - d*f^2*g^2
+ (e*f*g^3 - d*g^4)*x^2 + 2*(e*f^2*g^2 - d*f*g^3)*x), -2/3*(2*(b*e*g^2*n*x
^2 + 2*b*e*f*g*n*x + b*e*f^2*n)*sqrt(-e/(e*f - d*g))*arctan(-(e*f - d*g)*s
qrt(g*x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) - (2*b*e*g*n*x + 2*b*e*f*
n - a*e*f + a*d*g - (b*e*f - b*d*g)*n*log(e*x + d) - (b*e*f - b*d*g)*log(c
))*sqrt(g*x + f))/(e*f^3*g - d*f^2*g^2 + (e*f*g^3 - d*g^4)*x^2 + 2*(e*f^2*
g^2 - d*f*g^3)*x)]
```

3.142.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = \int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(5/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(f + g*x)**(5/2), x)
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = \frac{4\,be^2n\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{3\sqrt{-e^2f+deg}(efg-dg^2)} - \frac{2\,bn\log\left((gx+f)e-ef+dg\right)}{3\,(gx+f)^{\frac{3}{2}}g} + \frac{2\,(befn\log\left(g\right)-bdgn\log\left(g\right)+2\,(gx+f)ben-bef\log\left(c\right)+bdg\log\left(c\right)-aef+adg)}{3\left((gx+f)^{\frac{3}{2}}efg-(gx+f)^{\frac{3}{2}}dg^2\right)}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(5/2),x, algorithm="giac")
```

output
```
4/3*b*e^2*n*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*
e*g)*(e*f*g - d*g^2)) - 2/3*b*n*log((g*x + f)*e - e*f + d*g)/((g*x + f)^(3
/2)*g) + 2/3*(b*e*f*n*log(g) - b*d*g*n*log(g) + 2*(g*x + f)*b*e*n - b*e*f*
log(c) + b*d*g*log(c) - a*e*f + a*d*g)/((g*x + f)^(3/2)*e*f*g - (g*x + f)^
(3/2)*d*g^2)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(5/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(5/2), x)
```

3.143 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{7/2}}\,dx$

3.143.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx=\frac{4ben}{15g(ef-dg)(f+gx)^{3/2}}+\frac{4be^2n}{5g(ef-dg)^2\sqrt{f+gx}}-\frac{4be^{5/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5g(ef-dg)^{5/2}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g(f+gx)^{5/2}}$$

output
```
4/15*b*e*n/g/(-d*g+e*f)/(g*x+f)^(3/2)-4/5*b*e^(5/2)*n*arctanh(e^(1/2)*(g*x
+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*g+e*f)^(5/2)-2/5*(a+b*ln(c*(e*x+d)^n))/g
/(g*x+f)^(5/2)+4/5*b*e^2*n/g/(-d*g+e*f)^2/(g*x+f)^(1/2)
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx=\frac{2\left(\frac{2ben(f+gx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{e(f+gx)}{ef-dg}\right)}{ef-dg}-3\left(a+b\log\left(c(d+ex)^n\right)\right)\right)}{15g(f+gx)^{5/2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(7/2),x]
```

output
```
(2*((2*b*e*n*(f + g*x)*Hypergeometric2F1[-3/2, 1, -1/2, (e*(f + g*x))/(e*f
 - d*g)])/(e*f - d*g) - 3*(a + b*Log[c*(d + e*x)^n])))/(15*g*(f + g*x)^(5/
2))
```

3.143.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2842, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2ben\int \frac{1}{(d+ex)(f+gx)^{5/2}}dx}{5g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2ben\left(\frac{e\int \frac{1}{(d+ex)(f+gx)^{3/2}}dx}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{5g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2ben\left(\frac{e\left(\frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}}dx}{ef-dg}+\frac{2}{\sqrt{f+gx}(ef-dg)}\right)}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{5g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2ben\left(\frac{e\left(\frac{2e\int \frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{g(ef-dg)}+\frac{2}{\sqrt{f+gx}(ef-dg)}\right)}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{5g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2ben\left(\frac{e\left(\frac{2}{\sqrt{f+gx}(ef-dg)}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{5g}-\frac{2(a+b\log\left(c(d+ex)^n\right))}{5g(f+gx)^{5/2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(7/2),x]
```

output
```
(2*b*e*n*(2/(3*(e*f - d*g)*(f + g*x)^(3/2)) + (e*(2/((e*f - d*g)*Sqrt[f +
g*x]) - (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e*f
- d*g)^(3/2)))/(e*f - d*g)))/(5*g) - (2*(a + b*Log[c*(d + e*x)^n]))/(5*g*(
f + g*x)^(5/2))
```

3.143.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.143. $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{7/2}}\,dx$

3.143.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{(gx+f)^{\frac{7}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(7/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(7/2),x)
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(121) = 242.

Time = 0.36 (sec) , antiderivative size = 789, normalized size of antiderivative = 5.44

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(f+gx)^{7/2}}\,dx = \left[\frac{2\left(3\left(be^{2}g^{3}nx^{3}+3\,be^{2}fg^{2}nx^{2}+3\,be^{2}f^{2}gnx+be^{2}f^{3}n\right)\sqrt{\frac{e}{ef-dg}}\log\left(\frac{egx+2\,ef-}{}\right.\right.}{}\right.$$

$$-\frac{2\left(6\left(be^{2}g^{3}nx^{3}+3\,be^{2}fg^{2}nx^{2}+3\,be^{2}f^{2}gnx+be^{2}f^{3}n\right)\sqrt{-\frac{e}{ef-dg}}\arctan\left(-\frac{(ef-dg)\sqrt{gx+f}\sqrt{-\frac{e}{ef-dg}}}{egx+ef}\right)-(6\,be^{2}\right.}{15\left(e^{2}f^{5}g-2\,def^{4}g^{2}+d^{2}f^{3}g^{3}\right.}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(7/2),x, algorithm="fricas")
```

output
```
[2/15*(3*(b*e^2*g^3*n*x^3 + 3*b*e^2*f*g^2*n*x^2 + 3*b*e^2*f^2*g*n*x + b*e^
2*f^3*n)*sqrt(e/(e*f - d*g))*log((e*g*x + 2*e*f - d*g - 2*(e*f - d*g)*sqrt
(g*x + f)*sqrt(e/(e*f - d*g)))/(e*x + d)) + (6*b*e^2*g^2*n*x^2 - 3*a*e^2*f
^2 + 6*a*d*e*f*g - 3*a*d^2*g^2 + 2*(7*b*e^2*f*g - b*d*e*g^2)*n*x - 3*(b*e^
2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*log(e*x + d) + 2*(4*b*e^2*f^2 - b*d*e*f
*g)*n - 3*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*log(c))*sqrt(g*x + f))/(e^
2*f^5*g - 2*d*e*f^4*g^2 + d^2*f^3*g^3 + (e^2*f^2*g^4 - 2*d*e*f*g^5 + d^2*g
^6)*x^3 + 3*(e^2*f^3*g^3 - 2*d*e*f^2*g^4 + d^2*f*g^5)*x^2 + 3*(e^2*f^4*g^2
 - 2*d*e*f^3*g^3 + d^2*f^2*g^4)*x), -2/15*(6*(b*e^2*g^3*n*x^3 + 3*b*e^2*f*
g^2*n*x^2 + 3*b*e^2*f^2*g*n*x + b*e^2*f^3*n)*sqrt(-e/(e*f - d*g))*arctan(-
(e*f - d*g)*sqrt(g*x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) - (6*b*e^2*g
^2*n*x^2 - 3*a*e^2*f^2 + 6*a*d*e*f*g - 3*a*d^2*g^2 + 2*(7*b*e^2*f*g - b*d*
e*g^2)*n*x - 3*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*log(e*x + d) + 2*(4
*b*e^2*f^2 - b*d*e*f*g)*n - 3*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*log(c)
)*sqrt(g*x + f))/(e^2*f^5*g - 2*d*e*f^4*g^2 + d^2*f^3*g^3 + (e^2*f^2*g^4 -
 2*d*e*f*g^5 + d^2*g^6)*x^3 + 3*(e^2*f^3*g^3 - 2*d*e*f^2*g^4 + d^2*f*g^5)*
x^2 + 3*(e^2*f^4*g^2 - 2*d*e*f^3*g^3 + d^2*f^2*g^4)*x)]
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(7/2),x)
```

output
```
Timed out
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^{7/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.143.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x + f)^(7/2), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx=\int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{(f+g\,x)^{7/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(7/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(7/2), x)
```

3.144 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{9/2}}\,dx$

3.144.1 Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx=\frac{4ben}{35g(ef-dg)(f+gx)^{5/2}}+\frac{4be^2n}{21g(ef-dg)^2(f+gx)^{3/2}}+\frac{4be^3n}{7g(ef-dg)^3\sqrt{f+gx}}-\frac{4be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{7g(ef-dg)^{7/2}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{7g(f+gx)^{7/2}}$$

output
```
4/35*b*e*n/g/(-d*g+e*f)/(g*x+f)^(5/2)+4/21*b*e^2*n/g/(-d*g+e*f)^2/(g*x+f)^
(3/2)-4/7*b*e^(7/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-
d*g+e*f)^(7/2)-2/7*(a+b*ln(c*(e*x+d)^n))/g/(g*x+f)^(7/2)+4/7*b*e^3*n/g/(-d
*g+e*f)^3/(g*x+f)^(1/2)
```

3.144.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.44

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx=\frac{2\left(\frac{2ben(f+gx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{e(f+gx)}{ef-dg}\right)}{ef-dg}-5\left(a+b\log\left(c(d+ex)^n\right)\right)\right)}{35g(f+gx)^{7/2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(9/2),x]
```

output
```
(2*((2*b*e*n*(f + g*x)*Hypergeometric2F1[-5/2, 1, -3/2, (e*(f + g*x))/(e*f
 - d*g)])/(e*f - d*g) - 5*(a + b*Log[c*(d + e*x)^n])))/(35*g*(f + g*x)^(7/
2))
```

3.144.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2842, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2ben\int \frac{1}{(d+ex)(f+gx)^{7/2}}dx}{7g}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2ben\left(\frac{e\int \frac{1}{(d+ex)(f+gx)^{5/2}}dx}{ef-dg}+\frac{2}{5(f+gx)^{5/2}(ef-dg)}\right)}{7g}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2ben\left(\frac{e\left(\frac{e\int \frac{1}{(d+ex)(f+gx)^{3/2}}dx}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{ef-dg}+\frac{2}{5(f+gx)^{5/2}(ef-dg)}\right)}{7g}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2ben\left(\frac{e\left(\frac{e\left(\frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}}dx}{ef-dg}+\frac{2}{\sqrt{f+gx}(ef-dg)}\right)}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{ef-dg}+\frac{2}{5(f+gx)^{5/2}(ef-dg)}\right)}{7g}-\frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{2ben\left(\frac{e\left(\frac{e\left(\frac{2e\int \frac{1}{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}d\sqrt{f+gx}}{g(ef-dg)}+\frac{2}{\sqrt{f+gx}(ef-dg)}\right)}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{ef-dg}+\frac{2}{5(f+gx)^{5/2}(ef-dg)}\right)}{7g}-\frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{2ben\left(\frac{e\left(\frac{e\left(\frac{2}{\sqrt{f+gx}(ef-dg)}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{ef-dg}+\frac{2}{3(f+gx)^{3/2}(ef-dg)}\right)}{ef-dg}+\frac{2}{5(f+gx)^{5/2}(ef-dg)}\right)}{7g}-\frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(9/2),x]
```

3.144. $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{9/2}}\,dx$

output
```
(2*b*e*n*(2/(5*(e*f - d*g)*(f + g*x)^(5/2)) + (e*(2/(3*(e*f - d*g)*(f + g*
x)^(3/2)) + (e*(2/((e*f - d*g)*Sqrt[f + g*x]) - (2*Sqrt[e]*ArcTanh[(Sqrt[e
]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e*f - d*g)^(3/2)))/(e*f - d*g)))/(e*f
- d*g)))/(7*g) - (2*(a + b*Log[c*(d + e*x)^n]))/(7*g*(f + g*x)^(7/2))
```

3.144.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.144.4 Maple [F]

$$\int \frac{a+b\ln \left(c(ex+d)^n\right)}{(gx+f)^{\frac{9}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(9/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(9/2),x)
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(148) = 296$.

Time = 0.43 (sec) , antiderivative size = 1252, normalized size of antiderivative = 7.11

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(9/2),x, algorithm="fricas")
```

output
```
[-2/105*(15*(b*e^3*g^4*n*x^4 + 4*b*e^3*f*g^3*n*x^3 + 6*b*e^3*f^2*g^2*n*x^2
 + 4*b*e^3*f^3*g*n*x + b*e^3*f^4*n)*sqrt(e/(e*f - d*g))*log((e*g*x + 2*e*f
 - d*g + 2*(e*f - d*g)*sqrt(g*x + f)*sqrt(e/(e*f - d*g)))/(e*x + d)) - (30
*b*e^3*g^3*n*x^3 - 15*a*e^3*f^3 + 45*a*d*e^2*f^2*g - 45*a*d^2*e*f*g^2 + 15
*a*d^3*g^3 + 10*(10*b*e^3*f*g^2 - b*d*e^2*g^3)*n*x^2 + 2*(58*b*e^3*f^2*g -
 16*b*d*e^2*f*g^2 + 3*b*d^2*e*g^3)*n*x - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g +
 3*b*d^2*e*f*g^2 - b*d^3*g^3)*n*log(e*x + d) + 2*(23*b*e^3*f^3 - 11*b*d*e^
2*f^2*g + 3*b*d^2*e*f*g^2)*n - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e
*f*g^2 - b*d^3*g^3)*log(c))*sqrt(g*x + f))/(e^3*f^7*g - 3*d*e^2*f^6*g^2 +
3*d^2*e*f^5*g^3 - d^3*f^4*g^4 + (e^3*f^3*g^5 - 3*d*e^2*f^2*g^6 + 3*d^2*e*f
*g^7 - d^3*g^8)*x^4 + 4*(e^3*f^4*g^4 - 3*d*e^2*f^3*g^5 + 3*d^2*e*f^2*g^6 -
 d^3*f*g^7)*x^3 + 6*(e^3*f^5*g^3 - 3*d*e^2*f^4*g^4 + 3*d^2*e*f^3*g^5 - d^3
*f^2*g^6)*x^2 + 4*(e^3*f^6*g^2 - 3*d*e^2*f^5*g^3 + 3*d^2*e*f^4*g^4 - d^3*f
^3*g^5)*x), -2/105*(30*(b*e^3*g^4*n*x^4 + 4*b*e^3*f*g^3*n*x^3 + 6*b*e^3*f^
2*g^2*n*x^2 + 4*b*e^3*f^3*g*n*x + b*e^3*f^4*n)*sqrt(-e/(e*f - d*g))*arctan
(-(e*f - d*g)*sqrt(g*x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) - (30*b*e^
3*g^3*n*x^3 - 15*a*e^3*f^3 + 45*a*d*e^2*f^2*g - 45*a*d^2*e*f*g^2 + 15*a*d^
3*g^3 + 10*(10*b*e^3*f*g^2 - b*d*e^2*g^3)*n*x^2 + 2*(58*b*e^3*f^2*g - 16*b
*d*e^2*f*g^2 + 3*b*d^2*e*g^3)*n*x - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*
d^2*e*f*g^2 - b*d^3*g^3)*n*log(e*x + d) + 2*(23*b*e^3*f^3 - 11*b*d*e^2*...
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(9/2),x)
```

output
```
Timed out
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.144.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x + f)^(9/2), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{9/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(9/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(9/2), x)
```

3.145 $\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n))^2 \, dx$

3.145.1 Optimal result

Integrand size = 26, antiderivative size = 590

$$\int (f+gx)^{3/2} \left(a+b\log\left(c(d+ex)^n\right)\right)^2 \, dx = \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g} - \frac{368b^2(ef-dg)^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g} - \frac{8b^2(ef-dg)^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g} - \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g} - \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg} - \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g} + \frac{8b(ef-dg)^{5/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g} + \frac{16b^2(ef-dg)^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g} + \frac{8b^2(ef-dg)^{5/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g}$$

output
```
128/225*b^2*(-d*g+e*f)*n^2*(g*x+f)^(3/2)/e/g+16/125*b^2*n^2*(g*x+f)^(5/2)/
g-368/75*b^2*(-d*g+e*f)^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)
^(1/2))/e^(5/2)/g-8/5*b^2*(-d*g+e*f)^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/
2)/(-d*g+e*f)^(1/2))^2/e^(5/2)/g-8/15*b*(-d*g+e*f)*n*(g*x+f)^(3/2)*(a+b*ln
(c*(e*x+d)^n))/e/g-8/25*b*n*(g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/g+8/5*b*(-
d*g+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c
*(e*x+d)^n))/e^(5/2)/g+2/5*(g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))^2/g+16/5*b^
2*(-d*g+e*f)^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(
2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(5/2)/g+8/5*b^2*(-d*g+e*f)
^(5/2)*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(5/
2)/g+368/75*b^2*(-d*g+e*f)^2*n^2*(g*x+f)^(1/2)/e^2/g-8/5*b*(-d*g+e*f)^2*n*
(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e^2/g
```

3.145.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.45

$$\int (f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2\,dx = \frac{2\left((f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2 - \frac{bn\left(900a\sqrt{e}(ef-dg)^2\sqrt{f+gx}-1800b\sqrt{e}(ef-dg)^2n\sqrt{f\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(2*((f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])^2 - (b*n*(900*a*Sqrt[e]*(e*
f - d*g)^2*Sqrt[f + g*x] - 1800*b*Sqrt[e]*(e*f - d*g)^2*n*Sqrt[f + g*x] +
1800*b*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
] - 200*b*(e*f - d*g)*n*(Sqrt[e]*Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) - 3
*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]) - 24*
b*n*(3*e^(5/2)*(f + g*x)^(5/2) + 5*(e*f - d*g)*(Sqrt[e]*Sqrt[f + g*x]*(4*e
*f - 3*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/
Sqrt[e*f - d*g]])) + 900*b*Sqrt[e]*(e*f - d*g)^2*Sqrt[f + g*x]*Log[c*(d +
e*x)^n] + 300*e^(3/2)*(e*f - d*g)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n
]) + 180*e^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]) + 450*(e*f - d
*g)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f
+ g*x]] - 450*(e*f - d*g)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f -
d*g] + Sqrt[e]*Sqrt[f + g*x]] - 225*b*(e*f - d*g)^(5/2)*n*(Log[Sqrt[e*f -
d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]
] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2,
 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 225*b*(e*f - d*g)^(
5/2)*n*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g]
+ Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f
 - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]
)))/(225*e^(5/2))))/(5*g)
```

3.145.3 Rubi [A] (verified)

Time = 4.35 (sec) , antiderivative size = 1007, normalized size of antiderivative = 1.71, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {2845, 2858, 2788, 2756, 60, 60, 60, 73, 221, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4ben\int\frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}dx}{5g}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{5g}$$

$\Big\downarrow$ 2788

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\left(\frac{g\int\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)\right)}{5g}$$

$\Big\downarrow$ 2756

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}{d+ex}d(d+ex)}{5g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c\right.}{d+ex}}{5g}$$

$\Big\downarrow$ 60

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}{d+ex}d(d+ex)+\frac{2}{5}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}\right)}{5g}\right)}{e}+\left(f-\frac{dg}{e}\right)\right.}{5g}$$

$\Big\downarrow$ 60

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\right)+\frac{2}{5}\left(\frac{g(d+ex)}{e}-\right.\right.}{5g}\right)}{e}}{5g}$$

$\downarrow$ 60

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)+\frac{2}{3}\right.\right.\right.}{5g}\right)}{e}}{5g}$$

$\downarrow$ 73

3.145. $\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{g(\cdots}\right.\right.}{5g}\right.}{e}\right.}{5\cdots}$$

$\downarrow$ 221

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$\frac{4bn\left(\left(f-\frac{dg}{e}\right)\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(f-\frac{dg}{e}\right)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{5g}$$

$\downarrow$ 2788

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\int\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)\right.\right.$$

$\downarrow$ 2756

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}{d+ex}d(d+ex)}{3g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex}{e}}}{d}\right.\right.$$

$\downarrow$ 60

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\right)}{3g}\right)}{e}\right)+\left(\right.\right.$$

$\downarrow$ 60

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+\right.\right.\right.}{3g}\right.}{e}\right.\right.$$

$\downarrow$ 73

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+\cdots\right.\right.}{3g}\cdots\right.}{e}\cdots\right.\right.$$

$\Big\downarrow$ 221

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}d(d+ex)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\Big\downarrow$ 2788

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)\right)+\frac{g\left(\frac{2e\left(\frac{g(d+ex}{e}\right.}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 2756

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{a+b\log}{(d+ex)\sqrt{f}}\right.\right.\right.$$

$\downarrow$ 60

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}}\right.}{g}\right.}{e}\right.\right.\right.$$

$\downarrow$ 73

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}\right.}{g}\right.}{e}\right.\right.\right.$$

$\downarrow$ 221

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$\downarrow$ 2790

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\cdots\right.\right.\right.}{5g}\right.}{e}\right.$$

$\downarrow$ 27

3.145. $\int(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+e\cdots}{e}}\cdots\right.\right.\right.}{5g}\cdots\right.}{e}\cdots\right.$$

$\downarrow$ 7267

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+e\cdots}{e}}\cdots\right.\right.\right.}{5g}\cdots\right.}{e}\cdots\right.$$

$\downarrow$ 2092

3.145. $\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} -$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\right.\right.\right.}{5g}\right.}{e}\right.$$

$\downarrow$ 6546

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} -$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\right.\right.\right.}{5g}\right.}{e}\right.$$

$\downarrow$ 6470

3.145. $\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\right.\right.\right.}{5g}\right)}{e}\right.$$

$\downarrow$ 2849

3.145. $\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\cdots\right.\right.\right.}{5g}\cdots\right.}{e}\cdots\right.}{\cdots}$$

$\downarrow$ 2752

3.145. $\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\right.\right.\right.}{5g}\right.}{e}\right.$$

input
```
Int[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

3.145. $\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

output
```
(2*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])^2)/(5*g) - (4*b*n*((g*((-2*b
*e*n*((2*(f - (d*g)/e + (g*(d + e*x))/e)^(5/2))/5 + (f - (d*g)/e)*((2*(f -
 (d*g)/e + (g*(d + e*x))/e)^(3/2))/3 + (f - (d*g)/e)*(2*Sqrt[f - (d*g)/e +
 (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*
g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g]))))/(5*g) + (2*
e*(f - (d*g)/e + (g*(d + e*x))/e)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(5*g))
)/e + (f - (d*g)/e)*((g*((-2*b*e*n*((2*(f - (d*g)/e + (g*(d + e*x))/e)^(3/
2))/3 + (f - (d*g)/e)*(2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*
(f - (d*g)/e)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e
*f - d*g]])/Sqrt[e*f - d*g])))/(3*g) + (2*e*(f - (d*g)/e + (g*(d + e*x))/e
)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(3*g)))/e + (f - (d*g)/e)*((g*((-2*b*e
*n*(2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTa
nh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*
f - d*g]))/g + (2*e*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]*(a + b*Log[c*(d +
e*x)^n]))/g))/e + (f - (d*g)/e)*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*
g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt
[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d +
e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sq
rt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sq
rt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e] + (Sqrt[e...
```

3.145.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

3.145. $\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n))^2 \, dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.145.4 Maple [F]

$$\int (gx+f)^{\frac{3}{2}} \left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{2}dx$$

input
```
int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.145.5 Fricas [F]

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int (gx+f)^{\frac{3}{2}}\left(b\log\left((ex+d)^n c\right)+a\right)^2\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral((b^2*g*x + b^2*f)*sqrt(g*x + f)*log((e*x + d)^n*c)^2 + 2*(a*b*g*x
 + a*b*f)*sqrt(g*x + f)*log((e*x + d)^n*c) + (a^2*g*x + a^2*f)*sqrt(g*x +
f), x)
```

3.145.6 Sympy [F]

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int \left(a+b\log\left(c(d+ex)^n\right)\right)^2(f+gx)^{\frac{3}{2}}\,dx$$

input
```
integrate((g*x+f)**(3/2)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2*(f + g*x)**(3/2), x)
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.145.8 Giac [F]

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int (gx+f)^{\frac{3}{2}}\left(b\log\left((ex+d)^nc\right)+a\right)^2\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a)^2, x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int (f+g\,x)^{3/2}\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2\,dx$$

input
```
int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^2, x)
```

3.146 $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$

3.146.1 Optimal result

Integrand size = 26, antiderivative size = 510

$$\begin{aligned}
&\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx \\
&= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg}+\frac{16b^2n^2(f+gx)^{3/2}}{27g}-\frac{64b^2(ef-dg)^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} \\
&-\frac{8b^2(ef-dg)^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g}-\frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} \\
&-\frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} \\
&+\frac{8b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} \\
&+\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} \\
&+\frac{16b^2(ef-dg)^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} \\
&+\frac{8b^2(ef-dg)^{3/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g}
\end{aligned}$$

output
```
16/27*b^2*n^2*(g*x+f)^(3/2)/g-64/9*b^2*(-d*g+e*f)^(3/2)*n^2*arctanh(e^(1/2
)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/g-8/3*b^2*(-d*g+e*f)^(3/2)*n^2*a
rctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(3/2)/g-8/9*b*n*(g*x+f)
^(3/2)*(a+b*ln(c*(e*x+d)^n))/g+8/3*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g
*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/e^(3/2)/g+2/3*(g*x+f)^
(3/2)*(a+b*ln(c*(e*x+d)^n))^2/g+16/3*b^2*(-d*g+e*f)^(3/2)*n^2*arctanh(e^(1
/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*
f)^(1/2)))/e^(3/2)/g+8/3*b^2*(-d*g+e*f)^(3/2)*n^2*polylog(2,1-2/(1-e^(1/2)
*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(3/2)/g+64/9*b^2*(-d*g+e*f)*n^2*(g*x+f
)^(1/2)/e/g-8/3*b*(-d*g+e*f)*n*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e/g
```

3.146.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.33

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{2\left((f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2 - \frac{bn\left(36a\sqrt{e}(ef-dg)\sqrt{f+gx}-72b\sqrt{e}(ef-dg)n\sqrt{f+gx}-8b\sqrt{e}n\sqrt{f+gx}(4ef-3dg+egx)+\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(2*((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2 - (b*n*(36*a*Sqrt[e]*(e*f
 - d*g)*Sqrt[f + g*x] - 72*b*Sqrt[e]*(e*f - d*g)*n*Sqrt[f + g*x] - 8*b*Sqr
t[e]*n*Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) + 96*b*(e*f - d*g)^(3/2)*n*Ar
cTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 36*b*Sqrt[e]*(e*f - d*g)*
Sqrt[f + g*x]*Log[c*(d + e*x)^n] + 12*e^(3/2)*(f + g*x)^(3/2)*(a + b*Log[c
*(d + e*x)^n]) + 18*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[
e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 18*(e*f - d*g)^(3/2)*(a + b*Log[c*(d
 + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 9*b*(e*f - d*g)
^(3/2)*n*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g]
 + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*
f - d*g])]) - 9*b*(e*f - d*g)^(3/2)*n*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f
 + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[
e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) - 18*b*(e*f - d*g)^(3/2)*n*PolyLog[
2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])] + 18*b*(e*f - d*g)^(
3/2)*n*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]))/(9*e^
(3/2))))/(3*g)
```

3.146.3 Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.50, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {2845, 2858, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g}-\frac{4ben\int\frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}dx}{3g}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g}-\frac{4bn\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{3g}$$

$$\Big\downarrow \text{2788}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g}-\frac{4bn\left(\frac{g\int\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)\right)}{3g}$$

$$\Big\downarrow \text{2756}$$

$$\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}{d+ex}d(d+ex)}{3g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+$$

$$3g$$

$$\Big\downarrow \text{60}$$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\right)}{3g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \cdots\right)}{3g}$$

$$\Big\downarrow \; 60$$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)+\frac{2}{3}\left(\frac{g(d+ex)}{e} \cdots\right)\right)}{3g}\right)}{e}\cdots\right)}{3g}$$

$$\Big\downarrow \; 73$$

3.146. $\int \sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-\frac{4bn\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}\cdots}\cdots\right)\cdots\right)}{3g}\cdots\right)}{e}\cdots\right)}{3g}$$

$$\Big\downarrow \; 221$$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-\frac{4bn\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(2\sqrt{\cdots}\cdots\right)\cdots\right)}{\cdots}\right)}{\cdots}\right)}{3g}$$

$$\Big\downarrow \; 2788$$

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$\frac{4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\int\frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)\right)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a}{3g}\right.}{}\right.}{3g}$$

$\downarrow$ 2756

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}\right.\right.$$

$\downarrow$ 60

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}\right)+\left(f\right.\right.$$

$\downarrow$ 73

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{2ben\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\right.}{g}\right.}{e}\right.\right.$$

$\downarrow$ 221

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}\cdots}\right.}{\cdots}\right.}{e}\right.\right.$$

$\downarrow$ 2790

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(-bn\int-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a\cdots}{\sqrt{ef-dg}}\right.\right.\right.$$

$\downarrow$ 27

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{2b\sqrt{e}n\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d}{\sqrt{ef-dg}}\right.\right.\right.$$

$\downarrow$ 7267

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\ldots}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 2092

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.\right.\right.$$

↓ 6546

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\ldots\right.\right.\right.$$

↓ 6470

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\left(f-\frac{dg}{e}\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\cdots}}{\cdots}\right)}{\sqrt{ef-dg}}\right.\right.}{3g}\right.}{e}\right.$$

$\downarrow$ 2849

3.146. $\int \sqrt{f+gx}(a+b\log\left(c(d+ex)^n\right))^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{2ben\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\left(f-\frac{dg}{e}\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-}}{\sqrt{}}\right.}{\sqrt{ef-dg}}\right.\right.}{3g}\right.}{e}\right.$$

$\downarrow$ 2752

3.146. $\int\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

$$\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+e}{e}}}{\sqrt{ef}}}\right.}{\sqrt{e}}}{\sqrt{e}\sqrt{}}\right.}{\sqrt{ef-dg}}\right.\right.\right.$$

input
```
Int[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(2*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2)/(3*g) - (4*b*n*((g*((-2*b
*e*n*((2*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2))/3 + (f - (d*g)/e)*(2*Sqrt[
f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(Sqrt[e]
*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])))
/(3*g) + (2*e*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2)*(a + b*Log[c*(d + e*x)
^n]))/(3*g)))/e + (f - (d*g)/e)*((g*((-2*b*e*n*(2*Sqrt[f - (d*g)/e + (g*(d
 + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e +
 (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g]))/g + (2*e*Sqrt[f - (
d*g)/e + (g*(d + e*x))/e]*(a + b*Log[c*(d + e*x)^n]))/g))/e + (f - (d*g)/e
)*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[
e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(
ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(
2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))
/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))
/e])/Sqrt[e*f - d*g])])/Sqrt[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (
Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]
))/(Sqrt[e]*Sqrt[e*f - d*g])))/Sqrt[e*f - d*g]))))/(3*g)
```

3.146. $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.146. $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

3.146. $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.146.4 Maple [F]

$$\int \sqrt{gx+f}\,(a+b\ln(c(ex+d)^n))^2dx$$

input
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.146.5 Fricas [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx = \int \sqrt{gx+f}(b\log((ex+d)^nc)+a)^2\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log(
(e*x + d)^n*c) + sqrt(g*x + f)*a^2, x)
```

3.146.6 Sympy [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx = \int (a+b\log(c(d+ex)^n))^2\sqrt{f+gx}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2*sqrt(f + g*x), x)
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.146.8 Giac [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx = \int \sqrt{gx+f}(b\log((ex+d)^n c)+a)^2\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a)^2, x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx = \int \sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2\,dx$$

input
```
int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^2, x)
```

3.147 $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

3.147.1 Optimal result

Integrand size = 26, antiderivative size = 418

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{16b^2\sqrt{ef-dg}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{8b^2\sqrt{ef-dg}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} + \frac{16b^2\sqrt{ef-dg}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g} + \frac{8b^2\sqrt{ef-dg}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g}$$

output
```
-16*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(-d*g+e*f)^(1/
2)/g/e^(1/2)-8*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*(
-d*g+e*f)^(1/2)/g/e^(1/2)+8*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(
1/2))*(a+b*ln(c*(e*x+d)^n))*(-d*g+e*f)^(1/2)/g/e^(1/2)+16*b^2*n^2*arctanh(
e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*
g+e*f)^(1/2)))*(-d*g+e*f)^(1/2)/g/e^(1/2)+8*b^2*n^2*polylog(2,1-2/(1-e^(1/
2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*(-d*g+e*f)^(1/2)/g/e^(1/2)+16*b^2*n^2*
(g*x+f)^(1/2)/g-8*b*n*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/g+2*(a+b*ln(c*(e
*x+d)^n))^2*(g*x+f)^(1/2)/g
```

3.147.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \frac{2\left(\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2 - \frac{bn\left(4a\sqrt{e}\sqrt{f+gx}-8b\sqrt{e}n\sqrt{f+gx}+8b\sqrt{ef-dg}n\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4b\sqrt{e}\sqrt{f+gx}\log(c\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/Sqrt[f + g*x],x]
```

output
```
(2*(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2 - (b*n*(4*a*Sqrt[e]*Sqrt[f
+ g*x] - 8*b*Sqrt[e]*n*Sqrt[f + g*x] + 8*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt
[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 4*b*Sqrt[e]*Sqrt[f + g*x]*Log[c*(d +
 e*x)^n] + 2*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g
] - Sqrt[e]*Sqrt[f + g*x]] - 2*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n])*
Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - b*Sqrt[e*f - d*g]*n*(Log[Sq
rt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt
[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*P
olyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + b*Sqrt[e*f
 - d*g]*n*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*
g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[
e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])
/2])))/Sqrt[e]))/g
```

3.147.3 Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.32, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {2845, 2858, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-\frac{4ben\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}dx}{g}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-\frac{4bn\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \text{2788}$$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-\frac{4bn\left(\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)\right)}{g}$$

$$\Big\downarrow \text{2756}$$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-\frac{4bn\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)\right)}{g}$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-$$

$$\frac{4bn\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\ }{(d+}\right.}{g}$$

$\downarrow$ 73

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-$$

$$\frac{4bn\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}+\left(\right.\right.}{g}$$

$\downarrow$ 221

3.147. $\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{4bn\left(\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex) + \frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g} - \frac{2ben\left(2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}-\frac{2\sqrt{e}\left(f-\frac{dg}{e}\right)}{}\right.}{g}\right.}{e}\right.}{g}$$

$\downarrow$ 2790

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{4bn\left(\left(f-\frac{dg}{e}\right)\left(-bn\int-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex) - \frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)}{\sqrt{ef-dg}}\right.\right.}{g}$$

$\downarrow$ 27

3.147. $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-\frac{4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{2b\sqrt{e}n\int\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)+\right.}{g}$$

$\downarrow$ 7267

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-\frac{4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.\right.}{}$$

$\downarrow$ 2092

3.147. $\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.\right.$$

$\downarrow$ 6546

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}}{}\right.\right.$$

$\downarrow$ 6470

3.147. $\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}-$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int}{\sqrt{e}\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.\right.$$

$\downarrow$ 2849

3.147. $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{4bn\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e} - \frac{\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}} + \frac{\sqrt{ef-dg}\ldots}{\ldots}}{\sqrt{e}\sqrt{ef-dg}\ldots}\right.}{\sqrt{ef-dg}\ldots}\right.\right.}{\ldots}$$

$\downarrow$ 2752

3.147. $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g} -$$

$$4bn\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\cdots}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}\cdots$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2)/g - (4*b*n*((g*((-2*b*e*n*(
2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(
Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f -
d*g]))/g + (2*e*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]*(a + b*Log[c*(d + e*x)
^n]))/g))/e + (f - (d*g)/e)*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e
 + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt[e*f
 - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x)
)/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f
 - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f
 - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e] + (Sqrt[e*f - d*
g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqr
t[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f - d*g])))/Sqrt[e*f - d*g]))
)/g
```

3.147. $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.147. $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

3.147. $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.147.4 Maple [F]

$$\int \frac{(a + b\ln(c(ex + d)^n))^2}{\sqrt{gx + f}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x)
```

3.147.5 Fricas [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{\sqrt{f + gx}}\,dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{\sqrt{gx + f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log
((e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g*x + f), x)
```

3.147.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/sqrt(f + g*x), x)
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.147.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/sqrt(g*x + f), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{\sqrt{f+g\,x}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(1/2), x)
```

3.148 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$

3.148.1 Optimal result

Integrand size = 26, antiderivative size = 312

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \frac{8b^2\sqrt{e}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{8b\sqrt{e}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{16b^2\sqrt{e}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}} - \frac{8b^2\sqrt{e}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}}$$

output
```
8*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*e^(1/2)/g/(-d*
g+e*f)^(1/2)-8*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln
(c*(e*x+d)^n))*e^(1/2)/g/(-d*g+e*f)^(1/2)-16*b^2*n^2*arctanh(e^(1/2)*(g*x+
f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))
)*e^(1/2)/g/(-d*g+e*f)^(1/2)-8*b^2*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1
/2)/(-d*g+e*f)^(1/2)))*e^(1/2)/g/(-d*g+e*f)^(1/2)-2*(a+b*ln(c*(e*x+d)^n))^
2/g/(g*x+f)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\log (c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(-\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}+\frac{b\sqrt{e}n\left(2(a+b\log(c(d+ex)^n))\log\left(\sqrt{ef-dg}-\sqrt{e}\sqrt{f+gx}\right)-2(a+b\log(c\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(3/2),x]
```

output
```
(2*(-((a + b*Log[c*(d + e*x)^n])^2/Sqrt[f + g*x]) + (b*Sqrt[e]*n*(2*(a + b
*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 2*(a +
 b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - b*n*
(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[
e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]
) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + b*n
*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt
[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g
])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/Sq
rt[e*f - d*g]))/g
```

3.148.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2845, 2858, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log (c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{4ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}dx}{g}-\frac{2(a+b\log (c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

$$\Big\downarrow \text{2858}$$

$$\frac{4bn\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{g}-\frac{2(a+b\log (c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

$$\Big\downarrow \text{2790}$$

$$\frac{4bn\left(-bn\int -\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{g}-\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{g\sqrt{f+gx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4bn\left(\frac{2b\sqrt{e}n\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{g}-\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{g\sqrt{f+gx}}$$

$$\Big\downarrow \text{7267}$$

$$\frac{4bn\left(\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ ...}{\sqrt{ef-dg}}\right)}{g}$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{g\sqrt{f+gx}}$$

$$\Big\downarrow \text{2092}$$

3.148. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$

$$\frac{4bn\left(\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+}{\sqrt{ef-dg}}\right.}{g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

$\downarrow$ 6546

$$\frac{4bn\left(\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}}}{\sqrt{ef-d}}\right.}{\sqrt{ef}}\right.}{g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

$\downarrow$ 6470

3.148. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$

$$\frac{4bn\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\frac{\cdots}{1-\frac{\sqrt{e}\sqrt{f-\cdots}}{\sqrt{\cdots}}}\right)}{1-\frac{e\left(f-\frac{dg}{e}\cdots\right)}{ef\cdots}}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

$\big\downarrow$ 2849

3.148. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$

$$4bn\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\cdots}}{\cdots}\right.\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right.}{g}\cdots$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

$\downarrow$ 2752

3.148. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$

$$\frac{4bn\left(\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\,\text{PolyLog}\left(\cdots\right.}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\cdots\right.}{\sqrt{ef-dg}}\cdots\right.}{g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(3/2),x]
```

output
```
(-2*(a + b*Log[c*(d + e*x)^n])^2)/(g*Sqrt[f + g*x]) + (4*b*n*((-2*Sqrt[e]*
ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a
+ b*Log[c*(d + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e
]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e
*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f -
 d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f -
 d*g])])/Sqrt[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f
- (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqr
t[e*f - d*g])))/Sqrt[e*f - d*g]))/g
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.148. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.148.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{(gx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x)
```

3.148.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log
((e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.148.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(3/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**(3/2), x)
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.148.8 Giac [F]

$$\int \frac{(a + b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \int \frac{(b\log((ex+d)^n c) + a)^2}{(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(3/2), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \int \frac{(a + b\,\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(3/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(3/2), x)
```

3.149 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

3.149.1 Optimal result

Integrand size = 26, antiderivative size = 423

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx &= \frac{16b^2e^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} \\ &+ \frac{8b^2e^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} \\ &- \frac{8be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} \\ &- \frac{16b^2e^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}} \\ &- \frac{8b^2e^{3/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}}\end{aligned}$$

output
```
16/3*b^2*e^(3/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d
*g+e*f)^(3/2)+8/3*b^2*e^(3/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)
^(1/2))^2/g/(-d*g+e*f)^(3/2)-8/3*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^(3/2)-2/3*(a+b*ln(c*
(e*x+d)^n))^2/g/(g*x+f)^(3/2)-16/3*b^2*e^(3/2)*n^2*arctanh(e^(1/2)*(g*x+f)
^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/
g/(-d*g+e*f)^(3/2)-8/3*b^2*e^(3/2)*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1
/2)/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^(3/2)+8/3*b*e*n*(a+b*ln(c*(e*x+d)^n))/
g/(-d*g+e*f)/(g*x+f)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \frac{2\left(-(a+b\log(c(d+ex)^n))^2+\frac{ben(f+gx)\left(8b\sqrt{e}n\sqrt{f+gx}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4\sqrt{ef}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(5/2),x]
```

output
```
(2*(-(a + b*Log[c*(d + e*x)^n])^2 + (b*e*n*(f + g*x)*(8*b*Sqrt[e]*n*Sqrt[f
 + g*x]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 4*Sqrt[e*f - d*
g]*(a + b*Log[c*(d + e*x)^n]) + 2*Sqrt[e]*Sqrt[f + g*x]*(a + b*Log[c*(d +
e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 2*Sqrt[e]*Sqrt[f +
 g*x]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*
x]] - b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*
x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sq
rt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g
*x])/(2*Sqrt[e*f - d*g])]) + b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[Sqrt[e*f - d*g
] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] +
 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2,
(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/(e*f - d*g)^(3/2)))/(3
*g*(f + g*x)^(3/2))
```

3.149.3 Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2845, 2858, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{4ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}dx}{3g}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{2858}$$

$$\frac{4bn\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{3g}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{2789}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}\right)}{3g}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{2756}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{g}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{3g}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{4be^2n\int \frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g^2}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{3g}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}\right)}{ef-dg}\right)}{3g}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow 2790$$

$$\frac{4bn\left(\frac{e\left(-bn\int -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log}{g\sqrt{\frac{g(d+e}{e}}}\right.}{}\right.}{3g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow 27$$

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{2b\sqrt{e}n\int\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+\ldots}{g\sqrt{\frac{g(d+ex)}{e}-\ldots}}\ldots\right.}{\ldots}\right)$$

$$\overline{\qquad 3g \qquad}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$\downarrow$ 7267

$$4bn\left(\frac{e\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)\ldots}{\sqrt{ef-dg}}\ldots\right.}{ef-dg}\ldots\right.$$

$$\overline{\qquad 3g \qquad}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$\downarrow$ 2092

3.149. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)}{\sqrt{ef-dg}}\right.}{ef-dg}\right.$$

$$\overline{\qquad\qquad 3g \qquad\qquad}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{6546}$$

3.149. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

$$\frac{4bn\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\cdots}}{\sqrt{ef-dg}}\right.}{\sqrt{ef-\cdots}}\right.}{ef-dg}\right.}{3g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \text{ 6470}$$

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\frac{\cdots}{1-\frac{\sqrt{e}\sqrt{\cdots}}{\cdots}}\right)}{1-\frac{e\left(f-\cdots\right)}{\cdots}}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right)\cdots$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$$

$$\Big\downarrow \; 2849$$

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{\sqrt{e}\sqrt{ef-dg}}}{\sqrt{ef-dg}}\right.}{ef-dg}\right.$$

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$ $\quad \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}$

$$\Big\downarrow \; 2752$$

$$4bn\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{PolyLog}\ldots}{\ldots}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}\ldots\right)}{ef-dg}\ldots\right)$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{3g(f+gx)^{3/2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(5/2),x]
```

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

output
```
(-2*(a + b*Log[c*(d + e*x)^n])^2)/(3*g*(f + g*x)^(3/2)) + (4*b*n*(-((g*((-
4*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e
*f - d*g]])/(g*Sqrt[e*f - d*g]) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt
[f - (d*g)/e + (g*(d + e*x))/e])))/(e*f - d*g)) + (e*((-2*Sqrt[e]*ArcTanh[
(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[
c*(d + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f
 - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g
]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*L
og[2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])
/Sqrt[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/
e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f -
d*g])))/Sqrt[e*f - d*g]))/(e*f - d*g)))/(3*g)
```

3.149.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.149. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.149.4 Maple [F]

$$\int \frac{(a+b\ln (c(ex+d)^n))^2}{(gx+f)^{\frac{5}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x)
```

3.149.5 Fricas [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \int \frac{(b\log ((ex+d)^n c)+a)^2}{(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log
((e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x +
f^3), x)
```

3.149.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(5/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.149.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.149.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(5/2), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^{5/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(5/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(5/2), x)
```

3.150 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

3.150.1 Optimal result

Integrand size = 26, antiderivative size = 503

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = &-\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} \\ &+\frac{64b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}}+\frac{8b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}} \\ &+\frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}}+\frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}} \\ &-\frac{8be^{5/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} \\ &-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}-\frac{16b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} \\ &-\frac{8b^2e^{5/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}}\end{aligned}$$

output
```
64/15*b^2*e^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-
d*g+e*f)^(5/2)+8/5*b^2*e^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f
)^(1/2))^2/g/(-d*g+e*f)^(5/2)+8/15*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f
)/(g*x+f)^(3/2)-8/5*b*e^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(
1/2))*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^(5/2)-2/5*(a+b*ln(c*(e*x+d)^n))^2
/g/(g*x+f)^(5/2)-16/5*b^2*e^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+
e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^
(5/2)-8/5*b^2*e^(5/2)*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f
)^(1/2)))/g/(-d*g+e*f)^(5/2)-16/15*b^2*e^2*n^2/g/(-d*g+e*f)^2/(g*x+f)^(1/2
)+8/5*b*e^2*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^2/(g*x+f)^(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.22 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \frac{2\left(-3(a+b\log(c(d+ex)^n))^2 + \frac{ben(f+gx)\left(24be^{3/2}n(f+gx)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\right.}{\quad}\right.}{\quad}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(7/2),x]
```

output
```
(2*(-3*(a + b*Log[c*(d + e*x)^n])^2 + (b*e*n*(f + g*x)*(24*b*e^(3/2)*n*(f
+ g*x)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] - 8*b*e*Sqrt
[e*f - d*g]*n*(f + g*x)*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f
 - d*g)] + 4*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^n]) + 12*e*Sqrt[e*f
- d*g]*(f + g*x)*(a + b*Log[c*(d + e*x)^n]) + 6*e^(3/2)*(f + g*x)^(3/2)*(a
 + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 6*
e^(3/2)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + S
qrt[e]*Sqrt[f + g*x]] - 3*b*e^(3/2)*n*(f + g*x)^(3/2)*(Log[Sqrt[e*f - d*g]
 - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] +
2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2
 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 3*b*e^(3/2)*n*(f + g*x)
^(3/2)*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g]
+ Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f
 - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]
)))/(e*f - d*g)^(5/2)))/(15*g*(f + g*x)^(5/2))
```

3.150.3 Rubi [A] (verified)

Time = 3.14 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.41, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.731, Rules used = {2845, 2858, 2789, 2756, 61, 73, 221, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{4ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}dx}{5g}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{4bn\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{5g}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 2789}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}\right)}{5g}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 2756}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{3g}-\frac{2e(a+b\log(c(d+ex)^n))}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}}\right)}{ef-dg}\right)}{5g}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{3g}-\frac{2e(a+b\log(c(d+ex}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right.}\right.}{ef-dg}\right.}{5g}$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 73

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e^2\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g(ef-dg)}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{3g}-\frac{2}{3g}\right.}{ef-dg}\right.}{5g}$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 221

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{3g}-\frac{2e(a+b\log(c(\ldots}{3g\left(\frac{g(d+ex)}{e}-\ldots\right.}\right.}{ef-dg}\right.}{5g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 2789

$$\frac{4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\ldots}}{\ldots}\right)}{(ef-dg)^{3\ldots}}\right.}{3g}\right.}{ef-dg}\right.}{5g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 2756

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$\frac{4bn\left(\frac{e\left(\frac{e\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{g}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}\cdots}}\right.\right.}{}\right.}{5g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 73

$$\frac{4bn\left(\frac{e\left(\frac{e\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{4be^2n\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g^2}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(2ben\left(\cdots\right.\right.}{}\right.}{5g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 221

3.150. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$\frac{4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{4be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\cdots\right.}{(ef-dg)\sqrt{g\cdots}}\cdots\right.}{\cdots}\cdots\right)}{5g}\cdots$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 2790

$$\frac{4bn\left(\frac{e\left(\frac{e\left(-bn\int -\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(\cdots}{g\sqrt{\frac{g(d+ex)}{e}\cdots}}\cdots\right.}{\cdots}\right)}{ef-dg}\cdots\right)}{\cdots}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 27

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{2b\sqrt{e}n\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+\cdots}{g\sqrt{\frac{g(d+ex)}{e}-\cdots}}\cdots\right.}{\cdots}}{ef-dg}\cdots\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\Big\downarrow$ 7267

3.150. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+}{\sqrt{ef-dg}}\right.}{ef-dg}\right.}{ef-dg}\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 2092}$$

3.150. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+\cdots}{\sqrt{ef-dg}}\right.}{ef-dg}\right.}{ef-dg}\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$

$\downarrow$ 6546

3.150. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex}{e}}}{\sqrt{ef-}}\right.}{\sqrt{e}}\right.}{ef-dg}\right.}{ef-dg}\right.$$

3.150 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$\downarrow 6470$$

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\frac{\cdots}{1-\frac{\sqrt{e}\cdots}{\cdots}}\right)}{1-\frac{e\left(f\cdots\right)}{\cdots}}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right)}{\cdots}\cdots\right)$$

3.150 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$\downarrow \text{2849}$$

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\int\cdots}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right)\cdots\right.$$

3.150 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$\downarrow 2752$$

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{Poly}\cdots}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right)}{\cdots}\cdots\right.$$

3.150 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(7/2),x]
```

output
```
(-2*(a + b*Log[c*(d + e*x)^n])^2)/(5*g*(f + g*x)^(5/2)) + (4*b*n*(-((g*((2
*b*e*n*((2*e)/((e*f - d*g)*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]) - (2*e^(3/
2)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])
/(e*f - d*g)^(3/2)))/(3*g) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(3*g*(f - (d
*g)/e + (g*(d + e*x))/e)^(3/2))))/(e*f - d*g)) + (e*(-((g*((-4*b*e^(3/2)*n
*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/(
g*Sqrt[e*f - d*g]) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt[f - (d*g)/e
+ (g*(d + e*x))/e])))/(e*f - d*g)) + (e*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt
[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n
]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e +
(g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sq
rt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sq
rt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e] + (S
qrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e
*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f - d*g])))/Sqrt[
e*f - d*g]))/(e*f - d*g)))/(e*f - d*g)))/(5*g)
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

3.150. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.150.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{(gx+f)^{\frac{7}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x)
```

3.150.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log
((e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^
2 + 4*f^3*g*x + f^4), x)
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(7/2),x)
```

output
```
Timed out
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.150.8 Giac [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)^{7/2}}\,dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{(gx + f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(7/2), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)^{7/2}}\,dx = \int \frac{(a + b\,\ln(c\,(d + e\,x)^n))^2}{(f + g\,x)^{7/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(7/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(7/2), x)
```

3.151 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

3.151.1 Optimal result

Integrand size = 26, antiderivative size = 583

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx &= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} \\ &- \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} \\ &+ \frac{8b^2e^{7/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} \\ &+ \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} \\ &- \frac{8be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} \\ &- \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{16b^2e^{7/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} \\ &- \frac{8b^2e^{7/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}}\end{aligned}$$

output
```
-16/105*b^2*e^2*n^2/g/(-d*g+e*f)^2/(g*x+f)^(3/2)+368/105*b^2*e^(7/2)*n^2*a
rctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*g+e*f)^(7/2)+8/7*b^2*
e^(7/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/g/(-d*g+e*f)
^(7/2)+8/35*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)/(g*x+f)^(5/2)+8/21*b*
e^2*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^2/(g*x+f)^(3/2)-8/7*b*e^(7/2)*n*a
rctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/g/(-d
*g+e*f)^(7/2)-2/7*(a+b*ln(c*(e*x+d)^n))^2/g/(g*x+f)^(7/2)-16/7*b^2*e^(7/2)
*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+
f)^(1/2)/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^(7/2)-8/7*b^2*e^(7/2)*n^2*polylog
(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^(7/2)-128/
105*b^2*e^3*n^2/g/(-d*g+e*f)^3/(g*x+f)^(1/2)+8/7*b*e^3*n*(a+b*ln(c*(e*x+d)
^n))/g/(-d*g+e*f)^3/(g*x+f)^(1/2)
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.21 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \frac{2\left(-15(a+b\log(c(d+ex)^n))^2 + \frac{ben(f+gx)\left(120be^{5/2}n(f+gx)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+g}}{\sqrt{ef-dg}}\right.\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(9/2),x]
```

output
```
(2*(-15*(a + b*Log[c*(d + e*x)^n])^2 + (b*e*n*(f + g*x)*(120*b*e^(5/2)*n*(
f + g*x)^(5/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] - 8*b*e*(e
*f - d*g)^(3/2)*n*(f + g*x)*Hypergeometric2F1[-3/2, 1, -1/2, (e*(f + g*x))
/(e*f - d*g)] - 40*b*e^2*Sqrt[e*f - d*g]*n*(f + g*x)^2*Hypergeometric2F1[-
1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*g)] + 12*(e*f - d*g)^(5/2)*(a + b*Log[
c*(d + e*x)^n]) + 20*e*(e*f - d*g)^(3/2)*(f + g*x)*(a + b*Log[c*(d + e*x)^
n]) + 60*e^2*Sqrt[e*f - d*g]*(f + g*x)^2*(a + b*Log[c*(d + e*x)^n]) + 30*e
^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sq
rt[e]*Sqrt[f + g*x]] - 30*e^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n
])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - 15*b*e^(5/2)*n*(f + g*x)
^(5/2)*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g]
- Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d
*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]
) + 15*b*e^(5/2)*n*(f + g*x)^(5/2)*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f +
 g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e
]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f
 + g*x])/Sqrt[e*f - d*g])/2])))/(e*f - d*g)^(7/2)))/(105*g*(f + g*x)^(7/2)
)
```

3.151.3 Rubi [A] (verified)

Time = 4.25 (sec) , antiderivative size = 940, normalized size of antiderivative = 1.61, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.962$, Rules used = {2845, 2858, 2789, 2756, 61, 61, 73, 221, 2789, 2756, 61, 73, 221, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{4ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{7/2}}dx}{7g}-\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \text{2858}$$

$$\frac{4bn\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{7/2}}d(d+ex)}{7g}-\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \text{2789}$$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{7/2}}d(d+ex)}{ef-dg}\right)}{7g}-\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 2756

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{5g}-\frac{2e(a+b\log(c(d+ex)^n))}{5g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}}\right)}{ef-dg}\right)}{7g}-\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 61

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\int \frac{1}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}+\frac{2e}{3(ef-dg)\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}}\right)}{5g}-\frac{2e(a+b\,\mathrm{l}\cdots}{5g\left(\frac{g(d+e\cdots}{e}\right.}}{ef-dg}}{7g}-\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 61

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\left(\frac{e\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}+\frac{2e\cdots}{3(ef-dg)\left(\frac{g(d+e\cdots}{e}\cdots\right.}\cdots\right)}{5g}\cdots\right)}{ef-dg}\cdots\right)}{7g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow \text{73}$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\left(\frac{2e^2\int \frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g(ef-dg)}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}+\cdots\right)}{5g}}{ef-dg}\right)}{7g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 221

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\frac{4bn\left(\frac{e\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{ef-dg}+\frac{\cdots}{3(ef-dg)\left(\frac{g\cdots}{\cdots}\right)\cdots}\right)}{5g}\cdots\right)}{ef-dg}\cdots\right)}{7g}\cdots$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{7g(f+gx)^{7/2}}$$

$\Big\downarrow$ 2789

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\frac{4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}}{(}\right)}{ef-dg}\right)}{}\right)}{}\right)}{7g}$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 2756

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\frac{4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{3g}-\frac{2e(a+b\log(c(d+ex)^n))}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(e\left(\frac{\cdots}{(e\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{7g\cdots}$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 61

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{3g}-\frac{2e(a+b\log(c(d+ex)}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right.}\right.}{ef-dg}\right)}{ef-dg}\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 73

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e^2\int \frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g(ef-dg)}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{3g}-\frac{2e}{3g(\cdots}\right)}{ef-dg}\right)}{ef-dg}\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 221

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{3g}-\frac{2e(a+b\log(c(d}{3g\left(\frac{g(d+ex)}{e}-\frac{d}{e}\right.}\right.}{ef-dg}\right)}{ef-dg}\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 2789

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\int\frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\cdots}}{\cdots}\right)}{(ef-dg)^{3\cdots}}\cdots\right)}{3g}\cdots\right)}{ef-dg}\cdots\right)}{ef-dg}\cdots\right)$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 2756

3.151. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{g}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+e\cdots}{e}}}\cdots\right.\right.}{\cdots}}{ef-dg}\cdots\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$$\downarrow 73$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\int\frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{4be^2n\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g^2}-\frac{2e\left(a+b\log\left(c(d+ex)^n\right)\right)}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(2ben\left(\cdots\right.\right.}{\cdots}}{ef-dg}\cdots\right.$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 221

3.151. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{4be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben}{(ef-dg)\sqrt{g\cdots}}\cdots\right.}{\cdots}\right)}{ef-dg}\cdots\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$\downarrow$ 2790

3.151. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\left(-bn\int -\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log}{g\sqrt{\frac{g(d+}{e}}}\right.}{}}{ef-dg}}{}\right.$$

$$\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}$$

$$\Big\downarrow 27$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\left(\frac{2b\sqrt{e}n\int\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(\cdots}{g\sqrt{\frac{g(d+ex)}{e}\cdots}}\cdots\right.}{\cdots}\cdots\right)}{ef-dg}\cdots\right)}{\cdots}\cdots\right.$$

$$\frac{2(a+b\log\left(c(d+ex)^n\right))^2}{7g(f+gx)^{7/2}}$$

$$\downarrow 7267$$

3.151. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(\frac{e\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)(a+b\log(c(}{\sqrt{ef-dg}}\right.}{ef-dg}\right.}{ef-dg}\right.}{}\right.$$

3.151. $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\downarrow 2092$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$4bn\left(e\left(e\left(e\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)(a+b\log(c(}{\sqrt{ef-dg}}\right.\right.\right.\right.$$

$$\frac{\quad}{ef-dg}\qquad\frac{\quad}{ef-dg}$$

3.151 $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\Big\downarrow \text{6546}$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\Bigg(e \Bigg(e \Bigg(e \Bigg(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}} - \frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\cdots}}{\sqrt{\cdots}}\right.\cdots}{\cdots} \cdots \Bigg) \Big/ (ef-dg) \cdots \Bigg) \Big/ (ef-dg) \cdots$$

3.151 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

3.150. $\downarrow$ 6470

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$e\left(\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\cdots\right)}{1-\cdots}}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right)}{\cdots}\cdots\right)$$

3.151 $\int\frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\Big\downarrow \text{2849}$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\left(\left(\frac{\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\ \cdots}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\cdots}{e\sqrt{ef-dg}}\cdots\right)}{e(ef-dg)}\cdots}{e}\cdots\right.\right.$$

3.151 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$\downarrow 2752$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

$$e\left(e\left(e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\ldots}{\ldots}}{\sqrt{e}\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.}{ef-dg}\right.}{}\right.$$

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(9/2),x]
```

output
```
(-2*(a + b*Log[c*(d + e*x)^n])^2)/(7*g*(f + g*x)^(7/2)) + (4*b*n*(-((g*((2
*b*e*n*((2*e)/(3*(e*f - d*g)*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2)) + (e*(
(2*e)/((e*f - d*g)*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]) - (2*e^(3/2)*ArcTa
nh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/(e*f -
d*g)^(3/2)))/(e*f - d*g)))/(5*g) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(5*g*(
f - (d*g)/e + (g*(d + e*x))/e)^(5/2))))/(e*f - d*g)) + (e*(-((g*((2*b*e*n*
((2*e)/((e*f - d*g)*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]) - (2*e^(3/2)*ArcT
anh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/(e*f -
 d*g)^(3/2)))/(3*g) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(3*g*(f - (d*g)/e +
 (g*(d + e*x))/e)^(3/2))))/(e*f - d*g)) + (e*(-((g*((-4*b*e^(3/2)*n*ArcTan
h[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/(g*Sqrt[
e*f - d*g]) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt[f - (d*g)/e + (g*(d
 + e*x))/e])))/(e*f - d*g)) + (e*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f - (d
*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/Sqr
t[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d +
 e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*S
qrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*S
qrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e] + (Sqrt[e*f
 - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]
)/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f - d*g])))/Sqrt[e*f ...
```

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.151. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$

3.151.4 Maple [F]

$$\int \frac{\left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{2}}{\left(gx+f\right)^{\frac{9}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x)
```

3.151.5 Fricas [F]

$$\int \frac{\left(a+b\log \left(c(d+ex)^{n}\right)\right)^{2}}{(f+gx)^{9/2}}\,dx=\int \frac{\left(b\log \left((ex+d)^{n}c\right)+a\right)^{2}}{\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log
((e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x
^3 + 10*f^3*g^2*x^2 + 5*f^4*g*x + f^5), x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log \left(c(d+ex)^{n}\right)\right)^{2}}{(f+gx)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(9/2),x)
```

output
```
Timed out
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.151.8 Giac [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)^{9/2}}\,dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{(gx + f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(9/2), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)^{9/2}}\,dx = \int \frac{(a + b\,\ln(c\,(d + e\,x)^n))^2}{(f + g\,x)^{9/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(9/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(9/2), x)
```

3.152 $\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx$

3.152.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Int}\left(\frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)},x\right)$$

output
```
Unintegrable((g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.152.2 Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

input
```
Integrate[(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
Integrate[(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]), x]
```

3.152.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx$$
$$\Big\downarrow \; 2867$$
$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}dx$$

input
```
Int[(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
$Aborted
```

3.152.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.152.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(gx+f)^{\frac{3}{2}}}{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

input
```
int((g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.152.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral((g*x + f)^(3/2)/(b*log((e*x + d)^n*c) + a), x)
```

3.152.6 Sympy [N/A]

Not integrable

Time = 21.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(f+gx)^{\frac{3}{2}}}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)**(3/2)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((f + g*x)**(3/2)/(a + b*log(c*(d + e*x)**n)), x)
```

3.152.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 7.85

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
2/5*(g^2*x^2 + 2*f*g*x + f^2)*sqrt(g*x + f)/(b*g*log((e*x + d)^n) + b*g*lo
g(c) + a*g) + integrate(2/5*(b*e*g^2*n*x^2 + 2*b*e*f*g*n*x + b*e*f^2*n)*sq
rt(g*x + f)/(b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c) + a^2*d*g + (b^2*e*g*x +
b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 + 2*a*b*e*g*log(c) + a^2*e
*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log(c) + a*b*e*g)*x)*log((e
*x + d)^n)), x)
```

3.152.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)/(b*log((e*x + d)^n*c) + a), x)
```

3.152.9 Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(f+g\,x)^{3/2}}{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int((f + g*x)^(3/2)/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^(3/2)/(a + b*log(c*(d + e*x)^n)), x)
```

3.153 $\int \frac{\sqrt{f+gx}}{a+b\log(c(d+ex)^n)}\,dx$

3.153.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\text{Int}\left(\frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)},x\right)$$

output
```
Unintegrable((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.153.2 Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

input
```
Integrate[Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
Integrate[Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]), x]
```

3.153.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$\Big\downarrow \ 2867$$
$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}dx$$

input
```
Int[Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
$Aborted
```

3.153.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.153.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{gx+f}}{a+b\ln\left(c\,(ex+d)^{n}\right)}dx$$

input
```
int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.153.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{gx+f}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)/(b*log((e*x + d)^n*c) + a), x)
```

3.153.6 Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{f+gx}}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(sqrt(f + g*x)/(a + b*log(c*(d + e*x)**n)), x)
```

3.153.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 6.69

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{gx+f}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
2/3*(g*x + f)^(3/2)/(b*g*log((e*x + d)^n) + b*g*log(c) + a*g) + integrate(
2/3*(b*e*g*n*x + b*e*f*n)*sqrt(g*x + f)/(b^2*d*g*log(c)^2 + 2*a*b*d*g*log(
c) + a^2*d*g + (b^2*e*g*x + b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^
2 + 2*a*b*e*g*log(c) + a^2*e*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g
*log(c) + a*b*e*g)*x)*log((e*x + d)^n)), x)
```

3.153.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{gx+f}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(b*log((e*x + d)^n*c) + a), x)
```

3.153.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{f+g\,x}}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n)), x)
```

3.154 $\int \frac{1}{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}\,dx$

3.154.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\text{Int}\left(\frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))},x\right)$$

output
```
Unintegrable(1/(a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x)
```

3.154.2 Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx$$

input
```
Integrate[1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])), x]
```

3.154.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx$$

$$\Big\downarrow \ 2867$$

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}dx$$

input
```
Int[1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.154.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.154.4 Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+b\ln(c(ex+d)^n))\sqrt{gx+f}}dx$$

input
```
int(1/(a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x)
```

output
```
int(1/(a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x)
```

3.154.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x
)
```

3.154.6 Sympy [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(e*x+d)**n))/(g*x+f)**(1/2),x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))*sqrt(f + g*x)), x)
```

3.154.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 6.69

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(g*x + f)/(b*g*log((e*x + d)^n) + b*g*log(c) + a*g) + integrate(2*(b
*e*g*n*x + b*e*f*n)/((b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c) + a^2*d*g + (b^2
*e*g*x + b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 + 2*a*b*e*g*log(c
) + a^2*e*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log(c) + a*b*e*g)*
x)*log((e*x + d)^n))*sqrt(g*x + f)), x)
```

3.154.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\int \frac{1}{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.154.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))), x)
```

3.155 $\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))} dx$

3.155.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))} dx = \text{Int}\left(\frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}, x\right)$$

output
```
Unintegrable(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))} dx = \int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))} dx$$

input
```
Integrate[1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])), x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^{3/2} (a+b\log (c(d+ex)^n))} \, dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{1}{(f+gx)^{3/2} (a+b\log (c(d+ex)^n))}dx$$

input
```
Int[1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.155.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)^{\frac{3}{2}} (a+b\ln (c (ex+d)^{n}))}dx$$

input
```
int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}(b\log((ex+d)^nc)+a)}\,dx$$

input
```
integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)/(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f
*g*x + b*f^2)*log((e*x + d)^n*c)), x)
```

3.155.6 Sympy [N/A]

Not integrable

Time = 7.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(g*x+f)**(3/2)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)**(3/2)), x)
```

3.155.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 6.38

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}(b\log((ex+d)^nc)+a)}\,dx$$

input
```
integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
-2*b*e*n*integrate(1/((b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c) + a^2*d*g + (b^
2*e*g*x + b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 + 2*a*b*e*g*log(
c) + a^2*e*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log(c) + a*b*e*g)
*x)*log((e*x + d)^n))*sqrt(g*x + f)), x) - 2/((b*g*log((e*x + d)^n) + b*g*
log(c) + a*g)*sqrt(g*x + f))
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a)), x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))), x)
```

3.156 $\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$

3.156.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{2(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3g}-\frac{ben\text{Int}\left(\frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{3g}$$

output
```
2/3*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/g-1/3*b*e*n*Unintegrable((g*
x+f)^(3/2)/(e*x+d)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

3.156.2 Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

input
```
Integrate[Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
Integrate[Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

3.156.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2845, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3g}-\frac{ben\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}dx}{3g}$$

$$\Big\downarrow \text{2867}$$

$$\frac{2(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3g}-\frac{ben\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}dx}{3g}$$

input
```
Int[Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
$Aborted
```

3.156.3.1 Defintions of rubi rules used

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.156.4 Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \sqrt{gx+f}\,\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

input
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.156.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^{n}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.156.6 Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^{n}\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}\sqrt{f+gx}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))*sqrt(f + g*x), x)
```

3.156.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{gx+f}\sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.156.8 Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{gx+f}\sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.156.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{f+g\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.157 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx$

3.157.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}}{g} - \frac{ben\text{Int}\left(\frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{g}$$

output
```
2*(g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/g-b*e*n*Unintegrable((g*x+f)^(
1/2)/(e*x+d)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

3.157.2 Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/Sqrt[f + g*x],x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/Sqrt[f + g*x], x]
```

3.157.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2845, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g}-\frac{ben\int\frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}dx}{g}$$

$$\Big\downarrow \text{ 2867}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g}-\frac{ben\int\frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}dx}{g}$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/Sqrt[f + g*x],x]
```

output
```
$Aborted
```

3.157.3.1 Defintions of rubi rules used

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.157.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}{\sqrt{gx+f}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x)
```

3.157.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}{\sqrt{f+gx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.157.6 Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))/sqrt(f + g*x), x)
```

3.157.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/sqrt(g*x + f), x)
```

3.157.8 Giac [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/sqrt(g*x + f), x)
```

3.157.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{\sqrt{f+g\,x}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(1/2), x)
```

3.158 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^{3/2}}\,dx$

3.158.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx=-\frac{2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g\sqrt{f+gx}}+\frac{ben\text{Int}\left(\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{g}$$

output
```
-2*(a+b*ln(c*(e*x+d)^n))^(1/2)/g/(g*x+f)^(1/2)+b*e*n*Unintegrable(1/(e*x+d
)/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

3.158.2 Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^(3/2),x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^(3/2), x]
```

3.158.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2845, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{ben\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}}dx}{g}-\frac{2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g\sqrt{f+gx}}$$

$$\Big\downarrow \text{2867}$$

$$\frac{ben\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}}dx}{g}-\frac{2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g\sqrt{f+gx}}$$

input
```
Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^(3/2),x]
```

output
```
$Aborted
```

3.158.3.1 Defintions of rubi rules used

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.158.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}{\left(gx+f\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x)
```

3.158.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}{(f+gx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.158.6 Sympy [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}{\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**(3/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x)**(3/2), x)
```

3.158.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^(3/2), x)
```

3.158.8 Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^(3/2), x)
```

3.158.9 Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{{\left(f+g\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(3/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(3/2), x)
```

3.159 $\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.159.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Int}\left(\frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

output
```
Unintegrable((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.159.2 Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

input
```
Integrate[Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
Integrate[Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

3.159.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log(c(d+ex)^n)}}dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log(c(d+ex)^n)}}dx$$

input
```
Int[Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
$Aborted
```

3.159.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.159.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{gx+f}}{\sqrt{a+b\ln(c(ex+d)^n)}}dx$$

input
```
int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.159.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.159.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.159.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.159.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.159.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

input
```
int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.160 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.160.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Int}\left(\frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

output
```
Unintegrable(1/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.160.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

input
```
Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

output
```
Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

3.160.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

$$\Big\downarrow \; 2867$$

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}dx$$

input
```
Int[1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

output
```
$Aborted
```

3.160.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.160.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{gx+f}\,\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

input
```
int(1/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int(1/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.160.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.160.6 Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(g*x+f)**(1/2)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*log(c*(d + e*x)**n))*sqrt(f + g*x)), x)
```

3.160.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{gx+f}\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

3.160.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{gx+f}\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

3.160.9 Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2)), x)
```

3.161 $\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.161.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Int}\left(\frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

output
```
Unintegrable(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.161.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

input
```
Integrate[1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

output
```
Integrate[1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

3.161.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$\Big\downarrow \ 2867$$
$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}dx$$

input
```
Int[1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

output
```
$Aborted
```

3.161.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.161.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{(gx+f)^{\frac{3}{2}}\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

input
```
int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.161.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.161.6 Sympy [N/A]

Not integrable

Time = 26.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(g*x+f)**(3/2)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x)**(3/2)), x)
```

3.161.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}\sqrt{b\log\left((ex+d)^{n}c\right)+a}}\,dx$$

input
```
integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^(3/2)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

3.161.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^(3/2)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

3.161.9 Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^(1/2)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^(1/2)), x)
```

3.162 $\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx$

3.162.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \frac{ben(f+gx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)(1+m)(2+m)} + \frac{(f+gx)^{1+m}(a+b\log(c(d+ex)^n))}{g(1+m)}$$

output
```
b*e*n*(g*x+f)^(2+m)*hypergeom([1, 2+m],[3+m],e*(g*x+f)/(-d*g+e*f))/g/(-d*g
+e*f)/(1+m)/(2+m)+(g*x+f)^(1+m)*(a+b*ln(c*(e*x+d)^n))/g/(1+m)
```

3.162.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \frac{(f+gx)^{1+m}\left(a+\frac{ben(f+gx)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{e(f+gx)}{ef-dg}\right)}{(ef-dg)(2+m)}+b\log(c(d+ex)^n)\right)}{g(1+m)}$$

input
```
Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
((f + g*x)^(1 + m)*(a + (b*e*n*(f + g*x)*Hypergeometric2F1[1, 2 + m, 3 + m
, (e*(f + g*x))/(e*f - d*g)])/((e*f - d*g)*(2 + m)) + b*Log[c*(d + e*x)^n]
))/(g*(1 + m))
```

3.162.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2842, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \; 2842$$

$$\frac{(f+gx)^{m+1}(a+b\log(c(d+ex)^n))}{g(m+1)}-\frac{ben\int \frac{(f+gx)^{m+1}}{d+ex}dx}{g(m+1)}$$

$$\Big\downarrow \; 78$$

$$\frac{(f+gx)^{m+1}(a+b\log(c(d+ex)^n))}{g(m+1)}+\frac{ben(f+gx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{e(f+gx)}{ef-dg}\right)}{g(m+1)(m+2)(ef-dg)}$$

input
```
Int[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(b*e*n*(f + g*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (e*(f + g*x))/
(e*f - d*g)])/(g*(e*f - d*g)*(1 + m)*(2 + m)) + ((f + g*x)^(1 + m)*(a + b*
Log[c*(d + e*x)^n]))/(g*(1 + m))
```

3.162.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.162.4 Maple [F]

$$\int (gx+f)^m (a+b\ln (c(ex+d)^n))\,dx$$

input
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n)),x)
```

3.162.5 Fricas [F]

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))\ dx = \int (b\log ((ex+d)^n c)+a)(gx+f)^m\,dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral((g*x + f)^m*b*log((e*x + d)^n*c) + (g*x + f)^m*a, x)
```

3.162.6 Sympy [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.162.7 Maxima [F]

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)(gx+f)^m\,dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
b*((g*x + f)*(g*x + f)^m*log((e*x + d)^n)/(g*(m + 1)) + integrate((d*g*(m
+ 1)*log(c) - e*f*n + (e*g*(m + 1)*log(c) - e*g*n)*x)*(g*x + f)^m/(e*g*(m
+ 1)*x + d*g*(m + 1)), x)) + (g*x + f)^(m + 1)*a/(g*(m + 1))
```

3.162.8 Giac [F]

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)(gx+f)^m\,dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))\,dx=\int (f+g\,x)^m\,\left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

input
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n)), x)
```

3.163 $\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx$

3.163.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Int}\left(\frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)},x\right)$$

output
```
Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n)),x)
```

3.163.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

input
```
Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]), x]
```

3.163.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$\Big\downarrow \ 2867$$
$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}dx$$

input
```
Int[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
$Aborted
```

3.163.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.163.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^m}{a+b\ln\left(c\left(ex+d\right)^n\right)}dx$$

input
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n)),x)
```

3.163.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^m}{b\log((ex+d)^n c)+a}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral((g*x + f)^m/(b*log((e*x + d)^n*c) + a), x)
```

3.163.6 Sympy [N/A]

Not integrable

Time = 31.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx$$

input
```
integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((f + g*x)**m/(a + b*log(c*(d + e*x)**n)), x)
```

3.163.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^m}{b\log((ex+d)^n c)+a}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a), x)
```

3.163.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(gx+f)^m}{b\log\left((ex+d)^nc\right)+a}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a), x)
```

3.163.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+g\,x)^m}{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n)), x)
```

3.164 $\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2} dx$

3.164.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2} dx = \text{Int}\left(\frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}, x\right)$$

output
```
Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.164.2 Mathematica [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2} dx = \int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2} dx$$

input
```
Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2, x]
```

3.164.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}dx$$

input
```
Int[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
$Aborted
```

3.164.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.164.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^m}{(a+b\ln(c(ex+d)^n))^2}dx$$

input
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.164.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^nc)+a)^2}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral((g*x + f)^m/(b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c)
+ a^2), x)
```

3.164.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.164.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.46

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^nc)+a)^2}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*x + d)*(g*x + f)^m/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*
n) + integrate((e*g*(m + 1)*x + d*g*m + e*f)*(g*x + f)^m/(b^2*e*f*n*log(c)
 + a*b*e*f*n + (b^2*e*g*n*log(c) + a*b*e*g*n)*x + (b^2*e*g*n*x + b^2*e*f*n
)*log((e*x + d)^n)), x)
```

3.164.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a)^2, x)
```

3.164.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+g\,x)^m}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

input
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.165 $\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx$

3.165.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx = \text{Int}\left((f+gx)^m (a+b\log (c(d+ex)^n))^{3/2},x\right)$$

output
```
Unintegrable((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.165.2 Mathematica [N/A]

Not integrable

Time = 9.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx = \int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx$$

input
```
Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2), x]
```

3.165.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx$$

$$\Big\downarrow \ 2867$$

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2}dx$$

input
```
Int[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
$Aborted
```

3.165.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.165.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (gx+f)^m (a+b\ln (c(ex+d)^n))^{\frac{3}{2}}dx$$

input
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.165.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx = \int (b\log ((ex+d)^n c) + a)^{\frac{3}{2}} (gx+f)^m \, dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(((g*x + f)^m*b*log((e*x + d)^n*c) + (g*x + f)^m*a)*sqrt(b*log((e*
x + d)^n*c) + a), x)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Timed out
```

3.165.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^{3/2} \, dx = \int (b\log ((ex+d)^n c) + a)^{\frac{3}{2}} (gx+f)^m \, dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)*(g*x + f)^m, x)
```

3.165.8 Giac [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^{3/2} \, dx = \int (b\log((ex+d)^n c)+a)^{\frac{3}{2}} (gx+f)^m \, dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^(3/2)*(g*x + f)^m, x)
```

3.165.9 Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^{3/2} \, dx = \int (f+g\,x)^m \left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^{3/2} dx$$

input
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.166 $\int (f+gx)^m \sqrt{a+b\log(c(d+ex)^n)}\,dx$

3.166.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (f+gx)^m \sqrt{a+b\log(c(d+ex)^n)}\,dx = \text{Int}\left((f+gx)^m \sqrt{a+b\log(c(d+ex)^n)},x\right)$$

output
```
Unintegrable((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.166.2 Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (f+gx)^m \sqrt{a+b\log(c(d+ex)^n)}\,dx = \int (f+gx)^m \sqrt{a+b\log(c(d+ex)^n)}\,dx$$

input
```
Integrate[(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
Integrate[(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

3.166.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m \sqrt{a+b\log \left(c(d+ex)^n\right)} \, dx$$

$$\Big\downarrow \; 2867$$

$$\int (f+gx)^m \sqrt{a+b\log \left(c(d+ex)^n\right)}dx$$

input
```
Int[(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
$Aborted
```

3.166.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.166.4 Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \left(g x +f \right)^{m} \sqrt{a +b \ln \left(c \left(e x +d \right)^{n}\right)}dx$$

input
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.166.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{b\log\left((ex+d)^nc\right)+a}(gx+f)^m\,dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

3.166.6 Sympy [F(-2)]

Exception generated.

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.166.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{b\log\left((ex+d)^nc\right)+a}(gx+f)^m\,dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

3.166.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int\sqrt{b\log\left((ex+d)^nc\right)+a}(gx+f)^m\,dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

3.166.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (f+g\,x)^m\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

input
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.167 $\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$

3.167.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Int}\left(\frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

output
```
Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.167.2 Mathematica [N/A]

Not integrable

Time = 7.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

input
```
Integrate[(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
Integrate[(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

3.167.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}dx$$
$$\Big\downarrow \ 2867$$
$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}dx$$

input
```
Int[(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

output
```
$Aborted
```

3.167.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.167.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(gx+f)^m}{\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}}dx$$

input
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

output
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

3.167.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(gx+f)^m}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

output
```
integral((g*x + f)^m/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.167.6 Sympy [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c\,(d+ex)^n\right)}}\,dx$$

input
```
integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

output
```
Integral((f + g*x)**m/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

3.167.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(gx+f)^m}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^m/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.167.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(gx+f)^m}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^m/sqrt(b*log((e*x + d)^n*c) + a), x)
```

3.167.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(f+g\,x)^m}{\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

input
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(1/2),x)
```

output
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

3.168 $\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$

3.168.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \mathrm{Int}\left(\frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}},x\right)$$

output
```
Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.168.2 Mathematica [N/A]

Not integrable

Time = 8.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

input
```
Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2), x]
```

3.168.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 2867}$$
$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}dx$$

input
```
Int[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

output
```
$Aborted
```

3.168.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.168.4 Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(gx+f)^m}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

output
```
int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

3.168.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m/(b^2*log((e*x + d)^n*c
)^2 + 2*a*b*log((e*x + d)^n*c) + a^2), x)
```

3.168.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.168.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.168.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

3.168.9 Mupad [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+g\,x)^m}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

input
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(3/2),x)
```

output
```
int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

3.169 $\int (f+gx)^m (a+b\log(c(d+ex)^n))^n\,dx$

3.169.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n\,dx = \mathrm{Int}((f+gx)^m (a+b\log(c(d+ex)^n))^n, x)$$

output
```
Unintegrable((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^n,x)
```

3.169.2 Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n\,dx = \int (f+gx)^m (a+b\log(c(d+ex)^n))^n\,dx$$

input
```
Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n, x]
```

3.169.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n\,dx$$
$$\Big\downarrow \; 2867$$
$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n\,dx$$

input
```
Int[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
$Aborted
```

3.169.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.169.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (gx+f)^m (a+b\ln(c(ex+d)^n))^n dx$$

input
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^n,x)
```

output
```
int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^n,x)
```

3.169.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \int (gx+f)^m (b\log((ex+d)^n c)+a)^n \, dx$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
```

output
```
integral((g*x + f)^m*(b*log((e*x + d)^n*c) + a)^n, x)
```

3.169.6 Sympy [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.169.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.169.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^n \,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,7,4,0,5,0,3,5,0,0,0]%%%}+%%%{5,[0,0,6,4,0,4,1,3,5,0,0,0]%%%}+%%%{2,[0,0
,6,3,1,5,0,3
```

3.169.9 Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^n \,dx = \int (f+g\,x)^m\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^n \,dx$$

input
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^n,x)
```

output
```
int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^n, x)
```

3.170 $\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx$

3.170.1 Optimal result

Integrand size = 24, antiderivative size = 474

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx = \frac{4^{-1-n}e^{-\frac{4a}{bn}}g^3(d+ex)^4(c(d+ex)^n)^{-4/n}\Gamma\left(1+n,-\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(\ldots}{bn}\right.}{e^4} + \frac{3^{-n}e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3(c(d+ex)^n)^{-3/n}\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(\ldots\right.}{e^4} + \frac{3\,2^{-1-n}e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2(c(d+ex)^n)^{-2/n}\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n)\ldots}{e^4} + \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)(c(d+ex)^n)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(\ldots}{\ldots}\right.}{e^4}$$

output
```
4^(-1-n)*g^3*(e*x+d)^4*GAMMA(1+n,-4*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(
e*x+d)^n))^n/e^4/exp(4*a/b/n)/((c*(e*x+d)^n)^(4/n))/(((-a-b*ln(c*(e*x+d)^n
))/b/n)^n)+g^2*(-d*g+e*f)*(e*x+d)^3*GAMMA(1+n,-3*(a+b*ln(c*(e*x+d)^n))/b/n
)*(a+b*ln(c*(e*x+d)^n))^n/(3^n)/e^4/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/(((
-a-b*ln(c*(e*x+d)^n))/b/n)^n)+3*2^(-1-n)*g*(-d*g+e*f)^2*(e*x+d)^2*GAMMA(1+
n,-2*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e^4/exp(2*a/b/n)/(
(c*(e*x+d)^n)^(2/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)+(-d*g+e*f)^3*(e*x+d)
*GAMMA(1+n,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e^4/exp(a/b
/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)
```

3.170.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.72

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^n \, dx$$
$$= \frac{3^{-n}4^{-1-n}e^{-\frac{4a}{bn}}(d+ex)\,(c(d+ex)^n)^{-4/n}\left(3^n g^3(d+ex)^3\Gamma\left(1+n,-\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)+2^{1+n}e^{\frac{a}{bn}}(ef-dg)\right.}{}$$

input
```
Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
(4^(-1 - n)*(d + e*x)*(3^n*g^3*(d + e*x)^3*Gamma[1 + n, (-4*(a + b*Log[c*(
d + e*x)^n]))/(b*n)] + 2^(1 + n)*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n
^(-1)*(2^(1 + n)*g^2*(d + e*x)^2*Gamma[1 + n, (-3*(a + b*Log[c*(d + e*x)^n
]))/(b*n)] + 3^n*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*(3*g*(d +
e*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 2^(1 + n)*E^(a/
(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Gamma[1 + n, -((a + b*Log[c*(d +
 e*x)^n])/(b*n))])))*(a + b*Log[c*(d + e*x)^n])^n)/(3^n*e^4*E^((4*a)/(b*n)
)*(c*(d + e*x)^n)^(4/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

3.170.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^n \, dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{3g^2(d+ex)^2(ef-dg)\,(a+b\log (c(d+ex)^n))^n}{e^3}+\frac{(ef-dg)^3\,(a+b\log (c(d+ex)^n))^n}{e^3}+\frac{3g(d+ex)(ef-dg}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2 3^{-n} e^{-\frac{3a}{bn}} (d+ex)^3 (ef-dg) \left(c(d+ex)^n\right)^{-3/n} \left(a+b\log\left(c(d+ex)^n\right)\right)^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1, -\frac{3(a+b\log}{}}{e^4}$$
$$\frac{3g 2^{-n-1} e^{-\frac{2a}{bn}} (d+ex)^2 (ef-dg)^2 \left(c(d+ex)^n\right)^{-2/n} \left(a+b\log\left(c(d+ex)^n\right)\right)^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1, -\frac{2(a}{}}{e^4}$$
$$\frac{e^{-\frac{a}{bn}} (d+ex) (ef-dg)^3 \left(c(d+ex)^n\right)^{-1/n} \left(a+b\log\left(c(d+ex)^n\right)\right)^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1, -\frac{a+b\log(c(d+ex}{bn}}{e^4}$$
$$\frac{g^3 4^{-n-1} e^{-\frac{4a}{bn}} (d+ex)^4 \left(c(d+ex)^n\right)^{-4/n} \left(a+b\log\left(c(d+ex)^n\right)\right)^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1, -\frac{4(a+b\log(c(d+ex}{bn}}{e^4}$$

input
```
Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
(4^(-1 - n)*g^3*(d + e*x)^4*Gamma[1 + n, (-4*(a + b*Log[c*(d + e*x)^n]))/(
b*n)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^4*E^((4*a)/(b*n))*(c*(d + e*x)^n)^(
4/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n) + (g^2*(e*f - d*g)*(d + e*x)
^3*Gamma[1 + n, (-3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e
*x)^n])^n)/(3^n*e^4*E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)*(-((a + b*Log[c*
(d + e*x)^n])/(b*n)))^n) + (3*2^(-1 - n)*g*(e*f - d*g)^2*(d + e*x)^2*Gamma
[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n])^
n)/(e^4*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(-((a + b*Log[c*(d + e*x)^n]
)/(b*n)))^n) + ((e*f - d*g)^3*(d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d + e
*x)^n])/(b*n))]*(a + b*Log[c*(d + e*x)^n])^n)/(e^4*E^(a/(b*n))*(c*(d + e*x
)^n)^n^(-1)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.170.4 Maple [F]

$$\int (gx+f)^3 (a+b\ln(c(ex+d)^n))^n dx$$

input
```
int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^n,x)
```

output
```
int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^n,x)
```

3.170.5 Fricas [F]

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n \, dx = \int (gx+f)^3 (b\log((ex+d)^n c)+a)^n \, dx$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
```

output
```
integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(b*log((e*x + d)^n*c) +
 a)^n, x)
```

3.170.6 Sympy [F]

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n \, dx = \int (a+b\log(c(d+ex)^n))^n (f+gx)^3 \, dx$$

input
```
integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n))**n,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**n*(f + g*x)**3, x)
```

3.170.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.170.8 Giac [F]

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^n \, dx = \int (gx+f)^3 (b\log ((ex+d)^n c)+a)^n \, dx$$

input
```
integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

output
```
integrate((g*x + f)^3*(b*log((e*x + d)^n*c) + a)^n, x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^n \, dx = \int (f+g\,x)^3 \,(a+b\,\ln (c\,(d+e\,x)^n))^n \, dx$$

input
```
int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^n,x)
```

output
```
int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^n, x)
```

3.171 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^n \, dx$

3.171.1 Optimal result

Integrand size = 24, antiderivative size = 348

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^n \, dx$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{bn}}g^2(d+ex)^3 (c(d+ex)^n)^{-3/n}\,\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n \left(-\frac{a+b\log(c(\cdots}{bn}\right.}{e^3}$$
$$+\frac{2^{-n}e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2 (c(d+ex)^n)^{-2/n}\,\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n \left(-\cdots\right.}{e^3}$$
$$+\frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)(c(d+ex)^n)^{-1/n}\,\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n \left(-\frac{a+b\log(\cdots}{\cdots}\right.}{e^3}$$

output
```
3^(-1-n)*g^2*(e*x+d)^3*GAMMA(1+n,-3*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(
e*x+d)^n))^n/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/(((-a-b*ln(c*(e*x+d)^n
))/b/n)^n)+g*(-d*g+e*f)*(e*x+d)^2*GAMMA(1+n,-2*(a+b*ln(c*(e*x+d)^n))/b/n)*
(a+b*ln(c*(e*x+d)^n))^n/(2^n)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/(((-a
-b*ln(c*(e*x+d)^n))/b/n)^n)+(-d*g+e*f)^2*(e*x+d)*GAMMA(1+n,(-a-b*ln(c*(e*x
+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/
(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)
```

3.171.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.75

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx$$
$$= \frac{2^{-n}3^{-1-n}e^{-\frac{3a}{bn}}(d+ex)\,(c(d+ex)^n)^{-3/n}\left(2^n g^2(d+ex)^2\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)+3^{1+n}e^{\frac{a}{bn}}(ef-dg)\right.}{}$$

input
```
Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
(3^(-1 - n)*(d + e*x)*(2^n*g^2*(d + e*x)^2*Gamma[1 + n, (-3*(a + b*Log[c*(
d + e*x)^n]))/(b*n)] + 3^(1 + n)*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n
^(-1)*(g*(d + e*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 2
^n*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Gamma[1 + n, -((a + b*Lo
g[c*(d + e*x)^n])/(b*n))]))*(a + b*Log[c*(d + e*x)^n])^n)/(2^n*e^3*E^((3*a
)/(b*n))*(c*(d + e*x)^n)^(3/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

3.171.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(ef-dg)^2 (a+b\log (c(d+ex)^n))^n}{e^2}+\frac{2g(d+ex)(ef-dg) (a+b\log (c(d+ex)^n))^n}{e^2}+\frac{g^2(d+ex)^2 (a+b\log}{e^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{g2^{-n}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\Big(n+1,-\frac{2(a+b\log}{}}{e^3}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\Big(n+1,-\frac{a+b\log(c(d+ex}{bn}}{e^3}$$

$$\frac{g^2 3^{-n-1}e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\Big(n+1,-\frac{3(a+b\log(c(d+ex}{bn}}{e^3}$$

input
```
Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
(3^(-1 - n)*g^2*(d + e*x)^3*Gamma[1 + n, (-3*(a + b*Log[c*(d + e*x)^n]))/(
b*n)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^3*E^((3*a)/(b*n))*(c*(d + e*x)^n)^(
3/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n) + (g*(e*f - d*g)*(d + e*x)^2
*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x
)^n])^n)/(2^n*e^3*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(-((a + b*Log[c*(d
 + e*x)^n])/(b*n)))^n) + ((e*f - d*g)^2*(d + e*x)*Gamma[1 + n, -((a + b*Lo
g[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(d + e*x)^n])^n)/(e^3*E^(a/(b*n))*(
c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.171.4 Maple [F]

$$\int (gx+f)^2\left(a+b\ln\left(c(ex+d)^n\right)\right)^n dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^n,x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^n,x)
```

3.171.5 Fricas [F]

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx = \int (gx+f)^2 (b\log ((ex+d)^n c)+a)^n \, dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*(b*log((e*x + d)^n*c) + a)^n, x)
```

3.171.6 Sympy [F]

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx = \int (a+b\log (c(d+ex)^n))^n (f+gx)^2 \, dx$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**n,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**n*(f + g*x)**2, x)
```

3.171.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.171.8 Giac [F]

$$\int (f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx=\int (gx+f)^2\left(b\log\left((ex+d)^nc\right)+a\right)^n\,dx$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^n, x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx=\int (f+g\,x)^2\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^n\,dx$$

input
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^n,x)
```

output
```
int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^n, x)
```

3.172 $\int (f+gx)(a+b\log(c(d+ex)^n))^n\,dx$

3.172.1 Optimal result

Integrand size = 22, antiderivative size = 225

$$\int (f+gx)(a+b\log(c(d+ex)^n))^n\,dx = \frac{2^{-1-n}e^{-\frac{2a}{bn}}g(d+ex)^2(c(d+ex)^n)^{-2/n}\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d\cdots}{bn}\right.}{e^2} + \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)(c(d+ex)^n)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c\cdots}{b\cdots}\right.}{e^2}$$

output

```
2^(-1-n)*g*(e*x+d)^2*GAMMA(1+n,-2*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*
x+d)^n))^n/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/(((-a-b*ln(c*(e*x+d)^n))
/b/n)^n)+(-d*g+e*f)*(e*x+d)*GAMMA(1+n,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(
c*(e*x+d)^n))^n/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^
n))/b/n)^n)
```

3.172.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

$$\int (f+gx)(a+b\log(c(d+ex)^n))^n\,dx = \frac{2^{-1-n}e^{-\frac{2a}{bn}}(d+ex)(c(d+ex)^n)^{-2/n}\left(g(d+ex)\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)+2^{1+n}e^{\frac{a}{bn}}(ef-dg)(c(d+\cdots\right.}{e^2}$$

input
```
Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
(2^(-1 - n)*(d + e*x)*(g*(d + e*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)
^n]))/(b*n)] + 2^(1 + n)*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Ga
mma[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))])*(a + b*Log[c*(d + e*x)^n]
)^n)/(e^2*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(-((a + b*Log[c*(d + e*x)^
n])/(b*n)))^n)
```

3.172.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx$$

$$\Big\downarrow \text{ 2848}$$

$$\int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\log(c(d+ex)^n)}{bn}\right.}{e^2}$$
$$\frac{g2^{-n-1}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\log(c(d+ex)}{bn}\right.}{e^2}$$

input
```
Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
(2^(-1 - n)*g*(d + e*x)^2*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*
n)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^2*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/
n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n) + ((e*f - d*g)*(d + e*x)*Gamma
[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(d + e*x)^n])^n)
/(e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d + e*x)^n])/(b*
n)))^n)
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

3.172.4 Maple [F]

$$\int (gx+f)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{n}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^n,x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^n,x)
```

3.172.5 Fricas [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{n}\,dx=\int (gx+f)(b\log\left((ex+d)^{n}c\right)+a)^{n}\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
```

output
```
integral((g*x + f)*(b*log((e*x + d)^n*c) + a)^n, x)
```

3.172.6 Sympy [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{n}\,dx=\int \left(a+b\log\left(c(d+ex)^{n}\right)\right)^{n}(f+gx)\,dx$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**n,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**n*(f + g*x), x)
```

3.172.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.172.8 Giac [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx = \int (gx+f)(b\log\left((ex+d)^nc\right)+a)^n\,dx$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^n, x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx = \int (f+g\,x)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^n\,dx$$

input
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^n,x)
```

output
```
int((f + g*x)*(a + b*log(c*(d + e*x)^n))^n, x)
```

3.173 $\int (a+b\log (c(d+ex)^n))^n \, dx$

3.173.1 Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (a+b\log (c(d+ex)^n))^n \, dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log (c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}}{e}$$

output
```
(e*x+d)*GAMMA(1+n,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e/ex
p(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)
```

3.173.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int (a+b\log (c(d+ex)^n))^n \, dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log (c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}}{e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
((d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*
(d + e*x)^n])^n)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d
+ e*x)^n])/(b*n)))^n)
```

3.173.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2836, 2737, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^n\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(c(d+ex)^n))^n\,d(d+ex)}{e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \left(c(d+ex)^n\right)^{\frac{1}{n}}(a+b\log(c(d+ex)^n))^n\,d\log(c(d+ex)^n)}{en}$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^n,x]
```

output
```
((d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*
(d + e*x)^n])^n)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d
+ e*x)^n])/(b*n)))^n)
```

3.173.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.173.4 Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^n dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^n,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^n,x)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int (a+b\log(c(d+ex)^n))^n\,dx = \frac{e^{\left(-\frac{bn^2\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(n+1,-\frac{bn\log(ex+d)+b\log(c)+a}{bn}\right)}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
```

output
```
e^(-(b*n^2*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(n + 1, -(b*n*log(e*x
 + d) + b*log(c) + a)/(b*n))/e
```

3.173.6 Sympy [F]

$$\int (a+b\log(c(d+ex)^n))^n\,dx=\int (a+b\log(c(d+ex)^n))^n\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**n,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**n, x)
```

3.173.7 Maxima [F(-2)]

Exception generated.

$$\int (a+b\log(c(d+ex)^n))^n\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.173.8 Giac [F]

$$\int (a+b\log(c(d+ex)^n))^n\,dx=\int (b\log((ex+d)^n c)+a)^n\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^n, x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^n\,dx=\int (a+b\,\ln(c\,(d+e\,x)^n))^n\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^n,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^n, x)
```

3.174 $\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx} dx$

3.174.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx} dx = \operatorname{Int}\left(\frac{(a+b\log(c(d+ex)^n))^n}{f+gx},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^n/(g*x+f),x)
```

3.174.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx} dx = \int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx} dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^n/(f + g*x),x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])^n/(f + g*x), x]
```

3.174.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}dx$$

$$\Big\downarrow \text{2867}$$

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}dx$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^n/(f + g*x),x]
```

output
```
$Aborted
```

3.174.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.174.4 Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^n}{gx+f}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^n/(g*x+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^n/(g*x+f),x)
```

3.174.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^n}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^n/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)^n/(g*x + f), x)
```

3.174.6 Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**n/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**n/(f + g*x), x)
```

3.174.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^n/(g*x+f),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.174.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^n}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^n/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^n/(g*x + f), x)
```

3.174.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^n}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^n/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^n/(f + g*x), x)
```

3.175 $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.175.1 Optimal result

Integrand size = 30, antiderivative size = 315

$$\begin{aligned}\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = &-\frac{4bi(fh-ei)^3x}{df^4}-\frac{3bi^2(fh-ei)^2(e+fx)^2}{2df^5}\\ &-\frac{4bi^3(fh-ei)(e+fx)^3}{9df^5}-\frac{bi^4(e+fx)^4}{16df^5}\\ &-\frac{b(fh-ei)^4\log^2(e+fx)}{2df^5}\\ &+\frac{4i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5}\\ &+\frac{3i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5}\\ &+\frac{4i^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5}\\ &+\frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5}\\ &+\frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5}\end{aligned}$$

output
```
-4*b*i*(-e*i+f*h)^3*x/d/f^4-3/2*b*i^2*(-e*i+f*h)^2*(f*x+e)^2/d/f^5-4/9*b*i
^3*(-e*i+f*h)*(f*x+e)^3/d/f^5-1/16*b*i^4*(f*x+e)^4/d/f^5-1/2*b*(-e*i+f*h)^
4*ln(f*x+e)^2/d/f^5+4*i*(-e*i+f*h)^3*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^5+3*i
^2*(-e*i+f*h)^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))/d/f^5+4/3*i^3*(-e*i+f*h)*(f*
x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^5+1/4*i^4*(f*x+e)^4*(a+b*ln(c*(f*x+e)))/d/f
^5+(-e*i+f*h)^4*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^5
```

3.175.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.87

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= \frac{72a^2f^4h^4 - 288a^2ef^3h^3i + 432a^2e^2f^2h^2i^2 - 288a^2e^3fhi^3 + 72a^2e^4i^4 + 576abf^4h^3ix - 576b^2f^4h^3ix - 864}{}$$

input
```
Integrate[((h + i*x)^4*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(72*a^2*f^4*h^4 - 288*a^2*e*f^3*h^3*i + 432*a^2*e^2*f^2*h^2*i^2 - 288*a^2*
e^3*f*h*i^3 + 72*a^2*e^4*i^4 + 576*a*b*f^4*h^3*i*x - 576*b^2*f^4*h^3*i*x -
 864*a*b*e*f^3*h^2*i^2*x + 1296*b^2*e*f^3*h^2*i^2*x + 576*a*b*e^2*f^2*h*i^
3*x - 1056*b^2*e^2*f^2*h*i^3*x - 144*a*b*e^3*f*i^4*x + 300*b^2*e^3*f*i^4*x
 + 432*a*b*f^4*h^2*i^2*x^2 - 216*b^2*f^4*h^2*i^2*x^2 - 288*a*b*e*f^3*h*i^3
*x^2 + 240*b^2*e*f^3*h*i^3*x^2 + 72*a*b*e^2*f^2*i^4*x^2 - 78*b^2*e^2*f^2*i
^4*x^2 + 192*a*b*f^4*h*i^3*x^3 - 64*b^2*f^4*h*i^3*x^3 - 48*a*b*e*f^3*i^4*x
^3 + 28*b^2*e*f^3*i^4*x^3 + 36*a*b*f^4*i^4*x^4 - 9*b^2*f^4*i^4*x^4 - 12*b^
2*e^2*i^2*(36*f^2*h^2 - 40*e*f*h*i + 13*e^2*i^2)*Log[e + f*x] + 12*b*(12*a
*(f*h - e*i)^4 + b*i*(-12*e^4*i^3 - 12*e^3*f*i^2*(-4*h + i*x) + 6*e^2*f^2*
i*(-12*h^2 + 8*h*i*x + i^2*x^2) + 4*e*f^3*(12*h^3 - 18*h^2*i*x - 6*h*i^2*x
^2 - i^3*x^3) + f^4*x*(48*h^3 + 36*h^2*i*x + 16*h*i^2*x^2 + 3*i^3*x^3)))*L
og[c*(e + f*x)] + 72*b^2*(f*h - e*i)^4*Log[c*(e + f*x)]^2)/(144*b*d*f^5)
```

3.175.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$\Big\downarrow \; 2858$$
$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^4(a+b\log(c(e+fx)))}{df^4(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{df^5}$$

$$\Big\downarrow 2772$$

$$-b\int \left(\frac{1}{4}(e+fx)^3i^4+\frac{4}{3}(fh-ei)(e+fx)^2i^3+3(fh-ei)^2(e+fx)i^2+4(fh-ei)^3i+\frac{(fh-ei)^4\log(e+fx)}{e+fx}\right)d(e+f$$

$$\Big\downarrow 2009$$

$$\frac{4}{3}i^3(e+fx)^3(fh-ei)(a+b\log(c(e+fx)))+3i^2(e+fx)^2(fh-ei)^2(a+b\log(c(e+fx)))+(fh-ei)^4\log(e+$$

input
```
Int[((h + i*x)^4*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(-(b*(4*i*(f*h - e*i)^3*(e + f*x) + (3*i^2*(f*h - e*i)^2*(e + f*x)^2)/2 +
(4*i^3*(f*h - e*i)*(e + f*x)^3)/9 + (i^4*(e + f*x)^4)/16 + ((f*h - e*i)^4*
Log[e + f*x]^2)/2)) + 4*i*(f*h - e*i)^3*(e + f*x)*(a + b*Log[c*(e + f*x)])
 + 3*i^2*(f*h - e*i)^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)]) + (4*i^3*(f*h
- e*i)*(e + f*x)^3*(a + b*Log[c*(e + f*x)]))/3 + (i^4*(e + f*x)^4*(a + b*L
og[c*(e + f*x)]))/4 + (f*h - e*i)^4*Log[e + f*x]*(a + b*Log[c*(e + f*x)]))
/(d*f^5)
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

3.175. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.175.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.76

method	result
norman	$\frac{(12a e^4 i^4-48a e^3 f h i^3+72a e^2 f^2 h^2 i^2-48ae f^3 h^3 i+12a h^4 f^4-25b e^4 i^4+88b e^3 f h i^3-108b e^2 f^2 h^2 i^2+48be f^3 h^3 i) \ln(c(}{12d f^5}$
parts	$\frac{a\left(\frac{i\left(\frac{f^3 i^3 x^4}{4}+\frac{((-ei+2fh)f^2 i^2+2f^3 i^2 h)x^3}{3}+\frac{(2(-ei+2fh)f^2 hi+fi(e^2 i^2-2efhi+2f^2 h^2))x^2}{2}+x(-ei+2fh)(e^2 i^2-2efhi+2f^2 h\right.}{f^4}\right.}{d}$
risch	$-\frac{9\ln(fx+e)b e^2 h^2 i^2}{d f^3}+\frac{4\ln(fx+e)be h^3 i}{d f^2}-\frac{2b\ln(c(fx+e))^2 e^3 h i^3}{d f^4}+\frac{3b\ln(c(fx+e))^2 e^2 h^2 i^2}{d f^3}-\frac{2b\ln(c(fx+e))^2 e}{d f^2}$
parallelrisch	$\frac{144\ln(c(fx+e))a e^4 i^4+144\ln(c(fx+e))a f^4 h^4+1872b e^3 f h i^3-2376b e^2 f^2 h^2 i^2+1152be f^3 h^3 i-288x^2\ln(c(fx+e))be f^3}{}$
derivativedivides	$\frac{-\frac{4ca e^3 h i^3\ln(cfx+ce)}{f^3 d}-\frac{4cae h^3 i\ln(cfx+ce)}{fd}+\frac{6ca e^2 h^2 i^2\ln(cfx+ce)}{f^2 d}-\frac{6aeh i^3(cfx+ce)^2}{c f^3 d}-\frac{12beh i^3\left(\frac{(cfx+ce)^2\ln(cfx+ce)}{2}-\frac{(c}{}\right.}{c f^3 d}}{}$
default	$\frac{-\frac{4ca e^3 h i^3\ln(cfx+ce)}{f^3 d}-\frac{4cae h^3 i\ln(cfx+ce)}{fd}+\frac{6ca e^2 h^2 i^2\ln(cfx+ce)}{f^2 d}-\frac{6aeh i^3(cfx+ce)^2}{c f^3 d}-\frac{12beh i^3\left(\frac{(cfx+ce)^2\ln(cfx+ce)}{2}-\frac{(c}{}\right.}{c f^3 d}}{}$

input
```
int((i*x+h)^4*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
1/12*(12*a*e^4*i^4-48*a*e^3*f*h*i^3+72*a*e^2*f^2*h^2*i^2-48*a*e*f^3*h^3*i+
12*a*f^4*h^4-25*b*e^4*i^4+88*b*e^3*f*h*i^3-108*b*e^2*f^2*h^2*i^2+48*b*e*f^
3*h^3*i)/d/f^5*ln(c*(f*x+e))+1/2*b*(e^4*i^4-4*e^3*f*h*i^3+6*e^2*f^2*h^2*i^
2-4*e*f^3*h^3*i+f^4*h^4)/d/f^5*ln(c*(f*x+e))^2-1/12*i*(12*a*e^3*i^3-48*a*e
^2*f*h*i^2+72*a*e*f^2*h^2*i-48*a*f^3*h^3-25*b*e^3*i^3+88*b*e^2*f*h*i^2-108
*b*e*f^2*h^2*i+48*b*f^3*h^3)/d/f^4*x+1/24*i^2*(12*a*e^2*i^2-48*a*e*f*h*i+7
2*a*f^2*h^2-13*b*e^2*i^2+40*b*e*f*h*i-36*b*f^2*h^2)/d/f^3*x^2-1/36*i^3*(12
*a*e*i-48*a*f*h-7*b*e*i+16*b*f*h)/d/f^2*x^3+1/16*i^4*(4*a-b)/d/f*x^4+1/4*b
*i^4/d/f*x^4*ln(c*(f*x+e))-b*i*(e^3*i^3-4*e^2*f*h*i^2+6*e*f^2*h^2*i-4*f^3*
h^3)/f^4/d*x*ln(c*(f*x+e))+1/2*b*i^2*(e^2*i^2-4*e*f*h*i+6*f^2*h^2)/d/f^3*x
^2*ln(c*(f*x+e))-1/3*b*i^3*(e*i-4*f*h)/d/f^2*x^3*ln(c*(f*x+e))
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.52

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= \frac{9\,(4\,a-b)f^4i^4x^4+4\,(16\,(3\,a-b)f^4hi^3-(12\,a-7\,b)ef^3i^4)x^3+6\,(36\,(2\,a-b)f^4h^2i^2-8\,(6\,a-5\,b)ef^3hi}{}$$

input
```
integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas"
)
```

output
```
1/144*(9*(4*a - b)*f^4*i^4*x^4 + 4*(16*(3*a - b)*f^4*h*i^3 - (12*a - 7*b)*
e*f^3*i^4)*x^3 + 6*(36*(2*a - b)*f^4*h^2*i^2 - 8*(6*a - 5*b)*e*f^3*h*i^3 +
 (12*a - 13*b)*e^2*f^2*i^4)*x^2 + 72*(b*f^4*h^4 - 4*b*e*f^3*h^3*i + 6*b*e^
2*f^2*h^2*i^2 - 4*b*e^3*f*h*i^3 + b*e^4*i^4)*log(c*f*x + c*e)^2 + 12*(48*(
a - b)*f^4*h^3*i - 36*(2*a - 3*b)*e*f^3*h^2*i^2 + 8*(6*a - 11*b)*e^2*f^2*h
*i^3 - (12*a - 25*b)*e^3*f*i^4)*x + 12*(3*b*f^4*i^4*x^4 + 12*a*f^4*h^4 - 4
8*(a - b)*e*f^3*h^3*i + 36*(2*a - 3*b)*e^2*f^2*h^2*i^2 - 8*(6*a - 11*b)*e^
3*f*h*i^3 + (12*a - 25*b)*e^4*i^4 + 4*(4*b*f^4*h*i^3 - b*e*f^3*i^4)*x^3 +
6*(6*b*f^4*h^2*i^2 - 4*b*e*f^3*h*i^3 + b*e^2*f^2*i^4)*x^2 + 12*(4*b*f^4*h^
3*i - 6*b*e*f^3*h^2*i^2 + 4*b*e^2*f^2*h*i^3 - b*e^3*f*i^4)*x)*log(c*f*x +
c*e))/(d*f^5)
```

3.175.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(289) = 578$.

Time = 0.84 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.17

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= x^4\left(\frac{ai^4}{4df}-\frac{bi^4}{16df}\right)+x^3\left(-\frac{aei^4}{3df^2}+\frac{4ahi^3}{3df}+\frac{7bei^4}{36df^2}-\frac{4bhi^3}{9df}\right)$$
$$+x^2\left(\frac{ae^2i^4}{2df^3}-\frac{2aehi^3}{df^2}+\frac{3ah^2i^2}{df}-\frac{13be^2i^4}{24df^3}+\frac{5behi^3}{3df^2}-\frac{3bh^2i^2}{2df}\right)$$
$$+x\left(-\frac{ae^3i^4}{df^4}+\frac{4ae^2hi^3}{df^3}-\frac{6aeh^2i^2}{df^2}+\frac{4ah^3i}{df}+\frac{25be^3i^4}{12df^4}-\frac{22be^2hi^3}{3df^3}+\frac{9beh^2i^2}{df^2}-\frac{4bh^3i}{df}\right)$$
$$+\frac{(-12be^3i^4x+48be^2fhi^3x+6be^2fi^4x^2-72bef^2h^2i^2x-24bef^2hi^3x^2-4bef^2i^4x^3+48bf^3h^3ix+36bf^3}{12df^4}$$
$$+\frac{(be^4i^4-4be^3fhi^3+6be^2f^2h^2i^2-4bef^3h^3i+bf^4h^4)\log(c(e+fx))^2}{2df^5}$$
$$+\frac{(12ae^4i^4-48ae^3fhi^3+72ae^2f^2h^2i^2-48aef^3h^3i+12af^4h^4-25be^4i^4+88be^3fhi^3-108be^2f^2h^2i^2+}{12df^5}$$

input
```
integrate((i*x+h)**4*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

output
```
x**4*(a*i**4/(4*d*f) - b*i**4/(16*d*f)) + x**3*(-a*e*i**4/(3*d*f**2) + 4*a
*h*i**3/(3*d*f) + 7*b*e*i**4/(36*d*f**2) - 4*b*h*i**3/(9*d*f)) + x**2*(a*e
**2*i**4/(2*d*f**3) - 2*a*e*h*i**3/(d*f**2) + 3*a*h**2*i**2/(d*f) - 13*b*e
**2*i**4/(24*d*f**3) + 5*b*e*h*i**3/(3*d*f**2) - 3*b*h**2*i**2/(2*d*f)) +
x*(-a*e**3*i**4/(d*f**4) + 4*a*e**2*h*i**3/(d*f**3) - 6*a*e*h**2*i**2/(d*f
**2) + 4*a*h**3*i/(d*f) + 25*b*e**3*i**4/(12*d*f**4) - 22*b*e**2*h*i**3/(3
*d*f**3) + 9*b*e*h**2*i**2/(d*f**2) - 4*b*h**3*i/(d*f)) + (-12*b*e**3*i**4
*x + 48*b*e**2*f*h*i**3*x + 6*b*e**2*f*i**4*x**2 - 72*b*e*f**2*h**2*i**2*x
 - 24*b*e*f**2*h*i**3*x**2 - 4*b*e*f**2*i**4*x**3 + 48*b*f**3*h**3*i*x + 3
6*b*f**3*h**2*i**2*x**2 + 16*b*f**3*h*i**3*x**3 + 3*b*f**3*i**4*x**4)*log(
c*(e + f*x))/(12*d*f**4) + (b*e**4*i**4 - 4*b*e**3*f*h*i**3 + 6*b*e**2*f**
2*h**2*i**2 - 4*b*e*f**3*h**3*i + b*f**4*h**4)*log(c*(e + f*x))**2/(2*d*f*
*5) + (12*a*e**4*i**4 - 48*a*e**3*f*h*i**3 + 72*a*e**2*f**2*h**2*i**2 - 48
*a*e*f**3*h**3*i + 12*a*f**4*h**4 - 25*b*e**4*i**4 + 88*b*e**3*f*h*i**3 -
108*b*e**2*f**2*h**2*i**2 + 48*b*e*f**3*h**3*i)*log(e + f*x)/(12*d*f**5)
```

3.175. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.175.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(303) = 606$.

Time = 0.23 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.40

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = 4\,bh^3i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce) + \frac{1}{12}\,bi^4\left(\frac{12\,e^4\log(fx+e)}{df^5}+\frac{3\,f^3x^4-4\,ef^2x^3+6\,e^2fx^2-12\,e^3x}{df^4}\right)\log(cfx+ce) - \frac{2}{3}\,bhi^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)\log(cfx+ce) + 3\,bh^2i^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce) - \frac{1}{2}\,bh^4\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right) + 4\,ah^3i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right) + \frac{1}{12}\,ai^4\left(\frac{12\,e^4\log(fx+e)}{df^5}+\frac{3\,f^3x^4-4\,ef^2x^3+6\,e^2fx^2-12\,e^3x}{df^4}\right) - \frac{2}{3}\,ahi^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right) + 3\,ah^2i^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right) + \frac{bh^4\log(cfx+ce)\log(dfx+de)}{df} + \frac{ah^4\log(dfx+de)}{df} + \frac{2\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)bh^3i}{df^2} - \frac{3\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)bh^2i^2}{2\,df^3} - \frac{\left(4\,f^3x^3-15\,ef^2x^2-18\,e^3\log(fx+e)^2+66\,e^2fx-66\,e^3\log(fx+e)\right)bhi^3}{9\,df^4} - \frac{\left(9\,f^4x^4-28\,ef^3x^3+78\,e^2f^2x^2+72\,e^4\log(fx+e)^2-300\,e^3fx+300\,e^4\log(fx+e)\right)bi^4}{144\,df^5}$$

input
```
integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima"
)
```

output
```
4*b*h^3*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + 1/12*b*i^4
*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 1
2*e^3*x)/(d*f^4))*log(c*f*x + c*e) - 2/3*b*h*i^3*(6*e^3*log(f*x + e)/(d*f^
4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))*log(c*f*x + c*e) + 3*b*h^2
*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x + c*
e) - 1/2*b*h^4*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^
2 + 2*log(f*x + e)*log(c))/(d*f)) + 4*a*h^3*i*(x/(d*f) - e*log(f*x + e)/(d
*f^2)) + 1/12*a*i^4*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x^
3 + 6*e^2*f*x^2 - 12*e^3*x)/(d*f^4)) - 2/3*a*h*i^3*(6*e^3*log(f*x + e)/(d*
f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3*a*h^2*i^2*(2*e^2*log
(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + b*h^4*log(c*f*x + c*e)*log(
d*f*x + d*e)/(d*f) + a*h^4*log(d*f*x + d*e)/(d*f) + 2*(e*log(f*x + e)^2 -
2*f*x + 2*e*log(f*x + e))*b*h^3*i/(d*f^2) - 3/2*(f^2*x^2 + 2*e^2*log(f*x +
 e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))*b*h^2*i^2/(d*f^3) - 1/9*(4*f^3*x^3 -
 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x + e))*
b*h*i^3/(d*f^4) - 1/144*(9*f^4*x^4 - 28*e*f^3*x^3 + 78*e^2*f^2*x^2 + 72*e^
4*log(f*x + e)^2 - 300*e^3*f*x + 300*e^4*log(f*x + e))*b*i^4/(d*f^5)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.82

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{(4\,ai^4-bi^4)x^4}{16\,df}$$
$$+\frac{1}{12}\left(\frac{3\,bi^4x^4}{df}+\frac{4\,(4\,bfhi^3-bei^4)x^3}{df^2}+\frac{6\,(6\,bf^2h^2i^2-4\,befhi^3+be^2i^4)x^2}{df^3}+\frac{12\,(4\,bf^3h^3i-6\,bef^2h^2i^2+}{df^4}\right.$$
$$\left.+ce\right)+\frac{(48\,afhi^3-16\,bfhi^3-12\,aei^4+7\,bei^4)x^3}{36\,df^2}$$
$$+\frac{(72\,af^2h^2i^2-36\,bf^2h^2i^2-48\,aefhi^3+40\,befhi^3+12\,ae^2i^4-13\,be^2i^4)x^2}{24\,df^3}$$
$$+\frac{(48\,af^3h^3i-48\,bf^3h^3i-72\,aef^2h^2i^2+108\,bef^2h^2i^2+48\,ae^2fhi^3-88\,be^2fhi^3-12\,ae^3i^4+25\,be^3i^4)x}{12\,df^4}$$
$$+\frac{(bf^4h^4-4\,bef^3h^3i+6\,be^2f^2h^2i^2-4\,be^3fhi^3+be^4i^4)\log(cfx+ce)^2}{2\,df^5}$$
$$+\frac{(12\,af^4h^4-48\,aef^3h^3i+48\,bef^3h^3i+72\,ae^2f^2h^2i^2-108\,be^2f^2h^2i^2-48\,ae^3fhi^3+88\,be^3fhi^3+12\,ae}{12\,df^5}$$

input
```
integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

output
```
1/16*(4*a*i^4 - b*i^4)*x^4/(d*f) + 1/12*(3*b*i^4*x^4/(d*f) + 4*(4*b*f*h*i^
3 - b*e*i^4)*x^3/(d*f^2) + 6*(6*b*f^2*h^2*i^2 - 4*b*e*f*h*i^3 + b*e^2*i^4)
*x^2/(d*f^3) + 12*(4*b*f^3*h^3*i - 6*b*e*f^2*h^2*i^2 + 4*b*e^2*f*h*i^3 - b
*e^3*i^4)*x/(d*f^4))*log(c*f*x + c*e) + 1/36*(48*a*f*h*i^3 - 16*b*f*h*i^3
- 12*a*e*i^4 + 7*b*e*i^4)*x^3/(d*f^2) + 1/24*(72*a*f^2*h^2*i^2 - 36*b*f^2*
h^2*i^2 - 48*a*e*f*h*i^3 + 40*b*e*f*h*i^3 + 12*a*e^2*i^4 - 13*b*e^2*i^4)*x
^2/(d*f^3) + 1/12*(48*a*f^3*h^3*i - 48*b*f^3*h^3*i - 72*a*e*f^2*h^2*i^2 +
108*b*e*f^2*h^2*i^2 + 48*a*e^2*f*h*i^3 - 88*b*e^2*f*h*i^3 - 12*a*e^3*i^4 +
 25*b*e^3*i^4)*x/(d*f^4) + 1/2*(b*f^4*h^4 - 4*b*e*f^3*h^3*i + 6*b*e^2*f^2*
h^2*i^2 - 4*b*e^3*f*h*i^3 + b*e^4*i^4)*log(c*f*x + c*e)^2/(d*f^5) + 1/12*(
12*a*f^4*h^4 - 48*a*e*f^3*h^3*i + 48*b*e*f^3*h^3*i + 72*a*e^2*f^2*h^2*i^2
- 108*b*e^2*f^2*h^2*i^2 - 48*a*e^3*f*h*i^3 + 88*b*e^3*f*h*i^3 + 12*a*e^4*i
^4 - 25*b*e^4*i^4)*log(f*x + e)/(d*f^5)
```

3.175. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.175.9 Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.10

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = x^3\left(\frac{i^3\,(12\,a\,f\,h+b\,e\,i-4\,b\,f\,h)}{9\,d\,f^2}-\frac{e\,i^4\,(4\,a-b)}{12\,d\,f^2}\right) - x^2\left(\frac{e\left(\frac{i^3\,(12\,a\,f\,h+b\,e\,i-4\,b\,f\,h)}{3\,d\,f^2}-\frac{e\,i^4\,(4\,a-b)}{4\,d\,f^2}\right)}{2\,f} - \frac{i^2\,(12\,a\,f^2\,h^2-b\,e^2\,i^2-6\,b\,f^2\,h^2+4\,b\,e\,f\,h\,i)}{4\,d\,f^3}\right) + x\left(\frac{12\,b\,e^3\,i^4+48\,a\,f^3\,h^3\,i-48\,b\,f^3\,h^3\,i-48\,b\,e^2\,f\,h\,i^3+72\,b\,e\,f^2\,h^2\,i^2}{12\,d\,f^4} + \frac{e\left(\frac{e\left(\frac{i^3\,(12\,a\,f\,h+b\,e\,i-4\,b\,f\,h)}{3\,d\,f^2}-\frac{e\,i^4\,(4\,a-b)}{4\,d\,f^2}\right)}{f}-\frac{i^2\,(12\,a\,f^2\,h^2-b\,e^2\,i^2-6\,b\,f^2\,h^2+4\,b\,e\,f\,h\,i)}{2\,d\,f^3}\right)}{f}\right) + f\,\ln\left(c\,(e+f\,x)\right)\left(\frac{b\,i^4\,x^4}{4\,d\,f^2}+\frac{b\,i^2\,x^2\,(e^2\,i^2-4\,e\,f\,h\,i+6\,f^2\,h^2)}{2\,d\,f^4}-\frac{b\,i^3\,x^3\,(e\,i-4\,f\,h)}{3\,d\,f^3}-\frac{b\,i\,x\,(e^3\,i^3-4\,e^2\,f\,h\,i^2+6\,e\,f^2\,h^2\,i-4\,f^3\,h^3)}{d\,f^5}\right) + \frac{\ln\left(e+f\,x\right)\,(12\,a\,e^4\,i^4+12\,a\,f^4\,h^4-25\,b\,e^4\,i^4-48\,a\,e\,f^3\,h^3\,i-48\,a\,e^3\,f\,h\,i^3+48\,b\,e\,f^3\,h^3\,i+88\,b\,e^3\,f\,h}{12\,d\,f^5} + \frac{b\ln\left(c\,(e+f\,x)\right)^2\,(e^4\,i^4-4\,e^3\,f\,h\,i^3+6\,e^2\,f^2\,h^2\,i^2-4\,e\,f^3\,h^3\,i+f^4\,h^4)}{2\,d\,f^5} + \frac{i^4\,x^4\,(4\,a-b)}{16\,d\,f}$$

input
```
int(((h + i*x)^4*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

output
```
x^3*((i^3*(12*a*f*h + b*e*i - 4*b*f*h))/(9*d*f^2) - (e*i^4*(4*a - b))/(12*
d*f^2)) - x^2*((e*((i^3*(12*a*f*h + b*e*i - 4*b*f*h))/(3*d*f^2) - (e*i^4*(
4*a - b))/(4*d*f^2)))/(2*f) - (i^2*(12*a*f^2*h^2 - b*e^2*i^2 - 6*b*f^2*h^2
 + 4*b*e*f*h*i))/(4*d*f^3)) + x*((12*b*e^3*i^4 + 48*a*f^3*h^3*i - 48*b*f^3
*h^3*i - 48*b*e^2*f*h*i^3 + 72*b*e*f^2*h^2*i^2)/(12*d*f^4) + (e*((e*((i^3*
(12*a*f*h + b*e*i - 4*b*f*h))/(3*d*f^2) - (e*i^4*(4*a - b))/(4*d*f^2)))/f
- (i^2*(12*a*f^2*h^2 - b*e^2*i^2 - 6*b*f^2*h^2 + 4*b*e*f*h*i))/(2*d*f^3)))
/f) + f*log(c*(e + f*x))*((b*i^4*x^4)/(4*d*f^2) + (b*i^2*x^2*(e^2*i^2 + 6*
f^2*h^2 - 4*e*f*h*i))/(2*d*f^4) - (b*i^3*x^3*(e*i - 4*f*h))/(3*d*f^3) - (b
*i*x*(e^3*i^3 - 4*f^3*h^3 + 6*e*f^2*h^2*i - 4*e^2*f*h*i^2))/(d*f^5)) + (lo
g(e + f*x)*(12*a*e^4*i^4 + 12*a*f^4*h^4 - 25*b*e^4*i^4 - 48*a*e*f^3*h^3*i
- 48*a*e^3*f*h*i^3 + 48*b*e*f^3*h^3*i + 88*b*e^3*f*h*i^3 + 72*a*e^2*f^2*h^
2*i^2 - 108*b*e^2*f^2*h^2*i^2))/(12*d*f^5) + (b*log(c*(e + f*x))^2*(e^4*i^
4 + f^4*h^4 + 6*e^2*f^2*h^2*i^2 - 4*e*f^3*h^3*i - 4*e^3*f*h*i^3))/(2*d*f^5
) + (i^4*x^4*(4*a - b))/(16*d*f)
```

3.175. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.176 $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.176.1 Optimal result

Integrand size = 30, antiderivative size = 244

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx = -\frac{3bi(fh-ei)^2x}{df^3}-\frac{3bi^2(fh-ei)(e+fx)^2}{4df^4}-\frac{bi^3(e+fx)^3}{9df^4}-\frac{b(fh-ei)^3\log^2(e+fx)}{2df^4}+\frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}+\frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}+\frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4}+\frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4}$$

output
```
-3*b*i*(-e*i+f*h)^2*x/d/f^3-3/4*b*i^2*(-e*i+f*h)*(f*x+e)^2/d/f^4-1/9*b*i^3
*(f*x+e)^3/d/f^4-1/2*b*(-e*i+f*h)^3*ln(f*x+e)^2/d/f^4+3*i*(-e*i+f*h)^2*(f*
x+e)*(a+b*ln(c*(f*x+e)))/d/f^4+3/2*i^2*(-e*i+f*h)*(f*x+e)^2*(a+b*ln(c*(f*x
+e)))/d/f^4+1/3*i^3*(f*x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^4+(-e*i+f*h)^3*ln(f*
x+e)*(a+b*ln(c*(f*x+e)))/d/f^4
```

3.176.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.54

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= \frac{18a^2f^3h^3 - 54a^2efh^2i + 54a^2e^2fhi^2 - 18a^2e^3i^3 + 108abf^3h^2ix - 108b^2f^3h^2ix - 108abef^2hi^2x + 162b^2e\cdots}{}$$

input
```
Integrate[((h + i*x)^3*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(18*a^2*f^3*h^3 - 54*a^2*e*f^2*h^2*i + 54*a^2*e^2*f*h*i^2 - 18*a^2*e^3*i^3
 + 108*a*b*f^3*h^2*i*x - 108*b^2*f^3*h^2*i*x - 108*a*b*e*f^2*h*i^2*x + 162
*b^2*e*f^2*h*i^2*x + 36*a*b*e^2*f*i^3*x - 66*b^2*e^2*f*i^3*x + 54*a*b*f^3*
h*i^2*x^2 - 27*b^2*f^3*h*i^2*x^2 - 18*a*b*e*f^2*i^3*x^2 + 15*b^2*e*f^2*i^3
*x^2 + 12*a*b*f^3*i^3*x^3 - 4*b^2*f^3*i^3*x^3 + 6*b^2*e^2*i^2*(-9*f*h + 5*
e*i)*Log[e + f*x] + 6*b*(6*a*(f*h - e*i)^3 + b*i*(6*e^3*i^2 + 6*e^2*f*i*(-
3*h + i*x) + 3*e*f^2*(6*h^2 - 6*h*i*x - i^2*x^2) + f^3*x*(18*h^2 + 9*h*i*x
 + 2*i^2*x^2)))*Log[c*(e + f*x)] + 18*b^2*(f*h - e*i)^3*Log[c*(e + f*x)]^2
)/(36*b*d*f^4)
```

3.176.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$\Big\downarrow \text{2858}$$
$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^3(a+b\log(c(e+fx)))}{df^3(e+fx)}d(e+fx)}{f}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{df^4}$$

$$\Big\downarrow \text{2772}$$

$$-b\int \left(\frac{1}{3}(e+fx)^2i^3+\frac{3}{2}(fh-ei)(e+fx)i^2+3(fh-ei)^2i+\frac{(fh-ei)^3\log(e+fx)}{e+fx}\right)d(e+fx)+\frac{3}{2}i^2(e+fx)^2(fh-ei$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}i^2(e+fx)^2(fh-ei)(a+b\log(c(e+fx)))+(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))+3i(e+fx)(fh-ei)$$

input
```
Int[((h + i*x)^3*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(-(b*(3*i*(f*h - e*i)^2*(e + f*x) + (3*i^2*(f*h - e*i)*(e + f*x)^2)/4 + (i
^3*(e + f*x)^3)/9 + ((f*h - e*i)^3*Log[e + f*x]^2)/2)) + 3*i*(f*h - e*i)^2
*(e + f*x)*(a + b*Log[c*(e + f*x)]) + (3*i^2*(f*h - e*i)*(e + f*x)^2*(a +
b*Log[c*(e + f*x)]))/2 + (i^3*(e + f*x)^3*(a + b*Log[c*(e + f*x)]))/3 + (f
*h - e*i)^3*Log[e + f*x]*(a + b*Log[c*(e + f*x)]))/(d*f^4)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.176.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.46

method	result
norman	$\frac{bi(e^2i^2-3efhi+3f^2h^2)x\ln(c(fx+e))}{d f^3} - \frac{(6a e^3 i^3-18a e^2 f h i^2+18ae f^2h^2 i-6a f^3h^3-11b e^3i^3+27b e^2 f h i^2-18be f^2 h \ldots}{6d f^4}$
parts	$\frac{a\left(\frac{i\left(\frac{1}{3}f^2i^2x^3-\frac{1}{2}efi^2x^2+\frac{3}{2}f^2hix^2+xe^2i^2-3xefhi+3xf^2h^2\right)}{f^3}+\frac{\left(-e^3i^3+3e^2fhi^2-3ef^2h^2i+f^3h^3\right)\ln(fx+e)}{f^4}\right)}{d}+\frac{b\left(-\frac{ce^3i \ldots}{}\right.}{}$
risch	$-\frac{b\ln(c(fx+e))^2e^3i^3}{2df^4}+\frac{3b\ln(c(fx+e))^2e^2hi^2}{2df^3}-\frac{3b\ln(c(fx+e))^2eh^2i}{2df^2}+\frac{b\ln(c(fx+e))^2h^3}{2df}+\frac{bix(2f^2i^2x^2-3ef \ldots}{}$
parallelrisch	$\frac{-108x\ln(c(fx+e))bef^2hi^2+12af^3i^3x^3-4bf^3i^3x^3+108\ln(c(fx+e))ae^2fhi^2-108\ln(c(fx+e))aef^2h^2i-162\ln(c(fx \ldots}{}$
derivativedivides	$-\frac{cae^3i^3\ln(cfx+ce)}{f^3d}+\frac{3cae^2hi^2\ln(cfx+ce)}{f^2d}-\frac{3caeh^2i\ln(cfx+ce)}{fd}+\frac{cah^3\ln(cfx+ce)}{d}+\frac{3ae^2i^3(cfx+ce)}{f^3d}-\frac{6aehi^2(cfx+ce)}{f^2d}+\frac{3a \ldots}{}$
default	$-\frac{cae^3i^3\ln(cfx+ce)}{f^3d}+\frac{3cae^2hi^2\ln(cfx+ce)}{f^2d}-\frac{3caeh^2i\ln(cfx+ce)}{fd}+\frac{cah^3\ln(cfx+ce)}{d}+\frac{3ae^2i^3(cfx+ce)}{f^3d}-\frac{6aehi^2(cfx+ce)}{f^2d}+\frac{3a \ldots}{}$

input
```
int((i*x+h)^3*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
b*i*(e^2*i^2-3*e*f*h*i+3*f^2*h^2)/d/f^3*x*ln(c*(f*x+e))-1/6*(6*a*e^3*i^3-1
8*a*e^2*f*h*i^2+18*a*e*f^2*h^2*i-6*a*f^3*h^3-11*b*e^3*i^3+27*b*e^2*f*h*i^2
-18*b*e*f^2*h^2*i)/d/f^4*ln(c*(f*x+e))-1/2*b*(e^3*i^3-3*e^2*f*h*i^2+3*e*f^
2*h^2*i-f^3*h^3)/d/f^4*ln(c*(f*x+e))^2+1/6*i*(6*a*e^2*i^2-18*a*e*f*h*i+18*
a*f^2*h^2-11*b*e^2*i^2+27*b*e*f*h*i-18*b*f^2*h^2)/d/f^3*x-1/12*i^2*(6*a*e*
i-18*a*f*h-5*b*e*i+9*b*f*h)/d/f^2*x^2+1/9*i^3*(3*a-b)/d/f*x^3+1/3*b*i^3/d/
f*x^3*ln(c*(f*x+e))-1/2*b*i^2*(e*i-3*f*h)/d/f^2*x^2*ln(c*(f*x+e))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= \frac{4\,(3\,a-b)f^3i^3x^3+3\,(9\,(2\,a-b)f^3hi^2-(6\,a-5\,b)ef^2i^3)x^2+18\,(bf^3h^3-3\,bef^2h^2i+3\,be^2fhi^2-be^3i^3)\ldots}{}$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas"
)
```

output
```
1/36*(4*(3*a - b)*f^3*i^3*x^3 + 3*(9*(2*a - b)*f^3*h*i^2 - (6*a - 5*b)*e*f
^2*i^3)*x^2 + 18*(b*f^3*h^3 - 3*b*e*f^2*h^2*i + 3*b*e^2*f*h*i^2 - b*e^3*i^
3)*log(c*f*x + c*e)^2 + 6*(18*(a - b)*f^3*h^2*i - 9*(2*a - 3*b)*e*f^2*h*i^
2 + (6*a - 11*b)*e^2*f*i^3)*x + 6*(2*b*f^3*i^3*x^3 + 6*a*f^3*h^3 - 18*(a -
 b)*e*f^2*h^2*i + 9*(2*a - 3*b)*e^2*f*h*i^2 - (6*a - 11*b)*e^3*i^3 + 3*(3*
b*f^3*h*i^2 - b*e*f^2*i^3)*x^2 + 6*(3*b*f^3*h^2*i - 3*b*e*f^2*h*i^2 + b*e^
2*f*i^3)*x)*log(c*f*x + c*e))/(d*f^4)
```

3.176.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.75

$$\begin{aligned}
&\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx \\
&= x^3\left(\frac{ai^3}{3df}-\frac{bi^3}{9df}\right)+x^2\left(-\frac{aei^3}{2df^2}+\frac{3ahi^2}{2df}+\frac{5bei^3}{12df^2}-\frac{3bhi^2}{4df}\right) \\
&\quad +x\left(\frac{ae^2i^3}{df^3}-\frac{3aehi^2}{df^2}+\frac{3ah^2i}{df}-\frac{11be^2i^3}{6df^3}+\frac{9behi^2}{2df^2}-\frac{3bh^2i}{df}\right) \\
&\quad +\frac{(6be^2i^3x-18befhi^2x-3befi^3x^2+18bf^2h^2ix+9bf^2hi^2x^2+2bf^2i^3x^3)\log\left(c(e+fx)\right)}{6df^3} \\
&\quad +\frac{(-be^3i^3+3be^2fhi^2-3bef^2h^2i+bf^3h^3)\log\left(c(e+fx)\right)^2}{2df^4} \\
&\quad -\frac{(6ae^3i^3-18ae^2fhi^2+18aef^2h^2i-6af^3h^3-11be^3i^3+27be^2fhi^2-18bef^2h^2i)\log\left(e+fx\right)}{6df^4}
\end{aligned}$$

input
```
integrate((i*x+h)**3*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

output
```
x**3*(a*i**3/(3*d*f) - b*i**3/(9*d*f)) + x**2*(-a*e*i**3/(2*d*f**2) + 3*a*
h*i**2/(2*d*f) + 5*b*e*i**3/(12*d*f**2) - 3*b*h*i**2/(4*d*f)) + x*(a*e**2*
i**3/(d*f**3) - 3*a*e*h*i**2/(d*f**2) + 3*a*h**2*i/(d*f) - 11*b*e**2*i**3/
(6*d*f**3) + 9*b*e*h*i**2/(2*d*f**2) - 3*b*h**2*i/(d*f)) + (6*b*e**2*i**3*
x - 18*b*e*f*h*i**2*x - 3*b*e*f*i**3*x**2 + 18*b*f**2*h**2*i*x + 9*b*f**2*
h*i**2*x**2 + 2*b*f**2*i**3*x**3)*log(c*(e + f*x))/(6*d*f**3) + (-b*e**3*i
**3 + 3*b*e**2*f*h*i**2 - 3*b*e*f**2*h**2*i + b*f**3*h**3)*log(c*(e + f*x)
)**2/(2*d*f**4) - (6*a*e**3*i**3 - 18*a*e**2*f*h*i**2 + 18*a*e*f**2*h**2*i
 - 6*a*f**3*h**3 - 11*b*e**3*i**3 + 27*b*e**2*f*h*i**2 - 18*b*e*f**2*h**2*
i)*log(e + f*x)/(6*d*f**4)
```

3.176.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(234) = 468$.

Time = 0.23 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.21

$$
\begin{aligned}
&\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx \\
&= 3\,bh^2i\left(\frac{x}{df}-\frac{e\log\left(fx+e\right)}{df^2}\right)\log\left(cfx+ce\right) \\
&\quad -\frac{1}{6}\,bi^3\left(\frac{6\,e^3\log\left(fx+e\right)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)\log\left(cfx+ce\right) \\
&\quad +\frac{3}{2}\,bhi^2\left(\frac{2\,e^2\log\left(fx+e\right)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log\left(cfx+ce\right) \\
&\quad -\frac{1}{2}\,bh^3\left(\frac{2\,\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}-\frac{\log\left(fx+e\right)^2+2\,\log\left(fx+e\right)\log\left(c\right)}{df}\right) \\
&\quad +3\,ah^2i\left(\frac{x}{df}-\frac{e\log\left(fx+e\right)}{df^2}\right)-\frac{1}{6}\,ai^3\left(\frac{6\,e^3\log\left(fx+e\right)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right) \\
&\quad +\frac{3}{2}\,ahi^2\left(\frac{2\,e^2\log\left(fx+e\right)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)+\frac{bh^3\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df} \\
&\quad +\frac{ah^3\log\left(dfx+de\right)}{df}+\frac{3\left(e\log\left(fx+e\right)^2-2\,fx+2\,e\log\left(fx+e\right)\right)bh^2i}{2\,df^2} \\
&\quad -\frac{3\left(f^2x^2+2\,e^2\log\left(fx+e\right)^2-6\,efx+6\,e^2\log\left(fx+e\right)\right)bhi^2}{4\,df^3} \\
&\quad -\frac{\left(4\,f^3x^3-15\,ef^2x^2-18\,e^3\log\left(fx+e\right)^2+66\,e^2fx-66\,e^3\log\left(fx+e\right)\right)bi^3}{36\,df^4}
\end{aligned}
$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima"
)
```

output
```
3*b*h^2*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - 1/6*b*i^3*
(6*e^3*log(f*x + e)/(d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))*l
og(c*f*x + c*e) + 3/2*b*h*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x
)/(d*f^2))*log(c*f*x + c*e) - 1/2*b*h^3*(2*log(c*f*x + c*e)*log(d*f*x + d*
e)/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(c))/(d*f)) + 3*a*h^2*i*(x/
(d*f) - e*log(f*x + e)/(d*f^2)) - 1/6*a*i^3*(6*e^3*log(f*x + e)/(d*f^4) -
(2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3/2*a*h*i^2*(2*e^2*log(f*x +
e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + b*h^3*log(c*f*x + c*e)*log(d*f*x +
 d*e)/(d*f) + a*h^3*log(d*f*x + d*e)/(d*f) + 3/2*(e*log(f*x + e)^2 - 2*f*x
 + 2*e*log(f*x + e))*b*h^2*i/(d*f^2) - 3/4*(f^2*x^2 + 2*e^2*log(f*x + e)^2
 - 6*e*f*x + 6*e^2*log(f*x + e))*b*h*i^2/(d*f^3) - 1/36*(4*f^3*x^3 - 15*e*
f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x + e))*b*i^3/
(d*f^4)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.51

$$
\begin{aligned}
\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx &= \frac{(3\,ai^3-bi^3)x^3}{9\,df} \\
&+ \frac{1}{6}\left(\frac{2\,bi^3x^3}{df}+\frac{3\,(3\,bfhi^2-bei^3)x^2}{df^2}+\frac{6\,(3\,bf^2h^2i-3\,befhi^2+be^2i^3)x}{df^3}\right)\log\left(cfx+ce\right) \\
&+ \frac{(18\,afhi^2-9\,bfhi^2-6\,aei^3+5\,bei^3)x^2}{12\,df^2} \\
&+ \frac{(18\,af^2h^2i-18\,bf^2h^2i-18\,aefhi^2+27\,befhi^2+6\,ae^2i^3-11\,be^2i^3)x}{6\,df^3} \\
&+ \frac{(bf^3h^3-3\,bef^2h^2i+3\,be^2fhi^2-be^3i^3)\log\left(cfx+ce\right)^2}{2\,df^4} \\
&+ \frac{(6\,af^3h^3-18\,aef^2h^2i+18\,bef^2h^2i+18\,ae^2fhi^2-27\,be^2fhi^2-6\,ae^3i^3+11\,be^3i^3)\log\left(fx+e\right)}{6\,df^4}
\end{aligned}
$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

output
```
1/9*(3*a*i^3 - b*i^3)*x^3/(d*f) + 1/6*(2*b*i^3*x^3/(d*f) + 3*(3*b*f*h*i^2
- b*e*i^3)*x^2/(d*f^2) + 6*(3*b*f^2*h^2*i - 3*b*e*f*h*i^2 + b*e^2*i^3)*x/(
d*f^3))*log(c*f*x + c*e) + 1/12*(18*a*f*h*i^2 - 9*b*f*h*i^2 - 6*a*e*i^3 +
5*b*e*i^3)*x^2/(d*f^2) + 1/6*(18*a*f^2*h^2*i - 18*b*f^2*h^2*i - 18*a*e*f*h
*i^2 + 27*b*e*f*h*i^2 + 6*a*e^2*i^3 - 11*b*e^2*i^3)*x/(d*f^3) + 1/2*(b*f^3
*h^3 - 3*b*e*f^2*h^2*i + 3*b*e^2*f*h*i^2 - b*e^3*i^3)*log(c*f*x + c*e)^2/(
d*f^4) + 1/6*(6*a*f^3*h^3 - 18*a*e*f^2*h^2*i + 18*b*e*f^2*h^2*i + 18*a*e^2
*f*h*i^2 - 27*b*e^2*f*h*i^2 - 6*a*e^3*i^3 + 11*b*e^3*i^3)*log(f*x + e)/(d*
f^4)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.61

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx = x^2\left(\frac{i^2\,(6\,a\,f\,h+b\,e\,i-3\,b\,f\,h)}{4\,d\,f^2}-\frac{e\,i^3\,(3\,a-b)}{6\,d\,f^2}\right)$$
$$-x\left(\frac{e\left(\frac{i^2\,(6\,a\,f\,h+b\,e\,i-3\,b\,f\,h)}{2\,d\,f^2}-\frac{e\,i^3\,(3\,a-b)}{3\,d\,f^2}\right)}{f}-\frac{i\,(3\,a\,f^2\,h^2-b\,e^2\,i^2-3\,b\,f^2\,h^2+3\,b\,e\,f\,h\,i)}{d\,f^3}\right)$$
$$+f\,\ln\left(c\,(e+f\,x)\right)\left(\frac{b\,i^3\,x^3}{3\,d\,f^2}+\frac{b\,i\,x\,(e^2\,i^2-3\,e\,f\,h\,i+3\,f^2\,h^2)}{d\,f^4}-\frac{b\,i^2\,x^2\,(e\,i-3\,f\,h)}{2\,d\,f^3}\right)$$
$$+\frac{\ln\left(e+f\,x\right)\,\left(6\,a\,f^3\,h^3-6\,a\,e^3\,i^3+11\,b\,e^3\,i^3-18\,a\,e\,f^2\,h^2\,i+18\,a\,e^2\,f\,h\,i^2+18\,b\,e\,f^2\,h^2\,i-27\,b\,e^2\,f\,h\,i^2\right)}{6\,d\,f^4}$$
$$+\frac{i^3\,x^3\,(3\,a-b)}{9\,d\,f}-\frac{b\ln\left(c\,(e+f\,x)\right)^2\,(e^3\,i^3-3\,e^2\,f\,h\,i^2+3\,e\,f^2\,h^2\,i-f^3\,h^3)}{2\,d\,f^4}$$

input
```
int(((h + i*x)^3*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

output
```
x^2*((i^2*(6*a*f*h + b*e*i - 3*b*f*h))/(4*d*f^2) - (e*i^3*(3*a - b))/(6*d*
f^2)) - x*((e*((i^2*(6*a*f*h + b*e*i - 3*b*f*h))/(2*d*f^2) - (e*i^3*(3*a -
 b))/(3*d*f^2)))/f - (i*(3*a*f^2*h^2 - b*e^2*i^2 - 3*b*f^2*h^2 + 3*b*e*f*h
*i))/(d*f^3)) + f*log(c*(e + f*x))*((b*i^3*x^3)/(3*d*f^2) + (b*i*x*(e^2*i^
2 + 3*f^2*h^2 - 3*e*f*h*i))/(d*f^4) - (b*i^2*x^2*(e*i - 3*f*h))/(2*d*f^3))
 + (log(e + f*x)*(6*a*f^3*h^3 - 6*a*e^3*i^3 + 11*b*e^3*i^3 - 18*a*e*f^2*h^
2*i + 18*a*e^2*f*h*i^2 + 18*b*e*f^2*h^2*i - 27*b*e^2*f*h*i^2))/(6*d*f^4) +
 (i^3*x^3*(3*a - b))/(9*d*f) - (b*log(c*(e + f*x))^2*(e^3*i^3 - f^3*h^3 +
3*e*f^2*h^2*i - 3*e^2*f*h*i^2))/(2*d*f^4)
```

3.177 $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.177.1 Optimal result

Integrand size = 30, antiderivative size = 157

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx = -\frac{b(4fh-3ei+fix)^2}{4df^3} - \frac{b(fh-ei)^2\log^2(e+fx)}{2df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3}$$

output
```
-1/4*b*(f*i*x-3*e*i+4*f*h)^2/d/f^3-1/2*b*(-e*i+f*h)^2*ln(f*x+e)^2/d/f^3+2*
i*(-e*i+f*h)*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^3+1/2*i^2*(f*x+e)^2*(a+b*ln(c
*(f*x+e)))/d/f^3+(-e*i+f*h)^2*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^3
```

3.177.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.36

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{2a^2f^2h^2 - 4a^2efhi + 2a^2e^2i^2 + 8abf^2hix - 8b^2f^2hix - 4abefi^2x + 6b^2efi^2x + 2abf^2i^2x^2 - b^2f^2i^2x^2 - 2}{}$$

input
```
Integrate[((h + i*x)^2*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(2*a^2*f^2*h^2 - 4*a^2*e*f*h*i + 2*a^2*e^2*i^2 + 8*a*b*f^2*h*i*x - 8*b^2*f
^2*h*i*x - 4*a*b*e*f*i^2*x + 6*b^2*e*f*i^2*x + 2*a*b*f^2*i^2*x^2 - b^2*f^2
*i^2*x^2 - 2*b^2*e^2*i^2*Log[e + f*x] + 2*b*(2*a*(f*h - e*i)^2 + b*i*(-2*e
^2*i + e*f*(4*h - 2*i*x) + f^2*x*(4*h + i*x)))*Log[c*(e + f*x)] + 2*b^2*(f
*h - e*i)^2*Log[c*(e + f*x)]^2)/(4*b*d*f^3)
```

3.177.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^2(a+b\log(c(e+fx)))}{df^2(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{df^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-b\int \left(\frac{\log(e+fx)(fh-ei)^2}{e+fx}+\frac{1}{2}i(4(fh-ei)+i(e+fx))\right)d(e+fx)+(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))+}{df^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))+2i(e+fx)(fh-ei)(a+b\log(c(e+fx)))+\frac{1}{2}i^2(e+fx)^2(a+b\log}{df^3}$$

input
```
Int[((h + i*x)^2*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(-(b*((4*(f*h - e*i) + i*(e + f*x))^2/4 + ((f*h - e*i)^2*Log[e + f*x]^2)/2
)) + 2*i*(f*h - e*i)*(e + f*x)*(a + b*Log[c*(e + f*x)]) + (i^2*(e + f*x)^2
*(a + b*Log[c*(e + f*x)]))/2 + (f*h - e*i)^2*Log[e + f*x]*(a + b*Log[c*(e
+ f*x)]))/(d*f^3)
```

3.177.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.177. $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.177.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
norman	$\frac{(2a\,e^2i^2-4aefhi+2a\,f^2h^2-3b\,e^2i^2+4befhi)\ln(c(fx+e))}{2d\,f^3}+\frac{b(e^2i^2-2efhi+f^2h^2)\ln(c(fx+e))^2}{2d\,f^3}-\frac{i(2aei-4afh-3}{2d\,f^2}$
parts	$\frac{a\left(\frac{i\left(\frac{1}{2}fi\,x^2-xei+2xfh\right)}{f^2}+\frac{\left(e^2i^2-2efhi+f^2h^2\right)\ln(fx+e)}{f^3}\right)}{d}+\frac{b\left(\frac{c\,e^2i^2\ln(cfx+ce)^2}{2f^2}-\frac{cehi\ln(cfx+ce)^2}{f}+\frac{c\,h^2\ln(cfx+ce)^2}{2}-\right.}{}$
risch	$\frac{b\ln(c(fx+e))^2e^2i^2}{2d\,f^3}-\frac{b\ln(c(fx+e))^2ehi}{d\,f^2}+\frac{b\ln(c(fx+e))^2h^2}{2df}-\frac{bix(-fix+2ei-4fh)\ln(c(fx+e))}{2d\,f^2}+\frac{a\,i^2x^2}{2df}-\frac{b}{}$
parallelrisch	$6a\,e^2i^2-11b\,e^2i^2-16aefhi+16befhi-4aef\,i^2x+8a\,f^2hix+6bef\,i^2x-8b\,f^2hix+2a\,f^2i^2x^2-b\,f^2i^2x^2-4x\ln(c(fx+e))be$
derivativedivides	$\frac{ca\,e^2i^2\ln(cfx+ce)}{f^2d}-\frac{2caehi\ln(cfx+ce)}{fd}+\frac{ca\,h^2\ln(cfx+ce)}{d}-\frac{2ae\,i^2(cfx+ce)}{f^2d}+\frac{2ahi(cfx+ce)}{fd}+\frac{a\,i^2(cfx+ce)^2}{2c\,f^2d}+\frac{cb\,e^2i^2\ln(cfx+ce)}{2f^2d}$
default	$\frac{ca\,e^2i^2\ln(cfx+ce)}{f^2d}-\frac{2caehi\ln(cfx+ce)}{fd}+\frac{ca\,h^2\ln(cfx+ce)}{d}-\frac{2ae\,i^2(cfx+ce)}{f^2d}+\frac{2ahi(cfx+ce)}{fd}+\frac{a\,i^2(cfx+ce)^2}{2c\,f^2d}+\frac{cb\,e^2i^2\ln(cfx+ce)}{2f^2d}$

input
```
int((i*x+h)^2*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a*e^2*i^2-4*a*e*f*h*i+2*a*f^2*h^2-3*b*e^2*i^2+4*b*e*f*h*i)/d/f^3*ln
(c*(f*x+e))+1/2*b*(e^2*i^2-2*e*f*h*i+f^2*h^2)/d/f^3*ln(c*(f*x+e))^2-1/2*i*
(2*a*e*i-4*a*f*h-3*b*e*i+4*b*f*h)/d/f^2*x+1/4*i^2*(2*a-b)/d/f*x^2+1/2*b*i^
2/d/f*x^2*ln(c*(f*x+e))-b*i*(e*i-2*f*h)/d/f^2*x*ln(c*(f*x+e))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{(2\,a-b)f^2i^2x^2+2\,(bf^2h^2-2\,befhi+be^2i^2)\log{(cfx+ce)}^2+2\,(4\,(a-b)f^2hi-(2\,a-3\,b)efi^2)x+2\,(b}{4\,df^3}$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas"
)
```

output
```
1/4*((2*a - b)*f^2*i^2*x^2 + 2*(b*f^2*h^2 - 2*b*e*f*h*i + b*e^2*i^2)*log(c
*f*x + c*e)^2 + 2*(4*(a - b)*f^2*h*i - (2*a - 3*b)*e*f*i^2)*x + 2*(b*f^2*i
^2*x^2 + 2*a*f^2*h^2 - 4*(a - b)*e*f*h*i + (2*a - 3*b)*e^2*i^2 + 2*(2*b*f^
2*h*i - b*e*f*i^2)*x)*log(c*f*x + c*e))/(d*f^3)
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.44

$$\begin{aligned}
&\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx \\
&= x^2\left(\frac{ai^2}{2df}-\frac{bi^2}{4df}\right)+x\left(-\frac{aei^2}{df^2}+\frac{2ahi}{df}+\frac{3bei^2}{2df^2}-\frac{2bhi}{df}\right) \\
&\quad+\frac{(-2bei^2x+4bfhix+bfi^2x^2)\log\left(c(e+fx)\right)}{2df^2} \\
&\quad+\frac{(be^2i^2-2befhi+bf^2h^2)\log\left(c(e+fx)\right)^2}{2df^3} \\
&\quad+\frac{(2ae^2i^2-4aefhi+2af^2h^2-3be^2i^2+4befhi)\log\left(e+fx\right)}{2df^3}
\end{aligned}$$

input
```
integrate((i*x+h)**2*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

output
```
x**2*(a*i**2/(2*d*f) - b*i**2/(4*d*f)) + x*(-a*e*i**2/(d*f**2) + 2*a*h*i/(
d*f) + 3*b*e*i**2/(2*d*f**2) - 2*b*h*i/(d*f)) + (-2*b*e*i**2*x + 4*b*f*h*i
*x + b*f*i**2*x**2)*log(c*(e + f*x))/(2*d*f**2) + (b*e**2*i**2 - 2*b*e*f*h
*i + b*f**2*h**2)*log(c*(e + f*x))**2/(2*d*f**3) + (2*a*e**2*i**2 - 4*a*e*
f*h*i + 2*a*f**2*h**2 - 3*b*e**2*i**2 + 4*b*e*f*h*i)*log(e + f*x)/(2*d*f**
3)
```

3.177.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(151) = 302.

Time = 0.23 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.24

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= 2\,bhi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce)$$
$$+\frac{1}{2}\,bi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce)$$
$$-\frac{1}{2}\,bh^2\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right)$$
$$+2\,ahi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{1}{2}\,ai^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)$$
$$+\frac{bh^2\log(cfx+ce)\log(dfx+de)}{df}+\frac{ah^2\log(dfx+de)}{df}$$
$$+\frac{\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)bhi}{df^2}$$
$$-\frac{\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)bi^2}{4\,df^3}$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima"
)
```

output
```
2*b*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + 1/2*b*i^2*(2
*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x + c*e) - 1/
2*b*h^2*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*l
og(f*x + e)*log(c))/(d*f)) + 2*a*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) +
1/2*a*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + b*h^2*l
og(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a*h^2*log(d*f*x + d*e)/(d*f) + (e
*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*b*h*i/(d*f^2) - 1/4*(f^2*x^2 +
 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))*b*i^2/(d*f^3)
```

3.177. $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.177.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.33

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{1}{2}\left(\frac{bi^2x^2}{df}+\frac{2\,(2\,bfhi-bei^2)x}{df^2}\right)\log\left(cfx+ce\right)+\frac{(2\,ai^2-bi^2)x^2}{4\,df}$$
$$+\frac{(4\,afhi-4\,bfhi-2\,aei^2+3\,bei^2)x}{2\,df^2}+\frac{(bf^2h^2-2\,befhi+be^2i^2)\log\left(cfx+ce\right)^2}{2\,df^3}$$
$$+\frac{(2\,af^2h^2-4\,aefhi+4\,befhi+2\,ae^2i^2-3\,be^2i^2)\log\left(fx+e\right)}{2\,df^3}$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

output
```
1/2*(b*i^2*x^2/(d*f) + 2*(2*b*f*h*i - b*e*i^2)*x/(d*f^2))*log(c*f*x + c*e)
 + 1/4*(2*a*i^2 - b*i^2)*x^2/(d*f) + 1/2*(4*a*f*h*i - 4*b*f*h*i - 2*a*e*i^
2 + 3*b*e*i^2)*x/(d*f^2) + 1/2*(b*f^2*h^2 - 2*b*e*f*h*i + b*e^2*i^2)*log(c
*f*x + c*e)^2/(d*f^3) + 1/2*(2*a*f^2*h^2 - 4*a*e*f*h*i + 4*b*e*f*h*i + 2*a
*e^2*i^2 - 3*b*e^2*i^2)*log(f*x + e)/(d*f^3)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=x\left(\frac{i\,(2\,a\,f\,h+b\,e\,i-2\,b\,f\,h)}{d\,f^2}-\frac{e\,i^2\,(2\,a-b)}{2\,d\,f^2}\right)$$
$$+f\,\ln\left(c\,(e+f\,x)\right)\left(\frac{b\,i^2\,x^2}{2\,d\,f^2}-\frac{b\,i\,x\,(e\,i-2\,f\,h)}{d\,f^3}\right)$$
$$+\frac{\ln\left(e+f\,x\right)\,(2\,a\,e^2\,i^2+2\,a\,f^2\,h^2-3\,b\,e^2\,i^2-4\,a\,e\,f\,h\,i+4\,b\,e\,f\,h\,i)}{2\,d\,f^3}$$
$$+\frac{b\ln\left(c\,(e+f\,x)\right)^2\,(e^2\,i^2-2\,e\,f\,h\,i+f^2\,h^2)}{2\,d\,f^3}+\frac{i^2\,x^2\,(2\,a-b)}{4\,d\,f}$$

input
```
int(((h + i*x)^2*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

output
```
x*((i*(2*a*f*h + b*e*i - 2*b*f*h))/(d*f^2) - (e*i^2*(2*a - b))/(2*d*f^2))
+ f*log(c*(e + f*x))*((b*i^2*x^2)/(2*d*f^2) - (b*i*x*(e*i - 2*f*h))/(d*f^3
)) + (log(e + f*x)*(2*a*e^2*i^2 + 2*a*f^2*h^2 - 3*b*e^2*i^2 - 4*a*e*f*h*i
+ 4*b*e*f*h*i))/(2*d*f^3) + (b*log(c*(e + f*x))^2*(e^2*i^2 + f^2*h^2 - 2*e
*f*h*i))/(2*d*f^3) + (i^2*x^2*(2*a - b))/(4*d*f)
```

3.177. $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.178 $\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx$

3.178.1 Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{aix}{df}-\frac{bix}{df}+\frac{bi(e+fx)\log(c(e+fx))}{df^2}+\frac{(fh-ei)(a+b\log(c(e+fx)))^2}{2bdf^2}$$

output
```
a*i*x/d/f-b*i*x/d/f+b*i*(f*x+e)*ln(c*(f*x+e))/d/f^2+1/2*(-e*i+f*h)*(a+b*ln
(c*(f*x+e)))^2/b/d/f^2
```

3.178.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{2afix-2bfix+2bi(e+fx)\log(c(e+fx))+\frac{(fh-ei)(a+b\log(c(e+fx)))^2}{b}}{2df^2}$$

input
```
Integrate[((h + i*x)*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(2*a*f*i*x - 2*b*f*i*x + 2*b*i*(e + f*x)*Log[c*(e + f*x)] + ((f*h - e*i)*(
a + b*Log[c*(e + f*x)])^2)/b)/(2*d*f^2)
```

3.178.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2858, 27, 2788, 2009, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)(a+b\log(c(e+fx)))}{df(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{df^2}$$

$$\Big\downarrow \text{2788}$$

$$\frac{(fh-ei)\int \frac{a+b\log(c(e+fx))}{e+fx}d(e+fx)+i\int (a+b\log(c(e+fx)))d(e+fx)}{df^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fh-ei)\int \frac{a+b\log(c(e+fx))}{e+fx}d(e+fx)+i(a(e+fx)+b(e+fx)\log(c(e+fx))-b(e+fx))}{df^2}$$

$$\Big\downarrow \text{2738}$$

$$\frac{\frac{(fh-ei)(a+b\log(c(e+fx)))^2}{2b}+i(a(e+fx)+b(e+fx)\log(c(e+fx))-b(e+fx))}{df^2}$$

input
```
Int[((h + i*x)*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

output
```
(((f*h - e*i)*(a + b*Log[c*(e + f*x)])^2)/(2*b) + i*(a*(e + f*x) - b*(e +
f*x) + b*(e + f*x)*Log[c*(e + f*x)]))/(d*f^2)
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.178.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

method	result
norman	$\frac{i(a-b)x}{df}+\frac{bix\ln(c(fx+e))}{df}-\frac{(aei-afh-bei)\ln(c(fx+e))}{d\,f^{2}}-\frac{b(ei-fh)\ln(c(fx+e))^{2}}{2d\,f^{2}}$
parts	$\frac{a\left(\frac{xi}{f}+\frac{(-ei+fh)\ln(fx+e)}{f^{2}}\right)}{d}+\frac{b\left(-\frac{cei\ln(cfx+ce)^{2}}{2f}+\frac{ch\ln(cfx+ce)^{2}}{2}+\frac{i((cfx+ce)\ln(cfx+ce)-cfx-ce)}{f}\right)}{dcf}$
parallelrisch	$\frac{2x\ln(c(fx+e))b\,e^{2}fi-\ln(c(fx+e))^{2}b\,e^{3}i+\ln(c(fx+e))^{2}b\,e^{2}fh+2xa\,e^{2}fi-2xb\,e^{2}fi-2\ln(c(fx+e))a\,e^{3}i+2\ln(c(fx+e))}{2d\,e^{2}f^{2}}$
risch	$-\frac{b\ln(c(fx+e))^{2}ei}{2d\,f^{2}}+\frac{b\ln(c(fx+e))^{2}h}{2df}+\frac{bix\ln(c(fx+e))}{df}-\frac{\ln(fx+e)aei}{d\,f^{2}}+\frac{\ln(fx+e)ah}{df}+\frac{\ln(fx+e)bei}{d\,f^{2}}+\frac{ai}{df}$
derivativedivides	$\frac{-\frac{acei\ln(cfx+ce)}{fd}+\frac{ahc\ln(cfx+ce)}{d}+\frac{ai(cfx+ce)}{fd}-\frac{bcei\ln(cfx+ce)^{2}}{2fd}+\frac{bhc\ln(cfx+ce)^{2}}{2d}+\frac{bi((cfx+ce)\ln(cfx+ce)-cfx-ce)}{fd}}{cf}$
default	$\frac{-\frac{acei\ln(cfx+ce)}{fd}+\frac{ahc\ln(cfx+ce)}{d}+\frac{ai(cfx+ce)}{fd}-\frac{bcei\ln(cfx+ce)^{2}}{2fd}+\frac{bhc\ln(cfx+ce)^{2}}{2d}+\frac{bi((cfx+ce)\ln(cfx+ce)-cfx-ce)}{fd}}{cf}$

input
```
int((i*x+h)*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
i*(a-b)/d/f*x+b*i*x/d/f*ln(c*(f*x+e))-(a*e*i-a*f*h-b*e*i)/d/f^2*ln(c*(f*x+
e))-1/2*b*(e*i-f*h)/d/f^2*ln(c*(f*x+e))^2
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{2(a-b)fix+(bfh-bei)\log(cfx+ce)^2+2(bfix+afh-(a-b)ei)\log(cfx+ce)}{2\,df^2}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
```

output
```
1/2*(2*(a - b)*f*i*x + (b*f*h - b*e*i)*log(c*f*x + c*e)^2 + 2*(b*f*i*x + a
*f*h - (a - b)*e*i)*log(c*f*x + c*e))/(d*f^2)
```

3.178.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{bix\log(c(e+fx))}{df}+x\left(\frac{ai}{df}-\frac{bi}{df}\right) + \frac{(-bei+bfh)\log(c(e+fx))^2}{2df^2} - \frac{(aei-afh-bei)\log(e+fx)}{df^2}$$

input
```
integrate((i*x+h)*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

output
```
b*i*x*log(c*(e + f*x))/(d*f) + x*(a*i/(d*f) - b*i/(d*f)) + (-b*e*i + b*f*h
)*log(c*(e + f*x))**2/(2*d*f**2) - (a*e*i - a*f*h - b*e*i)*log(e + f*x)/(d
*f**2)
```

3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(77) = 154.

Time = 0.23 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.54

$$\begin{aligned}\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx &= bi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce)\\ &\quad-\frac{1}{2}bh\left(\frac{2\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\log(fx+e)\log(c)}{df}\right)\\ &\quad+ai\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{bh\log(cfx+ce)\log(dfx+de)}{df}\\ &\quad+\frac{ah\log(dfx+de)}{df}+\frac{\left(e\log(fx+e)^2-2fx+2e\log(fx+e)\right)bi}{2df^2}\end{aligned}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
```

output
```
b*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - 1/2*b*h*(2*log(c
*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(
c))/(d*f)) + a*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) + b*h*log(c*f*x + c*e)
*log(d*f*x + d*e)/(d*f) + a*h*log(d*f*x + d*e)/(d*f) + 1/2*(e*log(f*x + e)
^2 - 2*f*x + 2*e*log(f*x + e))*b*i/(d*f^2)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx &= \frac{bix\log(cfx+ce)}{df}+\frac{(ai-bi)x}{df}\\ &\quad+\frac{(bfh-bei)\log(cfx+ce)^2}{2df^2}\\ &\quad+\frac{(afh-aei+bei)\log(fx+e)}{df^2}\end{aligned}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

output
```
b*i*x*log(c*f*x + c*e)/(d*f) + (a*i - b*i)*x/(d*f) + 1/2*(b*f*h - b*e*i)*l
og(c*f*x + c*e)^2/(d*f^2) + (a*f*h - a*e*i + b*e*i)*log(f*x + e)/(d*f^2)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{2\,a\,f\,i\,x-2\,b\,f\,i\,x-b\,e\,i\ln\left(c\,e+c\,f\,x\right)^2+b\,f\,h\ln\left(c\,e+c\,f\,x\right)^2-2\,a\,e\,i\,\ln\left(e+f\,x\right)+2\,a\,f\,h\,\ln\left(e+f\,x\right)}{2\,d\,f^2}$$

input
```
int(((h + i*x)*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

output
```
(2*a*f*i*x - 2*b*f*i*x - b*e*i*log(c*e + c*f*x)^2 + b*f*h*log(c*e + c*f*x)
^2 - 2*a*e*i*log(e + f*x) + 2*a*f*h*log(e + f*x) + 2*b*e*i*log(e + f*x) +
2*b*f*i*x*log(c*e + c*f*x))/(2*d*f^2)
```

3.179 $\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx$

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx=\frac{(a+b\log(c(e+fx)))^2}{2bdf}$$

output
```
1/2*(a+b*ln(c*(f*x+e)))^2/b/d/f
```

3.179.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx=\frac{(a+b\log(c(e+fx)))^2}{2bdf}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^2/(2*b*d*f)
```

3.179.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2837, 27, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx$$

$$\Big\downarrow \; 2837$$

$$\frac{\int \frac{a+b\log(c(e+fx))}{d(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a+b\log(c(e+fx))}{e+fx}d(e+fx)}{df}$$

$$\Big\downarrow \; 2738$$

$$\frac{(a+b\log(c(e+fx)))^2}{2bdf}$$

input
```
Int[(a + b*Log[c*(e + f*x)])/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^2/(2*b*d*f)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.179.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{b\ln(c(fx+e))^2}{2df}+\frac{a\ln(fx+e)}{df}$	35
parallelrisch	$\frac{\ln(c(fx+e))^2bf+2\ln(c(fx+e))af}{2d\,f^2}$	35
parts	$\frac{b\ln(c(fx+e))^2}{2df}+\frac{a\ln(fx+e)}{df}$	35
norman	$\frac{a\ln(c(fx+e))}{df}+\frac{b\ln(c(fx+e))^2}{2df}$	37
derivativedivides	$\frac{\frac{ca\ln(cfx+ce)}{d}+\frac{cb\ln(cfx+ce)^2}{2d}}{cf}$	42
default	$\frac{\frac{ca\ln(cfx+ce)}{d}+\frac{cb\ln(cfx+ce)^2}{2d}}{cf}$	42

input
```
int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d/f*ln(c*(f*x+e))^2+a/d/f*ln(f*x+e)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx=\frac{b\log\left(cfx+ce\right)^2+2\,a\log\left(cfx+ce\right)}{2\,df}$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
```

output
```
1/2*(b*log(c*f*x + c*e)^2 + 2*a*log(c*f*x + c*e))/(d*f)
```

3.179.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{a\log{(de+dfx)}}{df} + \frac{b\log{(c(e+fx))}^2}{2df}$$

input
```
integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

output
```
a*log(d*e + d*f*x)/(d*f) + b*log(c*(e + f*x))**2/(2*d*f)
```

3.179.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx$$
$$= -\frac{1}{2}b\left(\frac{2\,\log{(cfx+ce)}\log{(dfx+de)}}{df} - \frac{\log{(fx+e)}^2 + 2\,\log{(fx+e)}\log{(c)}}{df}\right) + \frac{b\log{(cfx+ce)}\log{(dfx+de)}}{df} + \frac{a\log{(dfx+de)}}{df}$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
```

output
```
-1/2*b*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*lo
g(f*x + e)*log(c))/(d*f)) + b*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a*
log(d*f*x + d*e)/(d*f)
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{b\log{(cfx+ce)}^2}{2\,df} + \frac{a\log{(fx+e)}}{df}$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

output
```
1/2*b*log(c*f*x + c*e)^2/(d*f) + a*log(f*x + e)/(d*f)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx=\frac{b\ln\left(c\,e+c\,f\,x\right)^2+2\,a\,\ln\left(e+f\,x\right)}{2\,d\,f}$$

input
```
int((a + b*log(c*(e + f*x)))/(d*e + d*f*x),x)
```

output
```
(2*a*log(e + f*x) + b*log(c*e + c*f*x)^2)/(2*d*f)
```

3.180 $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx$

3.180.1 Optimal result

Integrand size = 30, antiderivative size = 87

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx = -\frac{(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{b\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

output
```
-(a+b*ln(c*(f*x+e)))*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)+b*polylog(2,(
e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)
```

3.180.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx = \frac{(a+b\log(c(e+fx)))\left(a+b\log(c(e+fx))-2b\log\left(\frac{f(h+ix)}{fh-ei}\right)\right)-2b^2\,\mathrm{PolyLog}\left(2,\frac{i(e+fx)}{-fh+ei}\right)}{2bd(fh-ei)}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)),x]
```

output
```
((a + b*Log[c*(e + f*x)])*(a + b*Log[c*(e + f*x)] - 2*b*Log[(f*(h + i*x))/
(f*h - e*i)]) - 2*b^2*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)])/(2*b*d*(f*
h - e*i))
```

3.180.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2858, 27, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(e+fx))}{(h+ix)(de+dfx)}dx$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{\int \frac{f(a+b\log(c(e+fx)))}{d(e+fx)\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)}d(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{d}$$

$$\Big\downarrow \text{ 2779}$$

$$\frac{\frac{b\int \frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)}{e+fx}d(e+fx)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{fh-ei}}{d}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{b\,\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{fh-ei}}{d}$$

input
```
Int[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)),x]
```

output
```
(-(((a + b*Log[c*(e + f*x)])*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(f*h - e*
i)) + (b*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))])/(f*h - e*i))/d
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.180.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(86) = 172$.

Time = 0.97 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.03

method	result
parts	$\frac{a\left(\frac{\ln(ix+h)}{ei-fh}-\frac{\ln(fx+e)}{ei-fh}\right)}{d}-\frac{b\ln(cfx+ce)^2}{2d(ei-fh)}+\frac{b\,\mathrm{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}+\frac{b\ln(cfx+ce)\ln\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right.}{d(ei-fh)}$
risch	$\frac{a\ln(ix+h)}{d(ei-fh)}-\frac{a\ln(fx+e)}{d(ei-fh)}-\frac{b\ln(cfx+ce)^2}{2d(ei-fh)}+\frac{b\,\mathrm{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}+\frac{b\ln(cfx+ce)\ln\left(\frac{-cei+hcf+i(cfx+c}{-cei+hcf}\right.}{d(ei-fh)}$
derivativedivides	$\frac{-\frac{cfa\ln(cfx+ce)}{d(ei-fh)}+\frac{cfa\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2fb\left(\frac{\ln(cfx+ce)^2}{2c(ei-fh)}-\frac{i\left(\frac{\mathrm{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{i}+\frac{\ln(cfx+ce)\ln\left(\frac{-cei+h}{-}\right.}{i}\right.}{c(ei-fh)}\right.}{d}}{cf}$
default	$\frac{-\frac{cfa\ln(cfx+ce)}{d(ei-fh)}+\frac{cfa\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2fb\left(\frac{\ln(cfx+ce)^2}{2c(ei-fh)}-\frac{i\left(\frac{\mathrm{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{i}+\frac{\ln(cfx+ce)\ln\left(\frac{-cei+h}{-}\right.}{i}\right.}{c(ei-fh)}\right.}{d}}{cf}$

input
```
int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x,method=_RETURNVERBOSE)
```

output
```
a/d*(1/(e*i-f*h)*ln(i*x+h)-1/(e*i-f*h)*ln(f*x+e))-1/2*b/d/(e*i-f*h)*ln(c*f
*x+c*e)^2+b/d/(e*i-f*h)*dilog((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))
+b/d/(e*i-f*h)*ln(c*f*x+c*e)*ln((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h
))
```

3.180.5 Fricas [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx=\int \frac{b\log((fx+e)c)+a}{(dfx+de)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x, algorithm="fricas")
```

output
```
integral((b*log(c*f*x + c*e) + a)/(d*f*i*x^2 + d*e*h + (d*f*h + d*e*i)*x),
 x)
```

3.180.6 Sympy [F]

$$\int \frac{a + b\log(c(e + fx))}{(de + dfx)(h + ix)}\,dx = \frac{\int \frac{a}{eh+eix+fhx+fix^2}\,dx + \int \frac{b\log(ce+cfx)}{eh+eix+fhx+fix^2}\,dx}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x)
```

output
```
(Integral(a/(e*h + e*i*x + f*h*x + f*i*x**2), x) + Integral(b*log(c*e + c*
f*x)/(e*h + e*i*x + f*h*x + f*i*x**2), x))/d
```

3.180.7 Maxima [F]

$$\int \frac{a + b\log(c(e + fx))}{(de + dfx)(h + ix)}\,dx = \int \frac{b\log((fx + e)c) + a}{(dfx + de)(ix + h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x, algorithm="maxima")
```

output
```
a*(log(f*x + e)/(d*f*h - d*e*i) - log(i*x + h)/(d*f*h - d*e*i)) + b*integr
ate((log(f*x + e) + log(c))/(d*f*i*x^2 + d*e*h + (f*h + e*i)*d*x), x)
```

3.180.8 Giac [F]

$$\int \frac{a + b\log(c(e + fx))}{(de + dfx)(h + ix)}\,dx = \int \frac{b\log((fx + e)c) + a}{(dfx + de)(ix + h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x, algorithm="giac")
```

output
```
integrate((b*log((f*x + e)*c) + a)/((d*f*x + d*e)*(i*x + h)), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx = \int \frac{a+b\,\ln\left(c\,(e+f\,x)\right)}{(h+i\,x)\,\left(d\,e+d\,f\,x\right)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))/((h + i*x)*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))/((h + i*x)*(d*e + d*f*x)), x)
```

3.181 $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx$

3.181.1 Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = -\frac{i(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^2(h+ix)} + \frac{bf\log(h+ix)}{d(fh-ei)^2} - \frac{f(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{bf\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}$$

output
```
-i*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/(-e*i+f*h)^2/(i*x+h)+b*f*ln(i*x+h)/d/(-e*
i+f*h)^2-f*(a+b*ln(c*(f*x+e)))*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^2+b
*f*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^2
```

3.181.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx$$
$$= \frac{\frac{2(fh-ei)(a+b\log(c(e+fx)))}{h+ix} + \frac{f(a+b\log(c(e+fx)))^2}{b} + 2bf(-\log(e+fx)+\log(h+ix)) - 2f(a+b\log(c(e+fx)))}{2d(fh-ei)^2}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^2),x]
```

output
```
((2*(f*h - e*i)*(a + b*Log[c*(e + f*x)]))/(h + i*x) + (f*(a + b*Log[c*(e +
 f*x)])^2)/b + 2*b*f*(-Log[e + f*x] + Log[h + i*x]) - 2*f*(a + b*Log[c*(e
+ f*x)])*Log[(f*(h + i*x))/(f*h - e*i)] - 2*b*f*PolyLog[2, (i*(e + f*x))/(
-(f*h) + e*i)])/(2*d*(f*h - e*i)^2)
```

3.181.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(e+fx))}{(h+ix)^2(de+dfx)}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{f^2(a+b\log(c(e+fx)))}{d(e+fx)\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^2}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{d}$$

$$\Big\downarrow \text{2789}$$

$$\frac{f\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{a+b\log(c(e+fx))}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{2751}$$

$$\frac{f\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\int \frac{1}{fh-ei+i(e+fx)}d(e+fx)}{fh-ei}\right)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{f\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\log(i(e+fx)-ei+fh)}{i(fh-ei)}\right)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \; 2779$$

$$\frac{f\left(\frac{\frac{b\int \frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)}{e+fx}d(e+fx)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\log(i(e+fx)-ei+fh)}{i(fh-ei)}\right)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \; 2838$$

$$\frac{f\left(\frac{\frac{b\,\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\log(i(e+fx)-ei+fh)}{i(fh-ei)}\right)}{fh-ei}\right)}{d}$$

input
```
Int[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^2),x]
```

output
```
(f*(-((i*(((e + f*x)*(a + b*Log[c*(e + f*x)]))/((f*h - e*i)*(f*h - e*i + i
*(e + f*x))) - (b*Log[f*h - e*i + i*(e + f*x)])/(i*(f*h - e*i))))/(f*h - e
*i)) + (-(((a + b*Log[c*(e + f*x)])*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(f
*h - e*i)) + (b*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))])/(f*h - e*i))/(f*
h - e*i)))/d
```

3.181.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.181.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(150) = 300$.

Time = 1.07 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.10

method	result
parts	$\frac{a\left(-\frac{1}{(ei-fh)(ix+h)}-\frac{f\ln(ix+h)}{(ei-fh)^2}+\frac{f\ln(fx+e)}{(ei-fh)^2}\right)}{d}+\frac{b\left(\frac{cf^2\ln(cfx+ce)^2}{2(ei-fh)^2}-\frac{cf^2 i\left(\frac{\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{i}+\frac{\ln(cfx+ce)\ln(}{}\right.}{(ei-fh)^2}\right.}{}$
risch	$-\frac{a}{d(ei-fh)(ix+h)}-\frac{af\ln(ix+h)}{d(ei-fh)^2}+\frac{af\ln(fx+e)}{d(ei-fh)^2}+\frac{bf\ln(cfx+ce)^2}{2d(ei-fh)^2}-\frac{bf\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)^2}-bf\ln($
derivativedivides	$\frac{c^3f^2a\left(\frac{1}{c(ei-fh)(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^2(ei-fh)^2}+\frac{\ln(cfx+ce)}{c^2(ei-fh)^2}\right)}{d}+\frac{c^3f^2b\left(\frac{i\left(\frac{\ln(cei-hcf-i(cfx+ce))}{c(ei-fh)i}+\frac{\ln(c}{c(ei-fh)(}\right.}{c(ei-fh)}\right.}{cf}$
default	$\frac{c^3f^2a\left(\frac{1}{c(ei-fh)(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^2(ei-fh)^2}+\frac{\ln(cfx+ce)}{c^2(ei-fh)^2}\right)}{d}+\frac{c^3f^2b\left(\frac{i\left(\frac{\ln(cei-hcf-i(cfx+ce))}{c(ei-fh)i}+\frac{\ln(c}{c(ei-fh)(}\right.}{c(ei-fh)}\right.}{cf}$

input
```
int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d*(-1/(e*i-f*h)/(i*x+h)-f/(e*i-f*h)^2*ln(i*x+h)+f/(e*i-f*h)^2*ln(f*x+e))
+b/d/c/f*(1/2*c*f^2/(e*i-f*h)^2*ln(c*f*x+c*e)^2-c*f^2/(e*i-f*h)^2*i*(dilog
((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))/i+ln(c*f*x+c*e)*ln((-c*e*i+h
*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))/i)+c^2*f^2/(e*i-f*h)*i*(1/c/(e*i-f*h)*
ln(-c*e*i+h*c*f+i*(c*f*x+c*e))/i-ln(c*f*x+c*e)*(c*f*x+c*e)/c/(e*i-f*h)/(-c
*e*i+h*c*f+i*(c*f*x+c*e))))
```

3.181.5 Fricas [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx=\int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="fricas"
)
```

output
```
integral((b*log(c*f*x + c*e) + a)/(d*f*i^2*x^3 + d*e*h^2 + (2*d*f*h*i + d*
e*i^2)*x^2 + (d*f*h^2 + 2*d*e*h*i)*x), x)
```

3.181.6 Sympy [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \frac{\int \frac{a}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx + \int \frac{b\log(ce+cfx)}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)**2,x)
```

output
```
(Integral(a/(e*h**2 + 2*e*h*i*x + e*i**2*x**2 + f*h**2*x + 2*f*h*i*x**2 +
f*i**2*x**3), x) + Integral(b*log(c*e + c*f*x)/(e*h**2 + 2*e*h*i*x + e*i**
2*x**2 + f*h**2*x + 2*f*h*i*x**2 + f*i**2*x**3), x))/d
```

3.181.7 Maxima [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \int \frac{b\log((fx+e)c)+a}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="maxima"
)
```

output
```
a*(f*log(f*x + e)/(d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) - f*log(i*x + h)/(
d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) + 1/(d*f*h^2 - d*e*h*i + (d*f*h*i - d
*e*i^2)*x)) + b*integrate((log(f*x + e) + log(c))/(d*f*i^2*x^3 + d*e*h^2 +
 (2*f*h*i + e*i^2)*d*x^2 + (f*h^2 + 2*e*h*i)*d*x), x)
```

3.181.8 Giac [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \int \frac{b\log((fx+e)c)+a}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="giac")
```

output
```
integrate((b*log((f*x + e)*c) + a)/((d*f*x + d*e)*(i*x + h)^2), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \int \frac{a+b\,\ln\left(c\,(e+f\,x)\right)}{(h+i\,x)^2\,(d\,e+d\,f\,x)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))/((h + i*x)^2*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))/((h + i*x)^2*(d*e + d*f*x)), x)
```

3.182 $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$

3.182.1 Optimal result

Integrand size = 30, antiderivative size = 250

$$\begin{aligned}\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx =& -\frac{bf}{2d(fh-ei)^2(h+ix)}-\frac{bf^2\log(e+fx)}{2d(fh-ei)^3}\\ &+\frac{a+b\log(c(e+fx))}{2d(fh-ei)(h+ix)^2}-\frac{fi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)}\\ &+\frac{3bf^2\log(h+ix)}{2d(fh-ei)^3}-\frac{f^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\\ &+\frac{bf^2\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\end{aligned}$$

output
```
-1/2*b*f/d/(-e*i+f*h)^2/(i*x+h)-1/2*b*f^2*ln(f*x+e)/d/(-e*i+f*h)^3+1/2*(a+
b*ln(c*(f*x+e)))/d/(-e*i+f*h)/(i*x+h)^2-f*i*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/
(-e*i+f*h)^3/(i*x+h)+3/2*b*f^2*ln(i*x+h)/d/(-e*i+f*h)^3-f^2*(a+b*ln(c*(f*x
+e)))*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^3+b*f^2*polylog(2,(e*i-f*h)/
i/(f*x+e))/d/(-e*i+f*h)^3
```

3.182.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$$
$$= \frac{\frac{(fh-ei)^2(a+b\log(c(e+fx)))}{(h+ix)^2}+\frac{2f(fh-ei)(a+b\log(c(e+fx)))}{h+ix}+\frac{f^2(a+b\log(c(e+fx)))^2}{b}+2bf^2(-\log(e+fx)+\log(h+ix))}{2d(}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^3),x]
```

output
```
(((f*h - e*i)^2*(a + b*Log[c*(e + f*x)]))/(h + i*x)^2 + (2*f*(f*h - e*i)*(
a + b*Log[c*(e + f*x)]))/(h + i*x) + (f^2*(a + b*Log[c*(e + f*x)])^2)/b +
2*b*f^2*(-Log[e + f*x] + Log[h + i*x]) - (b*f*(f*h - e*i + f*(h + i*x)*Log
[e + f*x] - f*(h + i*x)*Log[h + i*x]))/(h + i*x) - 2*f^2*(a + b*Log[c*(e +
 f*x)])*Log[(f*(h + i*x))/(f*h - e*i)] - 2*b*f^2*PolyLog[2, (i*(e + f*x))/
(-(f*h) + e*i)])/(2*d*(f*h - e*i)^3)
```

3.182.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(e+fx))}{(h+ix)^3(de+dfx)}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{f^3(a+b\log(c(e+fx)))}{d(e+fx)\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^3}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^3}d(e+fx)}{d}$$

$$\Big\downarrow \text{2789}$$

$$\frac{f^2\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}-\frac{i\int \frac{a+b\log(c(e+fx))}{(fh-ei+i(e+fx))^3}d(e+fx)}{fh-ei}\right)}{d}$$

$\downarrow$ 2756

$$\frac{f^2\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}-\frac{i\left(\frac{b\int \frac{1}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{2i}-\frac{a+b\log(c(e+fx))}{2i(i(e+fx)-ei+fh)^2}\right)}{fh-ei}\right)}{d}$$

$\downarrow$ 54

$$f^2\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}-\frac{i\left(\frac{b\int\left(-\frac{i}{(fh-ei)^2(fh-ei+i(e+fx))}-\frac{i}{(fh-ei)(fh-ei+i(e+fx))^2}+\frac{1}{(fh-ei)^2(e+fx)}\right)d(e+fx)}{2i}-\frac{a+b\log(c(e\cdots}{2i(i(e+fx)-\cdots}\right.}{fh-ei}\cdots\right.$$

d

$\downarrow$ 2009

$$\frac{f^2\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}-\frac{i\left(\frac{b\left(\frac{1}{(fh-ei)(i(e+fx)-ei+fh)}+\frac{\log(e+fx)}{(fh-ei)^2}-\frac{\log(i(e+fx)-ei+fh)}{(fh-ei)^2}\right)}{2i}-\frac{a+b\log(c(e+fx))}{2i(i(e+fx)-ei+fh)^2}\right)}{fh-ei}\right)}{d}$$

$\downarrow$ 2789

$$f^2\left(\frac{\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{a+b\log(c(e+fx))}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{1}{(fh-ei)(i(e+fx)-ei+fh)}+\frac{\log(e+fx)}{(fh-ei)^2}-\frac{\log(i(e+fx)-ei+fh)}{(fh-ei)^2}\right)}{2i}-\frac{a+b\,\cdots}{2i(i(e+\cdots}\right.}{fh-ei}\cdots\right.$$

d

$\downarrow$ 2751

$$f^2\left(\frac{\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\int\frac{1}{fh-ei+i(e+fx)}d(e+fx)}{fh-ei}\right)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{1}{(fh-ei)(i(e+fx)-ei+fh)}+\frac{\log(e+fx)}{(fh-ei)^2}-\text{lo}\cdots\right.}{2i}\right.}{fh-\cdots}\cdots\right.$$

d

$\downarrow$ 16

3.182. $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$

$$\frac{f^2\left(\frac{\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\log(i(e+fx)-ei+fh)}{i(fh-ei)}\right)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{1}{(fh-ei)(i(e+fx)-ei+fh)}+\frac{\log(e+fx)}{(fh-ei)^2}-\frac{\log(i(e+\cdots}{(f\cdots}\right.}{2i}\right.}{fh-ei}\right.}{d}$$

$\downarrow$ 2779

$$\frac{f^2\left(\frac{\frac{\frac{b\int \frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)}{e+fx}d(e+fx)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\log(i(e+fx)-ei+fh)}{i(fh-ei)}\right)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{1}{(fh-ei)(i(e+f\cdots}\right.}{}\right.}{}\right.}{d}$$

$\downarrow$ 2838

$$\frac{f^2\left(\frac{\frac{\frac{b\,\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b\log(i(e+fx)-ei+fh)}{i(fh-ei)}\right)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{1}{(fh-ei)(i(e+fx)-e\cdots}\right.}{}\right.}{}\right.}{d}$$

input
```
Int[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^3),x]
```

output
```
(f^2*(-((i*(-1/2*(a + b*Log[c*(e + f*x)])/(i*(f*h - e*i + i*(e + f*x))^2)
+ (b*(1/((f*h - e*i)*(f*h - e*i + i*(e + f*x))) + Log[e + f*x]/(f*h - e*i)
^2 - Log[f*h - e*i + i*(e + f*x)]/(f*h - e*i)^2))/(2*i)))/(f*h - e*i)) + (
-((i*(((e + f*x)*(a + b*Log[c*(e + f*x)]))/((f*h - e*i)*(f*h - e*i + i*(e
+ f*x))) - (b*Log[f*h - e*i + i*(e + f*x)])/(i*(f*h - e*i))))/(f*h - e*i))
 + (-(((a + b*Log[c*(e + f*x)])*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(f*h -
 e*i)) + (b*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))])/(f*h - e*i))/(f*h -
e*i))/(f*h - e*i)))/d
```

3.182. $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$

3.182.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

3.182. $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.182.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(241) = 482$.

Time = 1.20 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.04

method	result
parts	$\frac{a\left(-\frac{1}{2(ei-fh)(ix+h)^2}+\frac{f^2\ln(ix+h)}{(ei-fh)^3}+\frac{f}{(ei-fh)^2(ix+h)}-\frac{f^2\ln(fx+e)}{(ei-fh)^3}\right)}{d}+\frac{b\left(-\frac{c f^3\ln(cfx+ce)^2}{2(ei-fh)^3}+\frac{c^3f^3i\left(-\frac{\ln(-cei+hcf+i(c}{i}\right.}{}\right.}{}$
derivativedivides	$-\frac{c^4f^3a\left(\frac{\ln(cfx+ce)}{c^3(ei-fh)^3}+\frac{1}{c^2(ei-fh)^2(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^3(ei-fh)^3}+\frac{1}{2c(ei-fh)(cei-hcf-i(cfx+ce))^2}\right)}{d}-\frac{c^4f^3b}{}$
default	$-\frac{c^4f^3a\left(\frac{\ln(cfx+ce)}{c^3(ei-fh)^3}+\frac{1}{c^2(ei-fh)^2(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^3(ei-fh)^3}+\frac{1}{2c(ei-fh)(cei-hcf-i(cfx+ce))^2}\right)}{d}-\frac{c^4f^3b}{}$
risch	$-\frac{a}{2d(ei-fh)(ix+h)^2}+\frac{a f^2\ln(ix+h)}{d(ei-fh)^3}+\frac{af}{d(ei-fh)^2(ix+h)}-\frac{a f^2\ln(fx+e)}{d(ei-fh)^3}-\frac{b f^2\ln(cfx+ce)^2}{2d(ei-fh)^3}-\frac{3b f^2\ln(-c}{2c}$

input
```
int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d*(-1/2/(e*i-f*h)/(i*x+h)^2+f^2/(e*i-f*h)^3*ln(i*x+h)+f/(e*i-f*h)^2/(i*x
+h)-f^2/(e*i-f*h)^3*ln(f*x+e))+b/d/c/f*(-1/2*c*f^3/(e*i-f*h)^3*ln(c*f*x+c*
e)^2+c^3*f^3/(e*i-f*h)*i*(-1/2/c^2/(e*i-f*h)^2*(ln(-c*e*i+h*c*f+i*(c*f*x+c
*e))/i+c*(e*i-f*h)/i/(-c*e*i+h*c*f+i*(c*f*x+c*e)))+1/2*ln(c*f*x+c*e)*(-2*c
*e*i+2*h*c*f+i*(c*f*x+c*e))*(c*f*x+c*e)/(-c*e*i+h*c*f+i*(c*f*x+c*e))^2/c^2
/(e*i-f*h)^2)+c*f^3/(e*i-f*h)^3*i*(dilog((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*
e*i+c*f*h))/i+ln(c*f*x+c*e)*ln((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h)
)/i)-c^2*f^3/(e*i-f*h)^2*i*(1/c/(e*i-f*h)*ln(-c*e*i+h*c*f+i*(c*f*x+c*e))/i
-ln(c*f*x+c*e)*(c*f*x+c*e)/c/(e*i-f*h)/(-c*e*i+h*c*f+i*(c*f*x+c*e))))
```

3.182.5 Fricas [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx=\int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="fricas"
)
```

output
```
integral((b*log(c*f*x + c*e) + a)/(d*f*i^3*x^4 + d*e*h^3 + (3*d*f*h*i^2 +
d*e*i^3)*x^3 + 3*(d*f*h^2*i + d*e*h*i^2)*x^2 + (d*f*h^3 + 3*d*e*h^2*i)*x),
 x)
```

3.182.6 Sympy [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$$
$$=\frac{\int \frac{a}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}\,dx+\int \frac{b\log(ce+cfx)}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)**3,x)
```

output
```
(Integral(a/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**3 + f*h**
3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x) + Integral(b*lo
g(c*e + c*f*x)/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**3 + f*
h**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x))/d
```

3.182.7 Maxima [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = \int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="maxima"
)
```

output
```
1/2*(2*f^2*log(f*x + e)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*i^2 - d
*e^3*i^3) - 2*f^2*log(i*x + h)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*
i^2 - d*e^3*i^3) + (2*f*i*x + 3*f*h - e*i)/(d*f^2*h^4 - 2*d*e*f*h^3*i + d*
e^2*h^2*i^2 + (d*f^2*h^2*i^2 - 2*d*e*f*h*i^3 + d*e^2*i^4)*x^2 + 2*(d*f^2*h
^3*i - 2*d*e*f*h^2*i^2 + d*e^2*h*i^3)*x))*a + b*integrate((log(f*x + e) +
log(c))/(d*f*i^3*x^4 + d*e*h^3 + (3*f*h*i^2 + e*i^3)*d*x^3 + 3*(f*h^2*i +
e*h*i^2)*d*x^2 + (f*h^3 + 3*e*h^2*i)*d*x), x)
```

3.182.8 Giac [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = \int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="giac")
```

output
```
integrate((b*log((f*x + e)*c) + a)/((d*f*x + d*e)*(i*x + h)^3), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = \int \frac{a+b\,\ln\left(c\,(e+f\,x)\right)}{(h+i\,x)^3\,(d\,e+d\,f\,x)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))/((h + i*x)^3*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))/((h + i*x)^3*(d*e + d*f*x)), x)
```

3.183 $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.183.1 Optimal result

Integrand size = 32, antiderivative size = 579

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = -\frac{4abi(fh-ei)^3x}{df^4} + \frac{8b^2i(fh-ei)^3x}{df^4} + \frac{3b^2i^2(fh-ei)^2(e+fx)^2}{2df^5} + \frac{8b^2i^3(fh-ei)(e+fx)^3}{27df^5} + \frac{b^2i^4(e+fx)^4}{32df^5} + \frac{7b^2(fh-ei)^4\log^2(e+fx)}{12df^5} - \frac{4b^2i(fh-ei)^3(e+fx)\log(c(e+fx))}{df^5} - \frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} - \frac{3bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} - \frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5} - \frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5} - \frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5} + \frac{2i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2}{df^5} + \frac{i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^5} + \frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2} + \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df} + \frac{(fh-ei)^4(a+b\log(c(e+fx)))^3}{3bdf^5}$$

output
```
-4*a*b*i*(-e*i+f*h)^3*x/d/f^4+8*b^2*i*(-e*i+f*h)^3*x/d/f^4+3/2*b^2*i^2*(-e
*i+f*h)^2*(f*x+e)^2/d/f^5+8/27*b^2*i^3*(-e*i+f*h)*(f*x+e)^3/d/f^5+1/32*b^2
*i^4*(f*x+e)^4/d/f^5+7/12*b^2*(-e*i+f*h)^4*ln(f*x+e)^2/d/f^5-4*b^2*i*(-e*i
+f*h)^3*(f*x+e)*ln(c*(f*x+e))/d/f^5-4*b*i*(-e*i+f*h)^3*(f*x+e)*(a+b*ln(c*(
f*x+e)))/d/f^5-3*b*i^2*(-e*i+f*h)^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))/d/f^5-8/
9*b*i^3*(-e*i+f*h)*(f*x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^5-1/8*b*i^4*(f*x+e)^4
*(a+b*ln(c*(f*x+e)))/d/f^5-7/6*b*(-e*i+f*h)^4*ln(f*x+e)*(a+b*ln(c*(f*x+e))
)/d/f^5+2*i*(-e*i+f*h)^3*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/f^5+1/2*i^2*(-e*i
+f*h)^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))^2/d/f^5+1/3*(-e*i+f*h)*(i*x+h)^3*(a+
b*ln(c*(f*x+e)))^2/d/f^2+1/4*(i*x+h)^4*(a+b*ln(c*(f*x+e)))^2/d/f+1/3*(-e*i
+f*h)^4*(a+b*ln(c*(f*x+e)))^3/b/d/f^5
```

3.183.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.65

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{3456i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2+2592i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2+11}{}$$

input
```
Integrate[((h + i*x)^4*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

output
```
(3456*i*(f*h - e*i)^3*(e + f*x)*(a + b*Log[c*(e + f*x)])^2 + 2592*i^2*(f*h
 - e*i)^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2 + 1152*i^3*(f*h - e*i)*(e
 + f*x)^3*(a + b*Log[c*(e + f*x)])^2 + 216*i^4*(e + f*x)^4*(a + b*Log[c*(e
 + f*x)])^2 + (288*(f*h - e*i)^4*(a + b*Log[c*(e + f*x)])^3)/b - 6912*b*i*
(f*h - e*i)^3*((a - b)*f*x + b*(e + f*x)*Log[c*(e + f*x)]) + 1296*b*i^2*(f
*h - e*i)^2*(b*f*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])) +
 256*b*i^3*(f*h - e*i)*(b*f*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*
(a + b*Log[c*(e + f*x)])) + 27*b*i^4*(b*f*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x
^2 + f^3*x^3) - 4*(e + f*x)^4*(a + b*Log[c*(e + f*x)])))/(864*d*f^5)
```

3.183.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.45, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2858, 27, 2788, 2756, 2772, 2009, 2788, 2756, 2772, 2009, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^4(a+b\log(c(e+fx)))^2}{df^4(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)}{df^5}$$

$$\Big\downarrow \text{2788}$$

$$\frac{i\int(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))^2d(e+fx)+(fh-ei)\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx}{df^5}$$

$$\Big\downarrow \text{2756}$$

$$\frac{i\left(\frac{(i(e+fx)-ei+fh)^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\int \frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{2i}\right)+(fh-ei)\int \frac{(fh-ei+i(e+fx))^3(a+b\log(}{e+fx}}{df^5}$$

$$\Big\downarrow \text{2772}$$

$$i\left(\frac{(i(e+fx)-ei+fh)^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\left(-b\int\left(\frac{1}{4}(e+fx)^3i^4+\frac{4}{3}(fh-ei)(e+fx)^2i^3+3(fh-ei)^2(e+fx)i^2+4(fh-ei)^3i+\frac{(fh-ei)^4\log(e+f}{e+fx}\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(fh-ei)\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left(\frac{(i(e+fx)-ei+fh)^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b(\frac{4}{3}i^3(e+fx)^3(fh-ei)(a+}{}\right.$$

$\Big\downarrow$ 2788

$$(fh-ei)\left(i\int (fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^2d(e+fx)+(fh-ei)\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))}{e+fx}\right.$$

$\Big\downarrow$ 2756

$$(fh-ei)\left(i\left(\frac{(i(e+fx)-ei+fh)^3(a+b\log(c(e+fx)))^2}{3i}-\frac{2b\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{3i}\right)+(fh-ei)\int \frac{(fh-ei+i(e-}{}\right.$$

$\Big\downarrow$ 2772

$$(fh-ei)\left(i\left(\frac{(i(e+fx)-ei+fh)^3(a+b\log(c(e+fx)))^2}{3i}-\frac{2b\left(-b\int\left(\frac{1}{3}(e+fx)^2i^3+\frac{3}{2}(fh-ei)(e+fx)i^2+3(fh-ei)^2i+\frac{(fh-ei)^3\log(e+fx)}{e+fx}\right)d(e\right.}{}\right.\right.$$

$\Big\downarrow$ 2009

$$(fh-ei)\left((fh-ei)\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left(\frac{(i(e+fx)-ei+fh)^3(a+b\log(c(e+fx)))^2}{3i}-\frac{2b(\frac{3}{2}i^2(e+}{}\right.\right.$$

$\Big\downarrow$ 2788

$$(fh-ei)\left((fh-ei)\left(i\int (fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2d(e+fx)+(fh-ei)\int \frac{(fh-ei+i(e+fx))(a+b\log}{e+fx}\right.\right.$$

$\Big\downarrow$ 2767

$$(fh-ei)\left((fh-ei)\left((fh-ei)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\int\left(fh\left(1-\frac{ei}{fh}\right)(a+b\log(c(e+fx\right.\right.\right.\right.$$

$\Big\downarrow$ 2009

$$i\left(\frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\left(\log(e+fx)(a+b\log(c(e+fx)))(fh-ei)^4+4i(e+fx)(a+b\log(c(e+fx)))(fh-ei)^3+3i^2(e+fx)^2(a-\right.}{}\right.$$

$\Big\downarrow$ 2788

3.183. $\displaystyle\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

$$i\Bigg(\frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\big(\log(e+fx)(a+b\log(c(e+fx)))(fh-ei)^4+4i(e+fx)(a+b\log(c(e+fx)))(fh-ei)^3+3i^2(e+fx)^2(a-$$

$\downarrow$ 2733

$$i\Bigg(\frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\big(\log(e+fx)(a+b\log(c(e+fx)))(fh-ei)^4+4i(e+fx)(a+b\log(c(e+fx)))(fh-ei)^3+3i^2(e+fx)^2(a-$$

$\downarrow$ 2009

$$i\Bigg(\frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\big(\log(e+fx)(a+b\log(c(e+fx)))(fh-ei)^4+4i(e+fx)(a+b\log(c(e+fx)))(fh-ei)^3+3i^2(e+fx)^2(a-$$

$\downarrow$ 2739

$$i\Bigg(\frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\big(\log(e+fx)(a+b\log(c(e+fx)))(fh-ei)^4+4i(e+fx)(a+b\log(c(e+fx)))(fh-ei)^3+3i^2(e+fx)^2(a-$$

$\downarrow$ 15

$$i\Bigg(\frac{(fh-ei+i(e+fx))^4(a+b\log(c(e+fx)))^2}{4i}-\frac{b\big(\log(e+fx)(a+b\log(c(e+fx)))(fh-ei)^4+4i(e+fx)(a+b\log(c(e+fx)))(fh-ei)^3+3i^2(e+fx)^2(a-$$

input
```
Int[((h + i*x)^4*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

3.183. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

output
```
(i*(((f*h - e*i + i*(e + f*x))^4*(a + b*Log[c*(e + f*x)])^2)/(4*i) - (b*(-
(b*(4*i*(f*h - e*i)^3*(e + f*x) + (3*i^2*(f*h - e*i)^2*(e + f*x)^2)/2 + (4
*i^3*(f*h - e*i)*(e + f*x)^3)/9 + (i^4*(e + f*x)^4)/16 + ((f*h - e*i)^4*Lo
g[e + f*x]^2)/2)) + 4*i*(f*h - e*i)^3*(e + f*x)*(a + b*Log[c*(e + f*x)]) +
 3*i^2*(f*h - e*i)^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)]) + (4*i^3*(f*h -
e*i)*(e + f*x)^3*(a + b*Log[c*(e + f*x)]))/3 + (i^4*(e + f*x)^4*(a + b*Log
[c*(e + f*x)]))/4 + (f*h - e*i)^4*Log[e + f*x]*(a + b*Log[c*(e + f*x)])))/
(2*i)) + (f*h - e*i)*(i*(((f*h - e*i + i*(e + f*x))^3*(a + b*Log[c*(e + f*
x)])^2)/(3*i) - (2*b*(-(b*(3*i*(f*h - e*i)^2*(e + f*x) + (3*i^2*(f*h - e*i
)*(e + f*x)^2)/4 + (i^3*(e + f*x)^3)/9 + ((f*h - e*i)^3*Log[e + f*x]^2)/2)
) + 3*i*(f*h - e*i)^2*(e + f*x)*(a + b*Log[c*(e + f*x)]) + (3*i^2*(f*h - e
*i)*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))/2 + (i^3*(e + f*x)^3*(a + b*Log[
c*(e + f*x)]))/3 + (f*h - e*i)^3*Log[e + f*x]*(a + b*Log[c*(e + f*x)])))/(
3*i)) + (f*h - e*i)*(i*(-2*a*b*(f*h - e*i)*(e + f*x) + 2*b^2*(f*h - e*i)*(
e + f*x) + (b^2*i*(e + f*x)^2)/4 - 2*b^2*(f*h - e*i)*(e + f*x)*Log[c*(e +
f*x)] - (b*i*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))/2 + (f*h - e*i)*(e + f*
x)*(a + b*Log[c*(e + f*x)])^2 + (i*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2)
/2) + (f*h - e*i)*(((f*h - e*i)*(a + b*Log[c*(e + f*x)])^3)/(3*b) + i*((e
+ f*x)*(a + b*Log[c*(e + f*x)])^2 - 2*b*(a*(e + f*x) - b*(e + f*x) + b*(e
+ f*x)*Log[c*(e + f*x)]))))))/(d*f^5)
```

3.183.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.183. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.183. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.183.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(557) = 1114.

Time = 0.92 (sec) , antiderivative size = 1135, normalized size of antiderivative = 1.96

method	result	size
norman	Expression too large to display	1135
risch	Expression too large to display	1475
parts	Expression too large to display	1541
derivativedivides	Expression too large to display	1891
default	Expression too large to display	1891
parallelrisch	Expression too large to display	1912

input
```
int((i*x+h)^4*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
1/72*(72*a^2*e^4*i^4-288*a^2*e^3*f*h*i^3+432*a^2*e^2*f^2*h^2*i^2-288*a^2*e
*f^3*h^3*i+72*a^2*f^4*h^4-300*a*b*e^4*i^4+1056*a*b*e^3*f*h*i^3-1296*a*b*e^
2*f^2*h^2*i^2+576*a*b*e*f^3*h^3*i+415*b^2*e^4*i^4-1360*b^2*e^3*f*h*i^3+151
2*b^2*e^2*f^2*h^2*i^2-576*b^2*e*f^3*h^3*i)/d/f^5*ln(c*(f*x+e))+1/12*b*(12*
a*e^4*i^4-48*a*e^3*f*h*i^3+72*a*e^2*f^2*h^2*i^2-48*a*e*f^3*h^3*i+12*a*f^4*
h^4-25*b*e^4*i^4+88*b*e^3*f*h*i^3-108*b*e^2*f^2*h^2*i^2+48*b*e*f^3*h^3*i)/
d/f^5*ln(c*(f*x+e))^2+1/3*b^2*(e^4*i^4-4*e^3*f*h*i^3+6*e^2*f^2*h^2*i^2-4*e
*f^3*h^3*i+f^4*h^4)/d/f^5*ln(c*(f*x+e))^3-1/72*i*(72*a^2*e^3*i^3-288*a^2*e
^2*f*h*i^2+432*a^2*e*f^2*h^2*i-288*a^2*f^3*h^3-300*a*b*e^3*i^3+1056*a*b*e^
2*f*h*i^2-1296*a*b*e*f^2*h^2*i+576*a*b*f^3*h^3+415*b^2*e^3*i^3-1360*b^2*e^
2*f*h*i^2+1512*b^2*e*f^2*h^2*i-576*b^2*f^3*h^3)/d/f^4*x+1/144*i^2*(72*a^2*
e^2*i^2-288*a^2*e*f*h*i+432*a^2*f^2*h^2-156*a*b*e^2*i^2+480*a*b*e*f*h*i-43
2*a*b*f^2*h^2+115*b^2*e^2*i^2-304*b^2*e*f*h*i+216*b^2*f^2*h^2)/d/f^3*x^2-1
/216*i^3*(72*a^2*e*i-288*a^2*f*h-84*a*b*e*i+192*a*b*f*h+37*b^2*e*i-64*b^2*
f*h)/f^2/d*x^3+1/32*i^4*(8*a^2-4*a*b+b^2)/d/f*x^4+1/4*b^2*i^4/d/f*x^4*ln(c
*(f*x+e))^2-1/6*b*i*(12*a*e^3*i^3-48*a*e^2*f*h*i^2+72*a*e*f^2*h^2*i-48*a*f
^3*h^3-25*b*e^3*i^3+88*b*e^2*f*h*i^2-108*b*e*f^2*h^2*i+48*b*f^3*h^3)/d/f^4
*x*ln(c*(f*x+e))+1/12*b*i^2*(12*a*e^2*i^2-48*a*e*f*h*i+72*a*f^2*h^2-13*b*e
^2*i^2+40*b*e*f*h*i-36*b*f^2*h^2)/d/f^3*x^2*ln(c*(f*x+e))-1/18*b*i^3*(12*a
*e*i-48*a*f*h-7*b*e*i+16*b*f*h)/d/f^2*x^3*ln(c*(f*x+e))+1/8*b*i^4*(4*a-...
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.62

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{27\,(8\,a^2-4\,ab+b^2)f^4i^4x^4+4\,(32\,(9\,a^2-6\,ab+2\,b^2)f^4hi^3-(72\,a^2-84\,ab+37\,b^2)ef^3i^4)x^3+288\,(b^2f^4}{}$$

input
```
integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="frica
s")
```

output
```
1/864*(27*(8*a^2 - 4*a*b + b^2)*f^4*i^4*x^4 + 4*(32*(9*a^2 - 6*a*b + 2*b^2
)*f^4*h*i^3 - (72*a^2 - 84*a*b + 37*b^2)*e*f^3*i^4)*x^3 + 288*(b^2*f^4*h^4
 - 4*b^2*e*f^3*h^3*i + 6*b^2*e^2*f^2*h^2*i^2 - 4*b^2*e^3*f*h*i^3 + b^2*e^4
*i^4)*log(c*f*x + c*e)^3 + 6*(216*(2*a^2 - 2*a*b + b^2)*f^4*h^2*i^2 - 16*(
18*a^2 - 30*a*b + 19*b^2)*e*f^3*h*i^3 + (72*a^2 - 156*a*b + 115*b^2)*e^2*f
^2*i^4)*x^2 + 72*(3*b^2*f^4*i^4*x^4 + 12*a*b*f^4*h^4 - 48*(a*b - b^2)*e*f^
3*h^3*i + 36*(2*a*b - 3*b^2)*e^2*f^2*h^2*i^2 - 8*(6*a*b - 11*b^2)*e^3*f*h*
i^3 + (12*a*b - 25*b^2)*e^4*i^4 + 4*(4*b^2*f^4*h*i^3 - b^2*e*f^3*i^4)*x^3
+ 6*(6*b^2*f^4*h^2*i^2 - 4*b^2*e*f^3*h*i^3 + b^2*e^2*f^2*i^4)*x^2 + 12*(4*
b^2*f^4*h^3*i - 6*b^2*e*f^3*h^2*i^2 + 4*b^2*e^2*f^2*h*i^3 - b^2*e^3*f*i^4)
*x)*log(c*f*x + c*e)^2 + 12*(288*(a^2 - 2*a*b + 2*b^2)*f^4*h^3*i - 216*(2*
a^2 - 6*a*b + 7*b^2)*e*f^3*h^2*i^2 + 16*(18*a^2 - 66*a*b + 85*b^2)*e^2*f^2
*h*i^3 - (72*a^2 - 300*a*b + 415*b^2)*e^3*f*i^4)*x + 12*(9*(4*a*b - b^2)*f
^4*i^4*x^4 + 72*a^2*f^4*h^4 - 288*(a^2 - 2*a*b + 2*b^2)*e*f^3*h^3*i + 216*
(2*a^2 - 6*a*b + 7*b^2)*e^2*f^2*h^2*i^2 - 16*(18*a^2 - 66*a*b + 85*b^2)*e^
3*f*h*i^3 + (72*a^2 - 300*a*b + 415*b^2)*e^4*i^4 + 4*(16*(3*a*b - b^2)*f^4
*h*i^3 - (12*a*b - 7*b^2)*e*f^3*i^4)*x^3 + 6*(36*(2*a*b - b^2)*f^4*h^2*i^2
 - 8*(6*a*b - 5*b^2)*e*f^3*h*i^3 + (12*a*b - 13*b^2)*e^2*f^2*i^4)*x^2 + 12
*(48*(a*b - b^2)*f^4*h^3*i - 36*(2*a*b - 3*b^2)*e*f^3*h^2*i^2 + 8*(6*a*b -
 11*b^2)*e^2*f^2*h*i^3 - (12*a*b - 25*b^2)*e^3*f*i^4)*x)*log(c*f*x + c*...
```

3.183.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(534) = 1068.

Time = 1.88 (sec) , antiderivative size = 1479, normalized size of antiderivative = 2.55

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \text{Too large to display}$$

input
```
integrate((i*x+h)**4*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

output
```
x**4*(a**2*i**4/(4*d*f) - a*b*i**4/(8*d*f) + b**2*i**4/(32*d*f)) + x**3*(-
a**2*e*i**4/(3*d*f**2) + 4*a**2*h*i**3/(3*d*f) + 7*a*b*e*i**4/(18*d*f**2)
- 8*a*b*h*i**3/(9*d*f) - 37*b**2*e*i**4/(216*d*f**2) + 8*b**2*h*i**3/(27*d
*f)) + x**2*(a**2*e**2*i**4/(2*d*f**3) - 2*a**2*e*h*i**3/(d*f**2) + 3*a**2
*h**2*i**2/(d*f) - 13*a*b*e**2*i**4/(12*d*f**3) + 10*a*b*e*h*i**3/(3*d*f**
2) - 3*a*b*h**2*i**2/(d*f) + 115*b**2*e**2*i**4/(144*d*f**3) - 19*b**2*e*h
*i**3/(9*d*f**2) + 3*b**2*h**2*i**2/(2*d*f)) + x*(-a**2*e**3*i**4/(d*f**4)
 + 4*a**2*e**2*h*i**3/(d*f**3) - 6*a**2*e*h**2*i**2/(d*f**2) + 4*a**2*h**3
*i/(d*f) + 25*a*b*e**3*i**4/(6*d*f**4) - 44*a*b*e**2*h*i**3/(3*d*f**3) + 1
8*a*b*e*h**2*i**2/(d*f**2) - 8*a*b*h**3*i/(d*f) - 415*b**2*e**3*i**4/(72*d
*f**4) + 170*b**2*e**2*h*i**3/(9*d*f**3) - 21*b**2*e*h**2*i**2/(d*f**2) +
8*b**2*h**3*i/(d*f)) + (-144*a*b*e**3*i**4*x + 576*a*b*e**2*f*h*i**3*x + 7
2*a*b*e**2*f*i**4*x**2 - 864*a*b*e*f**2*h**2*i**2*x - 288*a*b*e*f**2*h*i**
3*x**2 - 48*a*b*e*f**2*i**4*x**3 + 576*a*b*f**3*h**3*i*x + 432*a*b*f**3*h*
*2*i**2*x**2 + 192*a*b*f**3*h*i**3*x**3 + 36*a*b*f**3*i**4*x**4 + 300*b**2
*e**3*i**4*x - 1056*b**2*e**2*f*h*i**3*x - 78*b**2*e**2*f*i**4*x**2 + 1296
*b**2*e*f**2*h**2*i**2*x + 240*b**2*e*f**2*h*i**3*x**2 + 28*b**2*e*f**2*i*
*4*x**3 - 576*b**2*f**3*h**3*i*x - 216*b**2*f**3*h**2*i**2*x**2 - 64*b**2*
f**3*h*i**3*x**3 - 9*b**2*f**3*i**4*x**4)*log(c*(e + f*x))/(72*d*f**4) + (
b**2*e**4*i**4 - 4*b**2*e**3*f*h*i**3 + 6*b**2*e**2*f**2*h**2*i**2 - 4*...
```

3.183.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1427 vs. 2(557) = 1114.

Time = 0.27 (sec) , antiderivative size = 1427, normalized size of antiderivative = 2.46

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \text{Too large to display}$$

input
```
integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxim
a")
```

output
```
8*a*b*h^3*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + 1/6*a*b*
i^4*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2
- 12*e^3*x)/(d*f^4))*log(c*f*x + c*e) - 4/3*a*b*h*i^3*(6*e^3*log(f*x + e)/
(d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))*log(c*f*x + c*e) + 6*
a*b*h^2*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f
*x + c*e) - a*b*h^4*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x
+ e)^2 + 2*log(f*x + e)*log(c))/(d*f)) + 4*a^2*h^3*i*(x/(d*f) - e*log(f*x
+ e)/(d*f^2)) + 1/12*a^2*i^4*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4
*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/(d*f^4)) - 2/3*a^2*h*i^3*(6*e^3*log(f
*x + e)/(d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3*a^2*h^2*i
^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + 1/3*b^2*h^4*lo
g(c*f*x + c*e)^3/(d*f) + 2*a*b*h^4*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f)
 + a^2*h^4*log(d*f*x + d*e)/(d*f) + 4*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(
f*x + e))*a*b*h^3*i/(d*f^2) - 3*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x
+ 6*e^2*log(f*x + e))*a*b*h^2*i^2/(d*f^3) - 4/3*(c^2*e*log(c*f*x + c*e)^3
- 3*(c*f*x + c*e)*(c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2
*h^3*i/(c^2*d*f^2) - 2/9*(4*f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2
 + 66*e^2*f*x - 66*e^3*log(f*x + e))*a*b*h*i^3/(d*f^4) - 1/72*(9*f^4*x^4 -
 28*e*f^3*x^3 + 78*e^2*f^2*x^2 + 72*e^4*log(f*x + e)^2 - 300*e^3*f*x + 300
*e^4*log(f*x + e))*a*b*i^4/(d*f^5) + 1/2*(4*c^3*e^2*log(c*f*x + c*e)^3 ...
```

3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(557) = 1114$.

Time = 0.33 (sec) , antiderivative size = 1213, normalized size of antiderivative = 2.09

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \text{Too large to display}$$

input
```
integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac"
)
```

output
```
1/32*(8*a^2*i^4 - 4*a*b*i^4 + b^2*i^4)*x^4/(d*f) + 1/12*(3*b^2*i^4*x^4/(d*
f) + 4*(4*b^2*f*h*i^3 - b^2*e*i^4)*x^3/(d*f^2) + 6*(6*b^2*f^2*h^2*i^2 - 4*
b^2*e*f*h*i^3 + b^2*e^2*i^4)*x^2/(d*f^3) + 12*(4*b^2*f^3*h^3*i - 6*b^2*e*f
^2*h^2*i^2 + 4*b^2*e^2*f*h*i^3 - b^2*e^3*i^4)*x/(d*f^4) + (12*a*b*f^4*h^4
- 48*a*b*e*f^3*h^3*i + 48*b^2*e*f^3*h^3*i + 72*a*b*e^2*f^2*h^2*i^2 - 108*b
^2*e^2*f^2*h^2*i^2 - 48*a*b*e^3*f*h*i^3 + 88*b^2*e^3*f*h*i^3 + 12*a*b*e^4*
i^4 - 25*b^2*e^4*i^4)/(d*f^5))*log(c*f*x + c*e)^2 + 1/72*(9*(4*a*b*i^4 - b
^2*i^4)*x^4/(d*f) + 4*(48*a*b*f*h*i^3 - 16*b^2*f*h*i^3 - 12*a*b*e*i^4 + 7*
b^2*e*i^4)*x^3/(d*f^2) + 6*(72*a*b*f^2*h^2*i^2 - 36*b^2*f^2*h^2*i^2 - 48*a
*b*e*f*h*i^3 + 40*b^2*e*f*h*i^3 + 12*a*b*e^2*i^4 - 13*b^2*e^2*i^4)*x^2/(d*
f^3) + 12*(48*a*b*f^3*h^3*i - 48*b^2*f^3*h^3*i - 72*a*b*e*f^2*h^2*i^2 + 10
8*b^2*e*f^2*h^2*i^2 + 48*a*b*e^2*f*h*i^3 - 88*b^2*e^2*f*h*i^3 - 12*a*b*e^3
*i^4 + 25*b^2*e^3*i^4)*x/(d*f^4))*log(c*f*x + c*e) + 1/216*(288*a^2*f*h*i^
3 - 192*a*b*f*h*i^3 + 64*b^2*f*h*i^3 - 72*a^2*e*i^4 + 84*a*b*e*i^4 - 37*b^
2*e*i^4)*x^3/(d*f^2) + 1/144*(432*a^2*f^2*h^2*i^2 - 432*a*b*f^2*h^2*i^2 +
216*b^2*f^2*h^2*i^2 - 288*a^2*e*f*h*i^3 + 480*a*b*e*f*h*i^3 - 304*b^2*e*f*
h*i^3 + 72*a^2*e^2*i^4 - 156*a*b*e^2*i^4 + 115*b^2*e^2*i^4)*x^2/(d*f^3) +
1/3*(b^2*f^4*h^4 - 4*b^2*e*f^3*h^3*i + 6*b^2*e^2*f^2*h^2*i^2 - 4*b^2*e^3*f
*h*i^3 + b^2*e^4*i^4)*log(c*f*x + c*e)^3/(d*f^5) + 1/72*(288*a^2*f^3*h^3*i
 - 576*a*b*f^3*h^3*i + 576*b^2*f^3*h^3*i - 432*a^2*e*f^2*h^2*i^2 + 1296...
```

3.183. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.183.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 1346, normalized size of antiderivative = 2.32

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \ln\left(c\left(e+f\,x\right)\right)^2\left(f\left(\frac{b^2\,i^4\,x^4}{4\,d\,f^2}-\frac{b^2\,i^3\,x^3\,\left(e\,i-4\,f\,h\right)}{3\,d\,f^3}-\frac{b^2\,i\,x\,\left(e^3\,i^3-4\,e^2\,f\,h\,i^2+6\,e\,f^2\,h^2\,i-4\,f^3\,h^3\right)}{d\,f^5}+\frac{b^2\,i^2\,x^2\,\left(e^2\,i^2-4\,e\,f\,h\,i+6\,f^2\,h^2\right)}{2\,d\,f^4}\right)\right.$$

$$+\frac{-25\,b^2\,e^4\,i^4+88\,b^2\,e^3\,f\,h\,i^3-108\,b^2\,e^2\,f^2\,h^2\,i^2+48\,b^2\,e\,f^3\,h^3\,i+12\,a\,b\,e^4\,i^4-48\,a\,b\,e^3\,f\,h\,i^3+72\,a\,b\,e^2}{12\,d\,f^5}$$

$$-x^2\left(\frac{e\left(\frac{i^3\,\left(72\,a^2\,f\,h-7\,b^2\,e\,i+16\,b^2\,f\,h+12\,a\,b\,e\,i-48\,a\,b\,f\,h\right)}{18\,d\,f^2}-\frac{e\,i^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{8\,d\,f^2}\right)}{2\,f}\right.$$

$$\left.-\frac{i^2\,\left(72\,a^2\,f^2\,h^2-12\,a\,b\,e^2\,i^2+48\,a\,b\,e\,f\,h\,i-72\,a\,b\,f^2\,h^2+13\,b^2\,e^2\,i^2-40\,b^2\,e\,f\,h\,i+36\,b^2\,f^2\,h^2\right)}{24\,d\,f^3}\right)$$

$$+x^3\left(\frac{i^3\,\left(72\,a^2\,f\,h-7\,b^2\,e\,i+16\,b^2\,f\,h+12\,a\,b\,e\,i-48\,a\,b\,f\,h\right)}{54\,d\,f^2}-\frac{e\,i^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{24\,d\,f^2}\right)$$

$$+x\left(\frac{288\,a^2\,f^3\,h^3\,i+144\,a\,b\,e^3\,i^4-576\,a\,b\,e^2\,f\,h\,i^3+864\,a\,b\,e\,f^2\,h^2\,i^2-576\,a\,b\,f^3\,h^3\,i-300\,b^2\,e^3\,i^4+105}{72\,d\,f^4}\right.$$

$$+\frac{e\left(\frac{e\left(\frac{i^3\,\left(72\,a^2\,f\,h-7\,b^2\,e\,i+16\,b^2\,f\,h+12\,a\,b\,e\,i-48\,a\,b\,f\,h\right)}{18\,d\,f^2}-\frac{e\,i^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{8\,d\,f^2}\right)}{f}-\frac{i^2\,\left(72\,a^2\,f^2\,h^2-12\,a\,b\,e^2\,i^2+48\,a\,b\,e\,f\,h\,i-72\,a\,b\,f^2\,h^2+13\right.}{12\,d\,f^3}\right.}{f}$$

$$+f\,\ln\left(c\left(e+f\,x\right)\right)\left(\frac{x^3\,\left(7\,e\,b^2\,i^4-16\,f\,h\,b^2\,i^3-12\,a\,e\,b\,i^4+48\,a\,f\,h\,b\,i^3\right)}{18\,d\,f^3}\right.$$

$$-\frac{x^2\,\left(13\,b^2\,e^2\,i^4-40\,b^2\,e\,f\,h\,i^3+36\,b^2\,f^2\,h^2\,i^2-12\,a\,b\,e^2\,i^4+48\,a\,b\,e\,f\,h\,i^3-72\,a\,b\,f^2\,h^2\,i^2\right)}{12\,d\,f^4}$$

$$+\frac{x\,\left(25\,b^2\,e^3\,i^4-88\,b^2\,e^2\,f\,h\,i^3+108\,b^2\,e\,f^2\,h^2\,i^2-48\,b^2\,f^3\,h^3\,i-12\,a\,b\,e^3\,i^4+48\,a\,b\,e^2\,f\,h\,i^3-72\,a\,b\,e\,f^2}{6\,d\,f^5}$$

$$\left.+\frac{b\,i^4\,x^4\,\left(4\,a-b\right)}{8\,d\,f^2}\right)$$

$$+\frac{\ln\left(e+f\,x\right)\,\left(72\,a^2\,e^4\,i^4-288\,a^2\,e^3\,f\,h\,i^3+432\,a^2\,e^2\,f^2\,h^2\,i^2-288\,a^2\,e\,f^3\,h^3\,i+72\,a^2\,f^4\,h^4-300\,a\,b\,e^4\,i}{}$$

$$+\frac{b^2\,\ln\left(c\left(e+f\,x\right)\right)^3\,\left(e^4\,i^4-4\,e^3\,f\,h\,i^3+6\,e^2\,f^2\,h^2\,i^2-4\,e\,f^3\,h^3\,i+f^4\,h^4\right)}{3\,d\,f^5}$$

$$+\frac{i^4\,x^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{32\,d\,f}$$

input
```
int(((h + i*x)^4*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

output
```
log(c*(e + f*x))^2*(f*((b^2*i^4*x^4)/(4*d*f^2) - (b^2*i^3*x^3*(e*i - 4*f*h
))/(3*d*f^3) - (b^2*i*x*(e^3*i^3 - 4*f^3*h^3 + 6*e*f^2*h^2*i - 4*e^2*f*h*i
^2))/(d*f^5) + (b^2*i^2*x^2*(e^2*i^2 + 6*f^2*h^2 - 4*e*f*h*i))/(2*d*f^4))
+ (12*a*b*e^4*i^4 - 25*b^2*e^4*i^4 + 12*a*b*f^4*h^4 - 108*b^2*e^2*f^2*h^2*
i^2 + 48*b^2*e*f^3*h^3*i + 88*b^2*e^3*f*h*i^3 + 72*a*b*e^2*f^2*h^2*i^2 - 4
8*a*b*e*f^3*h^3*i - 48*a*b*e^3*f*h*i^3)/(12*d*f^5)) - x^2*((e*((i^3*(72*a^
2*f*h - 7*b^2*e*i + 16*b^2*f*h + 12*a*b*e*i - 48*a*b*f*h))/(18*d*f^2) - (e
*i^4*(8*a^2 - 4*a*b + b^2))/(8*d*f^2)))/(2*f) - (i^2*(72*a^2*f^2*h^2 + 13*
b^2*e^2*i^2 + 36*b^2*f^2*h^2 - 12*a*b*e^2*i^2 - 72*a*b*f^2*h^2 - 40*b^2*e*
f*h*i + 48*a*b*e*f*h*i))/(24*d*f^3)) + x^3*((i^3*(72*a^2*f*h - 7*b^2*e*i +
 16*b^2*f*h + 12*a*b*e*i - 48*a*b*f*h))/(54*d*f^2) - (e*i^4*(8*a^2 - 4*a*b
 + b^2))/(24*d*f^2)) + x*((288*a^2*f^3*h^3*i - 300*b^2*e^3*i^4 + 576*b^2*f
^3*h^3*i + 144*a*b*e^3*i^4 - 576*a*b*f^3*h^3*i + 1056*b^2*e^2*f*h*i^3 - 12
96*b^2*e*f^2*h^2*i^2 - 576*a*b*e^2*f*h*i^3 + 864*a*b*e*f^2*h^2*i^2)/(72*d*
f^4) + (e*((e*((i^3*(72*a^2*f*h - 7*b^2*e*i + 16*b^2*f*h + 12*a*b*e*i - 48
*a*b*f*h))/(18*d*f^2) - (e*i^4*(8*a^2 - 4*a*b + b^2))/(8*d*f^2)))/f - (i^2
*(72*a^2*f^2*h^2 + 13*b^2*e^2*i^2 + 36*b^2*f^2*h^2 - 12*a*b*e^2*i^2 - 72*a
*b*f^2*h^2 - 40*b^2*e*f*h*i + 48*a*b*e*f*h*i))/(12*d*f^3)))/f) + f*log(c*(
e + f*x))*((x^3*(7*b^2*e*i^4 - 12*a*b*e*i^4 - 16*b^2*f*h*i^3 + 48*a*b*f*h*
i^3))/(18*d*f^3) - (x^2*(13*b^2*e^2*i^4 + 36*b^2*f^2*h^2*i^2 - 12*a*b*e...
```

3.183. $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.184 $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.184.1 Optimal result

Integrand size = 32, antiderivative size = 464

$$\begin{aligned}\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = & -\frac{4abi(fh-ei)^2x}{df^3}+\frac{6b^2i(fh-ei)^2x}{df^3} \\ & +\frac{3b^2i^2(fh-ei)(e+fx)^2}{4df^4}+\frac{2b^2i^3(e+fx)^3}{27df^4} \\ & +\frac{b^2(fh-ei)^3\log^2(e+fx)}{3df^4} \\ & -\frac{4b^2i(fh-ei)^2(e+fx)\log(c(e+fx))}{df^4} \\ & -\frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} \\ & -\frac{3bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4} \\ & -\frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} \\ & -\frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} \\ & +\frac{2i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2}{df^4} \\ & +\frac{i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^4} \\ & +\frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} \\ & +\frac{(fh-ei)^3(a+b\log(c(e+fx)))^3}{3bdf^4}\end{aligned}$$

output
```
-4*a*b*i*(-e*i+f*h)^2*x/d/f^3+6*b^2*i*(-e*i+f*h)^2*x/d/f^3+3/4*b^2*i^2*(-e
*i+f*h)*(f*x+e)^2/d/f^4+2/27*b^2*i^3*(f*x+e)^3/d/f^4+1/3*b^2*(-e*i+f*h)^3*
ln(f*x+e)^2/d/f^4-4*b^2*i*(-e*i+f*h)^2*(f*x+e)*ln(c*(f*x+e))/d/f^4-2*b*i*(
-e*i+f*h)^2*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^4-3/2*b*i^2*(-e*i+f*h)*(f*x+e)
^2*(a+b*ln(c*(f*x+e)))/d/f^4-2/9*b*i^3*(f*x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^4
-2/3*b*(-e*i+f*h)^3*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^4+2*i*(-e*i+f*h)^2*(
f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/f^4+1/2*i^2*(-e*i+f*h)*(f*x+e)^2*(a+b*ln(c*
(f*x+e)))^2/d/f^4+1/3*(i*x+h)^3*(a+b*ln(c*(f*x+e)))^2/d/f+1/3*(-e*i+f*h)^3
*(a+b*ln(c*(f*x+e)))^3/b/d/f^4
```

3.184.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.58

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{324i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2+162i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2+36i^3(e\cdots}{}$$

input
```
Integrate[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

output
```
(324*i*(f*h - e*i)^2*(e + f*x)*(a + b*Log[c*(e + f*x)])^2 + 162*i^2*(f*h -
 e*i)*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2 + 36*i^3*(e + f*x)^3*(a + b*L
og[c*(e + f*x)])^2 + (36*(f*h - e*i)^3*(a + b*Log[c*(e + f*x)])^3)/b - 648
*b*i*(f*h - e*i)^2*((a - b)*f*x + b*(e + f*x)*Log[c*(e + f*x)]) + 81*b*i^2
*(f*h - e*i)*(b*f*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))
+ 8*b*i^3*(b*f*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*(a + b*Log[c*
(e + f*x)])))/(108*d*f^4)
```

3.184.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2858, 27, 2788, 2756, 2772, 2009, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^3(a+b\log(c(e+fx)))^2}{df^3(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)}{df^4}$$

$$\Big\downarrow \text{2788}$$

$$\frac{i\int (fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^2d(e+fx)+(fh-ei)\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx}{df^4}$$

$$\Big\downarrow \text{2756}$$

$$\frac{i\left(\frac{(i(e+fx)-ei+fh)^3(a+b\log(c(e+fx)))^2}{3i}-\frac{2b\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))}{e+fx}d(e+fx)}{3i}\right)+(fh-ei)\int \frac{(fh-ei+i(e+fx))^2(a+b\log}{e+fx}}{df^4}$$

$$\Big\downarrow \text{2772}$$

$$i\left(\frac{(i(e+fx)-ei+fh)^3(a+b\log(c(e+fx)))^2}{3i}-\frac{2b\left(-b\int\left(\frac{1}{3}(e+fx)^2i^3+\frac{3}{2}(fh-ei)(e+fx)i^2+3(fh-ei)^2i+\frac{(fh-ei)^3\log(e+fx)}{e+fx}\right)d(e+fx)+\frac{3}{2}i^2(e+}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(fh-ei)\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left(\frac{(i(e+fx)-ei+fh)^3(a+b\log(c(e+fx)))^2}{3i}-\frac{2b\left(\frac{3}{2}i^2(e+fx)^2(fh-ei)(a\right.}{}\right.$$

$$\Big\downarrow \text{2788}$$

$$(fh-ei)\left(i\int (fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2d(e+fx)+(fh-ei)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}\right.$$

$$\Big\downarrow \text{2767}$$

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

$$(fh-ei)\left(\left(fh-ei\right)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\int \left(fh\left(1-\frac{ei}{fh}\right)(a+b\log(c(e+fx)))^2+i(e\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(fh-ei)\left(\left(fh-ei\right)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(fh-ei)(a+b\log(c(e+fx)))^2-\frac{1}{2}\right.\right.$$

$$\Big\downarrow \text{ 2788}$$

$$(fh-ei)\left(\left(fh-ei\right)\left(\left(fh-ei\right)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\int(a+b\log(c(e+fx)))^2d(e+fx)\right)+i\left((e+fx)\right.\right.$$

$$\Big\downarrow \text{ 2733}$$

$$(fh-ei)\left(\left(fh-ei\right)\left(\left(fh-ei\right)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b\int(a+b\log\right.\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(fh-ei)\left(\left(fh-ei\right)\left(\left(fh-ei\right)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+fx)\right.\right.\right.$$

$$\Big\downarrow \text{ 2739}$$

$$(fh-ei)\left(\left(fh-ei\right)\left(\frac{(fh-ei)\int(a+b\log(c(e+fx)))^2d(a+b\log(c(e+fx)))}{b}+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+f\right.\right.\right.$$

$$\Big\downarrow \text{ 15}$$

$$(fh-ei)\left(i\left((e+fx)(fh-ei)(a+b\log(c(e+fx)))^2-\frac{1}{2}bi(e+fx)^2(a+b\log(c(e+fx)))+\frac{1}{2}i(e+fx)^2(a+b\log\right.\right.$$

input
```
Int[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

output
```
(i*(((f*h - e*i + i*(e + f*x))^3*(a + b*Log[c*(e + f*x)])^2)/(3*i) - (2*b*
(-(b*(3*i*(f*h - e*i)^2*(e + f*x) + (3*i^2*(f*h - e*i)*(e + f*x)^2)/4 + (i
^3*(e + f*x)^3)/9 + ((f*h - e*i)^3*Log[e + f*x]^2)/2)) + 3*i*(f*h - e*i)^2
*(e + f*x)*(a + b*Log[c*(e + f*x)]) + (3*i^2*(f*h - e*i)*(e + f*x)^2*(a +
b*Log[c*(e + f*x)]))/2 + (i^3*(e + f*x)^3*(a + b*Log[c*(e + f*x)]))/3 + (f
*h - e*i)^3*Log[e + f*x]*(a + b*Log[c*(e + f*x)])))/(3*i)) + (f*h - e*i)*(
i*(-2*a*b*(f*h - e*i)*(e + f*x) + 2*b^2*(f*h - e*i)*(e + f*x) + (b^2*i*(e
+ f*x)^2)/4 - 2*b^2*(f*h - e*i)*(e + f*x)*Log[c*(e + f*x)] - (b*i*(e + f*x
)^2*(a + b*Log[c*(e + f*x)]))/2 + (f*h - e*i)*(e + f*x)*(a + b*Log[c*(e +
f*x)])^2 + (i*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2)/2) + (f*h - e*i)*(((
f*h - e*i)*(a + b*Log[c*(e + f*x)])^3)/(3*b) + i*((e + f*x)*(a + b*Log[c*(
e + f*x)])^2 - 2*b*(a*(e + f*x) - b*(e + f*x) + b*(e + f*x)*Log[c*(e + f*x
)])))))/(d*f^4)
```

3.184.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.184.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.55

method	result
norman	$\frac{b^2 i(e^2 i^2-3efhi+3f^2h^2)x\ln(c(fx+e))^2}{d\,f^3}-\frac{(18a^2e^3i^3-54a^2e^2fh\,i^2+54a^2e\,f^2h^2i-18a^2f^3h^3-66ab\,e^3i^3+162ab\,e^2fh}{18d\,f}$
risch	$-\frac{b(-2b\,f^3i^3x^3+3be\,f^2i^3x^2-9b\,f^3h\,i^2x^2-6b\,e^2f\,i^3x+18be\,f^2h\,i^2x-18b\,f^3h^2ix+6a\,e^3i^3-18a\,e^2fh\,i^2+18ae\,f^2h^2i-6a}{6d\,f^4}$
parts	$\frac{a^2\left(\frac{i\left(\frac{1}{3}f^2i^2x^3-\frac{1}{2}ef\,i^2x^2+\frac{3}{2}f^2hi\,x^2+x\,e^2i^2-3xefhi+3x\,f^2h^2\right)}{f^3}+\frac{\left(-e^3i^3+3e^2fh\,i^2-3e\,f^2h^2i+f^3h^3\right)\ln(fx+e)}{f^4}\right)}{d}+\frac{b^2\left(-\frac{ce}{}\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((i*x+h)^3*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
b^2*i*(e^2*i^2-3*e*f*h*i+3*f^2*h^2)/d/f^3*x*ln(c*(f*x+e))^2-1/18*(18*a^2*e
^3*i^3-54*a^2*e^2*f*h*i^2+54*a^2*e*f^2*h^2*i-18*a^2*f^3*h^3-66*a*b*e^3*i^3
+162*a*b*e^2*f*h*i^2-108*a*b*e*f^2*h^2*i+85*b^2*e^3*i^3-189*b^2*e^2*f*h*i^
2+108*b^2*e*f^2*h^2*i)/d/f^4*ln(c*(f*x+e))-1/6*b*(6*a*e^3*i^3-18*a*e^2*f*h
*i^2+18*a*e*f^2*h^2*i-6*a*f^3*h^3-11*b*e^3*i^3+27*b*e^2*f*h*i^2-18*b*e*f^2
*h^2*i)/d/f^4*ln(c*(f*x+e))^2-1/3*b^2*(e^3*i^3-3*e^2*f*h*i^2+3*e*f^2*h^2*i
-f^3*h^3)/d/f^4*ln(c*(f*x+e))^3+1/18*i*(18*a^2*e^2*i^2-54*a^2*e*f*h*i+54*a
^2*f^2*h^2-66*a*b*e^2*i^2+162*a*b*e*f*h*i-108*a*b*f^2*h^2+85*b^2*e^2*i^2-1
89*b^2*e*f*h*i+108*b^2*f^2*h^2)/d/f^3*x-1/36*i^2*(18*a^2*e*i-54*a^2*f*h-30
*a*b*e*i+54*a*b*f*h+19*b^2*e*i-27*b^2*f*h)/d/f^2*x^2+1/27*i^3*(9*a^2-6*a*b
+2*b^2)/d/f*x^3+1/3*b^2*i^3/d/f*x^3*ln(c*(f*x+e))^2+1/3*b*i*(6*a*e^2*i^2-1
8*a*e*f*h*i+18*a*f^2*h^2-11*b*e^2*i^2+27*b*e*f*h*i-18*b*f^2*h^2)/d/f^3*x*l
n(c*(f*x+e))-1/6*b*i^2*(6*a*e*i-18*a*f*h-5*b*e*i+9*b*f*h)/d/f^2*x^2*ln(c*(
f*x+e))+2/9*b*i^3*(3*a-b)/d/f*x^3*ln(c*(f*x+e))-1/2*b^2*i^2*(e*i-3*f*h)/d/
f^2*x^2*ln(c*(f*x+e))^2
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.31

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{4\,(9\,a^2-6\,ab+2\,b^2)f^3i^3x^3+36\,(b^2f^3h^3-3\,b^2ef^2h^2i+3\,b^2e^2fhi^2-b^2e^3i^3)\log\left(cfx+ce\right)^3+3\,(27\,(2\,a^2}{}$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="frica
s")
```

output
```
1/108*(4*(9*a^2 - 6*a*b + 2*b^2)*f^3*i^3*x^3 + 36*(b^2*f^3*h^3 - 3*b^2*e*f
^2*h^2*i + 3*b^2*e^2*f*h*i^2 - b^2*e^3*i^3)*log(c*f*x + c*e)^3 + 3*(27*(2*
a^2 - 2*a*b + b^2)*f^3*h*i^2 - (18*a^2 - 30*a*b + 19*b^2)*e*f^2*i^3)*x^2 +
 18*(2*b^2*f^3*i^3*x^3 + 6*a*b*f^3*h^3 - 18*(a*b - b^2)*e*f^2*h^2*i + 9*(2
*a*b - 3*b^2)*e^2*f*h*i^2 - (6*a*b - 11*b^2)*e^3*i^3 + 3*(3*b^2*f^3*h*i^2
- b^2*e*f^2*i^3)*x^2 + 6*(3*b^2*f^3*h^2*i - 3*b^2*e*f^2*h*i^2 + b^2*e^2*f*
i^3)*x)*log(c*f*x + c*e)^2 + 6*(54*(a^2 - 2*a*b + 2*b^2)*f^3*h^2*i - 27*(2
*a^2 - 6*a*b + 7*b^2)*e*f^2*h*i^2 + (18*a^2 - 66*a*b + 85*b^2)*e^2*f*i^3)*
x + 6*(4*(3*a*b - b^2)*f^3*i^3*x^3 + 18*a^2*f^3*h^3 - 54*(a^2 - 2*a*b + 2*
b^2)*e*f^2*h^2*i + 27*(2*a^2 - 6*a*b + 7*b^2)*e^2*f*h*i^2 - (18*a^2 - 66*a
*b + 85*b^2)*e^3*i^3 + 3*(9*(2*a*b - b^2)*f^3*h*i^2 - (6*a*b - 5*b^2)*e*f^
2*i^3)*x^2 + 6*(18*(a*b - b^2)*f^3*h^2*i - 9*(2*a*b - 3*b^2)*e*f^2*h*i^2 +
 (6*a*b - 11*b^2)*e^2*f*i^3)*x)*log(c*f*x + c*e))/(d*f^4)
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(428) = 856$.

Time = 1.10 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.98

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = x^3\left(\frac{a^2i^3}{3df}-\frac{2abi^3}{9df}+\frac{2b^2i^3}{27df}\right)$$
$$+x^2\left(-\frac{a^2ei^3}{2df^2}+\frac{3a^2hi^2}{2df}+\frac{5abei^3}{6df^2}-\frac{3abhi^2}{2df}-\frac{19b^2ei^3}{36df^2}+\frac{3b^2hi^2}{4df}\right)+x\left(\frac{a^2e^2i^3}{df^3}-\frac{3a^2ehi^2}{df^2}\right.$$
$$\left.+\frac{3a^2h^2i}{df}-\frac{11abe^2i^3}{3df^3}+\frac{9abehi^2}{df^2}-\frac{6abh^2i}{df}+\frac{85b^2e^2i^3}{18df^3}-\frac{21b^2ehi^2}{2df^2}+\frac{6b^2h^2i}{df}\right)$$
$$+\frac{(36abe^2i^3x-108abefhi^2x-18abefi^3x^2+108abf^2h^2ix+54abf^2hi^2x^2+12abf^2i^3x^3-66b^2e^2i^3x+16}{18df^3}$$
$$+\frac{(-b^2e^3i^3+3b^2e^2fhi^2-3b^2ef^2h^2i+b^2f^3h^3)\log(c(e+fx))^3}{3df^4}$$
$$-\frac{(18a^2e^3i^3-54a^2e^2fhi^2+54a^2ef^2h^2i-18a^2f^3h^3-66abe^3i^3+162abe^2fhi^2-108abef^2h^2i+85b^2e^3i^3}{18df^4}$$
$$+\frac{(-6abe^3i^3+18abe^2fhi^2-18abef^2h^2i+6abf^3h^3+11b^2e^3i^3-27b^2e^2fhi^2+6b^2e^2fi^3x+18b^2ef^2h^2i-}{6df^4}$$

input
```
integrate((i*x+h)**3*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

output
```
x**3*(a**2*i**3/(3*d*f) - 2*a*b*i**3/(9*d*f) + 2*b**2*i**3/(27*d*f)) + x**
2*(-a**2*e*i**3/(2*d*f**2) + 3*a**2*h*i**2/(2*d*f) + 5*a*b*e*i**3/(6*d*f**
2) - 3*a*b*h*i**2/(2*d*f) - 19*b**2*e*i**3/(36*d*f**2) + 3*b**2*h*i**2/(4*
d*f)) + x*(a**2*e**2*i**3/(d*f**3) - 3*a**2*e*h*i**2/(d*f**2) + 3*a**2*h**
2*i/(d*f) - 11*a*b*e**2*i**3/(3*d*f**3) + 9*a*b*e*h*i**2/(d*f**2) - 6*a*b*
h**2*i/(d*f) + 85*b**2*e**2*i**3/(18*d*f**3) - 21*b**2*e*h*i**2/(2*d*f**2)
 + 6*b**2*h**2*i/(d*f)) + (36*a*b*e**2*i**3*x - 108*a*b*e*f*h*i**2*x - 18*
a*b*e*f*i**3*x**2 + 108*a*b*f**2*h**2*i*x + 54*a*b*f**2*h*i**2*x**2 + 12*a
*b*f**2*i**3*x**3 - 66*b**2*e**2*i**3*x + 162*b**2*e*f*h*i**2*x + 15*b**2*
e*f*i**3*x**2 - 108*b**2*f**2*h**2*i*x - 27*b**2*f**2*h*i**2*x**2 - 4*b**2
*f**2*i**3*x**3)*log(c*(e + f*x))/(18*d*f**3) + (-b**2*e**3*i**3 + 3*b**2*
e**2*f*h*i**2 - 3*b**2*e*f**2*h**2*i + b**2*f**3*h**3)*log(c*(e + f*x))**3
/(3*d*f**4) - (18*a**2*e**3*i**3 - 54*a**2*e**2*f*h*i**2 + 54*a**2*e*f**2*
h**2*i - 18*a**2*f**3*h**3 - 66*a*b*e**3*i**3 + 162*a*b*e**2*f*h*i**2 - 10
8*a*b*e*f**2*h**2*i + 85*b**2*e**3*i**3 - 189*b**2*e**2*f*h*i**2 + 108*b**
2*e*f**2*h**2*i)*log(e + f*x)/(18*d*f**4) + (-6*a*b*e**3*i**3 + 18*a*b*e**
2*f*h*i**2 - 18*a*b*e*f**2*h**2*i + 6*a*b*f**3*h**3 + 11*b**2*e**3*i**3 -
27*b**2*e**2*f*h*i**2 + 6*b**2*e**2*f*i**3*x + 18*b**2*e*f**2*h**2*i - 18*
b**2*e*f**2*h*i**2*x - 3*b**2*e*f**2*i**3*x**2 + 18*b**2*f**3*h**2*i*x + 9
*b**2*f**3*h*i**2*x**2 + 2*b**2*f**3*i**3*x**3)*log(c*(e + f*x))**2/(6*...
```

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.184.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(446) = 892$.

Time = 0.28 (sec) , antiderivative size = 964, normalized size of antiderivative = 2.08

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = 6\,abh^2i\left(\frac{x}{df}-\frac{e\log\left(fx+e\right)}{df^2}\right)\log\left(cfx+ce\right)$$
$$-\frac{1}{3}\,abi^3\left(\frac{6\,e^3\log\left(fx+e\right)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)\log\left(cfx+ce\right)$$
$$+3\,abhi^2\left(\frac{2\,e^2\log\left(fx+e\right)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log\left(cfx+ce\right)$$
$$-abh^3\left(\frac{2\,\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}-\frac{\log\left(fx+e\right)^2+2\,\log\left(fx+e\right)\log\left(c\right)}{df}\right)$$
$$+3\,a^2h^2i\left(\frac{x}{df}-\frac{e\log\left(fx+e\right)}{df^2}\right)-\frac{1}{6}\,a^2i^3\left(\frac{6\,e^3\log\left(fx+e\right)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)$$
$$+\frac{3}{2}\,a^2hi^2\left(\frac{2\,e^2\log\left(fx+e\right)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)+\frac{b^2h^3\log\left(cfx+ce\right)^3}{3\,df}$$
$$+\frac{2\,abh^3\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}+\frac{a^2h^3\log\left(dfx+de\right)}{df}$$
$$+\frac{3\,\left(e\log\left(fx+e\right)^2-2\,fx+2\,e\log\left(fx+e\right)\right)abh^2i}{df^2}$$
$$-\frac{3\,\left(f^2x^2+2\,e^2\log\left(fx+e\right)^2-6\,efx+6\,e^2\log\left(fx+e\right)\right)abhi^2}{2\,df^3}$$
$$-\frac{\left(c^2e\log\left(cfx+ce\right)^3-3\,(cfx+ce)\left(c\log\left(cfx+ce\right)^2-2\,c\log\left(cfx+ce\right)+2\,c\right)\right)b^2h^2i}{c^2df^2}$$
$$-\frac{\left(4\,f^3x^3-15\,ef^2x^2-18\,e^3\log\left(fx+e\right)^2+66\,e^2fx-66\,e^3\log\left(fx+e\right)\right)abi^3}{18\,df^4}$$
$$+\frac{\left(4\,c^3e^2\log\left(cfx+ce\right)^3+3\,(cfx+ce)^2\left(2\,c\log\left(cfx+ce\right)^2-2\,c\log\left(cfx+ce\right)+c\right)-24\,\left(c^2e\log\left(cfx+\right.\right.}{4\,c^3df^3}$$
$$-\frac{\left(36\,c^4e^3\log\left(cfx+ce\right)^3-4\,(cfx+ce)^3\left(9\,c\log\left(cfx+ce\right)^2-6\,c\log\left(cfx+ce\right)+2\,c\right)+81\,\left(2\,c^2e\log\left(c\right.\right.}{}$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxim
a")
```

output
```
6*a*b*h^2*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - 1/3*a*b*
i^3*(6*e^3*log(f*x + e)/(d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3
))*log(c*f*x + c*e) + 3*a*b*h*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2
*e*x)/(d*f^2))*log(c*f*x + c*e) - a*b*h^3*(2*log(c*f*x + c*e)*log(d*f*x +
d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(c))/(d*f)) + 3*a^2*h^2*i
*(x/(d*f) - e*log(f*x + e)/(d*f^2)) - 1/6*a^2*i^3*(6*e^3*log(f*x + e)/(d*f
^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3/2*a^2*h*i^2*(2*e^2*lo
g(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + 1/3*b^2*h^3*log(c*f*x + c*
e)^3/(d*f) + 2*a*b*h^3*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a^2*h^3*l
og(d*f*x + d*e)/(d*f) + 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*a*
b*h^2*i/(d*f^2) - 3/2*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*lo
g(f*x + e))*a*b*h*i^2/(d*f^3) - (c^2*e*log(c*f*x + c*e)^3 - 3*(c*f*x + c*e
)*(c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2*h^2*i/(c^2*d*f^
2) - 1/18*(4*f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x -
 66*e^3*log(f*x + e))*a*b*i^3/(d*f^4) + 1/4*(4*c^3*e^2*log(c*f*x + c*e)^3
+ 3*(c*f*x + c*e)^2*(2*c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + c) -
24*(c^2*e*log(c*f*x + c*e)^2 - 2*c^2*e*log(c*f*x + c*e) + 2*c^2*e)*(c*f*x
+ c*e))*b^2*h*i^2/(c^3*d*f^3) - 1/108*(36*c^4*e^3*log(c*f*x + c*e)^3 - 4*(
c*f*x + c*e)^3*(9*c*log(c*f*x + c*e)^2 - 6*c*log(c*f*x + c*e) + 2*c) + 81*
(2*c^2*e*log(c*f*x + c*e)^2 - 2*c^2*e*log(c*f*x + c*e) + c^2*e)*(c*f*x ...
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.66

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,b^2i^3x^3}{df} + \frac{3\,(3\,b^2fhi^2 - b^2ei^3)x^2}{df^2} + \frac{6\,(3\,b^2f^2h^2i - 3\,b^2efhi^2 + b^2e^2i^3)x}{df^3} + \frac{6\,abf^3h^3 - 18\,abef^2h^2i + 1\cdots}{\cdots}\right.$$
$$+ ce)^2 + \frac{(9\,a^2i^3 - 6\,abi^3 + 2\,b^2i^3)x^3}{27\,df}$$
$$+ \frac{1}{18}\left(\frac{4\,(3\,abi^3 - b^2i^3)x^3}{df} + \frac{3\,(18\,abfhi^2 - 9\,b^2fhi^2 - 6\,abei^3 + 5\,b^2ei^3)x^2}{df^2} + \frac{6\,(18\,abf^2h^2i - 18\,b^2f^2h^2i - \cdots}{\cdots}\right.$$
$$+ ce) + \frac{(54\,a^2fhi^2 - 54\,abfhi^2 + 27\,b^2fhi^2 - 18\,a^2ei^3 + 30\,abei^3 - 19\,b^2ei^3)x^2}{36\,df^2}$$
$$+ \frac{(b^2f^3h^3 - 3\,b^2ef^2h^2i + 3\,b^2e^2fhi^2 - b^2e^3i^3)\log(cfx+ce)^3}{3\,df^4}$$
$$+ \frac{(54\,a^2f^2h^2i - 108\,abf^2h^2i + 108\,b^2f^2h^2i - 54\,a^2efhi^2 + 162\,abefhi^2 - 189\,b^2efhi^2 + 18\,a^2e^2i^3 - 66\,ab\cdots}{18\,df^3}$$
$$+ \frac{(18\,a^2f^3h^3 - 54\,a^2ef^2h^2i + 108\,abef^2h^2i - 108\,b^2ef^2h^2i + 54\,a^2e^2fhi^2 - 162\,abe^2fhi^2 + 189\,b^2e^2fhi^2 \cdots}{18\,df^4}$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac"
)
```

output
```
1/6*(2*b^2*i^3*x^3/(d*f) + 3*(3*b^2*f*h*i^2 - b^2*e*i^3)*x^2/(d*f^2) + 6*(
3*b^2*f^2*h^2*i - 3*b^2*e*f*h*i^2 + b^2*e^2*i^3)*x/(d*f^3) + (6*a*b*f^3*h^
3 - 18*a*b*e*f^2*h^2*i + 18*b^2*e*f^2*h^2*i + 18*a*b*e^2*f*h*i^2 - 27*b^2*
e^2*f*h*i^2 - 6*a*b*e^3*i^3 + 11*b^2*e^3*i^3)/(d*f^4))*log(c*f*x + c*e)^2
+ 1/27*(9*a^2*i^3 - 6*a*b*i^3 + 2*b^2*i^3)*x^3/(d*f) + 1/18*(4*(3*a*b*i^3
- b^2*i^3)*x^3/(d*f) + 3*(18*a*b*f*h*i^2 - 9*b^2*f*h*i^2 - 6*a*b*e*i^3 + 5
*b^2*e*i^3)*x^2/(d*f^2) + 6*(18*a*b*f^2*h^2*i - 18*b^2*f^2*h^2*i - 18*a*b*
e*f*h*i^2 + 27*b^2*e*f*h*i^2 + 6*a*b*e^2*i^3 - 11*b^2*e^2*i^3)*x/(d*f^3))*
log(c*f*x + c*e) + 1/36*(54*a^2*f*h*i^2 - 54*a*b*f*h*i^2 + 27*b^2*f*h*i^2
- 18*a^2*e*i^3 + 30*a*b*e*i^3 - 19*b^2*e*i^3)*x^2/(d*f^2) + 1/3*(b^2*f^3*h
^3 - 3*b^2*e*f^2*h^2*i + 3*b^2*e^2*f*h*i^2 - b^2*e^3*i^3)*log(c*f*x + c*e)
^3/(d*f^4) + 1/18*(54*a^2*f^2*h^2*i - 108*a*b*f^2*h^2*i + 108*b^2*f^2*h^2*
i - 54*a^2*e*f*h*i^2 + 162*a*b*e*f*h*i^2 - 189*b^2*e*f*h*i^2 + 18*a^2*e^2*
i^3 - 66*a*b*e^2*i^3 + 85*b^2*e^2*i^3)*x/(d*f^3) + 1/18*(18*a^2*f^3*h^3 -
54*a^2*e*f^2*h^2*i + 108*a*b*e*f^2*h^2*i - 108*b^2*e*f^2*h^2*i + 54*a^2*e^
2*f*h*i^2 - 162*a*b*e^2*f*h*i^2 + 189*b^2*e^2*f*h*i^2 - 18*a^2*e^3*i^3 + 6
6*a*b*e^3*i^3 - 85*b^2*e^3*i^3)*log(f*x + e)/(d*f^4)
```

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.184.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.73

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$

$$= x^2\left(\frac{i^2\,(18\,a^2\,f\,h-5\,b^2\,e\,i+9\,b^2\,f\,h+6\,a\,b\,e\,i-18\,a\,b\,f\,h)}{12\,d\,f^2}-\frac{e\,i^3\,(9\,a^2-6\,a\,b+2\,b^2)}{18\,d\,f^2}\right)$$
$$+\ln(c\,(e+f\,x))^2\left(f\left(\frac{b^2\,i^3\,x^3}{3\,d\,f^2}-\frac{b^2\,i^2\,x^2\,(e\,i-3\,f\,h)}{2\,d\,f^3}+\frac{b^2\,i\,x\,(e^2\,i^2-3\,e\,f\,h\,i+3\,f^2\,h^2)}{d\,f^4}\right)\right.$$
$$\left.+\frac{11\,b^2\,e^3\,i^3-27\,b^2\,e^2\,f\,h\,i^2+18\,b^2\,e\,f^2\,h^2\,i-6\,a\,b\,e^3\,i^3+18\,a\,b\,e^2\,f\,h\,i^2-18\,a\,b\,e\,f^2\,h^2\,i+6\,a\,b\,f^3\,h^3}{6\,d\,f^4}\right)$$
$$+x\left(\frac{54\,a^2\,f^2\,h^2\,i-36\,a\,b\,e^2\,i^3+108\,a\,b\,e\,f\,h\,i^2-108\,a\,b\,f^2\,h^2\,i+66\,b^2\,e^2\,i^3-162\,b^2\,e\,f\,h\,i^2+108\,b^2\,f^2\,h}{18\,d\,f^3}\right.$$
$$\left.-\frac{e\left(\frac{i^2\,(18\,a^2\,f\,h-5\,b^2\,e\,i+9\,b^2\,f\,h+6\,a\,b\,e\,i-18\,a\,b\,f\,h)}{6\,d\,f^2}-\frac{e\,i^3\,(9\,a^2-6\,a\,b+2\,b^2)}{9\,d\,f^2}\right)}{f}\right)$$
$$+f\,\ln(c\,(e+f\,x))\left(\frac{x^2\,(5\,e\,b^2\,i^3-9\,f\,h\,b^2\,i^2-6\,a\,e\,b\,i^3+18\,a\,f\,h\,b\,i^2)}{6\,d\,f^3}\right.$$
$$-\frac{x\,(11\,b^2\,e^2\,i^3-27\,b^2\,e\,f\,h\,i^2+18\,b^2\,f^2\,h^2\,i-6\,a\,b\,e^2\,i^3+18\,a\,b\,e\,f\,h\,i^2-18\,a\,b\,f^2\,h^2\,i)}{3\,d\,f^4}$$
$$\left.+\frac{2\,b\,i^3\,x^3\,(3\,a-b)}{9\,d\,f^2}\right)$$
$$-\frac{\ln(e+f\,x)\,(18\,a^2\,e^3\,i^3-54\,a^2\,e^2\,f\,h\,i^2+54\,a^2\,e\,f^2\,h^2\,i-18\,a^2\,f^3\,h^3-66\,a\,b\,e^3\,i^3+162\,a\,b\,e^2\,f\,h\,i^2-1}{18\,d\,f^4}$$
$$+\frac{i^3\,x^3\,(9\,a^2-6\,a\,b+2\,b^2)}{27\,d\,f}-\frac{b^2\ln(c\,(e+f\,x))^3\,(e^3\,i^3-3\,e^2\,f\,h\,i^2+3\,e\,f^2\,h^2\,i-f^3\,h^3)}{3\,d\,f^4}$$

input
```
int(((h + i*x)^3*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

output
```
x^2*((i^2*(18*a^2*f*h - 5*b^2*e*i + 9*b^2*f*h + 6*a*b*e*i - 18*a*b*f*h))/(
12*d*f^2) - (e*i^3*(9*a^2 - 6*a*b + 2*b^2))/(18*d*f^2)) + log(c*(e + f*x))
^2*(f*((b^2*i^3*x^3)/(3*d*f^2) - (b^2*i^2*x^2*(e*i - 3*f*h))/(2*d*f^3) + (
b^2*i*x*(e^2*i^2 + 3*f^2*h^2 - 3*e*f*h*i))/(d*f^4)) + (11*b^2*e^3*i^3 - 6*
a*b*e^3*i^3 + 6*a*b*f^3*h^3 + 18*b^2*e*f^2*h^2*i - 27*b^2*e^2*f*h*i^2 - 18
*a*b*e*f^2*h^2*i + 18*a*b*e^2*f*h*i^2)/(6*d*f^4)) + x*((66*b^2*e^2*i^3 + 5
4*a^2*f^2*h^2*i + 108*b^2*f^2*h^2*i - 36*a*b*e^2*i^3 - 108*a*b*f^2*h^2*i -
 162*b^2*e*f*h*i^2 + 108*a*b*e*f*h*i^2)/(18*d*f^3) - (e*((i^2*(18*a^2*f*h
- 5*b^2*e*i + 9*b^2*f*h + 6*a*b*e*i - 18*a*b*f*h))/(6*d*f^2) - (e*i^3*(9*a
^2 - 6*a*b + 2*b^2))/(9*d*f^2)))/f) + f*log(c*(e + f*x))*((x^2*(5*b^2*e*i^
3 - 6*a*b*e*i^3 - 9*b^2*f*h*i^2 + 18*a*b*f*h*i^2))/(6*d*f^3) - (x*(11*b^2*
e^2*i^3 + 18*b^2*f^2*h^2*i - 6*a*b*e^2*i^3 - 18*a*b*f^2*h^2*i - 27*b^2*e*f
*h*i^2 + 18*a*b*e*f*h*i^2))/(3*d*f^4) + (2*b*i^3*x^3*(3*a - b))/(9*d*f^2))
 - (log(e + f*x)*(18*a^2*e^3*i^3 - 18*a^2*f^3*h^3 + 85*b^2*e^3*i^3 - 66*a*
b*e^3*i^3 + 54*a^2*e*f^2*h^2*i - 54*a^2*e^2*f*h*i^2 + 108*b^2*e*f^2*h^2*i
- 189*b^2*e^2*f*h*i^2 - 108*a*b*e*f^2*h^2*i + 162*a*b*e^2*f*h*i^2))/(18*d*
f^4) + (i^3*x^3*(9*a^2 - 6*a*b + 2*b^2))/(27*d*f) - (b^2*log(c*(e + f*x))^
3*(e^3*i^3 - f^3*h^3 + 3*e*f^2*h^2*i - 3*e^2*f*h*i^2))/(3*d*f^4)
```

3.184. $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.185 $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.185.1 Optimal result

Integrand size = 32, antiderivative size = 238

$$\begin{aligned}\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = & -\frac{4abi(fh-ei)x}{df^2}+\frac{4b^2i(fh-ei)x}{df^2}+\frac{b^2i^2(e+fx)^2}{4df^3}\\ & -\frac{4b^2i(fh-ei)(e+fx)\log(c(e+fx))}{df^3}\\ & -\frac{bi^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3}\\ & +\frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2}{df^3}\\ & +\frac{i^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^3}\\ & +\frac{(fh-ei)^2(a+b\log(c(e+fx)))^3}{3bdf^3}\end{aligned}$$

output
```
-4*a*b*i*(-e*i+f*h)*x/d/f^2+4*b^2*i*(-e*i+f*h)*x/d/f^2+1/4*b^2*i^2*(f*x+e)
^2/d/f^3-4*b^2*i*(-e*i+f*h)*(f*x+e)*ln(c*(f*x+e))/d/f^3-1/2*b*i^2*(f*x+e)^
2*(a+b*ln(c*(f*x+e)))/d/f^3+2*i*(-e*i+f*h)*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d
/f^3+1/2*i^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))^2/d/f^3+1/3*(-e*i+f*h)^2*(a+b*l
n(c*(f*x+e)))^3/b/d/f^3
```

3.185.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{24i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2+6i^2(e+fx)^2(a+b\log(c(e+fx)))^2+\frac{4(fh-ei)^2(a+b\log(c(e+fx$$

input
```
Integrate[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

output
```
(24*i*(f*h - e*i)*(e + f*x)*(a + b*Log[c*(e + f*x)])^2 + 6*i^2*(e + f*x)^2
*(a + b*Log[c*(e + f*x)])^2 + (4*(f*h - e*i)^2*(a + b*Log[c*(e + f*x)])^3)
/b - 48*b*i*(f*h - e*i)*((a - b)*f*x + b*(e + f*x)*Log[c*(e + f*x)]) + 3*b
*i^2*(b*f*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])))/(12*d*f
^3)
```

3.185.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2858, 27, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^2(a+b\log(c(e+fx)))^2}{df^2(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)}{df^3}$$

$$\Big\downarrow \text{2788}$$

$$\frac{i\int (fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2d(e+fx)+(fh-ei)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)}{df^3}$$

$\big\downarrow$ 2767

$$\frac{(fh-ei)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\int \left(fh\left(1-\frac{ei}{fh}\right)(a+b\log(c(e+fx)))^2+i(e+fx)(a+b\right.}{df^3}$$

$\big\downarrow$ 2009

$$\frac{(fh-ei)\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(fh-ei)(a+b\log(c(e+fx)))^2-\frac{1}{2}bi(e+fx)^2\right.}{}$$

$\big\downarrow$ 2788

$$\frac{(fh-ei)\left((fh-ei)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\int (a+b\log(c(e+fx)))^2d(e+fx)\right)+i\left((e+fx)(fh-ei)(a\right.}{}$$

$\big\downarrow$ 2733

$$\frac{(fh-ei)\left((fh-ei)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b\int (a+b\log(c(e+fx))\right.\right.}{}$$

$\big\downarrow$ 2009

$$\frac{(fh-ei)\left((fh-ei)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+fx)+b(e+fx)\right.\right.}{}$$

$\big\downarrow$ 2739

$$\frac{(fh-ei)\left(\frac{(fh-ei)\int (a+b\log(c(e+fx)))^2d(a+b\log(c(e+fx)))}{b}+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+fx)+b(e+\right.\right.}{}$$

$\big\downarrow$ 15

$$\frac{i\left((e+fx)(fh-ei)(a+b\log(c(e+fx)))^2-\frac{1}{2}bi(e+fx)^2(a+b\log(c(e+fx)))+\frac{1}{2}i(e+fx)^2(a+b\log(c(e+fx\right.}{}$$

input
```
Int[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

3.185. $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

output
```
(i*(-2*a*b*(f*h - e*i)*(e + f*x) + 2*b^2*(f*h - e*i)*(e + f*x) + (b^2*i*(e
 + f*x)^2)/4 - 2*b^2*(f*h - e*i)*(e + f*x)*Log[c*(e + f*x)] - (b*i*(e + f*
x)^2*(a + b*Log[c*(e + f*x)]))/2 + (f*h - e*i)*(e + f*x)*(a + b*Log[c*(e +
 f*x)])^2 + (i*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2)/2) + (f*h - e*i)*((
(f*h - e*i)*(a + b*Log[c*(e + f*x)])^3)/(3*b) + i*((e + f*x)*(a + b*Log[c*
(e + f*x)])^2 - 2*b*(a*(e + f*x) - b*(e + f*x) + b*(e + f*x)*Log[c*(e + f*
x)]))))/(d*f^3)
```

3.185.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

3.185. $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.185.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.67

method	result
norman	$\frac{(2a^2e^2i^2-4a^2efhi+2a^2f^2h^2-6abe^2i^2+8abefhi+7b^2e^2i^2-8b^2efhi)\ln(c(fx+e))}{2df^3}+\frac{b(2ae^2i^2-4aefhi+2af^2h^2-3b\cdots}{2df^3\cdots}$
risch	$\frac{b^2\ln(c(fx+e))^3e^2i^2}{3df^3}-\frac{2b^2\ln(c(fx+e))^3ehi}{3df^2}+\frac{b^2\ln(c(fx+e))^3h^2}{3df}+\frac{b(bf^2i^2x^2-2befi^2x+4bf^2hix+2ae^2i^2-4ae\cdots}{2d\cdots}$
parts	$\frac{a^2\left(\frac{i\left(\frac{1}{2}fix^2-xei+2xfh\right)}{f^2}+\frac{\left(e^2i^2-2efhi+f^2h^2\right)\ln(fx+e)}{f^3}\right)}{d}+\frac{b^2\left(\frac{ce^2i^2\ln(cfx+ce)^3}{3f^2}-\frac{2cehi\ln(cfx+ce)^3}{3f}+\frac{ch^2\ln(cfx+ce)^3}{3}\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{ca^2e^2i^2\ln(cfx+ce)}{f^2d}-\frac{2ca^2ehi\ln(cfx+ce)}{fd}+\frac{ca^2h^2\ln(cfx+ce)}{d}-\frac{2a^2ei^2(cfx+ce)}{f^2d}+\frac{2a^2hi(cfx+ce)}{fd}+\frac{a^2i^2(cfx+ce)^2}{2cf^2d}+\frac{cabe^2i^2\ln\cdots}{f^2\cdots}}{\cdots}$
default	$\frac{\frac{ca^2e^2i^2\ln(cfx+ce)}{f^2d}-\frac{2ca^2ehi\ln(cfx+ce)}{fd}+\frac{ca^2h^2\ln(cfx+ce)}{d}-\frac{2a^2ei^2(cfx+ce)}{f^2d}+\frac{2a^2hi(cfx+ce)}{fd}+\frac{a^2i^2(cfx+ce)^2}{2cf^2d}+\frac{cabe^2i^2\ln\cdots}{f^2\cdots}}{\cdots}$
parallelrisch	$\frac{-66abe^2i^2-12a^2efi^2x+24a^2f^2hix-42b^2efi^2x+48b^2f^2hix-48a^2efhi+36abefi^2x+4\ln(c(fx+e))^3b^2e^2i^2+4\ln(c(fx\cdots}{\cdots}$

input
```
int((i*x+h)^2*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^2*e^2*i^2-4*a^2*e*f*h*i+2*a^2*f^2*h^2-6*a*b*e^2*i^2+8*a*b*e*f*h*i
+7*b^2*e^2*i^2-8*b^2*e*f*h*i)/d/f^3*ln(c*(f*x+e))+1/2*b*(2*a*e^2*i^2-4*a*e
*f*h*i+2*a*f^2*h^2-3*b*e^2*i^2+4*b*e*f*h*i)/d/f^3*ln(c*(f*x+e))^2+1/3*b^2*
(e^2*i^2-2*e*f*h*i+f^2*h^2)/d/f^3*ln(c*(f*x+e))^3-1/2*i*(2*a^2*e*i-4*a^2*f
*h-6*a*b*e*i+8*a*b*f*h+7*b^2*e*i-8*b^2*f*h)/d/f^2*x+1/4*i^2*(2*a^2-2*a*b+b
^2)/d/f*x^2+1/2*b^2*i^2/d/f*x^2*ln(c*(f*x+e))^2-b*i*(2*a*e*i-4*a*f*h-3*b*e
*i+4*b*f*h)/d/f^2*x*ln(c*(f*x+e))+1/2*b*i^2*(2*a-b)/d/f*x^2*ln(c*(f*x+e))-
b^2*i*(e*i-2*f*h)/d/f^2*x*ln(c*(f*x+e))^2
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.41

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{3\left(2a^2-2ab+b^2\right)f^2i^2x^2+4\left(b^2f^2h^2-2b^2efhi+b^2e^2i^2\right)\log\left(cfx+ce\right)^3+6\left(b^2f^2i^2x^2+2abf^2h^2-4\left(a\right.\right.}{}$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="frica
s")
```

output
```
1/12*(3*(2*a^2 - 2*a*b + b^2)*f^2*i^2*x^2 + 4*(b^2*f^2*h^2 - 2*b^2*e*f*h*i
 + b^2*e^2*i^2)*log(c*f*x + c*e)^3 + 6*(b^2*f^2*i^2*x^2 + 2*a*b*f^2*h^2 -
4*(a*b - b^2)*e*f*h*i + (2*a*b - 3*b^2)*e^2*i^2 + 2*(2*b^2*f^2*h*i - b^2*e
*f*i^2)*x)*log(c*f*x + c*e)^2 + 6*(4*(a^2 - 2*a*b + 2*b^2)*f^2*h*i - (2*a^
2 - 6*a*b + 7*b^2)*e*f*i^2)*x + 6*((2*a*b - b^2)*f^2*i^2*x^2 + 2*a^2*f^2*h
^2 - 4*(a^2 - 2*a*b + 2*b^2)*e*f*h*i + (2*a^2 - 6*a*b + 7*b^2)*e^2*i^2 + 2
*(4*(a*b - b^2)*f^2*h*i - (2*a*b - 3*b^2)*e*f*i^2)*x)*log(c*f*x + c*e))/(d
*f^3)
```

3.185.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(218) = 436.

Time = 0.60 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.99

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= x^2\left(\frac{a^2i^2}{2df}-\frac{abi^2}{2df}+\frac{b^2i^2}{4df}\right)+x\left(-\frac{a^2ei^2}{df^2}+\frac{2a^2hi}{df}+\frac{3abei^2}{df^2}-\frac{4abhi}{df}-\frac{7b^2ei^2}{2df^2}+\frac{4b^2hi}{df}\right)$$
$$+\frac{\left(-4abei^2x+8abfhix+2abfi^2x^2+6b^2ei^2x-8b^2fhix-b^2fi^2x^2\right)\log\left(c(e+fx)\right)}{2df^2}$$
$$+\frac{\left(b^2e^2i^2-2b^2efhi+b^2f^2h^2\right)\log\left(c(e+fx)\right)^3}{3df^3}$$
$$+\frac{\left(2a^2e^2i^2-4a^2efhi+2a^2f^2h^2-6abe^2i^2+8abefhi+7b^2e^2i^2-8b^2efhi\right)\log\left(e+fx\right)}{2df^3}$$
$$+\frac{\left(2abe^2i^2-4abefhi+2abf^2h^2-3b^2e^2i^2+4b^2efhi-2b^2efi^2x+4b^2f^2hix+b^2f^2i^2x^2\right)\log\left(c(e+fx)\right)^2}{2df^3}$$

input
```
integrate((i*x+h)**2*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

output
```
x**2*(a**2*i**2/(2*d*f) - a*b*i**2/(2*d*f) + b**2*i**2/(4*d*f)) + x*(-a**2
*e*i**2/(d*f**2) + 2*a**2*h*i/(d*f) + 3*a*b*e*i**2/(d*f**2) - 4*a*b*h*i/(d
*f) - 7*b**2*e*i**2/(2*d*f**2) + 4*b**2*h*i/(d*f)) + (-4*a*b*e*i**2*x + 8*
a*b*f*h*i*x + 2*a*b*f*i**2*x**2 + 6*b**2*e*i**2*x - 8*b**2*f*h*i*x - b**2*
f*i**2*x**2)*log(c*(e + f*x))/(2*d*f**2) + (b**2*e**2*i**2 - 2*b**2*e*f*h*
i + b**2*f**2*h**2)*log(c*(e + f*x))**3/(3*d*f**3) + (2*a**2*e**2*i**2 - 4
*a**2*e*f*h*i + 2*a**2*f**2*h**2 - 6*a*b*e**2*i**2 + 8*a*b*e*f*h*i + 7*b**
2*e**2*i**2 - 8*b**2*e*f*h*i)*log(e + f*x)/(2*d*f**3) + (2*a*b*e**2*i**2 -
 4*a*b*e*f*h*i + 2*a*b*f**2*h**2 - 3*b**2*e**2*i**2 + 4*b**2*e*f*h*i - 2*b
**2*e*f*i**2*x + 4*b**2*f**2*h*i*x + b**2*f**2*i**2*x**2)*log(c*(e + f*x))
**2/(2*d*f**3)
```

3.185.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(230) = 460$.

Time = 0.24 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.46

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = 4\,abhi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce)$$
$$+ abi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce)$$
$$- abh^2\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right)$$
$$+ 2\,a^2hi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{1}{2}\,a^2i^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)$$
$$+ \frac{b^2h^2\log(cfx+ce)^3}{3\,df}+\frac{2\,abh^2\log(cfx+ce)\log(dfx+de)}{df}$$
$$+ \frac{a^2h^2\log(dfx+de)}{df}+\frac{2\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)abhi}{df^2}$$
$$- \frac{\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)abi^2}{2\,df^3}$$
$$- \frac{2\left(c^2e\log(cfx+ce)^3-3\,(cfx+ce)\left(c\log(cfx+ce)^2-2\,c\log(cfx+ce)+2\,c\right)\right)b^2hi}{3\,c^2df^2}$$
$$+ \frac{\left(4\,c^3e^2\log(cfx+ce)^3+3\,(cfx+ce)^2\left(2\,c\log(cfx+ce)^2-2\,c\log(cfx+ce)+c\right)-24\,\left(c^2e\log(cfx+\right.\right.}{12\,c^3df^3}$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxim
a")
```

output
```
4*a*b*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + a*b*i^2*(2
*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x + c*e) - a*
b*h^2*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log
(f*x + e)*log(c))/(d*f)) + 2*a^2*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) +
1/2*a^2*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + 1/3*b
^2*h^2*log(c*f*x + c*e)^3/(d*f) + 2*a*b*h^2*log(c*f*x + c*e)*log(d*f*x + d
*e)/(d*f) + a^2*h^2*log(d*f*x + d*e)/(d*f) + 2*(e*log(f*x + e)^2 - 2*f*x +
 2*e*log(f*x + e))*a*b*h*i/(d*f^2) - 1/2*(f^2*x^2 + 2*e^2*log(f*x + e)^2 -
 6*e*f*x + 6*e^2*log(f*x + e))*a*b*i^2/(d*f^3) - 2/3*(c^2*e*log(c*f*x + c*
e)^3 - 3*(c*f*x + c*e)*(c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c)
)*b^2*h*i/(c^2*d*f^2) + 1/12*(4*c^3*e^2*log(c*f*x + c*e)^3 + 3*(c*f*x + c*
e)^2*(2*c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + c) - 24*(c^2*e*log(c
*f*x + c*e)^2 - 2*c^2*e*log(c*f*x + c*e) + 2*c^2*e)*(c*f*x + c*e))*b^2*i^2
/(c^3*d*f^3)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.76

$$
\begin{aligned}
&\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx \\
&= \frac{1}{2}\left(\frac{b^2i^2x^2}{df} + \frac{2\,(2\,b^2fhi - b^2ei^2)x}{df^2} + \frac{2\,abf^2h^2 - 4\,abefhi + 4\,b^2efhi + 2\,abe^2i^2 - 3\,b^2e^2i^2}{df^3}\right)\log\left(cfx\right. \\
&\quad \left.+ce\right)^2 + \frac{1}{2}\left(\frac{(2\,abi^2 - b^2i^2)x^2}{df} + \frac{2\,(4\,abfhi - 4\,b^2fhi - 2\,abei^2 + 3\,b^2ei^2)x}{df^2}\right)\log\left(cfx+ce\right) \\
&\quad + \frac{(2\,a^2i^2 - 2\,abi^2 + b^2i^2)x^2}{4\,df} + \frac{(b^2f^2h^2 - 2\,b^2efhi + b^2e^2i^2)\log\left(cfx+ce\right)^3}{3\,df^3} \\
&\quad + \frac{(4\,a^2fhi - 8\,abfhi + 8\,b^2fhi - 2\,a^2ei^2 + 6\,abei^2 - 7\,b^2ei^2)x}{2\,df^2} \\
&\quad + \frac{(2\,a^2f^2h^2 - 4\,a^2efhi + 8\,abefhi - 8\,b^2efhi + 2\,a^2e^2i^2 - 6\,abe^2i^2 + 7\,b^2e^2i^2)\log\left(fx+e\right)}{2\,df^3}
\end{aligned}
$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac"
)
```

output
```
1/2*(b^2*i^2*x^2/(d*f) + 2*(2*b^2*f*h*i - b^2*e*i^2)*x/(d*f^2) + (2*a*b*f^
2*h^2 - 4*a*b*e*f*h*i + 4*b^2*e*f*h*i + 2*a*b*e^2*i^2 - 3*b^2*e^2*i^2)/(d*
f^3))*log(c*f*x + c*e)^2 + 1/2*((2*a*b*i^2 - b^2*i^2)*x^2/(d*f) + 2*(4*a*b
*f*h*i - 4*b^2*f*h*i - 2*a*b*e*i^2 + 3*b^2*e*i^2)*x/(d*f^2))*log(c*f*x + c
*e) + 1/4*(2*a^2*i^2 - 2*a*b*i^2 + b^2*i^2)*x^2/(d*f) + 1/3*(b^2*f^2*h^2 -
 2*b^2*e*f*h*i + b^2*e^2*i^2)*log(c*f*x + c*e)^3/(d*f^3) + 1/2*(4*a^2*f*h*
i - 8*a*b*f*h*i + 8*b^2*f*h*i - 2*a^2*e*i^2 + 6*a*b*e*i^2 - 7*b^2*e*i^2)*x
/(d*f^2) + 1/2*(2*a^2*f^2*h^2 - 4*a^2*e*f*h*i + 8*a*b*e*f*h*i - 8*b^2*e*f*
h*i + 2*a^2*e^2*i^2 - 6*a*b*e^2*i^2 + 7*b^2*e^2*i^2)*log(f*x + e)/(d*f^3)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.71

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= x\left(\frac{i\,(2\,a^2\,f\,h-3\,b^2\,e\,i+4\,b^2\,f\,h+2\,a\,b\,e\,i-4\,a\,b\,f\,h)}{d\,f^2}-\frac{e\,i^2\,(2\,a^2-2\,a\,b+b^2)}{2\,d\,f^2}\right)$$
$$+\ln\left(c\,(e+f\,x)\right)^2\left(f\left(\frac{b^2\,i^2\,x^2}{2\,d\,f^2}-\frac{b^2\,i\,x\,(e\,i-2\,f\,h)}{d\,f^3}\right)+\frac{-3\,b^2\,e^2\,i^2+4\,b^2\,e\,f\,h\,i+2\,a\,b\,e^2\,i^2-4\,a\,b\,e\,f\,h\,i+2\,a\,b\,f^2\,h^2}{2\,d\,f^3}\right)$$
$$+f\,\ln\left(c\,(e+f\,x)\right)\left(\frac{x\,(3\,e\,b^2\,i^2-4\,f\,h\,b^2\,i-2\,a\,e\,b\,i^2+4\,a\,f\,h\,b\,i)}{d\,f^3}+\frac{b\,i^2\,x^2\,(2\,a-b)}{2\,d\,f^2}\right)$$
$$+\frac{\ln\left(e+f\,x\right)\,(2\,a^2\,e^2\,i^2-4\,a^2\,e\,f\,h\,i+2\,a^2\,f^2\,h^2-6\,a\,b\,e^2\,i^2+8\,a\,b\,e\,f\,h\,i+7\,b^2\,e^2\,i^2-8\,b^2\,e\,f\,h\,i)}{2\,d\,f^3}$$
$$+\frac{b^2\ln\left(c\,(e+f\,x)\right)^3\,(e^2\,i^2-2\,e\,f\,h\,i+f^2\,h^2)}{3\,d\,f^3}+\frac{i^2\,x^2\,(2\,a^2-2\,a\,b+b^2)}{4\,d\,f}$$

input
```
int(((h + i*x)^2*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

output
```
x*((i*(2*a^2*f*h - 3*b^2*e*i + 4*b^2*f*h + 2*a*b*e*i - 4*a*b*f*h))/(d*f^2)
 - (e*i^2*(2*a^2 - 2*a*b + b^2))/(2*d*f^2)) + log(c*(e + f*x))^2*(f*((b^2*
i^2*x^2)/(2*d*f^2) - (b^2*i*x*(e*i - 2*f*h))/(d*f^3)) + (2*a*b*e^2*i^2 - 3
*b^2*e^2*i^2 + 2*a*b*f^2*h^2 + 4*b^2*e*f*h*i - 4*a*b*e*f*h*i)/(2*d*f^3)) +
 f*log(c*(e + f*x))*((x*(3*b^2*e*i^2 - 2*a*b*e*i^2 - 4*b^2*f*h*i + 4*a*b*f
*h*i))/(d*f^3) + (b*i^2*x^2*(2*a - b))/(2*d*f^2)) + (log(e + f*x)*(2*a^2*e
^2*i^2 + 2*a^2*f^2*h^2 + 7*b^2*e^2*i^2 - 6*a*b*e^2*i^2 - 4*a^2*e*f*h*i - 8
*b^2*e*f*h*i + 8*a*b*e*f*h*i))/(2*d*f^3) + (b^2*log(c*(e + f*x))^3*(e^2*i^
2 + f^2*h^2 - 2*e*f*h*i))/(3*d*f^3) + (i^2*x^2*(2*a^2 - 2*a*b + b^2))/(4*d
*f)
```

3.185. $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.186 $\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.186.1 Optimal result

Integrand size = 30, antiderivative size = 113

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = -\frac{2abix}{df}+\frac{2b^2ix}{df}-\frac{2b^2i(e+fx)\log(c(e+fx))}{df^2}+\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{df^2}+\frac{(fh-ei)(a+b\log(c(e+fx)))^3}{3bdf^2}$$

output
```
-2*a*b*i*x/d/f+2*b^2*i*x/d/f-2*b^2*i*(f*x+e)*ln(c*(f*x+e))/d/f^2+i*(f*x+e)
*(a+b*ln(c*(f*x+e)))^2/d/f^2+1/3*(-e*i+f*h)*(a+b*ln(c*(f*x+e)))^3/b/d/f^2
```

3.186.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{-6(a-b)bfix-6b^2i(e+fx)\log(c(e+fx))+3i(e+fx)(a+b\log(c(e+fx)))^2+\frac{(fh-ei)(a+b\log(c(e+fx)))}{b}}{3df^2}$$

input
```
Integrate[((h + i*x)*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

output
```
(-6*(a - b)*b*f*i*x - 6*b^2*i*(e + f*x)*Log[c*(e + f*x)] + 3*i*(e + f*x)*(
a + b*Log[c*(e + f*x)])^2 + ((f*h - e*i)*(a + b*Log[c*(e + f*x)])^3)/b)/(3
*d*f^2)
```

3.186.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2858, 27, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)(a+b\log(c(e+fx)))^2}{df(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)}{df^2}$$

$$\Big\downarrow \text{2788}$$

$$\frac{(fh-ei)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\int (a+b\log(c(e+fx)))^2d(e+fx)}{df^2}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(fh-ei)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b\int (a+b\log(c(e+fx)))d(e+fx)\right.}{df^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fh-ei)\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+fx)+b(e+fx)\log(c(e+\right.}{df^2}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\frac{(fh-ei)\int (a+b\log(c(e+fx)))^2 d(a+b\log(c(e+fx)))}{b}+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+fx)+b(e+fx)\log(c(e-\right.}{df^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{(fh-ei)(a+b\log(c(e+fx)))^3}{3b}+i\left((e+fx)(a+b\log(c(e+fx)))^2-2b(a(e+fx)+b(e+fx)\log(c(e+fx))-b(e+fx\right.}{df^2}$$

input
```
Int[((h + i*x)*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

output
```
(((f*h - e*i)*(a + b*Log[c*(e + f*x)])^3)/(3*b) + i*((e + f*x)*(a + b*Log[
c*(e + f*x)])^2 - 2*b*(a*(e + f*x) - b*(e + f*x) + b*(e + f*x)*Log[c*(e +
f*x)])))/(d*f^2)
```

3.186.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.186.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.56

method	result
norman	$\frac{i(a^2-2ab+2b^2)x}{df}+\frac{b^2ix\ln(c(fx+e))^2}{df}-\frac{(a^2ei-a^2fh-2abei+2b^2ei)\ln(c(fx+e))}{d\,f^2}-\frac{b(aei-afh-bei)\ln(c(fx+e))}{d\,f^2}$
parts	$\frac{a^2\left(\frac{xi}{f}+\frac{(-ei+fh)\ln(fx+e)}{f^2}\right)}{d}+\frac{b^2\left(-\frac{cei\ln(cfx+ce)^3}{3f}+\frac{ch\ln(cfx+ce)^3}{3}+\frac{i\left((cfx+ce)\ln(cfx+ce)^2-2(cfx+ce)\ln(cfx+ce)+2cf\right.}{f}\right.}{dcf}$
risch	$-\frac{b^2\ln(c(fx+e))^3ei}{3d\,f^2}+\frac{b^2\ln(c(fx+e))^3h}{3df}-\frac{b(-bfix+aei-afh-bei)\ln(c(fx+e))^2}{d\,f^2}+\frac{2bi(a-b)x\ln(c(fx+e))}{df}-$
parallelrisch	$3x\ln(c(fx+e))^2b^2fi-\ln(c(fx+e))^3b^2ei+\ln(c(fx+e))^3b^2fh+6x\ln(c(fx+e))abfi-6x\ln(c(fx+e))b^2fi-3\ln(c(fx+e))$
derivativedivides	$\frac{-\frac{a^2cei\ln(cfx+ce)}{fd}+\frac{a^2hc\ln(cfx+ce)}{d}+\frac{a^2i(cfx+ce)}{fd}-\frac{abcei\ln(cfx+ce)^2}{fd}+\frac{abhc\ln(cfx+ce)^2}{d}+\frac{2abi((cfx+ce)\ln(cfx+ce)-cfx-c}{fd}}{cf}$
default	$\frac{-\frac{a^2cei\ln(cfx+ce)}{fd}+\frac{a^2hc\ln(cfx+ce)}{d}+\frac{a^2i(cfx+ce)}{fd}-\frac{abcei\ln(cfx+ce)^2}{fd}+\frac{abhc\ln(cfx+ce)^2}{d}+\frac{2abi((cfx+ce)\ln(cfx+ce)-cfx-c}{fd}}{cf}$

input
```
int((i*x+h)*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
i*(a^2-2*a*b+2*b^2)/d/f*x+b^2*i*x/d/f*ln(c*(f*x+e))^2-(a^2*e*i-a^2*f*h-2*a
*b*e*i+2*b^2*e*i)/d/f^2*ln(c*(f*x+e))-b*(a*e*i-a*f*h-b*e*i)/d/f^2*ln(c*(f*
x+e))^2-1/3*b^2*(e*i-f*h)/d/f^2*ln(c*(f*x+e))^3+2*b*i*(a-b)/d/f*x*ln(c*(f*
x+e))
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{3\left(a^2-2\,ab+2\,b^2\right)fix+\left(b^2fh-b^2ei\right)\log\left(cfx+ce\right)^3+3\left(b^2fix+abfh-\left(ab-b^2\right)ei\right)\log\left(cfx+ce\right)^2+}{3\,df^2}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas"
)
```

output
```
1/3*(3*(a^2 - 2*a*b + 2*b^2)*f*i*x + (b^2*f*h - b^2*e*i)*log(c*f*x + c*e)^
3 + 3*(b^2*f*i*x + a*b*f*h - (a*b - b^2)*e*i)*log(c*f*x + c*e)^2 + 3*(a^2*
f*h + 2*(a*b - b^2)*f*i*x - (a^2 - 2*a*b + 2*b^2)*e*i)*log(c*f*x + c*e))/(
d*f^2)
```

3.186.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.55

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = x\left(\frac{a^2i}{df}-\frac{2abi}{df}+\frac{2b^2i}{df}\right) + \frac{(2abix-2b^2ix)\log\left(c(e+fx)\right)}{df} + \frac{(-b^2ei+b^2fh)\log\left(c(e+fx)\right)^3}{3df^2} - \frac{(a^2ei-a^2fh-2abei+2b^2ei)\log\left(e+fx\right)}{df^2} + \frac{(-abei+abfh+b^2ei+b^2fix)\log\left(c(e+fx)\right)^2}{df^2}$$

input
```
integrate((i*x+h)*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

output
```
x*(a**2*i/(d*f) - 2*a*b*i/(d*f) + 2*b**2*i/(d*f)) + (2*a*b*i*x - 2*b**2*i*
x)*log(c*(e + f*x))/(d*f) + (-b**2*e*i + b**2*f*h)*log(c*(e + f*x))**3/(3*
d*f**2) - (a**2*e*i - a**2*f*h - 2*a*b*e*i + 2*b**2*e*i)*log(e + f*x)/(d*f
**2) + (-a*b*e*i + a*b*f*h + b**2*e*i + b**2*f*i*x)*log(c*(e + f*x))**2/(d
*f**2)
```

3.186.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(111) = 222$.

Time = 0.23 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.69

$$\begin{aligned}
&\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx \\
&= 2\,abi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce) \\
&\quad - abh\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right) \\
&\quad + a^2i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{b^2h\log(cfx+ce)^3}{3\,df}+\frac{2\,abh\log(cfx+ce)\log(dfx+de)}{df} \\
&\quad + \frac{a^2h\log(dfx+de)}{df}+\frac{\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)abi}{df^2} \\
&\quad - \frac{\left(c^2e\log(cfx+ce)^3-3\,(cfx+ce)\left(c\log(cfx+ce)^2-2\,c\log(cfx+ce)+2\,c\right)\right)b^2i}{3\,c^2df^2}
\end{aligned}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima"
)
```

output
```
2*a*b*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - a*b*h*(2*log
(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*lo
g(c))/(d*f)) + a^2*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) + 1/3*b^2*h*log(c*
f*x + c*e)^3/(d*f) + 2*a*b*h*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a^2
*h*log(d*f*x + d*e)/(d*f) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*
a*b*i/(d*f^2) - 1/3*(c^2*e*log(c*f*x + c*e)^3 - 3*(c*f*x + c*e)*(c*log(c*f
*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2*i/(c^2*d*f^2)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.58

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \left(\frac{b^2ix}{df}+\frac{abfh-abei+b^2ei}{df^2}\right)\log\left(cfx+ce\right)^2 + \frac{2\,(abi-b^2i)x\log\left(cfx+ce\right)}{df} + \frac{(b^2fh-b^2ei)\log\left(cfx+ce\right)^3}{3\,df^2} + \frac{(a^2i-2\,abi+2\,b^2i)x}{df} + \frac{(a^2fh-a^2ei+2\,abei-2\,b^2ei)\log\left(fx+e\right)}{df^2}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

output
```
(b^2*i*x/(d*f) + (a*b*f*h - a*b*e*i + b^2*e*i)/(d*f^2))*log(c*f*x + c*e)^2
 + 2*(a*b*i - b^2*i)*x*log(c*f*x + c*e)/(d*f) + 1/3*(b^2*f*h - b^2*e*i)*lo
g(c*f*x + c*e)^3/(d*f^2) + (a^2*i - 2*a*b*i + 2*b^2*i)*x/(d*f) + (a^2*f*h
- a^2*e*i + 2*a*b*e*i - 2*b^2*e*i)*log(f*x + e)/(d*f^2)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \ln\left(c\left(e+f\,x\right)\right)^2\left(\frac{b\,(a\,f\,h-a\,e\,i+b\,e\,i)}{d\,f^2}+\frac{b^2\,i\,x}{d\,f}\right) - \frac{\ln\left(e+f\,x\right)\,\left(a^2\,e\,i-a^2\,f\,h+2\,b^2\,e\,i-2\,a\,b\,e\,i\right)}{d\,f^2} + \frac{i\,x\,(a^2-2\,a\,b+2\,b^2)}{d\,f} - \frac{b^2\ln\left(c\left(e+f\,x\right)\right)^3\left(e\,i-f\,h\right)}{3\,d\,f^2} + \frac{2\,b\,i\,x\,\ln\left(c\left(e+f\,x\right)\right)\,\left(a-b\right)}{d\,f}$$

input
```
int(((h + i*x)*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

output
```
log(c*(e + f*x))^2*((b*(a*f*h - a*e*i + b*e*i))/(d*f^2) + (b^2*i*x)/(d*f))
 - (log(e + f*x)*(a^2*e*i - a^2*f*h + 2*b^2*e*i - 2*a*b*e*i))/(d*f^2) + (i
*x*(a^2 - 2*a*b + 2*b^2))/(d*f) - (b^2*log(c*(e + f*x))^3*(e*i - f*h))/(3*
d*f^2) + (2*b*i*x*log(c*(e + f*x))*(a - b))/(d*f)
```

3.186. $\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.187 $\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

3.187.1 Optimal result

Integrand size = 25, antiderivative size = 27

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^3}{3bdf}$$

output
```
1/3*(a+b*ln(c*(f*x+e)))^3/b/d/f
```

3.187.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^3}{3bdf}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^2/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^3/(3*b*d*f)
```

3.187.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2837, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}dx$$

$$\Big\downarrow \; 2837$$

$$\frac{\int \frac{(a+b\log(c(e+fx)))^2}{d(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\log(c(e+fx)))^2}{e+fx}d(e+fx)}{df}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int (a+b\log(c(e+fx)))^2d(a+b\log(c(e+fx)))}{bdf}$$

$$\Big\downarrow \; 15$$

$$\frac{(a+b\log(c(e+fx)))^3}{3bdf}$$

input
```
Int[(a + b*Log[c*(e + f*x)])^2/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^3/(3*b*d*f)
```

3.187.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.187.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

method	result	size
parallelrisch	$\frac{\ln(c(fx+e))^3 b^2 f+3\ln(c(fx+e))^2 abf+3\ln(c(fx+e))a^2 f}{3d\,f^2}$	54
risch	$\frac{b^2\ln(c(fx+e))^3}{3df}+\frac{ab\ln(c(fx+e))^2}{df}+\frac{a^2\ln(fx+e)}{df}$	58
parts	$\frac{b^2\ln(c(fx+e))^3}{3df}+\frac{ab\ln(c(fx+e))^2}{df}+\frac{a^2\ln(fx+e)}{df}$	58
norman	$\frac{a^2\ln(c(fx+e))}{df}+\frac{ab\ln(c(fx+e))^2}{df}+\frac{b^2\ln(c(fx+e))^3}{3df}$	60
derivativedivides	$\frac{\frac{c\,a^2\ln(cfx+ce)}{d}+\frac{cab\ln(cfx+ce)^2}{d}+\frac{c\,b^2\ln(cfx+ce)^3}{3d}}{cf}$	64
default	$\frac{\frac{c\,a^2\ln(cfx+ce)}{d}+\frac{cab\ln(cfx+ce)^2}{d}+\frac{c\,b^2\ln(cfx+ce)^3}{3d}}{cf}$	64

input
```
int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
1/3*(ln(c*(f*x+e))^3*b^2*f+3*ln(c*(f*x+e))^2*a*b*f+3*ln(c*(f*x+e))*a^2*f)/
d/f^2
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{b^2\log(cfx+ce)^3+3\,ab\log(cfx+ce)^2+3\,a^2\log(cfx+ce)}{3\,df}$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas")
```

output
```
1/3*(b^2*log(c*f*x + c*e)^3 + 3*a*b*log(c*f*x + c*e)^2 + 3*a^2*log(c*f*x +
 c*e))/(d*f)
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{a^2\log(de+dfx)}{df}+\frac{ab\log(c(e+fx))^2}{df}+\frac{b^2\log(c(e+fx))^3}{3df}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

output
```
a**2*log(d*e + d*f*x)/(d*f) + a*b*log(c*(e + f*x))**2/(d*f) + b**2*log(c*(
e + f*x))**3/(3*d*f)
```

3.187.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(25) = 50$.

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.74

$$\begin{aligned}&\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx\\&= -ab\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right)\\&\quad+\frac{b^2\log(cfx+ce)^3}{3\,df}+\frac{2\,ab\log(cfx+ce)\log(dfx+de)}{df}+\frac{a^2\log(dfx+de)}{df}\end{aligned}$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima")
```

output
```
-a*b*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(
f*x + e)*log(c))/(d*f)) + 1/3*b^2*log(c*f*x + c*e)^3/(d*f) + 2*a*b*log(c*f
*x + c*e)*log(d*f*x + d*e)/(d*f) + a^2*log(d*f*x + d*e)/(d*f)
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{(a + b\log(c(e + fx)))^2}{de + dfx}\,dx = \frac{b^2\log{(cfx + ce)}^3}{3\,df} + \frac{ab\log{(cfx + ce)}^2}{df} + \frac{a^2\log{(fx + e)}}{df}$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

output
```
1/3*b^2*log(c*f*x + c*e)^3/(d*f) + a*b*log(c*f*x + c*e)^2/(d*f) + a^2*log(
f*x + e)/(d*f)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{(a + b\log(c(e + fx)))^2}{de + dfx}\,dx = \frac{3\,\ln{(e + f\,x)}\,a^2 + 3\,a\,b\ln{(c\,e + c\,f\,x)}^2 + b^2\ln{(c\,e + c\,f\,x)}^3}{3\,d\,f}$$

input
```
int((a + b*log(c*(e + f*x)))^2/(d*e + d*f*x),x)
```

output
```
(b^2*log(c*e + c*f*x)^3 + 3*a^2*log(e + f*x) + 3*a*b*log(c*e + c*f*x)^2)/(
3*d*f)
```

3.188 $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx$

3.188.1 Optimal result

Integrand size = 32, antiderivative size = 142

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = -\frac{(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b(a+b\log(c(e+fx)))\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b^2\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

output
```
-(a+b*ln(c*(f*x+e)))^2*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)+2*b*(a+b*ln
(c*(f*x+e)))*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)+2*b^2*polylog(3,(
e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)
```

3.188.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx$$
$$= \frac{3a^2\log(e+fx)+3ab\log^2(c(e+fx))+b^2\log^3(c(e+fx))-3a^2\log(h+ix)-6ab\log(c(e+fx))\log\left(\frac{f(\cdots}{f\cdots}\right.\cdots}{3\cdots}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)),x]
```

output
```
(3*a^2*Log[e + f*x] + 3*a*b*Log[c*(e + f*x)]^2 + b^2*Log[c*(e + f*x)]^3 -
3*a^2*Log[h + i*x] - 6*a*b*Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h - e*i)]
 - 3*b^2*Log[c*(e + f*x)]^2*Log[(f*(h + i*x))/(f*h - e*i)] - 6*b*(a + b*Lo
g[c*(e + f*x)])*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)] + 6*b^2*PolyLog[3
, (i*(e + f*x))/(-(f*h) + e*i)])/(3*d*(f*h - e*i))
```

3.188.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2858, 27, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(h+ix)(de+dfx)}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{f(a+b\log(c(e+fx)))^2}{d(e+fx)\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{d}$$

$$\Big\downarrow \text{2779}$$

$$\frac{\frac{2b\int \frac{(a+b\log(c(e+fx)))\log\left(\frac{fh-ei}{i(e+fx)}+1\right)}{e+fx}d(e+fx)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{d}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{2b\left(\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))-b\int \frac{\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{d}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{2b\left(\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))+b\,\text{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{d}$$

input
```
Int[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)),x]
```

output
```
(-(((a + b*Log[c*(e + f*x)])^2*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(f*h -
e*i)) + (2*b*((a + b*Log[c*(e + f*x)])*PolyLog[2, -((f*h - e*i)/(i*(e + f*
x)))] + b*PolyLog[3, -((f*h - e*i)/(i*(e + f*x)))]))/(f*h - e*i))/d
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.188. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx$

3.188.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(140) = 280$.

Time = 0.94 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.35

method	result
parts	$\frac{a^2\left(\frac{\ln(ix+h)}{ei-fh}-\frac{\ln(fx+e)}{ei-fh}\right)}{d}+\frac{b^2c\left(-\frac{\ln(cfx+ce)^3}{3c(ei-fh)}+\frac{\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)+2\ln(cfx+ce)\,\mathrm{Li}_2\left(-\frac{i(cfx+ce)}{-cei+hcf}\right)-2\,\mathrm{Li}_3\left(-\right.}{c(ei-fh)}\right.}{d}$
risch	$\frac{a^2\ln(ix+h)}{d(ei-fh)}-\frac{a^2\ln(fx+e)}{d(ei-fh)}-\frac{b^2\ln(cfx+ce)^3}{3d(ei-fh)}+\frac{b^2\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}+\frac{2b^2\ln(cfx+ce)\,\mathrm{Li}_2\left(-\frac{i(cfx+}{-cei+}\right.}{d(ei-fh)}$
derivativedivides	$\frac{-\frac{cf\,a^2\ln(cfx+ce)}{d(ei-fh)}+\frac{cf\,a^2\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2f\,b^2\left(\frac{\ln(cfx+ce)^3}{3c(ei-fh)}-\frac{\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)+2\ln(cfx+ce)\,\mathrm{Li}_2\left(-\frac{i}{}\right.}{c(ei-fh)}\right.}{d}}{}$
default	$\frac{-\frac{cf\,a^2\ln(cfx+ce)}{d(ei-fh)}+\frac{cf\,a^2\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2f\,b^2\left(\frac{\ln(cfx+ce)^3}{3c(ei-fh)}-\frac{\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)+2\ln(cfx+ce)\,\mathrm{Li}_2\left(-\frac{i}{}\right.}{c(ei-fh)}\right.}{d}}{}$

input
```
int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/(e*i-f*h)*ln(i*x+h)-1/(e*i-f*h)*ln(f*x+e))+b^2/d*c*(-1/3/c/(e*i-f
*h)*ln(c*f*x+c*e)^3+1/c/(e*i-f*h)*(ln(c*f*x+c*e)^2*ln(1+i/(-c*e*i+c*f*h)*(
c*f*x+c*e))+2*ln(c*f*x+c*e)*polylog(2,-i/(-c*e*i+c*f*h)*(c*f*x+c*e))-2*pol
ylog(3,-i/(-c*e*i+c*f*h)*(c*f*x+c*e))))-a*b/d/(e*i-f*h)*ln(c*f*x+c*e)^2+2*
a*b/d/(e*i-f*h)*dilog((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))+2*a*b/d
/(e*i-f*h)*ln(c*f*x+c*e)*ln((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))
```

3.188.5 Fricas [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx=\int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x, algorithm="fricas"
)
```

output
```
integral((b^2*log(c*f*x + c*e)^2 + 2*a*b*log(c*f*x + c*e) + a^2)/(d*f*i*x^
2 + d*e*h + (d*f*h + d*e*i)*x), x)
```

3.188.6 Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = \frac{\int \frac{a^2}{eh+eix+fhx+fix^2}\,dx + \int \frac{b^2\log(ce+cfx)^2}{eh+eix+fhx+fix^2}\,dx + \int \frac{2ab\log(ce+cfx)}{eh+eix+fhx+fix^2}\,dx}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e)/(i*x+h),x)
```

output
```
(Integral(a**2/(e*h + e*i*x + f*h*x + f*i*x**2), x) + Integral(b**2*log(c*
e + c*f*x)**2/(e*h + e*i*x + f*h*x + f*i*x**2), x) + Integral(2*a*b*log(c*
e + c*f*x)/(e*h + e*i*x + f*h*x + f*i*x**2), x))/d
```

3.188.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(141) = 282.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.33

$$\begin{aligned}\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx &= a^2\left(\frac{\log(fx+e)}{dfh-dei} - \frac{\log(ix+h)}{dfh-dei}\right) \\ &\quad - \frac{\left(\log(fx+e)^2\log\left(\frac{fix+ei}{fh-ei}+1\right) + 2\,\mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\log(fx+e) - 2\,\mathrm{Li}_3(-\frac{fix+ei}{fh-ei})\right)b^2}{(fh-ei)d} \\ &\quad - \frac{2\,(b^2\log(c)+ab)\left(\log(fx+e)\log\left(\frac{fix+ei}{fh-ei}+1\right) + \mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\right)}{(fh-ei)d} \\ &\quad - \frac{\left(b^2\log(c)^2 + 2\,ab\log(c)\right)\log(ix+h)}{(fh-ei)d} \\ &\quad + \frac{b^2\log(fx+e)^3 + 3\,(b^2\log(c)+ab)\log(fx+e)^2 + 3\left(b^2\log(c)^2 + 2\,ab\log(c)\right)\log(fx+e)}{3\,(fh-ei)d}\end{aligned}$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x, algorithm="maxima"
)
```

output
```
a^2*(log(f*x + e)/(d*f*h - d*e*i) - log(i*x + h)/(d*f*h - d*e*i)) - (log(f
*x + e)^2*log((f*i*x + e*i)/(f*h - e*i) + 1) + 2*dilog(-(f*i*x + e*i)/(f*h
 - e*i))*log(f*x + e) - 2*polylog(3, -(f*i*x + e*i)/(f*h - e*i)))*b^2/((f*
h - e*i)*d) - 2*(b^2*log(c) + a*b)*(log(f*x + e)*log((f*i*x + e*i)/(f*h -
e*i) + 1) + dilog(-(f*i*x + e*i)/(f*h - e*i)))/((f*h - e*i)*d) - (b^2*log(
c)^2 + 2*a*b*log(c))*log(i*x + h)/((f*h - e*i)*d) + 1/3*(b^2*log(f*x + e)^
3 + 3*(b^2*log(c) + a*b)*log(f*x + e)^2 + 3*(b^2*log(c)^2 + 2*a*b*log(c))*
log(f*x + e))/((f*h - e*i)*d)
```

3.188.8 Giac [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x, algorithm="giac")
```

output
```
integrate((b*log((f*x + e)*c) + a)^2/((d*f*x + d*e)*(i*x + h)), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)^2}{\left(h+i\,x\right)\,\left(d\,e+d\,f\,x\right)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))^2/((h + i*x)*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))^2/((h + i*x)*(d*e + d*f*x)), x)
```

3.189 $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$

3.189.1 Optimal result

Integrand size = 32, antiderivative size = 273

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = -\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^2(h+ix)} + \frac{2bf(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^2} - \frac{f(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{2bf(a+b\log(c(e+fx)))\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{2b^2f\operatorname{PolyLog}\left(2,-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^2} + \frac{2b^2f\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}$$

output
```
-i*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/(-e*i+f*h)^2/(i*x+h)+2*b*f*(a+b*ln(c*(f
*x+e)))*ln(f*(i*x+h)/(-e*i+f*h))/d/(-e*i+f*h)^2-f*(a+b*ln(c*(f*x+e)))^2*ln
(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^2+2*b*f*(a+b*ln(c*(f*x+e)))*polylog(
2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^2+2*b^2*f*polylog(2,-i*(f*x+e)/(-e*i+f
*h))/d/(-e*i+f*h)^2+2*b^2*f*polylog(3,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^2
```

3.189.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$$
$$= \frac{3a^2(fh-ei)+3a^2f(h+ix)\log(e+fx)-3a^2f(h+ix)\log(h+ix)+3ab\Big(-2f(h+ix)\log(e+fx)+2}{}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^2),x]
```

output
```
(3*a^2*(f*h - e*i) + 3*a^2*f*(h + i*x)*Log[e + f*x] - 3*a^2*f*(h + i*x)*Lo
g[h + i*x] + 3*a*b*(-2*f*(h + i*x)*Log[e + f*x] + 2*(f*h - e*i)*Log[c*(e +
 f*x)] + f*(h + i*x)*Log[c*(e + f*x)]^2 + 2*f*(h + i*x)*Log[h + i*x] - 2*f
*(h + i*x)*(Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h - e*i)] + PolyLog[2, (
i*(e + f*x))/(-(f*h) + e*i)])) + b^2*(Log[c*(e + f*x)]*(f*(h + i*x)*Log[c*
(e + f*x)]^2 + 6*f*(h + i*x)*Log[(f*(h + i*x))/(f*h - e*i)] - 3*Log[c*(e +
 f*x)]*(i*(e + f*x) + f*(h + i*x)*Log[(f*(h + i*x))/(f*h - e*i)])) - 6*f*(
h + i*x)*(-1 + Log[c*(e + f*x)])*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)]
+ 6*f*(h + i*x)*PolyLog[3, (i*(e + f*x))/(-(f*h) + e*i)]))/(3*d*(f*h - e*i
)^2*(h + i*x))
```

3.189.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2858, 27, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(h+ix)^2(de+dfx)}\,dx$$
$$\Big\downarrow \text{2858}$$
$$\frac{\int \frac{f^2(a+b\log(c(e+fx)))^2}{d(e+fx)\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^2}d(e+fx)}{f}$$
$$\Big\downarrow \text{27}$$

$$\frac{f\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{d}$$

$$\Big\downarrow \text{ 2789}$$

$$\frac{f\left(\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{(a+b\log(c(e+fx)))^2}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{ 2755}$$

$$\frac{f\left(\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\int \frac{a+b\log(c(e+fx))}{fh-ei+i(e+fx)}d(e+fx)}{fh-ei}\right)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{ 2754}$$

$$\frac{f\left(\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\left(\frac{\log\left(\frac{i(e+fx)}{fh-ei}+1\right)(a+b\log(c(e+fx)))}{i}-\frac{b\int \frac{\log\left(\frac{i(e+fx)}{fh-ei}+1\right)}{e+fx}d(e+fx)}{i}\right)}{fh-ei}\right)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{ 2779}$$

3.189. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$

$$\frac{f\left(\frac{\frac{2b\int \frac{(a+b\log(c(e+fx)))\log\left(\frac{fh-ei}{i(e+fx)}+1\right)}{e+fx}d(e+fx)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\left(\frac{\log\left(\frac{i(e+fx)}{fh-ei}+1\right)(a+i}{i}\right.}{}\right.}{fh-ei}\right.}{d}$$

$\Big\downarrow$ 2821

$$\frac{f\left(\frac{\frac{2b\left(\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))-b\int \frac{\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+}{(fh-ei)(i(e+fx)-e}\right.}{}\right.}{d}$$

$\Big\downarrow$ 2838

$$\frac{f\left(\frac{\frac{2b\left(\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))-b\int \frac{\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+}{(fh-ei)(i(e+fx)-e}\right.}{}\right.}{d}$$

$\Big\downarrow$ 7143

3.189. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$

$$\frac{f\left(\frac{\frac{2b\left(\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))+b\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b}{\quad}\right.}{\quad}\right.}{d}$$

input
```
Int[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^2),x]
```

output
```
(f*(-((i*(((e + f*x)*(a + b*Log[c*(e + f*x)])^2)/((f*h - e*i)*(f*h - e*i +
 i*(e + f*x))) - (2*b*(((a + b*Log[c*(e + f*x)])*Log[1 + (i*(e + f*x))/(f*
h - e*i)])/i + (b*PolyLog[2, -((i*(e + f*x))/(f*h - e*i))])/i))/(f*h - e*i
)))/(f*h - e*i)) + (-(((a + b*Log[c*(e + f*x)])^2*Log[1 + (f*h - e*i)/(i*(
e + f*x))])/(f*h - e*i)) + (2*b*((a + b*Log[c*(e + f*x)])*PolyLog[2, -((f*
h - e*i)/(i*(e + f*x)))] + b*PolyLog[3, -((f*h - e*i)/(i*(e + f*x)))]))/(f
*h - e*i))/(f*h - e*i)))/d
```

3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.189. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$

3.189.4 Maple [F]

$$\int \frac{(a+b\ln(c(fx+e)))^2}{(dfx+de)(ix+h)^2}dx$$

input
```
int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x)
```

output
```
int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x)
```

3.189.5 Fricas [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log((fx+e)c)+a)^2}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="frica
s")
```

output
```
integral((b^2*log(c*f*x + c*e)^2 + 2*a*b*log(c*f*x + c*e) + a^2)/(d*f*i^2*
x^3 + d*e*h^2 + (2*d*f*h*i + d*e*i^2)*x^2 + (d*f*h^2 + 2*d*e*h*i)*x), x)
```

3.189.6 Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$$
$$= \frac{\int \frac{a^2}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx + \int \frac{b^2\log(ce+cfx)^2}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx + \int \frac{2ab\log(ce+cfx)}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2}}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e)/(i*x+h)**2,x)
```

output
```
(Integral(a**2/(e*h**2 + 2*e*h*i*x + e*i**2*x**2 + f*h**2*x + 2*f*h*i*x**2
 + f*i**2*x**3), x) + Integral(b**2*log(c*e + c*f*x)**2/(e*h**2 + 2*e*h*i*
x + e*i**2*x**2 + f*h**2*x + 2*f*h*i*x**2 + f*i**2*x**3), x) + Integral(2*
a*b*log(c*e + c*f*x)/(e*h**2 + 2*e*h*i*x + e*i**2*x**2 + f*h**2*x + 2*f*h*
i*x**2 + f*i**2*x**3), x))/d
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(271) = 542$.

Time = 0.30 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.28

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$$
$$= a^2\left(\frac{f\log(fx+e)}{df^2h^2-2\,defhi+de^2i^2}-\frac{f\log(ix+h)}{df^2h^2-2\,defhi+de^2i^2}+\frac{1}{dfh^2-dehi+(dfhi-dei^2)x}\right)$$
$$-\frac{\left(\log(fx+e)^2\log\left(\frac{fix+ei}{fh-ei}+1\right)+2\,\mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\log(fx+e)-2\,\mathrm{Li}_3(-\frac{fix+ei}{fh-ei})\right)b^2f}{(f^2h^2-2\,efhi+e^2i^2)d}$$
$$+\frac{3\,(fh-ei)b^2\log(c)^2+(b^2fix+b^2fh)\log(fx+e)^3+6\,(fh-ei)ab\log(c)+3\,(abfh+(fh\log(c)-e}{}$$
$$-\frac{2\,((f\log(c)-f)b^2+abf)\left(\log(fx+e)\log\left(\frac{fix+ei}{fh-ei}+1\right)+\mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\right)}{(f^2h^2-2\,efhi+e^2i^2)d}$$
$$-\frac{\left(2\,(f\log(c)-f)ab+\left(f\log(c)^2-2\,f\log(c)\right)b^2\right)\log(ix+h)}{(f^2h^2-2\,efhi+e^2i^2)d}$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="maxim
a")
```

output
```
a^2*(f*log(f*x + e)/(d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) - f*log(i*x + h)
/(d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) + 1/(d*f*h^2 - d*e*h*i + (d*f*h*i -
 d*e*i^2)*x)) - (log(f*x + e)^2*log((f*i*x + e*i)/(f*h - e*i) + 1) + 2*dil
og(-(f*i*x + e*i)/(f*h - e*i))*log(f*x + e) - 2*polylog(3, -(f*i*x + e*i)/
(f*h - e*i)))*b^2*f/((f^2*h^2 - 2*e*f*h*i + e^2*i^2)*d) + 1/3*(3*(f*h - e*
i)*b^2*log(c)^2 + (b^2*f*i*x + b^2*f*h)*log(f*x + e)^3 + 6*(f*h - e*i)*a*b
*log(c) + 3*(a*b*f*h + (f*h*log(c) - e*i)*b^2 + (a*b*f*i + (f*i*log(c) - f
*i)*b^2)*x)*log(f*x + e)^2 + 3*(2*(f*h*log(c) - e*i)*a*b + (f*h*log(c)^2 -
 2*e*i*log(c))*b^2 + (2*(f*i*log(c) - f*i)*a*b + (f*i*log(c)^2 - 2*f*i*log
(c))*b^2)*x)*log(f*x + e))/((f^2*h^2*i - 2*e*f*h*i^2 + e^2*i^3)*d*x + (f^2
*h^3 - 2*e*f*h^2*i + e^2*h*i^2)*d) - 2*((f*log(c) - f)*b^2 + a*b*f)*(log(f
*x + e)*log((f*i*x + e*i)/(f*h - e*i) + 1) + dilog(-(f*i*x + e*i)/(f*h - e
*i)))/((f^2*h^2 - 2*e*f*h*i + e^2*i^2)*d) - (2*(f*log(c) - f)*a*b + (f*log
(c)^2 - 2*f*log(c))*b^2)*log(i*x + h)/((f^2*h^2 - 2*e*f*h*i + e^2*i^2)*d)
```

3.189.8 Giac [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="giac"
)
```

output
```
integrate((b*log((f*x + e)*c) + a)^2/((d*f*x + d*e)*(i*x + h)^2), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(a+b\,\ln\left(c\,(e+f\,x)\right))^2}{(h+i\,x)^2\,(d\,e+d\,f\,x)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))^2/((h + i*x)^2*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))^2/((h + i*x)^2*(d*e + d*f*x)), x)
```

3.190 $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$

3.190.1 Optimal result

Integrand size = 32, antiderivative size = 485

$$\begin{aligned}\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx &= \frac{bfi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)}+\frac{(a+b\log(c(e+fx)))^2}{2d(fh-ei)(h+ix)^2}\\&-\frac{fi(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^3(h+ix)}-\frac{b^2f^2\log(h+ix)}{d(fh-ei)^3}\\&+\frac{2bf^2(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^3}\\&+\frac{bf^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\\&-\frac{f^2(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\\&-\frac{b^2f^2\,\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\\&+\frac{2bf^2(a+b\log(c(e+fx)))\,\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\\&+\frac{2b^2f^2\,\text{PolyLog}\left(2,-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^3}\\&+\frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\end{aligned}$$

output
```
b*f*i*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/(-e*i+f*h)^3/(i*x+h)+1/2*(a+b*ln(c*(f*
x+e)))^2/d/(-e*i+f*h)/(i*x+h)^2-f*i*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/(-e*i+
f*h)^3/(i*x+h)-b^2*f^2*ln(i*x+h)/d/(-e*i+f*h)^3+2*b*f^2*(a+b*ln(c*(f*x+e))
)*ln(f*(i*x+h)/(-e*i+f*h))/d/(-e*i+f*h)^3+b*f^2*(a+b*ln(c*(f*x+e)))*ln(1+(
-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^3-f^2*(a+b*ln(c*(f*x+e)))^2*ln(1+(-e*i+f
*h)/i/(f*x+e))/d/(-e*i+f*h)^3-b^2*f^2*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e
*i+f*h)^3+2*b*f^2*(a+b*ln(c*(f*x+e)))*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e
*i+f*h)^3+2*b^2*f^2*polylog(2,-i*(f*x+e)/(-e*i+f*h))/d/(-e*i+f*h)^3+2*b^2*
f^2*polylog(3,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^3
```

3.190.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$$
$$= \frac{3a^2(fh-ei)^2+6a^2f(fh-ei)(h+ix)+6a^2f^2(h+ix)^2\log(e+fx)-6a^2f^2(h+ix)^2\log(h+ix)+6ab}{}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^3),x]
```

output
```
(3*a^2*(f*h - e*i)^2 + 6*a^2*f*(f*h - e*i)*(h + i*x) + 6*a^2*f^2*(h + i*x)
^2*Log[e + f*x] - 6*a^2*f^2*(h + i*x)^2*Log[h + i*x] + 6*a*b*((f*h - e*i)^
2*Log[c*(e + f*x)] + f^2*(h + i*x)^2*Log[c*(e + f*x)]^2 - f*(h + i*x)*(f*h
 - e*i + f*(h + i*x)*Log[e + f*x] - f*(h + i*x)*Log[h + i*x]) - 2*f*(h + i
*x)*(f*(h + i*x)*Log[e + f*x] + (-(f*h) + e*i)*Log[c*(e + f*x)] - f*(h + i
*x)*Log[h + i*x]) - 2*f^2*(h + i*x)^2*(Log[c*(e + f*x)]*Log[(f*(h + i*x))/
(f*h - e*i)] + PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)])) + b^2*(2*f^2*(h
+ i*x)^2*Log[c*(e + f*x)]^3 - 6*f*(h + i*x)*(Log[c*(e + f*x)]*(i*(e + f*x)
*Log[c*(e + f*x)] - 2*f*(h + i*x)*Log[(f*(h + i*x))/(f*h - e*i)]) - 2*f*(h
 + i*x)*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)]) + 3*((f*h - e*i)^2*Log[c
*(e + f*x)]^2 + f*(h + i*x)*(2*f*(h + i*x)*Log[e + f*x] - 2*(f*h - e*i)*Lo
g[c*(e + f*x)] - f*(h + i*x)*Log[c*(e + f*x)]^2 - 2*f*(h + i*x)*Log[h + i*
x] + 2*f*(h + i*x)*Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h - e*i)] + 2*f*(
h + i*x)*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)])) - 6*f^2*(h + i*x)^2*(L
og[c*(e + f*x)]^2*Log[(f*(h + i*x))/(f*h - e*i)] + 2*Log[c*(e + f*x)]*Poly
Log[2, (i*(e + f*x))/(-(f*h) + e*i)] - 2*PolyLog[3, (i*(e + f*x))/(-(f*h)
+ e*i)])))/(6*d*(f*h - e*i)^3*(h + i*x)^2)
```

3.190.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2858, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(h+ix)^3(de+dfx)}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{f^3(a+b\log(c(e+fx)))^2}{d(e+fx)\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^3}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))^3}d(e+fx)}{d}$$

$$\Big\downarrow \text{2789}$$

$$\frac{f^2\left(\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}-\frac{i\int \frac{(a+b\log(c(e+fx)))^2}{(fh-ei+i(e+fx))^3}d(e+fx)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{2756}$$

$$\frac{f^2\left(\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}-\frac{i\left(\frac{b\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))^2}d(e+fx)}{i}-\frac{(a+b\log(c(e+fx)))^2}{2i(i(e+fx)-ei+fh)^2}\right)}{fh-ei}\right)}{d}$$

$$\Big\downarrow \text{2789}$$

$$\frac{f^2\left(\frac{\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{(a+b\log(c(e+fx)))^2}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{a+b\log(c(e+fx))}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}\right)}{i}\right.}{fh-ei}\right.}{d}$$

$\downarrow$ 2751

$$\frac{f^2\left(\frac{\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{(a+b\log(c(e+fx)))^2}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{b}{\cdots}\right.}{fh-e\cdots}\right.}{i}\right.}{fh-ei}\right.}{d}$$

$\downarrow$ 16

$$\frac{f^2\left(\frac{\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\int \frac{(a+b\log(c(e+fx)))^2}{(fh-ei+i(e+fx))^2}d(e+fx)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))}{(fh-ei)(i(e+fx)-ei+fh)}-b\cdots\right.}{fh-ei}\right.}{i}\right.}{fh-ei}\right.}{d}$$

$\downarrow$ 2755

$$\frac{f^2\left(\frac{\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\int \frac{a+b\log(c(e+fx))}{fh-ei+i(e+fx)}d(e+fx)}{fh-ei}\right)}{fh-ei}}{fh-ei}-\frac{i\left(\frac{b\left(\frac{\int \frac{a+b\log(c(e+fx))}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{d}$$

$\downarrow$ 2754

3.190. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$

$$\frac{f^2\left(\frac{\frac{\int \frac{(a+b\log(c(e+fx)))^2}{(e+fx)(fh-ei+i(e+fx))}d(e+fx)}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\left(\frac{\log\left(\frac{i(e+fx)}{fh-ei}+1\right)(a+b\log(c(e+fx)))}{i}-\frac{b\int \frac{\log\left(\frac{i(e+fx)}{fh-ei}+1\right)}{e+fx}d(e+fx)}{i}\right)}{fh-ei}\right)}{fh-ei}}{fh-ei} - \cdots\right)}{d}$$

$\downarrow$ 2779

$$\frac{f^2\left(\frac{\frac{\frac{2b\int \frac{(a+b\log(c(e+fx)))\log\left(\frac{fh-ei}{i(e+fx)}+1\right)}{e+fx}d(e+fx)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\left(\frac{\log\left(\frac{i(e+fx)}{fh-ei}+1\right)(a+b\cdots}{i}\cdots\right)}{\cdots}\right)}{fh-ei}}{fh-ei}\cdots\right)}{\cdots}$$

$\downarrow$ 2821

3.190. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$

$$f^2\left(\frac{\frac{\frac{2b\left(\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))-b\int\frac{\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+f}{(fh-ei)(i(e+fx)-ei}\right.}{}}{fh-ei}\right.$$

$\downarrow$ 2838

$$f^2\left(\frac{\frac{\frac{2b\left(\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))-b\int\frac{\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+f}{(fh-ei)(i(e+fx)-ei}\right.}{}}{fh-ei}\right.$$

$\downarrow$ 7143

3.190. $\int\frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$

$$f^2\left(\frac{\frac{\frac{2b\left(\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))+b\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)\right)}{fh-ei}-\frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{fh-ei}}{fh-ei}}{fh-ei}-\frac{i\left(\frac{(e+fx)(a+b\log(c(e+fx)))^2}{(fh-ei)(i(e+fx)-ei+fh)}-\frac{2b\left(\right.}{}\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^3),x]
```

output
```
(f^2*(-((i*(-1/2*(a + b*Log[c*(e + f*x)])^2/(i*(f*h - e*i + i*(e + f*x))^2
) + (b*(-((i*(((e + f*x)*(a + b*Log[c*(e + f*x)]))/((f*h - e*i)*(f*h - e*i
 + i*(e + f*x))) - (b*Log[f*h - e*i + i*(e + f*x)])/(i*(f*h - e*i))))/(f*h
 - e*i)) + (-(((a + b*Log[c*(e + f*x)])*Log[1 + (f*h - e*i)/(i*(e + f*x))]
)/(f*h - e*i)) + (b*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))])/(f*h - e*i))
/(f*h - e*i)))/i))/(f*h - e*i)) + (-((i*(((e + f*x)*(a + b*Log[c*(e + f*x)
])^2)/((f*h - e*i)*(f*h - e*i + i*(e + f*x))) - (2*b*(((a + b*Log[c*(e + f
*x)])*Log[1 + (i*(e + f*x))/(f*h - e*i)])/i + (b*PolyLog[2, -((i*(e + f*x)
)/(f*h - e*i))])/i))/(f*h - e*i)))/(f*h - e*i)) + (-(((a + b*Log[c*(e + f*
x)])^2*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(f*h - e*i)) + (2*b*((a + b*Log
[c*(e + f*x)])*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))] + b*PolyLog[3, -((
f*h - e*i)/(i*(e + f*x)))]))/(f*h - e*i))/(f*h - e*i))/(f*h - e*i)))/d
```

3.190.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

3.190. $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.190.4 Maple [F]

$$\int \frac{(a+b\ln(c(fx+e)))^2}{(dfx+de)(ix+h)^3}dx$$

input
```
int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x)
```

output
```
int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x)
```

3.190.5 Fricas [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log((fx+e)c)+a)^2}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="frica
s")
```

output
```
integral((b^2*log(c*f*x + c*e)^2 + 2*a*b*log(c*f*x + c*e) + a^2)/(d*f*i^3*
x^4 + d*e*h^3 + (3*d*f*h*i^2 + d*e*i^3)*x^3 + 3*(d*f*h^2*i + d*e*h*i^2)*x^
2 + (d*f*h^3 + 3*d*e*h^2*i)*x), x)
```

3.190.6 Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$$
$$= \frac{\int \frac{a^2}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}\,dx + \int \frac{b^2\log(ce+cfx)^2}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e)/(i*x+h)**3,x)
```

output
```
(Integral(a**2/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**3 + f*
h**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x) + Integral(b
**2*log(c*e + c*f*x)**2/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*
x**3 + f*h**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x) + I
ntegral(2*a*b*log(c*e + c*f*x)/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 +
e*i**3*x**3 + f*h**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4),
 x))/d
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1271 vs. 2(480) = 960.

Time = 0.38 (sec) , antiderivative size = 1271, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="maxim
a")
```

output
```
1/2*(2*f^2*log(f*x + e)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*i^2 - d
*e^3*i^3) - 2*f^2*log(i*x + h)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*
i^2 - d*e^3*i^3) + (2*f*i*x + 3*f*h - e*i)/(d*f^2*h^4 - 2*d*e*f*h^3*i + d*
e^2*h^2*i^2 + (d*f^2*h^2*i^2 - 2*d*e*f*h*i^3 + d*e^2*i^4)*x^2 + 2*(d*f^2*h
^3*i - 2*d*e*f*h^2*i^2 + d*e^2*h*i^3)*x))*a^2 - (log(f*x + e)^2*log((f*i*x
 + e*i)/(f*h - e*i) + 1) + 2*dilog(-(f*i*x + e*i)/(f*h - e*i))*log(f*x + e
) - 2*polylog(3, -(f*i*x + e*i)/(f*h - e*i)))*b^2*f^2/((f^3*h^3 - 3*e*f^2*
h^2*i + 3*e^2*f*h*i^2 - e^3*i^3)*d) + 1/6*(2*(b^2*f^2*i^2*x^2 + 2*b^2*f^2*
h*i*x + b^2*f^2*h^2)*log(f*x + e)^3 - 6*(f^2*h^2 - e*f*h*i - (3*f^2*h^2 -
4*e*f*h*i + e^2*i^2)*log(c))*a*b + 3*((3*f^2*h^2 - 4*e*f*h*i + e^2*i^2)*lo
g(c)^2 - 2*(f^2*h^2 - e*f*h*i)*log(c))*b^2 + 3*(2*a*b*f^2*h^2 + (2*f^2*h^2
*log(c) - 4*e*f*h*i + e^2*i^2)*b^2 + (2*a*b*f^2*i^2 + (2*f^2*i^2*log(c) -
3*f^2*i^2)*b^2)*x^2 + 2*(2*a*b*f^2*h*i + (2*f^2*h*i*log(c) - 2*f^2*h*i - e
*f*i^2)*b^2)*x)*log(f*x + e)^2 - 6*((f^2*h*i - e*f*i^2 - 2*(f^2*h*i - e*f*
i^2)*log(c))*a*b - ((f^2*h*i - e*f*i^2)*log(c)^2 - (f^2*h*i - e*f*i^2)*log
(c))*b^2)*x + 6*((2*f^2*h^2*log(c) - 4*e*f*h*i + e^2*i^2)*a*b + (f^2*h^2*l
og(c)^2 + e*f*h*i - (4*e*f*h*i - e^2*i^2)*log(c))*b^2 + ((2*f^2*i^2*log(c)
 - 3*f^2*i^2)*a*b + (f^2*i^2*log(c)^2 - 3*f^2*i^2*log(c) + f^2*i^2)*b^2)*x
^2 + (2*(2*f^2*h*i*log(c) - 2*f^2*h*i - e*f*i^2)*a*b + (2*f^2*h*i*log(c)^2
 + f^2*h*i + e*f*i^2 - 2*(2*f^2*h*i + e*f*i^2)*log(c))*b^2)*x)*log(f*x ...
```

3.190.8 Giac [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log((fx+e)c)+a)^2}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="giac"
)
```

output
```
integrate((b*log((f*x + e)*c) + a)^2/((d*f*x + d*e)*(i*x + h)^3), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,(e+f\,x)\right)\right)^2}{(h+i\,x)^3\,\left(d\,e+d\,f\,x\right)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))^2/((h + i*x)^3*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))^2/((h + i*x)^3*(d*e + d*f*x)), x)
```

3.191 $\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$

3.191.1 Optimal result

Integrand size = 32, antiderivative size = 230

$$\begin{aligned}\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx &= \frac{4e^{-\frac{a}{b}}i(fh-ei)^3\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^5}\\ &+\frac{6e^{-\frac{2a}{b}}i^2(fh-ei)^2\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^5}\\ &+\frac{4e^{-\frac{3a}{b}}i^3(fh-ei)\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^5}\\ &+\frac{e^{-\frac{4a}{b}}i^4\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(c(e+fx)))}{b}\right)}{bc^4df^5}+\frac{(fh-ei)^4\log(a+b\log(c(e+fx)))}{bdf^5}\end{aligned}$$

output

```
4*i*(-e*i+f*h)^3*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^5+6*i^2*(-e*i+
f*h)^2*Ei(2*(a+b*ln(c*(f*x+e)))/b)/b/c^2/d/exp(2*a/b)/f^5+4*i^3*(-e*i+f*h)
*Ei(3*(a+b*ln(c*(f*x+e)))/b)/b/c^3/d/exp(3*a/b)/f^5+i^4*Ei(4*(a+b*ln(c*(f*
x+e)))/b)/b/c^4/d/exp(4*a/b)/f^5+(-e*i+f*h)^4*ln(a+b*ln(c*(f*x+e)))/b/d/f^
5
```

3.191.2 Mathematica [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$

input
```
Integrate[(h + i*x)^4/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
Integrate[(h + i*x)^4/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])), x]
```

3.191.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^4}{df^4(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^4}{(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{df^5}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{(e+fx)^3i^4}{a+b\log(c(e+fx))}+\frac{4(fh-ei)(e+fx)^2i^3}{a+b\log(c(e+fx))}+\frac{6(fh-ei)^2(e+fx)i^2}{a+b\log(c(e+fx))}+\frac{4(fh-ei)^3i}{a+b\log(c(e+fx))}+\frac{(fh-ei)^4}{(e+fx)(a+b\log(c(e+fx)))}\right)d(e+fx)}{df^5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{i^4e^{-\frac{4a}{b}}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(c(e+fx)))}{b}\right)}{bc^4}+\frac{4i^3e^{-\frac{3a}{b}}(fh-ei)\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3}+\frac{6i^2e^{-\frac{2a}{b}}(fh-ei)^2\,\text{ExpIntegralEi}\left(\frac{2(a+\cdots}{\cdots}\right)}{bc^2}\cdots}{df^5}$$

input
```
Int[(h + i*x)^4/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
((4*i*(f*h - e*i)^3*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*E^(a/b
)) + (6*i^2*(f*h - e*i)^2*ExpIntegralEi[(2*(a + b*Log[c*(e + f*x)]))/b])/(
b*c^2*E^((2*a)/b)) + (4*i^3*(f*h - e*i)*ExpIntegralEi[(3*(a + b*Log[c*(e +
 f*x)]))/b])/(b*c^3*E^((3*a)/b)) + (i^4*ExpIntegralEi[(4*(a + b*Log[c*(e +
 f*x)]))/b])/(b*c^4*E^((4*a)/b)) + ((f*h - e*i)^4*Log[a + b*Log[c*(e + f*x
)]])/b)/(d*f^5)
```

3.191.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.191. $\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$

3.191.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(233) = 466$.

Time = 3.02 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.47

method	result
derivativedivides	$-\frac{i^4 e^{-\frac{4a}{b}} \operatorname{Ei}_1\left(-4\ln(cfx+ce)-\frac{4a}{b}\right)}{b}+\frac{c^4e^4i^4\ln(a+b\ln(cfx+ce))}{b}+\frac{c^4f^4h^4\ln(a+b\ln(cfx+ce))}{b}+\frac{4ce\,i^4e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\ln(cfx+ce)-\right.}{b}$
default	$-\frac{i^4 e^{-\frac{4a}{b}} \operatorname{Ei}_1\left(-4\ln(cfx+ce)-\frac{4a}{b}\right)}{b}+\frac{c^4e^4i^4\ln(a+b\ln(cfx+ce))}{b}+\frac{c^4f^4h^4\ln(a+b\ln(cfx+ce))}{b}+\frac{4ce\,i^4e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\ln(cfx+ce)-\right.}{b}$
risch	$\frac{e^4i^4\ln(a+b\ln(cfx+ce))}{f^5db}-\frac{4e^3h\,i^3\ln(a+b\ln(cfx+ce))}{f^4db}+\frac{6e^2h^2i^2\ln(a+b\ln(cfx+ce))}{f^3db}-\frac{4e\,h^3i\ln(a+b\ln(cfx+ce)}{f^2db}$

input
```
int((i*x+h)^4/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

output
```
1/c^4/f^5/d*(-i^4/b*exp(-4*a/b)*Ei(1,-4*ln(c*f*x+c*e)-4*a/b)+c^4*e^4*i^4*l
n(a+b*ln(c*f*x+c*e))/b+c^4*f^4*h^4*ln(a+b*ln(c*f*x+c*e))/b+4*c*e*i^4/b*exp
(-3*a/b)*Ei(1,-3*ln(c*f*x+c*e)-3*a/b)-6*c^2*e^2*i^4/b*exp(-2*a/b)*Ei(1,-2*
ln(c*f*x+c*e)-2*a/b)+4*c^3*e^3*i^4/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-4*
c*f*h*i^3/b*exp(-3*a/b)*Ei(1,-3*ln(c*f*x+c*e)-3*a/b)-6*c^2*f^2*h^2*i^2/b*e
xp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)-4*c^3*f^3*h^3*i/b*exp(-a/b)*Ei(1,-
ln(c*f*x+c*e)-a/b)-4*c^4*e*f^3*h^3*i*ln(a+b*ln(c*f*x+c*e))/b+6*c^4*e^2*f^2
*h^2*i^2*ln(a+b*ln(c*f*x+c*e))/b-4*c^4*e^3*f*h*i^3*ln(a+b*ln(c*f*x+c*e))/b
+12*c^2*e*f*h*i^3/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+12*c^3*e*f^2*
h^2*i^2/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-12*c^3*e^2*f*h*i^3/b*exp(-a/b
)*Ei(1,-ln(c*f*x+c*e)-a/b))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.74

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\left(i^4\operatorname{log\_integral}\left((c^4f^4x^4+4\,c^4ef^3x^3+6\,c^4e^2f^2x^2+4\,c^4e^3fx+c^4e^4)e^{\left(\frac{4a}{b}\right)}\right)+(c^4f^4h^4-4\,c^4ef^3h^3i+6\,c\right.}{}$$

input
```
integrate((i*x+h)^4/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas"
)
```

output
```
(i^4*log_integral((c^4*f^4*x^4 + 4*c^4*e*f^3*x^3 + 6*c^4*e^2*f^2*x^2 + 4*c
^4*e^3*f*x + c^4*e^4)*e^(4*a/b)) + (c^4*f^4*h^4 - 4*c^4*e*f^3*h^3*i + 6*c^
4*e^2*f^2*h^2*i^2 - 4*c^4*e^3*f*h*i^3 + c^4*e^4*i^4)*e^(4*a/b)*log(b*log(c
*f*x + c*e) + a) + 4*(c*f*h*i^3 - c*e*i^4)*e^(a/b)*log_integral((c^3*f^3*x
^3 + 3*c^3*e*f^2*x^2 + 3*c^3*e^2*f*x + c^3*e^3)*e^(3*a/b)) + 6*(c^2*f^2*h^
2*i^2 - 2*c^2*e*f*h*i^3 + c^2*e^2*i^4)*e^(2*a/b)*log_integral((c^2*f^2*x^2
 + 2*c^2*e*f*x + c^2*e^2)*e^(2*a/b)) + 4*(c^3*f^3*h^3*i - 3*c^3*e*f^2*h^2*
i^2 + 3*c^3*e^2*f*h*i^3 - c^3*e^3*i^4)*e^(3*a/b)*log_integral((c*f*x + c*e
)*e^(a/b)))*e^(-4*a/b)/(b*c^4*d*f^5)
```

3.191.6 Sympy [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{\int \frac{h^4}{ae+afx+be\log(ce+cfx)+bfx\log(ce+cfx)}\,dx + \int \frac{i^4x^4}{ae+afx+be\log(ce+cfx)+bfx\log(ce+cfx)}\,dx + \int \frac{4hi^3x^3}{ae+afx+be\log(ce+cfx)+bf}}{d}$$

input
```
integrate((i*x+h)**4/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

output
```
(Integral(h**4/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x
)), x) + Integral(i**4*x**4/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*lo
g(c*e + c*f*x)), x) + Integral(4*h*i**3*x**3/(a*e + a*f*x + b*e*log(c*e +
c*f*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(6*h**2*i**2*x**2/(a*e + a*
f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(4*h**3
*i*x/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x))/d
```

3.191.7 Maxima [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^4}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate((i*x+h)^4/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima"
)
```

output
```
h^4*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f) + integrate((i^4*x^4 +
4*h*i^3*x^3 + 6*h^2*i^2*x^2 + 4*h^3*i*x)/(b*d*e*log(c) + a*d*e + (b*d*f*lo
g(c) + a*d*f)*x + (b*d*f*x + b*d*e)*log(f*x + e)), x)
```

3.191.8 Giac [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^4}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate((i*x+h)^4/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

output
```
integrate((i*x + h)^4/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+i\,x)^4}{(d\,e+d\,f\,x)\,(a+b\,\ln(c\,(e+f\,x)))}\,dx$$

input
```
int((h + i*x)^4/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
int((h + i*x)^4/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

3.192 $\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$

3.192.1 Optimal result

Integrand size = 32, antiderivative size = 177

$$\begin{aligned}\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx &= \frac{3e^{-\frac{a}{b}}i(fh-ei)^2\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^4}\\ &+ \frac{3e^{-\frac{2a}{b}}i^2(fh-ei)\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^4}\\ &+ \frac{e^{-\frac{3a}{b}}i^3\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^4} + \frac{(fh-ei)^3\log(a+b\log(c(e+fx)))}{bdf^4}\end{aligned}$$

output
```
3*i*(-e*i+f*h)^2*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^4+3*i^2*(-e*i+
f*h)*Ei(2*(a+b*ln(c*(f*x+e)))/b)/b/c^2/d/exp(2*a/b)/f^4+i^3*Ei(3*(a+b*ln(c
*(f*x+e)))/b)/b/c^3/d/exp(3*a/b)/f^4+(-e*i+f*h)^3*ln(a+b*ln(c*(f*x+e)))/b/
d/f^4
```

3.192.2 Mathematica [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$

input
```
Integrate[(h + i*x)^3/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
Integrate[(h + i*x)^3/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])), x]
```

3.192.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$

$$\Big\downarrow \; 2858$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^3}{df^3(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^3}{(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{df^4}$$

$$\Big\downarrow \; 2795$$

$$\frac{\int \left(\frac{(e+fx)^2i^3}{a+b\log(c(e+fx))}+\frac{3(fh-ei)(e+fx)i^2}{a+b\log(c(e+fx))}+\frac{3(fh-ei)^2i}{a+b\log(c(e+fx))}+\frac{(fh-ei)^3}{(e+fx)(a+b\log(c(e+fx)))}\right)d(e+fx)}{df^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{i^3e^{-\frac{3a}{b}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3}+\frac{3i^2e^{-\frac{2a}{b}}(fh-ei)\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2}+\frac{3ie^{-\frac{a}{b}}(fh-ei)^2\text{ExpIntegralEi}\left(\frac{a+b\log(}{}\right.}{bc}}{df^4}$$

input
```
Int[(h + i*x)^3/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
((3*i*(f*h - e*i)^2*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*E^(a/b
)) + (3*i^2*(f*h - e*i)*ExpIntegralEi[(2*(a + b*Log[c*(e + f*x)]))/b])/(b*
c^2*E^((2*a)/b)) + (i^3*ExpIntegralEi[(3*(a + b*Log[c*(e + f*x)]))/b])/(b*
c^3*E^((3*a)/b)) + ((f*h - e*i)^3*Log[a + b*Log[c*(e + f*x)]])/b)/(d*f^4)
```

3.192.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.192.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(179) = 358$.

Time = 2.46 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.04

method	result
derivativedivides	$-\frac{\frac{i^3 e^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3\ln(cfx+ce)-\frac{3a}{b}\right)}{b}-\frac{c^3f^3h^3\ln(a+b\ln(cfx+ce))}{b}+\frac{c^3e^3i^3\ln(a+b\ln(cfx+ce))}{b}-\frac{3ce\,i^3 e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2\ln(cfx+ce)-\right.}{b}}{}$
default	$-\frac{\frac{i^3 e^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3\ln(cfx+ce)-\frac{3a}{b}\right)}{b}-\frac{c^3f^3h^3\ln(a+b\ln(cfx+ce))}{b}+\frac{c^3e^3i^3\ln(a+b\ln(cfx+ce))}{b}-\frac{3ce\,i^3 e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2\ln(cfx+ce)-\right.}{b}}{}$
risch	$-\frac{e^3i^3\ln(a+b\ln(cfx+ce))}{f^4db}+\frac{3e^2h\,i^2\ln(a+b\ln(cfx+ce))}{f^3db}-\frac{3e\,h^2i\ln(a+b\ln(cfx+ce))}{f^2db}+\frac{h^3\ln(a+b\ln(cfx+ce))}{fdb}-$

input
```
int((i*x+h)^3/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3/f^4/d*(i^3/b*exp(-3*a/b)*Ei(1,-3*ln(c*f*x+c*e)-3*a/b)-c^3*f^3*h^3*l
n(a+b*ln(c*f*x+c*e))/b+c^3*e^3*i^3*ln(a+b*ln(c*f*x+c*e))/b-3*c*e*i^3/b*exp
(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+3*c^2*e^2*i^3/b*exp(-a/b)*Ei(1,-ln(c
*f*x+c*e)-a/b)+3*c*f*h*i^2/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+3*c^
2*f^2*h^2*i/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)+3*c^3*e*f^2*h^2*i*ln(a+b*
ln(c*f*x+c*e))/b-3*c^3*e^2*f*h*i^2*ln(a+b*ln(c*f*x+c*e))/b-6*c^2*e*f*h*i^2
/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\left(i^3\,\mathrm{log\_integral}\left((c^3f^3x^3+3\,c^3ef^2x^2+3\,c^3e^2fx+c^3e^3)e^{\left(\frac{3\,a}{b}\right)}\right)+(c^3f^3h^3-3\,c^3ef^2h^2i+3\,c^3e^2fhi^2-c^3\right.}{}$$

input
```
integrate((i*x+h)^3/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas"
)
```

output
```
(i^3*log_integral((c^3*f^3*x^3 + 3*c^3*e*f^2*x^2 + 3*c^3*e^2*f*x + c^3*e^3
)*e^(3*a/b)) + (c^3*f^3*h^3 - 3*c^3*e*f^2*h^2*i + 3*c^3*e^2*f*h*i^2 - c^3*
e^3*i^3)*e^(3*a/b)*log(b*log(c*f*x + c*e) + a) + 3*(c*f*h*i^2 - c*e*i^3)*e
^(a/b)*log_integral((c^2*f^2*x^2 + 2*c^2*e*f*x + c^2*e^2)*e^(2*a/b)) + 3*(
c^2*f^2*h^2*i - 2*c^2*e*f*h*i^2 + c^2*e^2*i^3)*e^(2*a/b)*log_integral((c*f
*x + c*e)*e^(a/b)))*e^(-3*a/b)/(b*c^3*d*f^4)
```

3.192.6 Sympy [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{\int \frac{h^3}{ae+afx+be\log{(ce+cfx)}+bfx\log{(ce+cfx)}}\,dx + \int \frac{i^3x^3}{ae+afx+be\log{(ce+cfx)}+bfx\log{(ce+cfx)}}\,dx + \int \frac{3hi^2x^2}{ae+afx+be\log{(ce+cfx)}+bfx \cdots}}{d}$$

input
```
integrate((i*x+h)**3/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

output
```
(Integral(h**3/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x
)), x) + Integral(i**3*x**3/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*lo
g(c*e + c*f*x)), x) + Integral(3*h*i**2*x**2/(a*e + a*f*x + b*e*log(c*e +
c*f*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(3*h**2*i*x/(a*e + a*f*x +
b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x))/d
```

3.192.7 Maxima [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^3}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate((i*x+h)^3/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima"
)
```

output
```
h^3*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f) + integrate((i^3*x^3 +
3*h*i^2*x^2 + 3*h^2*i*x)/(b*d*e*log(c) + a*d*e + (b*d*f*log(c) + a*d*f)*x
+ (b*d*f*x + b*d*e)*log(f*x + e)), x)
```

3.192.8 Giac [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\int \frac{(ix+h)^3}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

input
```
integrate((i*x+h)^3/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

output
```
integrate((i*x + h)^3/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\int \frac{(h+i\,x)^3}{(d\,e+d\,f\,x)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

input
```
int((h + i*x)^3/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
int((h + i*x)^3/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

3.193 $\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$

3.193.1 Optimal result

Integrand size = 32, antiderivative size = 124

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{2e^{-\frac{a}{b}}i(fh-ei)\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^3} + \frac{e^{-\frac{2a}{b}}i^2\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^3} + \frac{(fh-ei)^2\log(a+b\log(c(e+fx)))}{bdf^3}$$

output
```
2*i*(-e*i+f*h)*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^3+i^2*Ei(2*(a+b*
ln(c*(f*x+e)))/b)/b/c^2/d/exp(2*a/b)/f^3+(-e*i+f*h)^2*ln(a+b*ln(c*(f*x+e))
)/b/d/f^3
```

3.193.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{e^{-\frac{2a}{b}}\left(2ce^{a/b}i(fh-ei)\,\text{ExpIntegralEi}\left(\frac{a}{b}+\log(c(e+fx))\right)+i^2\,\text{ExpIntegralEi}\left(2\left(\frac{a}{b}+\log(c(e+fx))\right)\right)+\right.}{bc^2df^3}$$

input
```
Integrate[(h + i*x)^2/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
(2*c*E^(a/b)*i*(f*h - e*i)*ExpIntegralEi[a/b + Log[c*(e + f*x)]] + i^2*Exp
IntegralEi[2*(a/b + Log[c*(e + f*x)])] + c^2*E^((2*a)/b)*(f*h - e*i)^2*Log
[a + b*Log[c*(e + f*x)]])/(b*c^2*d*E^((2*a)/b)*f^3)
```

3.193.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^2}{df^2(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fh-ei+i(e+fx))^2}{(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{df^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{(e+fx)i^2}{a+b\log(c(e+fx))}+\frac{2(fh-ei)i}{a+b\log(c(e+fx))}+\frac{(fh-ei)^2}{(e+fx)(a+b\log(c(e+fx)))}\right)d(e+fx)}{df^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{i^2e^{-\frac{2a}{b}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2}+\frac{2ie^{-\frac{a}{b}}(fh-ei)\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bc}+\frac{(fh-ei)^2\log(a+b\log(c(e+fx)))}{b}}{df^3}$$

input
```
Int[(h + i*x)^2/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
((2*i*(f*h - e*i)*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*E^(a/b))
 + (i^2*ExpIntegralEi[(2*(a + b*Log[c*(e + f*x)]))/b])/(b*c^2*E^((2*a)/b))
 + ((f*h - e*i)^2*Log[a + b*Log[c*(e + f*x)]])/b)/(d*f^3)
```

3.193.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.193.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{i^2 e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2\ln(cfx+ce)-\frac{2a}{b}\right)}{b}+\frac{c^2e^2i^2\ln(a+b\ln(cfx+ce))}{b}+\frac{c^2f^2h^2\ln(a+b\ln(cfx+ce))}{b}+\frac{2ce\, i^2 e^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{b}}{c^2f^3d}$
default	$\frac{-\frac{i^2 e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2\ln(cfx+ce)-\frac{2a}{b}\right)}{b}+\frac{c^2e^2i^2\ln(a+b\ln(cfx+ce))}{b}+\frac{c^2f^2h^2\ln(a+b\ln(cfx+ce))}{b}+\frac{2ce\, i^2 e^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{b}}{c^2f^3d}$
risch	$\frac{e^2i^2\ln(a+b\ln(cfx+ce))}{f^3db}-\frac{2ehi\ln(a+b\ln(cfx+ce))}{f^2db}+\frac{h^2\ln(a+b\ln(cfx+ce))}{fdb}+\frac{2e\, i^2 e^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{c\, f^3db}$

input
```
int((i*x+h)^2/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

output
```
1/c^2/f^3/d*(-i^2/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+c^2*e^2*i^2*l
n(a+b*ln(c*f*x+c*e))/b+c^2*f^2*h^2*ln(a+b*ln(c*f*x+c*e))/b+2*c*e*i^2/b*exp
(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-2*c*f*h*i/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-
a/b)-2*c^2*e*f*h*i*ln(a+b*ln(c*f*x+c*e))/b)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{\left((c^2f^2h^2 - 2\,c^2efhi + c^2e^2i^2)e^{\left(\frac{2\,a}{b}\right)}\log\left(b\log\left(cfx+ce\right)+a\right) + i^2\,\mathrm{log\_integral}\left((c^2f^2x^2 + 2\,c^2efx + c^2e^2\right)\right.}{bc^2df^3}$$

input
```
integrate((i*x+h)^2/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas"
)
```

output
```
((c^2*f^2*h^2 - 2*c^2*e*f*h*i + c^2*e^2*i^2)*e^(2*a/b)*log(b*log(c*f*x + c
*e) + a) + i^2*log_integral((c^2*f^2*x^2 + 2*c^2*e*f*x + c^2*e^2)*e^(2*a/b
)) + 2*(c*f*h*i - c*e*i^2)*e^(a/b)*log_integral((c*f*x + c*e)*e^(a/b)))*e^
(-2*a/b)/(b*c^2*d*f^3)
```

3.193.6 Sympy [F]

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{\int \frac{h^2}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx + \int \frac{i^2x^2}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx + \int \frac{2hix}{ae+afx+be\log\left(ce+cfx\right)+bf}}{d}$$

input
```
integrate((i*x+h)**2/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

output
```
(Integral(h**2/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x
)), x) + Integral(i**2*x**2/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*lo
g(c*e + c*f*x)), x) + Integral(2*h*i*x/(a*e + a*f*x + b*e*log(c*e + c*f*x)
 + b*f*x*log(c*e + c*f*x)), x))/d
```

3.193.7 Maxima [F]

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^2}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate((i*x+h)^2/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima"
)
```

output
```
h^2*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f) + integrate((i^2*x^2 +
2*h*i*x)/(b*d*e*log(c) + a*d*e + (b*d*f*log(c) + a*d*f)*x + (b*d*f*x + b*d
*e)*log(f*x + e)), x)
```

3.193.8 Giac [F]

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^2}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate((i*x+h)^2/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

output
```
integrate((i*x + h)^2/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+i\,x)^2}{(d\,e+d\,f\,x)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

input
```
int((h + i*x)^2/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
int((h + i*x)^2/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

3.194 $\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$

3.194.1 Optimal result

Integrand size = 30, antiderivative size = 71

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{e^{-\frac{a}{b}}i\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^2} + \frac{(fh-ei)\log(a+b\log(c(e+fx)))}{bdf^2}$$

output
```
i*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^2+(-e*i+f*h)*ln(a+b*ln(c*(f*x
+e)))/b/d/f^2
```

3.194.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{e^{-\frac{a}{b}}\left(i\,\text{ExpIntegralEi}\left(\frac{a}{b}+\log(c(e+fx))\right)+ce^{a/b}(fh-ei)\log(a+b\log(c(e+fx)))\right)}{bcdf^2}$$

input
```
Integrate[(h + i*x)/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
(i*ExpIntegralEi[a/b + Log[c*(e + f*x)]] + c*E^(a/b)*(f*h - e*i)*Log[a + b
*Log[c*(e + f*x)]])/(b*c*d*E^(a/b)*f^2)
```

3.194.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{f\left(h-\frac{ei}{f}\right)+i(e+fx)}{df(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{fh-ei+i(e+fx)}{(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{df^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{i}{a+b\log(c(e+fx))}+\frac{fh-ei}{(e+fx)(a+b\log(c(e+fx)))}\right)d(e+fx)}{df^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{ie^{-\frac{a}{b}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bc}+\frac{(fh-ei)\log(a+b\log(c(e+fx)))}{b}}{df^2}$$

input
```
Int[(h + i*x)/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
((i*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*E^(a/b)) + ((f*h - e*i
)*Log[a + b*Log[c*(e + f*x)]])/b)/(d*f^2)
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.194.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$-\frac{\frac{i\,{\mathrm e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\ln \left(c f x+c e\right)-\frac{a}{b}\right)}{b}-\frac{h c f \ln \left(a+b \ln \left(c f x+c e\right)\right)}{b}+\frac{c e i \ln \left(a+b \ln \left(c f x+c e\right)\right)}{b}}{c \,f^{2} d}$	88
default	$-\frac{\frac{i\,{\mathrm e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\ln \left(c f x+c e\right)-\frac{a}{b}\right)}{b}-\frac{h c f \ln \left(a+b \ln \left(c f x+c e\right)\right)}{b}+\frac{c e i \ln \left(a+b \ln \left(c f x+c e\right)\right)}{b}}{c \,f^{2} d}$	88
risch	$-\frac{e i \ln \left(a+b \ln \left(c f x+c e\right)\right)}{f^{2} d b}+\frac{h \ln \left(a+b \ln \left(c f x+c e\right)\right)}{f d b}-\frac{i\,{\mathrm e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\ln \left(c f x+c e\right)-\frac{a}{b}\right)}{c \,f^{2} d b}$	96

input
```
int((i*x+h)/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

output
```
-1/c/f^2/d*(i/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-h*c*f*ln(a+b*ln(c*f*x+c
*e))/b+c*e*i*ln(a+b*ln(c*f*x+c*e))/b)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\left((cfh-cei)e^{\frac{a}{b}}\log\left(b\log\left(cfx+ce\right)+a\right)+i\,\mathrm{log\_integral}\left((cfx+ce)e^{\frac{a}{b}}\right)\right)e^{\left(-\frac{a}{b}\right)}}{bcdf^2}$$

input
```
integrate((i*x+h)/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")
```

output
```
((c*f*h - c*e*i)*e^(a/b)*log(b*log(c*f*x + c*e) + a) + i*log_integral((c*f
*x + c*e)*e^(a/b)))*e^(-a/b)/(b*c*d*f^2)
```

3.194.6 Sympy [F]

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{h}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx+\int \frac{ix}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx}{d}$$

input
```
integrate((i*x+h)/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

output
```
(Integral(h/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)),
 x) + Integral(i*x/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c
*f*x)), x))/d
```

3.194.7 Maxima [F]

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{ix+h}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

input
```
integrate((i*x+h)/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
```

output
```
i*integrate(x/(b*d*e*log(c) + a*d*e + (b*d*f*log(c) + a*d*f)*x + (b*d*f*x
+ b*d*e)*log(f*x + e)), x) + h*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d
*f)
```

3.194.8 Giac [F]

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{ix+h}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate((i*x+h)/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

output
```
integrate((i*x + h)/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{h+i\,x}{(d\,e+d\,f\,x)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

input
```
int((h + i*x)/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
int((h + i*x)/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

3.195 $\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$

3.195.1 Optimal result

Integrand size = 25, antiderivative size = 23

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\frac{\log(a+b\log(c(e+fx)))}{bdf}$$

output
```
ln(a+b*ln(c*(f*x+e)))/b/d/f
```

3.195.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\frac{\log(a+b\log(c(e+fx)))}{bdf}$$

input
```
Integrate[1/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
Log[a + b*Log[c*(e + f*x)]]/(b*d*f)
```

3.195.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2837, 27, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}dx$$
$$\Big\downarrow \text{ 2837}$$
$$\frac{\int \frac{1}{d(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{f}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(e+fx)(a+b\log(c(e+fx)))}d(e+fx)}{df}$$
$$\Big\downarrow \text{ 2739}$$
$$\frac{\int \frac{1}{a+b\log(c(e+fx))}d(a+b\log(c(e+fx)))}{bdf}$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\log(a+b\log(c(e+fx)))}{bdf}$$

input
```
Int[1/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
Log[a + b*Log[c*(e + f*x)]]/(b*d*f)
```

3.195.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.195.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
norman	$\frac{\ln(a+b\ln(c(fx+e)))}{bdf}$	24
parallelrisch	$\frac{\ln(a+b\ln(c(fx+e)))}{bdf}$	24
derivativedivides	$\frac{\ln(a+b\ln(cfx+ce))}{fdb}$	25
default	$\frac{\ln(a+b\ln(cfx+ce))}{fdb}$	25
risch	$\frac{\ln\left(\ln(c(fx+e))+\frac{a}{b}\right)}{bdf}$	26

input
```
int(1/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*ln(c*(f*x+e)))/b/d/f
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log\left(b\log\left(cfx+ce\right)+a\right)}{bdf}$$

input
```
integrate(1/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")
```

output
```
log(b*log(c*f*x + c*e) + a)/(b*d*f)
```

3.195.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log\left(\frac{a}{b}+\log\left(c(e+fx)\right)\right)}{bdf}$$

input
```
integrate(1/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

output
```
log(a/b + log(c*(e + f*x)))/(b*d*f)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log\left(\frac{b\log(fx+e)+b\log(c)+a}{b}\right)}{bdf}$$

input
```
integrate(1/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
```

output
```
log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log\left(b\log\left(cfx+ce\right)+a\right)}{bdf}$$

input
```
integrate(1/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

output
```
log(b*log(c*f*x + c*e) + a)/(b*d*f)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\ln\left(a+b\,\ln\left(c\left(e+f\,x\right)\right)\right)}{b\,d\,f}$$

input
```
int(1/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
log(a + b*log(c*(e + f*x)))/(b*d*f)
```

3.196 $\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx$

3.196.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \frac{\log(a+b\log(c(e+fx)))}{bd(fh-ei)} - \frac{i\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(e+fx)))},x\right)}{d(fh-ei)}$$

output
```
ln(a+b*ln(c*(f*x+e)))/b/d/(-e*i+f*h)-i*Unintegrable(1/(i*x+h)/(a+b*ln(c*(f
*x+e))),x)/d/(-e*i+f*h)
```

3.196.2 Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx$$

input
```
Integrate[1/((d*e + d*f*x)*(h + i*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
Integrate[1/((d*e + d*f*x)*(h + i*x)*(a + b*Log[c*(e + f*x)])), x]
```

3.196.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(h+ix)(de+dfx)(a+b\log(c(e+fx)))}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{f}{d(e+fx)(fh-ei)(a+b\log(c(e+fx)))}-\frac{i}{d(h+ix)(fh-ei)(a+b\log(c(e+fx)))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(a+b\log(c(e+fx)))}{bd(fh-ei)}-\frac{i\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}dx}{d(fh-ei)}$$

input
```
Int[1/((d*e + d*f*x)*(h + i*x)*(a + b*Log[c*(e + f*x)])),x]
```

output
```
$Aborted
```

3.196.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.196.4 Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dfx + de)\,(ix + h)\,(a + b\ln(c\,(fx + e)))}dx$$

input
```
int(1/(d*f*x+d*e)/(i*x+h)/(a+b*ln(c*(f*x+e))),x)
```

output
```
int(1/(d*f*x+d*e)/(i*x+h)/(a+b*ln(c*(f*x+e))),x)
```

3.196.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{1}{(de + dfx)(h + ix)(a + b\log(c(e + fx)))}\,dx = \int \frac{1}{(dfx + de)(ix + h)(b\log((fx + e)c) + a)}\,dx$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*log(c*(f*x+e))),x, algorithm="fricas"
)
```

output
```
integral(1/(a*d*f*i*x^2 + a*d*e*h + (a*d*f*h + a*d*e*i)*x + (b*d*f*i*x^2 +
 b*d*e*h + (b*d*f*h + b*d*e*i)*x)*log(c*f*x + c*e)), x)
```

3.196.6 Sympy [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.09

$$\int \frac{1}{(de + dfx)(h + ix)(a + b\log(c(e + fx)))}\,dx = \frac{\int \frac{1}{aeh + aeix + afhx + afix^2 + beh\log(ce + cfx) + beix\log(ce + cfx) + bfhx\log(ce + cfx) + bfix^2\log(ce + cfx)}\,dx}{d}$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*ln(c*(f*x+e))),x)
```

output
```
Integral(1/(a*e*h + a*e*i*x + a*f*h*x + a*f*i*x**2 + b*e*h*log(c*e + c*f*x
) + b*e*i*x*log(c*e + c*f*x) + b*f*h*x*log(c*e + c*f*x) + b*f*i*x**2*log(c
*e + c*f*x)), x)/d
```

3.196.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*log(c*(f*x+e))),x, algorithm="maxima"
)
```

output
```
integrate(1/((d*f*x + d*e)*(i*x + h)*(b*log((f*x + e)*c) + a)), x)
```

3.196.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

output
```
integrate(1/((d*f*x + d*e)*(i*x + h)*(b*log((f*x + e)*c) + a)), x)
```

3.196.9 Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(h+i\,x)\,\left(d\,e+d\,f\,x\right)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

input
```
int(1/((h + i*x)*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
int(1/((h + i*x)*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

3.197 $\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$

3.197.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \frac{f\log(a+b\log(c(e+fx)))}{bd(fh-ei)^2} - \frac{i\text{Int}\left(\frac{1}{(h+ix)^2(a+b\log(c(e+fx)))},x\right)}{d(fh-ei)} - \frac{fi\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(e+fx)))},x\right)}{d(fh-ei)^2}$$

output
```
f*ln(a+b*ln(c*(f*x+e)))/b/d/(-e*i+f*h)^2-i*Unintegrable(1/(i*x+h)^2/(a+b*l
n(c*(f*x+e))),x)/d/(-e*i+f*h)-f*i*Unintegrable(1/(i*x+h)/(a+b*ln(c*(f*x+e)
)),x)/d/(-e*i+f*h)^2
```

3.197.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$$

input
```
Integrate[1/((d*e + d*f*x)*(h + i*x)^2*(a + b*Log[c*(e + f*x)])),x]
```

output
```
Integrate[1/((d*e + d*f*x)*(h + i*x)^2*(a + b*Log[c*(e + f*x)])), x]
```

3.197.3 Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(h+ix)^2(de+dfx)(a+b\log(c(e+fx)))}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{f^2}{d(e+fx)(fh-ei)^2(a+b\log(c(e+fx)))}-\frac{fi}{d(h+ix)(fh-ei)^2(a+b\log(c(e+fx)))}-\frac{}{d(h+ix)^2(fh-e}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(e+fx)))}dx}{d(fh-ei)}-\frac{fi\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}dx}{d(fh-ei)^2}+\frac{f\log(a+b\log(c(e+fx)))}{bd(fh-ei)^2}$$

input
```
Int[1/((d*e + d*f*x)*(h + i*x)^2*(a + b*Log[c*(e + f*x)])),x]
```

output
```
$Aborted
```

3.197.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.197.4 Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dfx+de)\,(ix+h)^2\,(a+b\ln{(c\,(fx+e))})}dx$$

input
```
int(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*ln(c*(f*x+e))),x)
```

output
```
int(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*ln(c*(f*x+e))),x)
```

3.197.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)^2(b\log\left((fx+e)c\right)+a)}\,dx$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*log(c*(f*x+e))),x, algorithm="frica
s")
```

output
```
integral(1/(a*d*f*i^2*x^3 + a*d*e*h^2 + (2*a*d*f*h*i + a*d*e*i^2)*x^2 + (a
*d*f*h^2 + 2*a*d*e*h*i)*x + (b*d*f*i^2*x^3 + b*d*e*h^2 + (2*b*d*f*h*i + b*
d*e*i^2)*x^2 + (b*d*f*h^2 + 2*b*d*e*h*i)*x)*log(c*f*x + c*e)), x)
```

3.197.6 Sympy [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.62

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{1}{aeh^2+2aehix+aei^2x^2+afh^2x+2afhix^2+afi^2x^3+beh^2\log(ce+cfx)+2behix\log(ce+cfx)+bei^2x^2\log(ce+cfx)+bfh^2x\log(ce+cfx)+2bfh}}{d}$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)**2/(a+b*ln(c*(f*x+e))),x)
```

output
```
Integral(1/(a*e*h**2 + 2*a*e*h*i*x + a*e*i**2*x**2 + a*f*h**2*x + 2*a*f*h*
i*x**2 + a*f*i**2*x**3 + b*e*h**2*log(c*e + c*f*x) + 2*b*e*h*i*x*log(c*e +
 c*f*x) + b*e*i**2*x**2*log(c*e + c*f*x) + b*f*h**2*x*log(c*e + c*f*x) + 2
*b*f*h*i*x**2*log(c*e + c*f*x) + b*f*i**2*x**3*log(c*e + c*f*x)), x)/d
```

3.197.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)^2(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*log(c*(f*x+e))),x, algorithm="maxim
a")
```

output
```
integrate(1/((d*f*x + d*e)*(i*x + h)^2*(b*log((f*x + e)*c) + a)), x)
```

3.197.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$$
$$=\int \frac{1}{(dfx+de)(ix+h)^2(b\log((fx+e)c)+a)}\,dx$$

input
```
integrate(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*log(c*(f*x+e))),x, algorithm="giac"
)
```

output
```
integrate(1/((d*f*x + d*e)*(i*x + h)^2*(b*log((f*x + e)*c) + a)), x)
```

3.197.9 Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$$
$$=\int \frac{1}{(h+i\,x)^2\,(d\,e+d\,f\,x)\,(a+b\,\ln(c\,(e+f\,x)))}\,dx$$

input
```
int(1/((h + i*x)^2*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
```

output
```
int(1/((h + i*x)^2*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

3.198 $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

3.198.1 Optimal result

Integrand size = 31, antiderivative size = 485

$$\begin{aligned}\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx &= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} \\ &- \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} \\ &+ \frac{92b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} \\ &+ \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} \\ &+ \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} \\ &+ \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} \\ &- \frac{2(ef-dg)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} \\ &- \frac{4b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} \\ &- \frac{2b(ef-dg)^{5/2}n\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}}\end{aligned}$$

output
```
-32/45*b*(-d*g+e*f)*n*(g*x+f)^(3/2)/e^2-4/25*b*n*(g*x+f)^(5/2)/e+92/15*b*(
-d*g+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(7/2)+
2*b*(-d*g+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e
^(7/2)+2/3*(-d*g+e*f)*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/e^2+2/5*(g*x+f)^
(5/2)*(a+b*ln(c*(e*x+d)^n))/e-2*(-d*g+e*f)^(5/2)*arctanh(e^(1/2)*(g*x+f)^(
1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/e^(7/2)-4*b*(-d*g+e*f)^(5/2)*
n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^
(1/2)/(-d*g+e*f)^(1/2)))/e^(7/2)-2*b*(-d*g+e*f)^(5/2)*n*polylog(2,1-2/(1-e
^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(7/2)-92/15*b*(-d*g+e*f)^2*n*(g*
x+f)^(1/2)/e^3+2*(-d*g+e*f)^2*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e^3
```

3.198.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \frac{900a\sqrt{e}(ef-dg)^2\sqrt{f+gx}-1800b\sqrt{e}(ef-dg)^2n\sqrt{f+gx}+}{}$$

input
```
Integrate[((f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

output
```
(900*a*Sqrt[e]*(e*f - d*g)^2*Sqrt[f + g*x] - 1800*b*Sqrt[e]*(e*f - d*g)^2*
n*Sqrt[f + g*x] + 1800*b*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x
])/Sqrt[e*f - d*g]] - 200*b*(e*f - d*g)*n*(Sqrt[e]*Sqrt[f + g*x]*(4*e*f -
3*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[
e*f - d*g]]) - 24*b*n*(3*e^(5/2)*(f + g*x)^(5/2) + 5*(e*f - d*g)*(Sqrt[e]*
Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[
e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])) + 900*b*Sqrt[e]*(e*f - d*g)^2*Sqrt[f
+ g*x]*Log[c*(d + e*x)^n] + 300*e^(3/2)*(e*f - d*g)*(f + g*x)^(3/2)*(a + b
*Log[c*(d + e*x)^n]) + 180*e^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^
n]) + 450*(e*f - d*g)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g]
 - Sqrt[e]*Sqrt[f + g*x]] - 450*(e*f - d*g)^(5/2)*(a + b*Log[c*(d + e*x)^n
])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - 225*b*(e*f - d*g)^(5/2)*
n*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqr
t[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/
2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2
25*b*(e*f - d*g)^(5/2)*n*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Lo
g[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f +
 g*x])/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/S
qrt[e*f - d*g])/2]))/(450*e^(7/2))
```

3.198.3 Rubi [A] (verified)

Time = 4.00 (sec) , antiderivative size = 969, normalized size of antiderivative = 2.00, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.871, Rules used = {2858, 2788, 2756, 60, 60, 60, 73, 221, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}dx$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2788}$$

$$\frac{\frac{g\int \left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2756}$$

$$\frac{\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}{d+ex}d(d+ex)}{5g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)}{d+ex}}{e}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}{d+ex}d(d+ex)+\frac{2}{5}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}\right)}{5g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{(f-}{}}{e}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\right)+\frac{2}{5}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right.\right.}{5g}\right.}{e}$$

$$e$$

$\Big\downarrow$ 60

$$\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)+\frac{2}{3}\left(\frac{g(d+\right.\right.\right.}{5g}\right.}{e}$$

$$e$$

$\Big\downarrow$ 73

$$\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\right.\right.\right.}{5g}\right.}{e}$$

$$e$$

$\Big\downarrow$ 221

$$\frac{\left(f-\frac{dg}{e}\right)\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\right.\right.}{}\right.}{}}{e}$$

$\Big\downarrow$ 2788

3.198. $\displaystyle\int\frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

g

$$\left(f-\frac{dg}{e}\right)\left(\frac{g\int\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)\right)+-$$

$\downarrow$ 2756

$$\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\int\frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}{d+ex}d(d+ex)}{3g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\,\mathrm{lo}}{d+ex}\right.$$

$\downarrow$ 60

$$\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\right)}{3g}\right)}{e}+\left(f-\frac{d}{e}\right.\right.$$

$\downarrow$ 60

3.198. $\int\frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)+\frac{2}{3}\left(\cdots\right.\right.}{3g}\cdots\right)}{e}\cdots\right.$$

$\downarrow$ 73

$$\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+\cdots}{e}}\cdots\right.\right.}{3g}\cdots\right)}{e}\cdots\right.$$

$\downarrow$ 221

3.198. $\displaystyle\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{d}{e}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 2788

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)\right)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

3.198. $\int\frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+e}{(d+ex)\sqrt{f-\frac{dg}{e}+}}\right.\right.$$

$\downarrow$ 60

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}\right.\right.$$

$\downarrow$ 73

3.198. $\int\frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\,(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(\cdots}{}}\cdots\right)}{g}\cdots\right)}{e}\cdots\right)\cdots\right)}{\cdots}$$

$\downarrow$ 221

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\,(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(2\sqrt{\frac{g(d+\cdots}{e}}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$$

$\downarrow$ 2790

3.198. $\int\frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(-bn\int -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}}}{\sqrt{ef-d}}\right)}{\sqrt{ef-d}}\right.\right.\right.$$

$\downarrow$ 27

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{2b\sqrt{e}n\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+}{\sqrt{ef-dg}}\right.\right.\right.$$

$\downarrow$ 7267

3.198. $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\cdots}{\cdots}\right.\right.\right.}{5g}\right.}{e}$$

$\downarrow$ 2092

$$\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\cdots}{\cdots}\right.\right.\right.}{5g}\right.}{e}$$

$\downarrow$ 6546

3.198. $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\cdots}{\cdots}\right)\right)\right)}{5g}\right)}{e}$$

$\Big\downarrow$ 6470

$$\frac{g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\cdots}{\cdots}\right)\right)\right)}{5g}\right)}{e}$$

$\Big\downarrow$ 2849

3.198. $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}}{}\right.\right.\right.}{5g}\right.$$
$$\overline{\qquad e \qquad}$$

$\downarrow$ 2752

$$g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}(a+b\log(c(d+ex)^n))}{5g}-\frac{2ben\left(\frac{2}{5}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}+\left(f-\frac{dg}{e}\right)\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}}{}\right.\right.\right.}{5g}\right.$$
$$\overline{\qquad e \qquad}$$

3.198. $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

input
```
Int[((f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

output
```
((g*((-2*b*e*n*((2*(f - (d*g)/e + (g*(d + e*x))/e)^(5/2))/5 + (f - (d*g)/e
)*((2*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2))/3 + (f - (d*g)/e)*(2*Sqrt[f -
 (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(Sqrt[e]*Sq
rt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g]))))/(
5*g) + (2*e*(f - (d*g)/e + (g*(d + e*x))/e)^(5/2)*(a + b*Log[c*(d + e*x)^n
]))/(5*g)))/e + (f - (d*g)/e)*((g*((-2*b*e*n*((2*(f - (d*g)/e + (g*(d + e*
x))/e)^(3/2))/3 + (f - (d*g)/e)*(2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (
2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/
e])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])))/(3*g) + (2*e*(f - (d*g)/e + (g*(d
 + e*x))/e)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(3*g)))/e + (f - (d*g)/e)*((
g*((-2*b*e*n*(2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g
)/e)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]
])/Sqrt[e*f - d*g]))/g + (2*e*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]*(a + b*L
og[c*(d + e*x)^n]))/g))/e + (f - (d*g)/e)*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sq
rt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)
^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e
+ (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(
Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (
Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e] +
(Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(...
```

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

3.198. $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.198.4 Maple [F]

$$\int \frac{(gx+f)^{\frac{5}{2}} (a+b\ln (c(ex+d)^n))}{ex+d} dx$$

input
```
int((g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

output
```
int((g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

3.198.5 Fricas [F]

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\int \frac{(gx+f)^{\frac{5}{2}}(b\log((ex+d)^nc)+a)}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(5/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="frica
s")
```

output
```
integral(((b*g^2*x^2 + 2*b*f*g*x + b*f^2)*sqrt(g*x + f)*log((e*x + d)^n*c)
 + (a*g^2*x^2 + 2*a*f*g*x + a*f^2)*sqrt(g*x + f))/(e*x + d), x)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**(5/2)*(a+b*ln(c*(e*x+d)**n))/(e*x+d),x)
```

output
```
Timed out
```

3.198.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(5/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.198.8 Giac [F]

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}(b\log((ex+d)^n c)+a)}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(5/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^(5/2)*(b*log((e*x + d)^n*c) + a)/(e*x + d), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(f+g\,x)^{5/2}\,(a+b\,\ln(c\,(d+e\,x)^n))}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^(5/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x),x)
```

output
```
int(((f + g*x)^(5/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x), x)
```

3.199 $\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex} dx$

3.199.1 Optimal result

Integrand size = 31, antiderivative size = 417

$$\begin{aligned}\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx &= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} \\ &- \frac{4bn(f+gx)^{3/2}}{9e} + \frac{16b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} \\ &+ \frac{2b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} \\ &+ \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} \\ &+ \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} \\ &- \frac{2(ef-dg)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} \\ &- \frac{4b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} \\ &- \frac{2b(ef-dg)^{3/2}n\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}}\end{aligned}$$

output
```
-4/9*b*n*(g*x+f)^(3/2)/e+16/3*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x+f)
^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)+2*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g
*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(5/2)+2/3*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d
)^n))/e-2*(-d*g+e*f)^(3/2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))
*(a+b*ln(c*(e*x+d)^n))/e^(5/2)-4*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x
+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
))/e^(5/2)-2*b*(-d*g+e*f)^(3/2)*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(
-d*g+e*f)^(1/2)))/e^(5/2)-16/3*b*(-d*g+e*f)*n*(g*x+f)^(1/2)/e^2+2*(-d*g+e*
f)*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e^2
```

3.199.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^{3/2}\,(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx = \frac{36a\sqrt{e}(ef-dg)\sqrt{f+gx}-72b\sqrt{e}(ef-dg)n\sqrt{f+gx}-8b\sqrt{e}}{}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

output
```
(36*a*Sqrt[e]*(e*f - d*g)*Sqrt[f + g*x] - 72*b*Sqrt[e]*(e*f - d*g)*n*Sqrt[
f + g*x] - 8*b*Sqrt[e]*n*Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) + 96*b*(e*f
 - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 36*b*Sq
rt[e]*(e*f - d*g)*Sqrt[f + g*x]*Log[c*(d + e*x)^n] + 12*e^(3/2)*(f + g*x)^
(3/2)*(a + b*Log[c*(d + e*x)^n]) + 18*(e*f - d*g)^(3/2)*(a + b*Log[c*(d +
e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 18*(e*f - d*g)^(3/
2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]
 + 9*b*(e*f - d*g)^(3/2)*n*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(L
og[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f
+ g*x])/(2*Sqrt[e*f - d*g])]) - 9*b*(e*f - d*g)^(3/2)*n*Log[Sqrt[e*f - d*g
] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] +
 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) - 18*b*(e*f - d*g
)^(3/2)*n*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])] +
18*b*(e*f - d*g)^(3/2)*n*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
- d*g])/2])/(18*e^(5/2))
```

3.199.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.74, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.645, Rules used = {2858, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$$

$$\Big\downarrow \ 2858$$

$$\frac{\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{e}$$

$$\Big\downarrow \ 2788$$

$$\frac{\frac{g\int \sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{e}$$

$$\Big\downarrow \ 2756$$

$$\frac{\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\int \frac{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}{d+ex}d(d+ex)}{3g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}\cdots}{e}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}\right)}{3g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{\sqrt{f-\frac{dg}{e}\cdots}}{\cdots}}{e}$$

$$\Big\downarrow \ 60$$

$$\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)+\frac{2}{3}\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right.\cdots}{3g}\right)}{e}$$

$$e$$

$\downarrow$ 73

$$\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)+\frac{2}{3}\cdots}{3g}\right)}{e}$$

$$e$$

$\downarrow$ 221

$$\frac{\left(f-\frac{dg}{e}\right)\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\left(2\sqrt{\frac{g(d+ex)}{e}}\cdots\right.\right.}{\cdots}\right)}{\cdots}}{e}$$

$\downarrow$ 2788

3.199. $\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\frac{\left(f-\frac{dg}{e}\right)\left(\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)\right)+\frac{g\left(\frac{2e\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}(a+b\log(c\cdots}{3g}\cdots\right)}{\cdots}}{e}$$

$\downarrow$ 2756

$$\frac{\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+\cdots\right)}{\cdots}$$

$\downarrow$ 60

$$\frac{\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\cdots\right)}{\cdots}$$

$\downarrow$ 73

3.199. $\displaystyle\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}\right.$$

$\downarrow$ 221

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}-\cdots\right.}{\cdots}\right.}{e}\right.$$

$\downarrow$ 2790

3.199. $\int\frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(-bn\int -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c}{\sqrt{ef-dg}}\right.\right.$$

$\downarrow$ 27

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{2b\sqrt{e}n\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n)}{\sqrt{ef-dg}}\right.\right.$$

$\downarrow$ 7267

3.199. $\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}}}{\sqrt{ef-d}}\right.}{\sqrt{ej}}\right.\right.$$

$$\Big\downarrow \text{ 2092}$$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}}}{\sqrt{ef-d}}\right.}{\sqrt{ej}}\right.\right.$$

$$\Big\downarrow \text{ 6546}$$

3.199. $\int\frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{ar}}{}\right.\right.$$

$\downarrow$ 6470

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}}}{\sqrt{ef-dg}}}\right.}{\sqrt{e}}}{\sqrt{e}\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.\right.$$

$\downarrow$ 2849

3.199. $\displaystyle\int\frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$g\left(\frac{2e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}(a+b\log(c(d+ex)^n))}{3g}-\frac{2ben\left(\frac{2}{3}\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}+\left(f-\frac{dg}{e}\right)\left(2\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}-\frac{2\sqrt{e}\left(f-\frac{dg}{e}\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+g(}}{\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.\right.}{3g}\right.$$
$$\overline{\qquad e \qquad}$$

$\downarrow$ 2752

$$\left(f-\frac{dg}{e}\right)\left(\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}}}{\sqrt{ef-dg}}}\right.}{\sqrt{e}}}{\sqrt{e}\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.\right.$$

3.199. $\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

input
```
Int[((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

output
```
((g*((-2*b*e*n*((2*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2))/3 + (f - (d*g)/e
)*(2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTan
h[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f
 - d*g])))/(3*g) + (2*e*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2)*(a + b*Log[c
*(d + e*x)^n]))/(3*g)))/e + (f - (d*g)/e)*((g*((-2*b*e*n*(2*Sqrt[f - (d*g)
/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d*g)/e)*ArcTanh[(Sqrt[e]*Sqrt[f -
 (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g]))/g + (2*e*
Sqrt[f - (d*g)/e + (g*(d + e*x))/e]*(a + b*Log[c*(d + e*x)^n]))/g))/e + (f
 - (d*g)/e)*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))
/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e
^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f -
 d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*
(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*
(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1
- 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/
(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f - d*g])))/Sqrt[e*f - d*g]))/e
```

3.199.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.199. $\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.199.4 Maple [F]

$$\int \frac{(gx+f)^{\frac{3}{2}} \left(a+b\ln \left(c(ex+d)^{n}\right)\right)}{ex+d}dx$$

input
```
int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

output
```
int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

3.199.5 Fricas [F]

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}(b\log((ex+d)^n c)+a)}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="frica
s")
```

output
```
integral(((b*g*x + b*f)*sqrt(g*x + f)*log((e*x + d)^n*c) + (a*g*x + a*f)*s
qrt(g*x + f))/(e*x + d), x)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(a+b*ln(c*(e*x+d)**n))/(e*x+d),x)
```

output
```
Timed out
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.199.8 Giac [F]

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}(b\log((ex+d)^n c)+a)}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a)/(e*x + d), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(f+g\,x)^{3/2}\,(a+b\,\ln(c\,(d+e\,x)^n))}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x),x)
```

output
```
int(((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x), x)
```

3.200 $\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

3.200.1 Optimal result

Integrand size = 31, antiderivative size = 349

$$\begin{aligned}\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx &= -\frac{4bn\sqrt{f+gx}}{e}+\frac{4b\sqrt{ef-dg}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}}\\
&+\frac{2b\sqrt{ef-dg}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}}+\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e}\\
&-\frac{2\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}}\\
&-\frac{4b\sqrt{ef-dg}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}\\
&-\frac{2b\sqrt{ef-dg}n\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}\end{aligned}$$

output
```
4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(-d*g+e*f)^(1/2)/e^(
3/2)+2*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*(-d*g+e*f)^(1
/2)/e^(3/2)-2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e
*x+d)^n))*(-d*g+e*f)^(1/2)/e^(3/2)-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*(-d*g+e*f)
^(1/2)/e^(3/2)-2*b*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/
2)))*(-d*g+e*f)^(1/2)/e^(3/2)-4*b*n*(g*x+f)^(1/2)/e+2*(a+b*ln(c*(e*x+d)^n)
)*(g*x+f)^(1/2)/e
```

3.200.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$$
$$= \frac{4a\sqrt{e}\sqrt{f+gx}-8b\sqrt{e}n\sqrt{f+gx}+8b\sqrt{ef-dg}n\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4b\sqrt{e}\sqrt{f+gx}\log(c(d+ex)^n)+2}{}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

output
```
(4*a*Sqrt[e]*Sqrt[f + g*x] - 8*b*Sqrt[e]*n*Sqrt[f + g*x] + 8*b*Sqrt[e*f -
d*g]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 4*b*Sqrt[e]*Sqrt
[f + g*x]*Log[c*(d + e*x)^n] + 2*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n]
)*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 2*Sqrt[e*f - d*g]*(a + b*
Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - b*Sqrt[
e*f - d*g]*n*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f -
 d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e
*f - d*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f -
d*g])]) + b*Sqrt[e*f - d*g]*n*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]
]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqr
t[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*
x])/Sqrt[e*f - d*g])/2]))/(2*e^(3/2))
```

3.200.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.49, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2858, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}(a+b\log(c(d+ex)^n))}{d+ex}d(d+ex)}{e}$$

$$\Big\downarrow \text{2788}$$

$$\frac{\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{d+ex}d(d+ex)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\left(f-\frac{dg}{e}\right)\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right)\int \frac{a+b\log(}{(d+ex)\sqrt{f}}}{e}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(\frac{2e\left(f-\frac{dg}{e}\right)\int\frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g}+2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}\right)}{g}\right)}{e}+\left(f-\frac{dg}{e}\right.}{e}$$

$\downarrow$ 221

$$\frac{\left(f-\frac{dg}{e}\right)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)+\frac{g\left(\frac{2e\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}(a+b\log(c(d+ex)^n))}{g}-\frac{2ben\left(2\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}-\frac{2\sqrt{e}\left(f-\frac{dg}{e}\right)\text{arcta}}{}\right.}{g}\right.}{e}}{e}$$

$\downarrow$ 2790

$$\frac{\left(f-\frac{dg}{e}\right)\left(-bn\int-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{e}$$

$\downarrow$ 27

3.200. $\int\frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\frac{\left(f-\frac{dg}{e}\right)\left(\frac{2b\sqrt{e}n\int\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)+g\left(\frac{2e\sqrt{\cdots}}{\cdots}\cdots\right)}{e}$$

$$\Big\downarrow \text{ 7267}$$

$$\frac{\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\cdots}{\sqrt{ef-dg}}\cdots\right)\cdots}{e}$$

$$\Big\downarrow \text{ 2092}$$

$$\frac{\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\int\frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\cdots}{\sqrt{ef-dg}}\cdots\right)\cdots}{e}$$

$$\Big\downarrow \text{ 6546}$$

3.200. $\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 6470

$$\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\ldots\right.}{1-\ldots}\ldots}{\sqrt{e}\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}\right.$$

$\downarrow$ 2849

3.200. $\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

$$\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{arctan}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right.$$

$\downarrow$ 2752

$$\left(f-\frac{dg}{e}\right)\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\,\cdots}{}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right.$$

3.200. $\int\frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$

input
```
Int[(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

output
```
((g*((-2*b*e*n*(2*Sqrt[f - (d*g)/e + (g*(d + e*x))/e] - (2*Sqrt[e]*(f - (d
*g)/e)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*
g]])/Sqrt[e*f - d*g]))/g + (2*e*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]*(a + b
*Log[c*(d + e*x)^n]))/g))/e + (f - (d*g)/e)*((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*
Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*
x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/
e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*ArcTanh
[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2/(1 -
 (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqrt[e]
+ (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d
 + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f - d*g])))/S
qrt[e*f - d*g]))/e
```

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.200. $\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex} dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.200.4 Maple [F]

$$\int \frac{\sqrt{gx+f}\,(a+b\ln(c(ex+d)^{n}))}{ex+d}dx$$

input
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

output
```
int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

3.200.5 Fricas [F]

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{\sqrt{gx+f}(b\log((ex+d)^nc)+a)}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="frica
s")
```

output
```
integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*x + d),
 x)
```

3.200.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\int \frac{(a+b\log(c(d+ex)^n))\sqrt{f+gx}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n))/(e*x+d),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*sqrt(f + g*x)/(d + e*x), x)
```

3.200.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.200.8 Giac [F]

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\int \frac{\sqrt{gx+f}(b\log((ex+d)^nc)+a)}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="giac"
)
```

output
```
integrate(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a)/(e*x + d), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx=\int \frac{\sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x),x)
```

output
```
int(((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x), x)
```

3.201 $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}\,dx$

3.201.1 Optimal result

Integrand size = 31, antiderivative size = 256

$$\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2bn\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{2\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} - \frac{4bn\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}}$$

output
```
2*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(1/2)/(-d*g+e*f)
^(1/2)-2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)
^n))/e^(1/2)/(-d*g+e*f)^(1/2)-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*
f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(1/2)/(-d*g+e
*f)^(1/2)-2*b*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/
e^(1/2)/(-d*g+e*f)^(1/2)
```

3.201.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \frac{-2a\sqrt{-ef+dg}\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+2b\sqrt{ef-dg}\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)\left(in\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)+\log\left(c(d+\right.\right.}{\sqrt{e}\sqrt{-(ef-dg)^2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(-2*a*Sqrt[-(e*f) + d*g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
+ 2*b*Sqrt[e*f - d*g]*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]*(
I*n*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]] + Log[c*(d + e*x)^n
] + 2*n*Log[(2*I)/(I - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g])]) + (2*
I)*b*Sqrt[e*f - d*g]*n*PolyLog[2, -((Sqrt[-(e*f) + d*g] - I*Sqrt[e]*Sqrt[f
 + g*x])/(Sqrt[-(e*f) + d*g] + I*Sqrt[e]*Sqrt[f + g*x]))])/(Sqrt[e]*Sqrt[-
(e*f - d*g)^2])
```

3.201.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.39, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2858, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 2858$$

$$\frac{\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{e}$$

$$\Big\downarrow \; 2790$$

$$\frac{-bn\int -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}}{e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2b\sqrt{e}n\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}}{e}$$

$$\Big\downarrow \; 7267$$

$$\frac{\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}}{e}$$

$$\Big\downarrow \; 2092$$

$$\frac{\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}}{e}$$

$$\Big\downarrow \; 6546$$

$$\frac{\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}}{e}$$

$$\Big\downarrow \; 6470$$

3.201. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}\,dx$

$$\frac{\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d\cdots}{}}}{\sqrt{ef-dg}}}\right)}{1-\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex\cdots}{e}\right)}{ef-dg}}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots}{e}$$

$\Big\downarrow$ 2849

$$\frac{\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}}+\frac{\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex\cdots}{e}}}{\sqrt{ef-\cdots}}\right.\cdots}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots}{e}$$

$\Big\downarrow$ 2752

$$\frac{\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\,\text{PolyLog}\left(2,1-\frac{\cdots}{1-\cdots}\right.\cdots}{2\sqrt{\cdots}}}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*Sqrt[f + g*x]),x]
```

3.201. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}\,dx$

output
```
((-2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*
f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(Ar
cTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*
e) - ((Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e
])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e
])/Sqrt[e*f - d*g])])/Sqrt[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sq
rt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))
/(Sqrt[e]*Sqrt[e*f - d*g])))/Sqrt[e*f - d*g])/e
```

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.201.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{(ex+d)\sqrt{gx+f}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x)
```

3.201.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*g*x^2 +
 d*f + (e*f + d*g)*x), x)
```

3.201.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\,\sqrt{f+gx}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/((d + e*x)*sqrt(f + g*x)), x)
```

3.201.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.201.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((e*x + d)*sqrt(g*x + f)), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{f+g\,x}\,\left(d+e\,x\right)} \,d x$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.202 $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$

3.202.1 Optimal result

Integrand size = 31, antiderivative size = 340

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx &= \frac{4b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}+\frac{2b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}}\\ &+\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)\sqrt{f+gx}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^{3/2}}\\ &-\frac{4b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}-\frac{2b\sqrt{e}n\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}\end{aligned}$$

output
```
4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*e^(1/2)/(-d*g+e*f)^(
3/2)+2*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*e^(1/2)/(-d*g
+e*f)^(3/2)-2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e
*x+d)^n))*e^(1/2)/(-d*g+e*f)^(3/2)-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*e^(1/2)/(-
d*g+e*f)^(3/2)-2*b*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/
2)))*e^(1/2)/(-d*g+e*f)^(3/2)+2*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)/(g*x+f)^(
1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.55

$$\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{8b\sqrt{e}n\sqrt{f+gx}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4\sqrt{ef-dg}(a+b\log(c(d+ex)^n))+2\sqrt{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(8*b*Sqrt[e]*n*Sqrt[f + g*x]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*
g]] + 4*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n]) + 2*Sqrt[e]*Sqrt[f + g*
x]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]
 - 2*Sqrt[e]*Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g]
+ Sqrt[e]*Sqrt[f + g*x]] - b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[Sqrt[e*f - d*g]
- Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2
*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2
- (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + b*Sqrt[e]*n*Sqrt[f + g*x
]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqr
t[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*
g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]))/(2
*(e*f - d*g)^(3/2)*Sqrt[f + g*x])
```

3.202.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.44, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2858, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2789}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}}{e}$$

$$\Big\downarrow \text{ 2756}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{g}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}}{e}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{4be^2n\int \frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g^2}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}}{e}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}\right)}{ef-dg}}{e}$$

$$\Big\downarrow \text{ 2790}$$

$$\frac{\frac{e\left(-bn\int -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}}}\right.}{}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{e\left(\frac{2b\sqrt{e}n\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right.}{}}{e}$$

$$\Big\downarrow \text{ 7267}$$

3.202. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$

$$\frac{\frac{e\left(\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-}{e}$$

$\Big\downarrow$ 2092

$$\frac{\frac{e\left(\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-}{e}$$

$\Big\downarrow$ 6546

$$\frac{\frac{e\left(\frac{4be^{3/2}n\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}\right.}{ef-dg}}{e}$$

$\Big\downarrow$ 6470

3.202. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$

$$\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\cdots}}{\sqrt{ef-d\cdots}}}\right)}{1-\frac{e\left(f-\frac{dg}{e}+\frac{g(d\cdots}{\cdots}\right)}{ef-dg}}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots$$

$\downarrow$ 2849

3.202. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$

$$e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+\cdots}{e}}}{\sqrt{e\cdots}}\right.\cdots}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\right.\cdots}{\sqrt{ef-dg}}\right.\cdots}{ef-dg}$$

$\downarrow$ 2752

$$e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\,\operatorname{PolyLog}\left(2,1-\cdots\right.}{\cdots}}{\sqrt{e}\sqrt{ef-dg}}\right.\cdots}{\sqrt{ef-dg}}\right.\cdots}{ef-dg}$$

3.202. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(-((g*((-4*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e
])/Sqrt[e*f - d*g]])/(g*Sqrt[e*f - d*g]) - (2*e*(a + b*Log[c*(d + e*x)^n])
)/(g*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])))/(e*f - d*g)) + (e*((-2*Sqrt[e]
*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a
 + b*Log[c*(d + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[
e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[
e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f
- d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f
- d*g])])/Sqrt[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f
 - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sq
rt[e*f - d*g])))/Sqrt[e*f - d*g]))/(e*f - d*g))/e
```

3.202.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

3.202. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.202.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{(ex+d)\left(gx+f\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x)
```

3.202.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*g^2*x^3
 + d*f^2 + (2*e*f*g + d*g^2)*x^2 + (e*f^2 + 2*d*f*g)*x), x)
```

3.202.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(e*x+d)/(g*x+f)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.202.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.202.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((e*x + d)*(g*x + f)^(3/2)), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(3/2)*(d + e*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(3/2)*(d + e*x)), x)
```

3.203 $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

3.203.1 Optimal result

Integrand size = 31, antiderivative size = 406

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx &= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}}+\frac{16be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}}\\
&+\frac{2be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}}+\frac{2(a+b\log\left(c(d+ex)^n\right))}{3(ef-dg)(f+gx)^{3/2}}\\
&+\frac{2e(a+b\log\left(c(d+ex)^n\right))}{(ef-dg)^2\sqrt{f+gx}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log\left(c(d+ex)^n\right))}{(ef-dg)^{5/2}}\\
&-\frac{4be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}}\\
&-\frac{2be^{3/2}n\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}}\end{aligned}$$

output
```
16/3*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/(-d*g+e*f
)^(5/2)+2*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/(-
d*g+e*f)^(5/2)+2/3*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)/(g*x+f)^(3/2)-2*e^(3/2
)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/(-
d*g+e*f)^(5/2)-4*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2
))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/(-d*g+e*f)^(5/2)-2*b*e
^(3/2)*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/(-d*g+e
*f)^(5/2)-4/3*b*e*n/(-d*g+e*f)^2/(g*x+f)^(1/2)+2*e*(a+b*ln(c*(e*x+d)^n))/(
-d*g+e*f)^2/(g*x+f)^(1/2)
```

3.203.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \frac{24be^{3/2}n(f+gx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)-8be\sqrt{ef-dg}n(f+gx)\,\text{Hypergeom}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(5/2)),x]
```

output
```
(24*b*e^(3/2)*n*(f + g*x)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g]] - 8*b*e*Sqrt[e*f - d*g]*n*(f + g*x)*Hypergeometric2F1[-1/2, 1, 1/2,
 (e*(f + g*x))/(e*f - d*g)] + 4*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^n
]) + 12*e*Sqrt[e*f - d*g]*(f + g*x)*(a + b*Log[c*(d + e*x)^n]) + 6*e^(3/2)
*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*
Sqrt[f + g*x]] - 6*e^(3/2)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[
Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - 3*b*e^(3/2)*n*(f + g*x)^(3/2)*(
Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e
]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])
 + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 3*b*
e^(3/2)*n*(f + g*x)^(3/2)*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(L
og[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f
+ g*x])/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/
Sqrt[e*f - d*g])/2]))/(6*(e*f - d*g)^(5/2)*(f + g*x)^(3/2))
```

3.203.3 Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.65, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.581, Rules used = {2858, 2789, 2756, 61, 73, 221, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{2789}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{5/2}}d(d+ex)}{ef-dg}}{e}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{3g}-\frac{2e(a+b\log(c(d+ex)^n))}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}}\right)}{ef-dg}}{e}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{e\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{3g}-\frac{2e(a+b\log(c(d+ex)^n))}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3/2}}\right)}{ef-dg}}{e}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e^2\int \frac{1}{\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}+d-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g(ef-dg)}+\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{3g}-\frac{2e(a+b\log\ldots}{3g\left(\frac{g(d+e\ldots}{e}\ldots\right.}\right.}{ef-dg}}{e}$$

$\downarrow$ 221

$$\frac{\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}\right)}{3g}-\frac{2e(a+b\log(c(d+ex)^n)\ldots}{3g\left(\frac{g(d+ex)}{e}-\frac{dg}{e}+f\right)^{3\ldots}}\right.}{ef-dg}}{e}$$

$\downarrow$ 2789

$$\frac{\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)^{3/2}}d(d+ex)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}\ldots}}{\sqrt{ef-d\ldots}}\right)}{(ef-dg)^{3/2}}\ldots\right.}{3g}\ldots\right.}{ef-dg}}{e}$$

$\downarrow$ 2756

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$$\frac{\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{2ben\int \frac{1}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{g}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+\cdots}}\cdots\right.\right.}{\cdots}}{e}$$

$\downarrow$ 73

$$\frac{\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(\frac{4be^2n\int \frac{1}{d+\frac{e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}{g}-\frac{ef}{g}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{g^2}-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{\cdots}{(ef-dg)\cdots}\right.}{\cdots}\right.}{\cdots}}{e}$$

$\downarrow$ 221

$$\frac{\frac{e\left(\frac{e\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}d(d+ex)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}-\frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}\right)}{ef-dg}\right)}{ef-dg}-\frac{g\left(\frac{2ben\left(\frac{2e}{(ef-dg)\sqrt{\frac{g(d+ex)}{e}\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}}{e}$$

$\downarrow$ 2790

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$$\frac{e\left(\frac{e\left(-bn\int -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}(d+ex)}d(d+ex)-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n\cdots}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\cdots\right.}{\cdots}\cdots\right.}{ef-dg}$$

$\downarrow$ 27

$$\frac{e\left(\frac{e\left(\frac{2b\sqrt{e}n\int \frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{d+ex}d(d+ex)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}-\frac{g\left(-\frac{2e(a+b\log(c(d+ex)^n))}{g\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}\cdots\right.}{\cdots}\cdots\right.}{ef-dg}$$

$\downarrow$ 7267

$$\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{dg-e\left(\frac{dg}{e}-\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}\cdots\right.}{ef-dg}$$

$\downarrow$ 2092

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$$\frac{e\left(\frac{e\left(\frac{4be^{3/2}n\int \frac{\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{-ef+dg+e\left(f-\frac{dg}{e}+\frac{g(d+ex)}{e}\right)}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{ef-dg}}\right)}{ef-dg}\right.}{ef-dg}$$

$\downarrow$ 6546

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$$e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}d\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{e}\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+\cdots}}{\sqrt{ef-dg}}\right.\cdots}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right)}{ef-dg}\cdots$$

$\big\downarrow$ 6470

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$$e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}-\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\cdots}}{\sqrt{ef\cdots}}}\right)}{1-\frac{e\left(f-\frac{dg}{e}+\cdots\right)}{ef-\cdots}}\cdots}{\sqrt{e}\sqrt{ef-dg}}\cdots\right)}{\sqrt{ef-dg}}\cdots\right)}{ef-dg}\cdots\right.$$

↓ 2849

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$$e\left(\frac{\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\sqrt{ef-dg}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}\right)}{1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}d\frac{1}{1-\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{g(d+ex)}{e}}}{\sqrt{ef-dg}}}}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{g}\cdots}{\cdots}\right.\cdots}{\sqrt{e}\sqrt{ef-dg}\cdots}\cdots\right.}{\sqrt{ef-dg}}\cdots\right.}{ef-dg}\cdots\right.}{\cdots}\cdots\right.$$

3.203. $\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

$\downarrow$ 2752

$$e\left(\frac{e\left(\frac{4be^{3/2}n\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)^2}{2e}-\frac{\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{\frac{g(d+ex)}{e}-\frac{dg}{e}+f}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}}+\frac{\sqrt{ef-dg}\operatorname{PolyLog}\left(2,\right.}{}}{\sqrt{e}\sqrt{ef-dg}}\right.}{\sqrt{ef-dg}}}{ef-dg}\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(5/2)),x]
```

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

output
```
(-((g*((2*b*e*n*((2*e)/((e*f - d*g)*Sqrt[f - (d*g)/e + (g*(d + e*x))/e]) -
 (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f
 - d*g]])/(e*f - d*g)^(3/2)))/(3*g) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(3*
g*(f - (d*g)/e + (g*(d + e*x))/e)^(3/2))))/(e*f - d*g)) + (e*(-((g*((-4*b*
e^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f -
 d*g]])/(g*Sqrt[e*f - d*g]) - (2*e*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt[f -
 (d*g)/e + (g*(d + e*x))/e])))/(e*f - d*g)) + (e*((-2*Sqrt[e]*ArcTanh[(Sqr
t[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d
 + e*x)^n]))/Sqrt[e*f - d*g] + (4*b*e^(3/2)*n*(ArcTanh[(Sqrt[e]*Sqrt[f - (
d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]^2/(2*e) - ((Sqrt[e*f - d*g]*Ar
cTanh[(Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g]]*Log[2
/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e + (g*(d + e*x))/e])/Sqrt[e*f - d*g])])/Sqr
t[e] + (Sqrt[e*f - d*g]*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f - (d*g)/e +
(g*(d + e*x))/e])/Sqrt[e*f - d*g])])/(2*Sqrt[e]))/(Sqrt[e]*Sqrt[e*f - d*g]
)))/Sqrt[e*f - d*g]))/(e*f - d*g)))/(e*f - d*g))/e
```

3.203.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.203. $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.203.4 Maple [F]

$$\int \frac{a + b\ln \left(c(ex+d)^{n}\right)}{(ex+d)\,(gx+f)^{\frac{5}{2}}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x)
```

3.203.5 Fricas [F]

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx=\int \frac{b\log \left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*g^3*x^4
 + d*f^3 + (3*e*f*g^2 + d*g^3)*x^3 + 3*(e*f^2*g + d*f*g^2)*x^2 + (e*f^3 +
3*d*f^2*g)*x), x)
```

3.203.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(e*x+d)/(g*x+f)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.203.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((e*x + d)*(g*x + f)^(5/2)), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{(f+g\,x)^{5/2}\,(d+e\,x)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(5/2)*(d + e*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(5/2)*(d + e*x)), x)
```

3.204 $\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

3.204.1 Optimal result

Integrand size = 23, antiderivative size = 381

$$\begin{aligned}\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx &= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} \\ &-\frac{4(d+ex)^{3/2}}{9b}+\frac{16(bd-ae)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} \\ &+\frac{2(bd-ae)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}}+\frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} \\ &+\frac{2(d+ex)^{3/2}\log(a+bx)}{3b}-\frac{2(bd-ae)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} \\ &-\frac{4(bd-ae)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}} \\ &-\frac{2(bd-ae)^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}}\end{aligned}$$

output
```
-4/9*(e*x+d)^(3/2)/b+16/3*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(
-a*e+b*d)^(1/2))/b^(5/2)+2*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/
(-a*e+b*d)^(1/2))^2/b^(5/2)+2/3*(e*x+d)^(3/2)*ln(b*x+a)/b-2*(-a*e+b*d)^(3/
2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)/b^(5/2)-4*(-a
*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1
/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(5/2)-2*(-a*e+b*d)^(3/2)*polylog(2,
1-2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(5/2)-16/3*(-a*e+b*d)*(e
*x+d)^(1/2)/b^2+2*(-a*e+b*d)*ln(b*x+a)*(e*x+d)^(1/2)/b^2
```

3.204.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \frac{-72\sqrt{b}(bd-ae)\sqrt{d+ex}-8\sqrt{b}\sqrt{d+ex}(4bd-3ae+bex)+96(bd-ae)^{3/2}}{}$$

input
```
Integrate[((d + e*x)^(3/2)*Log[a + b*x])/(a + b*x),x]
```

output
```
(-72*Sqrt[b]*(b*d - a*e)*Sqrt[d + e*x] - 8*Sqrt[b]*Sqrt[d + e*x]*(4*b*d -
3*a*e + b*e*x) + 96*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt
[b*d - a*e]] + 36*Sqrt[b]*(b*d - a*e)*Sqrt[d + e*x]*Log[a + b*x] + 12*b^(3
/2)*(d + e*x)^(3/2)*Log[a + b*x] + 18*(b*d - a*e)^(3/2)*Log[a + b*x]*Log[S
qrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - 18*(b*d - a*e)^(3/2)*Log[a + b*x
]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] - 9*(b*d - a*e)^(3/2)*(Log[
Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[b*d - a*e] - Sqrt[b]*Sq
rt[d + e*x]] + 2*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2]) + 2
*PolyLog[2, 1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]) + 9*(b*d -
 a*e)^(3/2)*(Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[b*d -
a*e] + Sqrt[b]*Sqrt[d + e*x]] + 2*Log[1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqr
t[b*d - a*e])]) + 2*PolyLog[2, (1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e
])/2]))/(18*b^(5/2))
```

3.204.3 Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.82, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.870, Rules used = {2858, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}\log(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{2788}$$

$$\frac{\frac{e\int \sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\log(a+bx)d(a+bx)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\log(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}{3e}-\frac{2b\int \frac{\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}{a+bx}d(a+bx)}{3e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\log(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}{3e}-\frac{2b\left(\left(d-\frac{ae}{b}\right)\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{a+bx}d(a+bx)+\frac{2}{3}\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}\right)}{3e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\log(}{a+bx}}{b}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}{3e}-\frac{2b\left(\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\int\frac{1}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)+2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}\right)+\frac{2}{3}\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}\right)}{3e}\right)}{b}+\cdots}{b}$$

$\downarrow$ 73

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}{3e}-\frac{2b\left(\left(d-\frac{ae}{b}\right)\left(\frac{2b\left(d-\frac{ae}{b}\right)\int\frac{1}{a+\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{e}-\frac{bd}{e}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{e}+2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}\right)+\frac{2}{3}\left(\frac{e(a+bx)}{b}-\frac{ae}{b}\cdots\right)\right)}{3e}\cdots\right)}{b}\cdots}{b}$$

$\downarrow$ 221

$$\frac{\left(d-\frac{ae}{b}\right)\int\frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\log(a+bx)}{a+bx}d(a+bx)+\frac{e\left(\frac{2b\log(a+bx)\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}{3e}-\frac{2b\left(\left(d-\frac{ae}{b}\right)\left(2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}-\frac{2\sqrt{b}\left(d-\frac{ae}{b}\right)\text{arc}\cdots}{\cdots}\right)\cdots\right)}{\cdots}\right)}{b}\cdots}{b}$$

$\downarrow$ 2788

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{e\int\frac{\log(a+bx)}{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{b}+\left(d-\frac{ae}{b}\right)\int\frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)\right)+\frac{e\left(\frac{2b\log(a+bx)\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}{3e}-\cdots\right)}{\cdots}}{b}$$

3.204. $\int\frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

$\Big\downarrow$ 2756

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\int\frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{a+bx}d(a+bx)}{e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int\frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)\right)+\frac{e}{\quad}}{b}$$

$\Big\downarrow$ 60

$$\left(d-\frac{ae}{b}\right)\left(\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\left(\left(d-\frac{ae}{b}\right)\int\frac{1}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)+2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}\right)}{e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int\frac{\log(}{(a+bx)\sqrt{d}}\right.$$

$\Big\downarrow$ 73

3.204. $\int\frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

$$\left(d-\frac{ae}{b}\right)\left(\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\left(\frac{2b\left(d-\frac{ae}{b}\right)\int\frac{1}{a+\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{e}-\frac{bd}{e}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{e}+2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}\right)}{e}\right)}{b}+\left(d-\frac{ae}{b}\right.\right.$$

$\downarrow$ 221

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\int\frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)+\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\left(2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}-\frac{2\sqrt{b}\left(d-\frac{ae}{b}\right)\text{arcta}}{}\right.}{e}\right.}{b}\right.$$

$\downarrow$ 2790

3.204. $\int\frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\left(-\int -\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}(a+bx)}d(a+bx)-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)+\right.$$

$$\downarrow 27$$

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\left(\frac{2\sqrt{b}\int \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{a+bx}d(a+bx)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)+\frac{e\left(\frac{2b\log\ldots}{\ldots}\right.}{\ldots}\right.$$

$$\downarrow 7267$$

3.204. $\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{ae-b\left(\frac{ae}{b}-\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a}{}}}{\sqrt{}}\right.}{\sqrt{bd-ae}}\right.\right.\right.$$

$\downarrow$ 2092

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{-bd+ae+b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a}{}}}{\sqrt{}}\right.}{\sqrt{bd-ae}}\right.\right.\right.$$

$\downarrow$ 6546

3.204. $\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+\cdots}{\cdots}\right.$$

$\downarrow$ 6470

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}-\cdots}{\sqrt{b}\sqrt{bd-ae}}\cdots\right.}{\sqrt{bd-ae}}\cdots\right.$$

$\downarrow$ 2849

3.204. $\int\frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}\right)}{1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}d\frac{1}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}{\sqrt{b}}+\frac{\sqrt{bd-a}}{}}{\sqrt{b}\sqrt{bd-a}}\right)}{\sqrt{bd-ae}}\right.\right.$$

$\downarrow$ 2752

$$\left(d-\frac{ae}{b}\right)\left(\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}+}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right.\right.$$

3.204. $\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

input
```
Int[((d + e*x)^(3/2)*Log[a + b*x])/(a + b*x),x]
```

output
```
((e*((-2*b*((2*(d - (a*e)/b + (e*(a + b*x))/b)^(3/2))/3 + (d - (a*e)/b)*(2
*Sqrt[d - (a*e)/b + (e*(a + b*x))/b] - (2*Sqrt[b]*(d - (a*e)/b)*ArcTanh[(S
qrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]])/Sqrt[b*d - a
*e])))/(3*e) + (2*b*(d - (a*e)/b + (e*(a + b*x))/b)^(3/2)*Log[a + b*x])/(3
*e)))/b + (d - (a*e)/b)*((e*((-2*b*(2*Sqrt[d - (a*e)/b + (e*(a + b*x))/b]
- (2*Sqrt[b]*(d - (a*e)/b)*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x
))/b])/Sqrt[b*d - a*e]])/Sqrt[b*d - a*e]))/e + (2*b*Sqrt[d - (a*e)/b + (e*
(a + b*x))/b]*Log[a + b*x])/e))/b + (d - (a*e)/b)*((-2*Sqrt[b]*ArcTanh[(Sq
rt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]]*Log[a + b*x])/
Sqrt[b*d - a*e] + (4*b^(3/2)*(ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a +
b*x))/b])/Sqrt[b*d - a*e]]^2/(2*b) - ((Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sq
rt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]]*Log[2/(1 - (Sqrt[b]*Sq
rt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e])])/Sqrt[b] + (Sqrt[b*d
- a*e]*PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])
/Sqrt[b*d - a*e])])/(2*Sqrt[b]))/(Sqrt[b]*Sqrt[b*d - a*e])))/Sqrt[b*d - a*
e])))/b
```

3.204.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.204. $\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.204.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{3}-\frac{4b\left(-\frac{-\frac{b(ex+d)^{\frac{3}{2}}}{3}+\sqrt{ex+d}\,ae-\sqrt{ex+d}\,bd}{b^{2}}+\frac{\left(a^{2}e^{2}-2adeb+b^{2}d^{2}\right)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b^{2}\sqrt{(ae-bd)b}}\right)}{3}}{b}-\cdots$
default	$\frac{\frac{2(ex+d)^{\frac{3}{2}}\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{3}-\frac{4b\left(-\frac{-\frac{b(ex+d)^{\frac{3}{2}}}{3}+\sqrt{ex+d}\,ae-\sqrt{ex+d}\,bd}{b^{2}}+\frac{\left(a^{2}e^{2}-2adeb+b^{2}d^{2}\right)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b^{2}\sqrt{(ae-bd)b}}\right)}{3}}{b}-\cdots$

input
```
int((e*x+d)^(3/2)*ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(1/3*(e*x+d)^(3/2)*ln(((e*x+d)*b+a*e-b*d)/e)-2/3*b*(-1/b^2*(-1/3*b*(e*x+
d)^(3/2)+(e*x+d)^(1/2)*a*e-(e*x+d)^(1/2)*b*d)+(a^2*e^2-2*a*b*d*e+b^2*d^2)/
b^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))))/b-2*
((e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*((e*x+d)^(1/2)/b+(-a*e+b*d)/b
/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))))*(a*e-b*
d)/b^2+2*Sum(1/2*(ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(
1/4/_alpha/b*ln((e*x+d)^(1/2)-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1
/2)-_alpha)*ln(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dil
og(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)))*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^3/_a
lpha,_alpha=RootOf(_Z^2*b+a*e-b*d))
```

3.204.5 Fricas [F]

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}\log(bx+a)}{bx+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)*log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^(3/2)*log(b*x + a)/(b*x + a), x)
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*ln(b*x+a)/(b*x+a),x)
```

output
```
Timed out
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*log(b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.204.8 Giac [F]

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}\log(bx+a)}{bx+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)*log(b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*log(b*x + a)/(b*x + a), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \int \frac{\ln(a+b\,x)\,(d+e\,x)^{3/2}}{a+b\,x}\,dx$$

input
```
int((log(a + b*x)*(d + e*x)^(3/2))/(a + b*x),x)
```

output
```
int((log(a + b*x)*(d + e*x)^(3/2))/(a + b*x), x)
```

3.205 $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

3.205.1 Optimal result

Integrand size = 23, antiderivative size = 323

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = -\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} + \frac{2\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}}+\frac{2\sqrt{d+ex}\log(a+bx)}{b} - \frac{2\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}} - \frac{4\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}} - \frac{2\sqrt{bd-ae}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}$$

output
```
4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*(-a*e+b*d)^(1/2)/b^(3/2)
+2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2*(-a*e+b*d)^(1/2)/b^(3
/2)-2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)*(-a*e+b*d)
^(1/2)/b^(3/2)-4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b
^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*(-a*e+b*d)^(1/2)/b^(3/2)-2*polylog
(2,1-2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*(-a*e+b*d)^(1/2)/b^(3/2
)-4*(e*x+d)^(1/2)/b+2*ln(b*x+a)*(e*x+d)^(1/2)/b
```

3.205.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$$
$$= \frac{-8\sqrt{b}\sqrt{d+ex}+8\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)+4\sqrt{b}\sqrt{d+ex}\log(a+bx)+2\sqrt{bd-ae}\log(a+bx)\log\Big(}{}$$

input
```
Integrate[(Sqrt[d + e*x]*Log[a + b*x])/(a + b*x),x]
```

output
```
(-8*Sqrt[b]*Sqrt[d + e*x] + 8*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*
x])/Sqrt[b*d - a*e]] + 4*Sqrt[b]*Sqrt[d + e*x]*Log[a + b*x] + 2*Sqrt[b*d -
 a*e]*Log[a + b*x]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - Sqrt[b*d
 - a*e]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]^2 - 2*Sqrt[b*d - a*e]
*Log[a + b*x]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] + Sqrt[b*d - a*
e]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]^2 + 2*Sqrt[b*d - a*e]*Log[
Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*Log[1/2 - (Sqrt[b]*Sqrt[d + e*x])
/(2*Sqrt[b*d - a*e])] - 2*Sqrt[b*d - a*e]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sq
rt[d + e*x]]*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2] - 2*Sqrt
[b*d - a*e]*PolyLog[2, 1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]
+ 2*Sqrt[b*d - a*e]*PolyLog[2, (1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e
])/2])/(2*b^(3/2))
```

3.205.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2858, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\log(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\downarrow \text{2788}$$

$$\frac{\frac{e\int \frac{\log(a+bx)}{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{b}$$

$$\downarrow \text{2756}$$

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{a+bx}d(a+bx)}{e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{b}$$

$$\downarrow \text{60}$$

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\left(\left(d-\frac{ae}{b}\right)\int \frac{1}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)+2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}\right)}{e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}}{b}$$

$$\downarrow \text{73}$$

$$\frac{\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\left(\frac{2b\left(d-\frac{ae}{b}\right)\int \frac{1}{a+\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{e}-\frac{bd}{e}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{e}+2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}\right)}{e}\right)}{b}+\left(d-\frac{ae}{b}\right)\int \frac{\text{lo}}{(a+bx)\sqrt{}}}{b}$$

$$\downarrow \text{221}$$

3.205. $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

$$\frac{\left(d-\frac{ae}{b}\right)\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)+\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{e}-\frac{2b\left(2\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}-\frac{2\sqrt{b}\left(d-\frac{ae}{b}\right)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+\cdots}{b}}}{\sqrt{b\cdots}}\right.}{\sqrt{bd-ae}}\cdots\right.}{e}\cdots\right.}{b}\cdots}{b}$$

$\downarrow$ 2790

$$\frac{\left(d-\frac{ae}{b}\right)\left(-\int -\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}(a+bx)}d(a+bx)-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)+\frac{e\left(\frac{2b\log(a+b\cdots}{\cdots}\cdots\right.}{\cdots}\cdots}{b}$$

$\downarrow$ 27

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{2\sqrt{b}\int \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{a+bx}d(a+bx)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)+\frac{e\left(\frac{2b\log(a+bx)\sqrt{\frac{e(a+b\cdots}{b}}}{e}\cdots\right.}{\cdots}\cdots}{b}$$

$\downarrow$ 7267

3.205. $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{ae-b\left(\frac{ae}{b}-\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right.}{b}$$

$\Big\downarrow$ 2092

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{-bd+ae+b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right.}{b}$$

$\Big\downarrow$ 6546

3.205. $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\operatorname{arctanh}\left(\cdots\right.}{\sqrt{bd-\cdots}}\cdots\right)}{b}$$

$\downarrow$ 6470

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}-\int\frac{\log\left(\frac{\cdots}{1-\frac{\sqrt{b}\sqrt{\cdots}}{\cdots}}\right)}{b\left(d-\cdots\right)\left(1-\cdots\right)}\cdots}{\sqrt{b}\sqrt{bd-ae}}\cdots\right)}{\sqrt{bd-ae}}\cdots\right)}{\cdots}$$

$\downarrow$ 2849

3.205. $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}\right)}{1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}d\frac{1}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}{\sqrt{b}}+\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\cdots\right.}{\cdots}}{\sqrt{b}\sqrt{bd-ae}}\right.}{\sqrt{bd-ae}}\cdots}{\cdots}$$

$\downarrow$ 2752

$$\frac{\left(d-\frac{ae}{b}\right)\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}+\frac{\sqrt{bd-ae}\operatorname{PolyLo}\cdots}{\cdots}}{\sqrt{b}\sqrt{bd-ae}}\right.}{\sqrt{bd-ae}}\cdots}{\cdots}$$

3.205. $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

input
```
Int[(Sqrt[d + e*x]*Log[a + b*x])/(a + b*x),x]
```

output
```
((e*((-2*b*(2*Sqrt[d - (a*e)/b + (e*(a + b*x))/b] - (2*Sqrt[b]*(d - (a*e)/
b)*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]])
/Sqrt[b*d - a*e]))/e + (2*b*Sqrt[d - (a*e)/b + (e*(a + b*x))/b]*Log[a + b*
x])/e))/b + (d - (a*e)/b)*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b +
 (e*(a + b*x))/b])/Sqrt[b*d - a*e]]*Log[a + b*x])/Sqrt[b*d - a*e] + (4*b^(
3/2)*(ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e
]]^2/(2*b) - ((Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a +
 b*x))/b])/Sqrt[b*d - a*e]]*Log[2/(1 - (Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a +
 b*x))/b])/Sqrt[b*d - a*e])])/Sqrt[b] + (Sqrt[b*d - a*e]*PolyLog[2, 1 - 2/
(1 - (Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e])])/(2*S
qrt[b]))/(Sqrt[b]*Sqrt[b*d - a*e])))/Sqrt[b*d - a*e]))/b
```

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.205. $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.205.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{b}-\frac{4\sqrt{ex+d}}{b}-\frac{4(-ae+bd)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b\sqrt{(ae-bd)b}}+2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\right.$
default	$\frac{2\sqrt{ex+d}\,\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{b}-\frac{4\sqrt{ex+d}}{b}-\frac{4(-ae+bd)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b\sqrt{(ae-bd)b}}+2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\right.$

input
```
int((e*x+d)^(1/2)*ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)/b-4*(e*x+d)^(1/2)/b-4*(-a*e+b*d)
/b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))+2*Sum(1
/2*(ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_alpha/b*l
n((e*x+d)^(1/2)-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_alpha)*ln
(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*((e*x+d
)^(1/2)+_alpha)/_alpha)))*(-a*e+b*d)/b^2/_alpha,_alpha=RootOf(_Z^2*b+a*e-b
*d))
```

3.205.5 Fricas [F]

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\sqrt{ex+d}\log\left(bx+a\right)}{bx+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)*log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*log(b*x + a)/(b*x + a), x)
```

3.205.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\sqrt{d+ex}\log\left(a+bx\right)}{a+bx}\,dx$$

input
```
integrate((e*x+d)**(1/2)*ln(b*x+a)/(b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)*log(a + b*x)/(a + b*x), x)
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*log(b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.205.8 Giac [F]

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\sqrt{ex+d}\log(bx+a)}{bx+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)*log(b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*log(b*x + a)/(b*x + a), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\ln(a+b\,x)\,\sqrt{d+e\,x}}{a+b\,x}\,dx$$

input
```
int((log(a + b*x)*(d + e*x)^(1/2))/(a + b*x),x)
```

output
```
int((log(a + b*x)*(d + e*x)^(1/2))/(a + b*x), x)
```

3.206 $\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$

3.206.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}}$$

output
```
2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2/b^(1/2)/(-a*e+b*d)^(1/
2)-2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)/b^(1/2)/(-a
*e+b*d)^(1/2)-4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^
(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(1/2)/(-a*e+b*d)^(1/2)-2*polylog(
2,1-2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(1/2)/(-a*e+b*d)^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.52

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$$
$$= \frac{2\log(a+bx)\log\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)-\log^2\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)-2\log(a+bx)\log\left(\sqrt{bd-\right.}$$

input
```
Integrate[Log[a + b*x]/((a + b*x)*Sqrt[d + e*x]),x]
```

output
```
(2*Log[a + b*x]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - Log[Sqrt[b*
d - a*e] - Sqrt[b]*Sqrt[d + e*x]]^2 - 2*Log[a + b*x]*Log[Sqrt[b*d - a*e] +
 Sqrt[b]*Sqrt[d + e*x]] + Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]^2 +
 2*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*Log[1/2 - (Sqrt[b]*Sqrt[d
+ e*x])/(2*Sqrt[b*d - a*e])] - 2*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*
x]]*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2] - 2*PolyLog[2, 1/
2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])] + 2*PolyLog[2, (1 + (Sqrt
[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2])/(2*Sqrt[b]*Sqrt[b*d - a*e])
```

3.206.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.43, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2858, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \; 2858$$
$$\frac{\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{b}$$
$$\Big\downarrow \; 2790$$

$$\frac{-\int -\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}(a+bx)}d(a+bx)-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\big\downarrow$ 27

$$\frac{\frac{2\sqrt{b}\int \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{a+bx}d(a+bx)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\big\downarrow$ 7267

$$\frac{\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{ae-b\left(\frac{ae}{b}-\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\big\downarrow$ 2092

$$\frac{\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{-bd+ae+b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\big\downarrow$ 6546

$$\frac{\frac{4b^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\int \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\ldots}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\big\downarrow$ 6470

3.206. $\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$

$$\frac{\frac{4b^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}-\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}\right)}{1-\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{bd-ae}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\Big\downarrow$ 2849

$$\frac{\frac{4b^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}\right)}{1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}d\frac{1}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}{\sqrt{b}}+\frac{\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\ldots}}{\sqrt{bd-ae}}\right.\ldots}{\ldots}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

$\Big\downarrow$ 2752

$$\frac{\frac{4b^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}+\frac{\sqrt{bd-ae}\,\text{PolyLog}\left(2,1-\frac{\ldots}{1-\frac{\sqrt{b}\ldots}{\ldots}}\right.\ldots}{2\sqrt{b}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}}{b}$$

input

```
Int[Log[a + b*x]/((a + b*x)*Sqrt[d + e*x]),x]
```

3.206. $\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$

output
```
((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*
d - a*e]]*Log[a + b*x])/Sqrt[b*d - a*e] + (4*b^(3/2)*(ArcTanh[(Sqrt[b]*Sqr
t[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]]^2/(2*b) - ((Sqrt[b*d -
a*e]*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]
]*Log[2/(1 - (Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]
)])/Sqrt[b] + (Sqrt[b*d - a*e]*PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d - (a*
e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e])])/(2*Sqrt[b]))/(Sqrt[b]*Sqrt[b*d
 - a*e])))/Sqrt[b*d - a*e])/b
```

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.206. $\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.206.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\frac{2\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)-b\left(\frac{\ln\left(\sqrt{ex+d}-\_\alpha\right)^2}{\_\alpha b}+\frac{2\_\alpha\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{\sqrt{ex+d}+\_\alpha}{2\_\alpha}\right)}{ae-bd}\right.}{\_\alpha}}{2b}$
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\frac{2\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)-b\left(\frac{\ln\left(\sqrt{ex+d}-\_\alpha\right)^2}{\_\alpha b}+\frac{2\_\alpha\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{\sqrt{ex+d}+\_\alpha}{2\_\alpha}\right)}{ae-bd}\right.}{\_\alpha}}{2b}$

input
```
int(ln(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*sum(1/_alpha*(2*ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-b
*(1/_alpha/b*ln((e*x+d)^(1/2)-_alpha)^2+2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2
)-_alpha)*ln(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+2*_alpha/(a*e-b*d)*dilog(1
/2*((e*x+d)^(1/2)+_alpha)/_alpha)),_alpha=RootOf(_Z^2*b+a*e-b*d))
```

3.206.5 Fricas [F]

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\log(bx+a)}{(bx+a)\sqrt{ex+d}}\,dx$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*log(b*x + a)/(b*e*x^2 + a*d + (b*d + a*e)*x), x)
```

3.206.6 Sympy [F]

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$$

input
```
integrate(ln(b*x+a)/(b*x+a)/(e*x+d)**(1/2),x)
```

output
```
Integral(log(a + b*x)/((a + b*x)*sqrt(d + e*x)), x)
```

3.206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.206.8 Giac [F]

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\log(bx+a)}{(bx+a)\sqrt{ex+d}}\,dx$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(b*x + a)/((b*x + a)*sqrt(e*x + d)), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\ln(a+b\,x)}{(a+b\,x)\,\sqrt{d+e\,x}}\,dx$$

input
```
int(log(a + b*x)/((a + b*x)*(d + e*x)^(1/2)),x)
```

output
```
int(log(a + b*x)/((a + b*x)*(d + e*x)^(1/2)), x)
```

3.207 $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 316

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \frac{4\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{4\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}} - \frac{2\sqrt{b}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}}$$

output
```
4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*b^(1/2)/(-a*e+b*d)^(3/2)
+2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2*b^(1/2)/(-a*e+b*d)^(3
/2)-2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)*b^(1/2)/(-
a*e+b*d)^(3/2)-4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b
^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*b^(1/2)/(-a*e+b*d)^(3/2)-2*polylog
(2,1-2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*b^(1/2)/(-a*e+b*d)^(3/2
)+2*ln(b*x+a)/(-a*e+b*d)/(e*x+d)^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.73

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = 2\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}}\right.$$
$$+\frac{\log(a+bx)}{(bd-ae)\sqrt{d+ex}}+\frac{\sqrt{b}\log(a+bx)\log\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)}{2(bd-ae)^{3/2}}$$
$$-\frac{\sqrt{b}\log(a+bx)\log\left(\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}\right)}{2(bd-ae)^{3/2}}$$
$$-\frac{\sqrt{b}\left(\log^2\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)+2\log\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)\log\left(\frac{\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}}{2\sqrt{bd-ae}}\right)+2\,\text{PolyLog}\right.}{4(bd-ae)^{3/2}}$$
$$+\frac{\sqrt{b}\left(2\log\left(\frac{\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}}{2\sqrt{bd-ae}}\right)\log\left(\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}\right)+\log^2\left(\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}\right)+2\,\text{PolyLog}\right.}{4(bd-ae)^{3/2}}$$

input
```
Integrate[Log[a + b*x]/((a + b*x)*(d + e*x)^(3/2)),x]
```

output
```
2*((2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e
)^(3/2) + Log[a + b*x]/((b*d - a*e)*Sqrt[d + e*x]) + (Sqrt[b]*Log[a + b*x]
*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]])/(2*(b*d - a*e)^(3/2)) - (Sq
rt[b]*Log[a + b*x]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]])/(2*(b*d -
 a*e)^(3/2)) - (Sqrt[b]*(Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]^2 +
2*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*Log[(Sqrt[b*d - a*e] + Sqrt
[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])] + 2*PolyLog[2, (Sqrt[b*d - a*e] -
Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]))/(4*(b*d - a*e)^(3/2)) + (Sqr
t[b]*(2*Log[(Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]
*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] + Log[Sqrt[b*d - a*e] + Sqrt
[b]*Sqrt[d + e*x]]^2 + 2*PolyLog[2, (Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*
x])/(2*Sqrt[b*d - a*e])]))/(4*(b*d - a*e)^(3/2)))
```

3.207.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2858, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{b}$$

$$\Big\downarrow \text{2789}$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\int \frac{\log(a+bx)}{\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}}{b}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{2b\int \frac{1}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{e}-\frac{2b\log(a+bx)}{e\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{bd-ae}}{b}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{4b^2\int \frac{1}{a+\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{e}-\frac{bd}{e}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{e^2}-\frac{2b\log(a+bx)}{e\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{bd-ae}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{e\sqrt{bd-ae}}-\frac{2b\log(a+bx)}{e\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{bd-ae}}{b}$$

$$\Big\downarrow \text{2790}$$

$$\frac{\frac{b\left(-\int -\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}(a+bx)}d(a+bx)-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}}}{\sqrt{bd-a}}\right.}{e\sqrt{bd-ae}}\right.}{bd-a}}{b}$$

$\downarrow$ 27

$$\frac{\frac{b\left(\frac{2\sqrt{b}\int \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{a+bx}d(a+bx)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}}}{\sqrt{bd-ae}}\right.}{e\sqrt{bd-ae}}\right.}{bd-ae}}{b}$$

$\downarrow$ 7267

$$\frac{\frac{b\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{ae-b\left(\frac{ae}{b}-\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}}{}\right.}{}}{b}$$

$\downarrow$ 2092

$$\frac{\frac{b\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{-bd+ae+b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}}{}\right.}{}}{b}$$

$\downarrow$ 6546

3.207. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$

$$\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\ldots}}{\sqrt{bd-ae}}\right.}{\sqrt{bd-ae}}\ldots}{bd-ae}\ldots}{b}$$

$\downarrow$ 6470

$$\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}-\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+\ldots}{b}}}{\sqrt{bd-ae}}}\right.}{1-\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right.}{bd-ae}}\ldots}{\sqrt{b}\sqrt{bd-ae}}\ldots}{\sqrt{bd-ae}}\ldots}{bd-ae}\ldots$$

$\downarrow$ 2849

3.207. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$

$$\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}\right)}{1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}d\frac{1}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}{\sqrt{b}}+\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}\cdots}}{\sqrt{bd-a\cdots}}\right.\cdots}{\cdots}}{\sqrt{b}\sqrt{bd-ae}}\cdots\right)}{\sqrt{bd-ae}}\cdots\right.}{bd-ae}\cdots$$

$$\Big\downarrow \text{2752}$$

$$\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}+\frac{\sqrt{bd-ae}\,\operatorname{PolyLog}\left(2,1-\frac{\cdots}{1-\cdots}\right.\cdots}{2\sqrt{b}\cdots}}{\sqrt{b}\sqrt{bd-ae}}\cdots\right)}{\sqrt{bd-ae}}\cdots\right.}{bd-ae}\cdots$$

3.207. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$

input
```
Int[Log[a + b*x]/((a + b*x)*(d + e*x)^(3/2)),x]
```

output
```
(-((e*((-4*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/S
qrt[b*d - a*e]])/(e*Sqrt[b*d - a*e]) - (2*b*Log[a + b*x])/(e*Sqrt[d - (a*e
)/b + (e*(a + b*x))/b])))/(b*d - a*e)) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*
Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]]*Log[a + b*x])/Sqrt[b
*d - a*e] + (4*b^(3/2)*(ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/
b])/Sqrt[b*d - a*e]]^2/(2*b) - ((Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d -
 (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]]*Log[2/(1 - (Sqrt[b]*Sqrt[d -
 (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e])])/Sqrt[b] + (Sqrt[b*d - a*e]
*PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[
b*d - a*e])])/(2*Sqrt[b]))/(Sqrt[b]*Sqrt[b*d - a*e])))/Sqrt[b*d - a*e]))/(
b*d - a*e))/b
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

3.207. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.207.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{2\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{4b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}}{ae-bd}+2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\left(-\frac{\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(ex+d)}{}\right.}{}\right.\right.$
default	$\frac{-\frac{2\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{4b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}}{ae-bd}+2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\left(-\frac{\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(ex+d)}{}\right.}{}\right.\right.$

input
```
int(ln(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1/(e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)+2*b/((a*e-b*d)*b)^(1/2)*arct
an(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2)))/(a*e-b*d)+2*Sum(-1/2*(ln((e*x+d)^
(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_alpha/b*ln((e*x+d)^(1/2)
-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_alpha)*ln(1/2*((e*x+d)^(
1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*((e*x+d)^(1/2)+_alpha)
/_alpha)))/(a*e-b*d)/_alpha,_alpha=RootOf(_Z^2*b+a*e-b*d))
```

3.207.5 Fricas [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \int \frac{\log\left(bx+a\right)}{(bx+a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*log(b*x + a)/(b*e^2*x^3 + a*d^2 + (2*b*d*e + a*e^2)
*x^2 + (b*d^2 + 2*a*d*e)*x), x)
```

3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(b*x+a)/(b*x+a)/(e*x+d)**(3/2),x)
```

output
```
Timed out
```

3.207.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.207.8 Giac [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \int \frac{\log(bx+a)}{(bx+a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(log(b*x + a)/((b*x + a)*(e*x + d)^(3/2)), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \int \frac{\ln(a+b\,x)}{(a+b\,x)\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(log(a + b*x)/((a + b*x)*(d + e*x)^(3/2)),x)
```

output
```
int(log(a + b*x)/((a + b*x)*(d + e*x)^(3/2)), x)
```

3.208 $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

3.208.1 Optimal result

Integrand size = 23, antiderivative size = 372

$$\begin{aligned}\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx &= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}}+\frac{16b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}}\\
&+\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}}+\frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}}\\
&+\frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}}-\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}}\\
&-\frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}}-\frac{2b^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}}\end{aligned}$$

output
```
16/3*b^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(5
/2)+2*b^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2/(-a*e+b*d)
^(5/2)+2/3*ln(b*x+a)/(-a*e+b*d)/(e*x+d)^(3/2)-2*b^(3/2)*arctanh(b^(1/2)*(e
*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)/(-a*e+b*d)^(5/2)-4*b^(3/2)*arctanh
(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a
*e+b*d)^(1/2)))/(-a*e+b*d)^(5/2)-2*b^(3/2)*polylog(2,1-2/(1-b^(1/2)*(e*x+d
)^(1/2)/(-a*e+b*d)^(1/2)))/(-a*e+b*d)^(5/2)-4/3*b/(-a*e+b*d)^2/(e*x+d)^(1/
2)+2*b*ln(b*x+a)/(-a*e+b*d)^2/(e*x+d)^(1/2)
```

3.208.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.53

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \frac{24b^{3/2}(d+ex)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)-8b\sqrt{bd-ae}(d+ex)\,\text{Hypergeometric2F1}}{}$$

input
```
Integrate[Log[a + b*x]/((a + b*x)*(d + e*x)^(5/2)),x]
```

output
```
(24*b^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e
]] - 8*b*Sqrt[b*d - a*e]*(d + e*x)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(d +
 e*x))/(b*d - a*e)] + 4*(b*d - a*e)^(3/2)*Log[a + b*x] + 12*b*Sqrt[b*d - a
*e]*(d + e*x)*Log[a + b*x] + 6*b^(3/2)*(d + e*x)^(3/2)*Log[a + b*x]*Log[Sq
rt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - 6*b^(3/2)*(d + e*x)^(3/2)*Log[a +
 b*x]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] - 3*b^(3/2)*(d + e*x)^(
3/2)*(Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[b*d - a*e] -
Sqrt[b]*Sqrt[d + e*x]] + 2*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e
])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])])
+ 3*b^(3/2)*(d + e*x)^(3/2)*(Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*
(Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] + 2*Log[1/2 - (Sqrt[b]*Sqrt[
d + e*x])/(2*Sqrt[b*d - a*e])]) + 2*PolyLog[2, (1 + (Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[b*d - a*e])/2]))/(6*(b*d - a*e)^(5/2)*(d + e*x)^(3/2))
```

3.208.3 Rubi [A] (verified)

Time = 2.64 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.72, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2858, 2789, 2756, 61, 73, 221, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 2858$$

$$\frac{\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{5/2}}d(a+bx)}{b}$$

$$\Big\downarrow \ 2789$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}-\frac{e\int \frac{\log(a+bx)}{\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{5/2}}d(a+bx)}{bd-ae}}{b}$$

$$\Big\downarrow \ 2756$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{2b\int \frac{1}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{3e}-\frac{2b\log(a+bx)}{3e\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}\right)}{bd-ae}}{b}$$

$$\Big\downarrow \ 61$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{2b\left(\frac{b\int \frac{1}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}+\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{3e}-\frac{2b\log(a+bx)}{3e\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}\right)}{bd-ae}}{b}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{2b\left(\frac{2b^2\int \frac{1}{a+\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{e}-\frac{bd}{e}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{e(bd-ae)}+\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{3e}-\frac{2b\log(a+bx)}{3e\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}\right)}{bd-ae}}{b}$$

$$\Big\downarrow \ 221$$

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$\frac{\frac{b\int \frac{\log(a+bx)}{(a+bx)\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{2b\left(\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}-\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}}\right)}{3e}-\frac{2b\log(a+bx)}{3e\left(\frac{e(a+bx)}{b}-\frac{ae}{b}+d\right)^{3/2}}\right)}{bd-ae}}{b}$$

$\downarrow$ 2789

$$\frac{\frac{b\left(\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\int \frac{\log(a+bx)}{\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)^{3/2}}d(a+bx)}{bd-ae}\right)}{bd-ae}-\frac{e\left(\frac{2b\left(\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}-\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{a}{}}}{\sqrt{bd-ae}}\right.}{(bd-ae)^{3/2}}\right.}{3e}\right.}{bd-ae}}{b}$$

$\downarrow$ 2756

$$\frac{\frac{b\left(\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{2b\int \frac{1}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{e}-\frac{2b\log(a+bx)}{e\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{bd-ae}\right)}{bd-ae}-\frac{e\left(\frac{2b\left(\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}-\frac{2b^{3/2}}{}\right.}{3}\right.}{}}{b}$$

$\downarrow$ 73

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$\frac{\frac{b\left(\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\left(\frac{4b^2\int \frac{1}{a+\frac{b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{e}-\frac{bd}{e}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{e^2}-\frac{2b\log(a+bx)}{e\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{bd-ae}\right)}{bd-ae}-\frac{e\Big(2b\Big(\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}\cdots}}\cdots}{\cdots}}{b}$$

$\downarrow$ 221

$$\frac{\frac{b\left(\frac{b\int \frac{\log(a+bx)}{(a+bx)\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}d(a+bx)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{e\sqrt{bd-ae}}-\frac{2b\log(a+bx)}{e\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}\right)}{bd-ae}\right)}{bd-ae}-\frac{e\Big(2b\Big(\frac{2b}{(bd-ae)\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}-\cdots}{\cdots}}{b}$$

$\downarrow$ 2790

$$\frac{\frac{b\left(\frac{b\left(-\int -\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}(a+bx)}d(a+bx)-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\Big(-\frac{4b^{3/2}\text{arctanh}\Big(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\cdots}}{\sqrt{bd-ae}}\cdots}{e\sqrt{bd-ae}}\cdots}{bd-ae}\cdots}{bd-ae}\cdots}{b}$$

$\downarrow$ 27

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$\frac{b\left(\frac{b\left(\frac{2\sqrt{b}\int\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{a+bx}d(a+bx)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}\cdots}}{\sqrt{bd-ae}}\right)}{e\sqrt{bd-ae}}\cdots\right)}{bd-ae}\cdots\right)}{bd-ae}\cdots}{b}$$

$\downarrow$ 7267

$$\frac{b\left(\frac{b\left(\frac{4b^{3/2}\int\frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{ae-b\left(\frac{ae}{b}-\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{bd-ae}\cdots$$

$\downarrow$ 2092

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$\frac{b\left(\frac{b\left(\frac{4b^{3/2}\int \frac{\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{-bd+ae+b\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}\right)}{bd-ae}-\frac{e\left(-\frac{4b^{3/2}\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{bd-ae}$$

$\downarrow$ 6546

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$b\left(\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}d\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{b}\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}}-\frac{2\sqrt{b}\log(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}}}{\sqrt{bd-a}}\right.}{\sqrt{bd-ae}}\right.}{bd-ae}\right.$$

$$\overline{\qquad bd-ae \qquad}$$

$\downarrow$ 6470

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$\frac{b\left(\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}-\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(\cdots}{\cdots}}}{\sqrt{bd-ae}}}\right)}{1-\frac{b\left(d-\frac{ae}{b}+\frac{e(a+\cdots}{b}\right)}{bd-ae}}\cdots}{\sqrt{b}\sqrt{bd-ae}}\cdots\right)}{\sqrt{bd-ae}}\cdots}{bd-ae}\cdots\right)}{bd\cdots}$$

$\downarrow$ 2849

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$$b\left(\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\sqrt{bd-ae}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}\right)}{1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}d\frac{1}{1-\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{e(a+bx)}{b}}}{\sqrt{bd-ae}}}}{\sqrt{b}}+\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+b}{b}}}{\sqrt{bd}}\right.}{\sqrt{b}\sqrt{bd-ae}}\right.}{\sqrt{bd-ae}}\right.}{bd-ae}\right.$$

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

$\downarrow$ 2752

$$\frac{b\left(\frac{b\left(\frac{4b^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)^2}{2b}-\frac{\frac{\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{\frac{e(a+bx)}{b}-\frac{ae}{b}+d}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}}}{\sqrt{b}\sqrt{bd-ae}}+\frac{\sqrt{bd-ae}\operatorname{PolyLog}\left(2,1-\cdots\right)}{2\cdots}\right.}{\sqrt{bd-ae}}}{bd-ae}}{bd-\cdots}$$

input
```
Int[Log[a + b*x]/((a + b*x)*(d + e*x)^(5/2)),x]
```

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

output
```
(-((e*((2*b*((2*b)/((b*d - a*e)*Sqrt[d - (a*e)/b + (e*(a + b*x))/b]) - (2*
b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a
*e]])/(b*d - a*e)^(3/2)))/(3*e) - (2*b*Log[a + b*x])/(3*e*(d - (a*e)/b + (
e*(a + b*x))/b)^(3/2))))/(b*d - a*e)) + (b*(-((e*((-4*b^(3/2)*ArcTanh[(Sqr
t[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]])/(e*Sqrt[b*d -
a*e]) - (2*b*Log[a + b*x])/(e*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])))/(b*d
- a*e)) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x)
)/b])/Sqrt[b*d - a*e]]*Log[a + b*x])/Sqrt[b*d - a*e] + (4*b^(3/2)*(ArcTanh
[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e]]^2/(2*b) -
((Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sq
rt[b*d - a*e]]*Log[2/(1 - (Sqrt[b]*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sq
rt[b*d - a*e])])/Sqrt[b] + (Sqrt[b*d - a*e]*PolyLog[2, 1 - 2/(1 - (Sqrt[b]
*Sqrt[d - (a*e)/b + (e*(a + b*x))/b])/Sqrt[b*d - a*e])])/(2*Sqrt[b]))/(Sqr
t[b]*Sqrt[b*d - a*e])))/Sqrt[b*d - a*e]))/(b*d - a*e)))/(b*d - a*e))/b
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.208. $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.208.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{2\left(-\frac{\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{2b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}\right)b}{a^2e^2-2adeb+b^2d^2}+2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\frac{\left(\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(e}{}\right.\right.}{}\right.$
default	$-\frac{2\left(-\frac{\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{2b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}\right)b}{a^2e^2-2adeb+b^2d^2}+2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)}\frac{\left(\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(e}{}\right.\right.}{}\right.$

input
```
int(ln(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/(e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)+2*b/((a*e-b*d)*b)^(1/2)*arc
tan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2)))*b/(a^2*e^2-2*a*b*d*e+b^2*d^2)+2*
Sum(1/2*(ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_alph
a/b*ln((e*x+d)^(1/2)-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_alph
a)*ln(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*((
e*x+d)^(1/2)+_alpha)/_alpha)))*b/(a*e-b*d)^2/_alpha,_alpha=RootOf(_Z^2*b+a
*e-b*d))+2*(-1/3/(e*x+d)^(3/2)*ln(((e*x+d)*b+a*e-b*d)/e)+2/3*b*(-1/(a*e-b*
d)/(e*x+d)^(1/2)-1/(a*e-b*d)*b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/
((a*e-b*d)*b)^(1/2))))/(a*e-b*d)
```

3.208.5 Fricas [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \int \frac{\log(bx+a)}{(bx+a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*log(b*x + a)/(b*e^3*x^4 + a*d^3 + (3*b*d*e^2 + a*e^
3)*x^3 + 3*(b*d^2*e + a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e)*x), x)
```

3.208.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(b*x+a)/(b*x+a)/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.208.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.208.8 Giac [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \int \frac{\log(bx+a)}{(bx+a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(log(b*x + a)/((b*x + a)*(e*x + d)^(5/2)), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \int \frac{\ln(a+b\,x)}{(a+b\,x)\,(d+e\,x)^{5/2}}\,dx$$

input
```
int(log(a + b*x)/((a + b*x)*(d + e*x)^(5/2)),x)
```

output
```
int(log(a + b*x)/((a + b*x)*(d + e*x)^(5/2)), x)
```

3.209 $\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$

3.209.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \text{Int}\left(\frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx},x\right)$$

output
```
Unintegrable((i*x+h)^q*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

3.209.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$

input
```
Integrate[((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
Integrate[((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x), x]
```

3.209.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}dx$$
$$\Big\downarrow \; 2867$$
$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}dx$$

input
```
Int[((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
$Aborted
```

3.209.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.209.4 Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(ix+h)^q\,(a+b\ln(c(fx+e)))^p}{dfx+de}dx$$

input
```
int((i*x+h)^q*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

output
```
int((i*x+h)^q*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

3.209.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\int \frac{(ix+h)^q(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^q*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="frica
s")
```

output
```
integral((i*x + h)^q*(b*log(c*f*x + c*e) + a)^p/(d*f*x + d*e), x)
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\text{Timed out}$$

input
```
integrate((i*x+h)**q*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

output
```
Timed out
```

3.209.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\int \frac{(ix+h)^q(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^q*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxim
a")
```

output
```
integrate((i*x + h)^q*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

3.209.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((i*x+h)^q*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,5,0,2,0,5,0,3,0,2,0]%%%}+%%%{5,[0,0,4,0,2,0,4,1,3,0,2,0]%%%}+%%%{10,[0,
0,3,0,2,0,3,
```

3.209.9 Mupad [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+i\,x)^q\,(a+b\,\ln(c\,(e+f\,x)))^p}{d\,e+d\,f\,x}\,dx$$

input
```
int(((h + i*x)^q*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
```

output
```
int(((h + i*x)^q*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

3.210 $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$

3.210.1 Optimal result

Integrand size = 32, antiderivative size = 305

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(fh-ei)^3(a+b\log(c(e+fx)))^{1+p}}{bdf^4(1+p)} + \frac{3^{-1-p}e^{-\frac{3a}{b}}i^3\Gamma\left(1+p,-\frac{3(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^3df^4} + \frac{3\,2^{-1-p}e^{-\frac{2a}{b}}i^2(fh-ei)\Gamma\left(1+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^2df^4} + \frac{3e^{-\frac{a}{b}}i(fh-ei)^2\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^4}$$

output
```
(-e*i+f*h)^3*(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f^4/(p+1)+3^(-1-p)*i^3*GAMMA(p+
1,-3*(a+b*ln(c*(f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c^3/d/exp(3*a/b)/f^4/(((
-a-b*ln(c*(f*x+e)))/b)^p)+3*2^(-1-p)*i^2*(-e*i+f*h)*GAMMA(p+1,-2*(a+b*ln(c
*(f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c^2/d/exp(2*a/b)/f^4/(((-a-b*ln(c*(f*x
+e)))/b)^p)+3*i*(-e*i+f*h)^2*GAMMA(p+1,(-a-b*ln(c*(f*x+e)))/b)*(a+b*ln(c*(
f*x+e)))^p/c/d/exp(a/b)/f^4/(((-a-b*ln(c*(f*x+e)))/b)^p)
```

3.210.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.79

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$= \frac{6^{-1-p}e^{-\frac{3a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\left(2^{1+p}bi^3\Gamma\left(2+p,-\frac{3(a+b\log(c(e+fx)))}{b}\right)+3^{1+p}ce^{a/b}\left(\right.\right.}{}$$

input
```
Integrate[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
(6^(-1 - p)*(a + b*Log[c*(e + f*x)])^p*(2^(1 + p)*b*i^3*Gamma[2 + p, (-3*(
a + b*Log[c*(e + f*x)]))/b] + 3^(1 + p)*c*E^(a/b)*(3*b*i^2*(f*h - e*i)*Gam
ma[2 + p, (-2*(a + b*Log[c*(e + f*x)]))/b] + 2^(1 + p)*c*E^(a/b)*(3*b*i*(f
*h - e*i)^2*Gamma[2 + p, -((a + b*Log[c*(e + f*x)])/b)] - b*c*E^(a/b)*f^3*
(h + i*x)^3*(-((a + b*Log[c*(e + f*x)])/b))^(1 + p)))))/(b*c^3*d*E^((3*a)/
b)*f^4*(1 + p)*(-((a + b*Log[c*(e + f*x)])/b))^p)
```

3.210.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$\Big\downarrow \; 2858$$
$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^3(a+b\log(c(e+fx)))^p}{df^3(e+fx)}d(e+fx)}{f}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{(fh-ei+i(e+fx))^3(a+b\log(c(e+fx)))^p}{e+fx}d(e+fx)}{df^4}$$
$$\Big\downarrow \; 2795$$

$$\frac{\int \left(3i(fh-ei)^2(a+b\log(c(e+fx)))^p+i^3(e+fx)^2(a+b\log(c(e+fx)))^p+3i^2(fh-ei)(e+fx)(a+b\log(c(e}{df^4}$$

$\big\downarrow$ 2009

$$\frac{i^3 3^{-p-1}e^{-\frac{3a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(c(e+fx)))}{b}\right)}{c^3}+\frac{3i^2 2^{-p-1}e^{-\frac{2a}{b}}(fh-ei)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log}{}\right.}{c^2}$$

input
```
Int[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
(((f*h - e*i)^3*(a + b*Log[c*(e + f*x)])^(1 + p))/(b*(1 + p)) + (3^(-1 - p
)*i^3*Gamma[1 + p, (-3*(a + b*Log[c*(e + f*x)]))/b]*(a + b*Log[c*(e + f*x)
])^p)/(c^3*E^((3*a)/b)*(-((a + b*Log[c*(e + f*x)])/b))^p) + (3*2^(-1 - p)*
i^2*(f*h - e*i)*Gamma[1 + p, (-2*(a + b*Log[c*(e + f*x)]))/b]*(a + b*Log[c
*(e + f*x)])^p)/(c^2*E^((2*a)/b)*(-((a + b*Log[c*(e + f*x)])/b))^p) + (3*i
*(f*h - e*i)^2*Gamma[1 + p, -((a + b*Log[c*(e + f*x)])/b)]*(a + b*Log[c*(e
 + f*x)])^p)/(c*E^(a/b)*(-((a + b*Log[c*(e + f*x)])/b))^p))/(d*f^4)
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.210.4 Maple [F]

$$\int \frac{(ix+h)^3 (a+b\ln (c(fx+e)))^p}{dfx+de}dx$$

input
```
int((i*x+h)^3*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

output
```
int((i*x+h)^3*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

3.210.5 Fricas [F]

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^3(b\log ((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="frica
s")
```

output
```
integral((i^3*x^3 + 3*h*i^2*x^2 + 3*h^2*i*x + h^3)*(b*log(c*f*x + c*e) + a
)^p/(d*f*x + d*e), x)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \text{Timed out}$$

input
```
integrate((i*x+h)**3*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

output
```
Timed out
```

3.210.7 Maxima [F]

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^3(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxim
a")
```

output
```
(b*c*log(c*f*x + c*e) + a*c)*(b*log(c*f*x + c*e) + a)^p*h^3/(b*c*d*f*(p +
1)) + integrate((i^3*x^3 + 3*h*i^2*x^2 + 3*h^2*i*x)*(b*log(f*x + e) + b*lo
g(c) + a)^p/(d*f*x + d*e), x)
```

3.210.8 Giac [F]

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^3(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac"
)
```

output
```
integrate((i*x + h)^3*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+i\,x)^3\,(a+b\,\ln(c\,(e+f\,x)))^p}{d\,e+d\,f\,x}\,dx$$

input
```
int(((h + i*x)^3*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
```

output
```
int(((h + i*x)^3*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

3.211 $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$

3.211.1 Optimal result

Integrand size = 32, antiderivative size = 210

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(fh-ei)^2(a+b\log(c(e+fx)))^{1+p}}{bdf^3(1+p)} + \frac{2^{-1-p}e^{-\frac{2a}{b}}i^2\Gamma\left(1+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^2df^3} + \frac{2e^{-\frac{a}{b}}i(fh-ei)\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^3}$$

output
```
(-e*i+f*h)^2*(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f^3/(p+1)+2^(-1-p)*i^2*GAMMA(p+
1,-2*(a+b*ln(c*(f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c^2/d/exp(2*a/b)/f^3/(((
-a-b*ln(c*(f*x+e)))/b)^p)+2*i*(-e*i+f*h)*GAMMA(p+1,(-a-b*ln(c*(f*x+e)))/b)
*(a+b*ln(c*(f*x+e)))^p/c/d/exp(a/b)/f^3/(((-a-b*ln(c*(f*x+e)))/b)^p)
```

3.211.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$= \frac{2^{-1-p}e^{-\frac{2a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\left(bi^2\Gamma\left(2+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)+2^{1+p}ce^{a/b}\left(2bi(\right.\right.}{bc^2df^3(1+p)}$$

input
```
Integrate[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
(2^(-1 - p)*(a + b*Log[c*(e + f*x)])^p*(b*i^2*Gamma[2 + p, (-2*(a + b*Log[
c*(e + f*x)]))/b] + 2^(1 + p)*c*E^(a/b)*(2*b*i*(f*h - e*i)*Gamma[2 + p, -(
(a + b*Log[c*(e + f*x)])/b)] - b*c*E^(a/b)*f^2*(h + i*x)^2*(-((a + b*Log[c
*(e + f*x)])/b))^(1 + p))))/(b*c^2*d*E^((2*a)/b)*f^3*(1 + p)*(-((a + b*Log
[c*(e + f*x)])/b))^p)
```

3.211.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$\Big\downarrow \ 2858$$
$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)^2(a+b\log(c(e+fx)))^p}{df^2(e+fx)}d(e+fx)}{f}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{(fh-ei+i(e+fx))^2(a+b\log(c(e+fx)))^p}{e+fx}d(e+fx)}{df^3}$$
$$\Big\downarrow \ 2795$$

$$\frac{\int \left(2i(fh-ei)(a+b\log(c(e+fx)))^p + i^2(e+fx)(a+b\log(c(e+fx)))^p + \frac{(fh-ei)^2(a+b\log(c(e+fx)))^p}{e+fx}\right) d(e+fx)}{df^3}$$

$\big\downarrow$ 2009

$$\frac{\frac{i^2 2^{-p-1} e^{-\frac{2a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(c(e+fx)))}{b}\right)}{c^2} + \frac{(fh-ei)^2(a+b\log(c(e+fx)))^{p+1}}{b(p+1)} + \frac{2ie^{-\frac{a}{b}}(fh-ei)(}{}}{df^3}$$

input
```
Int[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
(((f*h - e*i)^2*(a + b*Log[c*(e + f*x)])^(1 + p))/(b*(1 + p)) + (2^(-1 - p
)*i^2*Gamma[1 + p, (-2*(a + b*Log[c*(e + f*x)]))/b]*(a + b*Log[c*(e + f*x)
])^p)/(c^2*E^((2*a)/b)*(-((a + b*Log[c*(e + f*x)])/b))^p) + (2*i*(f*h - e*
i)*Gamma[1 + p, -((a + b*Log[c*(e + f*x)])/b)]*(a + b*Log[c*(e + f*x)])^p)
/(c*E^(a/b)*(-((a + b*Log[c*(e + f*x)])/b))^p))/(d*f^3)
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.211.4 Maple [F]

$$\int \frac{(ix+h)^2 (a+b\ln(c(fx+e)))^p}{dfx+de} dx$$

input
```
int((i*x+h)^2*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

output
```
int((i*x+h)^2*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

3.211.5 Fricas [F]

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\, dx = \int \frac{(ix+h)^2(b\log((fx+e)c)+a)^p}{dfx+de}\, dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="frica
s")
```

output
```
integral((i^2*x^2 + 2*h*i*x + h^2)*(b*log(c*f*x + c*e) + a)^p/(d*f*x + d*e
), x)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\, dx = \text{Timed out}$$

input
```
integrate((i*x+h)**2*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

output
```
Timed out
```

3.211.7 Maxima [F]

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^2(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxim
a")
```

output
```
(b*c*log(c*f*x + c*e) + a*c)*(b*log(c*f*x + c*e) + a)^p*h^2/(b*c*d*f*(p +
1)) + integrate((i^2*x^2 + 2*h*i*x)*(b*log(f*x + e) + b*log(c) + a)^p/(d*f
*x + d*e), x)
```

3.211.8 Giac [F]

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^2(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac"
)
```

output
```
integrate((i*x + h)^2*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+i\,x)^2\,(a+b\,\ln(c\,(e+f\,x)))^p}{d\,e+d\,f\,x}\,dx$$

input
```
int(((h + i*x)^2*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
```

output
```
int(((h + i*x)^2*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

3.212 $\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$

3.212.1 Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(fh-ei)(a+b\log(c(e+fx)))^{1+p}}{bdf^2(1+p)} + \frac{e^{-\frac{a}{b}}i\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^2}$$

output
```
(-e*i+f*h)*(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f^2/(p+1)+i*GAMMA(p+1,(-a-b*ln(c*
(f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c/d/exp(a/b)/f^2/(((-a-b*ln(c*(f*x+e)))
/b)^p)
```

3.212.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = (a+b\log(c(e+fx)))^p\left(\frac{(h+ix)(a+b\log(c(e+fx)))}{bdf+bdfp} + \frac{e^{-\frac{a}{b}}i\Gamma\left(2+p,-\frac{a+b\log(c(e+fx))}{b}\right)\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^2+cdf^2p}\right)$$

input
```
Integrate[((h + i*x)*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^p*(((h + i*x)*(a + b*Log[c*(e + f*x)]))/(b*d*f +
b*d*f*p) + (i*Gamma[2 + p, -((a + b*Log[c*(e + f*x)])/b)])/(E^(a/b)*(c*d*f
^2 + c*d*f^2*p)*(-((a + b*Log[c*(e + f*x)])/b))^p))
```

3.212.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2858, 27, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}dx$$
$$\Big\downarrow \text{2858}$$
$$\frac{\int \frac{\left(f\left(h-\frac{ei}{f}\right)+i(e+fx)\right)(a+b\log(c(e+fx)))^p}{df(e+fx)}d(e+fx)}{f}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(fh-ei+i(e+fx))(a+b\log(c(e+fx)))^p}{e+fx}d(e+fx)}{df^2}$$
$$\Big\downarrow \text{2795}$$
$$\frac{\int \left(i(a+b\log(c(e+fx)))^p+\frac{(fh-ei)(a+b\log(c(e+fx)))^p}{e+fx}\right)d(e+fx)}{df^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{(fh-ei)(a+b\log(c(e+fx)))^{p+1}}{b(p+1)}+\frac{ie^{-\frac{a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(c(e+fx))}{b}\right)}{c}}{df^2}$$

input
```
Int[((h + i*x)*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

output
```
(((f*h - e*i)*(a + b*Log[c*(e + f*x)])^(1 + p))/(b*(1 + p)) + (i*Gamma[1 +
 p, -((a + b*Log[c*(e + f*x)])/b)]*(a + b*Log[c*(e + f*x)])^p)/(c*E^(a/b)*
(-((a + b*Log[c*(e + f*x)])/b))^p))/(d*f^2)
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.212.4 Maple [F]

$$\int \frac{(ix+h)\,(a+b\ln{(c(fx+e))})^p}{dfx+de}dx$$

input
```
int((i*x+h)*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

output
```
int((i*x+h)*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$=\frac{(bip+bi)e^{\left(-\frac{bp\log\left(-\frac{1}{b}\right)+a}{b}\right)}\Gamma\left(p+1,-\frac{b\log(cfx+ce)+a}{b}\right)+(acfh-acei+(bcfh-bcei)\log(cfx+ce))(b\log(c}{bcdf^2p+bcdf^2}$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas"
)
```

output
```
((b*i*p + b*i)*e^(-(b*p*log(-1/b) + a)/b)*gamma(p + 1, -(b*log(c*f*x + c*e
) + a)/b) + (a*c*f*h - a*c*e*i + (b*c*f*h - b*c*e*i)*log(c*f*x + c*e))*(b*
log(c*f*x + c*e) + a)^p)/(b*c*d*f^2*p + b*c*d*f^2)
```

3.212.6 Sympy [F]

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\frac{\int \frac{h(a+b\log(ce+cfx))^p}{e+fx}\,dx+\int \frac{ix(a+b\log(ce+cfx))^p}{e+fx}\,dx}{d}$$

input
```
integrate((i*x+h)*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

output
```
(Integral(h*(a + b*log(c*e + c*f*x))**p/(e + f*x), x) + Integral(i*x*(a +
b*log(c*e + c*f*x))**p/(e + f*x), x))/d
```

3.212.7 Maxima [F]

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\int \frac{(ix+h)(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima"
)
```

output
```
i*integrate((b*log(f*x + e) + b*log(c) + a)^p*x/(d*f*x + d*e), x) + (b*c*l
og(c*f*x + c*e) + a*c)*(b*log(c*f*x + c*e) + a)^p*h/(b*c*d*f*(p + 1))
```

3.212.8 Giac [F]

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
```

output
```
integrate((i*x + h)*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+i\,x)\,\left(a+b\,\ln\left(c\,(e+f\,x)\right)\right)^p}{d\,e+d\,f\,x}\,dx$$

input
```
int(((h + i*x)*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
```

output
```
int(((h + i*x)*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

3.213 $\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$

3.213.1 Optimal result

Integrand size = 25, antiderivative size = 31

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\frac{(a+b\log(c(e+fx)))^{1+p}}{bdf(1+p)}$$

output
```
(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f/(p+1)
```

3.213.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\frac{(a+b\log(c(e+fx)))^{1+p}}{bdf(1+p)}$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^p/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^(1 + p)/(b*d*f*(1 + p))
```

3.213.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2837, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$

$$\Big\downarrow \; 2837$$

$$\frac{\int \frac{(a+b\log(c(e+fx)))^p}{d(e+fx)}d(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\log(c(e+fx)))^p}{e+fx}d(e+fx)}{df}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int (a+b\log(c(e+fx)))^p d(a+b\log(c(e+fx)))}{bdf}$$

$$\Big\downarrow \; 15$$

$$\frac{(a+b\log(c(e+fx)))^{p+1}}{bdf(p+1)}$$

input
```
Int[(a + b*Log[c*(e + f*x)])^p/(d*e + d*f*x),x]
```

output
```
(a + b*Log[c*(e + f*x)])^(1 + p)/(b*d*f*(1 + p))
```

3.213.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.213.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{(a+b\ln(c(fx+e)))^{p+1}}{bdf(p+1)}$	32
parallelrisch	$\frac{\ln(c(fx+e))(a+b\ln(c(fx+e)))^{p}bf+(a+b\ln(c(fx+e)))^{p}af}{d\,f^{2}b(p+1)}$	59
norman	$\frac{\ln(c(fx+e))e^{p\ln(a+b\ln(c(fx+e)))}}{df(p+1)}+\frac{a\,e^{p\ln(a+b\ln(c(fx+e)))}}{bdf(p+1)}$	70

input
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

output
```
(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f/(p+1)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(b\log(cfx+ce)+a)(b\log(cfx+ce)+a)^p}{bdfp+bdf}$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas")
```

output
```
(b*log(c*f*x + c*e) + a)*(b*log(c*f*x + c*e) + a)^p/(b*d*f*p + b*d*f)
```

3.213.6 Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{\int \frac{(a+b\log(ce+cfx))^p}{e+fx}\,dx}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

output
```
Integral((a + b*log(c*e + c*f*x))**p/(e + f*x), x)/d
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(b\log(cfx+ce)+a)^{p+1}}{bdf(p+1)}$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima")
```

output
```
(b*log(c*f*x + c*e) + a)^(p + 1)/(b*d*f*(p + 1))
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(b\log(cfx+ce)+a)^{p+1}}{bdf(p+1)}$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
```

output
```
(b*log(c*f*x + c*e) + a)^(p + 1)/(b*d*f*(p + 1))
```

3.213.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\frac{(a+b\,\ln\left(c\,(e+f\,x)\right))^{p+1}}{b\,d\,f\,\left(p+1\right)}$$

input
```
int((a + b*log(c*(e + f*x)))^p/(d*e + d*f*x),x)
```

output
```
(a + b*log(c*(e + f*x)))^(p + 1)/(b*d*f*(p + 1))
```

3.214 $\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx$

3.214.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \text{Int}\left(\frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)},x\right)$$

output
```
Unintegrable((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x)
```

3.214.2 Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)),x]
```

output
```
Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)), x]
```

3.214.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(h+ix)(de+dfx)}dx$$
$$\Big\downarrow \text{2867}$$
$$\int \frac{(a+b\log(c(e+fx)))^p}{(h+ix)(de+dfx)}dx$$

input
```
Int[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)),x]
```

output
```
$Aborted
```

3.214.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.214.4 Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(fx+e)))^p}{(dfx+de)(ix+h)}dx$$

input
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x)
```

output
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x)
```

3.214.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x, algorithm="fricas"
)
```

output
```
integral((b*log(c*f*x + c*e) + a)^p/(d*f*i*x^2 + d*e*h + (d*f*h + d*e*i)*x
), x)
```

3.214.6 Sympy [N/A]

Not integrable

Time = 154.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \frac{\int \frac{(a+b\log{(ce+cfx)})^p}{eh+eix+fhx+fix^2}\,dx}{d}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e)/(i*x+h),x)
```

output
```
Integral((a + b*log(c*e + c*f*x))**p/(e*h + e*i*x + f*h*x + f*i*x**2), x)/
d
```

3.214.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x, algorithm="maxima"
)
```

output
```
integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)), x)
```

3.214.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx=\int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x, algorithm="giac")
```

output
```
integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)), x)
```

3.214.9 Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)^p}{\left(h+i\,x\right)\,\left(d\,e+d\,f\,x\right)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))^p/((h + i*x)*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))^p/((h + i*x)*(d*e + d*f*x)), x)
```

3.215 $\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx$

3.215.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx=\text{Int}\left(\frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2},x\right)$$

output
```
Unintegrable((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x)
```

3.215.2 Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx=\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2),x]
```

output
```
Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2), x]
```

3.215.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(h+ix)^2(de+dfx)}\,dx$$

$$\Big\downarrow \; 2867$$

$$\int \frac{(a+b\log(c(e+fx)))^p}{(h+ix)^2(de+dfx)}dx$$

input
```
Int[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2),x]
```

output
```
$Aborted
```

3.215.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.215.4 Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(fx+e)))^p}{(dfx+de)(ix+h)^2}dx$$

input
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x)
```

output
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x)
```

3.215.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="frica
s")
```

output
```
integral((b*log(c*f*x + c*e) + a)^p/(d*f*i^2*x^3 + d*e*h^2 + (2*d*f*h*i +
d*e*i^2)*x^2 + (d*f*h^2 + 2*d*e*h*i)*x), x)
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e)/(i*x+h)**2,x)
```

output
```
Timed out
```

3.215.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="maxim
a")
```

output
```
integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)^2), x)
```

3.215.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0,0,0,0]%%%} / %%%{1,[0,0,1,1,1,0,0]%%%}+%%%{-1,[
0,0,0,1,0
```

3.215.9 Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(e+f\,x\right)\right)\right)^p}{\left(h+i\,x\right)^2\,\left(d\,e+d\,f\,x\right)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))^p/((h + i*x)^2*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))^p/((h + i*x)^2*(d*e + d*f*x)), x)
```

3.216 $\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx$

3.216.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \text{Int}\left(\frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3},x\right)$$

output
```
Unintegrable((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x)
```

3.216.2 Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx$$

input
```
Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3),x]
```

output
```
Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3), x]
```

3.216.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(h+ix)^3(de+dfx)}dx$$

$$\Big\downarrow \text{ 2867}$$

$$\int \frac{(a+b\log(c(e+fx)))^p}{(h+ix)^3(de+dfx)}dx$$

input
```
Int[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3),x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.216.4 Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(fx+e)))^p}{(dfx+de)(ix+h)^3}dx$$

input
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x)
```

output
```
int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x)
```

3.216.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx=\int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="frica
s")
```

output
```
integral((b*log(c*f*x + c*e) + a)^p/(d*f*i^3*x^4 + d*e*h^3 + (3*d*f*h*i^2
+ d*e*i^3)*x^3 + 3*(d*f*h^2*i + d*e*h*i^2)*x^2 + (d*f*h^3 + 3*d*e*h^2*i)*x
), x)
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e)/(i*x+h)**3,x)
```

output
```
Timed out
```

3.216.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx=\int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="maxim
a")
```

output
```
integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)^3), x)
```

3.216.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="giac"
)
```

output
```
integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)^3), x)
```

3.216.9 Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(a+b\,\ln\left(c\,(e+f\,x)\right))^p}{(h+i\,x)^3\,(d\,e+d\,f\,x)}\,dx$$

input
```
int((a + b*log(c*(e + f*x)))^p/((h + i*x)^3*(d*e + d*f*x)),x)
```

output
```
int((a + b*log(c*(e + f*x)))^p/((h + i*x)^3*(d*e + d*f*x)), x)
```

3.217 $\int \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

3.217.1 Optimal result

Integrand size = 29, antiderivative size = 402

$$\begin{aligned}\int \frac{(h+ix)^3\,(a+b\log\left(c(d+ex)^n\right))}{f+gx}\,dx &= \frac{ai(gh-fi)^2x}{g^3}-\frac{bi(eh-di)^2nx}{3e^2g}\\
&-\frac{bi(eh-di)(gh-fi)nx}{2eg^2}-\frac{bi(gh-fi)^2nx}{g^3}\\
&-\frac{b(eh-di)n(h+ix)^2}{6eg}-\frac{b(gh-fi)n(h+ix)^2}{4g^2}\\
&-\frac{bn(h+ix)^3}{9g}-\frac{b(eh-di)^3n\log(d+ex)}{3e^3g}\\
&-\frac{b(eh-di)^2(gh-fi)n\log(d+ex)}{2e^2g^2}\\
&+\frac{bi(gh-fi)^2(d+ex)\log\left(c(d+ex)^n\right)}{eg^3}\\
&+\frac{(gh-fi)(h+ix)^2\,(a+b\log\left(c(d+ex)^n\right))}{2g^2}\\
&+\frac{(h+ix)^3\,(a+b\log\left(c(d+ex)^n\right))}{3g}\\
&+\frac{(gh-fi)^3\,(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}\\
&+\frac{b(gh-fi)^3n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}\end{aligned}$$

output
```
a*i*(-f*i+g*h)^2*x/g^3-1/3*b*i*(-d*i+e*h)^2*n*x/e^2/g-1/2*b*i*(-d*i+e*h)*(
-f*i+g*h)*n*x/e/g^2-b*i*(-f*i+g*h)^2*n*x/g^3-1/6*b*(-d*i+e*h)*n*(i*x+h)^2/
e/g-1/4*b*(-f*i+g*h)*n*(i*x+h)^2/g^2-1/9*b*n*(i*x+h)^3/g-1/3*b*(-d*i+e*h)^
3*n*ln(e*x+d)/e^3/g-1/2*b*(-d*i+e*h)^2*(-f*i+g*h)*n*ln(e*x+d)/e^2/g^2+b*i*
(-f*i+g*h)^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g^3+1/2*(-f*i+g*h)*(i*x+h)^2*(a+b*l
n(c*(e*x+d)^n))/g^2+1/3*(i*x+h)^3*(a+b*ln(c*(e*x+d)^n))/g+(-f*i+g*h)^3*(a+
b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^4+b*(-f*i+g*h)^3*n*polylog(2
,-g*(e*x+d)/(-d*g+e*f))/g^4
```

3.217.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.94

$$\int \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$
$$= \frac{6bd^2g^2i^2(-9egh+3efi+2dgi)n\log(d+ex)+e\Big(gix(6ae^2(6f^2i^2-3fgi(6h+ix)+g^2(18h^2+9hix+2\ldots}{}$$

input
```
Integrate[((h + i*x)^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(6*b*d^2*g^2*i^2*(-9*e*g*h + 3*e*f*i + 2*d*g*i)*n*Log[d + e*x] + e*(g*i*x*
(6*a*e^2*(6*f^2*i^2 - 3*f*g*i*(6*h + i*x) + g^2*(18*h^2 + 9*h*i*x + 2*i^2*
x^2)) - b*n*(12*d^2*g^2*i^2 - 6*d*e*g*i*(9*g*h - 3*f*i + g*i*x) + e^2*(36*
f^2*i^2 - 9*f*g*i*(12*h + i*x) + g^2*(108*h^2 + 27*h*i*x + 4*i^2*x^2)))) +
 36*a*e^2*(g*h - f*i)^3*Log[(e*(f + g*x))/(e*f - d*g)] + 6*b*e*Log[c*(d +
e*x)^n]*(g*i*(6*d*(3*g^2*h^2 - 3*f*g*h*i + f^2*i^2) + e*x*(6*f^2*i^2 - 3*f
*g*i*(6*h + i*x) + g^2*(18*h^2 + 9*h*i*x + 2*i^2*x^2))) + 6*e*(g*h - f*i)^
3*Log[(e*(f + g*x))/(e*f - d*g)])) + 36*b*e^3*(g*h - f*i)^3*n*PolyLog[2, (
g*(d + e*x))/(-(e*f) + d*g)])/(36*e^3*g^4)
```

3.217.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.217. $\int \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx} \, dx$$

$$\Big\downarrow \; 2865$$

$$\int \left(\frac{(gh-fi)^3 (a+b\log(c(d+ex)^n))}{g^3(f+gx)} + \frac{i(gh-fi)^2 (a+b\log(c(d+ex)^n))}{g^3} + \frac{i(h+ix)(gh-fi) (a+b\log(c(d$$

$$\Big\downarrow \; 2009$$

$$\frac{(gh-fi)^3 \log\left(\frac{e(f+gx)}{ef-dg}\right) (a+b\log(c(d+ex)^n))}{g^4} + \frac{(h+ix)^2(gh-fi) (a+b\log(c(d+ex)^n))}{2g^2} + \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{3g} + \frac{aix(gh-fi)^2}{g^3} + \frac{bi(d+ex)(gh-fi)^2 \log(c(d+ex)^n)}{eg^3} - \frac{bn(eh-di)^3 \log(d+ex)}{3e^3g} - \frac{bn(eh-di)^2 \log(d+ex)(gh-fi)}{2e^2g^2} - \frac{binx(eh-di)^2}{3e^2g} + \frac{bn(gh-fi)^3 \operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4} - \frac{binx(eh-di)(gh-fi)}{2eg^2} - \frac{bn(h+ix)^2(eh-di)}{6eg} - \frac{binx(gh-fi)^2}{g^3} - \frac{bn(h+ix)^2(gh-fi)}{4g^2} - \frac{bn(h+ix)^3}{9g}$$

input
```
Int[((h + i*x)^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*i*(g*h - f*i)^2*x)/g^3 - (b*i*(e*h - d*i)^2*n*x)/(3*e^2*g) - (b*i*(e*h
- d*i)*(g*h - f*i)*n*x)/(2*e*g^2) - (b*i*(g*h - f*i)^2*n*x)/g^3 - (b*(e*h
- d*i)*n*(h + i*x)^2)/(6*e*g) - (b*(g*h - f*i)*n*(h + i*x)^2)/(4*g^2) - (b
*n*(h + i*x)^3)/(9*g) - (b*(e*h - d*i)^3*n*Log[d + e*x])/(3*e^3*g) - (b*(e
*h - d*i)^2*(g*h - f*i)*n*Log[d + e*x])/(2*e^2*g^2) + (b*i*(g*h - f*i)^2*(
d + e*x)*Log[c*(d + e*x)^n])/(e*g^3) + ((g*h - f*i)*(h + i*x)^2*(a + b*Log
[c*(d + e*x)^n]))/(2*g^2) + ((h + i*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*g)
 + ((g*h - f*i)^3*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)
])/g^4 + (b*(g*h - f*i)^3*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^4
```

3.217. $\int \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{f+gx} \, dx$

3.217.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.217.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 1208, normalized size of antiderivative = 3.00

method	result	size
risch	Expression too large to display	1208

input
```
int((i*x+h)^3*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
-49/36*b*n/g^4*i^3*f^3+3*b*n/g^2*i^2*f*h*x-1/3*b/e^2*n/g*i^3*d^2*x+1/6*b/e
*n/g*i^3*d*x^2-3*b*ln((e*x+d)^n)*i^2/g^2*x*f*h+3*b*ln((e*x+d)^n)/g^3*ln(g*
x+f)*f^2*h*i^2-3*b*ln((e*x+d)^n)/g^2*ln(g*x+f)*f*h^2*i-3*b*n/g^3*dilog(((g
*x+f)*e+d*g-e*f)/(d*g-e*f))*f^2*h*i^2+3*b*n/g^2*dilog(((g*x+f)*e+d*g-e*f)/
(d*g-e*f))*f*h^2*i+b*n/g^4*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f^3
*i^3+1/3*b/e^3*n/g*i^3*d^3*ln((g*x+f)*e+d*g-e*f)+15/4*b*n/g^3*i^2*f^2*h-3*
b*n/g^2*i*f*h^2+1/2*b/e^2*n/g^2*i^3*d^2*ln((g*x+f)*e+d*g-e*f)*f-3/2*b/e^2*
n/g*i^2*d^2*ln((g*x+f)*e+d*g-e*f)*h+b/e*n/g^3*i^3*d*ln((g*x+f)*e+d*g-e*f)*
f^2+3*b/e*n/g*i*d*ln((g*x+f)*e+d*g-e*f)*h^2-1/2*b/e*n/g^2*i^3*d*f*x+3/2*b/
e*n/g*i^2*d*h*x-3*b*n/g^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f^2*
h*i^2+3*b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*h^2*i+3/2*b/
e*n/g^2*i^2*d*f*h+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^
n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(i/g^
3*(1/3*i^2*x^3*g^2-1/2*x^2*f*g*i^2+3/2*x^2*g^2*h*i+x*f^2*i^2-3*x*f*g*h*i+3
*x*g^2*h^2)+(-f^3*i^3+3*f^2*g*h*i^2-3*f*g^2*h^2*i+g^3*h^3)/g^4*ln(g*x+f))-
1/3*b/e^2*n/g^2*i^3*d^2*f-2/3*b/e*n/g^3*i^3*d*f^2+1/4*b*n/g^2*i^3*f*x^2-b*
n/g^3*i^3*f^2*x-3/4*b*n/g*i^2*h*x^2-3*b*n/g*i*h^2*x+b*n/g^4*dilog(((g*x+f)
*e+d*g-e*f)/(d*g-e*f))*f^3*i^3-b*n/g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))*h^3+3*b*ln((e*x+d)^n)*i/g*x*h^2-b*ln((e*x+d)^n)/g^4*ln(g*x+f)*f^...
```

3.217.5 Fricas [F]

$$\int \frac{(h+ix)^3\,(a+b\log\left(c(d+ex)^n\right))}{f+gx}\,dx = \int \frac{(ix+h)^3(b\log\left((ex+d)^n c\right)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((a*i^3*x^3 + 3*a*h*i^2*x^2 + 3*a*h^2*i*x + a*h^3 + (b*i^3*x^3 + 3
*b*h*i^2*x^2 + 3*b*h^2*i*x + b*h^3)*log((e*x + d)^n*c))/(g*x + f), x)
```

3.217.6 Sympy [F]

$$\int \frac{(h+ix)^3\,(a+b\log(c(d+ex)^n))}{f+gx}\,dx=\int \frac{(a+b\log(c(d+ex)^n))\,(h+ix)^3}{f+gx}\,dx$$

input
```
integrate((i*x+h)**3*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(h + i*x)**3/(f + g*x), x)
```

3.217.7 Maxima [F]

$$\int \frac{(h+ix)^3\,(a+b\log(c(d+ex)^n))}{f+gx}\,dx=\int \frac{(ix+h)^3(b\log((ex+d)^nc)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
3*a*h^2*i*(x/g - f*log(g*x + f)/g^2) - 1/6*a*i^3*(6*f^3*log(g*x + f)/g^4 -
 (2*g^2*x^3 - 3*f*g*x^2 + 6*f^2*x)/g^3) + 3/2*a*h*i^2*(2*f^2*log(g*x + f)/
g^3 + (g*x^2 - 2*f*x)/g^2) + a*h^3*log(g*x + f)/g + integrate((b*i^3*x^3*l
og(c) + 3*b*h*i^2*x^2*log(c) + 3*b*h^2*i*x*log(c) + b*h^3*log(c) + (b*i^3*
x^3 + 3*b*h*i^2*x^2 + 3*b*h^2*i*x + b*h^3)*log((e*x + d)^n))/(g*x + f), x)
```

3.217.8 Giac [F]

$$\int \frac{(h+ix)^3\,(a+b\log(c(d+ex)^n))}{f+gx}\,dx=\int \frac{(ix+h)^3(b\log((ex+d)^nc)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)^3*(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(h+i\,x)^3 \left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)}{f+g\,x}\,dx$$

input
```
int(((h + i*x)^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

output
```
int(((h + i*x)^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

3.218 $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

3.218.1 Optimal result

Integrand size = 29, antiderivative size = 241

$$\begin{aligned}\int \frac{(h+ix)^2 (a+b\log (c(d+ex)^n))}{f+gx}\,dx =& \frac{ai(gh-fi)x}{g^2}-\frac{bi(eh-di)nx}{2eg}-\frac{bi(gh-fi)nx}{g^2}\\&-\frac{bn(h+ix)^2}{4g}-\frac{b(eh-di)^2n\log(d+ex)}{2e^2g}\\&+\frac{bi(gh-fi)(d+ex)\log (c(d+ex)^n)}{eg^2}\\&+\frac{(h+ix)^2 (a+b\log (c(d+ex)^n))}{2g}\\&+\frac{(gh-fi)^2 (a+b\log (c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}\\&+\frac{b(gh-fi)^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}\end{aligned}$$

output
```
a*i*(-f*i+g*h)*x/g^2-1/2*b*i*(-d*i+e*h)*n*x/e/g-b*i*(-f*i+g*h)*n*x/g^2-1/4
*b*n*(i*x+h)^2/g-1/2*b*(-d*i+e*h)^2*n*ln(e*x+d)/e^2/g+b*i*(-f*i+g*h)*(e*x+
d)*ln(c*(e*x+d)^n)/e/g^2+1/2*(i*x+h)^2*(a+b*ln(c*(e*x+d)^n))/g+(-f*i+g*h)^
2*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^3+b*(-f*i+g*h)^2*n*poly
log(2,-g*(e*x+d)/(-d*g+e*f))/g^3
```

3.218.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx}\,dx$$
$$= \frac{-2bd^2g^2i^2n\log(d+ex)+e\Big(gix(2ae(4gh-2fi+gix)+bn(2dgi-e(8gh-4fi+gix)))+4ae(gh-fi)}{}$$

input
```
Integrate[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(-2*b*d^2*g^2*i^2*n*Log[d + e*x] + e*(g*i*x*(2*a*e*(4*g*h - 2*f*i + g*i*x)
 + b*n*(2*d*g*i - e*(8*g*h - 4*f*i + g*i*x))) + 4*a*e*(g*h - f*i)^2*Log[(e
*(f + g*x))/(e*f - d*g)] + 2*b*Log[c*(d + e*x)^n]*(g*i*(d*(4*g*h - 2*f*i)
+ e*x*(4*g*h - 2*f*i + g*i*x)) + 2*e*(g*h - f*i)^2*Log[(e*(f + g*x))/(e*f
- d*g)])) + 4*b*e^2*(g*h - f*i)^2*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g
)])/(4*e^2*g^3)
```

3.218.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx}\,dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{(gh-fi)^2 (a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{i(gh-fi)(a+b\log(c(d+ex)^n))}{g^2}+\frac{i(h+ix)(a+b\log(c(d+ex)^n))}{g}\right)$$
$$\Big\downarrow \text{2009}$$

$$\frac{(gh-fi)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3}+\frac{(h+ix)^2(a+b\log(c(d+ex)^n))}{2g}+\frac{aix(gh-fi)}{g^2}+\frac{bi(d+ex)(gh-fi)\log(c(d+ex)^n)}{eg^2}-\frac{bn(eh-di)^2\log(d+ex)}{2e^2g}+\frac{bn(gh-fi)^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}-\frac{binx(eh-di)}{2eg}-\frac{binx(gh-fi)}{g^2}-\frac{bn(h+ix)^2}{4g}$$

input
```
Int[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*i*(g*h - f*i)*x)/g^2 - (b*i*(e*h - d*i)*n*x)/(2*e*g) - (b*i*(g*h - f*i)
*n*x)/g^2 - (b*n*(h + i*x)^2)/(4*g) - (b*(e*h - d*i)^2*n*Log[d + e*x])/(2*
e^2*g) + (b*i*(g*h - f*i)*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) + ((h + i*
x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*g) + ((g*h - f*i)^2*(a + b*Log[c*(d +
e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 + (b*(g*h - f*i)^2*n*PolyLog[
2, -((g*(d + e*x))/(e*f - d*g))])/g^3
```

3.218.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.218.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.99

method	result
risch	$\frac{b\ln((ex+d)^n)i^2x^2}{2g}-\frac{b\ln((ex+d)^n)i^2xf}{g^2}+\frac{2b\ln((ex+d)^n)ixh}{g}+\frac{b\ln((ex+d)^n)\ln(gx+f)f^2i^2}{g^3}-\frac{2b\ln((ex+d)^n)\ln(gx+f)fhi}{g^2}$

input
```
int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)*i^2/g*x^2-b*ln((e*x+d)^n)*i^2/g^2*x*f+2*b*ln((e*x+d)^n
)*i/g*x*h+b*ln((e*x+d)^n)/g^3*ln(g*x+f)*f^2*i^2-2*b*ln((e*x+d)^n)/g^2*ln(g
*x+f)*f*h*i+b*ln((e*x+d)^n)/g*ln(g*x+f)*h^2-b*n/g^3*dilog(((g*x+f)*e+d*g-e
*f)/(d*g-e*f))*f^2*i^2+2*b*n/g^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*h*
i-b*n/g*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h^2-b*n/g^3*ln(g*x+f)*ln(((g*
x+f)*e+d*g-e*f)/(d*g-e*f))*f^2*i^2+2*b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e
*f)/(d*g-e*f))*f*h*i-b*n/g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h^2
-1/4*b*n/g*i^2*x^2+b*n/g^2*i^2*f*x+5/4*b*n/g^3*i^2*f^2+1/2*b/e*n/g*i^2*d*x
+1/2*b/e*n/g^2*i^2*d*f-2*b*n/g*i*h*x-2*b*n/g^2*i*f*h-1/2*b/e^2*n/g*i^2*d^2
*ln((g*x+f)*e+d*g-e*f)-b/e*n/g^2*i^2*d*ln((g*x+f)*e+d*g-e*f)*f+2*b/e*n/g*i
*d*ln((g*x+f)*e+d*g-e*f)*h+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)
+a)*(i/g^2*(1/2*i*x^2*g-x*f*i+2*x*g*h)+(f^2*i^2-2*f*g*h*i+g^2*h^2)/g^3*ln(
g*x+f))
```

3.218.5 Fricas [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((a*i^2*x^2 + 2*a*h*i*x + a*h^2 + (b*i^2*x^2 + 2*b*h*i*x + b*h^2)*
log((e*x + d)^n*c))/(g*x + f), x)
```

3.218.6 Sympy [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))(h+ix)^2}{f+gx}\,dx$$

input
```
integrate((i*x+h)**2*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(h + i*x)**2/(f + g*x), x)
```

3.218.7 Maxima [F]

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)^2\left(b\log\left((ex+d)^nc\right)+a\right)}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
2*a*h*i*(x/g - f*log(g*x + f)/g^2) + 1/2*a*i^2*(2*f^2*log(g*x + f)/g^3 + (
g*x^2 - 2*f*x)/g^2) + a*h^2*log(g*x + f)/g + integrate((b*i^2*x^2*log(c) +
 2*b*h*i*x*log(c) + b*h^2*log(c) + (b*i^2*x^2 + 2*b*h*i*x + b*h^2)*log((e*
x + d)^n))/(g*x + f), x)
```

3.218.8 Giac [F]

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)^2\left(b\log\left((ex+d)^nc\right)+a\right)}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)^2*(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(h+i\,x)^2\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)}{f+g\,x}\,dx$$

input
```
int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

output
```
int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

3.219 $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

3.219.1 Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{(h+ix)\,(a+b\log\left(c(d+ex)^n\right))}{f+gx}\,dx = \frac{aix}{g}-\frac{binx}{g}+\frac{bi(d+ex)\log\left(c(d+ex)^n\right)}{eg} + \frac{(gh-fi)\,(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{b(gh-fi)n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

output
```
a*i*x/g-b*i*n*x/g+b*i*(e*x+d)*ln(c*(e*x+d)^n)/e/g+(-f*i+g*h)*(a+b*ln(c*(e*
x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^2+b*(-f*i+g*h)*n*polylog(2,-g*(e*x+d)/
(-d*g+e*f))/g^2
```

3.219.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(h+ix)\,(a+b\log\left(c(d+ex)^n\right))}{f+gx}\,dx = \frac{agix-bginx+\frac{bgi(d+ex)\log(c(d+ex)^n)}{e}+(gh-fi)\,(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)+b(gh-fi)n\,\mathrm{PolyL}}{g^2}$$

input
```
Integrate[((h + i*x)*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*g*i*x - b*g*i*n*x + (b*g*i*(d + e*x)*Log[c*(d + e*x)^n])/e + (g*h - f*i
)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + b*(g*h - f*i
)*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/g^2
```

3.219.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)(a+b\log(c(d+ex)^n))}{f+gx}dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{(gh-fi)(a+b\log(c(d+ex)^n))}{g(f+gx)}+\frac{i(a+b\log(c(d+ex)^n))}{g}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(gh-fi)\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{aix}{g}+\frac{bi(d+ex)\log(c(d+ex)^n)}{eg}+\frac{bn(gh-fi)\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{binx}{g}$$

input
```
Int[((h + i*x)*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*i*x)/g - (b*i*n*x)/g + (b*i*(d + e*x)*Log[c*(d + e*x)^n])/(e*g) + ((g*h
 - f*i)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (
b*(g*h - f*i)*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^2
```

3.219.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.219.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.31

method	result
risch	$\frac{b\ln((ex+d)^n)xi}{g} - \frac{b\ln((ex+d)^n)\ln(gx+f)fi}{g^2} + \frac{b\ln((ex+d)^n)\ln(gx+f)h}{g} - \frac{binx}{g} - \frac{bnif}{g^2} + \frac{bnid\ln((gx+f)e+dg-ef)}{eg} +$

input
```
int((i*x+h)*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)*x*i/g-b*ln((e*x+d)^n)/g^2*ln(g*x+f)*f*i+b*ln((e*x+d)^n)/g*
ln(g*x+f)*h-b*i*n*x/g-b*n/g^2*i*f+b/e*n/g*i*d*ln((g*x+f)*e+d*g-e*f)+b*n/g^
2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*i-b*n/g*dilog(((g*x+f)*e+d*g-e*f)
/(d*g-e*f))*h+b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*i-b*n/
g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h+(-1/2*I*b*Pi*csgn(I*c)*csg
n(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n
)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c
*(e*x+d)^n)^3+b*ln(c)+a)*(x*i/g+(-f*i+g*h)/g^2*ln(g*x+f))
```

3.219.5 Fricas [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx = \int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((a*i*x + a*h + (b*i*x + b*h)*log((e*x + d)^n*c))/(g*x + f), x)
```

3.219.6 Sympy [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)(h+ix)}{f+gx}\,dx$$

input
```
integrate((i*x+h)*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(h + i*x)/(f + g*x), x)
```

3.219.7 Maxima [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^nc\right)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
a*i*(x/g - f*log(g*x + f)/g^2) + a*h*log(g*x + f)/g + integrate((b*i*x*log
(c) + b*h*log(c) + (b*i*x + b*h)*log((e*x + d)^n))/(g*x + f), x)
```

3.219.8 Giac [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^nc\right)+a)}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)*(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)\,(a+b\log{(c(d+ex)^n)})}{f+gx}\,dx = \int \frac{(h+i\,x)\,\left(a+b\,\ln{\left(c\,{(d+e\,x)}^n\right)}\right)}{f+g\,x}\,dx$$

input
```
int(((h + i*x)*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

output
```
int(((h + i*x)*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

3.220 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx$

3.220.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output
```
(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g+b*n*polylog(2,-g*(e*x+d)/
(-d*g+e*f))/g
```

3.220.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyL
og[2, (g*(d + e*x))/(-(e*f) + d*g)])/g
```

3.220.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx$$

$$\Big\downarrow \ 2841$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \ 2840$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{bn\int \frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \ 2838$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyL
og[2, -((g*(d + e*x))/(e*f - d*g))])/g
```

3.220.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.220.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(gx+f)}{g} - \frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g} - \frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g} + \frac{\left(-\frac{ib\pi\,\mathrm{csgn}(ic(ex+d)^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}}{2}\right.}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)*ln(g*x+f)/g-b/g*n*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b/g
*n*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*
(e*x+d)^n)^3+b*ln(c)+a)*ln(g*x+f)/g
```

3.220.5 Fricas [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.220.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{a + b\log\left(c(d+ex)^n\right)}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.220.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(g*x + f), x) + a*log(g*x + f)/g
```

3.220.8 Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x), x)
```

3.221 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)}\,dx$

3.221.1 Optimal result

Integrand size = 29, antiderivative size = 155

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{bn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

output
```
(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)-(a+b*ln(c*(e*x+d
)^n))*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)+b*n*polylog(2,-g*(e*x+d)/(-d*g+e
*f))/(-f*i+g*h)-b*n*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)
```

3.221.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.72

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(\log\left(\frac{e(f+gx)}{ef-dg}\right)-\log\left(\frac{e(h+ix)}{eh-di}\right)\right)+bn\operatorname{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)-bn\operatorname{PolyLog}\left(2,\frac{i(d+e}{-eh+}\right.}{gh-fi}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*(Log[(e*(f + g*x))/(e*f - d*g)] - Log[(e*(h +
i*x))/(e*h - d*i)]) + b*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - b*n*P
olyLog[2, (i*(d + e*x))/(-(e*h) + d*i)])/(g*h - f*i)
```

3.221.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)}\,dx$$

$$\Big\downarrow \ 2865$$

$$\int \left(\frac{g(a+b\log(c(d+ex)^n))}{(f+gx)(gh-fi)}-\frac{i(a+b\log(c(d+ex)^n))}{(h+ix)(gh-fi)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{gh-fi}-\frac{\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{gh-fi}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi}-\frac{bn\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i) -
((a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i) +
(b*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i) - (b*n*PolyLog[
2, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)
```

3.221.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.221.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.44

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(ix+h)}{fi-gh}-\frac{b\ln((ex+d)^n)\ln(gx+f)}{fi-gh}-\frac{bn\,\mathrm{dilog}\left(\frac{(ix+h)e+di-eh}{di-eh}\right)}{fi-gh}-\frac{bn\ln(ix+h)\ln\left(\frac{(ix+h)e+di-eh}{di-eh}\right)}{fi-gh}+\frac{bn\,\mathrm{dilog}}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/(f*i-g*h)*ln(i*x+h)-b*ln((e*x+d)^n)/(f*i-g*h)*ln(g*x+f)-b*
n/(f*i-g*h)*dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))-b*n/(f*i-g*h)*ln(i*x+h)*l
n(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+b*n/(f*i-g*h)*dilog(((g*x+f)*e+d*g-e*f)/(
d*g-e*f))+b*n/(f*i-g*h)*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*
I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c
)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/(f*i-g*h)*ln(i*x+h)-1/(f*i
-g*h)*ln(g*x+f))
```

3.221.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

3.221.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)\,(h+ix)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)/(i*x+h),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/((f + g*x)*(h + i*x)), x)
```

3.221.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x, algorithm="maxima")
```

output
```
a*(log(g*x + f)/(g*h - f*i) - log(i*x + h)/(g*h - f*i)) + b*integrate((log
((e*x + d)^n) + log(c))/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

3.221.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*(i*x + h)), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\left(f+g\,x\right)\,\left(h+i\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)), x)
```

3.222 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^2}\,dx$

3.222.1 Optimal result

Integrand size = 29, antiderivative size = 252

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = &-\frac{ben\log(d+ex)}{(eh-di)(gh-fi)}+\frac{a+b\log\left(c(d+ex)^n\right)}{(gh-fi)(h+ix)}\\ &+\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}\\ &+\frac{ben\log(h+ix)}{(eh-di)(gh-fi)}-\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}\\ &+\frac{bgn\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}-\frac{bgn\,\mathrm{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}\end{aligned}$$

output
```
-b*e*n*ln(e*x+d)/(-d*i+e*h)/(-f*i+g*h)+(a+b*ln(c*(e*x+d)^n))/(-f*i+g*h)/(i
*x+h)+g*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^2+b*e*n*
ln(i*x+h)/(-d*i+e*h)/(-f*i+g*h)-g*(a+b*ln(c*(e*x+d)^n))*ln(e*(i*x+h)/(-d*i
+e*h))/(-f*i+g*h)^2+b*g*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2-b*
g*n*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2
```

3.222.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^2}\,dx$$
$$= \frac{\frac{(gh-fi)(a+b\log(c(d+ex)^n))}{h+ix}+g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)-\frac{be(gh-fi)n(\log(d+ex)-\log(h+ix))}{eh-di}-g(a+b\log \cdots}{(gh-fi)^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^2),x]
```

output
```
(((g*h - f*i)*(a + b*Log[c*(d + e*x)^n]))/(h + i*x) + g*(a + b*Log[c*(d + 
e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - (b*e*(g*h - f*i)*n*(Log[d + e*x]
 - Log[h + i*x]))/(e*h - d*i) - g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i
*x))/(e*h - d*i)] + b*g*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - b*g*n
*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)])/(g*h - f*i)^2
```

3.222.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^2}\,dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{g^2(a+b\log(c(d+ex)^n))}{(f+gx)(gh-fi)^2}-\frac{gi(a+b\log(c(d+ex)^n))}{(h+ix)(gh-fi)^2}-\frac{i(a+b\log(c(d+ex)^n))}{(h+ix)^2(gh-fi)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a+b\log\left(c(d+ex)^n\right)}{(h+ix)(gh-fi)}+\frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^2}-\frac{g\log\left(\frac{e(h+ix)}{eh-di}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^2}+\frac{bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}-\frac{bgn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}-\frac{ben\log(d+ex)}{(eh-di)(gh-fi)}+\frac{ben\log(h+ix)}{(eh-di)(gh-fi)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^2),x]
```

output
```
-((b*e*n*Log[d + e*x])/((e*h - d*i)*(g*h - f*i))) + (a + b*Log[c*(d + e*x)
^n])/((g*h - f*i)*(h + i*x)) + (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g
*x))/(e*f - d*g)])/(g*h - f*i)^2 + (b*e*n*Log[h + i*x])/((e*h - d*i)*(g*h
- f*i)) - (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)])/(g
*h - f*i)^2 + (b*g*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)
^2 - (b*g*n*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)^2
```

3.222.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.222.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{b\ln\left((ex+d)^n\right)}{(fi-gh)(ix+h)}-\frac{b\ln\left((ex+d)^n\right)g\ln(ix+h)}{(fi-gh)^2}+\frac{b\ln\left((ex+d)^n\right)g\ln(gx+f)}{(fi-gh)^2}-\frac{bng\operatorname{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{(fi-gh)^2}-\frac{bng\ln(gx+f)\ln\left(\right.}{(fi-}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x,method=_RETURNVERBOSE)
```

output
```
-b*ln((e*x+d)^n)/(f*i-g*h)/(i*x+h)-b*ln((e*x+d)^n)*g/(f*i-g*h)^2*ln(i*x+h)
+b*ln((e*x+d)^n)*g/(f*i-g*h)^2*ln(g*x+f)-b*n*g/(f*i-g*h)^2*dilog(((g*x+f)*
e+d*g-e*f)/(d*g-e*f))-b*n*g/(f*i-g*h)^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(
d*g-e*f))-b*e*n/(f*i-g*h)/(d*i-e*h)*ln(e*x+d)+b*e*n/(f*i-g*h)/(d*i-e*h)*ln
(i*x+h)+b*n*g/(f*i-g*h)^2*dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+b*n*g/(f*i-
g*h)^2*ln(i*x+h)*ln(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+(-1/2*I*b*Pi*csgn(I*c)*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d
)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(
I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/(f*i-g*h)/(i*x+h)-g/(f*i-g*h)^2*ln(i*x+h)+
g/(f*i-g*h)^2*ln(g*x+f))
```

3.222.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)
*x^2 + (g*h^2 + 2*f*h*i)*x), x)
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)/(i*x+h)**2,x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x, algorithm="maxima")
```

output
```
a*(g*log(g*x + f)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) - g*log(i*x + h)/(g^2*h^
2 - 2*f*g*h*i + f^2*i^2) + 1/(g*h^2 - f*h*i + (g*h*i - f*i^2)*x)) + b*inte
grate((log((e*x + d)^n) + log(c))/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x
^2 + (g*h^2 + 2*f*h*i)*x), x)
```

3.222.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*(i*x + h)^2), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{(f+g\,x)\,(h+i\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^2), x)
```

3.223 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^3}\,dx$

3.223.1 Optimal result

Integrand size = 29, antiderivative size = 402

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx = &-\frac{ben}{2(eh-di)(gh-fi)(h+ix)} \\ &-\frac{begn\log(d+ex)}{(eh-di)(gh-fi)^2}-\frac{be^2n\log(d+ex)}{2(eh-di)^2(gh-fi)} \\ &+\frac{a+b\log\left(c(d+ex)^n\right)}{2(gh-fi)(h+ix)^2}+\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^2(h+ix)} \\ &+\frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^3} \\ &+\frac{begn\log(h+ix)}{(eh-di)(gh-fi)^2}+\frac{be^2n\log(h+ix)}{2(eh-di)^2(gh-fi)} \\ &-\frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^3} \\ &+\frac{bg^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^3}-\frac{bg^2n\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^3}\end{aligned}$$

output
```
-1/2*b*e*n/(-d*i+e*h)/(-f*i+g*h)/(i*x+h)-b*e*g*n*ln(e*x+d)/(-d*i+e*h)/(-f*
i+g*h)^2-1/2*b*e^2*n*ln(e*x+d)/(-d*i+e*h)^2/(-f*i+g*h)+1/2*(a+b*ln(c*(e*x+
d)^n))/(-f*i+g*h)/(i*x+h)^2+g*(a+b*ln(c*(e*x+d)^n))/(-f*i+g*h)^2/(i*x+h)+g
^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^3+b*e*g*n*ln(
i*x+h)/(-d*i+e*h)/(-f*i+g*h)^2+1/2*b*e^2*n*ln(i*x+h)/(-d*i+e*h)^2/(-f*i+g*
h)-g^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)^3+b*g^2*n
*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^3-b*g^2*n*polylog(2,-i*(e*x+d
)/(-d*i+e*h))/(-f*i+g*h)^3
```

3.223.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^3}\,dx = \frac{\frac{(gh-fi)^2(a+b\log(c(d+ex)^n))}{(h+ix)^2}+\frac{2g(gh-fi)(a+b\log(c(d+ex)^n))}{h+ix}+2g^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)-\frac{2beg(gh-fi)n}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^3),x]
```

output
```
(((g*h - f*i)^2*(a + b*Log[c*(d + e*x)^n]))/(h + i*x)^2 + (2*g*(g*h - f*i)
*(a + b*Log[c*(d + e*x)^n]))/(h + i*x) + 2*g^2*(a + b*Log[c*(d + e*x)^n])*
Log[(e*(f + g*x))/(e*f - d*g)] - (2*b*e*g*(g*h - f*i)*n*(Log[d + e*x] - Lo
g[h + i*x]))/(e*h - d*i) - (b*e*(g*h - f*i)^2*n*(e*h - d*i + e*(h + i*x)*L
og[d + e*x] - e*(h + i*x)*Log[h + i*x]))/((e*h - d*i)^2*(h + i*x)) - 2*g^2
*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)] + 2*b*g^2*n*Pol
yLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*b*g^2*n*PolyLog[2, (i*(d + e*x))
/(-(e*h) + d*i)])/(2*(g*h - f*i)^3)
```

3.223.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx$$

$$\Big\downarrow \text{ 2865}$$

$$\int \left(\frac{g^3(a+b\log\left(c(d+ex)^n\right))}{(f+gx)(gh-fi)^3}-\frac{g^2i(a+b\log\left(c(d+ex)^n\right))}{(h+ix)(gh-fi)^3}-\frac{gi(a+b\log\left(c(d+ex)^n\right))}{(h+ix)^2(gh-fi)^2}-\frac{i(a+b\log\left(c(d+ex)^n\right.}{(h+ix)^3(gh-fi)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^2\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^3}-\frac{g^2\log\left(\frac{e(h+ix)}{eh-di}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^3}+\frac{g(a+b\log\left(c(d+ex)^n\right))}{(h+ix)(gh-fi)^2}+\frac{a+b\log\left(c(d+ex)^n\right)}{2(h+ix)^2(gh-fi)}-\frac{be^2n\log(d+ex)}{2(eh-di)^2(gh-fi)}+\frac{be^2n\log(h+ix)}{2(eh-di)^2(gh-fi)}+\frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^3}-\frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^3}-\frac{ben}{2(h+ix)(eh-di)(gh-fi)}-\frac{begn\log(d+ex)}{(eh-di)(gh-fi)^2}+\frac{begn\log(h+ix)}{(eh-di)(gh-fi)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^3),x]
```

output
```
-1/2*(b*e*n)/((e*h - d*i)*(g*h - f*i)*(h + i*x)) - (b*e*g*n*Log[d + e*x])/
((e*h - d*i)*(g*h - f*i)^2) - (b*e^2*n*Log[d + e*x])/(2*(e*h - d*i)^2*(g*h
 - f*i)) + (a + b*Log[c*(d + e*x)^n])/(2*(g*h - f*i)*(h + i*x)^2) + (g*(a
+ b*Log[c*(d + e*x)^n]))/((g*h - f*i)^2*(h + i*x)) + (g^2*(a + b*Log[c*(d
+ e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i)^3 + (b*e*g*n*Log[h
+ i*x])/((e*h - d*i)*(g*h - f*i)^2) + (b*e^2*n*Log[h + i*x])/(2*(e*h - d*i
)^2*(g*h - f*i)) - (g^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h
- d*i)])/(g*h - f*i)^3 + (b*g^2*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]
)/(g*h - f*i)^3 - (b*g^2*n*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/(g*h
- f*i)^3
```

3.223.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.223.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.17 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{b\ln((ex+d)^n)}{2(fi-gh)(ix+h)^2}+\frac{b\ln((ex+d)^n)g^2\ln(ix+h)}{(fi-gh)^3}+\frac{b\ln((ex+d)^n)g}{(fi-gh)^2(ix+h)}-\frac{b\ln((ex+d)^n)g^2\ln(gx+f)}{(fi-gh)^3}+\frac{ben\ln(ex+d)dgi}{(fi-gh)^2(di-eh)^2}+$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln((e*x+d)^n)/(f*i-g*h)/(i*x+h)^2+b*ln((e*x+d)^n)*g^2/(f*i-g*h)^3*l
n(i*x+h)+b*ln((e*x+d)^n)*g/(f*i-g*h)^2/(i*x+h)-b*ln((e*x+d)^n)*g^2/(f*i-g*
h)^3*ln(g*x+f)+b*e*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(e*x+d)*d*g*i+1/2*b*e^2*n/(
f*i-g*h)^2/(d*i-e*h)^2*ln(e*x+d)*f*i-3/2*b*e^2*n/(f*i-g*h)^2/(d*i-e*h)^2*l
n(e*x+d)*g*h-b*e*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(i*x+h)*d*g*i-1/2*b*e^2*n/(f*
i-g*h)^2/(d*i-e*h)^2*ln(i*x+h)*f*i+3/2*b*e^2*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(
i*x+h)*g*h-1/2*b*e*n/(f*i-g*h)^2/(d*i-e*h)/(i*x+h)*f*i+1/2*b*e*n/(f*i-g*h)
^2/(d*i-e*h)/(i*x+h)*g*h+b*n*g^2/(f*i-g*h)^3*dilog(((g*x+f)*e+d*g-e*f)/(d*
g-e*f))+b*n*g^2/(f*i-g*h)^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*
n*g^2/(f*i-g*h)^3*dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))-b*n*g^2/(f*i-g*h)^3
*ln(i*x+h)*ln(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2
+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e
*x+d)^n)^3+b*ln(c)+a)*(-1/2/(f*i-g*h)/(i*x+h)^2+g^2/(f*i-g*h)^3*ln(i*x+h)+
g/(f*i-g*h)^2/(i*x+h)-g^2/(f*i-g*h)^3*ln(g*x+f))
```

3.223.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*i^3*x^4 + f*h^3 + (3*g*h*i^2 + f*i^
3)*x^3 + 3*(g*h^2*i + f*h*i^2)*x^2 + (g*h^3 + 3*f*h^2*i)*x), x)
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)/(i*x+h)**3,x)
```

output
```
Timed out
```

3.223.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x, algorithm="maxima")
```

output
```
1/2*(2*g^2*log(g*x + f)/(g^3*h^3 - 3*f*g^2*h^2*i + 3*f^2*g*h*i^2 - f^3*i^3
) - 2*g^2*log(i*x + h)/(g^3*h^3 - 3*f*g^2*h^2*i + 3*f^2*g*h*i^2 - f^3*i^3)
 + (2*g*i*x + 3*g*h - f*i)/(g^2*h^4 - 2*f*g*h^3*i + f^2*h^2*i^2 + (g^2*h^2
*i^2 - 2*f*g*h*i^3 + f^2*i^4)*x^2 + 2*(g^2*h^3*i - 2*f*g*h^2*i^2 + f^2*h*i
^3)*x))*a + b*integrate((log((e*x + d)^n) + log(c))/(g*i^3*x^4 + f*h^3 + (
3*g*h*i^2 + f*i^3)*x^3 + 3*(g*h^2*i + f*h*i^2)*x^2 + (g*h^3 + 3*f*h^2*i)*x
), x)
```

3.223.8 Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)(ix+h)^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*(i*x + h)^3), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\left(f+g\,x\right)\,{\left(h+i\,x\right)}^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^3),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^3), x)
```

3.224 $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$

3.224.1 Optimal result

Integrand size = 31, antiderivative size = 469

$$\begin{aligned}
&\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx \\
&= -\frac{2abi(eh-di)nx}{eg}-\frac{2abi(gh-fi)nx}{g^2}+\frac{2b^2i(eh-di)n^2x}{eg}+\frac{2b^2i(gh-fi)n^2x}{g^2} \\
&\quad+\frac{b^2i^2n^2(d+ex)^2}{4e^2g}-\frac{2b^2i(eh-di)n(d+ex)\log\left(c(d+ex)^n\right)}{e^2g} \\
&\quad-\frac{2b^2i(gh-fi)n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g} \\
&\quad+\frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g} \\
&\quad+\frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}+\frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g} \\
&\quad+\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} \\
&\quad+\frac{2b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&\quad-\frac{2b^2(gh-fi)^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}
\end{aligned}$$

output
```
-2*a*b*i*(-d*i+e*h)*n*x/e/g-2*a*b*i*(-f*i+g*h)*n*x/g^2+2*b^2*i*(-d*i+e*h)*
n^2*x/e/g+2*b^2*i*(-f*i+g*h)*n^2*x/g^2+1/4*b^2*i^2*n^2*(e*x+d)^2/e^2/g-2*b
^2*i*(-d*i+e*h)*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g-2*b^2*i*(-f*i+g*h)*n*(e*x+
d)*ln(c*(e*x+d)^n)/e/g^2-1/2*b*i^2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2/g
+i*(-d*i+e*h)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g+i*(-f*i+g*h)*(e*x+d)*(
a+b*ln(c*(e*x+d)^n))^2/e/g^2+1/2*i^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2
/g+(-f*i+g*h)^2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g^3+2*b*(
-f*i+g*h)^2*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^3-2
*b^2*(-f*i+g*h)^2*n^2*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g^3
```

3.224.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.87

$$\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$$
$$= \frac{4e^2gi(2gh-fi)x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+2e^2g^2i^2x^2(a-bn\log(d+ex)+b\log(c(d+\cdots}{}$$

input
```
Integrate[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

output
```
(4*e^2*g*i*(2*g*h - f*i)*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2
 + 2*e^2*g^2*i^2*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 4*e
^2*(g*h - f*i)^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g
*x] + 8*b*e^2*g^2*h^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log
[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f
) + d*g)]) + 2*b*i^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(e*g*
(e*x*(4*f - g*x) + 2*d*(2*f + g*x)) - 2*Log[d + e*x]*(g*(d + e*x)*(2*e*f +
 d*g - e*g*x) - 2*e^2*f^2*Log[(e*(f + g*x))/(e*f - d*g)]) + 4*e^2*f^2*Poly
Log[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 16*b*e*g*h*i*n*(a - b*n*Log[d + e*
x] + b*Log[c*(d + e*x)^n])*(-(g*(d + e*x)*(-1 + Log[d + e*x])) + e*f*(Log[
d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f)
 + d*g)])) + 8*b^2*e*g*h*i*n^2*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d +
 e*x)*Log[d + e*x]^2) - e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)]
 + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3,
(g*(d + e*x))/(-(e*f) + d*g)])) - b^2*i^2*n^2*(4*e*f*g*(2*e*x - 2*(d + e*x
)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2) + g^2*(e*x*(6*d - e*x) + (-6*d^
2 - 4*d*e*x + 2*e^2*x^2)*Log[d + e*x] + 2*(d^2 - e^2*x^2)*Log[d + e*x]^2)
- 4*e^2*f^2*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x
]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-
(e*f) + d*g)])) + 4*b^2*e^2*g^2*h^2*n^2*(Log[d + e*x]^2*Log[(e*(f + g*x...
```

3.224.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{i(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^2}+\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^2(f+gx)}+\frac{i(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(d+ex)(eh-di)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g}-\frac{bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g}+\frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g}+\frac{2bn(gh-fi)^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^3}+\frac{(gh-fi)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^3}+\frac{i(d+ex)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}-\frac{2abinx(eh-di)}{eg}-\frac{2abinx(gh-fi)}{g^2}-\frac{2b^2in(d+ex)(eh-di)\log\left(c(d+ex)^n\right)}{e^2g}-\frac{2b^2in(d+ex)(gh-fi)\log\left(c(d+ex)^n\right)}{eg^2}+\frac{b^2i^2n^2(d+ex)^2}{4e^2g}-\frac{2b^2n^2(gh-fi)^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}+\frac{2b^2in^2x(eh-di)}{eg}+\frac{2b^2in^2x(gh-fi)}{g^2}$$

input
```
Int[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

output
```
(-2*a*b*i*(e*h - d*i)*n*x)/(e*g) - (2*a*b*i*(g*h - f*i)*n*x)/g^2 + (2*b^2*
i*(e*h - d*i)*n^2*x)/(e*g) + (2*b^2*i*(g*h - f*i)*n^2*x)/g^2 + (b^2*i^2*n^
2*(d + e*x)^2)/(4*e^2*g) - (2*b^2*i*(e*h - d*i)*n*(d + e*x)*Log[c*(d + e*x
)^n])/(e^2*g) - (2*b^2*i*(g*h - f*i)*n*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^
2) - (b*i^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g) + (i*(e*h
- d*i)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e^2*g) + (i*(g*h - f*i)*(d
 + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) + (i^2*(d + e*x)^2*(a + b*Lo
g[c*(d + e*x)^n])^2)/(2*e^2*g) + ((g*h - f*i)^2*(a + b*Log[c*(d + e*x)^n])
^2*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 + (2*b*(g*h - f*i)^2*n*(a + b*Log[c
*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^3 - (2*b^2*(g*h
 - f*i)^2*n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/g^3
```

3.224.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.224.4 Maple [F]

$$\int \frac{(ix+h)^2 (a+b\ln(c(ex+d)^n))^2}{gx+f}dx$$

input
```
int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)
```

output
```
int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)
```

3.224.5 Fricas [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas"
)
```

output
```
integral((a^2*i^2*x^2 + 2*a^2*h*i*x + a^2*h^2 + (b^2*i^2*x^2 + 2*b^2*h*i*x
 + b^2*h^2)*log((e*x + d)^n*c)^2 + 2*(a*b*i^2*x^2 + 2*a*b*h*i*x + a*b*h^2)
*log((e*x + d)^n*c))/(g*x + f), x)
```

3.224.6 Sympy [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2 (h+ix)^2}{f+gx}\,dx$$

input
```
integrate((i*x+h)**2*(a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2*(h + i*x)**2/(f + g*x), x)
```

3.224.7 Maxima [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima"
)
```

output
```
2*a^2*h*i*(x/g - f*log(g*x + f)/g^2) + 1/2*a^2*i^2*(2*f^2*log(g*x + f)/g^3
 + (g*x^2 - 2*f*x)/g^2) + a^2*h^2*log(g*x + f)/g + integrate((b^2*h^2*log(
c)^2 + 2*a*b*h^2*log(c) + (b^2*i^2*log(c)^2 + 2*a*b*i^2*log(c))*x^2 + (b^2
*i^2*x^2 + 2*b^2*h*i*x + b^2*h^2)*log((e*x + d)^n)^2 + 2*(b^2*h*i*log(c)^2
 + 2*a*b*h*i*log(c))*x + 2*(b^2*h^2*log(c) + a*b*h^2 + (b^2*i^2*log(c) + a
*b*i^2)*x^2 + 2*(b^2*h*i*log(c) + a*b*h*i)*x)*log((e*x + d)^n))/(g*x + f),
 x)
```

3.224.8 Giac [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)^2*(b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(h+i\,x)^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{f+g\,x}\,dx$$

input
```
int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x),x)
```

output
```
int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x), x)
```

3.225 $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$

3.225.1 Optimal result

Integrand size = 29, antiderivative size = 215

$$\begin{aligned}
\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx
&= -\frac{2abinx}{g}+\frac{2b^2in^2x}{g}-\frac{2b^2in(d+ex)\log\left(c(d+ex)^n\right)}{eg} \\
&\quad+\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg}+\frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} \\
&\quad+\frac{2b(gh-fi)n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2} \\
&\quad-\frac{2b^2(gh-fi)n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}
\end{aligned}$$

output
```
-2*a*b*i*n*x/g+2*b^2*i*n^2*x/g-2*b^2*i*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g+i*(e*
x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g+(-f*i+g*h)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(
g*x+f)/(-d*g+e*f))/g^2+2*b*(-f*i+g*h)*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-g
*(e*x+d)/(-d*g+e*f))/g^2-2*b^2*(-f*i+g*h)*n^2*polylog(3,-g*(e*x+d)/(-d*g+e
*f))/g^2
```

3.225.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 460 vs. $2(215) = 430$.

Time = 0.24 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.14

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx$$
$$= \frac{egix(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))^2+e(gh-fi)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2\log}{}$$

input
```
Integrate[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

output
```
(e*g*i*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + e*(g*h - f*i)*(
a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 2*b*e*g*h*n*
(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[(e*(f + g*
x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 2*b*i*n*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(-(g*(d + e*x)*(-1 + Log[d + e*
x])) + e*f*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d
 + e*x))/(-(e*f) + d*g)])) + b^2*i*n^2*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x
] + (d + e*x)*Log[d + e*x]^2) - e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f
 - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*Pol
yLog[3, (g*(d + e*x))/(-(e*f) + d*g)])) + b^2*e*g*h*n^2*(Log[d + e*x]^2*Lo
g[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(
e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*g^2)
```

3.225.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx$$
$$\Big\downarrow \; 2865$$

$$\int \left(\frac{(gh-fi)(a+b\log(c(d+ex)^n))^2}{g(f+gx)}+\frac{i(a+b\log(c(d+ex)^n))^2}{g}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bn(gh-fi)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{(gh-fi)\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{i(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}-\frac{2abinx}{g}-\frac{2b^2in(d+ex)\log(c(d+ex)^n)}{eg}-\frac{2b^2n^2(gh-fi)\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}+\frac{2b^2in^2x}{g}$$

input
```
Int[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

output
```
(-2*a*b*i*n*x)/g + (2*b^2*i*n^2*x)/g - (2*b^2*i*n*(d + e*x)*Log[c*(d + e*x
)^n])/(e*g) + (i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g) + ((g*h - f
*i)*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (2*
b*(g*h - f*i)*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f
 - d*g))])/g^2 - (2*b^2*(g*h - f*i)*n^2*PolyLog[3, -((g*(d + e*x))/(e*f -
d*g))])/g^2
```

3.225.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.225. $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$

3.225.4 Maple [F]

$$\int \frac{(ix+h)\left(a+b\ln\left(c(ex+d)^n\right)\right)^2}{gx+f}dx$$

input
```
int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)
```

output
```
int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)
```

3.225.5 Fricas [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((a^2*i*x + a^2*h + (b^2*i*x + b^2*h)*log((e*x + d)^n*c)^2 + 2*(a*
b*i*x + a*b*h)*log((e*x + d)^n*c))/(g*x + f), x)
```

3.225.6 Sympy [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2(h+ix)}{f+gx}\,dx$$

input
```
integrate((i*x+h)*(a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2*(h + i*x)/(f + g*x), x)
```

3.225.7 Maxima [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")
```

output
```
a^2*i*(x/g - f*log(g*x + f)/g^2) + a^2*h*log(g*x + f)/g + integrate((b^2*h
*log(c)^2 + 2*a*b*h*log(c) + (b^2*i*x + b^2*h)*log((e*x + d)^n)^2 + (b^2*i
*log(c)^2 + 2*a*b*i*log(c))*x + 2*(b^2*h*log(c) + a*b*h + (b^2*i*log(c) +
a*b*i)*x)*log((e*x + d)^n))/(g*x + f), x)
```

3.225.8 Giac [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)*(b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{(h+i\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}{f+g\,x}\,dx$$

input
```
int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x),x)
```

output
```
int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x), x)
```

3.226 $\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$

3.226.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output
```
(a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g+2*b*n*(a+b*ln(c*(e*x+d)
^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-2*b^2*n^2*polylog(3,-g*(e*x+d)/(-d
*g+e*f))/g
```

3.226.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx)+2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\log\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 2*b*n*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[(e*(f + g*x))/
(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^2*n^2*(Log[d
+ e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d
+ e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

3.226.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$$

$$\Big\downarrow \ 2843$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2ben\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \ 2881$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \ 2821$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}-\frac{2bn\left(bn\int\frac{\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right)}{g}$$

$$\Big\downarrow \ 7143$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g} - \frac{2bn\left(bn\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))\right)}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g - (2*b*n*(
-((a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]) + b
*n*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))]))/g
```

3.226.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.226.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.73 (sec) , antiderivative size = 737, normalized size of antiderivative = 6.64

method	result
risch	$\frac{b^2 \ln(g(ex+d)-dg+ef) \ln(ex+d)^2 n^2}{g} - \frac{2b^2 \ln(g(ex+d)-dg+ef) \ln((ex+d)^n) \ln(ex+d) n}{g} + \frac{b^2 \ln(g(ex+d)-dg+ef) \ln((ex+d)^n)^2}{g}$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^2*n^2-2*b^2*ln(g*(e*x+d)-d*g+e*f)/g*
ln((e*x+d)^n)*ln(e*x+d)*n+b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2+b^2*
n^2/g*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*b^2*n^2/g*ln(e*x+d)*polylog(
2,g*(e*x+d)/(d*g-e*f))-2*b^2*n^2/g*polylog(3,g*(e*x+d)/(d*g-e*f))-2*b^2*n^
2*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)+2*b^2*n*dilog((g*(e*x+
d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)-2*b^2*n^2*ln(e*x+d)^2*ln((g*(e*x+d
)-d*g+e*f)/(-d*g+e*f))/g+2*b^2*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*
f))/g*ln((e*x+d)^n)+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^
n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*
(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(ln((e*x+d)^n
)*ln(g*x+f)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln
(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e))+1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(
I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*
(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+
2*a)^2*ln(g*x+f)/g
```

3.226.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x
+ f), x)
```

3.226.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x), x)
```

3.226.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")
```

output
```
a^2*log(g*x + f)/g + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*
a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x + f), x)
```

3.226.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x), x)
```

3.227 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx$

3.227.1 Optimal result

Integrand size = 31, antiderivative size = 264

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

output
```
(a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)-(a+b*ln(c*(e*x
+d)^n))^2*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)+2*b*n*(a+b*ln(c*(e*x+d)^n))*
polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)-2*b*n*(a+b*ln(c*(e*x+d)^n))*po
lylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)-2*b^2*n^2*polylog(3,-g*(e*x+d)/(
-d*g+e*f))/(-f*i+g*h)+2*b^2*n^2*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h
)
```

3.227.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx)-(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(h-$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] - (a - b*n*L
og[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[h + i*x] + 2*b*n*(a - b*n*Log[d
+ e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*(Log[(e*(f + g*x))/(e*f - d*g
)] - Log[(e*(h + i*x))/(e*h - d*i)]) + PolyLog[2, (g*(d + e*x))/(-(e*f) +
d*g)] - PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]) + b^2*n^2*(Log[d + e*x]^
2*Log[(e*(f + g*x))/(e*f - d*g)] - Log[d + e*x]^2*Log[(e*(h + i*x))/(e*h -
 d*i)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*Log[d
 + e*x]*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)] - 2*PolyLog[3, (g*(d + e*
x))/(-(e*f) + d*g)] + 2*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)]))/(g*h -
f*i)
```

3.227.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx$$
$$\Big\downarrow \text{ 2865}$$
$$\int \left(\frac{g(a+b\log(c(d+ex)^n))^2}{(f+gx)(gh-fi)}-\frac{i(a+b\log(c(d+ex)^n))^2}{(h+ix)(gh-fi)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2bn \operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log\left(c(d+ex)^n\right))}{gh-fi} - \frac{2bn \operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log\left(c(d+ex)^n\right))}{gh-fi} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log\left(c(d+ex)^n\right))^2}{gh-fi} - \frac{\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log\left(c(d+ex)^n\right))^2}{gh-fi} - \frac{2b^2n^2 \operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} + \frac{2b^2n^2 \operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i)
- ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i
) + (2*b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*
g))])/(g*h - f*i) - (2*b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((i*(d +
 e*x))/(e*h - d*i))])/(g*h - f*i) - (2*b^2*n^2*PolyLog[3, -((g*(d + e*x))/
(e*f - d*g))])/(g*h - f*i) + (2*b^2*n^2*PolyLog[3, -((i*(d + e*x))/(e*h -
d*i))])/(g*h - f*i)
```

3.227.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.227.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.90 (sec) , antiderivative size = 1427, normalized size of antiderivative = 5.41

method	result	size
risch	Expression too large to display	1427

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x,method=_RETURNVERBOSE)
```

output
```
b^2/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)*ln(e*x+d)^2*n^2-2*b^2/(f*i-g*h)*ln(i*(
e*x+d)-d*i+e*h)*ln((e*x+d)^n)*ln(e*x+d)*n+b^2/(f*i-g*h)*ln(i*(e*x+d)-d*i+e
*h)*ln((e*x+d)^n)^2-b^2/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln(e*x+d)^2*n^2+2*
b^2/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)*ln(e*x+d)*n-b^2/(f*i-g*h
)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^2+b^2*n^2/(f*i-g*h)*ln(e*x+d)^2*ln(1
+i*(e*x+d)/(-d*i+e*h))+2*b^2*n^2/(f*i-g*h)*ln(e*x+d)*polylog(2,-i*(e*x+d)/
(-d*i+e*h))-2*b^2*n^2/(f*i-g*h)*polylog(3,-i*(e*x+d)/(-d*i+e*h))-b^2*n^2/(
f*i-g*h)*ln(e*x+d)^2*ln(1+g*(e*x+d)/(-d*g+e*f))-2*b^2*n^2/(f*i-g*h)*ln(e*x
+d)*polylog(2,-g*(e*x+d)/(-d*g+e*f))+2*b^2*n^2/(f*i-g*h)*polylog(3,-g*(e*x
+d)/(-d*g+e*f))-2*b^2*n^2/(f*i-g*h)*dilog((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*
ln(e*x+d)+2*b^2*n/(f*i-g*h)*dilog((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+
d)^n)-2*b^2*n^2/(f*i-g*h)*ln(e*x+d)^2*ln((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))+2
*b^2*n/(f*i-g*h)*ln(e*x+d)*ln((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n
)+2*b^2*n^2/(f*i-g*h)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln(e*x+d)-2*b^
2*n/(f*i-g*h)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)+2*b^2*n^
2/(f*i-g*h)*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))-2*b^2*n/(f*i-g*
h)*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)+(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*
x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(ln((e*x+d)^n)/(f*i-g*h)*ln(i*x+h)-ln((e*x...
```

3.227.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*i*
x^2 + f*h + (g*h + f*i)*x), x)
```

3.227.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)/(i*x+h),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/((f + g*x)*(h + i*x)), x)
```

3.227.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x, algorithm="maxima")
```

output
```
a^2*(log(g*x + f)/(g*h - f*i) - log(i*x + h)/(g*h - f*i)) + integrate((b^2
*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*l
og((e*x + d)^n))/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

3.227.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x + f)*(i*x + h)), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \int \frac{(a+b\ln(c(d+e\,x)^n))^2}{(f+g\,x)\,(h+i\,x)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)), x)
```

3.228 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx$

3.228.1 Optimal result

Integrand size = 31, antiderivative size = 427

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx &= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(eh-di)(gh-fi)(h+ix)} \\ &+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} \\ &+ \frac{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)} \\ &- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} \\ &+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} \\ &+ \frac{2b^2en^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} \\ &- \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} \\ &- \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} \\ &+ \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}\end{aligned}$$

output
```
-i*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/(-d*i+e*h)/(-f*i+g*h)/(i*x+h)+g*(a+b*ln
(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^2+2*b*e*n*(a+b*ln(c*(
e*x+d)^n))*ln(e*(i*x+h)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-g*(a+b*ln(c*(e*x
+d)^n))^2*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)^2+2*b*g*n*(a+b*ln(c*(e*x+d)^
n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2+2*b^2*e*n^2*polylog(2,-i
*(e*x+d)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-2*b*g*n*(a+b*ln(c*(e*x+d)^n))*p
olylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2-2*b^2*g*n^2*polylog(3,-g*(e*x
+d)/(-d*g+e*f))/(-f*i+g*h)^2+2*b^2*g*n^2*polylog(3,-i*(e*x+d)/(-d*i+e*h))/
(-f*i+g*h)^2
```

3.228.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.48

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \frac{(eh-di)(gh-fi)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2+g(eh-di)(h+ix)\left(a-bn\log(d+ex)+\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)^2),x]
```

output
```
((e*h - d*i)*(g*h - f*i)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 +
 g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*L
og[f + g*x] - g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d +
 e*x)^n])^2*Log[h + i*x] - 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)
^n])*((g*h - f*i)*(i*(d + e*x)*Log[d + e*x] - e*(h + i*x)*Log[h + i*x]) -
g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + Pol
yLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d +
e*x]*Log[(e*(h + i*x))/(e*h - d*i)] + PolyLog[2, (i*(d + e*x))/(-(e*h) + d
*i)])) - b^2*n^2*((g*h - f*i)*(Log[d + e*x]*(i*(d + e*x)*Log[d + e*x] - 2*
e*(h + i*x)*Log[(e*(h + i*x))/(e*h - d*i)]) - 2*e*(h + i*x)*PolyLog[2, (i*
(d + e*x))/(-(e*h) + d*i)]) - g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^2*Log[
(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*
f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(
h + i*x)*(Log[d + e*x]^2*Log[(e*(h + i*x))/(e*h - d*i)] + 2*Log[d + e*x]*P
olyLog[2, (i*(d + e*x))/(-(e*h) + d*i)] - 2*PolyLog[3, (i*(d + e*x))/(-(e*
h) + d*i)])))/((e*h - d*i)*(g*h - f*i)^2*(h + i*x))
```

3.228.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{g^2(a+b\log(c(d+ex)^n))^2}{(f+gx)(gh-fi)^2}-\frac{gi(a+b\log(c(d+ex)^n))^2}{(h+ix)(gh-fi)^2}-\frac{i(a+b\log(c(d+ex)^n))^2}{(h+ix)^2(gh-fi)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2}-\frac{2bgn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2}+\frac{2ben\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(eh-di)(gh-fi)}-\frac{i(d+ex)(a+b\log(c(d+ex)^n))^2}{(h+ix)(eh-di)(gh-fi)}+\frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2}-\frac{g\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2}+\frac{2b^2en^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)}-\frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}+\frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)^2),x]
```

output
```
-((i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/((e*h - d*i)*(g*h - f*i)*(h +
 i*x))) + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/
(g*h - f*i)^2 + (2*b*e*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h
 - d*i)])/((e*h - d*i)*(g*h - f*i)) - (g*(a + b*Log[c*(d + e*x)^n])^2*Log[
(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i)^2 + (2*b*g*n*(a + b*Log[c*(d + e*x
)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2 + (2*b^2*e*n
^2*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/((e*h - d*i)*(g*h - f*i)) - (
2*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))
])/(g*h - f*i)^2 - (2*b^2*g*n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/
(g*h - f*i)^2 + (2*b^2*g*n^2*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))])/(g*
h - f*i)^2
```

3.228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.228.4 Maple [F]

$$\int \frac{(a + b \ln (c(ex + d)^n))^2}{(gx + f)(ix + h)^2} dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x)
```

3.228.5 Fricas [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)(h + ix)^2} \, dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{(gx + f)(ix + h)^2} \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x, algorithm="fricas"
)
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*i^
2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (g*h^2 + 2*f*h*i)*x), x)
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)(h + ix)^2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)/(i*x+h)**2,x)
```

output
```
Timed out
```

3.228.7 Maxima [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx)(h + ix)^2} \, dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{(gx + f)(ix + h)^2} \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x, algorithm="maxima"
)
```

output
```
a^2*(g*log(g*x + f)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) - g*log(i*x + h)/(g^2*
h^2 - 2*f*g*h*i + f^2*i^2) + 1/(g*h^2 - f*h*i + (g*h*i - f*i^2)*x)) + inte
grate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c
) + a*b)*log((e*x + d)^n))/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (g
*h^2 + 2*f*h*i)*x), x)
```

3.228.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x + f)*(i*x + h)^2), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)\,(h+i\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)^2), x)
```

3.229 $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

3.229.1 Optimal result

Integrand size = 31, antiderivative size = 660

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$= \frac{6ab^2i(eh-di)n^2x}{eg} + \frac{6ab^2i(gh-fi)n^2x}{g^2} - \frac{6b^3i(eh-di)n^3x}{eg} - \frac{6b^3i(gh-fi)n^3x}{g^2}$$
$$- \frac{3b^3i^2n^3(d+ex)^2}{8e^2g} + \frac{6b^3i(eh-di)n^2(d+ex)\log(c(d+ex)^n)}{e^2g}$$
$$+ \frac{6b^3i(gh-fi)n^2(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{3b^2i^2n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2g}$$
$$- \frac{3bi(eh-di)n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g}$$
$$- \frac{3bi(gh-fi)n(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2}$$
$$- \frac{3bi^2n(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2g} + \frac{i(eh-di)(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2g}$$
$$+ \frac{i(gh-fi)(d+ex)(a+b\log(c(d+ex)^n))^3}{eg^2} + \frac{i^2(d+ex)^2(a+b\log(c(d+ex)^n))^3}{2e^2g}$$
$$+ \frac{(gh-fi)^2(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}$$
$$+ \frac{3b(gh-fi)^2n(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$- \frac{6b^2(gh-fi)^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$+ \frac{6b^3(gh-fi)^2n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

output
```
6*a*b^2*i*(-d*i+e*h)*n^2*x/e/g+6*a*b^2*i*(-f*i+g*h)*n^2*x/g^2-6*b^3*i*(-d*
i+e*h)*n^3*x/e/g-6*b^3*i*(-f*i+g*h)*n^3*x/g^2-3/8*b^3*i^2*n^3*(e*x+d)^2/e^
2/g+6*b^3*i*(-d*i+e*h)*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g+6*b^3*i*(-f*i+g*h
)*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+3/4*b^2*i^2*n^2*(e*x+d)^2*(a+b*ln(c*(e
*x+d)^n))/e^2/g-3*b*i*(-d*i+e*h)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g-3
*b*i*(-f*i+g*h)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g^2-3/4*b*i^2*n*(e*x+d
)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2/g+i*(-d*i+e*h)*(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^3/e^2/g+i*(-f*i+g*h)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e/g^2+1/2*i^2*(e*x
+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^2/g+(-f*i+g*h)^2*(a+b*ln(c*(e*x+d)^n))^3*l
n(e*(g*x+f)/(-d*g+e*f))/g^3+3*b*(-f*i+g*h)^2*n*(a+b*ln(c*(e*x+d)^n))^2*pol
ylog(2,-g*(e*x+d)/(-d*g+e*f))/g^3-6*b^2*(-f*i+g*h)^2*n^2*(a+b*ln(c*(e*x+d)
^n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g^3+6*b^3*(-f*i+g*h)^2*n^3*polylog(4
,-g*(e*x+d)/(-d*g+e*f))/g^3
```

3.229.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1521 vs. $2(660) = 1320$.

Time = 0.51 (sec) , antiderivative size = 1521, normalized size of antiderivative = 2.30

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

output
```
(8*e^2*g*i*(2*g*h - f*i)*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3
 + 4*e^2*g^2*i^2*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 + 8*e
^2*(g*h - f*i)^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g
*x] + 24*b*e^2*g^2*h^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(
Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(
e*f) + d*g)]) + 6*b*i^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*
(e*g*(e*x*(4*f - g*x) + 2*d*(2*f + g*x)) - 2*Log[d + e*x]*(g*(d + e*x)*(2*
e*f + d*g - e*g*x) - 2*e^2*f^2*Log[(e*(f + g*x))/(e*f - d*g)]) + 4*e^2*f^2
*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 48*b*e*g*h*i*n*(a - b*n*Log[d
 + e*x] + b*Log[c*(d + e*x)^n])^2*(-(g*(d + e*x)*(-1 + Log[d + e*x])) + e*
f*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/
(-(e*f) + d*g)])) + 48*b^2*e*g*h*i*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d
+ e*x)^n])*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2
) - e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*Po
lyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f
) + d*g)])) - 6*b^2*i^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*
(4*e*f*g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2) + g
^2*(e*x*(6*d - e*x) + (-6*d^2 - 4*d*e*x + 2*e^2*x^2)*Log[d + e*x] + 2*(d^2
 - e^2*x^2)*Log[d + e*x]^2) - 4*e^2*f^2*(Log[d + e*x]^2*Log[(e*(f + g*x))/
(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] ...
```

3.229.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx} \, dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{i(gh-fi)(a+b\log(c(d+ex)^n))^3}{g^2}+\frac{(gh-fi)^2(a+b\log(c(d+ex)^n))^3}{g^2(f+gx)}+\frac{i(h+ix)(a+b\log(c(d+ex)^n)}{g}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^2i^2n^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^2g}-\frac{6b^2n^2(gh-fi)^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^3}+\frac{6ab^2in^2x(eh-di)}{eg}+\frac{6ab^2in^2x(gh-fi)}{g^2}-\frac{3bin(d+ex)(eh-di)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g}+\frac{i(d+ex)(eh-di)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2g}-\frac{3bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2g}+\frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2g}+\frac{3bn(gh-fi)^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^3}+\frac{(gh-fi)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g^3}-\frac{3bin(d+ex)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}+\frac{i(d+ex)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2}+\frac{6b^3in^2(d+ex)(eh-di)\log\left(c(d+ex)^n\right)}{e^2g}+\frac{6b^3in^2(d+ex)(gh-fi)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{3b^3i^2n^3(d+ex)^2}{8e^2g}+\frac{6b^3n^3(gh-fi)^2\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}-\frac{6b^3in^3x(eh-di)}{eg}-\frac{6b^3in^3x(gh-fi)}{g^2}$$

input
```
Int[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

output
```
(6*a*b^2*i*(e*h - d*i)*n^2*x)/(e*g) + (6*a*b^2*i*(g*h - f*i)*n^2*x)/g^2 -
(6*b^3*i*(e*h - d*i)*n^3*x)/(e*g) - (6*b^3*i*(g*h - f*i)*n^3*x)/g^2 - (3*b
^3*i^2*n^3*(d + e*x)^2)/(8*e^2*g) + (6*b^3*i*(e*h - d*i)*n^2*(d + e*x)*Log
[c*(d + e*x)^n])/(e^2*g) + (6*b^3*i*(g*h - f*i)*n^2*(d + e*x)*Log[c*(d + e
*x)^n])/(e*g^2) + (3*b^2*i^2*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(
4*e^2*g) - (3*b*i*(e*h - d*i)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e
^2*g) - (3*b*i*(g*h - f*i)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^
2) - (3*b*i^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2*g) + (i*(
e*h - d*i)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(e^2*g) + (i*(g*h - f*i
)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(e*g^2) + (i^2*(d + e*x)^2*(a +
b*Log[c*(d + e*x)^n])^3)/(2*e^2*g) + ((g*h - f*i)^2*(a + b*Log[c*(d + e*x)
^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 + (3*b*(g*h - f*i)^2*n*(a + b*L
og[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^3 - (6*b^
2*(g*h - f*i)^2*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/
(e*f - d*g))])/g^3 + (6*b^3*(g*h - f*i)^2*n^3*PolyLog[4, -((g*(d + e*x))/(
e*f - d*g))])/g^3
```

3.229. $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

3.229.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.229.4 Maple [F]

$$\int \frac{(ix+h)^2 (a+b\ln(c(ex+d)^n))^3}{gx+f}dx$$

input
```
int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)
```

output
```
int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)
```

3.229.5 Fricas [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas"
)
```

output
```
integral((a^3*i^2*x^2 + 2*a^3*h*i*x + a^3*h^2 + (b^3*i^2*x^2 + 2*b^3*h*i*x
 + b^3*h^2)*log((e*x + d)^n*c)^3 + 3*(a*b^2*i^2*x^2 + 2*a*b^2*h*i*x + a*b^
2*h^2)*log((e*x + d)^n*c)^2 + 3*(a^2*b*i^2*x^2 + 2*a^2*b*h*i*x + a^2*b*h^2
)*log((e*x + d)^n*c))/(g*x + f), x)
```

3.229.6 Sympy [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3 (h+ix)^2}{f+gx}\,dx$$

input
```
integrate((i*x+h)**2*(a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3*(h + i*x)**2/(f + g*x), x)
```

3.229.7 Maxima [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima"
)
```

output
```
2*a^3*h*i*(x/g - f*log(g*x + f)/g^2) + 1/2*a^3*i^2*(2*f^2*log(g*x + f)/g^3
 + (g*x^2 - 2*f*x)/g^2) + a^3*h^2*log(g*x + f)/g + integrate((b^3*h^2*log(
c)^3 + 3*a*b^2*h^2*log(c)^2 + 3*a^2*b*h^2*log(c) + (b^3*i^2*x^2 + 2*b^3*h*
i*x + b^3*h^2)*log((e*x + d)^n)^3 + (b^3*i^2*log(c)^3 + 3*a*b^2*i^2*log(c)
^2 + 3*a^2*b*i^2*log(c))*x^2 + 3*(b^3*h^2*log(c) + a*b^2*h^2 + (b^3*i^2*lo
g(c) + a*b^2*i^2)*x^2 + 2*(b^3*h*i*log(c) + a*b^2*h*i)*x)*log((e*x + d)^n)
^2 + 2*(b^3*h*i*log(c)^3 + 3*a*b^2*h*i*log(c)^2 + 3*a^2*b*h*i*log(c))*x +
3*(b^3*h^2*log(c)^2 + 2*a*b^2*h^2*log(c) + a^2*b*h^2 + (b^3*i^2*log(c)^2 +
 2*a*b^2*i^2*log(c) + a^2*b*i^2)*x^2 + 2*(b^3*h*i*log(c)^2 + 2*a*b^2*h*i*l
og(c) + a^2*b*h*i)*x)*log((e*x + d)^n))/(g*x + f), x)
```

3.229.8 Giac [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

input
```
integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)^2*(b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{{\left(h+i\,x\right)}^2\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^3}{f+g\,x}\,dx$$

input
```
int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x),x)
```

output
```
int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x), x)
```

3.230 $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

3.230.1 Optimal result

Integrand size = 29, antiderivative size = 308

$$\begin{aligned}
&\int \frac{(h+ix)\,(a+b\log\left(c(d+ex)^n\right))^3}{f+gx}\,dx \\
&= \frac{6ab^2in^2x}{g} - \frac{6b^3in^3x}{g} + \frac{6b^3in^2(d+ex)\log\left(c(d+ex)^n\right)}{eg} \\
&\quad - \frac{3bin(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^2}{eg} + \frac{i(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^3}{eg} \\
&\quad + \frac{(gh-fi)\,(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} \\
&\quad + \frac{3b(gh-fi)n(a+b\log\left(c(d+ex)^n\right))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2} \\
&\quad - \frac{6b^2(gh-fi)n^2(a+b\log\left(c(d+ex)^n\right))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^2} \\
&\quad + \frac{6b^3(gh-fi)n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}
\end{aligned}$$

output
```
6*a*b^2*i*n^2*x/g-6*b^3*i*n^3*x/g+6*b^3*i*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g-
3*b*i*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g+i*(e*x+d)*(a+b*ln(c*(e*x+d)^n)
)^3/e/g+(-f*i+g*h)*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g^2+3*
b*(-f*i+g*h)*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^
2-6*b^2*(-f*i+g*h)*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-g*(e*x+d)/(-d*g+e*
f))/g^2+6*b^3*(-f*i+g*h)*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*f))/g^2
```

3.230.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 799 vs. $2(308) = 616$.

Time = 0.25 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.59

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx$$
$$= \frac{egix(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))^3+e(gh-fi)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^3\log(}{}$$

input
```
Integrate[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

output
```
(e*g*i*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 + e*(g*h - f*i)*(
a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] + 3*b*e*g*h*n*
(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*Log[(e*(f +
g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 3*b*i*n*(
a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(-(g*(d + e*x)*(-1 + Log[d
+ e*x])) + e*f*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (
g*(d + e*x))/(-(e*f) + d*g)])) + 3*b^2*i*n^2*(a - b*n*Log[d + e*x] + b*Log
[c*(d + e*x)^n])*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d +
e*x]^2) - e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e
*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/
(-(e*f) + d*g)])) + 6*b^2*e*g*h*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e
*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)])/2 + Log[d + e*x]*
PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - PolyLog[3, (g*(d + e*x))/(-(e*f
) + d*g)]) + b^3*e*g*h*n^3*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f - d*g)]
+ 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Log[d + e*
x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d + e*x))/(
-(e*f) + d*g)]) - b^3*i*n^3*(g*(6*e*x - 6*(d + e*x)*Log[d + e*x] + 3*(d +
e*x)*Log[d + e*x]^2 - (d + e*x)*Log[d + e*x]^3) + e*f*(Log[d + e*x]^3*Log[
(e*(f + g*x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(
e*f) + d*g)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] ...
```

3.230.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{(gh-fi)(a+b\log(c(d+ex)^n))^3}{g(f+gx)}+\frac{i(a+b\log(c(d+ex)^n))^3}{g}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^2n^2(gh-fi)\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{6ab^2in^2x}{g}+\frac{3bn(gh-fi)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{(gh-fi)\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g^2}-\frac{3bin(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}+\frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{eg}+\frac{6b^3in^2(d+ex)\log(c(d+ex)^n)}{eg}+\frac{6b^3n^3(gh-fi)\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{6b^3in^3x}{g}$$

input
```
Int[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

output
```
(6*a*b^2*i*n^2*x)/g - (6*b^3*i*n^3*x)/g + (6*b^3*i*n^2*(d + e*x)*Log[c*(d
+ e*x)^n])/(e*g) - (3*b*i*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g)
+ (i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(e*g) + ((g*h - f*i)*(a + b*L
og[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (3*b*(g*h - f*i
)*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])
/g^2 - (6*b^2*(g*h - f*i)*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(
d + e*x))/(e*f - d*g))])/g^2 + (6*b^3*(g*h - f*i)*n^3*PolyLog[4, -((g*(d +
 e*x))/(e*f - d*g))])/g^2
```

3.230.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.230.4 Maple [F]

$$\int \frac{(ix+h)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{3}}{gx+f}dx$$

input
```
int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)
```

output
```
int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)
```

3.230.5 Fricas [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{3}}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^{n}c\right)+a)^{3}}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")
```

output
```
integral((a^3*i*x + a^3*h + (b^3*i*x + b^3*h)*log((e*x + d)^n*c)^3 + 3*(a*
b^2*i*x + a*b^2*h)*log((e*x + d)^n*c)^2 + 3*(a^2*b*i*x + a^2*b*h)*log((e*x
 + d)^n*c))/(g*x + f), x)
```

3.230.6 Sympy [F]

$$\int \frac{(h+ix)\,(a+b\log{(c(d+ex)^n)})^3}{f+gx}\,dx=\int \frac{(a+b\log{(c(d+ex)^n)})^3\,(h+ix)}{f+gx}\,dx$$

input
```
integrate((i*x+h)*(a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3*(h + i*x)/(f + g*x), x)
```

3.230.7 Maxima [F]

$$\int \frac{(h+ix)\,(a+b\log{(c(d+ex)^n)})^3}{f+gx}\,dx=\int \frac{(ix+h)(b\log{((ex+d)^n c)}+a)^3}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")
```

output
```
a^3*i*(x/g - f*log(g*x + f)/g^2) + a^3*h*log(g*x + f)/g + integrate((b^3*h
*log(c)^3 + 3*a*b^2*h*log(c)^2 + 3*a^2*b*h*log(c) + (b^3*i*x + b^3*h)*log(
(e*x + d)^n)^3 + 3*(b^3*h*log(c) + a*b^2*h + (b^3*i*log(c) + a*b^2*i)*x)*l
og((e*x + d)^n)^2 + (b^3*i*log(c)^3 + 3*a*b^2*i*log(c)^2 + 3*a^2*b*i*log(c
))*x + 3*(b^3*h*log(c)^2 + 2*a*b^2*h*log(c) + a^2*b*h + (b^3*i*log(c)^2 +
2*a*b^2*i*log(c) + a^2*b*i)*x)*log((e*x + d)^n))/(g*x + f), x)
```

3.230.8 Giac [F]

$$\int \frac{(h+ix)\,(a+b\log{(c(d+ex)^n)})^3}{f+gx}\,dx=\int \frac{(ix+h)(b\log{((ex+d)^n c)}+a)^3}{gx+f}\,dx$$

input
```
integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
```

output
```
integrate((i*x + h)*(b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{(h+i\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^3}{f+g\,x}\,dx$$

input
```
int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x),x)
```

output
```
int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x), x)
```

3.231 $\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

3.231.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{6b^3n^3\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output
```
(a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g+3*b*n*(a+b*ln(c*(e*x+d)
^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-6*b^2*n^2*(a+b*ln(c*(e*x+d)^n))*
polylog(3,-g*(e*x+d)/(-d*g+e*f))/g+6*b^3*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*
f))/g
```

3.231.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(158) = 316.

Time = 0.07 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(f+gx)+3bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\left(\mathrm{lo}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] + 3*b*n*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*Log[(e*(f + g*x)
)/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 6*b^2*n^2*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*
x))/(e*f - d*g)])/2 + Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(Log[d + e*x]^3*Lo
g[(e*(f + g*x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(
-(e*f) + d*g)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] +
 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

3.231.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$\Big\downarrow \; 2843$$
$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3ben\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$
$$\Big\downarrow \; 2881$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\left(2bn\int\frac{(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2\right)}{g}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))-bn\int\frac{\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)\right)-\text{PolyLog}\left(2,-\frac{g}{e}\right.\cdots}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}-\frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))-bn\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)\right)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(\cdots}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g - (3*b*n*(
-((a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]) +
 2*b*n*((a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))
] - b*n*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])))/g
```

3.231. $\int\frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

3.231.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.231.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 1396, normalized size of antiderivative = 8.84

method	result	size
risch	Expression too large to display	1396

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
-b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^3*n^3+3*b^3*ln(g*(e*x+d)-d*g+e*f)/g
*ln((e*x+d)^n)*ln(e*x+d)^2*n^2-3*b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)
^2*ln(e*x+d)*n+b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^3-2*b^3*n^3/g*ln(
e*x+d)^3*ln(1-g*(e*x+d)/(d*g-e*f))-3*b^3*n^3/g*ln(e*x+d)^2*polylog(2,g*(e*
x+d)/(d*g-e*f))+6*b^3*n^3/g*polylog(4,g*(e*x+d)/(d*g-e*f))+3*b^3*n^3*dilog
((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)^2-6*b^3*n^2*dilog((g*(e*x+d)-
d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)*ln(e*x+d)+3*b^3*n*dilog((g*(e*x+d)-d*
g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)^2+3*b^3*n^3*ln(e*x+d)^3*ln((g*(e*x+d)-d
*g+e*f)/(-d*g+e*f))/g-6*b^3*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e
*f))/g*ln((e*x+d)^n)+3*b^3*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/
g*ln((e*x+d)^n)^2+3*b^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*
g-e*f))+6*b^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))
-6*b^3*n^2/g*ln((e*x+d)^n)*polylog(3,g*(e*x+d)/(d*g-e*f))+1/8*(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*
x+d)^n)^3*b+2*b*ln(c)+2*a)^3*ln(g*x+f)/g+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*
csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*cs
gn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*l
n(c)+2*a)*b^2*((ln((e*x+d)^n)-n*ln(e*x+d))^2*ln(g*(e*x+d)-d*g+e*f)/g+n^2/g
*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*n^2/g*ln(e*x+d)*polylog(2,g*(e...
```

3.231.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g*x + f), x)
```

3.231.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x), x)
```

3.231.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")
```

output
```
a^3*log(g*x + f)/g + integrate((b^3*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*
a*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^
2 + 3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log((e*x + d)^n))/(g*x + f),
 x)
```

3.231.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/(f + g*x), x)
```

3.232 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx$

3.232.1 Optimal result

Integrand size = 31, antiderivative size = 372

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx &= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} \\ &- \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} \\ &+ \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &- \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} \\ &- \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &+ \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} \\ &+ \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &- \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}\end{aligned}$$

output
```
(a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)-(a+b*ln(c*(e*x
+d)^n))^3*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)+3*b*n*(a+b*ln(c*(e*x+d)^n))^
2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)-3*b*n*(a+b*ln(c*(e*x+d)^n))^
2*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)-6*b^2*n^2*(a+b*ln(c*(e*x+d)^
n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)+6*b^2*n^2*(a+b*ln(c*(e*x+d
)^n))*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)+6*b^3*n^3*polylog(4,-g*(
e*x+d)/(-d*g+e*f))/(-f*i+g*h)-6*b^3*n^3*polylog(4,-i*(e*x+d)/(-d*i+e*h))/(
-f*i+g*h)
```

3.232.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(f+gx)-(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(h-$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] - (a - b*n*L
og[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[h + i*x] + 3*b*n*(a - b*n*Log[d
+ e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*(Log[(e*(f + g*x))/(e*f - d
*g)] - Log[(e*(h + i*x))/(e*h - d*i)]) + PolyLog[2, (g*(d + e*x))/(-(e*f)
+ d*g)] - PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]) + 6*b^2*n^2*(a - b*n*L
og[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*x))/(e*
f - d*g)])/2 - (Log[d + e*x]^2*Log[(e*(h + i*x))/(e*h - d*i)])/2 + Log[d +
 e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - Log[d + e*x]*PolyLog[2, (
i*(d + e*x))/(-(e*h) + d*i)] - PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] +
PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)]) + b^3*n^3*(Log[d + e*x]^3*Log[(e
*(f + g*x))/(e*f - d*g)] - Log[d + e*x]^3*Log[(e*(h + i*x))/(e*h - d*i)] +
 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 3*Log[d + e*x
]^2*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)] - 6*Log[d + e*x]*PolyLog[3, (
g*(d + e*x))/(-(e*f) + d*g)] + 6*Log[d + e*x]*PolyLog[3, (i*(d + e*x))/(-(
e*h) + d*i)] + 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)] - 6*PolyLog[4, (
i*(d + e*x))/(-(e*h) + d*i)]))/(g*h - f*i)
```

3.232.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{g(a+b\log(c(d+ex)^n))^3}{(f+gx)(gh-fi)}-\frac{i(a+b\log(c(d+ex)^n))^3}{(h+ix)(gh-fi)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{6b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{gh-fi}+\frac{6b^2n^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{gh-fi}+\frac{3bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{gh-fi}-\frac{3bn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{gh-fi}+\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{gh-fi}-\frac{\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^3}{gh-fi}+\frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi}-\frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i)
- ((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i
) + (3*b*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f -
d*g))])/(g*h - f*i) - (3*b*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((i*
(d + e*x))/(e*h - d*i))])/(g*h - f*i) - (6*b^2*n^2*(a + b*Log[c*(d + e*x)^
n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i) + (6*b^2*n^2*(a
+ b*Log[c*(d + e*x)^n])*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f
*i) + (6*b^3*n^3*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i) - (
6*b^3*n^3*PolyLog[4, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)
```

3.232.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.232.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.40 (sec) , antiderivative size = 2696, normalized size of antiderivative = 7.25

method	result	size
risch	Expression too large to display	2696

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x,method=_RETURNVERBOSE)
```

output
```
6*b^3*n^3/(f*i-g*h)*polylog(4,-i*(e*x+d)/(-d*i+e*h))-6*b^3*n^3/(f*i-g*h)*p
olylog(4,-g*(e*x+d)/(-d*g+e*f))+b^3/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)*ln((e*
x+d)^n)^3-b^3/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^3+1/8*(-I*b*Pi
*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(
e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c
*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^3*(1/(f*i-g*h)*ln(i*x+h)-1/(f*i-g*h)*ln(g*x
+f))+3/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csg
n(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^
2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2*b*(ln((e*x+d)^n)/(f*i-g*
h)*ln(i*x+h)-ln((e*x+d)^n)/(f*i-g*h)*ln(g*x+f)-e*n*(1/(f*i-g*h)*(dilog(((i
*x+h)*e+d*i-e*h)/(d*i-e*h))/e+ln(i*x+h)*ln(((i*x+h)*e+d*i-e*h)/(d*i-e*h))/
e)-1/(f*i-g*h)*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln(((g*x+
f)*e+d*g-e*f)/(d*g-e*f))/e)))-b^3/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)*ln(e*x+d
)^3*n^3+b^3/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln(e*x+d)^3*n^3+3*b^3*n^3/(f*i
-g*h)*dilog((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln(e*x+d)^2+3*b^3*n/(f*i-g*h)*
dilog((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n)^2+3*b^3*n^3/(f*i-g*h)*
ln(e*x+d)^3*ln((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))-3*b^3*n^3/(f*i-g*h)*dilog((
g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln(e*x+d)^2-3*b^3*n/(f*i-g*h)*dilog((g*(e*x
+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)^2-3*b^3*n^3/(f*i-g*h)*ln(e*x+d)^3*l
n((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))-6*b^3*n^2/(f*i-g*h)*polylog(3,-i*(e*x...
```

3.232.5 Fricas [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^3}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^3}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)/(i*x+h),x)
```

output
```
Timed out
```

3.232.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x, algorithm="maxima")
```

output
```
a^3*(log(g*x + f)/(g*h - f*i) - log(i*x + h)/(g*h - f*i)) + integrate((b^3
*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + 3
*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^2 + 3*(b^3*log(c)^2 + 2*a*b^2*log(c
) + a^2*b)*log((e*x + d)^n))/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

3.232.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3}{(gx+f)(ix+h)}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/((g*x + f)*(i*x + h)), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)\,(h+i\,x)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)), x)
```

3.233 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx$

3.233.1 Optimal result

Integrand size = 31, antiderivative size = 602

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx = -\frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} + \frac{3ben(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} + \frac{3bgn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} + \frac{6b^2en^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{3bgn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} - \frac{6b^2gn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{6b^3en^3\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} + \frac{6b^2gn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} + \frac{6b^3gn^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{6b^3gn^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

output
```
-i*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/(-d*i+e*h)/(-f*i+g*h)/(i*x+h)+g*(a+b*ln
(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^2+3*b*e*n*(a+b*ln(c*(
e*x+d)^n))^2*ln(e*(i*x+h)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-g*(a+b*ln(c*(e
*x+d)^n))^3*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)^2+3*b*g*n*(a+b*ln(c*(e*x+d
)^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2+6*b^2*e*n^2*(a+b*ln(
c*(e*x+d)^n))*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-3*b*g
*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2-6
*b^2*g*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*
h)^2-6*b^3*e*n^3*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)+6*
b^2*g*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h
)^2+6*b^3*g*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2-6*b^3*g*n^3*
polylog(4,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2
```

3.233.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.70

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx$$
$$=\frac{(eh-di)(gh-fi)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^3+g(eh-di)(h+ix)\left(a-bn\log(d+ex)+\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)^2),x]
```

output
```
((e*h - d*i)*(g*h - f*i)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 +
 g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*L
og[f + g*x] - g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d +
 e*x)^n])^3*Log[h + i*x] - 3*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)
^n])^2*((g*h - f*i)*(i*(d + e*x)*Log[d + e*x] - e*(h + i*x)*Log[h + i*x])
- g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + P
olyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d
+ e*x]*Log[(e*(h + i*x))/(e*h - d*i)] + PolyLog[2, (i*(d + e*x))/(-(e*h) +
 d*i)])) - 3*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((g*h -
 f*i)*(Log[d + e*x]*(i*(d + e*x)*Log[d + e*x] - 2*e*(h + i*x)*Log[(e*(h +
i*x))/(e*h - d*i)]) - 2*e*(h + i*x)*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i
)]) - g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g
)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3
, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^2
*Log[(e*(h + i*x))/(e*h - d*i)] + 2*Log[d + e*x]*PolyLog[2, (i*(d + e*x))/
(-(e*h) + d*i)] - 2*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)])) - b^3*n^3*(
(g*h - f*i)*(Log[d + e*x]^2*(i*(d + e*x)*Log[d + e*x] - 3*e*(h + i*x)*Log[
(e*(h + i*x))/(e*h - d*i)]) - 6*e*(h + i*x)*Log[d + e*x]*PolyLog[2, (i*(d
+ e*x))/(-(e*h) + d*i)] + 6*e*(h + i*x)*PolyLog[3, (i*(d + e*x))/(-(e*h) +
 d*i)]) - g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*...
```

3.233.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{g^2(a+b\log(c(d+ex)^n))^3}{(f+gx)(gh-fi)^2}-\frac{gi(a+b\log(c(d+ex)^n))^3}{(h+ix)(gh-fi)^2}-\frac{i(a+b\log(c(d+ex)^n))^3}{(h+ix)^2(gh-fi)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6b^2en^2\,\mathrm{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(eh-di)(gh-fi)}-\frac{6b^2gn^2\,\mathrm{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2}+\frac{6b^2gn^2\,\mathrm{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2}+\frac{3bgn\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2}-\frac{3bgn\,\mathrm{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2}+\frac{3ben\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{(eh-di)(gh-fi)}-\frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{(h+ix)(eh-di)(gh-fi)}+\frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{(gh-fi)^2}-\frac{g\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^3}{(gh-fi)^2}-\frac{6b^3en^3\,\mathrm{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)}+\frac{6b^3gn^3\,\mathrm{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}-\frac{6b^3gn^3\,\mathrm{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)^2),x]
```

output
```
-((i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/((e*h - d*i)*(g*h - f*i)*(h +
 i*x))) + (g*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/
(g*h - f*i)^2 + (3*b*e*n*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(h + i*x))/(e
*h - d*i)])/((e*h - d*i)*(g*h - f*i)) - (g*(a + b*Log[c*(d + e*x)^n])^3*Lo
g[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i)^2 + (3*b*g*n*(a + b*Log[c*(d + e
*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2 + (6*b^2
*e*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))]
)/((e*h - d*i)*(g*h - f*i)) - (3*b*g*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLo
g[2, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)^2 - (6*b^2*g*n^2*(a + b*Lo
g[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2
- (6*b^3*e*n^3*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))])/((e*h - d*i)*(g*h
 - f*i)) + (6*b^2*g*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((i*(d + e*
x))/(e*h - d*i))])/(g*h - f*i)^2 + (6*b^3*g*n^3*PolyLog[4, -((g*(d + e*x))
/(e*f - d*g))])/(g*h - f*i)^2 - (6*b^3*g*n^3*PolyLog[4, -((i*(d + e*x))/(e
*h - d*i))])/(g*h - f*i)^2
```

3.233. $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx$

3.233.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.233.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{3}}{\left(gx+f\right)\left(ix+h\right)^{2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x)
```

3.233.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{3}}{(gx+f)(ix+h)^{2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x, algorithm="fricas"
)
```

output
```
integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*
b*log((e*x + d)^n*c) + a^3)/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (
g*h^2 + 2*f*h*i)*x), x)
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d+ex)^n\right))^3}{(f+gx)(h+ix)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)/(i*x+h)**2,x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int \frac{(a+b\log\left(c(d+ex)^n\right))^3}{(f+gx)(h+ix)^2}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^3}{(gx+f)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x, algorithm="maxima"
)
```

output
```
a^3*(g*log(g*x + f)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) - g*log(i*x + h)/(g^2*
h^2 - 2*f*g*h*i + f^2*i^2) + 1/(g*h^2 - f*h*i + (g*h*i - f*i^2)*x)) + inte
grate((b^3*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*
log(c) + 3*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^2 + 3*(b^3*log(c)^2 + 2*a
*b^2*log(c) + a^2*b)*log((e*x + d)^n))/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i
^2)*x^2 + (g*h^2 + 2*f*h*i)*x), x)
```

3.233.8 Giac [F]

$$\int \frac{(a+b\log\left(c(d+ex)^n\right))^3}{(f+gx)(h+ix)^2}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^3}{(gx+f)(ix+h)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3/((g*x + f)*(i*x + h)^2), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)\,(h+i\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)^2), x)
```

3.234 $\int \frac{h+ix}{(f+gx)(a+b\log(c(d+ex)^n))} dx$

3.234.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)} dx = \frac{e^{-\frac{a}{bn}} i(d+ex)\left(c(d+ex)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{begn} + \frac{(gh-fi)\text{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}, x\right)}{g}$$

output
```
i*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e/exp(a/b/n)/g/n/((c*(e*x+d)^n)^
(1/n))+(-f*i+g*h)*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)/g
```

3.234.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)} dx = \int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)} dx$$

input
```
Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

3.234.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{h+ix}{(f+gx)(a+b\log(c(d+ex)^n))}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{gh-fi}{g(f+gx)(a+b\log(c(d+ex)^n))}+\frac{i}{g(a+b\log(c(d+ex)^n))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}dx}{g}+\frac{ie^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{begn}$$

input
```
Int[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.234.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.234.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{ix+h}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)}dx$$

input
```
int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.234.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^{n}c\right)+a)}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral((i*x + h)/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

3.234.6 Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{h+ix}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)(f+gx)}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral((h + i*x)/((a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

3.234.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate((i*x + h)/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.234.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((i*x + h)/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.234.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{h+i\,x}{(f+g\,x)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)}\,dx$$

input
```
int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))), x)
```

3.235 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))} dx$

3.235.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.235.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

3.235.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}dx$$
$$\Big\downarrow \ 2867$$
$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.235.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.235.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^n\right)\right)}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.235.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(1/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

3.235.6 Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

3.235.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.235.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

3.235.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))), x)
```

3.236 $\int \frac{1}{(f+gx)(h+ix)(a+b\log(c(d+ex)^n))}\,dx$

3.236.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \frac{g\mathrm{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))},x\right)}{gh-fi} - \frac{i\mathrm{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))},x\right)}{gh-fi}$$

output
```
g*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)-i*Unintegrabl
e(1/(i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)
```

3.236.2 Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

3.236.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{g}{(f+gx)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)}-\frac{i}{(h+ix)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{g\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}dx}{gh-fi}-\frac{i\int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))}dx}{gh-fi}$$

input
```
Int[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.236.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.236.4 Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)(ix+h)(a+b\ln(c(ex+d)^n))}dx$$

input
```
int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.236.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)(h+ix)(a+b\log(c(d+ex)^n))}dx = \int \frac{1}{(gx+f)(ix+h)(b\log((ex+d)^n c)+a)}dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(1/(a*g*i*x^2 + a*f*h + (a*g*h + a*f*i)*x + (b*g*i*x^2 + b*f*h + (
b*g*h + b*f*i)*x)*log((e*x + d)^n*c)), x)
```

3.236.6 Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{(f+gx)(h+ix)(a+b\log(c(d+ex)^n))}dx = \int \frac{1}{(a+b\log(c(d+ex)^n))(f+gx)(h+ix)}dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)*(h + i*x)), x)
```

3.236.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(i*x + h)*(b*log((e*x + d)^n*c) + a)), x)
```

3.236.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(i*x + h)*(b*log((e*x + d)^n*c) + a)), x)
```

3.236.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)\,\left(h+i\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))), x)
```

3.237 $\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))}\,dx$

3.237.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\begin{aligned}&\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx\\&=\frac{g^2\mathrm{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))},x\right)}{(gh-fi)^2}-\frac{i\mathrm{Int}\left(\frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))},x\right)}{gh-fi}\\&\quad-\frac{gi\mathrm{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))},x\right)}{(gh-fi)^2}\end{aligned}$$

output
```
g^2*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)^2-i*Uninteg
rable(1/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)-g*i*Unintegrable(1/(
i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)^2
```

3.237.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx\\&=\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx\end{aligned}$$

input
```
Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])), x]
```

3.237.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{g^2}{(f+gx)(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}-\frac{gi}{(h+ix)(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}-\frac{}{(h+ix)^2(gh-f}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{g^2\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}dx}{(gh-fi)^2}-\frac{gi\int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))}dx}{(gh-fi)^2}-\frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))}dx}{gh-fi}$$

input
```
Int[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
```

output
```
$Aborted
```

3.237.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.237.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(ix+h)^2\,(a+b\ln{(c\,(ex+d)^n)})}dx$$

input
```
int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

3.237.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.58

$$\int \frac{1}{(f+gx)(h+ix)^2\,(a+b\log{(c(d+ex)^n)})}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log{((ex+d)^n c)}+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas"
)
```

output
```
integral(1/(a*g*i^2*x^3 + a*f*h^2 + (2*a*g*h*i + a*f*i^2)*x^2 + (a*g*h^2 +
 2*a*f*h*i)*x + (b*g*i^2*x^3 + b*f*h^2 + (2*b*g*h*i + b*f*i^2)*x^2 + (b*g*
h^2 + 2*b*f*h*i)*x)*log((e*x + d)^n*c)), x)
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(i*x+h)**2/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Timed out
```

3.237.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima"
)
```

output
```
integrate(1/((g*x + f)*(i*x + h)^2*(b*log((e*x + d)^n*c) + a)), x)
```

3.237.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log((ex+d)^n c)+a)}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(i*x + h)^2*(b*log((e*x + d)^n*c) + a)), x)
```

3.237.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$
$$=\int \frac{1}{(f+g\,x)\,\left(h+i\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))),x)
```

output
```
int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))), x)
```

3.238 $\int \frac{h+ix}{(f+gx)(a+b\log(c(d+ex)^n))^2} dx$

3.238.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \frac{e^{-\frac{a}{bn}}i(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2egn^2} - \frac{i(d+ex)}{begn\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(gh-fi)\operatorname{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2},x\right)}{g}$$

output
```
i*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e/exp(a/b/n)/g/n^2/((c*(e*x+d)
^n)^(1/n))-i*(e*x+d)/b/e/g/n/(a+b*ln(c*(e*x+d)^n))+(-f*i+g*h)*Unintegrable
(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)/g
```

3.238.2 Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

input
```
Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

3.238.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{gh-fi}{g(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}+\frac{i}{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}dx}{g}+\frac{ie^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2egn^2}-\frac{i(d+ex)}{begn\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

input
```
Int[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
$Aborted
```

3.238.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.238.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{ix+h}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{2}}dx$$

input
```
int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.238.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^{n}c\right)+a)^{2}}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral((i*x + h)/(a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log((e*x + d)^n*c)
^2 + 2*(a*b*g*x + a*b*f)*log((e*x + d)^n*c)), x)
```

3.238.6 Sympy [N/A]

Not integrable

Time = 7.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}\,dx=\int \frac{h+ix}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{2}(f+gx)}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral((h + i*x)/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)), x)
```

3.238.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 7.69

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*i*x^2 + d*h + (e*h + d*i)*x)/(b^2*e*f*n*log(c) + a*b*e*f*n + (b^2*e*g*
n*log(c) + a*b*e*g*n)*x + (b^2*e*g*n*x + b^2*e*f*n)*log((e*x + d)^n)) + in
tegrate((e*g*i*x^2 + 2*e*f*i*x + e*f*h - (g*h - f*i)*d)/(b^2*e*f^2*n*log(c
) + a*b*e*f^2*n + (b^2*e*g^2*n*log(c) + a*b*e*g^2*n)*x^2 + 2*(b^2*e*f*g*n*
log(c) + a*b*e*f*g*n)*x + (b^2*e*g^2*n*x^2 + 2*b^2*e*f*g*n*x + b^2*e*f^2*n
)*log((e*x + d)^n)), x)
```

3.238.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate((i*x + h)/((g*x + f)*(b*log((e*x + d)^n*c) + a)^2), x)
```

3.238.9 Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{h+i\,x}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}\,dx$$

input
```
int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

3.238. $\int \frac{h+ix}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx$

3.239 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx$

3.239.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.239.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

3.239.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$
$$\Big\downarrow \; 2867$$
$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}dx$$

input
```
Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
$Aborted
```

3.239.3.1 Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

3.239.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.239.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log((e*x + d)^n*c)^2 + 2*(
a*b*g*x + a*b*f)*log((e*x + d)^n*c)), x)
```

3.239.6 Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)), x)
```

3.239.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 7.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
(e*f - d*g)*integrate(1/(b^2*e*f^2*n*log(c) + a*b*e*f^2*n + (b^2*e*g^2*n*l
og(c) + a*b*e*g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(c) + a*b*e*f*g*n)*x + (b^2*e
*g^2*n*x^2 + 2*b^2*e*f*g*n*x + b^2*e*f^2*n)*log((e*x + d)^n)), x) - (e*x +
 d)/(b^2*e*f*n*log(c) + a*b*e*f*n + (b^2*e*g*n*log(c) + a*b*e*g*n)*x + (b^
2*e*g*n*x + b^2*e*f*n)*log((e*x + d)^n))
```

3.239.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^2), x)
```

3.239.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

3.240 $\int \frac{1}{(f+gx)(h+ix)(a+b\log(c(d+ex)^n))^2}\,dx$

3.240.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \frac{g\text{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2},x\right)}{gh-fi}-\frac{i\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2},x\right)}{gh-fi}$$

output
```
g*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)-i*Unintegra
ble(1/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)
```

3.240.2 Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

3.240.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{g}{(f+gx)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}-\frac{i}{(h+ix)(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{g\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}dx}{gh-fi}-\frac{i\int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2}dx}{gh-fi}$$

input
```
Int[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
$Aborted
```

3.240.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.240.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(ix+h)\,(a+b\ln\left(c\,(ex+d)^{n}\right))^{2}}dx$$

input
```
int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.240.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.81

$$\int \frac{1}{(f+gx)(h+ix)\,(a+b\log\left(c(d+ex)^{n}\right))^{2}}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^{n}c\right)+a)^{2}}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas"
)
```

output
```
integral(1/(a^2*g*i*x^2 + a^2*f*h + (b^2*g*i*x^2 + b^2*f*h + (b^2*g*h + b^
2*f*i)*x)*log((e*x + d)^n*c)^2 + (a^2*g*h + a^2*f*i)*x + 2*(a*b*g*i*x^2 +
a*b*f*h + (a*b*g*h + a*b*f*i)*x)*log((e*x + d)^n*c)), x)
```

3.240.6 Sympy [N/A]

Not integrable

Time = 8.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{(f+gx)(h+ix)\,(a+b\log\left(c(d+ex)^{n}\right))^{2}}\,dx = \int \frac{1}{(a+b\log\left(c\,(d+ex)^{n}\right))^{2}\,(f+gx)\,(h+ix)}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)*(h + i*x)), x)
```

3.240.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 15.74

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima"
)
```

output
```
-(e*x + d)/(b^2*e*f*h*n*log(c) + a*b*e*f*h*n + (b^2*e*g*i*n*log(c) + a*b*e
*g*i*n)*x^2 + ((g*h*n + f*i*n)*b^2*e*log(c) + (g*h*n + f*i*n)*a*b*e)*x + (
b^2*e*g*i*n*x^2 + b^2*e*f*h*n + (g*h*n + f*i*n)*b^2*e*x)*log((e*x + d)^n))
 - integrate((e*g*i*x^2 + 2*d*g*i*x - e*f*h + (g*h + f*i)*d)/(b^2*e*f^2*h^
2*n*log(c) + a*b*e*f^2*h^2*n + (b^2*e*g^2*i^2*n*log(c) + a*b*e*g^2*i^2*n)*
x^4 + 2*((g^2*h*i*n + f*g*i^2*n)*b^2*e*log(c) + (g^2*h*i*n + f*g*i^2*n)*a*
b*e)*x^3 + ((g^2*h^2*n + 4*f*g*h*i*n + f^2*i^2*n)*b^2*e*log(c) + (g^2*h^2*
n + 4*f*g*h*i*n + f^2*i^2*n)*a*b*e)*x^2 + 2*((f*g*h^2*n + f^2*h*i*n)*b^2*e
*log(c) + (f*g*h^2*n + f^2*h*i*n)*a*b*e)*x + (b^2*e*g^2*i^2*n*x^4 + b^2*e*
f^2*h^2*n + 2*(g^2*h*i*n + f*g*i^2*n)*b^2*e*x^3 + (g^2*h^2*n + 4*f*g*h*i*n
 + f^2*i^2*n)*b^2*e*x^2 + 2*(f*g*h^2*n + f^2*h*i*n)*b^2*e*x)*log((e*x + d)
^n)), x)
```

3.240.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(i*x + h)*(b*log((e*x + d)^n*c) + a)^2), x)
```

3.240.9 Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,(h+i\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

3.241 $\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))^2}\,dx$

3.241.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \frac{g^2\text{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2},x\right)}{(gh-fi)^2}-\frac{i\text{Int}\left(\frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))^2},x\right)}{gh-fi}-\frac{gi\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2},x\right)}{(gh-fi)^2}$$

output
```
g^2*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)^2-i*Unint
egrable(1/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)-g*i*Unintegrable
(1/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)^2
```

3.241.2 Mathematica [N/A]

Not integrable

Time = 9.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2), x]
```

3.241.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(h+ix)^2 (a+b\log(c(d+ex)^n))^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{g^2}{(f+gx)(gh-fi)^2 (a+b\log(c(d+ex)^n))^2}-\frac{gi}{(h+ix)(gh-fi)^2 (a+b\log(c(d+ex)^n))^2}-\frac{}{(h+ix)^2(gh-}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}dx}{(gh-fi)^2}-\frac{gi\int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2}dx}{(gh-fi)^2}-\frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))^2}dx}{gh-fi}$$

input
```
Int[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
$Aborted
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.241.4 Maple [N/A]

Not integrable

Time = 122.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(ix+h)^2\,(a+b\ln(c\,(ex+d)^n))^2}dx$$

input
```
int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.241.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 6.55

$$\int \frac{1}{(f+gx)(h+ix)^2\,(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="frica
s")
```

output
```
integral(1/(a^2*g*i^2*x^3 + a^2*f*h^2 + (2*a^2*g*h*i + a^2*f*i^2)*x^2 + (b
^2*g*i^2*x^3 + b^2*f*h^2 + (2*b^2*g*h*i + b^2*f*i^2)*x^2 + (b^2*g*h^2 + 2*
b^2*f*h*i)*x)*log((e*x + d)^n*c)^2 + (a^2*g*h^2 + 2*a^2*f*h*i)*x + 2*(a*b*
g*i^2*x^3 + a*b*f*h^2 + (2*a*b*g*h*i + a*b*f*i^2)*x^2 + (a*b*g*h^2 + 2*a*b
*f*h*i)*x)*log((e*x + d)^n*c)), x)
```

3.241.6 Sympy [N/A]

Not integrable

Time = 42.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2\left(f+gx\right)\left(h+ix\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)**2/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)*(h + i*x)**2), x)
```

3.241.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 723, normalized size of antiderivative = 23.32

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxim
a")
```

output
```
-(e*x + d)/(b^2*e*f*h^2*n*log(c) + a*b*e*f*h^2*n + (b^2*e*g*i^2*n*log(c) +
 a*b*e*g*i^2*n)*x^3 + ((2*g*h*i*n + f*i^2*n)*b^2*e*log(c) + (2*g*h*i*n + f
*i^2*n)*a*b*e)*x^2 + ((g*h^2*n + 2*f*h*i*n)*b^2*e*log(c) + (g*h^2*n + 2*f*
h*i*n)*a*b*e)*x + (b^2*e*g*i^2*n*x^3 + b^2*e*f*h^2*n + (2*g*h*i*n + f*i^2*
n)*b^2*e*x^2 + (g*h^2*n + 2*f*h*i*n)*b^2*e*x)*log((e*x + d)^n)) - integrat
e((2*e*g*i*x^2 - e*f*h + (g*h + 2*f*i)*d + (e*f*i + 3*d*g*i)*x)/(b^2*e*f^2
*h^3*n*log(c) + a*b*e*f^2*h^3*n + (b^2*e*g^2*i^3*n*log(c) + a*b*e*g^2*i^3*
n)*x^5 + ((3*g^2*h*i^2*n + 2*f*g*i^3*n)*b^2*e*log(c) + (3*g^2*h*i^2*n + 2*
f*g*i^3*n)*a*b*e)*x^4 + ((3*g^2*h^2*i*n + 6*f*g*h*i^2*n + f^2*i^3*n)*b^2*e
*log(c) + (3*g^2*h^2*i*n + 6*f*g*h*i^2*n + f^2*i^3*n)*a*b*e)*x^3 + ((g^2*h
^3*n + 6*f*g*h^2*i*n + 3*f^2*h*i^2*n)*b^2*e*log(c) + (g^2*h^3*n + 6*f*g*h^
2*i*n + 3*f^2*h*i^2*n)*a*b*e)*x^2 + ((2*f*g*h^3*n + 3*f^2*h^2*i*n)*b^2*e*l
og(c) + (2*f*g*h^3*n + 3*f^2*h^2*i*n)*a*b*e)*x + (b^2*e*g^2*i^3*n*x^5 + b^
2*e*f^2*h^3*n + (3*g^2*h*i^2*n + 2*f*g*i^3*n)*b^2*e*x^4 + (3*g^2*h^2*i*n +
 6*f*g*h*i^2*n + f^2*i^3*n)*b^2*e*x^3 + (g^2*h^3*n + 6*f*g*h^2*i*n + 3*f^2
*h*i^2*n)*b^2*e*x^2 + (2*f*g*h^3*n + 3*f^2*h^2*i*n)*b^2*e*x)*log((e*x + d)
^n)), x)
```

3.241.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)*(i*x + h)^2*(b*log((e*x + d)^n*c) + a)^2), x)
```

3.241.9 Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$
$$=\int \frac{1}{(f+g\,x)\,\left(h+i\,x\right)^2\left(a+b\,\ln\left(c\,\left(d+e\,x\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2), x)
```

3.242 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx &= \frac{af^2x}{g^3}-\frac{bf^2nx}{g^3}-\frac{bdfnx}{2eg^2}-\frac{bd^2nx}{3e^2g}+\frac{bfnx^2}{4g^2}\\&+\frac{bdnx^2}{6eg}-\frac{bnx^3}{9g}+\frac{bd^2fn\log(d+ex)}{2e^2g^2}\\&+\frac{bd^3n\log(d+ex)}{3e^3g}+\frac{bf^2(d+ex)\log(c(d+ex)^n)}{eg^3}\\&-\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g}\\&-\frac{f^3(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}\\&-\frac{bf^3n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}\end{aligned}$$

output
```
a*f^2*x/g^3-b*f^2*n*x/g^3-1/2*b*d*f*n*x/e/g^2-1/3*b*d^2*n*x/e^2/g+1/4*b*f*
n*x^2/g^2+1/6*b*d*n*x^2/e/g-1/9*b*n*x^3/g+1/2*b*d^2*f*n*ln(e*x+d)/e^2/g^2+
1/3*b*d^3*n*ln(e*x+d)/e^3/g+b*f^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g^3-1/2*f*x^2*
(a+b*ln(c*(e*x+d)^n))/g^2+1/3*x^3*(a+b*ln(c*(e*x+d)^n))/g-f^3*(a+b*ln(c*(e
*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^4-b*f^3*n*polylog(2,-g*(e*x+d)/(-d*g+
e*f))/g^4
```

3.242.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$\int \frac{x^3(a+b\log (c(d+ex)^n))}{f+gx} \, dx$$
$$= \frac{6bd^2g^2(3ef+2dg)n\log(d+ex)+e\Big(gx(6ae^2(6f^2-3fgx+2g^2x^2)-bn(12d^2g^2-6deg(-3f+gx)+e^2(}{}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(6*b*d^2*g^2*(3*e*f + 2*d*g)*n*Log[d + e*x] + e*(g*x*(6*a*e^2*(6*f^2 - 3*f
*g*x + 2*g^2*x^2) - b*n*(12*d^2*g^2 - 6*d*e*g*(-3*f + g*x) + e^2*(36*f^2 -
 9*f*g*x + 4*g^2*x^2))) - 36*a*e^2*f^3*Log[(e*(f + g*x))/(e*f - d*g)] + 6*
b*e*Log[c*(d + e*x)^n]*(6*d*f^2*g + e*g*x*(6*f^2 - 3*f*g*x + 2*g^2*x^2) -
6*e*f^3*Log[(e*(f + g*x))/(e*f - d*g)])) - 36*b*e^3*f^3*n*PolyLog[2, (g*(d
 + e*x))/(-(e*f) + d*g)])/(36*e^3*g^4)
```

3.242.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log (c(d+ex)^n))}{f+gx} \, dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(-\frac{f^3(a+b\log (c(d+ex)^n))}{g^3(f+gx)}+\frac{f^2(a+b\log (c(d+ex)^n))}{g^3}-\frac{fx(a+b\log (c(d+ex)^n))}{g^2}+\frac{x^2(a+b\log (c(d+e}{g}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{f^3\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log\left(c(d+ex)^n\right))}{g^4}-\frac{fx^2(a+b\log\left(c(d+ex)^n\right))}{2g^2}+\frac{x^3(a+b\log\left(c(d+ex)^n\right))}{3g}+\frac{af^2x}{g^3}+\frac{bf^2(d+ex)\log\left(c(d+ex)^n\right)}{eg^3}+\frac{bd^3n\log(d+ex)}{3e^3g}+\frac{bd^2fn\log(d+ex)}{2e^2g^2}-\frac{bd^2nx}{3e^2g}-\frac{bf^3n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}-\frac{bdfnx}{2eg^2}+\frac{bdnx^2}{6eg}-\frac{bf^2nx}{g^3}+\frac{bfnx^2}{4g^2}-\frac{bnx^3}{9g}$$

input
```
Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*f^2*x)/g^3 - (b*f^2*n*x)/g^3 - (b*d*f*n*x)/(2*e*g^2) - (b*d^2*n*x)/(3*e
^2*g) + (b*f*n*x^2)/(4*g^2) + (b*d*n*x^2)/(6*e*g) - (b*n*x^3)/(9*g) + (b*d
^2*f*n*Log[d + e*x])/(2*e^2*g^2) + (b*d^3*n*Log[d + e*x])/(3*e^3*g) + (b*f
^2*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^3) - (f*x^2*(a + b*Log[c*(d + e*x)^n
]))/(2*g^2) + (x^3*(a + b*Log[c*(d + e*x)^n]))/(3*g) - (f^3*(a + b*Log[c*(
d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^4 - (b*f^3*n*PolyLog[2, -((
g*(d + e*x))/(e*f - d*g))])/g^4
```

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.242.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.80

method	result
risch	$\frac{b\ln((ex+d)^n)x^3}{3g}-\frac{b\ln((ex+d)^n)fx^2}{2g^2}+\frac{b\ln((ex+d)^n)xf^2}{g^3}-\frac{b\ln((ex+d)^n)f^3\ln(gx+f)}{g^4}-\frac{bnx^3}{9g}+\frac{bfnx^2}{4g^2}-\frac{bf^2nx}{g^3}-$ 4

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*ln((e*x+d)^n)/g*x^3-1/2*b*ln((e*x+d)^n)/g^2*f*x^2+b*ln((e*x+d)^n)/g^
3*x*f^2-b*ln((e*x+d)^n)*f^3/g^4*ln(g*x+f)-1/9*b*n*x^3/g+1/4*b*f*n*x^2/g^2-
b*f^2*n*x/g^3-49/36*b*n/g^4*f^3+1/6*b*d*n*x^2/e/g-1/2*b*d*f*n*x/e/g^2-2/3*
b/e*n/g^3*d*f^2-1/3*b*d^2*n*x/e^2/g-1/3*b/e^2*n/g^2*d^2*f+1/3*b/e^3*n/g*d^
3*ln((g*x+f)*e+d*g-e*f)+1/2*b/e^2*n/g^2*d^2*ln((g*x+f)*e+d*g-e*f)*f+b/e*n/
g^3*d*ln((g*x+f)*e+d*g-e*f)*f^2+b*n/g^4*f^3*dilog(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))+b*n/g^4*f^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*P
i*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csg
n(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*
I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^3*(1/3*g^2*x^3-1/2*f*g*x^2+f^
2*x)-f^3/g^4*ln(g*x+f))
```

3.242.5 Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g*x + f), x)
```

3.242.6 Sympy [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral(x**3*(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.242.7 Maxima [F]

$$\int \frac{x^3(a + b\log(c(d + ex)^n))}{f + gx} dx = \int \frac{(b\log((ex + d)^n c) + a)x^3}{gx + f} dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
-1/6*a*(6*f^3*log(g*x + f)/g^4 - (2*g^2*x^3 - 3*f*g*x^2 + 6*f^2*x)/g^3) +
b*integrate((x^3*log((e*x + d)^n) + x^3*log(c))/(g*x + f), x)
```

3.242.8 Giac [F]

$$\int \frac{x^3(a + b\log(c(d + ex)^n))}{f + gx} dx = \int \frac{(b\log((ex + d)^n c) + a)x^3}{gx + f} dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x + f), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\log(c(d + ex)^n))}{f + gx} dx = \int \frac{x^3 (a + b \ln(c (d + e x)^n))}{f + g x} dx$$

input
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

output
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

3.243 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

3.243.1 Optimal result

Integrand size = 25, antiderivative size = 181

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = -\frac{afx}{g^2}+\frac{bfnx}{g^2}+\frac{bdnx}{2eg}-\frac{bnx^2}{4g}-\frac{bd^2n\log(d+ex)}{2e^2g} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

output
```
-a*f*x/g^2+b*f*n*x/g^2+1/2*b*d*n*x/e/g-1/4*b*n*x^2/g-1/2*b*d^2*n*ln(e*x+d)
/e^2/g-b*f*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+1/2*x^2*(a+b*ln(c*(e*x+d)^n))/g+f
^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^3+b*f^2*n*polylog(2,-g
*(e*x+d)/(-d*g+e*f))/g^3
```

3.243.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = -\frac{afx}{g^2}+\frac{bfnx}{g^2}+\frac{bn\left(\frac{2dx}{e}-x^2-\frac{2d^2\log(d+ex)}{e^2}\right)}{4g} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{bf^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

input
```
Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
-((a*f*x)/g^2) + (b*f*n*x)/g^2 + (b*n*((2*d*x)/e - x^2 - (2*d^2*Log[d + e*
x])/e^2))/(4*g) - (b*f*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) + (x^2*(a + b
*Log[c*(d + e*x)^n]))/(2*g) + (f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f +
g*x))/(e*f - d*g)])/g^3 + (b*f^2*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))
])/g^3
```

3.243.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$

$$\Big\downarrow \; 2863$$

$$\int \left(\frac{f^2(a+b\log(c(d+ex)^n))}{g^2(f+gx)}-\frac{f(a+b\log(c(d+ex)^n))}{g^2}+\frac{x(a+b\log(c(d+ex)^n))}{g}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{f^2 \log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g}-\frac{afx}{g^2}-\frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}-\frac{bd^2n\log(d+ex)}{2e^2g}+\frac{bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}+\frac{bdnx}{2eg}+\frac{bfnx}{g^2}-\frac{bnx^2}{4g}$$

input
```
Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
-((a*f*x)/g^2) + (b*f*n*x)/g^2 + (b*d*n*x)/(2*e*g) - (b*n*x^2)/(4*g) - (b*
d^2*n*Log[d + e*x])/(2*e^2*g) - (b*f*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2)
 + (x^2*(a + b*Log[c*(d + e*x)^n]))/(2*g) + (f^2*(a + b*Log[c*(d + e*x)^n]
)*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 + (b*f^2*n*PolyLog[2, -((g*(d + e*x)
)/(e*f - d*g))])/g^3
```

3.243.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.243.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.14

method	result
risch	$\frac{b\ln((ex+d)^n)x^2}{2g}-\frac{b\ln((ex+d)^n)fx}{g^2}+\frac{b\ln((ex+d)^n)f^2\ln(gx+f)}{g^3}-\frac{bn\,f^2\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^3}-\frac{bn\,f^2\ln(gx+f)\ln\left(\frac{(gx+}{}\right.}{g^3}$

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)/g*x^2-b*ln((e*x+d)^n)/g^2*f*x+b*ln((e*x+d)^n)*f^2/g^3*
ln(g*x+f)-b*n/g^3*f^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n/g^3*f^2*ln(
g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-1/4*b*n*x^2/g+b*f*n*x/g^2+5/4*b*f
^2*n/g^3+1/2*b*d*n*x/e/g+1/2*b*d*f*n/e/g^2-1/2*b/e^2*n/g*d^2*ln((g*x+f)*e+
d*g-e*f)-b/e*n/g^2*d*ln((g*x+f)*e+d*g-e*f)*f+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2
+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e
*x+d)^n)^3+b*ln(c)+a)*(1/g^2*(1/2*g*x^2-f*x)+f^2/g^3*ln(g*x+f))
```

3.243.5 Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx=\int \frac{(b\log((ex+d)^n c)+a)x^2}{gx+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g*x + f), x)
```

3.243.6 Sympy [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx=\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.243.7 Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{gx+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
1/2*a*(2*f^2*log(g*x + f)/g^3 + (g*x^2 - 2*f*x)/g^2) + b*integrate((x^2*lo
g((e*x + d)^n) + x^2*log(c))/(g*x + f), x)
```

3.243.8 Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{gx+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x + f), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))}{f+g\,x}\,dx$$

input
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

output
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

3.244 $\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx$

3.244.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{ax}{g}-\frac{bnx}{g}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} - \frac{bfn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

output
```
a*x/g-b*n*x/g+b*(e*x+d)*ln(c*(e*x+d)^n)/e/g-f*(a+b*ln(c*(e*x+d)^n))*ln(e*(
g*x+f)/(-d*g+e*f))/g^2-b*f*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^2
```

3.244.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{agx-bgnx+\frac{bg(d+ex)\log(c(d+ex)^n)}{e}-f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)-bfn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g^2}$$

input
```
Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*g*x - b*g*n*x + (b*g*(d + e*x)*Log[c*(d + e*x)^n])/e - f*(a + b*Log[c*(
d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - b*f*n*PolyLog[2, (g*(d + e*x
))/(-(e*f) + d*g)])/g^2
```

3.244.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{a+b\log(c(d+ex)^n)}{g}-\frac{f(a+b\log(c(d+ex)^n))}{g(f+gx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{ax}{g}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg}-\frac{bfn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{bnx}{g}$$

input
```
Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

output
```
(a*x)/g - (b*n*x)/g + (b*(d + e*x)*Log[c*(d + e*x)^n])/(e*g) - (f*(a + b*L
og[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 - (b*f*n*PolyLog[2,
 -((g*(d + e*x))/(e*f - d*g))])/g^2
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.244.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.73

method	result
risch	$\frac{b\ln((ex+d)^n)x}{g} - \frac{b\ln((ex+d)^n)f\ln(gx+f)}{g^2} - \frac{bnx}{g} - \frac{bfn}{g^2} + \frac{bnd\ln((gx+f)e+dg-ef)}{eg} + \frac{bnf\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^2} + \frac{bn}{}$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/g*x-b*ln((e*x+d)^n)*f/g^2*ln(g*x+f)-b*n*x/g-b*f*n/g^2+b/e*
n/g*d*ln((g*x+f)*e+d*g-e*f)+b*n/g^2*f*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))
+b*n/g^2*f*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I
*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e
*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*c
sgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(x/g-f/g^2*ln(g*x+f))
```

3.244.5 Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*x*log((e*x + d)^n*c) + a*x)/(g*x + f), x)
```

3.244.6 Sympy [F]

$$\int \frac{x(a+b\log (c(d+ex)^n))}{f+gx}\,dx=\int \frac{x(a+b\log (c(d+ex)^n))}{f+gx}\,dx$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral(x*(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.244.7 Maxima [F]

$$\int \frac{x(a+b\log (c(d+ex)^n))}{f+gx}\,dx=\int \frac{(b\log ((ex+d)^nc)+a)x}{gx+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
a*(x/g - f*log(g*x + f)/g^2) + b*integrate((x*log((e*x + d)^n) + x*log(c))
/(g*x + f), x)
```

3.244.8 Giac [F]

$$\int \frac{x(a+b\log (c(d+ex)^n))}{f+gx}\,dx=\int \frac{(b\log ((ex+d)^nc)+a)x}{gx+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x/(g*x + f), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))}{f+g\,x}\,dx$$

input
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

output
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

3.245 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx$

3.245.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

output
```
(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g+b*n*polylog(2,-g*(e*x+d)/
(-d*g+e*f))/g
```

3.245.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyL
og[2, (g*(d + e*x))/(-(e*f) + d*g)])/g
```

3.245.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx$$

$$\Big\downarrow \; 2841$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \; 2840$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{bn\int \frac{\log\left(\frac{g(d+ex)}{ef-dg}+1\right)}{d+ex}d(d+ex)}{g}$$

$$\Big\downarrow \; 2838$$

$$\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyL
og[2, -((g*(d + e*x))/(e*f - d*g))])/g
```

3.245.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.245.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(gx+f)}{g} - \frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g} - \frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g} + \frac{\left(-\frac{ib\pi\,\mathrm{csgn}(ic(ex+d)^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}}{2}\right.}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)*ln(g*x+f)/g-b/g*n*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b/g
*n*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*
(e*x+d)^n)^3+b*ln(c)+a)*ln(g*x+f)/g
```

3.245.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.245.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

3.245.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(g*x + f), x) + a*log(g*x + f)/g
```

3.245.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{gx+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{f+g\,x}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x), x)
```

3.246 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)}\,dx$

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f} - \frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f} + \frac{bn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f}$$

output
```
ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f-(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*
g+e*f))/f-b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f+b*n*polylog(2,1+e*x/d)/f
```

3.246.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(\log\left(-\frac{ex}{d}\right)-\log\left(\frac{e(f+gx)}{ef-dg}\right)\right)-bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)+bn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*(Log[-((e*x)/d)] - Log[(e*(f + g*x))/(e*f - d*
g)]) - b*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] + b*n*PolyLog[2, 1 + (
e*x)/d])/f
```

3.246.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{x(f+gx)}\,dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{a+b\log \left(c(d+ex)^n\right)}{fx}-\frac{g\left(a+b\log \left(c(d+ex)^n\right)\right)}{f(f+gx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log \left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log \left(c(d+ex)^n\right)\right)}{f}+\frac{\log \left(-\frac{ex}{d}\right)\left(a+b\log \left(c(d+ex)^n\right)\right)}{f}-\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f}+\frac{bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)),x]
```

output
```
(Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f - ((a + b*Log[c*(d + e*x)^n
])*Log[(e*(f + g*x))/(e*f - d*g)])/f - (b*n*PolyLog[2, -((g*(d + e*x))/(e*
f - d*g))])/f + (b*n*PolyLog[2, 1 + (e*x)/d])/f
```

3.246.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.246.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.58

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(x)}{f}-\frac{b\ln((ex+d)^n)\ln(gx+f)}{f}-\frac{bn\,\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{f}-\frac{bn\ln(x)\ln\left(\frac{ex+d}{d}\right)}{f}+\frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{f}+$ bn ln

input
```
int((a+b*ln(c*(e*x+d)^n))/x/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/f*ln(x)-b*ln((e*x+d)^n)/f*ln(g*x+f)-b*n/f*dilog((e*x+d)/d)
-b*n/f*ln(x)*ln((e*x+d)/d)+b*n/f*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b*n/
f*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(
I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^
2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(
e*x+d)^n)^3+b*ln(c)+a)*(1/f*ln(x)-1/f*ln(g*x+f))
```

3.246.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^2 + f*x), x)
```

3.246.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x(f + gx)}\,dx = \int \frac{a + b\log\left(c(d + ex)^n\right)}{x\left(f + gx\right)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(x*(f + g*x)), x)
```

3.246.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x(f + gx)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx + f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f),x, algorithm="maxima")
```

output
```
-a*(log(g*x + f)/f - log(x)/f) + b*integrate((log((e*x + d)^n) + log(c))/(
g*x^2 + f*x), x)
```

3.246.8 Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x(f + gx)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx + f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*x), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,\left(f+g\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)), x)
```

3.247 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx)}\,dx$

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx = {} & \frac{ben\log(x)}{df}-\frac{ben\log(d+ex)}{df}-\frac{a+b\log\left(c(d+ex)^n\right)}{fx}\\ & -\frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}\\ & +\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2}\\ & +\frac{bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2}-\frac{bgn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}\end{aligned}$$

output
```
b*e*n*ln(x)/d/f-b*e*n*ln(e*x+d)/d/f+(-a-b*ln(c*(e*x+d)^n))/f/x-g*ln(-e*x/d
)*(a+b*ln(c*(e*x+d)^n))/f^2+g*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f
))/f^2+b*g*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f^2-b*g*n*polylog(2,1+e*x/d)
/f^2
```

3.247.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx$$
$$= \frac{\frac{befn(\log(x)-\log(d+ex))}{d} - \frac{f(a+b\log(c(d+ex)^n))}{x} - g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) + g(a+b\log\left(c(d+ex)^n\right))\,\mathrm{lo}}{f^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)),x]
```

output
```
((b*e*f*n*(Log[x] - Log[d + e*x]))/d - (f*(a + b*Log[c*(d + e*x)^n]))/x -
g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + g*(a + b*Log[c*(d + e*x)^n]
)*Log[(e*(f + g*x))/(e*f - d*g)] + b*g*n*PolyLog[2, (g*(d + e*x))/(-(e*f)
+ d*g)] - b*g*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

3.247.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx$$
$$\Big\downarrow \ 2863$$
$$\int \left(\frac{g^2(a+b\log\left(c(d+ex)^n\right))}{f^2(f+gx)} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{a+b\log\left(c(d+ex)^n\right)}{fx^2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{fx} + \frac{bgn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2} - \frac{bgn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2} + \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)),x]
```

output
```
(b*e*n*Log[x])/(d*f) - (b*e*n*Log[d + e*x])/(d*f) - (a + b*Log[c*(d + e*x)
^n])/(f*x) - (g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^2 + (g*(a +
b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^2 + (b*g*n*PolyLog
[2, -((g*(d + e*x))/(e*f - d*g))])/f^2 - (b*g*n*PolyLog[2, 1 + (e*x)/d])/f
^2
```

3.247.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.247.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.09

method	result
risch	$-\frac{b\ln\left((ex+d)^{n}\right)}{fx}-\frac{b\ln\left((ex+d)^{n}\right)g\ln(x)}{f^{2}}+\frac{b\ln\left((ex+d)^{n}\right)g\ln(gx+f)}{f^{2}}-\frac{bng\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{f^{2}}-\frac{bng\ln(gx+f)\ln\left(\frac{(gx+f}{d}\right.}{f^{2}}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
-b*ln((e*x+d)^n)/f/x-b*ln((e*x+d)^n)/f^2*g*ln(x)+b*ln((e*x+d)^n)/f^2*g*ln(
g*x+f)-b*n/f^2*g*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n/f^2*g*ln(g*x+f)*
ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*e*n*ln(e*x+d)/d/f+b*e*n*ln(x)/d/f+b*n/
f^2*g*dilog((e*x+d)/d)+b*n/f^2*g*ln(x)*ln((e*x+d)/d)+(-1/2*I*b*Pi*csgn(I*c
)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x
+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csg
n(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/f/x-1/f^2*g*ln(x)+1/f^2*g*ln(g*x+f))
```

3.247.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^3 + f*x^2), x)
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x+f),x)
```

output
```
Timed out
```

3.247.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f),x, algorithm="maxima")
```

output
```
a*(g*log(g*x + f)/f^2 - g*log(x)/f^2 - 1/(f*x)) + b*integrate((log((e*x +
d)^n) + log(c))/(g*x^3 + f*x^2), x)
```

3.247.8 Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^2(f + gx)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx + f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*x^2), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^2(f + gx)}\,dx = \int \frac{a + b\,\ln\left(c\,(d + e\,x)^n\right)}{x^2\,\left(f + g\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)), x)
```

3.248 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx)}\,dx$

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 250

$$\begin{aligned}\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx)}\,dx = &-\frac{ben}{2dfx}-\frac{be^2n\log(x)}{2d^2f}-\frac{begn\log(x)}{df^2}\\&+\frac{be^2n\log(d+ex)}{2d^2f}+\frac{begn\log(d+ex)}{df^2}\\&-\frac{a+b\log(c(d+ex)^n)}{2fx^2}+\frac{g(a+b\log(c(d+ex)^n))}{f^2x}\\&+\frac{g^2\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3}\\&-\frac{g^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3}\\&-\frac{bg^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3}+\frac{bg^2n\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3}\end{aligned}$$

output
```
-1/2*b*e*n/d/f/x-1/2*b*e^2*n*ln(x)/d^2/f-b*e*g*n*ln(x)/d/f^2+1/2*b*e^2*n*l
n(e*x+d)/d^2/f+b*e*g*n*ln(e*x+d)/d/f^2+1/2*(-a-b*ln(c*(e*x+d)^n))/f/x^2+g*
(a+b*ln(c*(e*x+d)^n))/f^2/x+g^2*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f^3-g^2*(
a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/f^3-b*g^2*n*polylog(2,-g*(e*
x+d)/(-d*g+e*f))/f^3+b*g^2*n*polylog(2,1+e*x/d)/f^3
```

3.248.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx =$$
$$-\frac{\frac{2befgn(\log(x)-\log(d+ex))}{d}+\frac{bef^2n(d+ex\log(x)-ex\log(d+ex))}{d^2x}+\frac{f^2(a+b\log(c(d+ex)^n))}{x^2}-\frac{2fg(a+b\log(c(d+ex)^n))}{x}-2g^2\log(\cdots}{\cdots}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)),x]
```

output
```
-1/2*((2*b*e*f*g*n*(Log[x] - Log[d + e*x]))/d + (b*e*f^2*n*(d + e*x*Log[x]
 - e*x*Log[d + e*x]))/(d^2*x) + (f^2*(a + b*Log[c*(d + e*x)^n]))/x^2 - (2*
f*g*(a + b*Log[c*(d + e*x)^n]))/x - 2*g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d
+ e*x)^n]) + 2*g^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g
)] + 2*b*g^2*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*b*g^2*n*PolyLo
g[2, 1 + (e*x)/d])/f^3
```

3.248.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(-\frac{g^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)}+\frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3x}-\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x^2}+\frac{a+b\log\left(c(d+ex)^n\right)}{fx^3}\cdots\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{g^2 \log \left(-\frac{ex}{d}\right) (a+b\log (c(d+ex)^n))}{f^3}-\frac{g^2 \log \left(\frac{e(f+gx)}{ef-dg}\right) (a+b\log (c(d+ex)^n))}{f^3}+ \frac{g(a+b\log (c(d+ex)^n))}{f^2x}-\frac{a+b\log (c(d+ex)^n)}{2fx^2}-\frac{be^2n\log(x)}{2d^2f}+\frac{be^2n\log(d+ex)}{2d^2f}- \frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3}+\frac{bg^2n\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3}-\frac{begn\log(x)}{df^2}+\frac{begn\log(d+ex)}{df^2}-\frac{ben}{2dfx}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)),x]
```

output
```
-1/2*(b*e*n)/(d*f*x) - (b*e^2*n*Log[x])/(2*d^2*f) - (b*e*g*n*Log[x])/(d*f^
2) + (b*e^2*n*Log[d + e*x])/(2*d^2*f) + (b*e*g*n*Log[d + e*x])/(d*f^2) - (
a + b*Log[c*(d + e*x)^n])/(2*f*x^2) + (g*(a + b*Log[c*(d + e*x)^n]))/(f^2*
x) + (g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^3 - (g^2*(a + b*Lo
g[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^3 - (b*g^2*n*PolyLog[2
, -((g*(d + e*x))/(e*f - d*g))])/f^3 + (b*g^2*n*PolyLog[2, 1 + (e*x)/d])/f
^3
```

3.248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.248.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{b\ln((ex+d)^n)}{2f\,x^2}+\frac{b\ln((ex+d)^n)g^2\ln(x)}{f^3}+\frac{b\ln((ex+d)^n)g}{f^2x}-\frac{b\ln((ex+d)^n)g^2\ln(gx+f)}{f^3}+\frac{begn\ln(ex+d)}{d\,f^2}+\frac{b\,e^2n\ln(ex+d)}{2d^2f}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln((e*x+d)^n)/f/x^2+b*ln((e*x+d)^n)/f^3*g^2*ln(x)+b*ln((e*x+d)^n)/f
^2*g/x-b*ln((e*x+d)^n)/f^3*g^2*ln(g*x+f)+b*e*g*n*ln(e*x+d)/d/f^2+1/2*b*e^2
*n*ln(e*x+d)/d^2/f-b*e*g*n*ln(x)/d/f^2-1/2*b*e^2*n*ln(x)/d^2/f-1/2*b*e*n/d
/f/x-b*n/f^3*g^2*dilog((e*x+d)/d)-b*n/f^3*g^2*ln(x)*ln((e*x+d)/d)+b*n/f^3*
g^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b*n/f^3*g^2*ln(g*x+f)*ln(((g*x+f)
*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(
e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*
(-1/2/f/x^2+1/f^3*g^2*ln(x)+1/f^2*g/x-1/f^3*g^2*ln(g*x+f))
```

3.248.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^4 + f*x^3), x)
```

3.248.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx\right)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(x**3*(f + g*x)), x)
```

3.248.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f),x, algorithm="maxima")
```

output
```
-1/2*a*(2*g^2*log(g*x + f)/f^3 - 2*g^2*log(x)/f^3 - (2*g*x - f)/(f^2*x^2))
 + b*integrate((log((e*x + d)^n) + log(c))/(g*x^4 + f*x^3), x)
```

3.248.8 Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*x^3), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx = \int \frac{a + b\,\ln\left(c\,(d+e\,x)^n\right)}{x^3\,\left(f+g\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)), x)
```

3.249 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 265

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = -\frac{2afx}{g^3}+\frac{2bfnx}{g^3}+\frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}-\frac{bd^2n\log(d+ex)}{2e^2g^2} - \frac{bef^3n\log(d+ex)}{g^4(ef-dg)}-\frac{2bf(d+ex)\log(c(d+ex)^n)}{eg^3} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)}+\frac{bef^3n\log(f+gx)}{g^4(ef-dg)} + \frac{3f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} + \frac{3bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}$$

output
```
-2*a*f*x/g^3+2*b*f*n*x/g^3+1/2*b*d*n*x/e/g^2-1/4*b*n*x^2/g^2-1/2*b*d^2*n*l
n(e*x+d)/e^2/g^2-b*e*f^3*n*ln(e*x+d)/g^4/(-d*g+e*f)-2*b*f*(e*x+d)*ln(c*(e*
x+d)^n)/e/g^3+1/2*x^2*(a+b*ln(c*(e*x+d)^n))/g^2+f^3*(a+b*ln(c*(e*x+d)^n))/
g^4/(g*x+f)+b*e*f^3*n*ln(g*x+f)/g^4/(-d*g+e*f)+3*f^2*(a+b*ln(c*(e*x+d)^n))
*ln(e*(g*x+f)/(-d*g+e*f))/g^4+3*b*f^2*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g
^4
```

3.249.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$
$$= \frac{-8afgx+8bfgnx-\frac{bg^2n(ex(-2d+ex)+2d^2\log(d+ex))}{e^2}-\frac{8bfg(d+ex)\log(c(d+ex)^n)}{e}+2g^2x^2(a+b\log(c(d+ex)^n))+}{}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

output
```
(-8*a*f*g*x + 8*b*f*g*n*x - (b*g^2*n*(e*x*(-2*d + e*x) + 2*d^2*Log[d + e*x
]))/e^2 - (8*b*f*g*(d + e*x)*Log[c*(d + e*x)^n])/e + 2*g^2*x^2*(a + b*Log[
c*(d + e*x)^n]) + (4*f^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) - (4*b*e*f^
3*n*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) + 12*f^2*(a + b*Log[c*(d +
e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + 12*b*f^2*n*PolyLog[2, (g*(d + e*
x))/(-(e*f) + d*g)])/(4*g^4)
```

3.249.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(-\frac{f^3(a+b\log(c(d+ex)^n))}{g^3(f+gx)^2}+\frac{3f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)}-\frac{2f(a+b\log(c(d+ex)^n))}{g^3}+\frac{x(a+b\log(c(d+e\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)}+\frac{3f^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^4}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{2afx}{g^3}-\frac{2bf(d+ex)\log(c(d+ex)^n)}{eg^3}-\frac{bd^2n\log(d+ex)}{2e^2g^2}-\frac{bef^3n\log(d+ex)}{g^4(ef-dg)}+\frac{bef^3n\log(f+gx)}{g^4(ef-dg)}+\frac{3bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}+\frac{bdnx}{2eg^2}+\frac{2bfnx}{g^3}-\frac{bnx^2}{4g^2}$$

input
```
Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

output
```
(-2*a*f*x)/g^3 + (2*b*f*n*x)/g^3 + (b*d*n*x)/(2*e*g^2) - (b*n*x^2)/(4*g^2)
 - (b*d^2*n*Log[d + e*x])/(2*e^2*g^2) - (b*e*f^3*n*Log[d + e*x])/(g^4*(e*f
 - d*g)) - (2*b*f*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^3) + (x^2*(a + b*Log[
c*(d + e*x)^n]))/(2*g^2) + (f^3*(a + b*Log[c*(d + e*x)^n]))/(g^4*(f + g*x)
) + (b*e*f^3*n*Log[f + g*x])/(g^4*(e*f - d*g)) + (3*f^2*(a + b*Log[c*(d +
e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^4 + (3*b*f^2*n*PolyLog[2, -((g*
(d + e*x))/(e*f - d*g))])/g^4
```

3.249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.249.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.08

method	result
risch	$\frac{b\ln((ex+d)^n)x^2}{2g^2}-\frac{2b\ln((ex+d)^n)xf}{g^3}+\frac{b\ln((ex+d)^n)f^3}{g^4(gx+f)}+\frac{3b\ln((ex+d)^n)f^2\ln(gx+f)}{g^4}-\frac{3bn\,f^2\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^4}-$

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)/g^2*x^2-2*b*ln((e*x+d)^n)/g^3*x*f+b*ln((e*x+d)^n)*f^3/
g^4/(g*x+f)+3*b*ln((e*x+d)^n)/g^4*f^2*ln(g*x+f)-3*b*n/g^4*f^2*dilog(((g*x+
f)*e+d*g-e*f)/(d*g-e*f))-3*b*n/g^4*f^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d
*g-e*f))-1/4*b*n*x^2/g^2+2*b*f*n*x/g^3+9/4*b*n/g^4*f^2+1/2*b*d*n*x/e/g^2+1
/2*b/e*n/g^3*d*f-b*e*n/g^4*f^3/(d*g-e*f)*ln(g*x+f)-1/2*b/e^2*n/g/(d*g-e*f)
*ln((g*x+f)*e+d*g-e*f)*d^3-3/2*b/e*n/g^2/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*d
^2*f+2*b*n/g^3/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*d*f^2+b*e*n/g^4/(d*g-e*f)*l
n((g*x+f)*e+d*g-e*f)*f^3+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a
)*(1/g^3*(1/2*g*x^2-2*f*x)+f^3/g^4/(g*x+f)+3/g^4*f^2*ln(g*x+f))
```

3.249.5 Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.249.6 Sympy [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

output
```
Integral(x**3*(a + b*log(c*(d + e*x)**n))/(f + g*x)**2, x)
```

3.249.7 Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
1/2*(2*f^3/(g^5*x + f*g^4) + 6*f^2*log(g*x + f)/g^4 + (g*x^2 - 4*f*x)/g^3)
*a + b*integrate((x^3*log((e*x + d)^n) + x^3*log(c))/(g^2*x^2 + 2*f*g*x +
f^2), x)
```

3.249.8 Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x + f)^2, x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))}{(f+g\,x)^2}\,dx$$

input
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2,x)
```

output
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2, x)
```

3.250 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$

3.250.1 Optimal result

Integrand size = 25, antiderivative size = 186

$$\begin{aligned}\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx &= \frac{ax}{g^2}-\frac{bnx}{g^2}+\frac{bef^2n\log(d+ex)}{g^3(ef-dg)}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg^2}\\ &\quad-\frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)}-\frac{bef^2n\log(f+gx)}{g^3(ef-dg)}\\ &\quad-\frac{2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}\\ &\quad-\frac{2bfn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}\end{aligned}$$

output
```
a*x/g^2-b*n*x/g^2+b*e*f^2*n*ln(e*x+d)/g^3/(-d*g+e*f)+b*(e*x+d)*ln(c*(e*x+d
)^n)/e/g^2-f^2*(a+b*ln(c*(e*x+d)^n))/g^3/(g*x+f)-b*e*f^2*n*ln(g*x+f)/g^3/(
-d*g+e*f)-2*f*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^3-2*b*f*n*p
olylog(2,-g*(e*x+d)/(-d*g+e*f))/g^3
```

3.250.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$
$$= \frac{agx-bgnx+\frac{bg(d+ex)\log(c(d+ex)^n)}{e}-\frac{f^2(a+b\log(c(d+ex)^n))}{f+gx}+\frac{bef^2n(\log(d+ex)-\log(f+gx))}{ef-dg}-2f(a+b\log(c(d+ex)^{}}{g^3}$$

input
```
Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

output
```
(a*g*x - b*g*n*x + (b*g*(d + e*x)*Log[c*(d + e*x)^n])/e - (f^2*(a + b*Log[
c*(d + e*x)^n]))/(f + g*x) + (b*e*f^2*n*(Log[d + e*x] - Log[f + g*x]))/(e*
f - d*g) - 2*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] -
 2*b*f*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/g^3
```

3.250.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$
$$\Big\downarrow \text{ 2863}$$
$$\int \left(\frac{f^2(a+b\log(c(d+ex)^n))}{g^2(f+gx)^2}-\frac{2f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{a+b\log(c(d+ex)^n)}{g^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)}-\frac{2f\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3}+\frac{ax}{g^2}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{bef^2n\log(d+ex)}{g^3(ef-dg)}-\frac{bef^2n\log(f+gx)}{g^3(ef-dg)}-\frac{2bfn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}-\frac{bnx}{g^2}$$

input
```
Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

output
```
(a*x)/g^2 - (b*n*x)/g^2 + (b*e*f^2*n*Log[d + e*x])/(g^3*(e*f - d*g)) + (b*
(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) - (f^2*(a + b*Log[c*(d + e*x)^n]))/(
g^3*(f + g*x)) - (b*e*f^2*n*Log[f + g*x])/(g^3*(e*f - d*g)) - (2*f*(a + b*
Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 - (2*b*f*n*PolyLog
[2, -((g*(d + e*x))/(e*f - d*g))])/g^3
```

3.250.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.250.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.34

method	result
risch	$\frac{b\ln((ex+d)^n)x}{g^2} - \frac{b\ln((ex+d)^n)f^2}{g^3(gx+f)} - \frac{2b\ln((ex+d)^n)f\ln(gx+f)}{g^3} - \frac{bnx}{g^2} - \frac{bfn}{g^3} + \frac{ben\,f^2\ln(gx+f)}{g^3(dg-ef)} + \frac{bn\ln((gx+f)e+dg-e}{eg(dg-ef)}$

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/g^2*x-b*ln((e*x+d)^n)/g^3*f^2/(g*x+f)-2*b*ln((e*x+d)^n)/g^
3*f*ln(g*x+f)-b*n*x/g^2-b*f*n/g^3+b*e*n/g^3*f^2/(d*g-e*f)*ln(g*x+f)+b/e*n/
g/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*d^2-b*n/g^2/(d*g-e*f)*ln((g*x+f)*e+d*g-e
*f)*d*f-b*e*n/g^3/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*f^2+2*b*n/g^3*f*dilog(((
g*x+f)*e+d*g-e*f)/(d*g-e*f))+2*b*n/g^3*f*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/
(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/
2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(x/g^2-1/g^3
*f^2/(g*x+f)-2/g^3*f*ln(g*x+f))
```

3.250.5 Fricas [F]

$$\int \frac{x^2(a+b\log{(c(d+ex)^n)})}{(f+gx)^2}\,dx=\int \frac{(b\log{((ex+d)^nc)}+a)x^2}{(gx+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.250.6 Sympy [F]

$$\int \frac{x^2(a+b\log{(c(d+ex)^n)})}{(f+gx)^2}\,dx=\int \frac{x^2(a+b\log{(c(d+ex)^n)})}{(f+gx)^2}\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e*x)**n))/(f + g*x)**2, x)
```

3.250.7 Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
-a*(f^2/(g^4*x + f*g^3) - x/g^2 + 2*f*log(g*x + f)/g^3) + b*integrate((x^2
*log((e*x + d)^n) + x^2*log(c))/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.250.8 Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x + f)^2, x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))}{(f+g\,x)^2}\,dx$$

input
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2,x)
```

output
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2, x)
```

3.251 $\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$

3.251.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = -\frac{befn\log(d+ex)}{g^2(ef-dg)}+\frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{befn\log(f+gx)}{g^2(ef-dg)}+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

output
```
-b*e*f*n*ln(e*x+d)/g^2/(-d*g+e*f)+f*(a+b*ln(c*(e*x+d)^n))/g^2/(g*x+f)+b*e*
f*n*ln(g*x+f)/g^2/(-d*g+e*f)+(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f)
)/g^2+b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^2
```

3.251.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \frac{\frac{f(a+b\log(c(d+ex)^n))}{f+gx}-\frac{befn(\log(d+ex)-\log(f+gx))}{ef-dg}+(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)+bn\operatorname{PolyLog}\left(2,\frac{g(d+ex}{-ef+d}\right.}{g^2}$$

input
```
Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

output
```
((f*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) - (b*e*f*n*(Log[d + e*x] - Log[f
 + g*x]))/(e*f - d*g) + (a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f
- d*g)] + b*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/g^2
```

3.251.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{a+b\log(c(d+ex)^n)}{g(f+gx)}-\frac{f(a+b\log(c(d+ex)^n))}{g(f+gx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{befn\log(d+ex)}{g^2(ef-dg)}+\frac{befn\log(f+gx)}{g^2(ef-dg)}$$

input
```
Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

output
```
-((b*e*f*n*Log[d + e*x])/(g^2*(e*f - d*g))) + (f*(a + b*Log[c*(d + e*x)^n]
))/(g^2*(f + g*x)) + (b*e*f*n*Log[f + g*x])/(g^2*(e*f - d*g)) + ((a + b*Lo
g[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (b*n*PolyLog[2, -(
(g*(d + e*x))/(e*f - d*g))])/g^2
```

3.251.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.251.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.25

method	result
risch	$\frac{b\ln\left((ex+d)^{n}\right)f}{g^{2}(gx+f)}+\frac{b\ln\left((ex+d)^{n}\right)\ln(gx+f)}{g^{2}}-\frac{bn\operatorname{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^{2}}-\frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^{2}}-\frac{benf\ln(gx+f}{g^{2}(dg-ef)}$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/g^2*f/(g*x+f)+b*ln((e*x+d)^n)/g^2*ln(g*x+f)-b*n/g^2*dilog(
((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d
*g-e*f))-b*e*n/g^2*f/(d*g-e*f)*ln(g*x+f)+b*e*n/g^2*f/(d*g-e*f)*ln((g*x+f)*
e+d*g-e*f)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/
2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^2*f/(g*
x+f)+1/g^2*ln(g*x+f))
```

3.251.5 Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^{n}))}{(f+gx)^{2}}\,dx=\int \frac{(b\log\left((ex+d)^{n}c\right)+a)x}{(gx+f)^{2}}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x*log((e*x + d)^n*c) + a*x)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

output
```
Timed out
```

3.251.7 Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{(gx+f)^2}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
a*(f/(g^3*x + f*g^2) + log(g*x + f)/g^2) + b*integrate((x*log((e*x + d)^n)
 + x*log(c))/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.251.8 Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{(gx+f)^2}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x/(g*x + f)^2, x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))}{(f+g\,x)^2}\,dx$$

input
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2,x)
```

output
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2, x)
```

3.252 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx$

3.252.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log(d+ex)}{g(ef-dg)} - \frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)} - \frac{ben\log(f+gx)}{g(ef-dg)}$$

output
```
b*e*n*ln(e*x+d)/g/(-d*g+e*f)+(-a-b*ln(c*(e*x+d)^n))/g/(g*x+f)-b*e*n*ln(g*x
+f)/g/(-d*g+e*f)
```

3.252.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{-\frac{a+b\log(c(d+ex)^n)}{f+gx} + \frac{ben(\log(d+ex)-\log(f+gx))}{ef-dg}}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

output
```
(-((a + b*Log[c*(d + e*x)^n])/(f + g*x)) + (b*e*n*(Log[d + e*x] - Log[f +
g*x]))/(e*f - d*g))/g
```

3.252.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2842, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{ben\int \frac{1}{(d+ex)(f+gx)}dx}{g}-\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}$$

$$\Big\downarrow \text{47}$$

$$\frac{ben\left(\frac{e\int \frac{1}{d+ex}dx}{ef-dg}-\frac{g\int \frac{1}{f+gx}dx}{ef-dg}\right)}{g}-\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{ben\left(\frac{\log(d+ex)}{ef-dg}-\frac{\log(f+gx)}{ef-dg}\right)}{g}-\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

output
```
-((a + b*Log[c*(d + e*x)^n])/(g*(f + g*x))) + (b*e*n*(Log[d + e*x]/(e*f -
d*g) - Log[f + g*x]/(e*f - d*g)))/g
```

3.252.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.252.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

method	result
parallelrisch	$-\frac{\ln(ex+d)xb\,e^2gn-\ln(gx+f)xb\,e^2gn+\ln(ex+d)b\,e^2fn-\ln(gx+f)b\,e^2fn+\ln(c(ex+d)^n)bdeg-\ln(c(ex+d)^n)b\,e^2f+adeg-a\,e^2}{(dg-ef)(gx+f)eg}$
risch	$-\frac{b\ln((ex+d)^n)}{g(gx+f)}-\frac{i\pi bef\ \text{csgn}(ic)\,\text{csgn}(i(ex+d)^n)\,\text{csgn}(ic(ex+d)^n)+i\pi bdg\ \text{csgn}(i(ex+d)^n)\,\text{csgn}(ic(ex+d)^n)^2-i\pi bef\ \text{csgn}(}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-(ln(e*x+d)*x*b*e^2*g*n-ln(g*x+f)*x*b*e^2*g*n+ln(e*x+d)*b*e^2*f*n-ln(g*x+f
)*b*e^2*f*n+ln(c*(e*x+d)^n)*b*d*e*g-ln(c*(e*x+d)^n)*b*e^2*f+a*d*e*g-a*e^2*
f)/(d*g-e*f)/(g*x+f)/e/g
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{aef-adg-(begnx+bdgn)\log\left(ex+d\right)+(begnx+befn)\log\left(gx+f\right)+(bef-bdg)\log\left(c\right)}{ef^2g-dfg^2+(efg^2-dg^3)x}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
-(a*e*f - a*d*g - (b*e*g*n*x + b*d*g*n)*log(e*x + d) + (b*e*g*n*x + b*e*f*
n)*log(g*x + f) + (b*e*f - b*d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 -
d*g^3)*x)
```

3.252.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(61) = 122.

Time = 3.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \begin{cases} \frac{ax+\frac{bd\log\left(c(d+ex)^n\right)}{e}-bnx+bx\log\left(c(d+ex)^n\right)}{f^2} \\ -\frac{a}{fg+g^2x}-\frac{bn}{fg+g^2x}-\frac{b\log\left(c\left(\frac{ef}{g}+ex\right)^n\right)}{fg+g^2x} \\ -\frac{adg}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{aef}{dfg^2+dg^3x-ef^2g-efg^2x}-\frac{bdg\log\left(c(d+ex)^n\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{befn\log\left(\frac{f}{g}+x\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{begnx\log\left(\frac{f}{g}\right.}{dfg^2+dg^3x-ef^2\ldots} \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

output
```
Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)*
*n))/f**2, Eq(g, 0)), (-a/(f*g + g**2*x) - b*n/(f*g + g**2*x) - b*log(c*(e
*f/g + e*x)**n)/(f*g + g**2*x), Eq(d, e*f/g)), (-a*d*g/(d*f*g**2 + d*g**3*
x - e*f**2*g - e*f*g**2*x) + a*e*f/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g
**2*x) - b*d*g*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g
**2*x) + b*e*f*n*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x
) + b*e*g*n*x*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x) -
 b*e*g*x*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x)
, True))
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = ben\left(\frac{\log\left(ex+d\right)}{efg-dg^2}-\frac{\log\left(gx+f\right)}{efg-dg^2}\right) - \frac{b\log\left((ex+d)^n c\right)}{g^2x+fg}-\frac{a}{g^2x+fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) - b*lo
g((e*x + d)^n*c)/(g^2*x + f*g) - a/(g^2*x + f*g)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log\left(ex+d\right)}{efg - dg^2} - \frac{ben\log\left(gx+f\right)}{efg - dg^2} - \frac{bn\log\left(ex+d\right)}{g^2x + fg} - \frac{b\log\left(c\right) + a}{g^2x + fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
b*e*n*log(e*x + d)/(e*f*g - d*g^2) - b*e*n*log(g*x + f)/(e*f*g - d*g^2) -
b*n*log(e*x + d)/(g^2*x + f*g) - (b*log(c) + a)/(g^2*x + f*g)
```

3.252.9 Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{a}{x\,g^2 + f\,g} - \frac{b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}{g\,\left(f + g\,x\right)} + \frac{b\,e\,n\,\mathrm{atan}\left(\frac{e\,f\,2{}\mathrm{i} + e\,g\,x\,2{}\mathrm{i}}{d\,g - e\,f} + 1{}\mathrm{i}\right)\,2{}\mathrm{i}}{g\,\left(d\,g - e\,f\right)}$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x)^2,x)
```

output
```
(b*e*n*atan((e*f*2i + e*g*x*2i)/(d*g - e*f) + 1i)*2i)/(g*(d*g - e*f)) - (b
*log(c*(d + e*x)^n))/(g*(f + g*x)) - a/(f*g + g^2*x)
```

3.253 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)^2}\,dx$

3.253.1 Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx=-\frac{ben\log(d+ex)}{f(ef-dg)}+\frac{a+b\log\left(c(d+ex)^n\right)}{f(f+gx)}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}+\frac{ben\log(f+gx)}{f(ef-dg)}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2}+\frac{bn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}$$

output
```
-b*e*n*ln(e*x+d)/f/(-d*g+e*f)+(a+b*ln(c*(e*x+d)^n))/f/(g*x+f)+ln(-e*x/d)*(
a+b*ln(c*(e*x+d)^n))/f^2+b*e*n*ln(g*x+f)/f/(-d*g+e*f)-(a+b*ln(c*(e*x+d)^n)
)*ln(e*(g*x+f)/(-d*g+e*f))/f^2-b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f^2+b*
n*polylog(2,1+e*x/d)/f^2
```

3.253.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx$$
$$=\frac{\frac{f(a+b\log(c(d+ex)^n))}{f+gx}+\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{befn(\log(d+ex)-\log(f+gx))}{ef-dg}-\left(a+b\log\left(c(d+ex)^n\right)\right)\log}{f^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)^2),x]
```

output
```
((f*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) + Log[-((e*x)/d)]*(a + b*Log[c*(
d + e*x)^n]) - (b*e*f*n*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) - (a +
b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - b*n*PolyLog[2, (g*(
d + e*x))/(-(e*f) + d*g)] + b*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

3.253.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(-\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2(f+gx)}+\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}-\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f(f+gx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}+\frac{a+b\log\left(c(d+ex)^n\right)}{f(f+gx)}-\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2}+\frac{bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2}-\frac{ben\log(d+ex)}{f(ef-dg)}+\frac{ben\log(f+gx)}{f(ef-dg)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)^2),x]
```

output
```
-((b*e*n*Log[d + e*x])/(f*(e*f - d*g))) + (a + b*Log[c*(d + e*x)^n])/(f*(f
 + g*x)) + (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^2 + (b*e*n*Log[f
 + g*x])/(f*(e*f - d*g)) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(
e*f - d*g)])/f^2 - (b*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/f^2 + (b
*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

3.253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.253.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.98

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(x)}{f^2} - \frac{b\ln((ex+d)^n)\ln(gx+f)}{f^2} + \frac{b\ln((ex+d)^n)}{f(gx+f)} - \frac{bn\,\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{f^2} - \frac{bn\ln(x)\ln\left(\frac{ex+d}{d}\right)}{f^2} + \frac{ben\ln(ex+d)}{f(dg-ef)} -$

input
```
int((a+b*ln(c*(e*x+d)^n))/x/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/f^2*ln(x)-b*ln((e*x+d)^n)/f^2*ln(g*x+f)+b*ln((e*x+d)^n)/f/
(g*x+f)-b*n/f^2*dilog((e*x+d)/d)-b*n/f^2*ln(x)*ln((e*x+d)/d)+b*e*n/f/(d*g-
e*f)*ln(e*x+d)-b*e*n/f/(d*g-e*f)*ln(g*x+f)+b*n/f^2*dilog(((g*x+f)*e+d*g-e*
f)/(d*g-e*f))+b*n/f^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*
b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*
csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1
/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/f^2*ln(x)-1/f^2*ln(g*x+f)+1/
f/(g*x+f))
```

3.253.5 Fricas [F]

$$\int \frac{a + b\log(c(d+ex)^n)}{x(f+gx)^2}\,dx = \int \frac{b\log((ex+d)^n c) + a}{(gx+f)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^3 + 2*f*g*x^2 + f^2*x), x)
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log(c(d+ex)^n)}{x(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x+f)**2,x)
```

output
```
Timed out
```

3.253.7 Maxima [F]

$$\int \frac{a + b\log(c(d+ex)^n)}{x(f+gx)^2}\,dx = \int \frac{b\log((ex+d)^n c) + a}{(gx+f)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f)^2,x, algorithm="maxima")
```

output
```
a*(1/(f*g*x + f^2) - log(g*x + f)/f^2 + log(x)/f^2) + b*integrate((log((e*
x + d)^n) + log(c))/(g^2*x^3 + 2*f*g*x^2 + f^2*x), x)
```

3.253.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)^2*x), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)^2), x)
```

3.254 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx)^2}\,dx$

3.254.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx = \frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}+\frac{begn\log(d+ex)}{f^2(ef-dg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2(f+gx)}-\frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}-\frac{begn\log(f+gx)}{f^2(ef-dg)}+\frac{2g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3}+\frac{2bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3}-\frac{2bgn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3}$$

output
```
b*e*n*ln(x)/d/f^2-b*e*n*ln(e*x+d)/d/f^2+b*e*g*n*ln(e*x+d)/f^2/(-d*g+e*f)+(
-a-b*ln(c*(e*x+d)^n))/f^2/x-g*(a+b*ln(c*(e*x+d)^n))/f^2/(g*x+f)-2*g*ln(-e*
x/d)*(a+b*ln(c*(e*x+d)^n))/f^3-b*e*g*n*ln(g*x+f)/f^2/(-d*g+e*f)+2*g*(a+b*l
n(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/f^3+2*b*g*n*polylog(2,-g*(e*x+d)/
(-d*g+e*f))/f^3-2*b*g*n*polylog(2,1+e*x/d)/f^3
```

3.254.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx$$
$$= \frac{\frac{befn(\log(x)-\log(d+ex))}{d} - \frac{f(a+b\log(c(d+ex)^n))}{x} - \frac{fg(a+b\log(c(d+ex)^n))}{f+gx} - 2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) + \frac{befgn}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)^2),x]
```

output
```
((b*e*f*n*(Log[x] - Log[d + e*x]))/d - (f*(a + b*Log[c*(d + e*x)^n]))/x -
(f*g*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) - 2*g*Log[-((e*x)/d)]*(a + b*Lo
g[c*(d + e*x)^n]) + (b*e*f*g*n*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g)
+ 2*g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + 2*b*g*n*
PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*b*g*n*PolyLog[2, 1 + (e*x)/d]
)/f^3
```

3.254.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx$$
$$\Big\downarrow \; 2863$$
$$\int \left(\frac{2g^2(a+b\log(c(d+ex)^n))}{f^3(f+gx)} - \frac{2g(a+b\log(c(d+ex)^n))}{f^3x} + \frac{g^2(a+b\log(c(d+ex)^n))}{f^2(f+gx)^2} + \frac{a+b\log(c(d+ex)^n)}{f^2x^2}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}+\frac{2g\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2(f+gx)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}+\frac{2bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3}-\frac{2bgn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3}+\frac{begn\log(d+ex)}{f^2(ef-dg)}-\frac{begn\log(f+gx)}{f^2(ef-dg)}+\frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)^2),x]
```

output
```
(b*e*n*Log[x])/(d*f^2) - (b*e*n*Log[d + e*x])/(d*f^2) + (b*e*g*n*Log[d + e
*x])/(f^2*(e*f - d*g)) - (a + b*Log[c*(d + e*x)^n])/(f^2*x) - (g*(a + b*Lo
g[c*(d + e*x)^n]))/(f^2*(f + g*x)) - (2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d
+ e*x)^n]))/f^3 - (b*e*g*n*Log[f + g*x])/(f^2*(e*f - d*g)) + (2*g*(a + b*L
og[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^3 + (2*b*g*n*PolyLog[
2, -((g*(d + e*x))/(e*f - d*g))])/f^3 - (2*b*g*n*PolyLog[2, 1 + (e*x)/d])/
f^3
```

3.254.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.254.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{b\ln\left((ex+d)^n\right)}{f^2x}-\frac{2b\ln\left((ex+d)^n\right)g\ln(x)}{f^3}-\frac{b\ln\left((ex+d)^n\right)g}{f^2(gx+f)}+\frac{2b\ln\left((ex+d)^n\right)g\ln(gx+f)}{f^3}+\frac{2bng\operatorname{dilog}\left(\frac{ex+d}{d}\right)}{f^3}+\frac{2bng\ln(x)}{f}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-b*ln((e*x+d)^n)/f^2/x-2*b*ln((e*x+d)^n)/f^3*g*ln(x)-b*ln((e*x+d)^n)/f^2*g
/(g*x+f)+2*b*ln((e*x+d)^n)/f^3*g*ln(g*x+f)+2*b*n/f^3*g*dilog((e*x+d)/d)+2*
b*n/f^3*g*ln(x)*ln((e*x+d)/d)-2*b*n/f^3*g*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e
*f))-2*b*n/f^3*g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-2*b*e*n/f^2/(
d*g-e*f)*ln(e*x+d)*g+b*e^2*n/f/(d*g-e*f)/d*ln(e*x+d)+b*e*n*ln(x)/d/f^2+b*e
*n/f^2*g/(d*g-e*f)*ln(g*x+f)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(
c)+a)*(-1/f^2/x-2/f^3*g*ln(x)-1/f^2*g/(g*x+f)+2/f^3*g*ln(g*x+f))
```

3.254.5 Fricas [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^4 + 2*f*g*x^3 + f^2*x^2), x)
```

3.254.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx = \int \frac{a + b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx\right)^2}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x+f)**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(x**2*(f + g*x)**2), x)
```

3.254.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
-a*((2*g*x + f)/(f^2*g*x^2 + f^3*x) - 2*g*log(g*x + f)/f^3 + 2*g*log(x)/f^
3) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^4 + 2*f*g*x^3 + f^2*x^
2), x)
```

3.254.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)^2*x^2), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x^2\,(f+g\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)^2), x)
```

3.255 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx)^2}\,dx$

3.255.1 Optimal result

Integrand size = 25, antiderivative size = 335

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx =& -\frac{ben}{2df^2x}-\frac{be^2n\log(x)}{2d^2f^2}-\frac{2begn\log(x)}{df^3}\\&+\frac{be^2n\log(d+ex)}{2d^2f^2}+\frac{2begn\log(d+ex)}{df^3}\\&-\frac{beg^2n\log(d+ex)}{f^3(ef-dg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2}\\&+\frac{2g(a+b\log\left(c(d+ex)^n\right))}{f^3x}+\frac{g^2(a+b\log\left(c(d+ex)^n\right))}{f^3(f+gx)}\\&+\frac{3g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4}+\frac{beg^2n\log(f+gx)}{f^3(ef-dg)}\\&-\frac{3g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^4}\\&-\frac{3bg^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^4}+\frac{3bg^2n\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^4}\end{aligned}$$

output
```
-1/2*b*e*n/d/f^2/x-1/2*b*e^2*n*ln(x)/d^2/f^2-2*b*e*g*n*ln(x)/d/f^3+1/2*b*e
^2*n*ln(e*x+d)/d^2/f^2+2*b*e*g*n*ln(e*x+d)/d/f^3-b*e*g^2*n*ln(e*x+d)/f^3/(
-d*g+e*f)+1/2*(-a-b*ln(c*(e*x+d)^n))/f^2/x^2+2*g*(a+b*ln(c*(e*x+d)^n))/f^3
/x+g^2*(a+b*ln(c*(e*x+d)^n))/f^3/(g*x+f)+3*g^2*ln(-e*x/d)*(a+b*ln(c*(e*x+d
)^n))/f^4+b*e*g^2*n*ln(g*x+f)/f^3/(-d*g+e*f)-3*g^2*(a+b*ln(c*(e*x+d)^n))*l
n(e*(g*x+f)/(-d*g+e*f))/f^4-3*b*g^2*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f^4
+3*b*g^2*n*polylog(2,1+e*x/d)/f^4
```

3.255.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx =$$
$$-\frac{\frac{4befgn(\log(x)-\log(d+ex))}{d}+\frac{bef^2n(d+ex\log(x)-ex\log(d+ex))}{d^2x}+\frac{f^2(a+b\log(c(d+ex)^n))}{x^2}-\frac{4fg(a+b\log(c(d+ex)^n))}{x}-\frac{2fg^2(a+b\log}{f\cdots}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)^2),x]
```

output
```
-1/2*((4*b*e*f*g*n*(Log[x] - Log[d + e*x]))/d + (b*e*f^2*n*(d + e*x*Log[x]
 - e*x*Log[d + e*x]))/(d^2*x) + (f^2*(a + b*Log[c*(d + e*x)^n]))/x^2 - (4*
f*g*(a + b*Log[c*(d + e*x)^n]))/x - (2*f*g^2*(a + b*Log[c*(d + e*x)^n]))/(
f + g*x) - 6*g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + (2*b*e*f*g^2
*n*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) + 6*g^2*(a + b*Log[c*(d + e*
x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + 6*b*g^2*n*PolyLog[2, (g*(d + e*x))
/(-(e*f) + d*g)] - 6*b*g^2*n*PolyLog[2, 1 + (e*x)/d])/f^4
```

3.255.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(-\frac{3g^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4(f+gx)}+\frac{3g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4x}-\frac{g^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)^2}-\frac{2g\left(a+b\log\left(c(d+\cdots\right.\right.}{f^3x^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{3g^2\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^4}-\frac{3g^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{f^4}+\frac{g^2(a+b\log(c(d+ex)^n))}{f^3(f+gx)}+\frac{2g(a+b\log(c(d+ex)^n))}{f^3x}-\frac{a+b\log(c(d+ex)^n)}{2f^2x^2}-\frac{be^2n\log(x)}{2d^2f^2}+\frac{be^2n\log(d+ex)}{2d^2f^2}-\frac{3bg^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^4}+\frac{3bg^2n\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^4}-\frac{beg^2n\log(d+ex)}{f^3(ef-dg)}+\frac{beg^2n\log(f+gx)}{f^3(ef-dg)}-\frac{2begn\log(x)}{df^3}+\frac{2begn\log(d+ex)}{df^3}-\frac{ben}{2df^2x}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)^2),x]
```

output
```
-1/2*(b*e*n)/(d*f^2*x) - (b*e^2*n*Log[x])/(2*d^2*f^2) - (2*b*e*g*n*Log[x])
/(d*f^3) + (b*e^2*n*Log[d + e*x])/(2*d^2*f^2) + (2*b*e*g*n*Log[d + e*x])/(
d*f^3) - (b*e*g^2*n*Log[d + e*x])/(f^3*(e*f - d*g)) - (a + b*Log[c*(d + e*
x)^n])/(2*f^2*x^2) + (2*g*(a + b*Log[c*(d + e*x)^n]))/(f^3*x) + (g^2*(a +
b*Log[c*(d + e*x)^n]))/(f^3*(f + g*x)) + (3*g^2*Log[-((e*x)/d)]*(a + b*Log
[c*(d + e*x)^n]))/f^4 + (b*e*g^2*n*Log[f + g*x])/(f^3*(e*f - d*g)) - (3*g^
2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^4 - (3*b*g^
2*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/f^4 + (3*b*g^2*n*PolyLog[2,
1 + (e*x)/d])/f^4
```

3.255.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.255.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.66 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.64

method	result
risch	$-\frac{b\ln((ex+d)^n)}{2f^2x^2}+\frac{3b\ln((ex+d)^n)g^2\ln(x)}{f^4}+\frac{2b\ln((ex+d)^n)g}{f^3x}-\frac{3b\ln((ex+d)^n)g^2\ln(gx+f)}{f^4}+\frac{b\ln((ex+d)^n)g^2}{f^3(gx+f)}+\frac{3ben\ln(e}{f^3(dg}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln((e*x+d)^n)/f^2/x^2+3*b*ln((e*x+d)^n)/f^4*g^2*ln(x)+2*b*ln((e*x+d
)^n)/f^3*g/x-3*b*ln((e*x+d)^n)/f^4*g^2*ln(g*x+f)+b*ln((e*x+d)^n)/f^3*g^2/(
g*x+f)+3*b*e*n/f^3/(d*g-e*f)*ln(e*x+d)*g^2-3/2*b*e^2*n/f^2/(d*g-e*f)/d*ln(
e*x+d)*g-1/2*b*e^3*n/f/(d*g-e*f)/d^2*ln(e*x+d)-2*b*e*g*n*ln(x)/d/f^3-1/2*b
*e^2*n*ln(x)/d^2/f^2-1/2*b*e*n/d/f^2/x-b*e*n/f^3*g^2/(d*g-e*f)*ln(g*x+f)-3
*b*n/f^4*g^2*dilog((e*x+d)/d)-3*b*n/f^4*g^2*ln(x)*ln((e*x+d)/d)+3*b*n/f^4*
g^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+3*b*n/f^4*g^2*ln(g*x+f)*ln(((g*x+
f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a
)*(-1/2/f^2/x^2+3/f^4*g^2*ln(x)+2/f^3*g/x-3/f^4*g^2*ln(g*x+f)+1/f^3*g^2/(g
*x+f))
```

3.255.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^5 + 2*f*g*x^4 + f^2*x^3), x)
```

3.255.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx\right)^2}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x+f)**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(x**3*(f + g*x)**2), x)
```

3.255.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx+f\right)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f)^2,x, algorithm="maxima")
```

output
```
1/2*a*((6*g^2*x^2 + 3*f*g*x - f^2)/(f^3*g*x^3 + f^4*x^2) - 6*g^2*log(g*x +
 f)/f^4 + 6*g^2*log(x)/f^4) + b*integrate((log((e*x + d)^n) + log(c))/(g^2
*x^5 + 2*f*g*x^4 + f^2*x^3), x)
```

3.255.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx+f\right)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)^2*x^3), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^3\,{\left(f+g\,x\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)^2), x)
```

3.256 $\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$

3.256.1 Optimal result

Integrand size = 27, antiderivative size = 397

$$\begin{aligned}\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = &-\frac{bdfnx}{2eg^2}+\frac{bd^3nx}{4e^3g}+\frac{bfnx^2}{4g^2}-\frac{bd^2nx^2}{8e^2g}+\frac{bdnx^3}{12eg}\\ &-\frac{bnx^4}{16g}+\frac{bd^2fn\log(d+ex)}{2e^2g^2}-\frac{bd^4n\log(d+ex)}{4e^4g}\\ &-\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{x^4(a+b\log(c(d+ex)^n))}{4g}\\ &+\frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}\\ &+\frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}\\ &+\frac{bf^2n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}\\ &+\frac{bf^2n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}\end{aligned}$$

output
```
-1/2*b*d*f*n*x/e/g^2+1/4*b*d^3*n*x/e^3/g+1/4*b*f*n*x^2/g^2-1/8*b*d^2*n*x^2
/e^2/g+1/12*b*d*n*x^3/e/g-1/16*b*n*x^4/g+1/2*b*d^2*f*n*ln(e*x+d)/e^2/g^2-1
/4*b*d^4*n*ln(e*x+d)/e^4/g-1/2*f*x^2*(a+b*ln(c*(e*x+d)^n))/g^2+1/4*x^4*(a+
b*ln(c*(e*x+d)^n))/g+1/2*f^2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1
/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^3+1/2*f^2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f
)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+1/2*b*f^2*n*polylog(2,-(e
*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+1/2*b*f^2*n*polylog(2,(e*x+d)*
g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3
```

3.256.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.83

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{\frac{12bfgn(ex(-2d+ex)+2d^2\log(d+ex))}{e^2} - \frac{bg^2n(ex(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+12d^4\log(d+ex))}{e^4} - 24fgx^2(a+b\log(c(d+ex}{}$$

input
```
Integrate[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
((12*b*f*g*n*(e*x*(-2*d + e*x) + 2*d^2*Log[d + e*x]))/e^2 - (b*g^2*n*(e*x*
(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 12*d^4*Log[d + e*x]))/e^
4 - 24*f*g*x^2*(a + b*Log[c*(d + e*x)^n]) + 12*g^2*x^4*(a + b*Log[c*(d + e
*x)^n]) + 24*f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))
/(e*Sqrt[-f] + d*Sqrt[g])] + 24*f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqr
t[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] + 24*b*f^2*n*PolyLog[2, -((S
qrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 24*b*f^2*n*PolyLog[2, (Sqrt
[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(48*g^3)
```

3.256.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{f^2x(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)}-\frac{fx(a+b\log(c(d+ex)^n))}{g^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{g}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{f^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^3}+\frac{f^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^3}-\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{x^4(a+b\log(c(d+ex)^n))}{4g}-\frac{bd^4n\log(d+ex)}{4e^4g}+\frac{bd^3nx}{4e^3g}+\frac{bd^2fn\log(d+ex)}{2e^2g^2}-\frac{bd^2nx^2}{8e^2g}+\frac{bf^2n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}+\frac{bf^2n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^3}-\frac{bdfnx}{2eg^2}+\frac{bdnx^3}{12eg}+\frac{bfnx^2}{4g^2}-\frac{bnx^4}{16g}$$

input
```
Int[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
-1/2*(b*d*f*n*x)/(e*g^2) + (b*d^3*n*x)/(4*e^3*g) + (b*f*n*x^2)/(4*g^2) - (
b*d^2*n*x^2)/(8*e^2*g) + (b*d*n*x^3)/(12*e*g) - (b*n*x^4)/(16*g) + (b*d^2*
f*n*Log[d + e*x])/(2*e^2*g^2) - (b*d^4*n*Log[d + e*x])/(4*e^4*g) - (f*x^2*
(a + b*Log[c*(d + e*x)^n]))/(2*g^2) + (x^4*(a + b*Log[c*(d + e*x)^n]))/(4*
g) + (f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqr
t[-f] + d*Sqrt[g])]])/(2*g^3) + (f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqr
t[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]])/(2*g^3) + (b*f^2*n*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^3) + (b*f^2*n*Po
lyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3)
```

3.256.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.256.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.38

method	result
risch	$\frac{b\ln((ex+d)^n)x^4}{4g} - \frac{b\ln((ex+d)^n)f x^2}{2g^2} + \frac{b\ln((ex+d)^n)f^2\ln(g x^2+f)}{2g^3} - \frac{bn f^2\ln(ex+d)\ln(g x^2+f)}{2g^3} + \frac{bn f^2\ln(ex+d)\ln\left(\frac{e\sqrt{-}}{}\right.}{2g^3}$

input
```
int(x^5*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*ln((e*x+d)^n)/g*x^4-1/2*b*ln((e*x+d)^n)/g^2*f*x^2+1/2*b*ln((e*x+d)^n
)*f^2/g^3*ln(g*x^2+f)-1/2*b*n*f^2/g^3*ln(e*x+d)*ln(g*x^2+f)+1/2*b*n*f^2/g^
3*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*
n*f^2/g^3*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g)
)+1/2*b*n*f^2/g^3*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g
))+1/2*b*n*f^2/g^3*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*
g))-1/16*b*n*x^4/g+1/12*b*d*n*x^3/e/g-1/8*b*d^2*n*x^2/e^2/g+1/4*b*f*n*x^2/
g^2+1/4*b*d^3*n*x/e^3/g-1/2*b*d*f*n*x/e/g^2-1/4*b*d^4*n*ln(e*x+d)/e^4/g+1/
2*b*d^2*f*n*ln(e*x+d)/e^2/g^2+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csg
n(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln
(c)+a)*(1/2/g^2*(1/2*g*x^4-f*x^2)+1/2*f^2/g^3*ln(g*x^2+f))
```

3.256.5 Fricas [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^5}{gx^2+f}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*x^5*log((e*x + d)^n*c) + a*x^5)/(g*x^2 + f), x)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
```

output
```
Timed out
```

3.256.7 Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^5}{gx^2+f}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/4*a*(2*f^2*log(g*x^2 + f)/g^3 + (g*x^4 - 2*f*x^2)/g^2) + b*integrate((x^
5*log((e*x + d)^n) + x^5*log(c))/(g*x^2 + f), x)
```

3.256.8 Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^5}{gx^2+f}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^5/(g*x^2 + f), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{x^5\,(a+b\,\ln(c\,(d+e\,x)^n))}{g\,x^2+f}\,dx$$

input
```
int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
```

output
```
int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

3.257 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2} dx$

3.257.1 Optimal result

Integrand size = 27, antiderivative size = 278

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2} dx = {} & \frac{bdnx}{2eg} - \frac{bnx^2}{4g} - \frac{bd^2n\log(d+ex)}{2e^2g} \\ & + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} \\ & - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} \\ & - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} \\ & - \frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} \\ & - \frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}\end{aligned}$$

output
```
1/2*b*d*n*x/e/g-1/4*b*n*x^2/g-1/2*b*d^2*n*ln(e*x+d)/e^2/g+1/2*x^2*(a+b*ln(
c*(e*x+d)^n))/g-1/2*f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e
*(-f)^(1/2)+d*g^(1/2)))/g^2-1/2*f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x
*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-1/2*b*f*n*polylog(2,-(e*x+d)*g^(1/
2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-1/2*b*f*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-
f)^(1/2)+d*g^(1/2)))/g^2
```

3.257.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = -\frac{\frac{bgn(ex(-2d+ex)+2d^2\log(d+ex))}{e^2} - 2gx^2(a+b\log(c(d+ex)^n)) + 2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
-1/4*((b*g*n*(e*x*(-2*d + e*x) + 2*d^2*Log[d + e*x]))/e^2 - 2*g*x^2*(a + b
*Log[c*(d + e*x)^n]) + 2*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - S
qrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] + 2*f*(a + b*Log[c*(d + e*x)^n])*Log[
(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] + 2*b*f*n*PolyLog[2,
-((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 2*b*f*n*PolyLog[2, (Sqr
t[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^2
```

3.257.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{x(a+b\log(c(d+ex)^n))}{g} - \frac{fx(a+b\log(c(d+ex)^n))}{g(f+gx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g}-\frac{bd^2n\log(d+ex)}{2e^2g}-\frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}-\frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^2}+\frac{bdnx}{2eg}-\frac{bnx^2}{4g}$$

input
```
Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
(b*d*n*x)/(2*e*g) - (b*n*x^2)/(4*g) - (b*d^2*n*Log[d + e*x])/(2*e^2*g) + (
x^2*(a + b*Log[c*(d + e*x)^n]))/(2*g) - (f*(a + b*Log[c*(d + e*x)^n])*Log[
(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2) - (f*(a + b*
Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g]
)])/(2*g^2) - (b*f*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt
[g]))])/(2*g^2) - (b*f*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sq
rt[g])])/(2*g^2)
```

3.257.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.257.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.55

method	result
risch	$\frac{b\ln((ex+d)^n)x^2}{2g}-\frac{b\ln((ex+d)^n)f\ln(gx^2+f)}{2g^2}-\frac{bn\,x^2}{4g}+\frac{bdnx}{2eg}-\frac{b\,d^2n\ln(ex+d)}{2e^2g}+\frac{bnf\ln(ex+d)\ln(gx^2+f)}{2g^2}-\frac{bnf\ln(ex}{}$

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)/g*x^2-1/2*b*ln((e*x+d)^n)*f/g^2*ln(g*x^2+f)-1/4*b*n*x^
2/g+1/2*b*d*n*x/e/g-1/2*b*d^2*n*ln(e*x+d)/e^2/g+1/2*b*n*f/g^2*ln(e*x+d)*ln
(g*x^2+f)-1/2*b*n*f/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f
*g)^(1/2)+d*g))-1/2*b*n*f/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/
(e*(-f*g)^(1/2)-d*g))-1/2*b*n*f/g^2*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(
e*(-f*g)^(1/2)+d*g))-1/2*b*n*f/g^2*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e
*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x
+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d
)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/
2*x^2/g-1/2*f/g^2*ln(g*x^2+f))
```

3.257.5 Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx^2+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g*x^2 + f), x)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx^2+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*a*(x^2/g - f*log(g*x^2 + f)/g^2) + b*integrate((x^3*log((e*x + d)^n) +
 x^3*log(c))/(g*x^2 + f), x)
```

3.257.8 Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx^2+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x^2 + f), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))}{g\,x^2+f}\,dx$$

input
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
```

output
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

3.258 $\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$

3.258.1 Optimal result

Integrand size = 25, antiderivative size = 203

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} + \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g}$$

output
```
1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1
/2)))/g+1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2
)-d*g^(1/2)))/g+1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)
))/g+1/2*b*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g
```

3.258.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{(a+b\log(c(d+ex)^n))\left(\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)+\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)+bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+bn\,\mathrm{Pol}}{2g}$$

input
```
Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
d*Sqrt[g])] + Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) +
b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*n*Poly
Log[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g)
```

3.258.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g}+\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g}$$

input
```
Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d
*Sqrt[g])])/(2*g) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]
*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g) + (b*n*PolyLog[2, -((Sqrt[g]*(d + e*
x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x)
)/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g)
```

3.258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.258.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.75

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2g} - \frac{bn\ln(ex+d)\ln(g\,x^2+f)}{2g} + \frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2g} + \frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}+g(ex+d}{e\sqrt{-fg}-dg}\right.}{2g}$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)/g*ln(g*x^2+f)-1/2*b/g*n*ln(e*x+d)*ln(g*x^2+f)+1/2*b/g*
n*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b/
g*n*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*
b/g*n*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b/g*n
*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*(-1/2*I*b*
Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*cs
gn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2
*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)/g*ln(g*x^2+f)
```

3.258.5 Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx^2+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*x*log((e*x + d)^n*c) + a*x)/(g*x^2 + f), x)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
```

output
```
Timed out
```

3.258.7 Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx^2+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
```

output
```
b*integrate((x*log((e*x + d)^n) + x*log(c))/(g*x^2 + f), x) + 1/2*a*log(g*
x^2 + f)/g
```

3.258.8 Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx^2+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x/(g*x^2 + f), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))}{g\,x^2+f}\,dx$$

input
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
```

output
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

3.259 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx^2)}\,dx$

3.259.1 Optimal result

Integrand size = 27, antiderivative size = 245

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx = \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} + \frac{bn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f}$$

output
```
ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1
/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*
((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f+b*n*polylog(2,1+e*x/d)/
f-1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f-1/2*b*n*p
olylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f
```

3.259.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx =$$
$$-\frac{-2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)+\left(a+b\log\left(c(d+ex\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)),x]
```

output
```
-1/2*(-2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + (a + b*Log[c*(d + e*
x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] + (a + b*L
og[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])
] + b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*n*
PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])] - 2*b*n*PolyLog[2
, 1 + (e*x)/d])/f
```

3.259.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx}-\frac{gx\left(a+b\log\left(c(d+ex)^n\right)\right)}{f\left(f+gx^2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2f}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f}+\frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f}-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f}+\frac{bn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)),x]
```

output
```
(Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f - ((a + b*Log[c*(d + e*x)^n
])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f) - ((a +
 b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt
[g])])/(2*f) - (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[
g]))])/(2*f) - (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g]
)])/(2*f) + (b*n*PolyLog[2, 1 + (e*x)/d])/f
```

3.259.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.259.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.69

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(x)}{f}-\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2f}-\frac{bn\operatorname{dilog}\left(\frac{ex+d}{d}\right)}{f}-\frac{bn\ln(x)\ln\left(\frac{ex+d}{d}\right)}{f}+\frac{bn\ln(ex+d)\ln(g\,x^2+f)}{2f}-\frac{bn\ln}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/f*ln(x)-1/2*b*ln((e*x+d)^n)/f*ln(g*x^2+f)-b*n/f*dilog((e*x
+d)/d)-b*n/f*ln(x)*ln((e*x+d)/d)+1/2*b*n/f*ln(e*x+d)*ln(g*x^2+f)-1/2*b*n/f
*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n
/f*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b
*n/f*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f*
dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*cs
gn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*
c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*
Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/f*ln(x)-1/2/f*ln(g*x^2+f))
```

3.259.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^3 + f*x), x)
```

3.259.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x**2+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(x*(f + g*x**2)), x)
```

3.259.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f),x, algorithm="maxima")
```

output
```
-1/2*a*(log(g*x^2 + f)/f - 2*log(x)/f) + b*integrate((log((e*x + d)^n) + l
og(c))/(g*x^3 + f*x), x)
```

3.259.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,\left(g\,x^2+f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)), x)
```

3.260 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)}\,dx$

3.260.1 Optimal result

Integrand size = 27, antiderivative size = 331

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx = -\frac{ben}{2dfx}-\frac{be^2n\log(x)}{2d^2f}+\frac{be^2n\log(d+ex)}{2d^2f}-\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2}-\frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}+\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}+\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}+\frac{bgn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}+\frac{bgn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}-\frac{bgn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}$$

output
```
-1/2*b*e*n/d/f/x-1/2*b*e^2*n*ln(x)/d^2/f+1/2*b*e^2*n*ln(e*x+d)/d^2/f+1/2*(
-a-b*ln(c*(e*x+d)^n))/f/x^2-g*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f^2+1/2*g*(
a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))
/f^2+1/2*g*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)
-d*g^(1/2)))/f^2-b*g*n*polylog(2,1+e*x/d)/f^2+1/2*b*g*n*polylog(2,-(e*x+d)
*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2+1/2*b*g*n*polylog(2,(e*x+d)*g^(1/2)
/(e*(-f)^(1/2)+d*g^(1/2)))/f^2
```

3.260.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx$$
$$= \frac{-\frac{befn(d+ex\log(x)-ex\log(d+ex))}{d^2x} - \frac{f(a+b\log(c(d+ex)^n))}{x^2} - 2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) + g(a+b\log\left(c(d\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)),x]
```

output
```
(-((b*e*f*n*(d + e*x*Log[x] - e*x*Log[d + e*x]))/(d^2*x)) - (f*(a + b*Log[
c*(d + e*x)^n]))/x^2 - 2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + g*
(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*
Sqrt[g])] + g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e
*Sqrt[-f] - d*Sqrt[g])] + b*g*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] - d*Sqrt[g]))] + b*g*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*S
qrt[g])] - 2*b*g*n*PolyLog[2, 1 + (e*x)/d])/(2*f^2)
```

3.260.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx$$
$$\Big\downarrow \; 2863$$
$$\int \left(\frac{g^2x(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{a+b\log\left(c(d+ex)^n\right)}{fx^3}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{g\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{f^2}+\frac{g\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{2f^2}+\frac{g\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{2f^2}-\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2}-\frac{be^2n\log(x)}{2d^2f}+\frac{be^2n\log(d+ex)}{2d^2f}+\frac{bgn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}+\frac{bgn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2}-\frac{bgn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2}-\frac{ben}{2dfx}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)),x]
```

output
```
-1/2*(b*e*n)/(d*f*x) - (b*e^2*n*Log[x])/(2*d^2*f) + (b*e^2*n*Log[d + e*x])
/(2*d^2*f) - (a + b*Log[c*(d + e*x)^n])/(2*f*x^2) - (g*Log[-((e*x)/d)]*(a
+ b*Log[c*(d + e*x)^n]))/f^2 + (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[
-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2) + (g*(a + b*Log[c*(d
+ e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^
2) + (b*g*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*f^2) + (b*g*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/
(2*f^2) - (b*g*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

3.260.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.260.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{b\ln\left((ex+d)^{n}\right)}{2f\,x^{2}}-\frac{b\ln\left((ex+d)^{n}\right)g\ln\left(x\right)}{f^{2}}+\frac{b\ln\left((ex+d)^{n}\right)g\ln\left(g\,x^{2}+f\right)}{2f^{2}}-\frac{bng\ln\left(ex+d\right)\ln\left(g\,x^{2}+f\right)}{2f^{2}}+\frac{bng\ln\left(ex+d\right)\ln\left(\frac{e\sqrt{-fg}\cdots}{e\sqrt{\cdots}}\right.}{2f^{2}}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln((e*x+d)^n)/f/x^2-b*ln((e*x+d)^n)/f^2*g*ln(x)+1/2*b*ln((e*x+d)^n)
*g/f^2*ln(g*x^2+f)-1/2*b*n/f^2*g*ln(e*x+d)*ln(g*x^2+f)+1/2*b*n/f^2*g*ln(e*
x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/f^2*g
*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n
/f^2*g*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/
f^2*g*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*e^2
*n*ln(e*x+d)/d^2/f-1/2*b*e*n/d/f/x-1/2*b*e^2*n*ln(x)/d^2/f+b*n/f^2*g*dilog
((e*x+d)/d)+b*n/f^2*g*ln(x)*ln((e*x+d)/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e
*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/
2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+
d)^n)^3+b*ln(c)+a)*(-1/2/f/x^2-1/f^2*g*ln(x)+1/2*g/f^2*ln(g*x^2+f))
```

3.260.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{x^{3}\left(f+gx^{2}\right)}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{(gx^{2}+f)x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^5 + f*x^3), x)
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x**2+f),x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*a*(g*log(g*x^2 + f)/f^2 - 2*g*log(x)/f^2 - 1/(f*x^2)) + b*integrate((l
og((e*x + d)^n) + log(c))/(g*x^5 + f*x^3), x)
```

3.260.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x^3), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^3\left(f + gx^2\right)}\,dx = \int \frac{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}{x^3\,\left(g\,x^2 + f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)), x)
```

3.261 $\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$

3.261.1 Optimal result

Integrand size = 27, antiderivative size = 369

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = -\frac{afx}{g^2}+\frac{bfnx}{g^2}-\frac{bd^2nx}{3e^2g}+\frac{bdnx^2}{6eg}-\frac{bnx^3}{9g}+\frac{bd^3n\log(d+ex)}{3e^3g} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g} + \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} - \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{b(-f)^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} + \frac{b(-f)^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$

output
```
-a*f*x/g^2+b*f*n*x/g^2-1/3*b*d^2*n*x/e^2/g+1/6*b*d*n*x^2/e/g-1/9*b*n*x^3/g
+1/3*b*d^3*n*ln(e*x+d)/e^3/g-b*f*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+1/3*x^3*(a+
b*ln(c*(e*x+d)^n))/g+1/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)
-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)-1/2*(-f)^(3/2)*(a+b*ln(c*(e*
x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)-1/2
*b*(-f)^(3/2)*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/
2)+1/2*b*(-f)^(3/2)*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/
g^(5/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.92

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$
$$= \frac{-18af\sqrt{g}x+18bf\sqrt{g}nx-\frac{bg^{3/2}n(ex(6d^2-3dex+2e^2x^2)-6d^3\log(d+ex))}{e^3}-\frac{18bf\sqrt{g}(d+ex)\log(c(d+ex)^n)}{e}+6g^{3/2}x^3(a+b}{}$$

input
```
Integrate[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
(-18*a*f*Sqrt[g]*x + 18*b*f*Sqrt[g]*n*x - (b*g^(3/2)*n*(e*x*(6*d^2 - 3*d*e
*x + 2*e^2*x^2) - 6*d^3*Log[d + e*x]))/e^3 - (18*b*f*Sqrt[g]*(d + e*x)*Log
[c*(d + e*x)^n])/e + 6*g^(3/2)*x^3*(a + b*Log[c*(d + e*x)^n]) + 9*(-f)^(3/
2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
 d*Sqrt[g])] + 9*Sqrt[-f]*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] +
Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] - 9*b*(-f)^(3/2)*n*PolyLog[2, -((Sqr
t[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 9*b*(-f)^(3/2)*n*PolyLog[2, (
Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(18*g^(5/2))
```

3.261.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{f^2(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))}{g^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{g}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^{5/2}} - \frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^{5/2}} + \frac{x^3(a+b\log(c(d+ex)^n))}{3g} - \frac{afx}{g^2} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{bd^3n\log(d+ex)}{3e^3g} - \frac{bd^2nx}{3e^2g} - \frac{b(-f)^{3/2}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} + \frac{b(-f)^{3/2}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^{5/2}} + \frac{bdnx^2}{6eg} + \frac{bfnx}{g^2} - \frac{bnx^3}{9g}$$

input
```
Int[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
-((a*f*x)/g^2) + (b*f*n*x)/g^2 - (b*d^2*n*x)/(3*e^2*g) + (b*d*n*x^2)/(6*e*
g) - (b*n*x^3)/(9*g) + (b*d^3*n*Log[d + e*x])/(3*e^3*g) - (b*f*(d + e*x)*L
og[c*(d + e*x)^n])/(e*g^2) + (x^3*(a + b*Log[c*(d + e*x)^n]))/(3*g) + ((-f
)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[
-f] + d*Sqrt[g])])/(2*g^(5/2)) - ((-f)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Lo
g[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(5/2)) - (b*(
-f)^(3/2)*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*g^(5/2)) + (b*(-f)^(3/2)*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/(2*g^(5/2))
```

3.261.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.261. $\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$

3.261.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.17 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.64

method	result
risch	$\frac{b\ln((ex+d)^n)x^3}{3g}+\frac{b\,d^3\ln((ex+d)^n)}{3e^3g}-\frac{b\ln((ex+d)^n)fx}{g^2}-\frac{bdf\ln((ex+d)^n)}{e\,g^2}-\frac{b\,f^2\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{g^2\sqrt{fg}}+\frac{b\,f^2}{}$

input
```
int(x^4*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*ln((e*x+d)^n)/g*x^3+1/3*b/e^3/g*d^3*ln((e*x+d)^n)-b*ln((e*x+d)^n)/g^
2*f*x-b/e/g^2*d*f*ln((e*x+d)^n)-b*f^2/g^2/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x
+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(e*x+d)+b*f^2/g^2/(f*g)^(1/2)*arctan(1/2*(2*
g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)-1/9*b*n*x^3/g+1/6*b*d*n*x^2/
e/g-1/3*b*d^2*n*x/e^2/g-11/18*b*d^3*n/e^3/g+b*f*n*x/g^2+b*d*f*n/e/g^2+1/2*
b*n*f^2/g^2*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-
f*g)^(1/2)+d*g))-1/2*b*n*f^2/g^2*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)
+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n*f^2/g^2/(-f*g)^(1/2)*dilog((
e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n*f^2/g^2/(-f*g)
^(1/2)*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*
b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*
csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1
/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^2*(1/3*g*x^3-f*x)+f^2/g^2/
(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

3.261.5 Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^4}{gx^2+f}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*x^4*log((e*x + d)^n*c) + a*x^4)/(g*x^2 + f), x)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
```

output
```
Timed out
```

3.261.7 Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^4}{gx^2+f}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/3*a*(3*f^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2) + (g*x^3 - 3*f*x)/g^2)
+ b*integrate((x^4*log((e*x + d)^n) + x^4*log(c))/(g*x^2 + f), x)
```

3.261.8 Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^4}{gx^2+f}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^4/(g*x^2 + f), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,(d+e\,x)^n))}{g\,x^2+f}\,dx$$

input
```
int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
```

output
```
int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

3.262 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$

3.262.1 Optimal result

Integrand size = 27, antiderivative size = 276

$$\begin{aligned}\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx &= \frac{ax}{g}-\frac{bnx}{g}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg}\\
&+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}\\
&-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}}\\
&-\frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}}\\
&+\frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}\end{aligned}$$

output
```
a*x/g-b*n*x/g+b*(e*x+d)*ln(c*(e*x+d)^n)/e/g+1/2*(a+b*ln(c*(e*x+d)^n))*ln(e
*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)-1/2*(
a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))
*(-f)^(1/2)/g^(3/2)-1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(
1/2)))*(-f)^(1/2)/g^(3/2)+1/2*b*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+
d*g^(1/2)))*(-f)^(1/2)/g^(3/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.95

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{2a\sqrt{g}x-2b\sqrt{g}nx+\frac{2b\sqrt{g}(d+ex)\log(c(d+ex)^n)}{e}+\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)-\sqrt{-f}(a+b}{2g}$$

input
```
Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
(2*a*Sqrt[g]*x - 2*b*Sqrt[g]*n*x + (2*b*Sqrt[g]*(d + e*x)*Log[c*(d + e*x)^
n])/e + Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))
/(e*Sqrt[-f] + d*Sqrt[g])] - Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(S
qrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] - b*Sqrt[-f]*n*PolyLog[2,
-((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*Sqrt[-f]*n*PolyLog[2,
 (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^(3/2))
```

3.262.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$

$$\Big\downarrow \; 2863$$

$$\int \left(\frac{a+b\log(c(d+ex)^n)}{g}-\frac{f(a+b\log(c(d+ex)^n))}{g(f+gx^2)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^{3/2}} - \frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^{3/2}} + \frac{ax}{g} + \frac{b(d+ex)\log(c(d+ex)^n)}{eg} - \frac{b\sqrt{-f}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} + \frac{b\sqrt{-f}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^{3/2}} - \frac{bnx}{g}$$

input
```
Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

output
```
(a*x)/g - (b*n*x)/g + (b*(d + e*x)*Log[c*(d + e*x)^n])/(e*g) + (Sqrt[-f]*(
a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*g^(3/2)) - (Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[
-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(3/2)) - (b*Sqrt[-f]*n*P
olyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^(3/2)) +
(b*Sqrt[-f]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2
*g^(3/2))
```

3.262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.262.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.78

method	result
risch	$\frac{b\ln((ex+d)^n)x}{g} + \frac{bd\ln((ex+d)^n)}{eg} + \frac{bf\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{g\sqrt{fg}} - \frac{bf\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^n)}{g\sqrt{fg}} - \frac{bnx}{g} -$

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/g*x+b/e/g*d*ln((e*x+d)^n)+b*f/g/(f*g)^(1/2)*arctan(1/2*(2*
g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(e*x+d)-b*f/g/(f*g)^(1/2)*arctan(1/2*(
2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)-b*n*x/g-b*d*n/e/g-1/2*b*n*
f/g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/
2)+d*g))+1/2*b*n*f/g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d
*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*n*f/g/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g
*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n*f/g/(-f*g)^(1/2)*dilog((e*(-f*
g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(
I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^
2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(
e*x+d)^n)^3+b*ln(c)+a)*(x/g-f/g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

3.262.5 Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^2}{gx^2+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g*x^2 + f), x)
```

3.262.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
```

output
```
Timed out
```

3.262.7 Maxima [F]

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx=\int \frac{(b\log\left((ex+d)^nc\right)+a)x^2}{gx^2+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
```

output
```
-a*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + b*integrate((x^2*log((e
*x + d)^n) + x^2*log(c))/(g*x^2 + f), x)
```

3.262.8 Giac [F]

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx=\int \frac{(b\log\left((ex+d)^nc\right)+a)x^2}{gx^2+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x^2 + f), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx=\int \frac{x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{g\,x^2+f}\,dx$$

input
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
```

output
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

3.263 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx$

3.263.1 Optimal result

Integrand size = 24, antiderivative size = 239

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

output
```
1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1
/2)))/(-f)^(1/2)/g^(1/2)-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1
/2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*b*n*polylog(2,-(e*x+
d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+1/2*b*n*polylog(2,
(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx = \frac{(a+b\log\left(c(d+ex)^n\right))\left(\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)-\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)-bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+bn\operatorname{Pol}}{2\sqrt{-f}\sqrt{g}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
d*Sqrt[g])] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) -
b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*n*Poly
Log[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

3.263.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx$$
$$\Big\downarrow \; 2856$$
$$\int \left(\frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{2\sqrt{-f}\sqrt{g}}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d
*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqr
t[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (b*n
*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*
Sqrt[g]) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/
(2*Sqrt[-f]*Sqrt[g])
```

3.263.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.263.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{b\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{\sqrt{fg}}+\frac{b\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln\left((ex+d)^n\right)}{\sqrt{fg}}+\frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2\sqrt{-fg}}-\frac{bn\ln$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-b/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(e*x+d)+b
/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)+1
/2*b*n*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^
(1/2)+d*g))-1/2*b*n*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*
g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x
+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+
g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b
*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)
^3+b*ln(c)+a)/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))
```

3.263.5 Fricas [F]

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{f+gx^2}\,dx=\int \frac{b\log \left((ex+d)^nc\right)+a}{gx^2+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^2 + f), x)
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{f+gx^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
```

output
```
Timed out
```

3.263.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{gx^2+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(g*x^2 + f), x) + a*arctan(g*x/sqr
t(f*g))/sqrt(f*g)
```

3.263.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{gx^2+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x^2 + f), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx=\int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{g\,x^2+f}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x^2), x)
```

3.264 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)}\,dx$

3.264.1 Optimal result

Integrand size = 27, antiderivative size = 290

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\frac{ben\log(x)}{df}-\frac{ben\log(d+ex)}{df}-\frac{a+b\log\left(c(d+ex)^n\right)}{fx}+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}-\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}+\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}$$

output
```
b*e*n*ln(x)/d/f-b*e*n*ln(e*x+d)/d/f+(-a-b*ln(c*(e*x+d)^n))/f/x+1/2*(a+b*ln
(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/
2)/(-f)^(3/2)-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f
)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)-1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/
(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)+1/2*b*n*polylog(2,(e*x+d)*g^(
1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97

$$\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)}\,dx$$
$$= \frac{f\Big(2be(-f)^{3/2}nx(\log(x)-\log(d+ex))+2d\sqrt{-f}f(a+b\log(c(d+ex)^n))+df\sqrt{g}x(a+b\log(c(d+ex)^n}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)),x]
```

output
```
(f*(2*b*e*(-f)^(3/2)*n*x*(Log[x] - Log[d + e*x]) + 2*d*Sqrt[-f]*f*(a + b*L
og[c*(d + e*x)^n]) + d*f*Sqrt[g]*x*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt
[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] - d*f*Sqrt[g]*x*(a + b*Log[c*
(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] - b
*d*f*Sqrt[g]*n*x*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g])
)] + b*d*f*Sqrt[g]*n*x*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt
[g])]))/(2*d*(-f)^(7/2)*x)
```

3.264.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)}\,dx$$
$$\Big\downarrow \; 2863$$
$$\int \left(\frac{a+b\log(c(d+ex)^n)}{fx^2}-\frac{g(a+b\log(c(d+ex)^n))}{f(f+gx^2)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{g}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2(-f)^{3/2}}-\frac{\sqrt{g}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2(-f)^{3/2}}-\frac{a+b\log(c(d+ex)^n)}{fx}-\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}+\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2(-f)^{3/2}}+\frac{ben\log(x)}{df}-\frac{ben\log(d+ex)}{df}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)),x]
```

output
```
(b*e*n*Log[x])/(d*f) - (b*e*n*Log[d + e*x])/(d*f) - (a + b*Log[c*(d + e*x)
^n])/(f*x) + (Sqrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g
]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2)) - (Sqrt[g]*(a + b*Log[c*(d
 + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*(
-f)^(3/2)) - (b*Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d
*Sqrt[g]))])/(2*(-f)^(3/2)) + (b*Sqrt[g]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/
(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2))
```

3.264.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.264.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{b\ln((ex+d)^n)}{fx}+\frac{bg\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{f\sqrt{fg}}-\frac{bg\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^n)}{f\sqrt{fg}}+\frac{ben\ln(ex)}{fd}-\frac{ben\ln(ex+}{df}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-b*ln((e*x+d)^n)/f/x+b/f*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f
*g)^(1/2))*n*ln(e*x+d)-b/f*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/
(f*g)^(1/2))*ln((e*x+d)^n)+b*e*n/f/d*ln(e*x)-b*e*n*ln(e*x+d)/d/f-1/2*b*n/f
*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2
)+d*g))+1/2*b*n/f*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*
g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*n/f*g/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*
(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/f*g/(-f*g)^(1/2)*dilog((e*(-f*g
)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2
+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e
*x+d)^n)^3+b*ln(c)+a)*(-1/f/x-1/f*g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

3.264.5 Fricas [F]

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{b\log \left((ex+d)^n c\right)+a}{(gx^2+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^4 + f*x^2), x)
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.264.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
-a*(g*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f) + 1/(f*x)) + b*integrate((log((e
*x + d)^n) + log(c))/(g*x^4 + f*x^2), x)
```

3.264.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x^2), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{a+b\,\ln\left(c\,{(d+e\,x)}^n\right)}{x^2\,\left(g\,x^2+f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)), x)
```

3.265 $\int \frac{a+b\log(c(d+ex)^n)}{x^4(f+gx^2)}\,dx$

3.265.1 Optimal result

Integrand size = 27, antiderivative size = 388

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx = & -\frac{ben}{6dfx^2}+\frac{be^2n}{3d^2fx}+\frac{be^3n\log(x)}{3d^3f}-\frac{begn\log(x)}{df^2}\\ & -\frac{be^3n\log(d+ex)}{3d^3f}+\frac{begn\log(d+ex)}{df^2}\\ & -\frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3}+\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x}\\ & +\frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}\\ & -\frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}\\ & -\frac{bg^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}\\ & +\frac{bg^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}\end{aligned}$$

output
```
-1/6*b*e*n/d/f/x^2+1/3*b*e^2*n/d^2/f/x+1/3*b*e^3*n*ln(x)/d^3/f-b*e*g*n*ln(
x)/d/f^2-1/3*b*e^3*n*ln(e*x+d)/d^3/f+b*e*g*n*ln(e*x+d)/d/f^2+1/3*(-a-b*ln(
c*(e*x+d)^n))/f/x^3+g*(a+b*ln(c*(e*x+d)^n))/f^2/x+1/2*g^(3/2)*(a+b*ln(c*(e
*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(5/2)
-1/2*g^(3/2)*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/
2)-d*g^(1/2)))/(-f)^(5/2)-1/2*b*g^(3/2)*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-
f)^(1/2)-d*g^(1/2)))/(-f)^(5/2)+1/2*b*g^(3/2)*n*polylog(2,(e*x+d)*g^(1/2)/
(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(5/2)
```

3.265.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{x^4 \left(f+gx^2\right)}\,dx = \frac{1}{6}\left(-\frac{6begn(\log(x)-\log(d+ex))}{df^2} - \frac{ben\left(d(d-2ex)-2e^2x^2\log(x)+2e^2x^2\log(d+ex)\right)}{d^3fx^2} - \frac{2\left(a+b\log \left(c(d+ex)^n\right)\right)}{fx^3} + \frac{6g\left(a+b\log \left(c(d+ex)^n\right)\right)}{f^2x} + \frac{3g^{3/2}\left(a+b\log \left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{3g^{3/2}\left(a+b\log \left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{3bg^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{3bg^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^4*(f + g*x^2)),x]
```

output
```
((-6*b*e*g*n*(Log[x] - Log[d + e*x]))/(d*f^2) - (b*e*n*(d*(d - 2*e*x) - 2*
e^2*x^2*Log[x] + 2*e^2*x^2*Log[d + e*x]))/(d^3*f*x^2) - (2*(a + b*Log[c*(d
 + e*x)^n]))/(f*x^3) + (6*g*(a + b*Log[c*(d + e*x)^n]))/(f^2*x) + (3*g^(3/
2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/(-f)^(5/2) - (3*g^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(S
qrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(-f)^(5/2) - (3*b*g^(3/2)
*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(5/2)
 + (3*b*g^(3/2)*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])]
)/(-f)^(5/2))/6
```

3.265.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2\left(f+gx^2\right)}-\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x^2}+\frac{a+b\log\left(c(d+ex)^n\right)}{fx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x}+\frac{g^{3/2}\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2(-f)^{5/2}}-\frac{g^{3/2}\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2(-f)^{5/2}}-\frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3}+\frac{be^3n\log(x)}{3d^3f}-\frac{be^3n\log(d+ex)}{3d^3f}+\frac{be^2n}{3d^2fx}-\frac{begn\log(x)}{df^2}+\frac{begn\log(d+ex)}{df^2}-\frac{bg^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}+\frac{bg^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2(-f)^{5/2}}-\frac{ben}{6dfx^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^4*(f + g*x^2)),x]
```

output
```
-1/6*(b*e*n)/(d*f*x^2) + (b*e^2*n)/(3*d^2*f*x) + (b*e^3*n*Log[x])/(3*d^3*f
) - (b*e*g*n*Log[x])/(d*f^2) - (b*e^3*n*Log[d + e*x])/(3*d^3*f) + (b*e*g*n
*Log[d + e*x])/(d*f^2) - (a + b*Log[c*(d + e*x)^n])/(3*f*x^3) + (g*(a + b*
Log[c*(d + e*x)^n]))/(f^2*x) + (g^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*
(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(5/2)) - (g^(3/
2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] -
 d*Sqrt[g])])/(2*(-f)^(5/2)) - (b*g^(3/2)*n*PolyLog[2, -((Sqrt[g]*(d + e*x
))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*(-f)^(5/2)) + (b*g^(3/2)*n*PolyLog[2, (S
qrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(5/2))
```

3.265.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.265.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.75 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{b\,g^2 \arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right) n \ln(ex+d)}{f^2\sqrt{fg}}+\frac{b\,g^2 \arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right) \ln((ex+d)^n)}{f^2\sqrt{fg}}-\frac{b\ln((ex+d)^n)}{3f\,x^3}+\frac{b\ln((ex+d)^n)g}{f^2x}-\frac{ebr}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^4/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-b*g^2/f^2/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(
e*x+d)+b*g^2/f^2/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))
*ln((e*x+d)^n)-1/3*b*ln((e*x+d)^n)/f/x^3+b*ln((e*x+d)^n)/f^2*g/x-e*b*n/f^2
*g/d*ln(e*x)+b*e*g*n*ln(e*x+d)/d/f^2-1/6*b*e*n/d/f/x^2+1/3*b*e^2*n/d^2/f/x
+1/3*e^3*b*n/f/d^3*ln(e*x)-1/3*b*e^3*n*ln(e*x+d)/d^3/f+1/2*b*n*g^2/f^2*ln(
e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g)
)-1/2*b*n*g^2/f^2*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)
/(e*(-f*g)^(1/2)-d*g))+1/2*b*n*g^2/f^2/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-
g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n*g^2/f^2/(-f*g)^(1/2)*dilog((e
*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d
)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(
I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/3/f/x^3+1/f^2*g/x+g^2/f^2/(f*g)^(1/2)*arct
an(g*x/(f*g)^(1/2)))
```

3.265.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^4/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g*x^6 + f*x^4), x)
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**4/(g*x**2+f),x)
```

output
```
Timed out
```

3.265.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^4/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/3*a*(3*g^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f^2) + (3*g*x^2 - f)/(f^2*x^
3)) + b*integrate((log((e*x + d)^n) + log(c))/(g*x^6 + f*x^4), x)
```

3.265.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^4/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x^4), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^4\,\left(g\,x^2+f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^4*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^4*(f + g*x^2)), x)
```

3.266 $\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$

3.266.1 Optimal result

Integrand size = 27, antiderivative size = 417

$$\begin{aligned}\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx =\ & \frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}+\frac{bdef^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{5/2}(e^2f+d^2g)}\\ & -\frac{bd^2n\log(d+ex)}{2e^2g^2}+\frac{be^2f^2n\log(d+ex)}{2g^3(e^2f+d^2g)}\\ & +\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)}\\ & -\frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}\\ & -\frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}\\ & -\frac{be^2f^2n\log(f+gx^2)}{4g^3(e^2f+d^2g)}-\frac{bfn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}\\ & -\frac{bfn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}\end{aligned}$$

output
```
1/2*b*d*n*x/e/g^2-1/4*b*n*x^2/g^2+1/2*b*d*e*f^(3/2)*n*arctan(x*g^(1/2)/f^(
1/2))/g^(5/2)/(d^2*g+e^2*f)-1/2*b*d^2*n*ln(e*x+d)/e^2/g^2+1/2*b*e^2*f^2*n*
ln(e*x+d)/g^3/(d^2*g+e^2*f)+1/2*x^2*(a+b*ln(c*(e*x+d)^n))/g^2-1/2*f^2*(a+b
*ln(c*(e*x+d)^n))/g^3/(g*x^2+f)-1/4*b*e^2*f^2*n*ln(g*x^2+f)/g^3/(d^2*g+e^2
*f)-f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^
(1/2)))/g^3-f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1
/2)-d*g^(1/2)))/g^3-b*f*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/
2)))/g^3-b*f*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3
```

3.266.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.27

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$= \frac{2gx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n)) - \frac{2f^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f+gx^2} - 4f(a-bn\log(d+ex)+$$

input
```
Integrate[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
(2*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) - (2*f^2*(a - b*n*L
og[d + e*x] + b*Log[c*(d + e*x)^n]))/(f + g*x^2) - 4*f*(a - b*n*Log[d + e*
x] + b*Log[c*(d + e*x)^n])*Log[f + g*x^2] + b*n*((g*(e*x*(2*d - e*x) - 2*(
d^2 - e^2*x^2)*Log[d + e*x]))/e^2 + (f^(3/2)*(I*Sqrt[g]*(d + e*x)*Log[d +
e*x] - e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f]
- I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*f^(3/2)*(-(Sqrt[g]*(d + e*x)*
Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e
*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 4*f*(Log[d + e*x]*Log[(
e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*S
qrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - 4*f*(Log[d + e*x]*Log[(e*(
Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]
*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])))/(4*g^3)
```

3.266.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{f^2x(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)^2}-\frac{2fx(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)}+\frac{x(a+b\log(c(d+ex)^n))}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)}-\frac{f\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^3}-\frac{f\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^3}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{bdef^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{5/2}(d^2g+e^2f)}-\frac{be^2f^2n\log(f+gx^2)}{4g^3(d^2g+e^2f)}+\frac{be^2f^2n\log(d+ex)}{2g^3(d^2g+e^2f)}-\frac{bd^2n\log(d+ex)}{2e^2g^2}-\frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}-\frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3}+\frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}$$

input
```
Int[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
(b*d*n*x)/(2*e*g^2) - (b*n*x^2)/(4*g^2) + (b*d*e*f^(3/2)*n*ArcTan[(Sqrt[g]
*x)/Sqrt[f]])/(2*g^(5/2)*(e^2*f + d^2*g)) - (b*d^2*n*Log[d + e*x])/(2*e^2*
g^2) + (b*e^2*f^2*n*Log[d + e*x])/(2*g^3*(e^2*f + d^2*g)) + (x^2*(a + b*Lo
g[c*(d + e*x)^n]))/(2*g^2) - (f^2*(a + b*Log[c*(d + e*x)^n]))/(2*g^3*(f +
g*x^2)) - (f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*
Sqrt[-f] + d*Sqrt[g])])/g^3 - (f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-
f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/g^3 - (b*e^2*f^2*n*Log[f + g*x
^2])/(4*g^3*(e^2*f + d^2*g)) - (b*f*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*
Sqrt[-f] - d*Sqrt[g]))])/g^3 - (b*f*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sq
rt[-f] + d*Sqrt[g])])/g^3
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.266.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.50

method	result
risch	$\frac{b\ln((ex+d)^n)x^2}{2g^2}-\frac{b\ln((ex+d)^n)f^2}{2g^3(g\,x^2+f)}-\frac{b\ln((ex+d)^n)f\ln(g\,x^2+f)}{g^3}-\frac{bn\,x^2}{4g^2}+\frac{bdnx}{2e\,g^2}-\frac{bn\ln(ex+d)d^4}{2e^2g(d^2g+f\,e^2)}-\frac{bn\ln(ex+d)d^2f}{2g^2(d^2g+f\,e^2)}$

input
```
int(x^5*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)/g^2*x^2-1/2*b*ln((e*x+d)^n)*f^2/g^3/(g*x^2+f)-b*ln((e*
x+d)^n)*f/g^3*ln(g*x^2+f)-1/4*b*n*x^2/g^2+1/2*b*d*n*x/e/g^2-1/2*b/e^2*n/g/
(d^2*g+e^2*f)*ln(e*x+d)*d^4-1/2*b*n/g^2/(d^2*g+e^2*f)*ln(e*x+d)*d^2*f+1/2*
b*e^2*f^2*n*ln(e*x+d)/g^3/(d^2*g+e^2*f)-1/4*b*e^2*f^2*n*ln(g*x^2+f)/g^3/(d
^2*g+e^2*f)+1/2*b*e*n/g^2*f^2/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(g*x/(f*g)
^(1/2))+b*n*f/g^3*ln(e*x+d)*ln(g*x^2+f)-b*n*f/g^3*ln(e*x+d)*ln((e*(-f*g)^(
1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-b*n*f/g^3*ln(e*x+d)*ln((e*(-f*g)
^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-b*n*f/g^3*dilog((e*(-f*g)^(1/2
)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-b*n*f/g^3*dilog((e*(-f*g)^(1/2)+g*(
e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)
*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi
*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+
b*ln(c)+a)*(1/2*x^2/g^2-1/2*f/g^2*(f/g/(g*x^2+f)+2/g*ln(g*x^2+f)))
```

3.266.5 Fricas [F]

$$\int \frac{x^5(a + b\log(c(d + ex)^n))}{(f + gx^2)^2}\,dx = \int \frac{(b\log((ex + d)^n c) + a)x^5}{(gx^2 + f)^2}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x^5*log((e*x + d)^n*c) + a*x^5)/(g^2*x^4 + 2*f*g*x^2 + f^2), x
)
```

3.266.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\log(c(d + ex)^n))}{(f + gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.266.7 Maxima [F]

$$\int \frac{x^5(a + b\log(c(d + ex)^n))}{(f + gx^2)^2}\,dx = \int \frac{(b\log((ex + d)^n c) + a)x^5}{(gx^2 + f)^2}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(f^2/(g^4*x^2 + f*g^3) - x^2/g^2 + 2*f*log(g*x^2 + f)/g^3) + b*inte
grate((x^5*log((e*x + d)^n) + x^5*log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.266.8 Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^5}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^5/(g*x^2 + f)^2, x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx=\int \frac{x^5\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

output
```
int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

3.267 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$

3.267.1 Optimal result

Integrand size = 27, antiderivative size = 344

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = &-\frac{bde\sqrt{f}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{3/2}(e^2f+d^2g)}-\frac{be^2fn\log(d+ex)}{2g^2(e^2f+d^2g)}\\&+\frac{f(a+b\log(c(d+ex)^n))}{2g^2(f+gx^2)}\\&+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}\\&+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}\\&+\frac{be^2fn\log(f+gx^2)}{4g^2(e^2f+d^2g)}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}\\&+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}\end{aligned}$$

output
```
-1/2*b*e^2*f*n*ln(e*x+d)/g^2/(d^2*g+e^2*f)+1/2*f*(a+b*ln(c*(e*x+d)^n))/g^2
/(g*x^2+f)+1/4*b*e^2*f*n*ln(g*x^2+f)/g^2/(d^2*g+e^2*f)+1/2*(a+b*ln(c*(e*x+
d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^2+1/2*(a+b*
ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2
+1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+1/2*b*n*
polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2-1/2*b*d*e*n*arctan
(x*g^(1/2)/f^(1/2))*f^(1/2)/g^(3/2)/(d^2*g+e^2*f)
```

3.267.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.32

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$= \frac{\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f+gx^2}+2(a-bn\log(d+ex)+b\log(c(d+ex)^n))\log(f+gx^2)+bn\left(\frac{\sqrt{f}(-i\sqrt{g}(d+ex\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
((2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/(f + g*x^2) + 2*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*Log[f + g*x^2] + b*n*((Sqrt[f]*((
-I)*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[
f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (S
qrt[f]*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I
*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)
) + 2*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[
g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2
*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])]
+ PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])))/(4*g^2)
```

3.267.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{x(a+b\log(c(d+ex)^n))}{g(f+gx^2)}-\frac{fx(a+b\log(c(d+ex)^n))}{g(f+gx^2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{f(a+b\log(c(d+ex)^n))}{2g^2(f+gx^2)}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^2}+$$
$$\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^2}-\frac{bde\sqrt{f}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{3/2}(d^2g+e^2f)}+\frac{be^2fn\log(f+gx^2)}{4g^2(d^2g+e^2f)}-$$
$$\frac{be^2fn\log(d+ex)}{2g^2(d^2g+e^2f)}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}+\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^2}$$

input
```
Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
-1/2*(b*d*e*Sqrt[f]*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(g^(3/2)*(e^2*f + d^2*g
)) - (b*e^2*f*n*Log[d + e*x])/(2*g^2*(e^2*f + d^2*g)) + (f*(a + b*Log[c*(d
 + e*x)^n]))/(2*g^2*(f + g*x^2)) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqr
t[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2) + ((a + b*Log[c*(d
+ e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^
2) + (b*e^2*f*n*Log[f + g*x^2])/(4*g^2*(e^2*f + d^2*g)) + (b*n*PolyLog[2,
-((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^2) + (b*n*PolyLog[2
, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2)
```

3.267.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.267.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.44

method	result
risch	$\frac{b\ln((ex+d)^n)f}{2g^2(gx^2+f)}+\frac{b\ln((ex+d)^n)\ln(gx^2+f)}{2g^2}-\frac{bn\ln(ex+d)\ln(gx^2+f)}{2g^2}+\frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2g^2}+$ bn ln(ex+d

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln((e*x+d)^n)*f/g^2/(g*x^2+f)+1/2*b*ln((e*x+d)^n)/g^2*ln(g*x^2+f)-1/
2*b*n/g^2*ln(e*x+d)*ln(g*x^2+f)+1/2*b*n/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g
*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/
2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n/g^2*dilog((e*(-f*g)^(1/2)-
g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/g^2*dilog((e*(-f*g)^(1/2)+g*(
e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*e^2*f*n*ln(e*x+d)/g^2/(d^2*g+e^2*f
)+1/4*b*e^2*f*n*ln(g*x^2+f)/g^2/(d^2*g+e^2*f)-1/2*b*e*n*f/g/(d^2*g+e^2*f)*
d/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d
)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*
b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n
)^3+b*ln(c)+a)*(1/2*f/g^2/(g*x^2+f)+1/2/g^2*ln(g*x^2+f))
```

3.267.5 Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g^2*x^4 + 2*f*g*x^2 + f^2), x
)
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.267.7 Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a*(f/(g^3*x^2 + f*g^2) + log(g*x^2 + f)/g^2) + b*integrate((x^3*log((e
*x + d)^n) + x^3*log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.267.8 Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x^2 + f)^2, x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

output
```
int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

3.268 $\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2} \, dx$

3.268.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2} \, dx = \frac{bden\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}(e^2f+d^2g)} + \frac{be^2n\log(d+ex)}{2g(e^2f+d^2g)} - \frac{a+b\log(c(d+ex)^n)}{2g(f+gx^2)} - \frac{be^2n\log(f+gx^2)}{4g(e^2f+d^2g)}$$

output
```
1/2*b*e^2*n*ln(e*x+d)/g/(d^2*g+e^2*f)+1/2*(-a-b*ln(c*(e*x+d)^n))/g/(g*x^2+
f)-1/4*b*e^2*n*ln(g*x^2+f)/g/(d^2*g+e^2*f)+1/2*b*d*e*n*arctan(x*g^(1/2)/f^
(1/2))/(d^2*g+e^2*f)/f^(1/2)/g^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.19

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2} \, dx = \frac{2bde\sqrt{g}n(f+gx^2)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) - \sqrt{f}(2ae^2f+2ad^2g-2be^2n(f+gx^2)\log(d+ex)+2b(e^2f+d^2g)\log}{4\sqrt{f}g(e^2f+d^2g)(f+gx^2)}$$

input
```
Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
(2*b*d*e*Sqrt[g]*n*(f + g*x^2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - Sqrt[f]*(2*a*
e^2*f + 2*a*d^2*g - 2*b*e^2*n*(f + g*x^2)*Log[d + e*x] + 2*b*(e^2*f + d^2*
g)*Log[c*(d + e*x)^n] + b*e^2*f*n*Log[f + g*x^2] + b*e^2*g*n*x^2*Log[f + g
*x^2]))/(4*Sqrt[f]*g*(e^2*f + d^2*g)*(f + g*x^2))
```

3.268.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2860, 479, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$

$$\Big\downarrow \text{2860}$$

$$\frac{ben\int \frac{1}{(d+ex)(gx^2+f)}dx}{2g}-\frac{a+b\log(c(d+ex)^n)}{2g(f+gx^2)}$$

$$\Big\downarrow \text{479}$$

$$\frac{ben\left(\frac{g\int \frac{d-ex}{gx^2+f}dx}{d^2g+e^2f}+\frac{e\log(d+ex)}{d^2g+e^2f}\right)}{2g}-\frac{a+b\log(c(d+ex)^n)}{2g(f+gx^2)}$$

$$\Big\downarrow \text{452}$$

$$\frac{ben\left(\frac{g\left(d\int \frac{1}{gx^2+f}dx-e\int \frac{x}{gx^2+f}dx\right)}{d^2g+e^2f}+\frac{e\log(d+ex)}{d^2g+e^2f}\right)}{2g}-\frac{a+b\log(c(d+ex)^n)}{2g(f+gx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{ben\left(\frac{g\left(\frac{d\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-e\int \frac{x}{gx^2+f}dx\right)}{d^2g+e^2f}+\frac{e\log(d+ex)}{d^2g+e^2f}\right)}{2g}-\frac{a+b\log(c(d+ex)^n)}{2g(f+gx^2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{ben\left(\frac{g\left(\frac{d\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-\frac{e\log\left(f+gx^2\right)}{2g}\right)}{d^2g+e^2f}+\frac{e\log(d+ex)}{d^2g+e^2f}\right)}{2g}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g\left(f+gx^2\right)}$$

input
```
Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n])/(g*(f + g*x^2)) + (b*e*n*((e*Log[d + e*x])
/(e^2*f + d^2*g) + (g*((d*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) -
 (e*Log[f + g*x^2])/(2*g)))/(e^2*f + d^2*g)))/(2*g)
```

3.268.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2860
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Simp[b*e*n*(p/(g*r*(q + 1)))
   Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && N
eQ[q, -1] && IGtQ[p, 0]
```

3.268.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.36 (sec) , antiderivative size = 969, normalized size of antiderivative = 6.97

method	result
risch	$-\frac{b\ln((ex+d)^n)}{2g(g\,x^2+f)} - \frac{i\pi b\,e^2 f^2 \operatorname{csgn}(i(ex+d)^n) \operatorname{csgn}(ic(ex+d)^n)^2 + i\pi b\,e^2 f^2 \operatorname{csgn}(ic) \operatorname{csgn}(ic(ex+d)^n)^2 + if g\pi b\,d^2 \operatorname{csgn}(ic) \operatorname{csgn}(ic(ex-$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/g/(g*x^2+f)*ln((e*x+d)^n)-1/4/f*(I*Pi*b*e^2*f^2*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)^2+I*Pi*b*e^2*f^2*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*f*g*
Pi*b*d^2*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*f*g*Pi*b*d^2*csgn(I*c)*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)-I*f*g*Pi*b*d^2*csgn(I*c*(e*x+d)^n)^3+I*f*g*P
i*b*d^2*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e^2*f^2*csgn(I*c)*c
sgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)-I*Pi*b*e^2*f^2*csgn(I*c*(e*x+d)^n)^3-
ln((-(-f*g)^(1/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x+4*(-f*g)^(1/2)*d
*e*f+d^2*f*g-3*e^2*f^2)*(-f*g)^(1/2)*b*d*e*g*n*x^2+ln(((-f*g)^(1/2)*d^2*g-
3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*
(-f*g)^(1/2)*b*d*e*g*n*x^2+ln((-(-f*g)^(1/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*
d*e*f*g)*x+4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*b*e^2*f*g*n*x^2+ln(((-f
*g)^(1/2)*d^2*g-3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2
*f*g-3*e^2*f^2)*b*e^2*f*g*n*x^2-2*ln(e*x+d)*b*e^2*f*g*n*x^2-ln((-(-f*g)^(1
/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x+4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3
*e^2*f^2)*(-f*g)^(1/2)*b*d*e*f*n+ln(((-f*g)^(1/2)*d^2*g-3*(-f*g)^(1/2)*e^2
*f+4*d*e*f*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*(-f*g)^(1/2)*b*d*e
*f*n+ln((-(-f*g)^(1/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x+4*(-f*g)^(1
/2)*d*e*f+d^2*f*g-3*e^2*f^2)*b*e^2*f^2*n+ln(((-f*g)^(1/2)*d^2*g-3*(-f*g)^(
1/2)*e^2*f+4*d*e*f*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*b*e^2*f^2*
n-2*b*e^2*f^2*n*ln(e*x+d)+2*ln(c)*b*d^2*f*g+2*ln(c)*b*e^2*f^2+2*a*d^2*f...
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.68

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$= \left[-\frac{2\,ae^2f^2+2\,ad^2fg+(bdegnx^2+bdefn)\sqrt{-fg}\log\left(\frac{gx^2-2\sqrt{-fg}x-f}{gx^2+f}\right)+(be^2fgnx^2+be^2f^2n)\log(gx^2+\right.}{4\,(e^2f^3g+d^2f^2g^2+(e^2f^2g^2+d^2fg^3)}$$
$$-\frac{2\,ae^2f^2+2\,ad^2fg-2\,(bdegnx^2+bdefn)\sqrt{fg}\arctan\left(\frac{\sqrt{fg}x}{f}\right)+(be^2fgnx^2+be^2f^2n)\log(gx^2+f)-2}{4\,(e^2f^3g+d^2f^2g^2+(e^2f^2g^2+d^2fg^3)x^2)}$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*e^2*f^2 + 2*a*d^2*f*g + (b*d*e*g*n*x^2 + b*d*e*f*n)*sqrt(-f*g)*
log((g*x^2 - 2*sqrt(-f*g)*x - f)/(g*x^2 + f)) + (b*e^2*f*g*n*x^2 + b*e^2*f
^2*n)*log(g*x^2 + f) - 2*(b*e^2*f*g*n*x^2 - b*d^2*f*g*n)*log(e*x + d) + 2*
(b*e^2*f^2 + b*d^2*f*g)*log(c))/(e^2*f^3*g + d^2*f^2*g^2 + (e^2*f^2*g^2 +
d^2*f*g^3)*x^2), -1/4*(2*a*e^2*f^2 + 2*a*d^2*f*g - 2*(b*d*e*g*n*x^2 + b*d*
e*f*n)*sqrt(f*g)*arctan(sqrt(f*g)*x/f) + (b*e^2*f*g*n*x^2 + b*e^2*f^2*n)*l
og(g*x^2 + f) - 2*(b*e^2*f*g*n*x^2 - b*d^2*f*g*n)*log(e*x + d) + 2*(b*e^2*
f^2 + b*d^2*f*g)*log(c))/(e^2*f^3*g + d^2*f^2*g^2 + (e^2*f^2*g^2 + d^2*f*g
^3)*x^2)]
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{1}{4}\,ben\left(\frac{e\log(gx^2+f)}{e^2fg+d^2g^2} - \frac{2\,e\log(ex+d)}{e^2fg+d^2g^2} - \frac{2\,d\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{(e^2f+d^2g)\sqrt{fg}}\right) - \frac{b\log((ex+d)^nc)}{2\,(g^2x^2+fg)} - \frac{a}{2\,(g^2x^2+fg)}$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/4*b*e*n*(e*log(g*x^2 + f)/(e^2*f*g + d^2*g^2) - 2*e*log(e*x + d)/(e^2*f
*g + d^2*g^2) - 2*d*arctan(g*x/sqrt(f*g))/((e^2*f + d^2*g)*sqrt(f*g))) - 1
/2*b*log((e*x + d)^n*c)/(g^2*x^2 + f*g) - 1/2*a/(g^2*x^2 + f*g)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.40

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{be^2n\log(gx^2+f)}{4\,(e^2fg+d^2g^2)} + \frac{be^2n\log(ex+d)}{2\,(e^2fg+d^2g^2)} + \frac{bden\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{2\,(e^2f+d^2g)\sqrt{fg}} - \frac{bn\log(ex+d)}{2\,(g^2x^2+fg)} - \frac{b\log(c)+a}{g^2x^2+fg} - \frac{be^2f\log(c)+bd^2g\log(c)+ae^2f+ad^2g}{2\,(e^2f+d^2g)(gx^2+f)g}$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
-1/4*b*e^2*n*log(g*x^2 + f)/(e^2*f*g + d^2*g^2) + 1/2*b*e^2*n*log(e*x + d)
/(e^2*f*g + d^2*g^2) + 1/2*b*d*e*n*arctan(g*x/sqrt(f*g))/((e^2*f + d^2*g)*
sqrt(f*g)) - 1/2*b*n*log(e*x + d)/(g^2*x^2 + f*g) - (b*log(c) + a)/(g^2*x^
2 + f*g) - 1/2*(b*e^2*f*log(c) + b*d^2*g*log(c) + a*e^2*f + a*d^2*g)/((e^2
*f + d^2*g)*(g*x^2 + f)*g)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.63

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \frac{b\,e^2\,n\,\ln(d+e\,x)}{2\,d^2\,g^2+2\,f\,e^2\,g}$$
$$-\frac{\ln\left(\frac{\left(b\,e^2\,f\,g\,n+b\,d\,e\,n\,\sqrt{-f\,g^3}\right)\left(x\,(2\,d^2\,e\,g^3-6\,e^3\,f\,g^2)-8\,d\,e^2\,f\,g^2\right)}{4\,(d^2\,f\,g^3+e^2\,f^2\,g^2)}+\frac{b\,d\,e^2\,g\,n}{2}+\frac{3\,b\,e^3\,g\,n\,x}{2}\right)\left(b\,e^2\,f\,g\,n+b\,d\,e\,n\,\sqrt{-f\,g^3}\right)}{4\,(d^2\,f\,g^3+e^2\,f^2\,g^2)}$$
$$-\frac{\ln\left(\frac{\left(b\,e^2\,f\,g\,n-b\,d\,e\,n\,\sqrt{-f\,g^3}\right)\left(x\,(2\,d^2\,e\,g^3-6\,e^3\,f\,g^2)-8\,d\,e^2\,f\,g^2\right)}{4\,(d^2\,f\,g^3+e^2\,f^2\,g^2)}+\frac{b\,d\,e^2\,g\,n}{2}+\frac{3\,b\,e^3\,g\,n\,x}{2}\right)\left(b\,e^2\,f\,g\,n-b\,d\,e\,n\,\sqrt{-f\,g^3}\right)}{4\,(d^2\,f\,g^3+e^2\,f^2\,g^2)}$$
$$-\frac{b\,\ln\left(c\,(d+e\,x)^n\right)}{2\,g\,(g\,x^2+f)}-\frac{a}{2\,g^2\,x^2+2\,f\,g}$$

input
```
int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

output
```
(b*e^2*n*log(d + e*x))/(2*d^2*g^2 + 2*e^2*f*g) - (log(((b*e^2*f*g*n + b*d*
e*n*(-f*g^3)^(1/2))*(x*(2*d^2*e*g^3 - 6*e^3*f*g^2) - 8*d*e^2*f*g^2))/(4*(d
^2*f*g^3 + e^2*f^2*g^2)) + (b*d*e^2*g*n)/2 + (3*b*e^3*g*n*x)/2)*(b*e^2*f*g
*n + b*d*e*n*(-f*g^3)^(1/2)))/(4*(d^2*f*g^3 + e^2*f^2*g^2)) - (log(((b*e^2
*f*g*n - b*d*e*n*(-f*g^3)^(1/2))*(x*(2*d^2*e*g^3 - 6*e^3*f*g^2) - 8*d*e^2*
f*g^2))/(4*(d^2*f*g^3 + e^2*f^2*g^2)) + (b*d*e^2*g*n)/2 + (3*b*e^3*g*n*x)/
2)*(b*e^2*f*g*n - b*d*e*n*(-f*g^3)^(1/2)))/(4*(d^2*f*g^3 + e^2*f^2*g^2)) -
 (b*log(c*(d + e*x)^n))/(2*g*(f + g*x^2)) - a/(2*f*g + 2*g^2*x^2)
```

3.269 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx^2)^2}\,dx$

3.269.1 Optimal result

Integrand size = 27, antiderivative size = 383

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx &= -\frac{bde\sqrt{g}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)}-\frac{be^2n\log(d+ex)}{2f\left(e^2f+d^2g\right)}\\&+\frac{a+b\log\left(c(d+ex)^n\right)}{2f\left(f+gx^2\right)}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}\\&-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}\\&-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}\\&+\frac{be^2n\log\left(f+gx^2\right)}{4f\left(e^2f+d^2g\right)}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}\\&-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}+\frac{bn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}\end{aligned}$$

output
```
-1/2*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)+1/2*(a+b*ln(c*(e*x+d)^n))/f/(g*x^2+
f)+ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f^2+1/4*b*e^2*n*ln(g*x^2+f)/f/(d^2*g+e
^2*f)-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+
d*g^(1/2)))/f^2-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(
-f)^(1/2)-d*g^(1/2)))/f^2+b*n*polylog(2,1+e*x/d)/f^2-1/2*b*n*polylog(2,-(e
*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2-1/2*b*n*polylog(2,(e*x+d)*g^(1
/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2-1/2*b*d*e*n*arctan(x*g^(1/2)/f^(1/2))*g^
(1/2)/f^(3/2)/(d^2*g+e^2*f)
```

3.269.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.36

$$\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx^2)^2}\,dx$$
$$= \frac{a-bn\log(d+ex)+b\log(c(d+ex)^n)}{2f^2+2fgx^2} + \frac{\log(x)\,(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f^2}$$
$$- \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))\log(f+gx^2)}{2f^2}$$
$$+ \frac{bn\left(\frac{\sqrt{f}(-i\sqrt{g}(d+ex)\log(d+ex)+e(\sqrt{f}+i\sqrt{g}x)\log(i\sqrt{f}-\sqrt{g}x))}{(e\sqrt{f}-id\sqrt{g})(\sqrt{f}+i\sqrt{g}x)} + \frac{\sqrt{f}(i\sqrt{g}(d+ex)\log(d+ex)+e(\sqrt{f}-i\sqrt{g}x)\log(i\sqrt{f}+\sqrt{g}x))}{(e\sqrt{f}+id\sqrt{g})(\sqrt{f}-i\sqrt{g}x)} - 2\left(\log\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)^2),x]
```

output
```
(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])/(2*f^2 + 2*f*g*x^2) + (Log[x
]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/f^2 - ((a - b*n*Log[d + e
*x] + b*Log[c*(d + e*x)^n])*Log[f + g*x^2])/(2*f^2) + (b*n*((Sqrt[f]*((-I)
*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f]
- Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt
[f]*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sq
rt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) -
 2*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])
] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - 2*(L
og[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + P
olyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) + 4*(Log[-((e*
x)/d)]*Log[d + e*x] + PolyLog[2, 1 + (e*x)/d])))/(4*f^2)
```

3.269.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{gx(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}-\frac{gx(a+b\log\left(c(d+ex)^n\right))}{f\left(f+gx^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{2f^2}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{2f^2}+\frac{\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{f^2}+\frac{a+b\log\left(c(d+ex)^n\right)}{2f\left(f+gx^2\right)}-\frac{bde\sqrt{g}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{3/2}\left(d^2g+e^2f\right)}+\frac{be^2n\log\left(f+gx^2\right)}{4f\left(d^2g+e^2f\right)}-\frac{be^2n\log(d+ex)}{2f\left(d^2g+e^2f\right)}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2}+\frac{bn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)^2),x]
```

output
```
-1/2*(b*d*e*Sqrt[g]*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(f^(3/2)*(e^2*f + d^2*g
)) - (b*e^2*n*Log[d + e*x])/(2*f*(e^2*f + d^2*g)) + (a + b*Log[c*(d + e*x)
^n])/(2*f*(f + g*x^2)) + (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^2
- ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/(2*f^2) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqr
t[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2) + (b*e^2*n*Log[f + g*x^2])/(4*
f*(e^2*f + d^2*g)) - (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d
*Sqrt[g]))])/(2*f^2) - (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d
*Sqrt[g])])/(2*f^2) + (b*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

3.269.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.269.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.45

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(x)}{f^2}+\frac{b\ln((ex+d)^n)}{2f(g\,x^2+f)}-\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2f^2}-\frac{b\,e^2n\ln(ex+d)}{2f(d^2g+f\,e^2)}+\frac{b\,e^2n\ln(g\,x^2+f)}{4f(d^2g+f\,e^2)}-\frac{bengd\arctan\left(\frac{gx}{\sqrt{f}}\right.}{2f(d^2g+f\,e^2)\sqrt{f}}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)/f^2*ln(x)+1/2*b*ln((e*x+d)^n)/f/(g*x^2+f)-1/2*b*ln((e*x+d)
^n)/f^2*ln(g*x^2+f)-1/2*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)+1/4*b*e^2*n*ln(g
*x^2+f)/f/(d^2*g+e^2*f)-1/2*b*e*n/f/(d^2*g+e^2*f)*g*d/(f*g)^(1/2)*arctan(g
*x/(f*g)^(1/2))-b*n/f^2*dilog((e*x+d)/d)-b*n/f^2*ln(x)*ln((e*x+d)/d)+1/2*b
*n/f^2*ln(e*x+d)*ln(g*x^2+f)-1/2*b*n/f^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e
*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)+
g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*n/f^2*dilog((e*(-f*g)^(1/2)-g*(
e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f^2*dilog((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*cs
gn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*cs
gn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*l
n(c)+a)*(1/f^2*ln(x)-1/2*g/f^2*(-f/g/(g*x^2+f)+1/g*ln(g*x^2+f)))
```

3.269.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.269.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + 2*log(x)/f^2) + b*integrat
e((log((e*x + d)^n) + log(c))/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

3.269.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)^2*x), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x\,{\left(g\,x^2+f\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)^2), x)
```

3.270 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)^2}\,dx$

3.270.1 Optimal result

Integrand size = 27, antiderivative size = 460

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx = &-\frac{ben}{2df^2x}+\frac{bdeg^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{5/2}\left(e^2f+d^2g\right)}-\frac{be^2n\log(x)}{2d^2f^2}\\ &+\frac{be^2n\log(d+ex)}{2d^2f^2}+\frac{be^2gn\log(d+ex)}{2f^2\left(e^2f+d^2g\right)}\\ &-\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{2f^2\left(f+gx^2\right)}\\ &-\frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}\\ &+\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}\\ &+\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}\\ &-\frac{be^2gn\log\left(f+gx^2\right)}{4f^2\left(e^2f+d^2g\right)}+\frac{bgn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}\\ &+\frac{bgn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}-\frac{2bgn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3}\end{aligned}$$

output
```
-1/2*b*e*n/d/f^2/x+1/2*b*d*e*g^(3/2)*n*arctan(x*g^(1/2)/f^(1/2))/f^(5/2)/(
d^2*g+e^2*f)-1/2*b*e^2*n*ln(x)/d^2/f^2+1/2*b*e^2*n*ln(e*x+d)/d^2/f^2+1/2*b
*e^2*g*n*ln(e*x+d)/f^2/(d^2*g+e^2*f)+1/2*(-a-b*ln(c*(e*x+d)^n))/f^2/x^2-1/
2*g*(a+b*ln(c*(e*x+d)^n))/f^2/(g*x^2+f)-2*g*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n
))/f^3-1/4*b*e^2*g*n*ln(g*x^2+f)/f^2/(d^2*g+e^2*f)+g*(a+b*ln(c*(e*x+d)^n))
*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^3+g*(a+b*ln(c*(e*
x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f^3-2*b*g*n
*polylog(2,1+e*x/d)/f^3+b*g*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g
^(1/2)))/f^3+b*g*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^3
```

3.270.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.30

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx = \frac{-\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{x^2}-\frac{2fg(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f+gx^2}-8g\log(x)\left(a-bn\log(d+ex)+b\log\left(c(\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)^2),x]
```

output
```
((-2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/x^2 - (2*f*g*(a - b*
n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/(f + g*x^2) - 8*g*Log[x]*(a - b*n*
Log[d + e*x] + b*Log[c*(d + e*x)^n]) + 4*g*(a - b*n*Log[d + e*x] + b*Log[c
*(d + e*x)^n])*Log[f + g*x^2] + b*n*((-2*f*(d*e*x + e^2*x^2*Log[x] + (d^2
- e^2*x^2)*Log[d + e*x]))/(d^2*x^2) + (I*Sqrt[f]*g*(Sqrt[g]*(d + e*x)*Log[
d + e*x] + I*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sq
rt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*Sqrt[f]*g*(-(Sqrt[g]*(d
 + e*x)*Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*
x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 4*g*(Log[d + e*
x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2,
 ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 4*g*(Log[d + e*x]*
Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I
*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) - 8*g*(Log[-((e*x)/d)]*Log
[d + e*x] + PolyLog[2, 1 + (e*x)/d])))/(4*f^3)
```

3.270.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{2g^2x(a+b\log\left(c(d+ex)^n\right))}{f^3\left(f+gx^2\right)}-\frac{2g(a+b\log\left(c(d+ex)^n\right))}{f^3x}+\frac{g^2x(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)^2}+\frac{a+b\log\left(c(d+ex\right.}{f^2x^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{f^3}+\frac{g\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{f^3}+\frac{g\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{f^3}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{2f^2\left(f+gx^2\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2}+\frac{bdeg^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{5/2}\left(d^2g+e^2f\right)}-\frac{be^2gn\log\left(f+gx^2\right)}{4f^2\left(d^2g+e^2f\right)}+\frac{be^2gn\log(d+ex)}{2f^2\left(d^2g+e^2f\right)}-\frac{be^2n\log(x)}{2d^2f^2}+\frac{be^2n\log(d+ex)}{2d^2f^2}+\frac{bgn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}+\frac{bgn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3}-\frac{2bgn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3}-\frac{ben}{2df^2x}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)^2),x]
```

output
```
-1/2*(b*e*n)/(d*f^2*x) + (b*d*e*g^(3/2)*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*
f^(5/2)*(e^2*f + d^2*g)) - (b*e^2*n*Log[x])/(2*d^2*f^2) + (b*e^2*n*Log[d +
 e*x])/(2*d^2*f^2) + (b*e^2*g*n*Log[d + e*x])/(2*f^2*(e^2*f + d^2*g)) - (a
 + b*Log[c*(d + e*x)^n])/(2*f^2*x^2) - (g*(a + b*Log[c*(d + e*x)^n]))/(2*f
^2*(f + g*x^2)) - (2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^3 + (
g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/f^3 + (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g
]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/f^3 - (b*e^2*g*n*Log[f + g*x^2])/(4*f^2*(
e^2*f + d^2*g)) + (b*g*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*
Sqrt[g]))])/f^3 + (b*g*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sq
rt[g])])/f^3 - (2*b*g*n*PolyLog[2, 1 + (e*x)/d])/f^3
```

3.270.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.270.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.91 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{b\ln((ex+d)^n)}{2f^2x^2}-\frac{2b\ln((ex+d)^n)g\ln(x)}{f^3}-\frac{b\ln((ex+d)^n)g}{2f^2(g\,x^2+f)}+\frac{b\ln((ex+d)^n)g\ln(g\,x^2+f)}{f^3}+\frac{2bng\,\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{f^3}+\frac{2bng\ln(x)}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln((e*x+d)^n)/f^2/x^2-2*b*ln((e*x+d)^n)/f^3*g*ln(x)-1/2*b*ln((e*x+d
)^n)*g/f^2/(g*x^2+f)+b*ln((e*x+d)^n)*g/f^3*ln(g*x^2+f)+2*b*n/f^3*g*dilog((
e*x+d)/d)+2*b*n/f^3*g*ln(x)*ln((e*x+d)/d)-b*n/f^3*g*ln(e*x+d)*ln(g*x^2+f)+
b*n/f^3*g*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g)
)+b*n/f^3*g*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*
g))+b*n/f^3*g*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+b
*n/f^3*g*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+b*e^2*
g*n*ln(e*x+d)/f^2/(d^2*g+e^2*f)+1/2*b*e^4*n/f/(d^2*g+e^2*f)/d^2*ln(e*x+d)-
1/2*b*e*n/d/f^2/x-1/2*b*e^2*n*ln(x)/d^2/f^2-1/4*b*e^2*g*n*ln(g*x^2+f)/f^2/
(d^2*g+e^2*f)+1/2*b*e*n/f^2/(d^2*g+e^2*f)*g^2*d/(f*g)^(1/2)*arctan(g*x/(f*
g)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2
*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(
I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/2/f^2/x^2
-2/f^3*g*ln(x)+1/2*g^2/f^3*(-f/g/(g*x^2+f)+2/g*ln(g*x^2+f)))
```

3.270.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.270.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((2*g*x^2 + f)/(f^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 4*g
*log(x)/f^3) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^7 + 2*f*g*x^
5 + f^2*x^3), x)
```

3.270.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)^2*x^3), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^3\,{\left(g\,x^2+f\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)^2), x)
```

3.271 $\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$

3.271.1 Optimal result

Integrand size = 27, antiderivative size = 534

$$\begin{aligned}\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx &= \frac{ax}{g^2}-\frac{bnx}{g^2}-\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}+\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}\\
&+\frac{b(d+ex)\log(c(d+ex)^n)}{eg^2}-\frac{f(a+b\log(c(d+ex)^n))}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}\\
&+\frac{f(a+b\log(c(d+ex)^n))}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{befn\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}\\
&+\frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}\\
&+\frac{befn\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}\\
&-\frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}\\
&-\frac{3b\sqrt{-f}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}\\
&+\frac{3b\sqrt{-f}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}\end{aligned}$$

output
```
a*x/g^2-b*n*x/g^2+b*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+3/4*(a+b*ln(c*(e*x+d)^n)
)*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)
-3/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(
1/2)))*(-f)^(1/2)/g^(5/2)-3/4*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)
-d*g^(1/2)))*(-f)^(1/2)/g^(5/2)+3/4*b*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^
(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)-1/4*b*e*f*n*ln(e*x+d)/g^(5/2)/(e*(-f)
^(1/2)-d*g^(1/2))+1/4*b*e*f*n*ln((-f)^(1/2)+x*g^(1/2))/g^(5/2)/(e*(-f)^(1/
2)-d*g^(1/2))+1/4*b*e*f*n*ln(e*x+d)/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/4*b
*e*f*n*ln((-f)^(1/2)-x*g^(1/2))/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/4*f*(a+
b*ln(c*(e*x+d)^n))/g^(5/2)/((-f)^(1/2)-x*g^(1/2))+1/4*f*(a+b*ln(c*(e*x+d)^
n))/g^(5/2)/((-f)^(1/2)+x*g^(1/2))
```

3.271.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.81

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$= \frac{4a\sqrt{g}x-4b\sqrt{g}nx+\frac{4b\sqrt{g}(d+ex)\log(c(d+ex)^n)}{e}-\frac{f(a+b\log(c(d+ex)^n))}{\sqrt{-f}-\sqrt{g}x}+\frac{f(a+b\log(c(d+ex)^n))}{\sqrt{-f}+\sqrt{g}x}+\frac{befn(\log(d+ex)-\log(\sqrt{-f}-}{e\sqrt{-f}+d\sqrt{g}}}{}$$

input
```
Integrate[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
(4*a*Sqrt[g]*x - 4*b*Sqrt[g]*n*x + (4*b*Sqrt[g]*(d + e*x)*Log[c*(d + e*x)^
n])/e - (f*(a + b*Log[c*(d + e*x)^n]))/(Sqrt[-f] - Sqrt[g]*x) + (f*(a + b*
Log[c*(d + e*x)^n]))/(Sqrt[-f] + Sqrt[g]*x) + (b*e*f*n*(Log[d + e*x] - Log
[Sqrt[-f] - Sqrt[g]*x]))/(e*Sqrt[-f] + d*Sqrt[g]) + 3*Sqrt[-f]*(a + b*Log[
c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] +
 (b*e*f*n*(Log[d + e*x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(-(e*Sqrt[-f]) + d*S
qrt[g]) - 3*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]
*x))/(e*Sqrt[-f] - d*Sqrt[g])] - 3*b*Sqrt[-f]*n*PolyLog[2, -((Sqrt[g]*(d +
 e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 3*b*Sqrt[-f]*n*PolyLog[2, (Sqrt[g]*(d
+ e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2))
```

3.271.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{f^2(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)^2}-\frac{2f(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)}+\frac{a+b\log(c(d+ex)^n)}{g^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{f(a+b\log(c(d+ex)^n))}{4g^{5/2}(\sqrt{-f}-\sqrt{g}x)}+\frac{f(a+b\log(c(d+ex)^n))}{4g^{5/2}(\sqrt{-f}+\sqrt{g}x)}+\frac{3\sqrt{-f}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{4g^{5/2}}-\frac{3\sqrt{-f}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{4g^{5/2}}+\frac{ax}{g^2}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg^2}-\frac{3b\sqrt{-f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}+\frac{3b\sqrt{-f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4g^{5/2}}-\frac{befn\log(d+ex)}{4g^{5/2}(e\sqrt{-f}-d\sqrt{g})}+\frac{befn\log(d+ex)}{4g^{5/2}(d\sqrt{g}+e\sqrt{-f})}-\frac{befn\log(\sqrt{-f}-\sqrt{g}x)}{4g^{5/2}(d\sqrt{g}+e\sqrt{-f})}+\frac{befn\log(\sqrt{-f}+\sqrt{g}x)}{4g^{5/2}(e\sqrt{-f}-d\sqrt{g})}-\frac{bnx}{g^2}$$

input
```
Int[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
(a*x)/g^2 - (b*n*x)/g^2 - (b*e*f*n*Log[d + e*x])/(4*(e*Sqrt[-f] - d*Sqrt[g
])*g^(5/2)) + (b*e*f*n*Log[d + e*x])/(4*(e*Sqrt[-f] + d*Sqrt[g])*g^(5/2))
+ (b*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) - (f*(a + b*Log[c*(d + e*x)^n])
)/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*(a + b*Log[c*(d + e*x)^n]))/(4*g
^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*f*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*(e
*Sqrt[-f] + d*Sqrt[g])*g^(5/2)) + (3*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*L
og[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2)) + (b*
e*f*n*Log[Sqrt[-f] + Sqrt[g]*x])/(4*(e*Sqrt[-f] - d*Sqrt[g])*g^(5/2)) - (3
*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqr
t[-f] - d*Sqrt[g])])/(4*g^(5/2)) - (3*b*Sqrt[-f]*n*PolyLog[2, -((Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(4*g^(5/2)) + (3*b*Sqrt[-f]*n*PolyLo
g[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2))
```

3.271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.271.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 1619, normalized size of antiderivative = 3.03

method	result	size
risch	Expression too large to display	1619

input
```
int(x^4*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-b*d*n/e/g^2-1/4*b*e^4*n/g*f^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(
-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2+1/
4*b*e^2*n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(
-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^2*d^2-1/4*b*e^2*n*f*ln(
e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(
e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*d^2+1/4*b*e^4*n/g*f^2*ln(e*x+d)/(d^2
*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)
/(e*(-f*g)^(1/2)+d*g))*x^2-3/4*b*n/g^2*f/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2
)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+3/4*b*n/g^2*f/(-f*g)^(1/2)*dilog((e
*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-3/2*b/g^2*f/(f*g)^(1/2)
*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)+1/4*b*e^2*n/g
*f^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(
1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d^2-1/4*b*e^2*n/g*f^2*ln(e*x+d)/
(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-
d*g)/(e*(-f*g)^(1/2)-d*g))*d^2+b*n/g^2*f*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*
g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*e^2/g^2*f*x/(e^2*g*x^2
+e^2*f)*ln((e*x+d)^n)+b*ln((e*x+d)^n)/g^2*x+b/e/g^2*d*ln((e*x+d)^n)-1/4*b*
e*n/g^2*f/(d^2*g+e^2*f)*d*ln(g*(e*x+d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)-1/2*b*
e^2*n/g^2*f^2/(d^2*g+e^2*f)/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(
f*g)^(1/2))-1/2*b*e^2/g^2*f*x/(e^2*g*x^2+e^2*f)*n*ln(e*x+d)+1/2*b*e^3*n...
```

3.271.5 Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^4}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*log((e*x + d)^n*c) + a*x^4)/(g^2*x^4 + 2*f*g*x^2 + f^2), x
)
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log\left(c(d+ex)^n\right))}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.271.7 Maxima [F]

$$\int \frac{x^4(a+b\log\left(c(d+ex)^n\right))}{\left(f+gx^2\right)^2}\,dx = \int \frac{(b\log\left((ex+d)^nc\right)+a)x^4}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a*(f*x/(g^3*x^2 + f*g^2) - 3*f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2) +
 2*x/g^2) + b*integrate((x^4*log((e*x + d)^n) + x^4*log(c))/(g^2*x^4 + 2*f
*g*x^2 + f^2), x)
```

3.271.8 Giac [F]

$$\int \frac{x^4(a+b\log\left(c(d+ex)^n\right))}{\left(f+gx^2\right)^2}\,dx = \int \frac{(b\log\left((ex+d)^nc\right)+a)x^4}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^4/(g*x^2 + f)^2, x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

output
```
int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

3.272 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{\left(f+gx^2\right)^2}\,dx$

3.272.1 Optimal result

Integrand size = 27, antiderivative size = 491

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{\left(f+gx^2\right)^2}\,dx = \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}-\frac{ben\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}+\frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}-\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$

output
```
1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1
/2)))/g^(3/2)/(-f)^(1/2)-1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1
/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/4*b*n*polylog(2,-(e*x+
d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)+1/4*b*n*polylog(2,
(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(3/2)/(-f)^(1/2)+1/4*b*e*n*ln(
e*x+d)/g^(3/2)/(e*(-f)^(1/2)-d*g^(1/2))-1/4*b*e*n*ln((-f)^(1/2)+x*g^(1/2))
/g^(3/2)/(e*(-f)^(1/2)-d*g^(1/2))-1/4*b*e*n*ln(e*x+d)/g^(3/2)/(e*(-f)^(1/2
)+d*g^(1/2))+1/4*b*e*n*ln((-f)^(1/2)-x*g^(1/2))/g^(3/2)/(e*(-f)^(1/2)+d*g^
(1/2))+1/4*(a+b*ln(c*(e*x+d)^n))/g^(3/2)/((-f)^(1/2)-x*g^(1/2))+1/4*(-a-b*
ln(c*(e*x+d)^n))/g^(3/2)/((-f)^(1/2)+x*g^(1/2))
```

3.272.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.78

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$= \frac{\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x} - \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x} + \frac{ben\left(-\log(d+ex)+\log\left(\sqrt{-f}-\sqrt{g}x\right)\right)}{e\sqrt{-f}+d\sqrt{g}} + \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}} + \frac{ben(\log(d\cdots}{}}{4g^{3/2}}$$

input
```
Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
((a + b*Log[c*(d + e*x)^n])/(Sqrt[-f] - Sqrt[g]*x) - (a + b*Log[c*(d + e*x
)^n])/(Sqrt[-f] + Sqrt[g]*x) + (b*e*n*(-Log[d + e*x] + Log[Sqrt[-f] - Sqrt
[g]*x]))/(e*Sqrt[-f] + d*Sqrt[g]) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sq
rt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/Sqrt[-f] + (b*e*n*(Log[d +
 e*x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(e*Sqrt[-f] - d*Sqrt[g]) + (f*(a + b*L
og[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])
])/(-f)^(3/2) + (b*f*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sq
rt[g]))])/(-f)^(3/2) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d
*Sqrt[g])])/Sqrt[-f])/(4*g^(3/2))
```

3.272.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{a+b\log(c(d+ex)^n)}{g(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))}{g(f+gx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{4\sqrt{-f}g^{3/2}}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{4\sqrt{-f}g^{3/2}}-\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}+\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4\sqrt{-f}g^{3/2}}+\frac{ben\log(d+ex)}{4g^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)}-\frac{ben\log(d+ex)}{4g^{3/2}\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4g^{3/2}\left(d\sqrt{g}+e\sqrt{-f}\right)}-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4g^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)}$$

input
```
Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

output
```
(b*e*n*Log[d + e*x])/(4*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - (b*e*n*Log[d +
 e*x])/(4*(e*Sqrt[-f] + d*Sqrt[g])*g^(3/2)) + (a + b*Log[c*(d + e*x)^n])/(
4*g^(3/2)*(Sqrt[-f] - Sqrt[g]*x)) - (a + b*Log[c*(d + e*x)^n])/(4*g^(3/2)*
(Sqrt[-f] + Sqrt[g]*x)) + (b*e*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*(e*Sqrt[-f]
 + d*Sqrt[g])*g^(3/2)) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sq
rt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2)) - (b*e*n*Log[Sqr
t[-f] + Sqrt[g]*x])/(4*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - ((a + b*Log[c*(
d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*
Sqrt[-f]*g^(3/2)) - (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*
Sqrt[g]))])/(4*Sqrt[-f]*g^(3/2)) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*
Sqrt[-f] + d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2))
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.272.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.31 (sec) , antiderivative size = 1521, normalized size of antiderivative = 3.10

method	result	size
risch	Expression too large to display	1521

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*e^2/(e^2*g*x^2+e^2*f)/g*x*n*ln(e*x+d)-1/2*b*e^2/(e^2*g*x^2+e^2*f)/g*
x*ln((e*x+d)^n)-1/2*b/g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)
^(1/2))*n*ln(e*x+d)+1/2*b/g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(
f*g)^(1/2))*ln((e*x+d)^n)+1/4*b*e*n/g/(d^2*g+e^2*f)*d*ln(g*(e*x+d)^2-2*(e*
x+d)*d*g+d^2*g+f*e^2)+1/2*b*e^2*n*f/g/(d^2*g+e^2*f)/(f*g)^(1/2)*arctan(1/2
*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))-1/4*b*e^2*n*g*ln(e*x+d)/(d^2*g+e^2*f)/
(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)
^(1/2)+d*g))*x^2*d^2-1/4*b*e^4*n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*
f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^
2+1/4*b*e^2*n*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln(
(e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*d^2+1/4*b*e^4*n*f
*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)
+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2-1/2*b*e^3*n*ln(e*x+d)/(d^2*g+e^2
*f)/(e^2*g*x^2+e^2*f)*x^2*d-1/4*b*e^2*n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x
^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d
*g))*d^2-1/4*b*e^4*n*f^2/g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g
)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/4*b*e^2*
n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1
/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*d^2+1/4*b*e^4*n*f^2/g*ln(e*x+d)/(
d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d...
```

3.272.5 Fricas [F]

$$\int \frac{x^2(a + b\log{(c(d+ex)^n)})}{(f+gx^2)^2}\,dx = \int \frac{(b\log{((ex+d)^n c)} + a)x^2}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g^2*x^4 + 2*f*g*x^2 + f^2), x
)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.272.7 Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(x/(g^2*x^2 + f*g) - arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g)) + b*integ
rate((x^2*log((e*x + d)^n) + x^2*log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.272.8 Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x^2 + f)^2, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

output
```
int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

3.273 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx^2)^2}\,dx$

3.273.1 Optimal result

Integrand size = 24, antiderivative size = 503

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx &= \frac{ben\log(d+ex)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}+\frac{ben\log(d+ex)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}\\
&\quad-\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\\
&\quad-\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}\\
&\quad-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}\\
&\quad-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}\\
&\quad+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}\\
&\quad+\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}-\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}\end{aligned}$$

output
```
-1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(
1/2)))/(-f)^(3/2)/g^(1/2)+1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(
1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/4*b*n*polylog(2,-(e*x
+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)-1/4*b*n*polylog(2
,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/4*b*e*n*ln
(e*x+d)/f/g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/4*b*e*n*ln((-f)^(1/2)-x*g^(1/
2))/f/g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))+1/4*b*e*n*ln(e*x+d)/g^(1/2)/(e*(-f)
^(3/2)+d*f*g^(1/2))-1/4*b*e*n*ln((-f)^(1/2)+x*g^(1/2))/g^(1/2)/(e*(-f)^(3/
2)+d*f*g^(1/2))+1/4*(-a-b*ln(c*(e*x+d)^n))/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2)
)+1/4*(a+b*ln(c*(e*x+d)^n))/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

3.273.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log (c(d+ex)^n)}{(f+gx^2)^2}\,dx = \frac{1}{4}\left(\frac{a+b\log (c(d+ex)^n)}{f\left(\sqrt{-f}\sqrt{g}+gx\right)}+\frac{a+b\log (c(d+ex)^n)}{(-f)^{3/2}\sqrt{g}+fgx}+\frac{ben\left(\log(d+ex)-\log\left(\sqrt{-f}-\sqrt{g}x\right)\right)}{e\sqrt{-f}f\sqrt{g}+dfg}+\frac{f(a+b\log (c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}\sqrt{g}}+\frac{ben\left(\log(d+ex)-\log\left(\sqrt{-f}+\sqrt{g}x\right)\right)}{e(-f)^{3/2}\sqrt{g}+dfg}+\frac{(a+b\log (c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}\sqrt{g}}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}\sqrt{g}}+\frac{bfn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}\sqrt{g}}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2)^2,x]
```

output
```
((a + b*Log[c*(d + e*x)^n])/(f*(Sqrt[-f]*Sqrt[g] + g*x)) + (a + b*Log[c*(d
 + e*x)^n])/((-f)^(3/2)*Sqrt[g] + f*g*x) + (b*e*n*(Log[d + e*x] - Log[Sqrt
[-f] - Sqrt[g]*x]))/(e*Sqrt[-f]*f*Sqrt[g] + d*f*g) + (f*(a + b*Log[c*(d +
e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/((-f)^(
5/2)*Sqrt[g]) + (b*e*n*(Log[d + e*x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(e*(-f)
^(3/2)*Sqrt[g] + d*f*g) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + S
qrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/((-f)^(3/2)*Sqrt[g]) + (b*n*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/((-f)^(3/2)*Sqrt[g])
+ (b*f*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/((-f)^(
5/2)*Sqrt[g]))/4
```

3.273.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx^2)^2}dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(-\frac{g(a+b\log\left(c(d+ex)^n\right))}{2f\left(-fg-g^2x^2\right)}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{4(-f)^{3/2}\sqrt{g}}+\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{4(-f)^{3/2}\sqrt{g}}+\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}-\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{3/2}\sqrt{g}}+\frac{ben\log(d+ex)}{4f\sqrt{g}\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{ben\log(d+ex)}{4\sqrt{g}\left(df\sqrt{g}+e(-f)^{3/2}\right)}-\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\sqrt{g}\left(d\sqrt{g}+e\sqrt{-f}\right)}-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\sqrt{g}\left(df\sqrt{g}+e(-f)^{3/2}\right)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2)^2,x]
```

output
```
(b*e*n*Log[d + e*x])/(4*f*(e*Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) + (b*e*n*Log[d
 + e*x])/(4*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) - (a + b*Log[c*(d + e*x)
^n])/(4*f*Sqrt[g]*(Sqrt[-f] - Sqrt[g]*x)) + (a + b*Log[c*(d + e*x)^n])/(4*
f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*f
*(e*Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(S
qrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g]) -
(b*e*n*Log[Sqrt[-f] + Sqrt[g]*x])/(4*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g])
 + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f]
- d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g]) + (b*n*PolyLog[2, -((Sqrt[g]*(d + e*
x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(4*(-f)^(3/2)*Sqrt[g]) - (b*n*PolyLog[2, (
Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g])
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.273.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.31 (sec) , antiderivative size = 1406, normalized size of antiderivative = 2.80

method	result	size
risch	Expression too large to display	1406

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*e^2/f/(e^2*g*x^2+e^2*f)*x*n*ln(e*x+d)+1/2*b*e^2/f/(e^2*g*x^2+e^2*f)
*x*ln((e*x+d)^n)-1/2*b/f/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g
)^(1/2))*n*ln(e*x+d)+1/2*b/f/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/
(f*g)^(1/2))*ln((e*x+d)^n)-1/4*b*e*n/f/(d^2*g+e^2*f)*d*ln(g*(e*x+d)^2-2*(e
*x+d)*d*g+d^2*g+f*e^2)-1/2*b*e^2*n/(d^2*g+e^2*f)/(f*g)^(1/2)*arctan(1/2*(2
*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))+1/4*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)/(e^
2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1
/2)+d*g))*x^2*d^2*g^2+1/4*b*e^4*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f
)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^2
*g-1/4*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln
((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*d^2*g^2-1/4*b*e^
4*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1
/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*g+1/2*b*e^3*n*ln(e*x+d)/f/(d^
2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x^2*d*g+1/4*b*e^2*n*ln(e*x+d)/(d^2*g+e^2*f)/(
e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^
(1/2)+d*g))*d^2*g+1/4*b*e^4*n*ln(e*x+d)*f/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/
(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/4*b
*e^2*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)
^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*d^2*g-1/4*b*e^4*n*ln(e*x+d)*f/
(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+...
```

3.273.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.273.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a*(x/(f*g*x^2 + f^2) + arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f)) + b*integr
ate((log((e*x + d)^n) + log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.273.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(g*x^2 + f)^2, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^2, x)
```

3.274 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)^2}\,dx$

3.274.1 Optimal result

Integrand size = 27, antiderivative size = 560

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx &= \frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}-\frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}\\
&-\frac{be\sqrt{g}n\log(d+ex)}{4f\left(e(-f)^{3/2}+df\sqrt{g}\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}\\
&+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}\\
&-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}\\
&-\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}}\\
&+\frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f\left(e(-f)^{3/2}+df\sqrt{g}\right)}\\
&+\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}\\
&+\frac{3b\sqrt{g}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}\\
&-\frac{3b\sqrt{g}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}}\end{aligned}$$

output
```
b*e*n*ln(x)/d/f^2-b*e*n*ln(e*x+d)/d/f^2+(-a-b*ln(c*(e*x+d)^n))/f^2/x-3/4*(
a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))
*g^(1/2)/(-f)^(5/2)+3/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/
(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(5/2)+3/4*b*n*polylog(2,-(e*x+d)*g^
(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(5/2)-3/4*b*n*polylog(2,(e*x+
d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(5/2)-1/4*b*e*n*ln(e*x+d
)*g^(1/2)/f^2/(e*(-f)^(1/2)+d*g^(1/2))+1/4*b*e*n*ln((-f)^(1/2)-x*g^(1/2))*
g^(1/2)/f^2/(e*(-f)^(1/2)+d*g^(1/2))-1/4*b*e*n*ln(e*x+d)*g^(1/2)/f/(e*(-f)
^(3/2)+d*f*g^(1/2))+1/4*b*e*n*ln((-f)^(1/2)+x*g^(1/2))*g^(1/2)/f/(e*(-f)^(
3/2)+d*f*g^(1/2))+1/4*(a+b*ln(c*(e*x+d)^n))*g^(1/2)/f^2/((-f)^(1/2)-x*g^(1
/2))-1/4*(a+b*ln(c*(e*x+d)^n))*g^(1/2)/f^2/((-f)^(1/2)+x*g^(1/2))
```

3.274.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\frac{1}{4}\left(\frac{4ben(\log(x)-\log(d+ex))}{df^2}-\frac{4(a+b\log\left(c(d+ex)^n\right))}{f^2x}+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{f^2\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{f^2\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{be\sqrt{g}n\left(-\log(d+ex)+\log\left(\sqrt{-f}-\sqrt{g}x\right)\right)}{f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}-\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}+\frac{be\sqrt{g}n\left(\log(d+ex)-\log\left(\sqrt{-f}+\sqrt{g}x\right)\right)}{f^2\left(e\sqrt{-f}-d\sqrt{g}\right)}+\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}+\frac{3b\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}-\frac{3b\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)^2),x]
```

output
```
((4*b*e*n*(Log[x] - Log[d + e*x]))/(d*f^2) - (4*(a + b*Log[c*(d + e*x)^n])
)/(f^2*x) + (Sqrt[g]*(a + b*Log[c*(d + e*x)^n]))/(f^2*(Sqrt[-f] - Sqrt[g]*
x)) - (Sqrt[g]*(a + b*Log[c*(d + e*x)^n]))/(f^2*(Sqrt[-f] + Sqrt[g]*x)) +
(b*e*Sqrt[g]*n*(-Log[d + e*x] + Log[Sqrt[-f] - Sqrt[g]*x]))/(f^2*(e*Sqrt[-
f] + d*Sqrt[g])) - (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f]
- Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2) + (b*e*Sqrt[g]*n*(Log[
d + e*x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(f^2*(e*Sqrt[-f] - d*Sqrt[g])) + (3
*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt
[-f] - d*Sqrt[g])])/(-f)^(5/2) + (3*b*Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(d +
 e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(5/2) - (3*b*Sqrt[g]*n*PolyLog[2,
(Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2))/4
```

3.274.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x^2}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{f\left(f+gx^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}-\frac{3\sqrt{g}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{4(-f)^{5/2}}+\frac{3\sqrt{g}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{4(-f)^{5/2}}-\frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}+\frac{3b\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}-\frac{3b\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{5/2}}-\frac{be\sqrt{g}n\log(d+ex)}{4f\left(df\sqrt{g}+e(-f)^{3/2}\right)}+\frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f\left(df\sqrt{g}+e(-f)^{3/2}\right)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)^2),x]
```

output
```
(b*e*n*Log[x])/(d*f^2) - (b*e*n*Log[d + e*x])/(d*f^2) - (b*e*Sqrt[g]*n*Log
[d + e*x])/(4*f^2*(e*Sqrt[-f] + d*Sqrt[g])) - (b*e*Sqrt[g]*n*Log[d + e*x])
/(4*f*(e*(-f)^(3/2) + d*f*Sqrt[g])) - (a + b*Log[c*(d + e*x)^n])/(f^2*x) +
 (Sqrt[g]*(a + b*Log[c*(d + e*x)^n]))/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sq
rt[g]*(a + b*Log[c*(d + e*x)^n]))/(4*f^2*(Sqrt[-f] + Sqrt[g]*x)) + (b*e*Sq
rt[g]*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*f^2*(e*Sqrt[-f] + d*Sqrt[g])) - (3*S
qrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-
f] + d*Sqrt[g])])/(4*(-f)^(5/2)) + (b*e*Sqrt[g]*n*Log[Sqrt[-f] + Sqrt[g]*x
])/(4*f*(e*(-f)^(3/2) + d*f*Sqrt[g])) + (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^
n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*(-f)^(5/2
)) + (3*b*Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[
g]))])/(4*(-f)^(5/2)) - (3*b*Sqrt[g]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*S
qrt[-f] + d*Sqrt[g])])/(4*(-f)^(5/2))
```

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.274.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.25 (sec) , antiderivative size = 1619, normalized size of antiderivative = 2.89

method	result	size
risch	Expression too large to display	1619

input
```
int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*b/f^2*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln
(e*x+d)-1/2*b*e^2/f^2*g*x/(e^2*g*x^2+e^2*f)*ln((e*x+d)^n)-b*ln((e*x+d)^n)/
f^2/x-1/2*b*e^3*n/f^2*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x^2*d-
1/2*b*e^2*n/f^2*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x*d^2-1/2*b*
n/f^2*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)
^(1/2)+d*g))+1/2*b*n/f^2*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*
x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/4*b*e*n/f^2*g/(d^2*g+e^2*f)*d*ln(g*(e*x+
d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)+1/2*b*e^2*n/f*g/(d^2*g+e^2*f)/(f*g)^(1/2)*
arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))+1/2*b*e^2/f^2*g*x/(e^2*g*x^2
+e^2*f)*n*ln(e*x+d)-1/2*b*e^3*n/f*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2
*f)*d-1/2*b*e^4*n/f*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x-1/4*b*e^
4*n*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^
(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/4*b*e^4*n*g*ln(e*x+d)/(d^2*g+
e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e
*(-f*g)^(1/2)-d*g))-1/4*b*e^4*n/f*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e
^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))
*x^2+1/4*b*e^4*n/f*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1
/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2-1/4*b*e^2*
n/f*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g
)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d^2+1/4*b*e^2*n/f*g^2*ln(e...
```

3.274.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.274.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((3*g*x^2 + 2*f)/(f^2*g*x^3 + f^3*x) + 3*g*arctan(g*x/sqrt(f*g))/(s
qrt(f*g)*f^2)) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^6 + 2*f*g*
x^4 + f^2*x^2), x)
```

3.274.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)^2*x^2), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^2\,{\left(g\,x^2+f\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)^2), x)
```

3.275 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx$

3.275.1 Optimal result

Integrand size = 26, antiderivative size = 326

$$\begin{aligned}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx &= \frac{bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}} \\ &- \frac{bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} \\ &- \frac{bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} \\ &+ \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} \\ &- \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} \\ &- \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}\end{aligned}$$

output
```
1/2*b*n*arcsinh(1/2*x*g^(1/2)*2^(1/2))^2/g^(1/2)+arcsinh(1/2*x*g^(1/2)*2^(
1/2))*(a+b*ln(c*(e*x+d)^n))/g^(1/2)-b*n*arcsinh(1/2*x*g^(1/2)*2^(1/2))*ln(
1+e*(1/2*x*g^(1/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(1/2)/(d*g^(1/2)-(d^2*
g+2*e^2)^(1/2)))/g^(1/2)-b*n*arcsinh(1/2*x*g^(1/2)*2^(1/2))*ln(1+e*(1/2*x*
g^(1/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(1/2)/(d*g^(1/2)+(d^2*g+2*e^2)^(1
/2)))/g^(1/2)-b*n*polylog(2,-e*(1/2*x*g^(1/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2
))*2^(1/2)/(d*g^(1/2)-(d^2*g+2*e^2)^(1/2)))/g^(1/2)-b*n*polylog(2,-e*(1/2*
x*g^(1/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(1/2)/(d*g^(1/2)+(d^2*g+2*e^2)^
(1/2)))/g^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx$$
$$= \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(2a+bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)-2bn\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)-2bn\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)\right.}{2\sqrt{g}}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[2 + g*x^2],x]
```

output
```
(ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*(2*a + b*n*ArcSinh[(Sqrt[g]*x)/Sqrt[2]] - 2*
b*n*Log[1 + (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] - Sqrt[2
*e^2 + d^2*g])] - 2*b*n*Log[1 + (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])
/(d*Sqrt[g] + Sqrt[2*e^2 + d^2*g])] + 2*b*Log[c*(d + e*x)^n]) - 2*b*n*Poly
Log[2, (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(-(d*Sqrt[g]) + Sqrt[2*e
^2 + d^2*g])] - 2*b*n*PolyLog[2, -((Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2
]])/(d*Sqrt[g] + Sqrt[2*e^2 + d^2*g]))])/(2*Sqrt[g])
```

3.275.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2851, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{gx^2+2}}\,dx$$

$$\Big\downarrow \text{2851}$$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}}-ben\int \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{\sqrt{g}(d+ex)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}}-\frac{ben\int \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{d+ex}dx}{\sqrt{g}}$$

$$\Big\downarrow \text{6242}$$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}}-\frac{ben\int \frac{\sqrt{\frac{gx^2}{2}+1}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{\frac{\sqrt{g}d}{\sqrt{2}}+\frac{e\sqrt{g}x}{\sqrt{2}}}d\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{\sqrt{g}}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}}-$$
$$\frac{ben\left(\int \frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{\frac{\sqrt{g}d}{\sqrt{2}}+ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}-\frac{\sqrt{gd^2+2e^2}}{\sqrt{2}}}d\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)+\int \frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{\frac{\sqrt{g}d}{\sqrt{2}}+ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}+\frac{\sqrt{gd^2+2e^2}}{\sqrt{2}}}d\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}}{\sqrt{}}\right.}{2e}\right.}{\sqrt{g}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}}-$$
$$\frac{ben\left(-\frac{\int \log\left(\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}e}{d\sqrt{g}-\sqrt{gd^2+2e^2}}+1\right)d\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{e}-\frac{\int \log\left(\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}e}{\sqrt{g}d+\sqrt{gd^2+2e^2}}+1\right)d\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{e}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}ee}{d\sqrt{}}\right.}{e}\right.}{\sqrt{g}}$$

$$\Big\downarrow \text{2715}$$

3.275. $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{ben\left(-\frac{\int e^{-\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}\log\left(\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}e}{d\sqrt{g}-\sqrt{gd^2+2e^2}}+1\right)de^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{e} - \frac{\int e^{-\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}\log\left(\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}e}{\sqrt{g}d+\sqrt{gd^2+2e^2}}+1\right)de^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{e}\right.}{\sqrt{g}}$$

$\big\downarrow$ 2838

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{ben\left(\frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{gd^2+2e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{\sqrt{g}d+\sqrt{gd^2+2e^2}}\right)}{e} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{d^2g+2e^2}}+1\right)}{e} + \frac{\operatorname{arcs}}{}\right.}{\sqrt{g}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/Sqrt[2 + g*x^2],x]
```

output
```
(ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt[g] - (b*e*n
*(-1/2*ArcSinh[(Sqrt[g]*x)/Sqrt[2]]^2/e + (ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*Lo
g[1 + (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] - Sqrt[2*e^2 +
 d^2*g])])/e + (ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*Log[1 + (Sqrt[2]*e*E^ArcSinh[
(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] + Sqrt[2*e^2 + d^2*g])])/e + PolyLog[2, -
((Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] - Sqrt[2*e^2 + d^2*
g]))]/e + PolyLog[2, -((Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[
g] + Sqrt[2*e^2 + d^2*g]))]/e))/Sqrt[g]
```

3.275. $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx$

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.275. $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx$

3.275.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{\sqrt{g\,x^{2}+2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x)
```

3.275.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{2+gx^{2}}}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{\sqrt{gx^{2}+2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x^2 + 2)*b*log((e*x + d)^n*c) + sqrt(g*x^2 + 2)*a)/(g*x^2
 + 2), x)
```

3.275.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{2+gx^{2}}}\,dx=\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{gx^{2}+2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+2)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/sqrt(g*x**2 + 2), x)
```

3.275.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx^2+2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/sqrt(g*x^2 + 2), x) + a*arcsinh(1/
2*sqrt(2)*sqrt(g)*x)/sqrt(g)
```

3.275.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx^2+2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/sqrt(g*x^2 + 2), x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{(d+e\,x)}^n\right)}{\sqrt{g\,x^2+2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(g*x^2 + 2)^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(g*x^2 + 2)^(1/2), x)
```

3.276 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx$

3.276.1 Optimal result

Integrand size = 26, antiderivative size = 506

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx = \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} + \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}}$$

output
```
1/2*b*n*arcsinh(x*g^(1/2)/f^(1/2))^2*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*
x^2+f)^(1/2)+arcsinh(x*g^(1/2)/f^(1/2))*(a+b*ln(c*(e*x+d)^n))*f^(1/2)*(1+g
*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-b*n*arcsinh(x*g^(1/2)/f^(1/2))*ln(1+
e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2)-(d^2*g+e^2*f)^(
1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-b*n*arcsinh(x*g^(
1/2)/f^(1/2))*ln(1+e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1
/2)+(d^2*g+e^2*f)^(1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2
)-b*n*polylog(2,-e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2
)-(d^2*g+e^2*f)^(1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-
b*n*polylog(2,-e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2)+
(d^2*g+e^2*f)^(1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)
```

3.276.2 Mathematica [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x^2],x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x^2], x]
```

3.276.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2853, 2851, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx$$

$$\Big\downarrow \ 2853$$

$$\frac{\sqrt{\frac{gx^2}{f}+1}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{\frac{gx^2}{f}+1}}dx}{\sqrt{f+gx^2}}$$

$\downarrow$ 2851

$$\frac{\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}-ben\int\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}(d+ex)}dx\right)}{\sqrt{f+gx^2}}$$

$\downarrow$ 27

$$\frac{\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}-\frac{be\sqrt{f}n\int\frac{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex}dx}{\sqrt{g}}\right)}{\sqrt{f+gx^2}}$$

$\downarrow$ 6242

$$\frac{\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}-\frac{be\sqrt{f}n\int\frac{\sqrt{\frac{gx^2}{f}+1}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\frac{\sqrt{g}d}{\sqrt{f}}+\frac{e\sqrt{g}x}{\sqrt{f}}}d\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}\right)}{\sqrt{f+gx^2}}$$

$\downarrow$ 6095

$$\frac{\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}-\frac{be\sqrt{f}n\left(\int\frac{e^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\frac{\sqrt{g}d}{\sqrt{f}}+ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}-\frac{\sqrt{gd^2+e^2f}}{\sqrt{f}}}d\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)+\int\frac{e^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}}{\frac{\sqrt{g}d}{\sqrt{f}}+ee^{\text{arcsinh}\left(\right.}}\right.}{\sqrt{g}}\right.}{\sqrt{f+gx^2}}$$

$\downarrow$ 2620

$$\frac{\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}-\frac{be\sqrt{f}n\left(-\frac{\int\log\left(\frac{e^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}e}{d\sqrt{g}-\sqrt{gd^2+e^2f}}+1\right)d\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e}-\frac{\int\log\left(\frac{e^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{}}{\sqrt{g}d+\sqrt{gd^2+e^2f}}\right.}{}\right.}{}\right.}{\sqrt{f}}$$

$\downarrow$ 2715

3.276. $\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx$

$$\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{be\sqrt{f}n\left(-\frac{\int e^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\log\left(\frac{e^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}e}{d\sqrt{g}-\sqrt{gd^2+e^2f}}+1\right)de^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}}{e} - \frac{\int e^{-\text{ar}}\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2838

$$\frac{\sqrt{\frac{gx^2}{f}+1}\left(\frac{\sqrt{f}\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{be\sqrt{f}n\left(\frac{\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{gd^2+e^2f}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{\sqrt{g}d+\sqrt{gd^2+e^2f}}\right)}{e}+\frac{\text{ar}\cdots}{\cdots}\right.}{\cdots}\right.}{\sqrt{f+gx^2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x^2],x]
```

output
```
(Sqrt[1 + (g*x^2)/f]*((Sqrt[f]*ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*(a + b*Log[c*(
d + e*x)^n]))/Sqrt[g] - (b*e*Sqrt[f]*n*(-1/2*ArcSinh[(Sqrt[g]*x)/Sqrt[f]]^
2/e + (ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + (e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[
f]]*Sqrt[f])/(d*Sqrt[g] - Sqrt[e^2*f + d^2*g])])/e + (ArcSinh[(Sqrt[g]*x)/
Sqrt[f]]*Log[1 + (e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*Sqrt[f])/(d*Sqrt[g] + S
qrt[e^2*f + d^2*g])])/e + PolyLog[2, -((e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*S
qrt[f])/(d*Sqrt[g] - Sqrt[e^2*f + d^2*g]))]/e + PolyLog[2, -((e*E^ArcSinh[
(Sqrt[g]*x)/Sqrt[f]]*Sqrt[f])/(d*Sqrt[g] + Sqrt[e^2*f + d^2*g]))]/e))/Sqrt
[g]))/Sqrt[f + g*x^2]
```

3.276. $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx$

3.276.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 2853
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> Simp[Sqrt[1 + (g/f)*x^2]/Sqrt[f + g*x^2]   Int[(a + b
*Log[c*(d + e*x)^n])/Sqrt[1 + (g/f)*x^2], x], x] /; FreeQ[{a, b, c, d, e, f
, g, n}, x] &&  !GtQ[f, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.276.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{\sqrt{g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x)
```

3.276.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f+gx^{2}}}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{\sqrt{gx^{2}+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x, algorithm="fricas")
```

output
```
integral((sqrt(g*x^2 + f)*b*log((e*x + d)^n*c) + sqrt(g*x^2 + f)*a)/(g*x^2
 + f), x)
```

3.276.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f+gx^{2}}}\,dx=\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f+gx^{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/sqrt(f + g*x**2), x)
```

3.276.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/sqrt(g*x^2 + f), x) + a*arcsinh(g*
x/sqrt(f*g))/sqrt(g)
```

3.276.8 Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/sqrt(g*x^2 + f), x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{a + b\,\ln\left(c\,(d+e\,x)^n\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^(1/2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^(1/2), x)
```

3.277 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx$

3.277.1 Optimal result

Integrand size = 34, antiderivative size = 278

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \frac{ibn\arcsin\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} + \frac{ibn\,\text{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{ibn\,\text{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g}$$

output
```
1/2*I*b*n*arcsin(1/2*g*x)^2/g+arcsin(1/2*g*x)*(a+b*ln(c*(e*x+d)^n))/g-b*n*
arcsin(1/2*g*x)*ln(1+2*e*(1/2*I*g*x+1/2*(-g^2*x^2+4)^(1/2))/(I*d*g-(-d^2*g
^2+4*e^2)^(1/2)))/g-b*n*arcsin(1/2*g*x)*ln(1+2*e*(1/2*I*g*x+1/2*(-g^2*x^2+
4)^(1/2))/(I*d*g+(-d^2*g^2+4*e^2)^(1/2)))/g+I*b*n*polylog(2,-2*e*(1/2*I*g*
x+1/2*(-g^2*x^2+4)^(1/2))/(I*d*g-(-d^2*g^2+4*e^2)^(1/2)))/g+I*b*n*polylog(
2,-2*e*(1/2*I*g*x+1/2*(-g^2*x^2+4)^(1/2))/(I*d*g+(-d^2*g^2+4*e^2)^(1/2)))/
g
```

3.277.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \frac{a\arcsin\left(\frac{gx}{2}\right)}{g}+\frac{ibn\arcsin\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{2}\right)}g}{\frac{1}{2}idg^2-\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{2}\right)}g}{\frac{1}{2}idg^2+\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{b\arcsin\left(\frac{gx}{2}\right)\log\left(c(d+ex)^n\right)}{g} + \frac{ibn\,\mathrm{PolyLog}\left(2,\frac{2iee^{i\arcsin\left(\frac{gx}{2}\right)}}{dg-i\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{ibn\,\mathrm{PolyLog}\left(2,\frac{2iee^{i\arcsin\left(\frac{gx}{2}\right)}}{dg+i\sqrt{4e^2-d^2g^2}}\right)}{g}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(Sqrt[2 - g*x]*Sqrt[2 + g*x]),x]
```

output
```
(a*ArcSin[(g*x)/2])/g + ((I/2)*b*n*ArcSin[(g*x)/2]^2)/g - (b*n*ArcSin[(g*x
)/2]*Log[1 + (e*E^(I*ArcSin[(g*x)/2])*g)/((I/2)*d*g^2 - (g*Sqrt[4*e^2 - d^
2*g^2])/2)])/g - (b*n*ArcSin[(g*x)/2]*Log[1 + (e*E^(I*ArcSin[(g*x)/2])*g)/
((I/2)*d*g^2 + (g*Sqrt[4*e^2 - d^2*g^2])/2)])/g + (b*ArcSin[(g*x)/2]*Log[c
*(d + e*x)^n])/g + (I*b*n*PolyLog[2, ((2*I)*e*E^(I*ArcSin[(g*x)/2]))/(d*g
- I*Sqrt[4*e^2 - d^2*g^2])])/g + (I*b*n*PolyLog[2, ((2*I)*e*E^(I*ArcSin[(g
*x)/2]))/(d*g + I*Sqrt[4*e^2 - d^2*g^2])])/g
```

3.277.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2852, 27, 5240, 5032, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{gx+2}}dx$$

$$\Big\downarrow \text{2852}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-ben\int\frac{\arcsin\left(\frac{gx}{2}\right)}{g(d+ex)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\int\frac{\arcsin\left(\frac{gx}{2}\right)}{d+ex}dx}{g}$$

$$\Big\downarrow \text{5240}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\int\frac{\sqrt{1-\frac{g^2x^2}{4}}\arcsin\left(\frac{gx}{2}\right)}{\frac{dg}{2}+\frac{exg}{2}}d\arcsin\left(\frac{gx}{2}\right)}{g}$$

$$\Big\downarrow \text{5032}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\left(i\int\frac{2e^{i\arcsin\left(\frac{gx}{2}\right)}\arcsin\left(\frac{gx}{2}\right)}{2e^{i\arcsin\left(\frac{gx}{2}\right)}e+idg-\sqrt{4e^2-d^2g^2}}d\arcsin\left(\frac{gx}{2}\right)+i\int\frac{2e^{i\arcsin\left(\frac{gx}{2}\right)}\arcsin\left(\frac{gx}{2}\right)}{2e^{i\arcsin\left(\frac{gx}{2}\right)}e+idg+\sqrt{4e^2-d^2g^2}}d\arcsin\left(\frac{gx}{2}\right)-\frac{i\arcsin\left(\frac{gx}{2}\right)^2}{2e}\right)}{g}$$

$$\Big\downarrow \text{27}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\left(2i\int\frac{e^{i\arcsin\left(\frac{gx}{2}\right)}\arcsin\left(\frac{gx}{2}\right)}{2e^{i\arcsin\left(\frac{gx}{2}\right)}e+idg-\sqrt{4e^2-d^2g^2}}d\arcsin\left(\frac{gx}{2}\right)+2i\int\frac{e^{i\arcsin\left(\frac{gx}{2}\right)}\arcsin\left(\frac{gx}{2}\right)}{2e^{i\arcsin\left(\frac{gx}{2}\right)}e+idg+\sqrt{4e^2-d^2g^2}}d\arcsin\left(\frac{gx}{2}\right)-\frac{i\arcsin\left(\frac{gx}{2}\right)^2}{2e}\right)}{g}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-\frac{ben\left(2i\left(\frac{i\int\log\left(\frac{2e^{i\arcsin\left(\frac{gx}{2}\right)}e}{idg-\sqrt{4e^2-d^2g^2}}+1\right)d\arcsin\left(\frac{gx}{2}\right)}{2e}-\frac{i\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{-\sqrt{4e^2-d^2g^2}+idg}\right)}{2e}\right)+2i\left(\frac{i\int\log\left(\frac{2e^{i\arcsin\left(\frac{gx}{2}\right)}e}{idg+\sqrt{4e^2-d^2g^2}}+1\right)d\arcs}{2e}\right.\right.}{g}$$

$$\Big\downarrow \text{2715}$$

3.277. $\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}dx$

$$\frac{\arcsin\left(\frac{gx}{2}\right)(a+b\log(c(d+ex)^n))}{g} - \frac{ben\left(2i\left(\frac{\int e^{-i\arcsin\left(\frac{gx}{2}\right)}\log\left(\frac{2e^{i\arcsin\left(\frac{gx}{2}\right)}e}{idg-\sqrt{4e^2-d^2g^2}}+1\right)de^{i\arcsin\left(\frac{gx}{2}\right)}}{2e} - \frac{i\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{-\sqrt{4e^2-d^2g^2}+idg}\right)}{2e}\right)+2i\left(\frac{\int e^{-i\arcsin\left(\frac{gx}{2}\right)}\log\right.\right.}{g}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\arcsin\left(\frac{gx}{2}\right)(a+b\log(c(d+ex)^n))}{g} - \frac{ben\left(2i\left(-\frac{\text{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{2e} - \frac{i\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{-\sqrt{4e^2-d^2g^2}+idg}\right)}{2e}\right)+2i\left(-\frac{\text{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{2e} - \frac{i\,\text{ar}}{}\right.\right.}{g}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(Sqrt[2 - g*x]*Sqrt[2 + g*x]),x]
```

output
```
(ArcSin[(g*x)/2]*(a + b*Log[c*(d + e*x)^n]))/g - (b*e*n*(((-1/2*I)*ArcSin[
(g*x)/2]^2)/e + (2*I)*(((-1/2*I)*ArcSin[(g*x)/2]*Log[1 + (2*e*E^(I*ArcSin[
(g*x)/2]))/(I*d*g - Sqrt[4*e^2 - d^2*g^2])])/e - PolyLog[2, (-2*e*E^(I*Arc
Sin[(g*x)/2]))/(I*d*g - Sqrt[4*e^2 - d^2*g^2])]/(2*e)) + (2*I)*(((-1/2*I)*
ArcSin[(g*x)/2]*Log[1 + (2*e*E^(I*ArcSin[(g*x)/2]))/(I*d*g + Sqrt[4*e^2 -
d^2*g^2])])/e - PolyLog[2, (-2*e*E^(I*ArcSin[(g*x)/2]))/(I*d*g + Sqrt[4*e^
2 - d^2*g^2])]/(2*e))))/g
```

3.277.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2852
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> With[{u = IntHide[1/Sqrt[
f1*f2 + g1*g2*x^2], x]}, Simp[u*(a + b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n
   Int[SimplifyIntegrand[u/(d + e*x), x], x], x]] /; FreeQ[{a, b, c, d, e,
f1, g1, f2, g2, n}, x] && EqQ[f2*g1 + f1*g2, 0] && GtQ[f1, 0] && GtQ[f2, 0]
```

rule 5032
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2,
2] + b*E^(I*(c + d*x)))), x], x] + Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x)
)/(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

rule 5240
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.277.4 Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{\sqrt{-gx+2}\,\sqrt{gx+2}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x)
```

3.277.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx+2}\sqrt{-gx+2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-(sqrt(g*x + 2)*sqrt(-g*x + 2)*b*log((e*x + d)^n*c) + sqrt(g*x +
2)*sqrt(-g*x + 2)*a)/(g^2*x^2 - 4), x)
```

3.277.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{-gx+2}\sqrt{gx+2}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(-g*x+2)**(1/2)/(g*x+2)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(sqrt(-g*x + 2)*sqrt(g*x + 2)), x)
```

3.277.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx+2}\sqrt{-gx+2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x, algorithm
="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(sqrt(g*x + 2)*sqrt(-g*x + 2)), x)
 + a*arcsin(1/2*g*x)/g
```

3.277.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx+2}\sqrt{-gx+2}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x, algorithm
="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(sqrt(g*x + 2)*sqrt(-g*x + 2)), x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{2-g\,x}\,\sqrt{g\,x+2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((2 - g*x)^(1/2)*(g*x + 2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((2 - g*x)^(1/2)*(g*x + 2)^(1/2)), x)
```

3.278 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx$

3.278.1 Optimal result

Integrand size = 34, antiderivative size = 510

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\operatorname{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\operatorname{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$

output
```
1/2*I*b*f*n*arcsin(g*x/f)^2*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1/2)/(g*x+f)
^(1/2)+f*arcsin(g*x/f)*(a+b*ln(c*(e*x+d)^n))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x
+f)^(1/2)/(g*x+f)^(1/2)-b*f*n*arcsin(g*x/f)*ln(1+e*(I*g*x/f+(1-g^2*x^2/f^2
)^(1/2))*f/(I*d*g-(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x
+f)^(1/2)/(g*x+f)^(1/2)-b*f*n*arcsin(g*x/f)*ln(1+e*(I*g*x/f+(1-g^2*x^2/f^2
)^(1/2))*f/(I*d*g+(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x
+f)^(1/2)/(g*x+f)^(1/2)+I*b*f*n*polylog(2,-e*(I*g*x/f+(1-g^2*x^2/f^2)^(1/2
))*f/(I*d*g-(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1
/2)/(g*x+f)^(1/2)+I*b*f*n*polylog(2,-e*(I*g*x/f+(1-g^2*x^2/f^2)^(1/2))*f/(
I*d*g+(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1/2)/(g
*x+f)^(1/2)
```

3.278.2 Mathematica [A] (warning: unable to verify)

Time = 10.18 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.24

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \frac{\arctan\left(\frac{gx}{\sqrt{f-gx}\sqrt{f+gx}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{g}$$

$$-\frac{bn\sqrt{f-gx}\left(2fg(d+ex)\sqrt{\frac{f+gx}{f-gx}}\arctan\left(\frac{1}{\sqrt{\frac{f+gx}{f-gx}}}\right)\log(d+ex)+(f+gx)\left(dg+ef\cos\left(2\arctan\left(\frac{1}{\sqrt{\frac{f}{f}}}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/(Sqrt[f - g*x]*Sqrt[f + g*x]),x]
```

output
```
(ArcTan[(g*x)/(Sqrt[f - g*x]*Sqrt[f + g*x])]*(a - b*n*Log[d + e*x] + b*Log
[c*(d + e*x)^n]))/g - (b*n*Sqrt[f - g*x]*(2*f*g*(d + e*x)*Sqrt[(f + g*x)/(
f - g*x)]*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]]*Log[d + e*x] + (f + g*x)*(d*
g + e*f*Cos[2*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]]])*Csc[2*ArcTan[1/Sqrt[(f
 + g*x)/(f - g*x)]]]*((2*I)*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]]^2 - (4*I)*
ArcSin[Sqrt[1 + (d*g)/(e*f)]/Sqrt[2]]*ArcTan[(-(e*f) + d*g)/(Sqrt[-(e^2*f^
2) + d^2*g^2]*Sqrt[(f + g*x)/(f - g*x)])] - 2*(ArcSin[Sqrt[1 + (d*g)/(e*f)
]/Sqrt[2]] + ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*Log[1 + (E^((2*I)*ArcTan
[1/Sqrt[(f + g*x)/(f - g*x)]])*(d*g - Sqrt[-(e^2*f^2) + d^2*g^2]))/(e*f)]
+ 2*(ArcSin[Sqrt[1 + (d*g)/(e*f)]/Sqrt[2]] - ArcTan[1/Sqrt[(f + g*x)/(f -
g*x)]])*Log[1 + (E^((2*I)*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*(d*g + Sqrt
[-(e^2*f^2) + d^2*g^2]))/(e*f)] + I*(PolyLog[2, (E^((2*I)*ArcTan[1/Sqrt[(f
 + g*x)/(f - g*x)]])*(-(d*g) + Sqrt[-(e^2*f^2) + d^2*g^2]))/(e*f)] + PolyL
og[2, -((E^((2*I)*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*(d*g + Sqrt[-(e^2*f
^2) + d^2*g^2]))/(e*f))])))/(f*g^2*(d + e*x)*Sqrt[f + g*x])
```

3.278.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2854, 2851, 27, 5240, 5032, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{2854}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{1-\frac{g^2x^2}{f^2}}}dx}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$\Big\downarrow \text{2851}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-ben\int \frac{f\arcsin\left(\frac{gx}{f}\right)}{g(d+ex)}dx\right)}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{befn\int\frac{\arcsin\left(\frac{gx}{f}\right)}{d+ex}dx}{g}\right)}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 5240}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{befn\int\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)}{\frac{exg}{f}+\frac{dg}{f}}d\arcsin\left(\frac{gx}{f}\right)}{g}\right)}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 5032}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{befn\left(i\int\frac{e^{i\arcsin\left(\frac{gx}{f}\right)}\arcsin\left(\frac{gx}{f}\right)}{e^{i\arcsin\left(\frac{gx}{f}\right)}e+\frac{idg-\sqrt{e^2f^2-d^2g^2}}{f}}d\arcsin\left(\frac{gx}{f}\right)+i\int\frac{e^{i\arcsin\left(\frac{gx}{f}\right)}\arcsin\left(\frac{gx}{f}\right)}{e^{i\arcsin\left(\frac{gx}{f}\right)}e+\frac{idg+\sqrt{e^2f^2-\cdots}}{f}}\cdots\right.}{g}\cdots\right.}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{befn\left(i\left(\frac{i\int\log\left(\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}+1\right)d\arcsin\left(\frac{gx}{f}\right)}{e}-\frac{i\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{efe^{i\arcsin\left(\frac{gx}{f}\right)}}{-\sqrt{e^2f^2-d^2g^2}+idg}\cdots\right.}{e}\cdots\right.\right.}{\cdots}\cdots\right.}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{befn\left(i\left(\frac{\int e^{-i\arcsin\left(\frac{gx}{f}\right)}\log\left(\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}+1\right)de^{i\arcsin\left(\frac{gx}{f}\right)}}{e}-\frac{i\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{e\cdots}{-\sqrt{\cdots}}\cdots\right.}{e}\cdots\right.\right.}{\cdots}\cdots\right.}{\sqrt{f-gx}\sqrt{\cdots}}$$

$$\Big\downarrow \text{ 2838}$$

3.278. $\displaystyle\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx$

$$\frac{\sqrt{1-\frac{g^2x^2}{f^2}}\left(\frac{f\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{befn\left(i\left(\left(-\frac{\operatorname{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{e}-\frac{i\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{efe^{i\arcsin\left(\frac{gx}{f}\right)}}{-\sqrt{e^2f^2-d^2g^2}+idg}\right)}{e}\right)+i \cdots}{\cdots}\right.}{\sqrt{f-gx}\sqrt{f+gx}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/(Sqrt[f - g*x]*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[1 - (g^2*x^2)/f^2]*((f*ArcSin[(g*x)/f]*(a + b*Log[c*(d + e*x)^n]))/g
 - (b*e*f*n*(((-1/2*I)*ArcSin[(g*x)/f]^2)/e + I*(((-I)*ArcSin[(g*x)/f]*Log
[1 + (e*E^(I*ArcSin[(g*x)/f])*f)/(I*d*g - Sqrt[e^2*f^2 - d^2*g^2])])/e - P
olyLog[2, -((e*E^(I*ArcSin[(g*x)/f])*f)/(I*d*g - Sqrt[e^2*f^2 - d^2*g^2]))
]/e) + I*(((-I)*ArcSin[(g*x)/f]*Log[1 + (e*E^(I*ArcSin[(g*x)/f])*f)/(I*d*g
 + Sqrt[e^2*f^2 - d^2*g^2])])/e - PolyLog[2, -((e*E^(I*ArcSin[(g*x)/f])*f)
/(I*d*g + Sqrt[e^2*f^2 - d^2*g^2]))]/e)))/g))/(Sqrt[f - g*x]*Sqrt[f + g*x]
)
```

3.278.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 2854
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> Simp[Sqrt[1 + g1*(g2/(f1*
f2))*x^2]/(Sqrt[f1 + g1*x]*Sqrt[f2 + g2*x])   Int[(a + b*Log[c*(d + e*x)^n]
)/Sqrt[1 + g1*(g2/(f1*f2))*x^2], x], x] /; FreeQ[{a, b, c, d, e, f1, g1, f2
, g2, n}, x] && EqQ[f2*g1 + f1*g2, 0]
```

rule 5032
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2,
2] + b*E^(I*(c + d*x)))), x], x] + Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x)
)/(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

rule 5240
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.278.4 Maple [F]

$$\int \frac{a + b\ln\left(c(ex+d)^{n}\right)}{\sqrt{-gx+f}\,\sqrt{gx+f}}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x)
```

3.278.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx+f}\sqrt{-gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-(sqrt(g*x + f)*sqrt(-g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x +
f)*sqrt(-g*x + f)*a)/(g^2*x^2 - f^2), x)
```

3.278.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(-g*x+f)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(sqrt(f - g*x)*sqrt(f + g*x)), x)
```

3.278.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx+f}\sqrt{-gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x, algorithm
="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(sqrt(g*x + f)*sqrt(-g*x + f)), x)
 + a*arcsin(g*x/f)/g
```

3.278.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\sqrt{gx+f}\sqrt{-gx+f}}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x, algorithm
="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/(sqrt(g*x + f)*sqrt(-g*x + f)), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{f+g\,x}\,\sqrt{f-g\,x}}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(f - g*x)^(1/2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(f - g*x)^(1/2)), x)
```

3.279 $\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$

3.279.1 Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

output
```
1/2*polylog(2,1-2*e/(f*x+e))/e/f
```

3.279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\text{PolyLog}\left(2,\frac{-e+fx}{e+fx}\right)}{2ef}$$

input
```
Integrate[Log[(2*e)/(e + f*x)]/(e^2 - f^2*x^2),x]
```

output
```
PolyLog[2, (-e + f*x)/(e + f*x)]/(2*e*f)
```

3.279.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$$
$$\Big\downarrow \; 2849$$
$$\frac{\int \frac{\log\left(\frac{2e}{e+fx}\right)}{1-\frac{2e}{e+fx}}d\frac{1}{e+fx}}{f}$$
$$\Big\downarrow \; 2752$$
$$\frac{\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

input
```
Int[Log[(2*e)/(e + f*x)]/(e^2 - f^2*x^2),x]
```

output
```
PolyLog[2, 1 - (2*e)/(e + f*x)]/(2*e*f)
```

3.279.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.279.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	20
default	$\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	20
risch	$\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	20
parts	$\frac{\ln\left(\frac{2e}{fx+e}\right)\ln(fx+e)}{2ef}-\frac{\ln\left(\frac{2e}{fx+e}\right)\ln(-fx+e)}{2ef}+\frac{f\left(\frac{\ln(fx+e)^2}{2e\,f^2}+\frac{-\operatorname{dilog}\left(-\frac{-fx-e}{2e}\right)-\ln(-fx+e)\ln\left(-\frac{-fx-e}{2e}\right)}{e\,f^2}\right)}{2}$	120

input
```
int(ln(2*e/(f*x+e))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/e*dilog(2*e/(f*x+e))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{\operatorname{Li}_2\left(-\frac{2\,e}{fx+e}+1\right)}{2\,ef}$$

input
```
integrate(log(2*e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="fricas")
```

output
```
1/2*dilog(-2*e/(f*x + e) + 1)/(e*f)
```

3.279.6 Sympy [F]

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\int \frac{\log(2)}{-e^2+f^2x^2}\,dx-\int \frac{\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

input
```
integrate(ln(2*e/(f*x+e))/(-f**2*x**2+e**2),x)
```

output
```
-Integral(log(2)/(-e**2 + f**2*x**2), x) - Integral(log(e/(e + f*x))/(-e**
2 + f**2*x**2), x)
```

3.279.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$$
$$=\frac{1}{4}f\left(\frac{\log(fx+e)^2-2\log(fx+e)\log(fx-e)}{ef^2}+\frac{2\left(\log(fx+e)\log\left(-\frac{fx+e}{2e}+1\right)+\mathrm{Li}_2\left(\frac{fx+e}{2e}\right)\right)}{ef^2}\right)$$
$$+\frac{1}{2}\left(\frac{\log(fx+e)}{ef}-\frac{\log(fx-e)}{ef}\right)\log\left(\frac{2e}{fx+e}\right)$$

input
```
integrate(log(2*e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="maxima")
```

output
```
1/4*f*((log(f*x + e)^2 - 2*log(f*x + e)*log(f*x - e))/(e*f^2) + 2*(log(f*x
 + e)*log(-1/2*(f*x + e)/e + 1) + dilog(1/2*(f*x + e)/e))/(e*f^2)) + 1/2*(
log(f*x + e)/(e*f) - log(f*x - e)/(e*f))*log(2*e/(f*x + e))
```

3.279.8 Giac [F]

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{\log\left(\frac{2e}{fx+e}\right)}{f^2x^2-e^2}\,dx$$

input
```
integrate(log(2*e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="giac")
```

output
```
integrate(-log(2*e/(f*x + e))/(f^2*x^2 - e^2), x)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{2\,e}{e+f\,x}\right)}{2\,e\,f}$$

input
```
int(log((2*e)/(e + f*x))/(e^2 - f^2*x^2),x)
```

output
```
dilog((2*e)/(e + f*x))/(2*e*f)
```

3.280 $\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx$

3.280.1 Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\frac{\operatorname{arctanh}\left(\frac{fx}{e}\right)\log(2)}{ef}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

output
```
-arctanh(f*x/e)*ln(2)/e/f+1/2*polylog(2,1-2*e/(f*x+e))/e/f
```

3.280.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.93

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\frac{\log\left(\frac{e-fx}{2e}\right)\log\left(\frac{e}{e+fx}\right)}{2ef}-\frac{\log^2\left(\frac{e}{e+fx}\right)}{4ef}+\frac{\operatorname{PolyLog}\left(2,\frac{e+fx}{2e}\right)}{2ef}$$

input
```
Integrate[Log[e/(e + f*x)]/(e^2 - f^2*x^2),x]
```

output
```
-1/2*(Log[(e - f*x)/(2*e)]*Log[e/(e + f*x)])/(e*f) - Log[e/(e + f*x)]^2/(4
*e*f) + PolyLog[2, (e + f*x)/(2*e)]/(2*e*f)
```

3.280.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2850, 221, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}dx$$

$$\Big\downarrow \text{2850}$$

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}dx-\log(2)\int \frac{1}{e^2-f^2x^2}dx$$

$$\Big\downarrow \text{221}$$

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}dx-\frac{\log(2)\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\int \frac{\log\left(\frac{2e}{e+fx}\right)}{1-\frac{2e}{e+fx}}d\frac{1}{e+fx}}{f}-\frac{\log(2)\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}-\frac{\log(2)\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

input
```
Int[Log[e/(e + f*x)]/(e^2 - f^2*x^2),x]
```

output
```
-((ArcTanh[(f*x)/e]*Log[2])/(e*f)) + PolyLog[2, 1 - (2*e)/(e + f*x)]/(2*e*
f)
```

3.280.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2850
```
Int[((a_.) + Log[(c_.)/((d_) + (e_.)*(x_))]*(b_.))/((f_) + (g_.)*(x_)^2), x
_Symbol] :> Simp[(a + b*Log[c/(2*d)])   Int[1/(f + g*x^2), x], x] + Simp[b
  Int[Log[2*(d/(d + e*x))]/(f + g*x^2), x], x] /; FreeQ[{a, b, c, d, e, f,
g}, x] && EqQ[e^2*f + d^2*g, 0] && GtQ[c/(2*d), 0]
```

3.280.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

method	result	siz
derivativedivides	$-\frac{\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}}{fe}$	62
default	$-\frac{\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}}{fe}$	62
risch	$-\frac{\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{e}{fx+e}\right)}{2ef}+\frac{\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{2e}{fx+e}\right)}{2ef}+\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	84
parts	$\frac{\ln\left(\frac{e}{fx+e}\right)\ln(fx+e)}{2ef}-\frac{\ln\left(\frac{e}{fx+e}\right)\ln(-fx+e)}{2ef}+\frac{f\left(\frac{\ln(fx+e)^2}{2e\,f^2}+\frac{-\operatorname{dilog}\left(-\frac{-fx-e}{2e}\right)-\ln(-fx+e)\ln\left(-\frac{-fx-e}{2e}\right)}{e\,f^2}\right)}{2}$	118

input
```
int(ln(e/(f*x+e))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/e*(1/2*(ln(e/(f*x+e))-ln(2*e/(f*x+e)))*ln(1-2*e/(f*x+e))-1/2*dilog(2*
e/(f*x+e)))
```

3.280.5 Fricas [F]

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{\log\left(\frac{e}{fx+e}\right)}{f^2x^2-e^2}\,dx$$

input
```
integrate(log(e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="fricas")
```

output
```
integral(-log(e/(f*x + e))/(f^2*x^2 - e^2), x)
```

3.280.6 Sympy [F]

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\int \frac{\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

input
```
integrate(ln(e/(f*x+e))/(-f**2*x**2+e**2),x)
```

output
```
-Integral(log(e/(e + f*x))/(-e**2 + f**2*x**2), x)
```

3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(39) = 78$.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx$$
$$=\frac{1}{4}f\left(\frac{\log(fx+e)^2-2\log(fx+e)\log(fx-e)}{ef^2}+\frac{2\left(\log(fx+e)\log\left(-\frac{fx+e}{2e}+1\right)+\mathrm{Li}_2\left(\frac{fx+e}{2e}\right)\right)}{ef^2}\right)$$
$$+\frac{1}{2}\left(\frac{\log(fx+e)}{ef}-\frac{\log(fx-e)}{ef}\right)\log\left(\frac{e}{fx+e}\right)$$

input
```
integrate(log(e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="maxima")
```

output
```
1/4*f*((log(f*x + e)^2 - 2*log(f*x + e)*log(f*x - e))/(e*f^2) + 2*(log(f*x
 + e)*log(-1/2*(f*x + e)/e + 1) + dilog(1/2*(f*x + e)/e))/(e*f^2)) + 1/2*(
log(f*x + e)/(e*f) - log(f*x - e)/(e*f))*log(e/(f*x + e))
```

3.280.8 Giac [F]

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int -\frac{\log\left(\frac{e}{fx+e}\right)}{f^2x^2-e^2}\,dx$$

input
```
integrate(log(e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="giac")
```

output
```
integrate(-log(e/(f*x + e))/(f^2*x^2 - e^2), x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int \frac{\ln\left(\frac{e}{e+f\,x}\right)}{e^2-f^2\,x^2}\,dx$$

input
```
int(log(e/(e + f*x))/(e^2 - f^2*x^2),x)
```

output
```
int(log(e/(e + f*x))/(e^2 - f^2*x^2), x)
```

3.281 $\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$

3.281.1 Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{a\text{arctanh}\left(\frac{fx}{e}\right)}{ef}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

output
```
a*arctanh(f*x/e)/e/f+1/2*b*polylog(2,1-2*e/(f*x+e))/e/f
```

3.281.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{-\left(\left(a+b\log\left(\frac{2e}{e+fx}\right)\right)\left(a+2b\log\left(\frac{e-fx}{2e}\right)+b\log\left(\frac{2e}{e+fx}\right)\right)\right)+2b^2\,\text{PolyLog}\left(2,\frac{e+fx}{2e}\right)}{4bef}$$

input
```
Integrate[(a + b*Log[(2*e)/(e + f*x)])/(e^2 - f^2*x^2),x]
```

output
```
(-((a + b*Log[(2*e)/(e + f*x)])*(a + 2*b*Log[(e - f*x)/(2*e)] + b*Log[(2*e
)/(e + f*x)])) + 2*b^2*PolyLog[2, (e + f*x)/(2*e)])/(4*b*e*f)
```

3.281.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2850, 221, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$$

$$\Big\downarrow \text{2850}$$

$$a\int \frac{1}{e^2-f^2x^2}dx+b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}dx+\frac{a\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

$$\Big\downarrow \text{2849}$$

$$\frac{b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{1-\frac{2e}{e+fx}}d\frac{1}{e+fx}}{f}+\frac{a\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

$$\Big\downarrow \text{2752}$$

$$\frac{a\text{arctanh}\left(\frac{fx}{e}\right)}{ef}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

input
```
Int[(a + b*Log[(2*e)/(e + f*x)])/(e^2 - f^2*x^2),x]
```

output
```
(a*ArcTanh[(f*x)/e])/(e*f) + (b*PolyLog[2, 1 - (2*e)/(e + f*x)])/(2*e*f)
```

3.281.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2850
```
Int[((a_.) + Log[(c_.)/((d_) + (e_.)*(x_))]*(b_.))/((f_) + (g_.)*(x_)^2), x
_Symbol] :> Simp[(a + b*Log[c/(2*d)])   Int[1/(f + g*x^2), x], x] + Simp[b
  Int[Log[2*(d/(d + e*x))]/(f + g*x^2), x], x] /; FreeQ[{a, b, c, d, e, f,
g}, x] && EqQ[e^2*f + d^2*g, 0] && GtQ[c/(2*d), 0]
```

3.281.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$-\frac{2e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{4e^{2}}-\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{4e^{2}}\right)}{f}$	44
default	$-\frac{2e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{4e^{2}}-\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{4e^{2}}\right)}{f}$	44
risch	$-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}+\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	54
parts	$-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}+\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	54

input
```
int((a+b*ln(2*e/(f*x+e)))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*e*(1/4/e^2*a*ln(2*e/(f*x+e)-1)-1/4/e^2*b*dilog(2*e/(f*x+e)))
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{b\mathrm{Li}_2\left(-\frac{2\,e}{fx+e}+1\right)+a\log\left(fx+e\right)-a\log\left(fx-e\right)}{2\,ef}$$

input
```
integrate((a+b*log(2*e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="fricas")
```

output
```
1/2*(b*dilog(-2*e/(f*x + e) + 1) + a*log(f*x + e) - a*log(f*x - e))/(e*f)
```

3.281.6 Sympy [F]

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = -\int \frac{a}{-e^2+f^2x^2}\,dx - \int \frac{b\log\left(2\right)}{-e^2+f^2x^2}\,dx - \int \frac{b\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

input
```
integrate((a+b*ln(2*e/(f*x+e)))/(-f**2*x**2+e**2),x)
```

output
```
-Integral(a/(-e**2 + f**2*x**2), x) - Integral(b*log(2)/(-e**2 + f**2*x**2
), x) - Integral(b*log(e/(e + f*x))/(-e**2 + f**2*x**2), x)
```

3.281.7 Maxima [F]

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int -\frac{b\log\left(\frac{2\,e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

input
```
integrate((a+b*log(2*e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="maxima")
```

output
```
1/2*a*(log(f*x + e)/(e*f) - log(f*x - e)/(e*f)) + b*integrate(-(log(2) - l
og(f*x + e) + log(e))/(f^2*x^2 - e^2), x)
```

3.281.8 Giac [F]

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{2\,e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

input
```
integrate((a+b*log(2*e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="giac")
```

output
```
integrate(-(b*log(2*e/(f*x + e)) + a)/(f^2*x^2 - e^2), x)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\frac{a\,\ln\left(f\,x-e\right)-b\,\mathrm{Li}_2\left(\frac{2\,e}{e+f\,x}\right)+a\,\ln\left(\frac{1}{e+f\,x}\right)}{2\,e\,f}$$

input
```
int((a + b*log((2*e)/(e + f*x)))/(e^2 - f^2*x^2),x)
```

output
```
-(a*log(f*x - e) - b*dilog((2*e)/(e + f*x)) + a*log(1/(e + f*x)))/(2*e*f)
```

3.282 $\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx$

3.282.1 Optimal result

Integrand size = 29, antiderivative size = 47

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{fx}{e}\right)(a-b\log(2))}{ef}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

output
```
arctanh(f*x/e)*(a-b*ln(2))/e/f+1/2*b*polylog(2,1-2*e/(f*x+e))/e/f
```

3.282.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{-\left(\left(a+b\log\left(\frac{e}{e+fx}\right)\right)\left(a+2b\log\left(\frac{e-fx}{2e}\right)+b\log\left(\frac{e}{e+fx}\right)\right)\right)+2b^2\,\text{PolyLog}\left(2,\frac{e+fx}{2e}\right)}{4bef}$$

input
```
Integrate[(a + b*Log[e/(e + f*x)])/(e^2 - f^2*x^2),x]
```

output
```
(-((a + b*Log[e/(e + f*x)])*(a + 2*b*Log[(e - f*x)/(2*e)] + b*Log[e/(e + f
*x)])) + 2*b^2*PolyLog[2, (e + f*x)/(2*e)])/(4*b*e*f)
```

3.282.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2850, 221, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}dx$$

$$\Big\downarrow \text{2850}$$

$$(a-b\log(2))\int \frac{1}{e^2-f^2x^2}dx+b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}dx+\frac{(a-b\log(2))\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

$$\Big\downarrow \text{2849}$$

$$\frac{b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{1-\frac{2e}{e+fx}}d\frac{1}{e+fx}}{f}+\frac{(a-b\log(2))\text{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(a-b\log(2))\text{arctanh}\left(\frac{fx}{e}\right)}{ef}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

input
```
Int[(a + b*Log[e/(e + f*x)])/(e^2 - f^2*x^2),x]
```

output
```
(ArcTanh[(f*x)/e]*(a - b*Log[2]))/(e*f) + (b*PolyLog[2, 1 - (2*e)/(e + f*x
)])/(2*e*f)
```

3.282.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2850
```
Int[((a_.) + Log[(c_.)/((d_) + (e_.)*(x_))]*(b_.))/((f_) + (g_.)*(x_)^2), x
_Symbol] :> Simp[(a + b*Log[c/(2*d)])   Int[1/(f + g*x^2), x], x] + Simp[b
  Int[Log[2*(d/(d + e*x))]/(f + g*x^2), x], x] /; FreeQ[{a, b, c, d, e, f,
g}, x] && EqQ[e^2*f + d^2*g, 0] && GtQ[c/(2*d), 0]
```

3.282.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$-\frac{e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{2e^2}+\frac{b\left(\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}\right)}{e^2}\right)}{f}$	85
default	$-\frac{e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{2e^2}+\frac{b\left(\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}\right)}{e^2}\right)}{f}$	85
parts	$-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}-\frac{b\left(\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}\right)}{fe}$	96
risch	$-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}-\frac{b\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{e}{fx+e}\right)}{2ef}+\frac{b\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{2e}{fx+e}\right)}{2ef}+\frac{b\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$	119

input
```
int((a+b*ln(e/(f*x+e)))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
```

3.282. $\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx$

output
```
-1/f*e*(1/2/e^2*a*ln(2*e/(f*x+e)-1)+1/e^2*b*(1/2*(ln(e/(f*x+e))-ln(2*e/(f*
x+e)))*ln(1-2*e/(f*x+e))-1/2*dilog(2*e/(f*x+e))))
```

3.282.5 Fricas [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

input
```
integrate((a+b*log(e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="fricas")
```

output
```
integral(-(b*log(e/(f*x + e)) + a)/(f^2*x^2 - e^2), x)
```

3.282.6 Sympy [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\int \frac{a}{-e^2+f^2x^2}\,dx-\int \frac{b\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

input
```
integrate((a+b*ln(e/(f*x+e)))/(-f**2*x**2+e**2),x)
```

output
```
-Integral(a/(-e**2 + f**2*x**2), x) - Integral(b*log(e/(e + f*x))/(-e**2 +
 f**2*x**2), x)
```

3.282.7 Maxima [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

input
```
integrate((a+b*log(e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="maxima")
```

output
```
1/2*a*(log(f*x + e)/(e*f) - log(f*x - e)/(e*f)) + b*integrate((log(f*x + e
) - log(e))/(f^2*x^2 - e^2), x)
```

3.282.8 Giac [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int -\frac{b\log\left(\frac{e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

input
```
integrate((a+b*log(e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="giac")
```

output
```
integrate(-(b*log(e/(f*x + e)) + a)/(f^2*x^2 - e^2), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int \frac{a+b\,\ln\left(\frac{e}{e+f\,x}\right)}{e^2-f^2\,x^2}\,dx$$

input
```
int((a + b*log(e/(e + f*x)))/(e^2 - f^2*x^2),x)
```

output
```
int((a + b*log(e/(e + f*x)))/(e^2 - f^2*x^2), x)
```

3.283 $\int \frac{x^5 \log(c+dx)}{a+bx^3} \, dx$

3.283.1 Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int \frac{x^5 \log(c+dx)}{a+bx^3} \, dx = &-\frac{c^2x}{3bd^2}+\frac{cx^2}{6bd}-\frac{x^3}{9b}+\frac{c^3\log(c+dx)}{3bd^3}+\frac{x^3\log(c+dx)}{3b} \\ &-\frac{a\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} \\ &-\frac{a\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} \\ &-\frac{a\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} \\ &-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2}-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2} \\ &-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}\end{aligned}$$

output
```
-1/3*c^2*x/b/d^2+1/6*c*x^2/b/d-1/9*x^3/b+1/3*c^3*ln(d*x+c)/b/d^3+1/3*x^3*l
n(d*x+c)/b-1/3*a*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c
)/b^2-1/3*a*ln(-d*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)*a^(
1/3)*d))*ln(d*x+c)/b^2-1/3*a*ln((-1)^(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x
)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))*ln(d*x+c)/b^2-1/3*a*polylog(2,b^(1/3)*
(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/b^2-1/3*a*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3
)*c+(-1)^(1/3)*a^(1/3)*d))/b^2-1/3*a*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-
(-1)^(2/3)*a^(1/3)*d))/b^2
```

3.283.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.01

$$\int \frac{x^5\log(c+dx)}{a+bx^3}\,dx = -\frac{c^2x}{3bd^2}+\frac{cx^2}{6bd}-\frac{x^3}{9b}+\frac{c^3\log(c+dx)}{3bd^3}+\frac{x^3\log(c+dx)}{3b} - \frac{a\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2} - \frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2} - \frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}$$

input
```
Integrate[(x^5*Log[c + d*x])/(a + b*x^3),x]
```

output
```
-1/3*(c^2*x)/(b*d^2) + (c*x^2)/(6*b*d) - x^3/(9*b) + (c^3*Log[c + d*x])/(3
*b*d^3) + (x^3*Log[c + d*x])/(3*b) - (a*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b
^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^2) - (a*Log[-(((-1)^(2/3)*d*(a^
(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c
+ d*x])/(3*b^2) - (a*Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(
b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*b^2) - (a*PolyLog[2, (
b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1
/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2
, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^2)
```

3.283.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \log(c+dx)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{x^2 \log(c+dx)}{b} - \frac{ax^2 \log(c+dx)}{b\,(a+bx^3)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a \operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2}-\frac{a \operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2}-\frac{a \operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}-\frac{a \log(c+dx) \log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2}-\frac{a \log(c+dx) \log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}-\frac{a \log(c+dx) \log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b^2}+\frac{c^3 \log(c+dx)}{3bd^3}-\frac{c^2 x}{3bd^2}+\frac{x^3 \log(c+dx)}{3b}+\frac{cx^2}{6bd}-\frac{x^3}{9b}$$

input
```
Int[(x^5*Log[c + d*x])/(a + b*x^3),x]
```

3.283. $\int \frac{x^5 \log(c+dx)}{a+bx^3} \, dx$

output
```
-1/3*(c^2*x)/(b*d^2) + (c*x^2)/(6*b*d) - x^3/(9*b) + (c^3*Log[c + d*x])/(3
*b*d^3) + (x^3*Log[c + d*x])/(3*b) - (a*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b
^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^2) - (a*Log[-((d*((-1)^(2/3)*a^
(1/3) + b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*
b^2) - (a*Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c +
 (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*(c
+ d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*(c + d*
x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*
(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^2)
```

3.283.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.283.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.41

method	result
risch	$-\frac{c^2 x}{3b\, d^2}-\frac{11c^3}{18d^3 b}+\frac{c\, x^2}{6bd}+\frac{x^3\ln(dx+c)}{3b}+\frac{c^3\ln(dx+c)}{3b\, d^3}-\frac{x^3}{9b}-\frac{a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\, c^2\_Z+a\, d^3-\right.}\right.}{}$
derivativedivides	$\frac{\frac{d^3\left(c^2((dx+c)\ln(dx+c)-dx-c)-2c\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)}{b}-\frac{a\, d^6\left(\_R1=\mathrm{RootOf}\left(b\_Z^3\right.\right.}{}}{d^6}$
default	$\frac{\frac{d^3\left(c^2((dx+c)\ln(dx+c)-dx-c)-2c\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)}{b}-\frac{a\, d^6\left(\_R1=\mathrm{RootOf}\left(b\_Z^3\right.\right.}{}}{d^6}$
parts	$\frac{x^3\ln(dx+c)}{3b}-\frac{\ln(dx+c)a\ln(b\, x^3+a)}{3b^2}-\frac{d\left(\frac{x^3}{3bd}-\frac{x^2 c}{2b\, d^2}+\frac{x\, c^2}{b\, d^3}-\frac{c^3\ln(dx+c)}{b\, d^4}-\frac{a\ln(dx+c)\ln(b\, x^3+a)}{b^2 d}+\frac{a\left(\_R1=\mathrm{RootOf}\left(b\right.\right.}{}\right.}{}$

input
```
int(x^5*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^2*x/b/d^2-11/18/d^3/b*c^3+1/6*c*x^2/b/d+1/3*x^3*ln(d*x+c)/b+1/3*c^3
*ln(d*x+c)/b/d^3-1/9*x^3/b-1/3*a/b^2*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+di
log((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)
```

3.283.5 Fricas [F]

$$\int \frac{x^5\log(c+dx)}{a+bx^3}\,dx=\int \frac{x^5\log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^5*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(x^5*log(d*x + c)/(b*x^3 + a), x)
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*ln(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.283.7 Maxima [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^5 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^5*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^5*log(d*x + c)/(b*x^3 + a), x)
```

3.283.8 Giac [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^5 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^5*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^5*log(d*x + c)/(b*x^3 + a), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^5\,\ln\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

input
```
int((x^5*log(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^5*log(c + d*x))/(a + b*x^3), x)
```

3.284 $\int \frac{x^2 \log(c+dx)}{a+bx^3} dx$

3.284.1 Optimal result

Integrand size = 19, antiderivative size = 292

$$\begin{aligned}\int \frac{x^2 \log(c+dx)}{a+bx^3} dx &= \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b} \\ &+ \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} \\ &+ \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} \\ &+ \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b} \\ &+ \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}\end{aligned}$$

output
```
1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/b+1/3*ln(-d
*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))*ln(d*x+c
)/b+1/3*ln((-1)^(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*c+(-1)^(1/
3)*a^(1/3)*d))*ln(d*x+c)/b+1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3
)*d))/b+1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b+
1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b
```

3.284.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02

$$\int \frac{x^2 \log(c+dx)}{a+bx^3} \, dx = \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}$$

input
```
Integrate[(x^2*Log[c + d*x])/(a + b*x^3),x]
```

output
```
(Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(
3*b) + (Log[-(((-1)^(2/3)*d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c -
 (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*b) + (Log[((-1)^(1/3)*d*(a^(1/3)
 + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x]
)/(3*b) + PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*b) +
PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(3*b) +
 PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(3*b)
```

3.284.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\log(c+dx)}{a+bx^3}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(c+dx)}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{\log(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b}+\frac{\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}+\frac{\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b}$$

input
```
Int[(x^2*Log[c + d*x])/(a + b*x^3),x]
```

output
```
(Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(
3*b) + (Log[-((d*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)
*a^(1/3)*d))]*Log[c + d*x])/(3*b) + (Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*b) + P
olyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*b) + PolyLog[2,
(b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(3*b) + PolyLog[2,
 (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(3*b)
```

3.284.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.284.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.26

method	result
derivativedivides	$\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\text{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{3b}$
default	$\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\text{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{3b}$
risch	$\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\text{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{3b}$
parts	$\frac{\ln(dx+c)\ln\left(b\,x^3+a\right)}{3b}-\frac{d\left(\frac{\ln(dx+c)\ln\left(b\,x^3+a\right)}{d}-\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+}{}\right.\right.}{d}\right.}{3b}$

input
```
int(x^2*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

3.284.5 Fricas [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^2*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(x^2*log(d*x + c)/(b*x^3 + a), x)
```

3.284.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.284.7 Maxima [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^2*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^2*log(d*x + c)/(b*x^3 + a), x)
```

3.284.8 Giac [F]

$$\int \frac{x^2 \log(c + dx)}{a + bx^3} \, dx = \int \frac{x^2 \log(dx + c)}{bx^3 + a} \, dx$$

input
```
integrate(x^2*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^2*log(d*x + c)/(b*x^3 + a), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c + dx)}{a + bx^3} \, dx = \int \frac{x^2 \ln(c + d\,x)}{b\,x^3 + a} \, dx$$

input
```
int((x^2*log(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^2*log(c + d*x))/(a + b*x^3), x)
```

3.285 $\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx$

3.285.1 Optimal result

Integrand size = 19, antiderivative size = 324

$$\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a} + \frac{\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)}{a}$$

output
```
ln(-d*x/c)*ln(d*x+c)/a-1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d)
)*ln(d*x+c)/a-1/3*ln(-d*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/
3)*a^(1/3)*d))*ln(d*x+c)/a-1/3*ln((-1)^(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)
*x)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))*ln(d*x+c)/a-1/3*polylog(2,b^(1/3)*(d
*x+c)/(b^(1/3)*c-a^(1/3)*d))/a-1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-
1)^(1/3)*a^(1/3)*d))/a-1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)
*a^(1/3)*d))/a+polylog(2,1+d*x/c)/a
```

3.285.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02

$$\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} + \frac{\text{PolyLog}\left(2,\frac{c+dx}{c}\right)}{a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}$$

input
```
Integrate[Log[c + d*x]/(x*(a + b*x^3)),x]
```

output
```
(Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/
3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a) - (Log[-(((-1)^(2/3)*d*(a^(1/3) -
(-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/
(3*a) - (Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c +
(-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*a) + PolyLog[2, (c + d*x)/c]/a - P
olyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*a) - PolyLog[2,
(b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(3*a) - PolyLog[2,
 (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(3*a)
```

3.285.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x(a+bx^3)}dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{\log(c+dx)}{ax}-\frac{bx^2\log(c+dx)}{a(a+bx^3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a}-\frac{\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}-\frac{\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3a}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}$$

input
```
Int[Log[c + d*x]/(x*(a + b*x^3)),x]
```

output
```
(Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/
3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a) - (Log[-((d*((-1)^(2/3)*a^(1/3) +
b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*a) - (Lo
g[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*
a^(1/3)*d)]*Log[c + d*x])/(3*a) - PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*
c - a^(1/3)*d)]/(3*a) - PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(
1/3)*a^(1/3)*d)]/(3*a) - PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^
(2/3)*a^(1/3)*d)]/(3*a) + PolyLog[2, 1 + (d*x)/c]/a
```

3.285.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.285.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\right.}{3a}$
default	$\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\right.}{3a}$
risch	$\frac{\ln\left(-\frac{xd}{c}\right)\ln(dx+c)}{a}+\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.\right.}{3a}$
parts	$\frac{\ln(dx+c)\ln(x)}{a}-\frac{\ln(dx+c)\ln(bx^3+a)}{3a}-\frac{d\left(\frac{3\operatorname{dilog}\left(\frac{dx+c}{c}\right)}{ad}+\frac{3\ln(x)\ln\left(\frac{dx+c}{c}\right)}{ad}-\frac{\ln(dx+c)\ln(bx^3+a)}{ad}+\frac{\_R1=\operatorname{RootOf}(b}{}\right.}{}$

input
```
int(ln(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(dilog(-x*d/c)+ln(d*x+c)*ln(-x*d/c))-1/3/a*sum(ln(d*x+c)*ln((-d*x+_R1-
c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*
d^3-b*c^3))
```

3.285.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x}\,dx$$

input
```
integrate(log(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^4 + a*x), x)
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\left(a+bx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x/(b*x**3+a),x)
```

output
```
Timed out
```

3.285.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x}\,dx$$

input
```
integrate(log(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x), x)
```

3.285.8 Giac [F]

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x}\,dx$$

input
```
integrate(log(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\ln\left(c+d\,x\right)}{x\,\left(b\,x^3+a\right)}\,dx$$

input
```
int(log(c + d*x)/(x*(a + b*x^3)),x)
```

output
```
int(log(c + d*x)/(x*(a + b*x^3)), x)
```

3.286 $\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx$

3.286.1 Optimal result

Integrand size = 19, antiderivative size = 414

$$\begin{aligned}\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx &= -\frac{d}{6acx^2}+\frac{d^2}{3ac^2x}+\frac{d^3\log(x)}{3ac^3}-\frac{d^3\log(c+dx)}{3ac^3}-\frac{\log(c+dx)}{3ax^3}\\
&\quad-\frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2}+\frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}\\
&\quad+\frac{b\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}\\
&\quad+\frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}\\
&\quad+\frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2}\\
&\quad+\frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}-\frac{b\,\mathrm{PolyLog}\left(2,1+\frac{dx}{c}\right)}{a^2}\end{aligned}$$

output
```
-1/6*d/a/c/x^2+1/3*d^2/a/c^2/x+1/3*d^3*ln(x)/a/c^3-1/3*d^3*ln(d*x+c)/a/c^3
-1/3*ln(d*x+c)/a/x^3-b*ln(-d*x/c)*ln(d*x+c)/a^2+1/3*b*ln(-d*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^2+1/3*b*ln(-d*((-1)^(2/3)*a^(1/3)
+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))*ln(d*x+c)/a^2+1/3*b*ln((-1)^
(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))*l
n(d*x+c)/a^2+1/3*b*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^2+1/
3*b*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/a^2+1/3*b*
polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/a^2-b*polylog(
2,1+d*x/c)/a^2
```

3.286.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.98

$$\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx = -\frac{\log(c+dx)}{3ax^3} - \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} + \frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} - \frac{d\left(\frac{1}{cx^2}-\frac{2d}{c^2x}-\frac{2d^2\log(x)}{c^3}+\frac{2d^2\log(c+dx)}{c^3}\right)}{6a} - \frac{b\,\mathrm{PolyLog}\left(2,\frac{c+dx}{c}\right)}{a^2} + \frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2} + \frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2} + \frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}$$

input
```
Integrate[Log[c + d*x]/(x^4*(a + b*x^3)),x]
```

output
```
-1/3*Log[c + d*x]/(a*x^3) - (b*Log[-((d*x)/c)]*Log[c + d*x])/a^2 + (b*Log[
-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a^2
) + (b*Log[-(((-1)^(2/3)*d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c -
(-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*a^2) + (b*Log[((-1)^(1/3)*d*(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d
*x])/(3*a^2) - (d*(1/(c*x^2) - (2*d)/(c^2*x) - (2*d^2*Log[x])/c^3 + (2*d^2
*Log[c + d*x])/c^3))/(6*a) - (b*PolyLog[2, (c + d*x)/c])/a^2 + (b*PolyLog[
2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*a^2) + (b*PolyLog[2, (
b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*a^2) + (b*PolyL
og[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*a^2)
```

3.286.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{b^2x^2\log(c+dx)}{a^2(a+bx^3)}-\frac{b\log(c+dx)}{a^2x}+\frac{\log(c+dx)}{ax^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}-\frac{b\,\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a^2}-\frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2}+\frac{b\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3a^2}+\frac{d^3\log(x)}{3ac^3}-\frac{d^3\log(c+dx)}{3ac^3}+\frac{d^2}{3ac^2x}-\frac{\log(c+dx)}{3ax^3}-\frac{d}{6acx^2}$$

input
```
Int[Log[c + d*x]/(x^4*(a + b*x^3)),x]
```

output
```
-1/6*d/(a*c*x^2) + d^2/(3*a*c^2*x) + (d^3*Log[x])/(3*a*c^3) - (d^3*Log[c +
 d*x])/(3*a*c^3) - Log[c + d*x]/(3*a*x^3) - (b*Log[-((d*x)/c)]*Log[c + d*x
])/a^2 + (b*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[
c + d*x])/(3*a^2) + (b*Log[-((d*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)
*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*a^2) + (b*Log[((-1)^(1/3)*d*
(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[
c + d*x])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)
*d)])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*
a^(1/3)*d)])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)
^(2/3)*a^(1/3)*d)])/(3*a^2) - (b*PolyLog[2, 1 + (d*x)/c])/a^2
```

3.286.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.286.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.73 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{d}{6acx^2}+\frac{d^2}{3a\,c^2x}+\frac{d^3\ln(-dx)}{3a\,c^3}-\frac{d^3\ln(dx+c)}{3a\,c^3}-\frac{\ln(dx+c)}{3a\,x^3}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-\right.}\right.}{}$
derivativedivides	$d^3\left(\frac{-\frac{1}{6c\,d^2x^2}+\frac{1}{3c^2dx}+\frac{\ln(-dx)}{3c^3}-\frac{\ln(dx+c)(dx+c)\left(3c^2-3c(dx+c)+(dx+c)^2\right)}{3c^3d^3x^3}}{a}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_\right.}\right.}{}\right.$
default	$d^3\left(\frac{-\frac{1}{6c\,d^2x^2}+\frac{1}{3c^2dx}+\frac{\ln(-dx)}{3c^3}-\frac{\ln(dx+c)(dx+c)\left(3c^2-3c(dx+c)+(dx+c)^2\right)}{3c^3d^3x^3}}{a}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_\right.}\right.}{}\right.$
parts	$-\frac{\ln(dx+c)}{3a\,x^3}-\frac{\ln(dx+c)b\ln(x)}{a^2}+\frac{\ln(dx+c)b\ln(b\,x^3+a)}{3a^2}-\frac{d\left(\frac{b\left(\frac{\ln(dx+c)\ln\left(b\,x^3+a\right)}{d}-\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2\right.}\right.}{}\right.}{}$

input
```
int(ln(d*x+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d/a/c/x^2+1/3*d^2/a/c^2/x+1/3*d^3/a/c^3*ln(-d*x)-1/3*d^3*ln(d*x+c)/a/
c^3-1/3*ln(d*x+c)/a/x^3+1/3*b/a^2*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog
((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-b
*ln(-x*d/c)*ln(d*x+c)/a^2-b/a^2*dilog(-x*d/c)
```

3.286.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x^4}\,dx$$

input
```
integrate(log(d*x+c)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^7 + a*x^4), x)
```

3.286.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x**4/(b*x**3+a),x)
```

output
```
Timed out
```

3.286.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x^4}\,dx$$

input
```
integrate(log(d*x+c)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x^4), x)
```

3.286.8 Giac [F]

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x^4}\,dx$$

input
```
integrate(log(d*x+c)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x^4), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx = \int \frac{\ln(c+d\,x)}{x^4\,(b\,x^3+a)}\,dx$$

input
```
int(log(c + d*x)/(x^4*(a + b*x^3)),x)
```

output
```
int(log(c + d*x)/(x^4*(a + b*x^3)), x)
```

3.287 $\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx$

3.287.1 Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx &= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2 \log(c+dx)}{2bd^2} + \frac{x^2 \log(c+dx)}{2b} \\ &+ \frac{a^{2/3} \log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right) \log(c+dx)}{3b^{5/3}} \\ &- \frac{\sqrt[3]{-1}a^{2/3} \log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right) \log(c+dx)}{3b^{5/3}} \\ &+ \frac{(-1)^{2/3}a^{2/3} \log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right) \log(c+dx)}{3b^{5/3}} \\ &+ \frac{a^{2/3} \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} \\ &+ \frac{(-1)^{2/3}a^{2/3} \operatorname{PolyLog}\left(2, \frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} \\ &- \frac{\sqrt[3]{-1}a^{2/3} \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{5/3}}\end{aligned}$$

output
```
1/2*c*x/b/d-1/4*x^2/b-1/2*c^2*ln(d*x+c)/b/d^2+1/2*x^2*ln(d*x+c)/b+1/3*a^(2
/3)*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/b^(5/3)-1/3
*(-1)^(1/3)*a^(2/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3
)*c+a^(1/3)*d))*ln(d*x+c)/b^(5/3)+1/3*(-1)^(2/3)*a^(2/3)*ln(-d*(a^(1/3)+(-
1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/b^(5/3)+1/
3*a^(2/3)*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/b^(5/3)+1/3*(-1
)^(2/3)*a^(2/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c
-a^(1/3)*d))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(
d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/b^(5/3)
```

3.287.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx$$

$$= \frac{6b^{2/3}cdx - 3b^{2/3}d^2x^2 - 6b^{2/3}c^2\log(c+dx) + 6b^{2/3}d^2x^2\log(c+dx) + 4a^{2/3}d^2\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+}{}$$

input
```
Integrate[(x^4*Log[c + d*x])/(a + b*x^3),x]
```

output
```
(6*b^(2/3)*c*d*x - 3*b^(2/3)*d^2*x^2 - 6*b^(2/3)*c^2*Log[c + d*x] + 6*b^(2
/3)*d^2*x^2*Log[c + d*x] + 4*a^(2/3)*d^2*Log[(d*(a^(1/3) + b^(1/3)*x))/(-(
b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - 4*(-1)^(1/3)*a^(2/3)*d^2*Log[(d*(a
^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c
+ d*x] + 4*(-1)^(2/3)*a^(2/3)*d^2*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))
/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] + 4*a^(2/3)*d^2*PolyL
og[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)] + 4*(-1)^(2/3)*a^(2/3)*
d^2*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1
/3)*d)] - 4*(-1)^(1/3)*a^(2/3)*d^2*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x
))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(12*b^(5/3)*d^2)
```

3.287.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{x\log(c+dx)}{b}-\frac{ax\log(c+dx)}{b(a+bx^3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{5/3}}+\frac{a^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}}-\frac{c^2\log(c+dx)}{2bd^2}+\frac{x^2\log(c+dx)}{2b}+\frac{cx}{2bd}-\frac{x^2}{4b}$$

input
```
Int[(x^4*Log[c + d*x])/(a + b*x^3),x]
```

output
```
(c*x)/(2*b*d) - x^2/(4*b) - (c^2*Log[c + d*x])/(2*b*d^2) + (x^2*Log[c + d*
x])/(2*b) + (a^(2/3)*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*
d))]*Log[c + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Log[(d*(a^(1/3) - (-1
)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*b
^(5/3)) + ((-1)^(2/3)*a^(2/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(
(-1)^(2/3)*b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^(5/3)) + (a^(2/3)*P
olyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^(5/3)) + ((-1
)^(2/3)*a^(2/3)*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1
/3)*c - a^(1/3)*d)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*PolyLog[2, ((-1)^(1
/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*b^(5/3))
```

3.287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.287.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.36

method	result
risch	$\frac{x^{2} \ln(dx+c)}{2b}-\frac{c^{2} \ln(dx+c)}{2b\,d^{2}}-\frac{x^{2}}{4b}+\frac{cx}{2db}+\frac{3c^{2}}{4d^{2}b}+\frac{da\left(\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\frac{\ln(dx+}{}\right.}{3b^{2}}$
derivativedivides	$\frac{-\frac{d^{3}\left(-\frac{(dx+c)^{2}\ln(dx+c)}{2}+\frac{(dx+c)^{2}}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}+\frac{a\,d^{6}\left(\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\right.}{3b^{2}}}{d^{5}}$
default	$\frac{-\frac{d^{3}\left(-\frac{(dx+c)^{2}\ln(dx+c)}{2}+\frac{(dx+c)^{2}}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}+\frac{a\,d^{6}\left(\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\right.}{3b^{2}}}{d^{5}}$

input
```
int(x^4*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(d*x+c)/b-1/2*c^2*ln(d*x+c)/b/d^2-1/4*x^2/b+1/2/d/b*c*x+3/4/d^2/
b*c^2+1/3*d*a/b^2*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d
*x+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

3.287.5 Fricas [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^4*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(x^4*log(d*x + c)/(b*x^3 + a), x)
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*ln(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.287.7 Maxima [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^4*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^4*log(d*x + c)/(b*x^3 + a), x)
```

3.287.8 Giac [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^4*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^4*log(d*x + c)/(b*x^3 + a), x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \ln(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int((x^4*log(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^4*log(c + d*x))/(a + b*x^3), x)
```

3.288 $\int \frac{x^3 \log(c+dx)}{a+bx^3} dx$

3.288.1 Optimal result

Integrand size = 19, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3 \log(c+dx)}{a+bx^3} dx =& -\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}}\\&-\frac{(-1)^{2/3}\sqrt[3]{a}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}}\\&+\frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}}\\&-\frac{\sqrt[3]{a}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}\\&+\frac{\sqrt[3]{-1}\sqrt[3]{a}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}\\&-\frac{(-1)^{2/3}\sqrt[3]{a}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{4/3}}\end{aligned}$$

output
```
-x/b+(d*x+c)*ln(d*x+c)/b/d-1/3*a^(1/3)*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*
c-a^(1/3)*d))*ln(d*x+c)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*ln(d*(a^(1/3)-(-1)^
(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))*ln(d*x+c)/b^(4/3)+1/3*(
-1)^(1/3)*a^(1/3)*ln(-d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)
*c-a^(1/3)*d))*ln(d*x+c)/b^(4/3)-1/3*a^(1/3)*polylog(2,b^(1/3)*(d*x+c)/(b^
(1/3)*c-a^(1/3)*d))/b^(4/3)+1/3*(-1)^(1/3)*a^(1/3)*polylog(2,(-1)^(2/3)*b^
(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))/b^(4/3)-1/3*(-1)^(2/3)*a^(
1/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d)
)/b^(4/3)
```

3.288.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.96

$$\int \frac{x^3\log(c+dx)}{a+bx^3}\,dx$$

$$= \frac{-3\sqrt[3]{b}dx+3\sqrt[3]{b}c\log(c+dx)+3\sqrt[3]{b}dx\log(c+dx)-\sqrt[3]{a}d\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)-(-1)^{2/3}\sqrt[3]{a}d}{}$$

input
```
Integrate[(x^3*Log[c + d*x])/(a + b*x^3),x]
```

output
```
(-3*b^(1/3)*d*x + 3*b^(1/3)*c*Log[c + d*x] + 3*b^(1/3)*d*x*Log[c + d*x] -
a^(1/3)*d*Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*Log[c
+ d*x] - (-1)^(2/3)*a^(1/3)*d*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-
1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x] + (-1)^(1/3)*a^(1/3)*d*Log[(
d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]
*Log[c + d*x] - a^(1/3)*d*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1
/3)*d)] + (-1)^(1/3)*a^(1/3)*d*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/(
(-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] - (-1)^(2/3)*a^(1/3)*d*PolyLog[2, ((-1)
^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*b^(4/3)*
d)
```

3.288.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} \, dx$$

$$\Big\downarrow \ 2863$$

$$\int \left(\frac{\log(c+dx)}{b} - \frac{a \log(c+dx)}{b\,(a+bx^3)}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\sqrt[3]{a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

input
```
Int[(x^3*Log[c + d*x])/(a + b*x^3),x]
```

output
```
-(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a^(1/3)*Log[-((d*(a^(1/3) + b^(
1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^(4/3)) - ((-1)^(2/3)
*a^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c +
a^(1/3)*d)]*Log[c + d*x])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*Log[-((d*(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d))]*Log[c +
d*x])/(3*b^(4/3)) - (a^(1/3)*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a
^(1/3)*d)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*PolyLog[2, ((-1)^(2/3)*b^(1/
3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(3*b^(4/3)) - ((-1)^(2/
3)*a^(1/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c
 + a^(1/3)*d)])/(3*b^(4/3))
```

3.288.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.288.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.33

method	result
derivativedivides	$\frac{\frac{d^3((dx+c)\ln(dx+c)-dx-c)}{b}-\frac{a\,d^6\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\right.}{\_R1^2-2\_R1c+c}\right.}{3b^2}}{d^4}$
default	$\frac{\frac{d^3((dx+c)\ln(dx+c)-dx-c)}{b}-\frac{a\,d^6\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\right.}{\_R1^2-2\_R1c+c}\right.}{3b^2}}{d^4}$
risch	$\frac{x\ln(dx+c)}{b}+\frac{\ln(dx+c)c}{db}-\frac{x}{b}-\frac{c}{db}-\frac{d^2a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+}{\_R}\right.}{\_R}\right.}{3b^2}$

input
```
int(x^3*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(d^3/b*((d*x+c)*ln(d*x+c)-d*x-c)-1/3*a*d^6/b^2*sum(1/(_R1^2-2*_R1*c+
c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

3.288.5 Fricas [F]

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^3 \log(dx+c)}{bx^3+a} \, dx$$

input
```
integrate(x^3*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(x^3*log(d*x + c)/(b*x^3 + a), x)
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} \, dx = \text{Timed out}$$

input
```
integrate(x**3*ln(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.288.7 Maxima [F]

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^3 \log(dx+c)}{bx^3+a} \, dx$$

input
```
integrate(x^3*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^3*log(d*x + c)/(b*x^3 + a), x)
```

3.288.8 Giac [F]

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^3 \log{(dx+c)}}{bx^3+a} \, dx$$

input
```
integrate(x^3*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^3*log(d*x + c)/(b*x^3 + a), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^3 \ln{(c+d\,x)}}{b\,x^3+a} \, dx$$

input
```
int((x^3*log(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^3*log(c + d*x))/(a + b*x^3), x)
```

3.289 $\int \frac{x\log(c+dx)}{a+bx^3}\,dx$

3.289.1 Optimal result

Integrand size = 17, antiderivative size = 359

$$\int \frac{x\log(c+dx)}{a+bx^3}\,dx = -\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}}-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}$$

output
```
-1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(1/3)/b^
(2/3)+1/3*(-1)^(1/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/
3)*c+a^(1/3)*d))*ln(d*x+c)/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*ln(-d*(a^(1/3)+(
-1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(1/3)/b
^(2/3)-1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(1/3)/b^(2/3
)-1/3*(-1)^(2/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*
c-a^(1/3)*d))/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*polylog(2,(-1)^(1/3)*b^(1/3)*
(d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/a^(1/3)/b^(2/3)
```

3.289.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.83

$$\int \frac{x\log(c+dx)}{a+bx^3}\,dx$$
$$= \frac{-\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)+\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)-(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}+(\cdots}{-(-1)^{2/3}\cdots}\right.}{\cdots}$$

input
```
Integrate[(x*Log[c + d*x])/(a + b*x^3),x]
```

output
```
(-(Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x])
 + (-1)^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)
*c + a^(1/3)*d)]*Log[c + d*x] - (-1)^(2/3)*Log[(d*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - PolyLog[2,
 (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)] - (-1)^(2/3)*PolyLog[2, ((-1
)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] + (-1)^(1/3
)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3
)*d)])/(3*a^(1/3)*b^(2/3))
```

3.289.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\log(c+dx)}{a+bx^3}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[(x*Log[c + d*x])/(a + b*x^3),x]
```

output
```
-1/3*(Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*
x])/(a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x
))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*a^(1/3)*b^(2/3)) -
 ((-1)^(2/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3
)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a^(1/3)*b^(2/3)) - PolyLog[2, (b^(1/3)
*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*a^(1/3)*b^(2/3)) - ((-1)^(2/3)*Pol
yLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)]
)/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*
x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(1/3)*b^(2/3))
```

3.289.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.289.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.24

method	result	size
derivativedivides	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3b}$	86
default	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3b}$	86
risch	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3b}$	86

input
```
int(x*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/b*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)
/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

3.289.5 Fricas [F]

$$\int \frac{x \log(c+dx)}{a+bx^3} \, dx = \int \frac{x \log(dx+c)}{bx^3+a} \, dx$$

input
```
integrate(x*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(x*log(d*x + c)/(b*x^3 + a), x)
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \log(c+dx)}{a+bx^3} \, dx = \text{Timed out}$$

input
```
integrate(x*ln(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.289.7 Maxima [F]

$$\int \frac{x \log(c+dx)}{a+bx^3} \, dx = \int \frac{x \log(dx+c)}{bx^3+a} \, dx$$

input
```
integrate(x*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x*log(d*x + c)/(b*x^3 + a), x)
```

3.289.8 Giac [F]

$$\int \frac{x\log(c+dx)}{a+bx^3}\,dx = \int \frac{x\log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x*log(d*x + c)/(b*x^3 + a), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(c+dx)}{a+bx^3}\,dx = \int \frac{x\,\ln(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int((x*log(c + d*x))/(a + b*x^3),x)
```

output
```
int((x*log(c + d*x))/(a + b*x^3), x)
```

3.290 $\int \frac{\log(c+dx)}{a+bx^3} dx$

3.290.1 Optimal result

Integrand size = 16, antiderivative size = 359

$$\int \frac{\log(c+dx)}{a+bx^3} dx = \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}$$

output
```
1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(2/3)/b^(
1/3)+1/3*(-1)^(2/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3
)*c+a^(1/3)*d))*ln(d*x+c)/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*ln(-d*(a^(1/3)+(-
1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(2/3)/b^
(1/3)+1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(2/3)/b^(1/3)
-1/3*(-1)^(1/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c
-a^(1/3)*d))/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(
d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/a^(2/3)/b^(1/3)
```

3.290.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.82

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx$$

$$= \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)+(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)-\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}+(-1\right.}{-(-1)^{2/3}\sqrt[3]{}}\right.}{}$$

input
```
Integrate[Log[c + d*x]/(a + b*x^3),x]
```

output
```
(Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] +
(-1)^(2/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c
+ a^(1/3)*d)]*Log[c + d*x] - (-1)^(1/3)*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] + PolyLog[2, (b
^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)] - (-1)^(1/3)*PolyLog[2, ((-1)^(
2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] + (-1)^(2/3)*P
olyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d
)])/(3*a^(2/3)*b^(1/3))
```

3.290.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(-\frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\log(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[Log[c + d*x]/(a + b*x^3),x]
```

output
```
(Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(
3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/
((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*a^(2/3)*b^(1/3)) - ((
-1)^(1/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c
 - a^(1/3)*d))]*Log[c + d*x])/(3*a^(2/3)*b^(1/3)) + PolyLog[2, (b^(1/3)*(c
 + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*a^(2/3)*b^(1/3)) - ((-1)^(1/3)*PolyLo
g[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(
3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))
/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(2/3)*b^(1/3))
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.290.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.26

method	result	size
derivativedivides	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{3b}$	94
default	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{3b}$	94
risch	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{3b}$	94

input
```
int(ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog(
(-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

3.290.5 Fricas [F]

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^3 + a), x)
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.290.7 Maxima [F]

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/(b*x^3 + a), x)
```

3.290.8 Giac [F]

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\log(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(log(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/(b*x^3 + a), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\ln(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int(log(c + d*x)/(a + b*x^3),x)
```

output
```
int(log(c + d*x)/(a + b*x^3), x)
```

3.291 $\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx$

3.291.1 Optimal result

Integrand size = 19, antiderivative size = 398

$$\begin{aligned}\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx ={}& \frac{d\log(x)}{ac}-\frac{d\log(c+dx)}{ac}-\frac{\log(c+dx)}{ax}\\
&+\frac{\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}}\\
&-\frac{\sqrt[3]{-1}\sqrt[3]{b}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}}\\
&+\frac{(-1)^{2/3}\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}}\\
&+\frac{\sqrt[3]{b}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}}\\
&+\frac{(-1)^{2/3}\sqrt[3]{b}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}}\\
&-\frac{\sqrt[3]{-1}\sqrt[3]{b}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{4/3}}\end{aligned}$$

output
```
d*ln(x)/a/c-d*ln(d*x+c)/a/c-ln(d*x+c)/a/x+1/3*b^(1/3)*ln(-d*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*ln(d
*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))*ln(d*x+c
)/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*ln(-d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/((-1
)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(4/3)+1/3*b^(1/3)*polylog(2,b^(1
/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*polylog(
2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))/a^(4/3)-1/3
*(-1)^(1/3)*b^(1/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)/((-1)^(1/3)*b^(1/
3)*c+a^(1/3)*d))/a^(4/3)
```

3.291.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.95

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx$$

$$= \frac{3\sqrt[3]{a}dx\log(x)-3\sqrt[3]{a}c\log(c+dx)-3\sqrt[3]{a}dx\log(c+dx)+\sqrt[3]{b}cx\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)-\sqrt[3]{-1}}{}$$

input
```
Integrate[Log[c + d*x]/(x^2*(a + b*x^3)),x]
```

output
```
(3*a^(1/3)*d*x*Log[x] - 3*a^(1/3)*c*Log[c + d*x] - 3*a^(1/3)*d*x*Log[c + d
*x] + b^(1/3)*c*x*Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)
]*Log[c + d*x] - (-1)^(1/3)*b^(1/3)*c*x*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/
3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x] + (-1)^(2/3)*b^(1/
3)*c*x*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((-1)^(2/3)*b^(1/3)*c) +
 a^(1/3)*d)]*Log[c + d*x] + b^(1/3)*c*x*PolyLog[2, (b^(1/3)*(c + d*x))/(b^
(1/3)*c - a^(1/3)*d)] + (-1)^(2/3)*b^(1/3)*c*x*PolyLog[2, ((-1)^(2/3)*b^(1
/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] - (-1)^(1/3)*b^(1/3)*c*
x*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3
)*d)])/(3*a^(4/3)*c*x)
```

3.291.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x^2 (a+bx^3)} \, dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{\log(c+dx)}{ax^2}-\frac{bx\log(c+dx)}{a(a+bx^3)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[3]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}}+\frac{d\log(x)}{ac}-\frac{d\log(c+dx)}{ac}-\frac{\log(c+dx)}{ax}$$

input
```
Int[Log[c + d*x]/(x^2*(a + b*x^3)),x]
```

output
```
(d*Log[x])/(a*c) - (d*Log[c + d*x])/(a*c) - Log[c + d*x]/(a*x) + (b^(1/3)*
Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3
*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(
(-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*a^(4/3)) + ((-1)^(2/3)
*b^(1/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c
- a^(1/3)*d))]*Log[c + d*x])/(3*a^(4/3)) + (b^(1/3)*PolyLog[2, (b^(1/3)*(c
 + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*PolyL
og[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/
(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x)
)/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(4/3))
```

3.291.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.291.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.31

method	result
derivativedivides	$d\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}}{3a}+\frac{\ln(-dx}{c}\right.$
default	$d\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}}{3a}+\frac{\ln(-dx}{c}\right.$
risch	$\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3a}+\frac{d\ln(-}{ac}$

input
```
int(ln(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d*(1/3/a*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)
/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(1/c*ln(-
d*x)-ln(d*x+c)*(d*x+c)/c/d/x))
```

3.291.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx=\int \frac{\log(dx+c)}{(bx^3+a)x^2}\,dx$$

input
```
integrate(log(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^5 + a*x^2), x)
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x**2/(b*x**3+a),x)
```

output
```
Timed out
```

3.291.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x^2}\,dx$$

input
```
integrate(log(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x^2), x)
```

3.291.8 Giac [F]

$$\int \frac{\log(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x^2}\,dx$$

input
```
integrate(log(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x^2), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2 (a+bx^3)}\,dx = \int \frac{\ln(c+d\,x)}{x^2\,\left(b\,x^3+a\right)}\,dx$$

input
```
int(log(c + d*x)/(x^2*(a + b*x^3)),x)
```

output
```
int(log(c + d*x)/(x^2*(a + b*x^3)), x)
```

3.292 $\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx$

3.292.1 Optimal result

Integrand size = 19, antiderivative size = 423

$$\begin{aligned}\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = &-\frac{d}{2acx}-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2}\\
&-\frac{b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}\\
&-\frac{(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}\\
&+\frac{\sqrt[3]{-1}b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}\\
&-\frac{b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}\\
&+\frac{\sqrt[3]{-1}b^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}\\
&-\frac{(-1)^{2/3}b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{5/3}}\end{aligned}$$

output
```
-1/2*d/a/c/x-1/2*d^2*ln(x)/a/c^2+1/2*d^2*ln(d*x+c)/a/c^2-1/2*ln(d*x+c)/a/x
^2-1/3*b^(2/3)*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/
a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(
1/3)*b^(1/3)*c+a^(1/3)*d))*ln(d*x+c)/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*ln(-d*
(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)
/a^(5/3)-1/3*b^(2/3)*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(5
/3)+1/3*(-1)^(1/3)*b^(2/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3
)*b^(1/3)*c-a^(1/3)*d))/a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*polylog(2,(-1)^(1/3
)*b^(1/3)*(d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/a^(5/3)
```

3.292.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.88

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx$$

$$= \frac{-\frac{3a^{2/3}\log(c+dx)}{x^2} - 2b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx) - 2(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+}{}$$

input
```
Integrate[Log[c + d*x]/(x^3*(a + b*x^3)),x]
```

output
```
((-3*a^(2/3)*Log[c + d*x])/x^2 - 2*b^(2/3)*Log[(d*(a^(1/3) + b^(1/3)*x))/(
-(b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - 2*(-1)^(2/3)*b^(2/3)*Log[(d*(a^(
1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c +
d*x] + 2*(-1)^(1/3)*b^(2/3)*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((-
1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - (3*a^(2/3)*d*(c + d*x*Log
[x] - d*x*Log[c + d*x]))/(c^2*x) - 2*b^(2/3)*PolyLog[2, (b^(1/3)*(c + d*x)
)/(b^(1/3)*c - a^(1/3)*d)] + 2*(-1)^(1/3)*b^(2/3)*PolyLog[2, ((-1)^(2/3)*b
^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] - 2*(-1)^(2/3)*b^(2/
3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/
3)*d)])/(6*a^(5/3))
```

3.292.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^3\right)} \, dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{\log(c+dx)}{ax^3}-\frac{b\log(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{5/3}}-\frac{b^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2}-\frac{d}{2acx}$$

input
```
Int[Log[c + d*x]/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*d/(a*c*x) - (d^2*Log[x])/(2*a*c^2) + (d^2*Log[c + d*x])/(2*a*c^2) - L
og[c + d*x]/(2*a*x^2) - (b^(2/3)*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*
c - a^(1/3)*d))]*Log[c + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Log[(d*(a
^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c
+ d*x])/(3*a^(5/3)) + ((-1)^(1/3)*b^(2/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b
^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a^(5/3))
- (b^(2/3)*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*a^(
5/3)) + ((-1)^(1/3)*b^(2/3)*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1
)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*PolyLog
[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3
*a^(5/3))
```

3.292.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.292.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.35

method	result
derivativedivides	$d^2\left(\frac{-\frac{\ln(-dx)}{2c^2}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^2d^2x^2}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+}{}\right.}{-}}{3a}\right.$
default	$d^2\left(\frac{-\frac{\ln(-dx)}{2c^2}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^2d^2x^2}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+}{}\right.}{-}}{3a}\right.$
risch	$-\frac{d^2\ln(-dx)}{2a\,c^2}-\frac{d}{2acx}+\frac{d^2\ln(dx+c)}{2a\,c^2}-\frac{\ln(dx+c)}{2a\,x^2}-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+}{}\right.}{3a}$

input
```
int(ln(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d^2*(1/a*(-1/2/c^2*ln(-d*x)-1/2/c/d/x-1/2*ln(d*x+c)*(d*x+c)*(-d*x+c)/c^2/d
^2/x^2)-1/3/a*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+di
log((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
)))
```

3.292.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x^3\,(a+bx^3)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x^3}\,dx$$

input
```
integrate(log(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^6 + a*x^3), x)
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x**3/(b*x**3+a),x)
```

output
```
Timed out
```

3.292.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\log(dx+c)}{(bx^3+a)x^3}\,dx$$

input
```
integrate(log(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x^3), x)
```

3.292.8 Giac [F]

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\log(dx+c)}{(bx^3+a)x^3}\,dx$$

input
```
integrate(log(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^3 + a)*x^3), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\ln(c+d\,x)}{x^3\,(b\,x^3+a)}\,dx$$

input
```
int(log(c + d*x)/(x^3*(a + b*x^3)),x)
```

output
```
int(log(c + d*x)/(x^3*(a + b*x^3)), x)
```

3.293 $\int \frac{x^7 \log(c+dx)}{a+bx^4} dx$

3.293.1 Optimal result

Integrand size = 19, antiderivative size = 498

$$\begin{aligned}\int \frac{x^7 \log(c+dx)}{a+bx^4} dx &= \frac{c^3x}{4bd^3} - \frac{c^2x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4\log(c+dx)}{4bd^4} \\ &+ \frac{x^4\log(c+dx)}{4b} - \frac{a\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} \\ &- \frac{a\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} \\ &- \frac{a\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} \\ &- \frac{a\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} \\ &- \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^2} \\ &- \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2}\end{aligned}$$

output
```
1/4*c^3*x/b/d^3-1/8*c^2*x^2/b/d^2+1/12*c*x^3/b/d-1/16*x^4/b-1/4*c^4*ln(d*x
+c)/b/d^4+1/4*x^4*ln(d*x+c)/b-1/4*a*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c
+(-a)^(1/4)*d))*ln(d*x+c)/b^2-1/4*a*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*
c-(-a)^(1/4)*d))*ln(d*x+c)/b^2-1/4*a*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/
2))^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^2-1/4*a*ln(d*x+c)*ln(d*(-b
^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^2-1/4*a
*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/b^2-1/4*a*polylog(2,b
^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^2-1/4*a*polylog(2,b^(1/4)*(d*x+
c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^2-1/4*a*polylog(2,b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^2
```

3.293.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{x^7\log(c+dx)}{a+bx^4}\,dx &= \frac{c^3x}{4bd^3}-\frac{c^2x^2}{8bd^2}+\frac{cx^3}{12bd}-\frac{x^4}{16b}-\frac{c^4\log(c+dx)}{4bd^4} \\ &\quad+\frac{x^4\log(c+dx)}{4b}-\frac{a\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} \\ &\quad-\frac{a\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} \\ &\quad-\frac{a\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} \\ &\quad-\frac{a\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} \\ &\quad-\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2}-\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4b^2} \\ &\quad-\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4b^2}-\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2}\end{aligned}$$

input
```
Integrate[(x^7*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(c^3*x)/(4*b*d^3) - (c^2*x^2)/(8*b*d^2) + (c*x^3)/(12*b*d) - x^4/(16*b) -
(c^4*Log[c + d*x])/(4*b*d^4) + (x^4*Log[c + d*x])/(4*b) - (a*Log[(d*(I*(-a
)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d*x])/(4*b^2)
- (a*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c +
d*x])/(4*b^2) - (a*Log[-((d*(I*(-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*(-a
)^(1/4)*d))]*Log[c + d*x])/(4*b^2) - (a*Log[-((d*((-a)^(1/4) + b^(1/4)*x))
/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b^2) - (a*PolyLog[2, (b^(1/
4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[2, (b^(1/4
)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[2, (b^(1/
4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[2, (b^(1
/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^2)
```

3.293.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \log(c+dx)}{a+bx^4} \, dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{x^3 \log(c+dx)}{b}-\frac{ax^3 \log(c+dx)}{b\left(a+bx^4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2}-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^2}-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2}-\frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2}-\frac{a\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b^2}-\frac{a\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^2}-\frac{a\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2}-\frac{a\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2}-\frac{c^4\log(c+dx)}{4bd^4}+\frac{c^3x}{4bd^3}-\frac{c^2x^2}{8bd^2}+\frac{x^4\log(c+dx)}{4b}+\frac{cx^3}{12bd}-\frac{x^4}{16b}$$

input
```
Int[(x^7*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(c^3*x)/(4*b*d^3) - (c^2*x^2)/(8*b*d^2) + (c*x^3)/(12*b*d) - x^4/(16*b) -
(c^4*Log[c + d*x])/(4*b*d^4) + (x^4*Log[c + d*x])/(4*b) - (a*Log[(d*(Sqrt[
-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4
*b^2) - (a*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Lo
g[c + d*x])/(4*b^2) - (a*Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*
c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*b^2) - (a*Log[-((d*((-a)^(1/4) +
 b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b^2) - (a*PolyL
og[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^2) - (a*P
olyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^2) -
(a*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^2) - (
a*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^2)
```

3.293.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.293.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{c^4\ln(dx+c)}{4b\,d^4}+\frac{c^3x}{4b\,d^3}+\frac{25c^4}{48d^4b}-\frac{c^2x^2}{8b\,d^2}+\frac{c\,x^3}{12bd}+\frac{x^4\ln(dx+c)}{4b}-\frac{x^4}{16b}-\frac{a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\cdots\right.}\cdots\right.}{\cdots}$
parts	$\frac{x^4\ln(dx+c)}{4b}-\frac{\ln(dx+c)a\ln(b\,x^4+a)}{4b^2}-\frac{d\left(\frac{\frac{\frac{1}{4}d^3x^4-\frac{1}{3}c\,x^3d^2+\frac{1}{2}d\,x^2c^2-x\,c^3}{d^4}+\frac{c^4\ln(dx+c)}{d^5}}{b}-\frac{a\ln(dx+c)\ln\left(b\,x^4+a\right)}{b^2d}+\frac{a\left(\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$-\frac{\frac{d^4\left(c^3((dx+c)\ln(dx+c)-dx-c)-3c^2\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+3c\left(\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)-\frac{(dx+c)^4\ln(dx+c)}{4}+\cdots\right.}{b}}{\cdots}$
default	$-\frac{\frac{d^4\left(c^3((dx+c)\ln(dx+c)-dx-c)-3c^2\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+3c\left(\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)-\frac{(dx+c)^4\ln(dx+c)}{4}+\cdots\right.}{b}}{\cdots}$

input
```
int(x^7*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*c^4*ln(d*x+c)/b/d^4+1/4*c^3*x/b/d^3+25/48/d^4/b*c^4-1/8*c^2*x^2/b/d^2
+1/12*c*x^3/b/d+1/4*x^4*ln(d*x+c)/b-1/16*x^4/b-1/4*a/b^2*sum(ln(d*x+c)*ln(
(-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_
Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4))
```

3.293.5 Fricas [F]

$$\int \frac{x^7\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^7\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^7*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^7*log(d*x + c)/(b*x^4 + a), x)
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**7*ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.293.7 Maxima [F]

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^7 \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^7*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^7*log(d*x + c)/(b*x^4 + a), x)
```

3.293.8 Giac [F]

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^7 \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^7*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^7*log(d*x + c)/(b*x^4 + a), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^7 \ln(c+d\,x)}{b\,x^4+a} \, dx$$

input
```
int((x^7*log(c + d*x))/(a + b*x^4),x)
```

output
```
int((x^7*log(c + d*x))/(a + b*x^4), x)
```

3.294 $\int \frac{x^3 \log(c+dx)}{a+bx^4}\,dx$

3.294.1 Optimal result

Integrand size = 19, antiderivative size = 401

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b}$$

output
```
1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/b+1/4*
ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)/b+1/4*ln(
d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1
/2)))/b+1/4*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(
-(-a)^(1/2))^(1/2)))/b+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)
*d))/b+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b+1/4*polyl
og(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b+1/4*polylog(2,b^
(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b
```

3.294.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \frac{\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b}$$

input
```
Integrate[(x^3*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(Log[(d*(I*(-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]*Log[c +
d*x])/(4*b) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)
]*Log[c + d*x])/(4*b) + (Log[-((d*(I*(-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c -
 I*(-a)^(1/4)*d))]*Log[c + d*x])/(4*b) + (Log[-((d*((-a)^(1/4) + b^(1/4)*x
))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b) + PolyLog[2, (b^(1/4)*
(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*
x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*x))/(
b^(1/4)*c + I*(-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/
4)*c + (-a)^(1/4)*d)]/(4*b)
```

3.294.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log(c+dx)}{a+bx^4} \, dx$$
$$\Big\downarrow \; 2863$$
$$\int \left(\frac{x \log(c+dx)}{2\left(bx^2-\sqrt{-a}\sqrt{b}\right)}+\frac{x \log(c+dx)}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b}$$

input
```
Int[(x^3*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Lo
g[c + d*x])/(4*b) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1
/4)*d)]*Log[c + d*x])/(4*b) + (Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^
(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*b) + (Log[-((d*((-a)^(1/4)
 + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b) + PolyLog[
2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*b) + PolyLog[2,
 (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]/(4*b) + PolyLog[2, (
b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*b)
```

3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.294.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.21

method	result
derivativedivides	$\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6bc^2\_Z^2-4bc^3\_Z+ad^4+bc^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{4b}$
default	$\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6bc^2\_Z^2-4bc^3\_Z+ad^4+bc^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{4b}$
risch	$\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6bc^2\_Z^2-4bc^3\_Z+ad^4+bc^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{4b}$
parts	$\frac{\ln(dx+c)\ln\left(bx^4+a\right)}{4b}-\frac{d\left(\frac{\ln(dx+c)\ln\left(bx^4+a\right)}{d}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6bc^2\_Z^2-4bc^3\_Z+ad^4+bc^4\right)}\left(\ln(dx+c)\right.}{d}\right.}{4b}$

input
```
int(x^3*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootO
f(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4))
```

3.294.5 Fricas [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^3\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^3*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^3*log(d*x + c)/(b*x^4 + a), x)
```

3.294.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx=\text{Timed out}$$

input
```
integrate(x**3*ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.294.7 Maxima [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \int \frac{x^3\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^3*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^3*log(d*x + c)/(b*x^4 + a), x)
```

3.294.8 Giac [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \int \frac{x^3\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^3*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^3*log(d*x + c)/(b*x^4 + a), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \int \frac{x^3\,\ln(c+d\,x)}{b\,x^4+a}\,dx$$

input
```
int((x^3*log(c + d*x))/(a + b*x^4),x)
```

output
```
int((x^3*log(c + d*x))/(a + b*x^4), x)
```

3.295 $\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx$

3.295.1 Optimal result

Integrand size = 19, antiderivative size = 433

$$\begin{aligned}\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx &= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} \\ &\quad - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} \\ &\quad - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} \\ &\quad - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a} \\ &\quad - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a} \\ &\quad - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a} + \frac{\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)}{a}\end{aligned}$$

output
```
ln(-d*x/c)*ln(d*x+c)/a-1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/
4)*d))*ln(d*x+c)/a-1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*
d))*ln(d*x+c)/a-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/
4)*c-d*(-(-a)^(1/2))^(1/2)))/a-1/4*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2)
)^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/a-1/4*polylog(2,b^(1/4)*(d*x+c
)/(b^(1/4)*c-(-a)^(1/4)*d))/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a
)^(1/4)*d))/a+polylog(2,1+d*x/c)/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*
c-d*(-(-a)^(1/2))^(1/2)))/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(
-a)^(1/2))^(1/2)))/a
```

3.295.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.96

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} + \frac{\text{PolyLog}\left(2,\frac{c+dx}{c}\right)}{a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a}$$

input
```
Integrate[Log[c + d*x]/(x*(a + b*x^4)),x]
```

output
```
(Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[(d*(I*(-a)^(1/4) - b^(1/4)*x))/(b^
(1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d*x])/(4*a) - (Log[(d*((-a)^(1/4) - b^(
1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*a) - (Log[-((d*(I*(-
a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*(-a)^(1/4)*d))]*Log[c + d*x])/(4*a)
- (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c +
 d*x])/(4*a) + PolyLog[2, (c + d*x)/c]/a - PolyLog[2, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - (-a)^(1/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4
)*c - I*(-a)^(1/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c +
 I*(-a)^(1/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)
^(1/4)*d)]/(4*a)
```

3.295.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log(c+dx)}{ax}-\frac{bx^3\log(c+dx)}{a\left(a+bx^4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4a}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}$$

input
```
Int[Log[c + d*x]/(x*(a + b*x^4)),x]
```

output
```
(Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/
(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*a) - (Log[(d*((-a)^(1/4)
 - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*a) - (Log[-((d
*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c +
d*x])/(4*a) - (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*
d))]*Log[c + d*x])/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqr
t[-Sqrt[-a]]*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1
/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]
/(4*a) + PolyLog[2, 1 + (d*x)/c]/a
```

3.295.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.295.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.26

method	result
derivativedivides	$\frac{\mathrm{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_}{-}\right.\right.}{4a}$
default	$\frac{\mathrm{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_}{-}\right.\right.}{4a}$
risch	$\frac{\ln\left(-\frac{xd}{c}\right)\ln(dx+c)}{a}+\frac{\mathrm{dilog}\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-d}{}\right.\right.}{4a}$
parts	$\frac{\ln(dx+c)\ln(x)}{a}-\frac{\ln(dx+c)\ln(b\,x^4+a)}{4a}-\frac{d\left(\frac{4\,\mathrm{dilog}\left(\frac{dx+c}{c}\right)}{ad}+\frac{4\ln(x)\ln\left(\frac{dx+c}{c}\right)}{ad}-\frac{\ln(dx+c)\ln\left(b\,x^4+a\right)}{ad}+\frac{\_R1=\mathrm{RootOf}\left(b\_\right.}{}\right.}{}$

input
```
int(ln(d*x+c)/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(dilog(-x*d/c)+ln(d*x+c)*ln(-x*d/c))-1/4/a*sum(ln(d*x+c)*ln((-d*x+_R1-
c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-
4*_Z*b*c^3+a*d^4+b*c^4))
```

3.295.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x\left(a+bx^4\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^4+a)x}\,dx$$

input
```
integrate(log(d*x+c)/x/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^5 + a*x), x)
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\,(a+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x/(b*x**4+a),x)
```

output
```
Timed out
```

3.295.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x\,(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x}\,dx$$

input
```
integrate(log(d*x+c)/x/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^4 + a)*x), x)
```

3.295.8 Giac [F]

$$\int \frac{\log(c+dx)}{x\,(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x}\,dx$$

input
```
integrate(log(d*x+c)/x/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^4 + a)*x), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\,(a+bx^4)}\,dx = \int \frac{\ln(c+d\,x)}{x\,(b\,x^4+a)}\,dx$$

input
```
int(log(c + d*x)/(x*(a + b*x^4)),x)
```

output
```
int(log(c + d*x)/(x*(a + b*x^4)), x)
```

3.296 $\int \frac{x^5 \log(c+dx)}{a+bx^4} dx$

3.296.1 Optimal result

Integrand size = 19, antiderivative size = 530

$$\begin{aligned}\int \frac{x^5\log(c+dx)}{a+bx^4}\,dx = {} & \frac{cx}{2bd}-\frac{x^2}{4b}-\frac{c^2\log(c+dx)}{2bd^2}+\frac{x^2\log(c+dx)}{2b} \\
& -\frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} \\
& +\frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} \\
& -\frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} \\
& +\frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} \\
& -\frac{\sqrt{-a}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} \\
& -\frac{\sqrt{-a}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} \\
& +\frac{\sqrt{-a}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}} \\
& +\frac{\sqrt{-a}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/2}}\end{aligned}$$

output
```
1/2*c*x/b/d-1/4*x^2/b-1/2*c^2*ln(d*x+c)/b/d^2+1/2*x^2*ln(d*x+c)/b+1/4*ln(d
*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)*(-a)^(1/2)/b^(
3/2)+1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)*
(-a)^(1/2)/b^(3/2)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^
(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*(-a)^(1/2)/b^(3/2)-1/4*ln(d*x+c)*ln(d*(-b^
(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))*(-a)^(1/2)
/b^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))*(-a)^(1/2
)/b^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))*(-a)^(1/
2)/b^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2))
)*(-a)^(1/2)/b^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/
2))^(1/2)))*(-a)^(1/2)/b^(3/2)
```

3.296.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.91

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx$$

$$= \frac{2\sqrt{b}cdx - \sqrt{b}d^2x^2 - 2\sqrt{b}c^2\log(c+dx) + 2\sqrt{b}d^2x^2\log(c+dx) + \sqrt{-a}d^2\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+d}{}$$

input
```
Integrate[(x^5*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(2*Sqrt[b]*c*d*x - Sqrt[b]*d^2*x^2 - 2*Sqrt[b]*c^2*Log[c + d*x] + 2*Sqrt[b
]*d^2*x^2*Log[c + d*x] + Sqrt[-a]*d^2*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^
(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - Sqrt[-a]*d^2*Log[(d*((-a)^(1/4) -
I*b^(1/4)*x))/(I*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - Sqrt[-a]*d^2*Lo
g[(d*((-a)^(1/4) + I*b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c +
d*x] + Sqrt[-a]*d^2*Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^
(1/4)*d)]*Log[c + d*x] + Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1
/4)*c - (-a)^(1/4)*d)] - Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1
/4)*c - I*(-a)^(1/4)*d)] - Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^
(1/4)*c + I*(-a)^(1/4)*d)] + Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(
b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/2)*d^2)
```

3.296.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \log(c+dx)}{a+bx^4} \, dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{x \log(c+dx)}{b}-\frac{ax \log(c+dx)}{b\,(a+bx^4)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}}-\frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}}+\frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}}+\frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/2}}-\frac{\sqrt{-a}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b^{3/2}}+\frac{\sqrt{-a}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{3/2}}-\frac{\sqrt{-a}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}}+\frac{\sqrt{-a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}}-\frac{c^2\log(c+dx)}{2bd^2}+\frac{x^2\log(c+dx)}{2b}+\frac{cx}{2bd}-\frac{x^2}{4b}$$

input
```
Int[(x^5*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(c*x)/(2*b*d) - x^2/(4*b) - (c^2*Log[c + d*x])/(2*b*d^2) + (x^2*Log[c + d*
x])/(2*b) - (Sqrt[-a]*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + S
qrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*b^(3/2)) + (Sqrt[-a]*Log[(d*((-a)^(1/4
) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*b^(3/2)) - (S
qrt[-a]*Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a
]]*d))]*Log[c + d*x])/(4*b^(3/2)) + (Sqrt[-a]*Log[-((d*((-a)^(1/4) + b^(1/
4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b^(3/2)) - (Sqrt[-a]*
PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/2
)) - (Sqrt[-a]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]
*d)])/(4*b^(3/2)) + (Sqrt[-a]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c -
(-a)^(1/4)*d)])/(4*b^(3/2)) + (Sqrt[-a]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^
(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/2))
```

3.296.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.296.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\frac{d^4\left(-\frac{(dx+c)^2\ln(dx+c)}{2}+\frac{(dx+c)^2}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}-\frac{a\,d^8\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a \ldots\right.}\ldots\right.}{\ldots}}{d^6}$
default	$-\frac{\frac{d^4\left(-\frac{(dx+c)^2\ln(dx+c)}{2}+\frac{(dx+c)^2}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}-\frac{a\,d^8\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a \ldots\right.}\ldots\right.}{\ldots}}{d^6}$
risch	$\frac{x^2\ln(dx+c)}{2b}-\frac{c^2\ln(dx+c)}{2b\,d^2}-\frac{x^2}{4b}+\frac{cx}{2db}+\frac{3c^2}{4d^2b}-\frac{d^2a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+\ldots\right.}\ldots\right.}{4b\ldots}$

input
```
int(x^5*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^6*(-d^4/b*(-1/2*(d*x+c)^2*ln(d*x+c)+1/4*(d*x+c)^2+c*((d*x+c)*ln(d*x+c)
-d*x-c))-1/4*a*d^8/b^2*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c
)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-
4*_Z*b*c^3+a*d^4+b*c^4)))
```

3.296.5 Fricas [F]

$$\int \frac{x^5\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^5\log{(dx+c)}}{bx^4+a}\,dx$$

input
```
integrate(x^5*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^5*log(d*x + c)/(b*x^4 + a), x)
```

3.296.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^4} \, dx = \text{Timed out}$$

input
```
integrate(x**5*ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.296.7 Maxima [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^5 \log(dx+c)}{bx^4+a} \, dx$$

input
```
integrate(x^5*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^5*log(d*x + c)/(b*x^4 + a), x)
```

3.296.8 Giac [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^5 \log(dx+c)}{bx^4+a} \, dx$$

input
```
integrate(x^5*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^5*log(d*x + c)/(b*x^4 + a), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^5\,\ln\left(c+d\,x\right)}{b\,x^4+a}\,dx$$

input
```
int((x^5*log(c + d*x))/(a + b*x^4),x)
```

output
```
int((x^5*log(c + d*x))/(a + b*x^4), x)
```

3.297 $\int \frac{x\log(c+dx)}{a+bx^4}\,dx$

3.297.1 Optimal result

Integrand size = 17, antiderivative size = 473

$$\begin{aligned}\int \frac{x\log(c+dx)}{a+bx^4}\,dx = & -\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} \\ & +\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} \\ & -\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} \\ & +\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} \\ & -\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}} \\ & +\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}\end{aligned}$$

output
```
1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(
1/2)/b^(1/2)+1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln
(d*x+c)/(-a)^(1/2)/b^(1/2)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1
/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*ln(d*x+c)*l
n(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/(-
a)^(1/2)/b^(1/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/(
-a)^(1/2)/b^(1/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/
(-a)^(1/2)/b^(1/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2)
)^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-
(-a)^(1/2))^(1/2)))/(-a)^(1/2)/b^(1/2)
```

3.297.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.74

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx$$
$$= \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)-\log\left(\frac{d\left(\sqrt[4]{-a}-i\sqrt[4]{b}x\right)}{i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)-\log\left(\frac{d\left(\sqrt[4]{-a}+i\sqrt[4]{b}x\right)}{-i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+}{}$$

input
```
Integrate[(x*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x]
 - Log[(d*((-a)^(1/4) - I*b^(1/4)*x))/(I*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c
+ d*x] - Log[(d*((-a)^(1/4) + I*b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*d
)]*Log[c + d*x] + Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1
/4)*d)]*Log[c + d*x] + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1
/4)*d)] - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)] - P
olyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)] + PolyLog[2, (
b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*Sqrt[-a]*Sqrt[b])
```

3.297.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-bx^2\right)}-\frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}$$

input
```
Int[(x*Log[c + d*x])/(a + b*x^4),x]
```

output
```
-1/4*(Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d
)]*Log[c + d*x])/(Sqrt[-a]*Sqrt[b]) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b
^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*Sqrt[-a]*Sqrt[b]) - (Log[-((d*(
Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*
x])/(4*Sqrt[-a]*Sqrt[b]) + (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c
- (-a)^(1/4)*d))]*Log[c + d*x])/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, (b^(1/4)
*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*Sqrt[-a]*Sqrt[b]) - PolyLo
g[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]/(4*Sqrt[-a]*Sqrt
[b]) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*Sqrt[
-a]*Sqrt[b]) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(4*Sqrt[-a]*Sqrt[b])
```

3.297.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.297.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.22

method	result
derivativedivides	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right.}{4b}$
default	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right.}{4b}$
risch	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right.}{4b}$

input
```
int(x*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog(
(-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*
d^4+b*c^4))
```

3.297.5 Fricas [F]

$$\int \frac{x \log(c+dx)}{a+bx^4}\,dx = \int \frac{x \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x*log(d*x + c)/(b*x^4 + a), x)
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.297.7 Maxima [F]

$$\int \frac{x \log(c+dx)}{a+bx^4}\,dx = \int \frac{x \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x*log(d*x + c)/(b*x^4 + a), x)
```

3.297.8 Giac [F]

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \int \frac{x\log\left(dx+c\right)}{bx^4+a}\,dx$$

input
```
integrate(x*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x*log(d*x + c)/(b*x^4 + a), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \int \frac{x\,\ln\left(c+d\,x\right)}{b\,x^4+a}\,dx$$

input
```
int((x*log(c + d*x))/(a + b*x^4),x)
```

output
```
int((x*log(c + d*x))/(a + b*x^4), x)
```

3.298 $\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx$

3.298.1 Optimal result

Integrand size = 19, antiderivative size = 537

$$\begin{aligned}
\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx &= -\frac{d}{2acx}-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2} \\
&\quad-\frac{\sqrt{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} \\
&\quad+\frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} \\
&\quad-\frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} \\
&\quad+\frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} \\
&\quad-\frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}} \\
&\quad+\frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}
\end{aligned}$$

output
```
-1/2*d/a/c/x-1/2*d^2*ln(x)/a/c^2+1/2*d^2*ln(d*x+c)/a/c^2-1/2*ln(d*x+c)/a/x
^2+1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)*b^(
1/2)/(-a)^(3/2)+1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))
*ln(d*x+c)*b^(1/2)/(-a)^(3/2)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))
^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*b^(1/2)/(-a)^(3/2)-1/4*ln(d*x+c
)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))
*b^(1/2)/(-a)^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d)
)*b^(1/2)/(-a)^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d
))*b^(1/2)/(-a)^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1
/2))^(1/2)))*b^(1/2)/(-a)^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d
*(-(-a)^(1/2))^(1/2)))*b^(1/2)/(-a)^(3/2)
```

3.298.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.94

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = -\frac{\log(c+dx)}{2ax^2} - \frac{\sqrt{b}\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{d\left(\frac{1}{cx}+\frac{d\log(x)}{c^2}-\frac{d\log(c+dx)}{c^2}\right)}{2a} + \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}$$

input
```
Integrate[Log[c + d*x]/(x^3*(a + b*x^4)),x]
```

3.298. $\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx$

output
```
-1/2*Log[c + d*x]/(a*x^2) - (Sqrt[b]*Log[(d*(I*(-a)^(1/4) - b^(1/4)*x))/(b
^(1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[(d
*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-
a)^(3/2)) - (Sqrt[b]*Log[-((d*(I*(-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*(
-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[-((d*((-a)^(1/4
) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/2))
- (d*(1/(c*x) + (d*Log[x])/c^2 - (d*Log[c + d*x])/c^2))/(2*a) + (Sqrt[b]*P
olyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*(-a)^(3/2))
- (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)])/(
4*(-a)^(3/2)) - (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a
)^(1/4)*d)])/(4*(-a)^(3/2)) + (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(
1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(3/2))
```

3.298.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx$$

$$\Big\downarrow \ 2863$$

$$\int \left(\frac{\log(c+dx)}{ax^3}-\frac{bx\log(c+dx)}{a(a+bx^4)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2}-\frac{d}{2acx}$$

input
```
Int[Log[c + d*x]/(x^3*(a + b*x^4)),x]
```

output
```
-1/2*d/(a*c*x) - (d^2*Log[x])/(2*a*c^2) + (d^2*Log[c + d*x])/(2*a*c^2) - L
og[c + d*x]/(2*a*x^2) - (Sqrt[b]*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^
(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[
(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*
(-a)^(3/2)) - (Sqrt[b]*Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c
- Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[-((d*((
-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)
^(3/2)) - (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[
-a]]*d)])/(4*(-a)^(3/2)) - (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4
)*c + Sqrt[-Sqrt[-a]]*d)])/(4*(-a)^(3/2)) + (Sqrt[b]*PolyLog[2, (b^(1/4)*(
c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*(-a)^(3/2)) + (Sqrt[b]*PolyLog[2
, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(3/2))
```

3.298.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.298. $\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx$

3.298.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.29

method	result
derivativedivides	$d^2\left(\frac{-\frac{\ln(-dx)}{2c^2}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^2d^2x^2}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c}{}}{4a}\right.$
default	$d^2\left(\frac{-\frac{\ln(-dx)}{2c^2}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^2d^2x^2}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c}{}}{4a}\right.$
risch	$-\frac{d^2\ln(-dx)}{2a\,c^2}-\frac{d}{2acx}+\frac{d^2\ln(dx+c)}{2a\,c^2}-\frac{\ln(dx+c)}{2a\,x^2}-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\right.}\right.}{4a}$

input
```
int(ln(d*x+c)/x^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d^2*(1/a*(-1/2/c^2*ln(-d*x)-1/2/c/d/x-1/2*ln(d*x+c)*(d*x+c)*(-d*x+c)/c^2/d
^2/x^2)-1/4/a*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+di
log((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^
3+a*d^4+b*c^4)))
```

3.298.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x^3}\,dx$$

input
```
integrate(log(d*x+c)/x^3/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^7 + a*x^3), x)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x**3/(b*x**4+a),x)
```

output
```
Timed out
```

3.298.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^4\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^4+a)x^3}\,dx$$

input
```
integrate(log(d*x+c)/x^3/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^4 + a)*x^3), x)
```

3.298.8 Giac [F]

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^4\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^4+a)x^3}\,dx$$

input
```
integrate(log(d*x+c)/x^3/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^4 + a)*x^3), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^4\right)}\,dx=\int \frac{\ln\left(c+d\,x\right)}{x^3\,\left(b\,x^4+a\right)}\,dx$$

input
```
int(log(c + d*x)/(x^3*(a + b*x^4)),x)
```

output
```
int(log(c + d*x)/(x^3*(a + b*x^4)), x)
```

3.299 $\int \frac{x^4 \log(c+dx)}{a+bx^4} \, dx$

3.299.1 Optimal result

Integrand size = 19, antiderivative size = 521

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = -\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd}+\frac{\sqrt{-\sqrt{-a}}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}+\frac{\sqrt[4]{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}-\frac{\sqrt{-\sqrt{-a}}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}-\frac{\sqrt[4]{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}-\frac{\sqrt{-\sqrt{-a}}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}+\frac{\sqrt{-\sqrt{-a}}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}-\frac{\sqrt[4]{-a}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}}+\frac{\sqrt[4]{-a}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{5/4}}$$

output
```
-x/b+(d*x+c)*ln(d*x+c)/b/d+1/4*(-a)^(1/4)*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(
1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/b^(5/4)-1/4*(-a)^(1/4)*ln(-d*((-a)^(1/4)+b
^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)/b^(5/4)-1/4*(-a)^(1/4)*polyl
og(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/b^(5/4)+1/4*(-a)^(1/4)*poly
log(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(5/4)-1/4*ln(d*x+c)*ln(-
d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*(-(-a
)^(1/2))^(1/2)/b^(5/4)+1/4*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))
/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))*(-(-a)^(1/2))^(1/2)/b^(5/4)-1/4*polylo
g(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*(-(-a)^(1/2))^(1/2)
/b^(5/4)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))*
(-(-a)^(1/2))^(1/2)/b^(5/4)
```

3.299.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.88

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx$$

$$= \frac{-4\sqrt[4]{b}dx + 4\sqrt[4]{b}c\log(c+dx) + 4\sqrt[4]{b}dx\log(c+dx) + \sqrt[4]{-a}d\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx) - i\sqrt[4]{-a}d\,\cdots}{\cdots}$$

input
```
Integrate[(x^4*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(-4*b^(1/4)*d*x + 4*b^(1/4)*c*Log[c + d*x] + 4*b^(1/4)*d*x*Log[c + d*x] +
(-a)^(1/4)*d*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*
Log[c + d*x] - I*(-a)^(1/4)*d*Log[(d*((-a)^(1/4) - I*b^(1/4)*x))/(I*b^(1/4
)*c + (-a)^(1/4)*d)]*Log[c + d*x] + I*(-a)^(1/4)*d*Log[(d*((-a)^(1/4) + I*
b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - (-a)^(1/4)*d*L
og[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*d)]*Log[c + d*x
] - (-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)
] - I*(-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4
)*d)] + I*(-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^
(1/4)*d)] + (-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^
(1/4)*d)])/(4*b^(5/4)*d)
```

3.299.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \log(c+dx)}{a+bx^4} \, dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log(c+dx)}{b}-\frac{a \log(c+dx)}{b \left(a+bx^4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{-\sqrt{-a}}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}+\frac{\sqrt{-\sqrt{-a}}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}-\frac{\sqrt[4]{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}}+\frac{\sqrt[4]{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{5/4}}+\frac{\sqrt{-\sqrt{-a}}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b^{5/4}}+\frac{\sqrt[4]{-a}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{5/4}}-\frac{\sqrt{-\sqrt{-a}}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}-\frac{\sqrt[4]{-a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

input
```
Int[(x^4*Log[c + d*x])/(a + b*x^4),x]
```

output
```
-(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) + (Sqrt[-Sqrt[-a]]*Log[(d*(Sqrt[-S
qrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*b
^(5/4)) + ((-a)^(1/4)*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(
1/4)*d)]*Log[c + d*x])/(4*b^(5/4)) - (Sqrt[-Sqrt[-a]]*Log[-((d*(Sqrt[-Sqrt
[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*b^(
5/4)) - ((-a)^(1/4)*Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(
1/4)*d))]*Log[c + d*x])/(4*b^(5/4)) - (Sqrt[-Sqrt[-a]]*PolyLog[2, (b^(1/4)
*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(5/4)) + (Sqrt[-Sqrt[-a
]]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(
5/4)) - ((-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)
*d)])/(4*b^(5/4)) + ((-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c
+ (-a)^(1/4)*d)])/(4*b^(5/4))
```

3.299.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.299.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.28

method	result
derivativedivides	$\frac{\frac{d^4((dx+c)\ln(dx+c)-dx-c)}{b}+\frac{a\,d^8\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.}{-\_R1^3+3c\_R}\right.}{4b^2}}{d^5}$
default	$\frac{\frac{d^4((dx+c)\ln(dx+c)-dx-c)}{b}+\frac{a\,d^8\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.}{-\_R1^3+3c\_R}\right.}{4b^2}}{d^5}$
risch	$\frac{x\ln(dx+c)}{b}+\frac{\ln(dx+c)c}{db}-\frac{x}{b}-\frac{c}{db}+\frac{d^3a\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln}{-}\right.}{4b^2}$

input
```
int(x^4*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^5*(d^4/b*((d*x+c)*ln(d*x+c)-d*x-c)+1/4*a*d^8/b^2*sum(1/(-_R1^3+3*_R1^2
*c-3*_R1*c^2+c^3)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1))
,_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4)))
```

3.299.5 Fricas [F]

$$\int \frac{x^4\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^4\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^4*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^4*log(d*x + c)/(b*x^4 + a), x)
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4*ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.299.7 Maxima [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^4 \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^4*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^4*log(d*x + c)/(b*x^4 + a), x)
```

3.299.8 Giac [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^4 \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^4*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^4*log(d*x + c)/(b*x^4 + a), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^4 \ln(c+d x)}{b x^4+a} \, dx$$

input
```
int((x^4*log(c + d*x))/(a + b*x^4),x)
```

output
```
int((x^4*log(c + d*x))/(a + b*x^4), x)
```

3.300 $\int \frac{x^2 \log(c+dx)}{a+bx^4} \, dx$

3.300.1 Optimal result

Integrand size = 19, antiderivative size = 497

$$\begin{aligned}\int \frac{x^2 \log(c+dx)}{a+bx^4} \, dx = & \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} \\ & + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}} \\ & - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} \\ & - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}} \\ & - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}} \\ & - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}\end{aligned}$$

output
```
1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(
1/4)/b^(3/4)-1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln
(d*x+c)/(-a)^(1/4)/b^(3/4)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(
1/4)*d))/(-a)^(1/4)/b^(3/4)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^
(1/4)*d))/(-a)^(1/4)/b^(3/4)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^
(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)+1/4*
ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4
)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)+1/4*polylog(2,b^(1
/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.93

$$\int \frac{x^2 \log(c+dx)}{a+bx^4} \, dx$$
$$= \frac{\sqrt[4]{-a} \log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right) \log(c+dx) + \sqrt{-\sqrt{-a}} \log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right) \log(c+dx) - \sqrt[4]{-a} \log\left(\frac{d(\cdots}{-\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^2*Log[c + d*x])/(a + b*x^4),x]
```

output
```
((-a)^(1/4)*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[
-a]]*d)]*Log[c + d*x] + Sqrt[-Sqrt[-a]]*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(
b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - (-a)^(1/4)*Log[(d*(Sqrt[-Sqrt[-a
]] + b^(1/4)*x))/(-(b^(1/4)*c) + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x] - Sqrt[-
Sqrt[-a]]*Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*d)]*
Log[c + d*x] - (-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt
[-Sqrt[-a]]*d)] + (-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + S
qrt[-Sqrt[-a]]*d)] - Sqrt[-Sqrt[-a]]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/
4)*c - (-a)^(1/4)*d)] + Sqrt[-Sqrt[-a]]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^
(1/4)*c + (-a)^(1/4)*d)])/(4*Sqrt[-Sqrt[-a]]*(-a)^(1/4)*b^(3/4))
```

3.300.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\log(c+dx)}{a+bx^4}\,dx$$

$$\Big\downarrow \ 2863$$

$$\int \left(\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x^2\right)}-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x^2\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4\sqrt[4]{-a}b^{3/4}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}$$

input
```
Int[(x^2*Log[c + d*x])/(a + b*x^4),x]
```

output
```
(Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Lo
g[c + d*x])/(4*Sqrt[-Sqrt[-a]]*b^(3/4)) + (Log[(d*((-a)^(1/4) - b^(1/4)*x)
)/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(1/4)*b^(3/4)) - (Log[
-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[
c + d*x])/(4*Sqrt[-Sqrt[-a]]*b^(3/4)) - (Log[-((d*((-a)^(1/4) + b^(1/4)*x)
)/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(1/4)*b^(3/4)) - Poly
Log[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*Sqrt[-Sqrt[
-a]]*b^(3/4)) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]
]*d)]/(4*Sqrt[-Sqrt[-a]]*b^(3/4)) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4
)*c - (-a)^(1/4)*d)]/(4*(-a)^(1/4)*b^(3/4)) + PolyLog[2, (b^(1/4)*(c + d*x
))/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*(-a)^(1/4)*b^(3/4))
```

3.300.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.300.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.19

method	result
derivativedivides	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{4}-4cb\_Z^{3}+6b\,c^{2}\_Z^{2}-4b\,c^{3}\_Z+a\,d^{4}+b\,c^{4}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{4b}$
default	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{4}-4cb\_Z^{3}+6b\,c^{2}\_Z^{2}-4b\,c^{3}\_Z+a\,d^{4}+b\,c^{4}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{4b}$
risch	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{4}-4cb\_Z^{3}+6b\,c^{2}\_Z^{2}-4b\,c^{3}\_Z+a\,d^{4}+b\,c^{4}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{4b}$

input
```
int(x^2*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/b*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)
/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4))
```

3.300.5 Fricas [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2 \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^2*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^2*log(d*x + c)/(b*x^4 + a), x)
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.300.7 Maxima [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2 \log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(x^2*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^2*log(d*x + c)/(b*x^4 + a), x)
```

3.300.8 Giac [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2 \log{(dx+c)}}{bx^4+a}\,dx$$

input
```
integrate(x^2*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^2*log(d*x + c)/(b*x^4 + a), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2\,\ln{(c+d\,x)}}{b\,x^4+a}\,dx$$

input
```
int((x^2*log(c + d*x))/(a + b*x^4),x)
```

output
```
int((x^2*log(c + d*x))/(a + b*x^4), x)
```

3.301 $\int \frac{\log(c+dx)}{a+bx^4}\,dx$

3.301.1 Optimal result

Integrand size = 16, antiderivative size = 497

$$\begin{aligned}\int \frac{\log(c+dx)}{a+bx^4}\,dx &= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} \\ &+ \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}} \\ &- \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} \\ &- \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}} \\ &- \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} \\ &- \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}\end{aligned}$$

output
```
1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(
3/4)/b^(1/4)-1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln
(d*x+c)/(-a)^(3/4)/b^(1/4)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(
1/4)*d))/(-a)^(3/4)/b^(1/4)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^
(1/4)*d))/(-a)^(3/4)/b^(1/4)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^
(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)+1/4*
ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4
)*c-d*(-(-a)^(1/2))^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)+1/4*polylog(2,b^(1
/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)
```

3.301.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.72

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx$$
$$= \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)-i\log\left(\frac{d\left(\sqrt[4]{-a}-i\sqrt[4]{b}x\right)}{i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)+i\log\left(\frac{d\left(\sqrt[4]{-a}+i\sqrt[4]{b}x\right)}{-i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(\cdots}{\cdots}$$

input
```
Integrate[Log[c + d*x]/(a + b*x^4),x]
```

output
```
(Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d]*Log[c + d*x]
 - I*Log[(d*((-a)^(1/4) - I*b^(1/4)*x))/(I*b^(1/4)*c + (-a)^(1/4)*d]*Log[
c + d*x] + I*Log[(d*((-a)^(1/4) + I*b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/
4)*d)]*Log[c + d*x] - Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a
)^(1/4)*d)]*Log[c + d*x] - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a
)^(1/4)*d)] - I*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d
)] + I*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)] + Poly
Log[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(3/4)*b^(1
/4))
```

3.301.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{a+bx^4} \, dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(\frac{\sqrt{-a}\log(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x^2\right)}+\frac{\sqrt{-a}\log(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4(-a)^{3/4}\sqrt[4]{b}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}$$

input
```
Int[Log[c + d*x]/(a + b*x^4),x]
```

output
```
(Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Lo
g[c + d*x])/(4*(-Sqrt[-a])^(3/2)*b^(1/4)) + (Log[(d*((-a)^(1/4) - b^(1/4)*
x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(3/4)*b^(1/4)) - (Lo
g[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Lo
g[c + d*x])/(4*(-Sqrt[-a])^(3/2)*b^(1/4)) - (Log[-((d*((-a)^(1/4) + b^(1/4
)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/4)*b^(1/4)) -
PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*(-Sqrt[
-a])^(3/2)*b^(1/4)) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sq
rt[-a]]*d)]/(4*(-Sqrt[-a])^(3/2)*b^(1/4)) - PolyLog[2, (b^(1/4)*(c + d*x))
/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*(-a)^(3/4)*b^(1/4)) + PolyLog[2, (b^(1/4)*
(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*(-a)^(3/4)*b^(1/4))
```

3.301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.301.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

method	result
derivativedivides	$-\frac{d^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\text{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1^3+3c\_R1^2-3c^2\_R1+c^3}\right)}{4b}$
default	$-\frac{d^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\text{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1^3+3c\_R1^2-3c^2\_R1+c^3}\right)}{4b}$
risch	$-\frac{d^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\text{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1^3+3c\_R1^2-3c^2\_R1+c^3}\right)}{4b}$

input
```
int(ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^3/b*sum(1/(-_R1^3+3*_R1^2*c-3*_R1*c^2+c^3)*(ln(d*x+c)*ln((-d*x+_R1-
c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2
-4*_Z*b*c^3+a*d^4+b*c^4))
```

3.301.5 Fricas [F]

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^4 + a), x)
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.301.7 Maxima [F]

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/(b*x^4 + a), x)
```

3.301.8 Giac [F]

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\log(dx+c)}{bx^4+a}\,dx$$

input
```
integrate(log(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/(b*x^4 + a), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\ln(c+d\,x)}{b\,x^4+a}\,dx$$

input
```
int(log(c + d*x)/(a + b*x^4),x)
```

output
```
int(log(c + d*x)/(a + b*x^4), x)
```

3.302 $\int \frac{\log(c+dx)}{x^2(a+bx^4)} \, dx$

3.302.1 Optimal result

Integrand size = 19, antiderivative size = 536

$$\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx = \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}} - \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}} - \frac{\sqrt[4]{b}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} + \frac{\sqrt[4]{b}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt[4]{b}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}} + \frac{\sqrt[4]{b}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}$$

output
```
d*ln(x)/a/c-d*ln(d*x+c)/a/c-ln(d*x+c)/a/x+1/4*b^(1/4)*ln(d*((-a)^(1/4)-b^(
1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(5/4)-1/4*b^(1/4)*ln(-d*(
(-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(5/4)-1/4*b
^(1/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/(-a)^(5/4)+1/4*
b^(1/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/(-a)^(5/4)-1/4
*b^(1/4)*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c-d*(-(-
a)^(1/2))^(1/2)))/(-(-a)^(1/2))^(5/2)+1/4*b^(1/4)*ln(d*x+c)*ln(d*(-b^(1/4)
*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/(-(-a)^(1/2))^(
5/2)-1/4*b^(1/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2
)))/(-(-a)^(1/2))^(5/2)+1/4*b^(1/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d
*(-(-a)^(1/2))^(1/2)))/(-(-a)^(1/2))^(5/2)
```

3.302.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00

$$\int \frac{\log(c+dx)}{x^2\left(a+bx^4\right)}\,dx$$

$$= \frac{\frac{a\sqrt[4]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{\sqrt{-\sqrt{-a}}} - \frac{-4adx\log(x)+4ac\log(c+dx)+4adx\log(c+dx)+\frac{a\sqrt[4]{b}cx\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{\sqrt{-\sqrt{-a}}}-(-a)^{3/4}}{}}{}$$

input
```
Integrate[Log[c + d*x]/(x^2*(a + b*x^4)),x]
```

output
```
((a*b^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)
])/Sqrt[-Sqrt[-a]] - (-4*a*d*x*Log[x] + 4*a*c*Log[c + d*x] + 4*a*d*x*Log[c
 + d*x] + (a*b^(1/4)*c*x*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c
+ Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/Sqrt[-Sqrt[-a]] - (-a)^(3/4)*b^(1/4)*c
*x*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*
x] - (a*b^(1/4)*c*x*Log[(d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(-(b^(1/4)*c) +
Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/Sqrt[-Sqrt[-a]] + (-a)^(3/4)*b^(1/4)*c*x
*Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*d)]*Log[c + d
*x] + (a*b^(1/4)*c*x*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqr
t[-a]]*d)])/Sqrt[-Sqrt[-a]] + (-a)^(3/4)*b^(1/4)*c*x*PolyLog[2, (b^(1/4)*(
c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)] - (-a)^(3/4)*b^(1/4)*c*x*PolyLog[2,
(b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(c*x))/(4*a^2)
```

3.302.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c+dx)}{x^2\left(a+bx^4\right)}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log(c+dx)}{ax^2} - \frac{bx^2\log(c+dx)}{a(a+bx^4)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}+\frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}-\frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}+\frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}+\frac{\sqrt[4]{b}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}+\frac{\sqrt[4]{b}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4(-a)^{5/4}}-\frac{\sqrt[4]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}-\frac{\sqrt[4]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}+\frac{d\log(x)}{ac}-\frac{d\log(c+dx)}{ac}-\frac{\log(c+dx)}{ax}$$

input
```
Int[Log[c + d*x]/(x^2*(a + b*x^4)),x]
```

output
```
(d*Log[x])/(a*c) - (d*Log[c + d*x])/(a*c) - Log[c + d*x]/(a*x) + (b^(1/4)*
Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log
[c + d*x])/(4*(-Sqrt[-a])^(5/2)) + (b^(1/4)*Log[(d*((-a)^(1/4) - b^(1/4)*x
))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(5/4)) - (b^(1/4)*Log
[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log
[c + d*x])/(4*(-Sqrt[-a])^(5/2)) - (b^(1/4)*Log[-((d*((-a)^(1/4) + b^(1/4)
*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(5/4)) - (b^(1/4)*
PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*(-Sqrt
[-a])^(5/2)) + (b^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-
Sqrt[-a]]*d)])/(4*(-Sqrt[-a])^(5/2)) - (b^(1/4)*PolyLog[2, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*(-a)^(5/4)) + (b^(1/4)*PolyLog[2, (b^
(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(5/4))
```

3.302. $\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx$

3.302.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.302.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.25

method	result
derivativedivides	$d\left(\frac{\frac{\ln(-dx)}{c}-\frac{\ln(dx+c)(dx+c)}{cdx}}{a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.}{-\_}}{4a}\right.$
default	$d\left(\frac{\frac{\ln(-dx)}{c}-\frac{\ln(dx+c)(dx+c)}{cdx}}{a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.}{-\_}}{4a}\right.$
risch	$\frac{d\ln(-dx)}{ac}-\frac{d\ln(dx+c)}{ac}-\frac{\ln(dx+c)}{ax}+\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\right.}{}\right.}{4a}$

input
```
int(ln(d*x+c)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d*(1/a*(1/c*ln(-d*x)-ln(d*x+c)*(d*x+c)/c/d/x)+1/4/a*sum(1/(-_R1+c)*(ln(d*x
+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3
*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4)))
```

3.302.5 Fricas [F]

$$\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x^2}\,dx$$

input
```
integrate(log(d*x+c)/x^2/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(log(d*x + c)/(b*x^6 + a*x^2), x)
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*x+c)/x**2/(b*x**4+a),x)
```

output
```
Timed out
```

3.302.7 Maxima [F]

$$\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x^2}\,dx$$

input
```
integrate(log(d*x+c)/x^2/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(log(d*x + c)/((b*x^4 + a)*x^2), x)
```

3.302.8 Giac [F]

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^4)}\,dx = \int \frac{\log\,(dx+c)}{(bx^4+a)x^2}\,dx$$

input
```
integrate(log(d*x+c)/x^2/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(log(d*x + c)/((b*x^4 + a)*x^2), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^4)}\,dx = \int \frac{\ln\,(c+d\,x)}{x^2\,(b\,x^4+a)}\,dx$$

input
```
int(log(c + d*x)/(x^2*(a + b*x^4)),x)
```

output
```
int(log(c + d*x)/(x^2*(a + b*x^4)), x)
```

3.303 $\int \left(f+\frac{g}{x}\right) x(a+b\log(c(d+ex)^n))\,dx$

3.303.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \left(f+\frac{g}{x}\right) x(a+b\log(c(d+ex)^n))\,dx = \frac{b(df-eg)nx}{2e}-\frac{bn(g+fx)^2}{4f}-\frac{b(df-eg)^2n\log(d+ex)}{2e^2f}+\frac{(g+fx)^2(a+b\log(c(d+ex)^n))}{2f}$$

output
```
1/2*b*(d*f-e*g)*n*x/e-1/4*b*n*(f*x+g)^2/f-1/2*b*(d*f-e*g)^2*n*ln(e*x+d)/e^
2/f+1/2*(f*x+g)^2*(a+b*ln(c*(e*x+d)^n))/f
```

3.303.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \left(f+\frac{g}{x}\right) x(a+b\log(c(d+ex)^n))\,dx = agx-bgnx+\frac{1}{2}afx^2-\frac{1}{2}bfn\left(-\frac{dx}{e}+\frac{x^2}{2}+\frac{d^2\log(d+ex)}{e^2}\right)+\frac{1}{2}bfx^2\log(c(d+ex)^n)+\frac{bg(d+ex)\log(c(d+ex)^n)}{e}$$

input
```
Integrate[(f + g/x)*x*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
a*g*x - b*g*n*x + (a*f*x^2)/2 - (b*f*n*(-((d*x)/e) + x^2/2 + (d^2*Log[d +
e*x])/e^2))/2 + (b*f*x^2*Log[c*(d + e*x)^n])/2 + (b*g*(d + e*x)*Log[c*(d +
 e*x)^n])/e
```

3.303.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2005, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(f+\frac{g}{x}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)dx$$

$$\Big\downarrow \text{2005}$$

$$\int (fx+g)\left(a+b\log\left(c(d+ex)^n\right)\right)dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(fx+g)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}-\frac{ben\int\frac{(g+fx)^2}{d+ex}dx}{2f}$$

$$\Big\downarrow \text{49}$$

$$\frac{(fx+g)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}-\frac{ben\int\left(\frac{(eg-df)^2}{e^2(d+ex)}+\frac{f(eg-df)}{e^2}+\frac{f(g+fx)}{e}\right)dx}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fx+g)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}-\frac{ben\left(\frac{(df-eg)^2\log(d+ex)}{e^3}-\frac{fx(df-eg)}{e^2}+\frac{(fx+g)^2}{2e}\right)}{2f}$$

input
```
Int[(f + g/x)*x*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/2*(b*e*n*(-((f*(d*f - e*g)*x)/e^2) + (g + f*x)^2/(2*e) + ((d*f - e*g)^2
*Log[d + e*x])/e^3))/f + ((g + f*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*f)
```

3.303.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.303.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

method	result
parts	$a\left(\frac{1}{2} f\,x^{2}+g x\right)+b\left(g \ln\left(c (e x +d)^{n}\right) x -g n x +\frac{g n d \ln(e x +d)}{e}+\frac{f\,x^{2} \ln\left(c\,{\mathrm e}^{n \ln(e x +d)}\right)}{2}-\frac{f n\,x^{2}}{4}-\frac{n\,d^{2} f}{}\right.$
default	$a g x +\frac{a f\,x^{2}}{2}+b g \ln\left(c (e x +d)^{n}\right) x -b g n x +\frac{b g n d \ln(e x +d)}{e}+\frac{b f\,x^{2} \ln\left(c\,{\mathrm e}^{n \ln(e x +d)}\right)}{2}-\frac{n b\,x^{2} f}{4}-\frac{n b\,d^{2} f \mathrm{l}}{2}$
parallelrisch	$-\frac{-2 x^{2} \ln\left(c (e x +d)^{n}\right) b\,e^{2} f +b\,e^{2} f n\,x^{2}+2 \ln(e x +d) b\,d^{2} f n -8 \ln(e x +d) b d e g n -2 a\,e^{2} f\,x^{2}-4 x \ln\left(c (e x +d)^{n}\right) b\,e^{2} g -2 b d e f n x +4 b e}{4 e^{2}}$
risch	$\frac{b x (f x +2 g) \ln\left((e x +d)^{n}\right)}{2}-\frac{i \pi b f\,x^{2} \mathrm{csgn}\left(i c (e x +d)^{n}\right)^{3}}{4}+\frac{i \pi b f\,x^{2} \mathrm{csgn}\left(i (e x +d)^{n}\right) \mathrm{csgn}\left(i c (e x +d)^{n}\right)^{2}}{4}+\frac{i \pi b g x \,\mathrm{csgn}(i c) \mathrm{csgn}}{2}$

input
```
int((f+g/x)*x*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+g*x)+b*(g*ln(c*(e*x+d)^n)*x-g*n*x+g/e*n*d*ln(e*x+d)+1/2*f*x^2
*ln(c*exp(n*ln(e*x+d)))-1/4*f*n*x^2-1/2*n*d^2*f/e^2*ln(e*x+d)+1/2*d*f*n/e*
x)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx = -\frac{(be^2fn-2\,ae^2f)x^2-2\left(2\,ae^2g+(bdef-2\,be^2g)n\right)x-2\left(be^2fnx^2+2\,be^2gnx-(bd^2f-2\,bdeg)n\right)\log}{4\,e^2}$$

input
```
integrate((f+g/x)*x*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/4*((b*e^2*f*n - 2*a*e^2*f)*x^2 - 2*(2*a*e^2*g + (b*d*e*f - 2*b*e^2*g)*n
)*x - 2*(b*e^2*f*n*x^2 + 2*b*e^2*g*n*x - (b*d^2*f - 2*b*d*e*g)*n)*log(e*x
+ d) - 2*(b*e^2*f*x^2 + 2*b*e^2*g*x)*log(c))/e^2
```

3.303.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx = \begin{cases} \frac{afx^2}{2}+agx-\frac{bd^2f\log\left(c(d+ex)^n\right)}{2e^2}+\frac{bdfnx}{2e}+\frac{bdg\log\left(c(d+ex)^n\right)}{e}-\frac{bfnx^2}{4}+\frac{bfx^2\log\left(c(d+ex)^n\right)}{2}-bgnx+bgx\log\left(c(d\right. \\ (a+b\log\left(cd^n\right))\left(\frac{fx^2}{2}+gx\right) \end{cases}$$

input
```
integrate((f+g/x)*x*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f*x**2/2 + a*g*x - b*d**2*f*log(c*(d + e*x)**n)/(2*e**2) + b*
d*f*n*x/(2*e) + b*d*g*log(c*(d + e*x)**n)/e - b*f*n*x**2/4 + b*f*x**2*log(
c*(d + e*x)**n)/2 - b*g*n*x + b*g*x*log(c*(d + e*x)**n), Ne(e, 0)), ((a +
b*log(c*d**n))*(f*x**2/2 + g*x), True))
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \left(f+\frac{g}{x}\right)x(a+b\log(c(d+ex)^n))\,dx = -begn\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) - \frac{1}{4}befn\left(\frac{2\,d^2\log(ex+d)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right) + \frac{1}{2}\,bfx^2\log\left((ex+d)^nc\right)+\frac{1}{2}\,afx^2 + bgx\log\left((ex+d)^nc\right)+agx$$

input
```
integrate((f+g/x)*x*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
-b*e*g*n*(x/e - d*log(e*x + d)/e^2) - 1/4*b*e*f*n*(2*d^2*log(e*x + d)/e^3
+ (e*x^2 - 2*d*x)/e^2) + 1/2*b*f*x^2*log((e*x + d)^n*c) + 1/2*a*f*x^2 + b*
g*x*log((e*x + d)^n*c) + a*g*x
```

3.303.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(83) = 166.

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.01

$$\int \left(f+\frac{g}{x}\right)x(a+b\log(c(d+ex)^n))\,dx = \frac{(ex+d)^2bfn\log(ex+d)}{2\,e^2} - \frac{(ex+d)bdfn\log(ex+d)}{e^2} + \frac{(ex+d)bgn\log(ex+d)}{e} - \frac{(ex+d)^2bfn}{4\,e^2} + \frac{(ex+d)bdfn}{e^2} - \frac{(ex+d)bgn}{e} + \frac{(ex+d)^2bf\log(c)}{2\,e^2} - \frac{(ex+d)bdf\log(c)}{e^2} + \frac{(ex+d)bg\log(c)}{e} + \frac{(ex+d)^2af}{2\,e^2} - \frac{(ex+d)adf}{e^2} + \frac{(ex+d)ag}{e}$$

input
```
integrate((f+g/x)*x*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
1/2*(e*x + d)^2*b*f*n*log(e*x + d)/e^2 - (e*x + d)*b*d*f*n*log(e*x + d)/e^
2 + (e*x + d)*b*g*n*log(e*x + d)/e - 1/4*(e*x + d)^2*b*f*n/e^2 + (e*x + d)
*b*d*f*n/e^2 - (e*x + d)*b*g*n/e + 1/2*(e*x + d)^2*b*f*log(c)/e^2 - (e*x +
 d)*b*d*f*log(c)/e^2 + (e*x + d)*b*g*log(c)/e + 1/2*(e*x + d)^2*a*f/e^2 -
(e*x + d)*a*d*f/e^2 + (e*x + d)*a*g/e
```

3.303.9 Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx = x\left(\frac{2\,a\,d\,f+2\,a\,e\,g-2\,b\,e\,g\,n}{2\,e}-\frac{d\,f\,(2\,a-b\,n)}{2\,e}\right) + \ln\left(c\,(d+e\,x)^n\right)\left(\frac{b\,f\,x^2}{2}+b\,g\,x\right) - \frac{\ln\left(d+e\,x\right)\,\left(b\,d^2\,f\,n-2\,b\,d\,e\,g\,n\right)}{2\,e^2} + \frac{f\,x^2\,(2\,a-b\,n)}{4}$$

input
```
int(x*(f + g/x)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x*((2*a*d*f + 2*a*e*g - 2*b*e*g*n)/(2*e) - (d*f*(2*a - b*n))/(2*e)) + log(
c*(d + e*x)^n)*(b*g*x + (b*f*x^2)/2) - (log(d + e*x)*(b*d^2*f*n - 2*b*d*e*
g*n))/(2*e^2) + (f*x^2*(2*a - b*n))/4
```

3.304 $\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx$

3.304.1 Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx=-\frac{b(df-eg)^2nx}{3e^2}+\frac{b(df-eg)n(g+fx)^2}{6ef}-\frac{bn(g+fx)^3}{9f}+\frac{b(df-eg)^3n\log(d+ex)}{3e^3f}+\frac{(g+fx)^3(a+b\log(c(d+ex)^n))}{3f}$$

output
```
-1/3*b*(d*f-e*g)^2*n*x/e^2+1/6*b*(d*f-e*g)*n*(f*x+g)^2/e/f-1/9*b*n*(f*x+g)
^3/f+1/3*b*(d*f-e*g)^3*n*ln(e*x+d)/e^3/f+1/3*(f*x+g)^3*(a+b*ln(c*(e*x+d)^n
))/f
```

3.304.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx=\frac{6bd^2f(df-3eg)n\log(d+ex)+e(x(6ae^2(3g^2+3fgx+f^2x^2)-bn(6d^2f^2-3def(6g+fx)+e^2(18g^2+}{18e^3}$$

input
```
Integrate[(f + g/x)^2*x^2*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(6*b*d^2*f*(d*f - 3*e*g)*n*Log[d + e*x] + e*(x*(6*a*e^2*(3*g^2 + 3*f*g*x +
 f^2*x^2) - b*n*(6*d^2*f^2 - 3*d*e*f*(6*g + f*x) + e^2*(18*g^2 + 9*f*g*x +
 2*f^2*x^2))) + 6*b*e*(3*d*g^2 + e*x*(3*g^2 + 3*f*g*x + f^2*x^2))*Log[c*(d
 + e*x)^n]))/(18*e^3)
```

3.304.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2005, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(f+\frac{g}{x}\right)^2(a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \ 2005$$

$$\int (fx+g)^2(a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \ 2842$$

$$\frac{(fx+g)^3(a+b\log(c(d+ex)^n))}{3f}-\frac{ben\int \frac{(g+fx)^3}{d+ex}dx}{3f}$$

$$\Big\downarrow \ 49$$

$$\frac{(fx+g)^3(a+b\log(c(d+ex)^n))}{3f}-\frac{ben\int \left(\frac{(eg-df)^3}{e^3(d+ex)}+\frac{f(eg-df)^2}{e^3}+\frac{f(g+fx)(eg-df)}{e^2}+\frac{f(g+fx)^2}{e}\right)dx}{3f}$$

$$\Big\downarrow \ 2009$$

$$\frac{(fx+g)^3(a+b\log(c(d+ex)^n))}{3f}-\frac{ben\left(-\frac{(df-eg)^3\log(d+ex)}{e^4}+\frac{fx(df-eg)^2}{e^3}-\frac{(fx+g)^2(df-eg)}{2e^2}+\frac{(fx+g)^3}{3e}\right)}{3f}$$

input
```
Int[(f + g/x)^2*x^2*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/3*(b*e*n*((f*(d*f - e*g)^2*x)/e^3 - ((d*f - e*g)*(g + f*x)^2)/(2*e^2) +
 (g + f*x)^3/(3*e) - ((d*f - e*g)^3*Log[d + e*x])/e^4))/f + ((g + f*x)^3*(
a + b*Log[c*(d + e*x)^n]))/(3*f)
```

3.304.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.304.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(110) = 220$.

Time = 0.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.21

method	result
parallelrisch	$\frac{6x^3\ln(c(ex+d)^n)bd\,e^3f^2-2x^3bd\,e^3f^2n+6x^3ad\,e^3f^2+18x^2\ln(c(ex+d)^n)bd\,e^3fg+3x^2b\,d^2e^2f^2n-9x^2bd\,e^3fgn+6\ln(ex+d)b\,d}{}$
risch	$-\frac{f\ln(ex+d)b\,d^2gn}{e^2}-\frac{fbgn\,x^2}{2}-\frac{f^2b\,d^2nx}{3e^2}+\frac{f^2\ln(ex+d)b\,d^3n}{3e^3}-\frac{if\pi bg\,x^2\operatorname{csgn}(ic)\operatorname{csgn}(i(ex+d)^n)\operatorname{csgn}(ic(ex+d)^n)}{2}$

input
```
int((f+g/x)^2*x^2*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*x^3*ln(c*(e*x+d)^n)*b*d*e^3*f^2-2*x^3*b*d*e^3*f^2*n+6*x^3*a*d*e^3*
f^2+18*x^2*ln(c*(e*x+d)^n)*b*d*e^3*f*g+3*x^2*b*d^2*e^2*f^2*n-9*x^2*b*d*e^3
*f*g*n+6*ln(e*x+d)*b*d^4*f^2*n-18*ln(e*x+d)*b*d^3*e*f*g*n+36*ln(e*x+d)*b*d
^2*e^2*g^2*n+18*x^2*a*d*e^3*f*g+18*x*ln(c*(e*x+d)^n)*b*d*e^3*g^2-6*x*b*d^3
*e*f^2*n+18*x*b*d^2*e^2*f*g*n-18*x*b*d*e^3*g^2*n+18*x*a*d*e^3*g^2-18*ln(c*
(e*x+d)^n)*b*d^2*e^2*g^2)/d/e^3
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.82

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = -\frac{2\,(be^3f^2n-3\,ae^3f^2)x^3-3\,(6\,ae^3fg+(bde^2f^2-3\,be^3fg)n)x^2-6\,(3\,ae^3g^2-(bd^2ef^2-3\,bde^2fg+3\,be$$

input
```
integrate((f+g/x)^2*x^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/18*(2*(b*e^3*f^2*n - 3*a*e^3*f^2)*x^3 - 3*(6*a*e^3*f*g + (b*d*e^2*f^2 -
 3*b*e^3*f*g)*n)*x^2 - 6*(3*a*e^3*g^2 - (b*d^2*e*f^2 - 3*b*d*e^2*f*g + 3*b
*e^3*g^2)*n)*x - 6*(b*e^3*f^2*n*x^3 + 3*b*e^3*f*g*n*x^2 + 3*b*e^3*g^2*n*x
+ (b*d^3*f^2 - 3*b*d^2*e*f*g + 3*b*d*e^2*g^2)*n)*log(e*x + d) - 6*(b*e^3*f
^2*x^3 + 3*b*e^3*f*g*x^2 + 3*b*e^3*g^2*x)*log(c))/e^3
```

3.304.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(102) = 204$.

Time = 4.50 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.10

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = \begin{cases} \frac{af^2x^3}{3}+afgx^2+ag^2x+\frac{bd^3f^2\log(c(d+ex)^n)}{3e^3}-\frac{bd^2f^2nx}{3e^2}-\frac{bd^2fg\log(c(d+ex)^n)}{e^2}+\frac{bdf^2nx^2}{6e}+\frac{bdfgnx}{e}+\frac{bdg^2\log(c(d+ex)}{e} \\ (a+b\log(cd^n))\left(\frac{f^2x^3}{3}+fgx^2+g^2x\right) \end{cases}$$

input
```
integrate((f+g/x)**2*x**2*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f**2*x**3/3 + a*f*g*x**2 + a*g**2*x + b*d**3*f**2*log(c*(d +
e*x)**n)/(3*e**3) - b*d**2*f**2*n*x/(3*e**2) - b*d**2*f*g*log(c*(d + e*x)*
*n)/e**2 + b*d*f**2*n*x**2/(6*e) + b*d*f*g*n*x/e + b*d*g**2*log(c*(d + e*x
)**n)/e - b*f**2*n*x**3/9 + b*f**2*x**3*log(c*(d + e*x)**n)/3 - b*f*g*n*x*
*2/2 + b*f*g*x**2*log(c*(d + e*x)**n) - b*g**2*n*x + b*g**2*x*log(c*(d + e
*x)**n), Ne(e, 0)), ((a + b*log(c*d**n))*(f**2*x**3/3 + f*g*x**2 + g**2*x)
, True))
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.56

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = \frac{1}{3}\,bf^2x^3\log\left((ex+d)^nc\right)+\frac{1}{3}\,af^2x^3-beg^2n\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) + \frac{1}{18}\,bef^2n\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right) - \frac{1}{2}\,befgn\left(\frac{2\,d^2\log(ex+d)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right) + bfgx^2\log\left((ex+d)^nc\right)+afgx^2+bg^2x\log\left((ex+d)^nc\right)+ag^2x$$

input
```
integrate((f+g/x)^2*x^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/3*b*f^2*x^3*log((e*x + d)^n*c) + 1/3*a*f^2*x^3 - b*e*g^2*n*(x/e - d*log(
e*x + d)/e^2) + 1/18*b*e*f^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*
e*x^2 + 6*d^2*x)/e^3) - 1/2*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2
*d*x)/e^2) + b*f*g*x^2*log((e*x + d)^n*c) + a*f*g*x^2 + b*g^2*x*log((e*x +
 d)^n*c) + a*g^2*x
```

3.304.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(110) = 220.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.53

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = \frac{(ex+d)^3bf^2n\log(ex+d)}{3\,e^3}-\frac{(ex+d)^2bdf^2n\log(ex+d)}{e^3}+\frac{(ex+d)bd^2f^2n\log(ex+d)}{e^3} + \frac{(ex+d)^2bfgn\log(ex+d)}{e^2}-\frac{2\,(ex+d)bdfgn\log(ex+d)}{e^2}+\frac{(ex+d)bg^2n\log(ex+d)}{e} - \frac{(ex+d)^3bf^2n}{9\,e^3}+\frac{(ex+d)^2bdf^2n}{2\,e^3}-\frac{(ex+d)bd^2f^2n}{e^3}-\frac{(ex+d)^2bfgn}{2\,e^2}+\frac{2\,(ex+d)bdfgn}{e^2} - \frac{(ex+d)bg^2n}{e}+\frac{(ex+d)^3bf^2\log(c)}{3\,e^3}-\frac{(ex+d)^2bdf^2\log(c)}{e^3}+\frac{(ex+d)bd^2f^2\log(c)}{e^3} + \frac{(ex+d)^2bfg\log(c)}{e^2}-\frac{2\,(ex+d)bdfg\log(c)}{e^2}+\frac{(ex+d)bg^2\log(c)}{e}+\frac{(ex+d)^3af^2}{3\,e^3} - \frac{(ex+d)^2adf^2}{e^3}+\frac{(ex+d)ad^2f^2}{e^3}+\frac{(ex+d)^2afg}{e^2}-\frac{2\,(ex+d)adfg}{e^2}+\frac{(ex+d)ag^2}{e}$$

input
```
integrate((f+g/x)^2*x^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
1/3*(e*x + d)^3*b*f^2*n*log(e*x + d)/e^3 - (e*x + d)^2*b*d*f^2*n*log(e*x +
 d)/e^3 + (e*x + d)*b*d^2*f^2*n*log(e*x + d)/e^3 + (e*x + d)^2*b*f*g*n*log
(e*x + d)/e^2 - 2*(e*x + d)*b*d*f*g*n*log(e*x + d)/e^2 + (e*x + d)*b*g^2*n
*log(e*x + d)/e - 1/9*(e*x + d)^3*b*f^2*n/e^3 + 1/2*(e*x + d)^2*b*d*f^2*n/
e^3 - (e*x + d)*b*d^2*f^2*n/e^3 - 1/2*(e*x + d)^2*b*f*g*n/e^2 + 2*(e*x + d
)*b*d*f*g*n/e^2 - (e*x + d)*b*g^2*n/e + 1/3*(e*x + d)^3*b*f^2*log(c)/e^3 -
 (e*x + d)^2*b*d*f^2*log(c)/e^3 + (e*x + d)*b*d^2*f^2*log(c)/e^3 + (e*x +
d)^2*b*f*g*log(c)/e^2 - 2*(e*x + d)*b*d*f*g*log(c)/e^2 + (e*x + d)*b*g^2*l
og(c)/e + 1/3*(e*x + d)^3*a*f^2/e^3 - (e*x + d)^2*a*d*f^2/e^3 + (e*x + d)*
a*d^2*f^2/e^3 + (e*x + d)^2*a*f*g/e^2 - 2*(e*x + d)*a*d*f*g/e^2 + (e*x + d
)*a*g^2/e
```

3.304.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\begin{aligned}
&\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log \left(c(d+ex)^n\right))\,dx \\
&= x^2\left(\frac{f\,(a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{2\,e}-\frac{d\,f^2\,(3\,a-b\,n)}{6\,e}\right) \\
&\quad + x\left(\frac{3\,a\,e\,g^2-3\,b\,e\,g^2\,n+6\,a\,d\,f\,g}{3\,e}-\frac{d\left(\frac{f\,(a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{e}-\frac{d\,f^2\,(3\,a-b\,n)}{3\,e}\right)}{e}\right) \\
&\quad + \ln\left(c\,(d+e\,x)^n\right)\left(\frac{b\,f^2\,x^3}{3}+b\,f\,g\,x^2+b\,g^2\,x\right)+\frac{f^2\,x^3\,(3\,a-b\,n)}{9} \\
&\quad + \frac{\ln\left(d+e\,x\right)\,\left(b\,n\,d^3\,f^2-3\,b\,n\,d^2\,e\,f\,g+3\,b\,n\,d\,e^2\,g^2\right)}{3\,e^3}
\end{aligned}$$

input
```
int(x^2*(f + g/x)^2*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x^2*((f*(a*d*f + 2*a*e*g - b*e*g*n))/(2*e) - (d*f^2*(3*a - b*n))/(6*e)) +
x*((3*a*e*g^2 - 3*b*e*g^2*n + 6*a*d*f*g)/(3*e) - (d*((f*(a*d*f + 2*a*e*g -
 b*e*g*n))/e - (d*f^2*(3*a - b*n))/(3*e)))/e) + log(c*(d + e*x)^n)*((b*f^2
*x^3)/3 + b*g^2*x + b*f*g*x^2) + (f^2*x^3*(3*a - b*n))/9 + (log(d + e*x)*(
b*d^3*f^2*n + 3*b*d*e^2*g^2*n - 3*b*d^2*e*f*g*n))/(3*e^3)
```

3.305 $\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx$

3.305.1 Optimal result

Integrand size = 27, antiderivative size = 149

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx = \frac{b(df-eg)^3nx}{4e^3}-\frac{b(df-eg)^2n(g+fx)^2}{8e^2f}+\frac{b(df-eg)n(g+fx)^3}{12ef}-\frac{bn(g+fx)^4}{16f}-\frac{b(df-eg)^4n\log(d+ex)}{4e^4f}+\frac{(g+fx)^4(a+b\log(c(d+ex)^n))}{4f}$$

output
```
1/4*b*(d*f-e*g)^3*n*x/e^3-1/8*b*(d*f-e*g)^2*n*(f*x+g)^2/e^2/f+1/12*b*(d*f-
e*g)*n*(f*x+g)^3/e/f-1/16*b*n*(f*x+g)^4/f-1/4*b*(d*f-e*g)^4*n*ln(e*x+d)/e^
4/f+1/4*(f*x+g)^4*(a+b*ln(c*(e*x+d)^n))/f
```

3.305.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.52

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx$$
$$= \frac{ex(12ae^3(4g^3+6fg^2x+4f^2gx^2+f^3x^3)+bn(12d^3f^3-6d^2ef^2(8g+fx)+4de^2f(18g^2+6fgx+f^2x^2)}{}$$

input
```
Integrate[(f + g/x)^3*x^3*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(e*x*(12*a*e^3*(4*g^3 + 6*f*g^2*x + 4*f^2*g*x^2 + f^3*x^3) + b*n*(12*d^3*f
^3 - 6*d^2*e*f^2*(8*g + f*x) + 4*d*e^2*f*(18*g^2 + 6*f*g*x + f^2*x^2) - e^
3*(48*g^3 + 36*f*g^2*x + 16*f^2*g*x^2 + 3*f^3*x^3))) - 12*b*d^2*f*(d^2*f^2
 - 4*d*e*f*g + 6*e^2*g^2)*n*Log[d + e*x] + 12*b*e^3*(4*d*g^3 + e*x*(4*g^3
+ 6*f*g^2*x + 4*f^2*g*x^2 + f^3*x^3))*Log[c*(d + e*x)^n])/(48*e^4)
```

3.305.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2005, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(f+\frac{g}{x}\right)^3(a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{2005}$$

$$\int (fx+g)^3(a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(fx+g)^4(a+b\log(c(d+ex)^n))}{4f}-\frac{ben\int\frac{(g+fx)^4}{d+ex}dx}{4f}$$

$$\Big\downarrow \text{49}$$

$$\frac{(fx+g)^4(a+b\log(c(d+ex)^n))}{4f}-\frac{ben\int\left(\frac{(eg-df)^4}{e^4(d+ex)}+\frac{f(eg-df)^3}{e^4}+\frac{f(g+fx)(eg-df)^2}{e^3}+\frac{f(g+fx)^2(eg-df)}{e^2}+\frac{f(g+fx)^3}{e}\right)dx}{4f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fx+g)^4(a+b\log(c(d+ex)^n))}{4f}-\frac{ben\left(\frac{(df-eg)^4\log(d+ex)}{e^5}-\frac{fx(df-eg)^3}{e^4}+\frac{(fx+g)^2(df-eg)^2}{2e^3}-\frac{(fx+g)^3(df-eg)}{3e^2}+\frac{(fx+g)^4}{4e}\right)}{4f}$$

input
```
Int[(f + g/x)^3*x^3*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
-1/4*(b*e*n*(-((f*(d*f - e*g)^3*x)/e^4) + ((d*f - e*g)^2*(g + f*x)^2)/(2*e
^3) - ((d*f - e*g)*(g + f*x)^3)/(3*e^2) + (g + f*x)^4/(4*e) + ((d*f - e*g)
^4*Log[d + e*x])/e^5))/f + ((g + f*x)^4*(a + b*Log[c*(d + e*x)^n]))/(4*f)
```

3.305.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.305.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(137) = 274$.

Time = 0.72 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.87

method	result
parallelrisch	$-\frac{-48bd\,e^3g^3n+12b\,d^4f^3n+36x^2b\,e^4f\,g^2n-48x^3\ln(c(ex+d)^n)b\,e^4f^2g-4x^3bd\,e^3f^3n+16x^3b\,e^4f^2gn+6x^2b\,d^2e^2f^3n-12xb\,d^3}{}$
risch	Expression too large to display

input
```
int((f+g/x)^3*x^3*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-48*b*d*e^3*g^3*n+12*b*d^4*f^3*n+36*x^2*b*e^4*f*g^2*n-48*x^3*ln(c*(
e*x+d)^n)*b*e^4*f^2*g-4*x^3*b*d*e^3*f^3*n+16*x^3*b*e^4*f^2*g*n+6*x^2*b*d^2
*e^2*f^3*n-12*x*b*d^3*e*f^3*n-96*ln(e*x+d)*b*d*e^3*g^3*n+72*b*d^2*e^2*f*g^
2*n-72*x^2*a*e^4*f*g^2-12*x^4*a*e^4*f^3-48*x*a*e^4*g^3-48*b*d^3*e*f^2*g*n+
48*a*d*g^3*e^3-24*x^2*b*d*e^3*f^2*g*n+48*x*b*d^2*e^2*f^2*g*n-48*ln(e*x+d)*
b*d^3*e*f^2*g*n+72*ln(e*x+d)*b*d^2*e^2*f*g^2*n-72*x^2*ln(c*(e*x+d)^n)*b*e^
4*f*g^2-72*x*b*d*e^3*f*g^2*n-48*x^3*a*e^4*f^2*g+12*ln(e*x+d)*b*d^4*f^3*n-1
2*x^4*ln(c*(e*x+d)^n)*b*e^4*f^3+48*ln(c*(e*x+d)^n)*b*d*e^3*g^3+48*x*b*e^4*
g^3*n+3*x^4*b*e^4*f^3*n-48*x*ln(c*(e*x+d)^n)*b*e^4*g^3)/e^4
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(137) = 274$.

Time = 0.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.26

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx = -\frac{3\left(be^4f^3n-4\,ae^4f^3\right)x^4-4\left(12\,ae^4f^2g+\left(bde^3f^3-4\,be^4f^2g\right)n\right)x^3-6\left(12\,ae^4fg^2-\left(bd^2e^2f^3-4\,bde^3f^2\right.\right.}{}$$

input
```
integrate((f+g/x)^3*x^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
-1/48*(3*(b*e^4*f^3*n - 4*a*e^4*f^3)*x^4 - 4*(12*a*e^4*f^2*g + (b*d*e^3*f^
3 - 4*b*e^4*f^2*g)*n)*x^3 - 6*(12*a*e^4*f*g^2 - (b*d^2*e^2*f^3 - 4*b*d*e^3
*f^2*g + 6*b*e^4*f*g^2)*n)*x^2 - 12*(4*a*e^4*g^3 + (b*d^3*e*f^3 - 4*b*d^2*
e^2*f^2*g + 6*b*d*e^3*f*g^2 - 4*b*e^4*g^3)*n)*x - 12*(b*e^4*f^3*n*x^4 + 4*
b*e^4*f^2*g*n*x^3 + 6*b*e^4*f*g^2*n*x^2 + 4*b*e^4*g^3*n*x - (b*d^4*f^3 - 4
*b*d^3*e*f^2*g + 6*b*d^2*e^2*f*g^2 - 4*b*d*e^3*g^3)*n)*log(e*x + d) - 12*(
b*e^4*f^3*x^4 + 4*b*e^4*f^2*g*x^3 + 6*b*e^4*f*g^2*x^2 + 4*b*e^4*g^3*x)*log
(c))/e^4
```

3.305.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(128) = 256$.

Time = 19.60 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.75

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx$$
$$=\begin{cases} \frac{af^3x^4}{4}+af^2gx^3+\frac{3afg^2x^2}{2}+ag^3x-\frac{bd^4f^3\log(c(d+ex)^n)}{4e^4}+\frac{bd^3f^3nx}{4e^3}+\frac{bd^3f^2g\log(c(d+ex)^n)}{e^3}-\frac{bd^2f^3nx^2}{8e^2}-\frac{bd^2f^2gnx}{e^2} - \\ (a+b\log(cd^n))\left(\frac{f^3x^4}{4}+f^2gx^3+\frac{3fg^2x^2}{2}+g^3x\right) \end{cases}$$

input
```
integrate((f+g/x)**3*x**3*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*f**3*x**4/4 + a*f**2*g*x**3 + 3*a*f*g**2*x**2/2 + a*g**3*x -
b*d**4*f**3*log(c*(d + e*x)**n)/(4*e**4) + b*d**3*f**3*n*x/(4*e**3) + b*d*
*3*f**2*g*log(c*(d + e*x)**n)/e**3 - b*d**2*f**3*n*x**2/(8*e**2) - b*d**2*
f**2*g*n*x/e**2 - 3*b*d**2*f*g**2*log(c*(d + e*x)**n)/(2*e**2) + b*d*f**3*
n*x**3/(12*e) + b*d*f**2*g*n*x**2/(2*e) + 3*b*d*f*g**2*n*x/(2*e) + b*d*g**
3*log(c*(d + e*x)**n)/e - b*f**3*n*x**4/16 + b*f**3*x**4*log(c*(d + e*x)**
n)/4 - b*f**2*g*n*x**3/3 + b*f**2*g*x**3*log(c*(d + e*x)**n) - 3*b*f*g**2*
n*x**2/4 + 3*b*f*g**2*x**2*log(c*(d + e*x)**n)/2 - b*g**3*n*x + b*g**3*x*l
og(c*(d + e*x)**n), Ne(e, 0)), ((a + b*log(c*d**n))*(f**3*x**4/4 + f**2*g*
x**3 + 3*f*g**2*x**2/2 + g**3*x), True))
```

3.305.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(137) = 274$.

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.91

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$=\frac{1}{4}\,bf^3x^4\log\left((ex+d)^nc\right)+\frac{1}{4}\,af^3x^4+bf^2gx^3\log\left((ex+d)^nc\right)$$
$$+af^2gx^3-beg^3n\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right)$$
$$-\frac{1}{48}\,bef^3n\left(\frac{12\,d^4\log\left(ex+d\right)}{e^5}+\frac{3\,e^3x^4-4\,de^2x^3+6\,d^2ex^2-12\,d^3x}{e^4}\right)$$
$$+\frac{1}{6}\,bef^2gn\left(\frac{6\,d^3\log\left(ex+d\right)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)$$
$$-\frac{3}{4}\,befg^2n\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)$$
$$+\frac{3}{2}\,bfg^2x^2\log\left((ex+d)^nc\right)+\frac{3}{2}\,afg^2x^2+bg^3x\log\left((ex+d)^nc\right)+ag^3x$$

input
```
integrate((f+g/x)^3*x^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/4*b*f^3*x^4*log((e*x + d)^n*c) + 1/4*a*f^3*x^4 + b*f^2*g*x^3*log((e*x +
d)^n*c) + a*f^2*g*x^3 - b*e*g^3*n*(x/e - d*log(e*x + d)/e^2) - 1/48*b*e*f^
3*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12
*d^3*x)/e^4) + 1/6*b*e*f^2*g*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*
e*x^2 + 6*d^2*x)/e^3) - 3/4*b*e*f*g^2*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 -
 2*d*x)/e^2) + 3/2*b*f*g^2*x^2*log((e*x + d)^n*c) + 3/2*a*f*g^2*x^2 + b*g^
3*x*log((e*x + d)^n*c) + a*g^3*x
```

3.305. $\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$

3.305.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(137) = 274$.

Time = 0.33 (sec) , antiderivative size = 770, normalized size of antiderivative = 5.17

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx = \frac{(ex+d)^4bf^3n\log(ex+d)}{4e^4} - \frac{(ex+d)^3bdf^3n\log(ex+d)}{e^4} + \frac{3(ex+d)^2bd^2f^3n\log(ex+d)}{2e^4} - \frac{(ex+d)bd^3f^3n\log(ex+d)}{e^4} + \frac{(ex+d)^3bf^2gn\log(ex+d)}{e^3} - \frac{3(ex+d)^2bdf^2gn\log(ex+d)}{e^3} + \frac{3(ex+d)bd^2f^2gn\log(ex+d)}{e^3} + \frac{3(ex+d)^2bfg^2n\log(ex+d)}{2e^2} - \frac{3(ex+d)bdfg^2n\log(ex+d)}{e^2} + \frac{(ex+d)bg^3n\log(ex+d)}{e} - \frac{(ex+d)^4bf^3n}{16e^4} + \frac{(ex+d)^3bdf^3n}{3e^4} - \frac{3(ex+d)^2bd^2f^3n}{4e^4} + \frac{(ex+d)bd^3f^3n}{e^4} - \frac{(ex+d)^3bf^2gn}{3e^3} + \frac{3(ex+d)^2bdf^2gn}{2e^3} - \frac{3(ex+d)bd^2f^2gn}{e^3} - \frac{3(ex+d)^2bfg^2n}{4e^2} + \frac{3(ex+d)bdfg^2n}{e^2} - \frac{(ex+d)bg^3n}{e} + \frac{(ex+d)^4bf^3\log(c)}{4e^4} - \frac{(ex+d)^3bdf^3\log(c)}{e^4} + \frac{3(ex+d)^2bd^2f^3\log(c)}{2e^4} - \frac{(ex+d)bd^3f^3\log(c)}{e^4} + \frac{(ex+d)^3bf^2g\log(c)}{e^3} - \frac{3(ex+d)^2bdf^2g\log(c)}{e^3} + \frac{3(ex+d)bd^2f^2g\log(c)}{e^3} + \frac{3(ex+d)^2bfg^2\log(c)}{2e^2} - \frac{3(ex+d)bdfg^2\log(c)}{e^2} + \frac{(ex+d)bg^3\log(c)}{e} + \frac{(ex+d)^4af^3}{4e^4} - \frac{(ex+d)^3adf^3}{e^4}$$

3.305. $\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$

input
```
integrate((f+g/x)^3*x^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
1/4*(e*x + d)^4*b*f^3*n*log(e*x + d)/e^4 - (e*x + d)^3*b*d*f^3*n*log(e*x +
 d)/e^4 + 3/2*(e*x + d)^2*b*d^2*f^3*n*log(e*x + d)/e^4 - (e*x + d)*b*d^3*f
^3*n*log(e*x + d)/e^4 + (e*x + d)^3*b*f^2*g*n*log(e*x + d)/e^3 - 3*(e*x +
d)^2*b*d*f^2*g*n*log(e*x + d)/e^3 + 3*(e*x + d)*b*d^2*f^2*g*n*log(e*x + d)
/e^3 + 3/2*(e*x + d)^2*b*f*g^2*n*log(e*x + d)/e^2 - 3*(e*x + d)*b*d*f*g^2*
n*log(e*x + d)/e^2 + (e*x + d)*b*g^3*n*log(e*x + d)/e - 1/16*(e*x + d)^4*b
*f^3*n/e^4 + 1/3*(e*x + d)^3*b*d*f^3*n/e^4 - 3/4*(e*x + d)^2*b*d^2*f^3*n/e
^4 + (e*x + d)*b*d^3*f^3*n/e^4 - 1/3*(e*x + d)^3*b*f^2*g*n/e^3 + 3/2*(e*x
+ d)^2*b*d*f^2*g*n/e^3 - 3*(e*x + d)*b*d^2*f^2*g*n/e^3 - 3/4*(e*x + d)^2*b
*f*g^2*n/e^2 + 3*(e*x + d)*b*d*f*g^2*n/e^2 - (e*x + d)*b*g^3*n/e + 1/4*(e*
x + d)^4*b*f^3*log(c)/e^4 - (e*x + d)^3*b*d*f^3*log(c)/e^4 + 3/2*(e*x + d)
^2*b*d^2*f^3*log(c)/e^4 - (e*x + d)*b*d^3*f^3*log(c)/e^4 + (e*x + d)^3*b*f
^2*g*log(c)/e^3 - 3*(e*x + d)^2*b*d*f^2*g*log(c)/e^3 + 3*(e*x + d)*b*d^2*f
^2*g*log(c)/e^3 + 3/2*(e*x + d)^2*b*f*g^2*log(c)/e^2 - 3*(e*x + d)*b*d*f*g
^2*log(c)/e^2 + (e*x + d)*b*g^3*log(c)/e + 1/4*(e*x + d)^4*a*f^3/e^4 - (e*
x + d)^3*a*d*f^3/e^4 + 3/2*(e*x + d)^2*a*d^2*f^3/e^4 - (e*x + d)*a*d^3*f^3
/e^4 + (e*x + d)^3*a*f^2*g/e^3 - 3*(e*x + d)^2*a*d*f^2*g/e^3 + 3*(e*x + d)
*a*d^2*f^2*g/e^3 + 3/2*(e*x + d)^2*a*f*g^2/e^2 - 3*(e*x + d)*a*d*f*g^2/e^2
 + (e*x + d)*a*g^3/e
```

3.305. $\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$

3.305.9 Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.36

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$= x\left(\frac{4\,a\,e\,g^3+12\,a\,d\,f\,g^2-4\,b\,e\,g^3\,n}{4\,e}+\frac{d\left(\frac{d\left(\frac{f^2\,(a\,d\,f+3\,a\,e\,g-b\,e\,g\,n)}{e}-\frac{d\,f^3\,(4\,a-b\,n)}{4\,e}\right)}{e}-\frac{3\,f\,g\,(2\,a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{2\,e}\right)}{e}\right)$$
$$+x^3\left(\frac{f^2\,(a\,d\,f+3\,a\,e\,g-b\,e\,g\,n)}{3\,e}-\frac{d\,f^3\,(4\,a-b\,n)}{12\,e}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)\left(\frac{b\,f^3\,x^4}{4}+b\,f^2\,g\,x^3+\frac{3\,b\,f\,g^2\,x^2}{2}+b\,g^3\,x\right)$$
$$-x^2\left(\frac{d\left(\frac{f^2\,(a\,d\,f+3\,a\,e\,g-b\,e\,g\,n)}{e}-\frac{d\,f^3\,(4\,a-b\,n)}{4\,e}\right)}{2\,e}-\frac{3\,f\,g\,(2\,a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{4\,e}\right)$$
$$-\frac{\ln\left(d+e\,x\right)\,(b\,n\,d^4\,f^3-4\,b\,n\,d^3\,e\,f^2\,g+6\,b\,n\,d^2\,e^2\,f\,g^2-4\,b\,n\,d\,e^3\,g^3)}{4\,e^4}$$
$$+\frac{f^3\,x^4\,(4\,a-b\,n)}{16}$$

input
```
int(x^3*(f + g/x)^3*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
x*((4*a*e*g^3 + 12*a*d*f*g^2 - 4*b*e*g^3*n)/(4*e) + (d*((d*((f^2*(a*d*f +
3*a*e*g - b*e*g*n))/e - (d*f^3*(4*a - b*n))/(4*e)))/e - (3*f*g*(2*a*d*f +
2*a*e*g - b*e*g*n))/(2*e)))/e) + x^3*((f^2*(a*d*f + 3*a*e*g - b*e*g*n))/(3
*e) - (d*f^3*(4*a - b*n))/(12*e)) + log(c*(d + e*x)^n)*((b*f^3*x^4)/4 + b*
g^3*x + (3*b*f*g^2*x^2)/2 + b*f^2*g*x^3) - x^2*((d*((f^2*(a*d*f + 3*a*e*g
- b*e*g*n))/e - (d*f^3*(4*a - b*n))/(4*e)))/(2*e) - (3*f*g*(2*a*d*f + 2*a*
e*g - b*e*g*n))/(4*e)) - (log(d + e*x)*(b*d^4*f^3*n - 4*b*d*e^3*g^3*n - 4*
b*d^3*e*f^2*g*n + 6*b*d^2*e^2*f*g^2*n))/(4*e^4) + (f^3*x^4*(4*a - b*n))/16
```

3.306 $\int \frac{a+b\log(c(d+ex)^n)}{(f+\frac{g}{x})x}\,dx$

3.306.1 Optimal result

Integrand size = 27, antiderivative size = 63

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)x}\,dx = \frac{(a+b\log(c(d+ex)^n))\log\left(-\frac{e(g+fx)}{df-eg}\right)}{f}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{f(d+ex)}{df-eg}\right)}{f}$$

output
```
(a+b*ln(c*(e*x+d)^n))*ln(-e*(f*x+g)/(d*f-e*g))/f+b*n*polylog(2,f*(e*x+d)/(
d*f-e*g))/f
```

3.306.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)x}\,dx=\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(g+fx)}{-df+eg}\right)}{f}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{f(d+ex)}{df-eg}\right)}{f}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g/x)*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[(e*(g + f*x))/(-(d*f) + e*g)])/f + (b*n*Po
lyLog[2, (f*(d + e*x))/(d*f - e*g)])/f
```

3.306.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2005, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(c(d+ex)^n)}{x\left(f+\frac{g}{x}\right)}dx$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{a+b\log(c(d+ex)^n)}{fx+g}dx$$

$$\Big\downarrow \text{2841}$$

$$\frac{\log\left(-\frac{e(fx+g)}{df-eg}\right)(a+b\log(c(d+ex)^n))}{f}-\frac{ben\int \frac{\log\left(-\frac{e(g+fx)}{df-eg}\right)}{d+ex}dx}{f}$$

$$\Big\downarrow \text{2840}$$

$$\frac{\log\left(-\frac{e(fx+g)}{df-eg}\right)(a+b\log(c(d+ex)^n))}{f}-\frac{bn\int \frac{\log\left(1-\frac{f(d+ex)}{df-eg}\right)}{d+ex}d(d+ex)}{f}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log\left(-\frac{e(fx+g)}{df-eg}\right)(a+b\log(c(d+ex)^n))}{f}+\frac{bn\,\text{PolyLog}\left(2,\frac{f(d+ex)}{df-eg}\right)}{f}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((f + g/x)*x),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])*Log[-((e*(g + f*x))/(d*f - e*g))])/f + (b*n*Po
lyLog[2, (f*(d + e*x))/(d*f - e*g)])/f
```

3.306.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.306.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

method	result
risch	$\frac{b\ln((ex+d)^n)\ln(fx+g)}{f} - \frac{bn\,\mathrm{dilog}\left(\frac{(fx+g)e+df-eg}{df-eg}\right)}{f} - \frac{bn\ln(fx+g)\ln\left(\frac{(fx+g)e+df-eg}{df-eg}\right)}{f} + \frac{\left(-\frac{ib\pi\ \mathrm{csgn}(ic(ex+d)^n)\ \mathrm{csgn}(ic)\ \mathrm{csgn}(}{2}\right.}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(f+g/x)/x,x,method=_RETURNVERBOSE)
```

output
```
b*ln((e*x+d)^n)*ln(f*x+g)/f-b/f*n*dilog(((f*x+g)*e+d*f-e*g)/(d*f-e*g))-b/f
*n*ln(f*x+g)*ln(((f*x+g)*e+d*f-e*g)/(d*f-e*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*
(e*x+d)^n)^3+b*ln(c)+a)*ln(f*x+g)/f
```

3.306.5 Fricas [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)x}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(f+\frac{g}{x}\right)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)/x,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)/(f*x + g), x)
```

3.306.6 Sympy [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)x}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{fx+g}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(f+g/x)/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))/(f*x + g), x)
```

3.306.7 Maxima [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)x}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(f+\frac{g}{x}\right)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)/x,x, algorithm="maxima")
```

output
```
b*integrate((log((e*x + d)^n) + log(c))/(f*x + g), x) + a*log(f*x + g)/f
```

3.306.8 Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)x}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(f+\frac{g}{x}\right)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)/((f + g/x)*x), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)x}\,dx=\int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,\left(f+\frac{g}{x}\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g/x)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))/(x*(f + g/x)), x)
```

3.307 $\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx$

3.307.1 Optimal result

Integrand size = 27, antiderivative size = 74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx=-\frac{ben\log(d+ex)}{f(df-eg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f(g+fx)}+\frac{ben\log(g+fx)}{f(df-eg)}$$

output
```
-b*e*n*ln(e*x+d)/f/(d*f-e*g)+(-a-b*ln(c*(e*x+d)^n))/f/(f*x+g)+b*e*n*ln(f*x
+g)/f/(d*f-e*g)
```

3.307.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx=\frac{-\frac{a+b\log(c(d+ex)^n)}{g+fx}+\frac{ben(\log(d+ex)-\log(g+fx))}{-df+eg}}{f}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^2*x^2),x]
```

output
```
(-((a + b*Log[c*(d + e*x)^n])/(g + f*x)) + (b*e*n*(Log[d + e*x] - Log[g +
f*x]))/(-(d*f) + e*g))/f
```

3.307.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2005, 2842, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+\frac{g}{x}\right)^2}dx$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(fx+g)^2}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{ben\int \frac{1}{(d+ex)(g+fx)}dx}{f}-\frac{a+b\log\left(c(d+ex)^n\right)}{f(fx+g)}$$

$$\Big\downarrow \text{47}$$

$$\frac{ben\left(\frac{f\int \frac{1}{g+fx}dx}{df-eg}-\frac{e\int \frac{1}{d+ex}dx}{df-eg}\right)}{f}-\frac{a+b\log\left(c(d+ex)^n\right)}{f(fx+g)}$$

$$\Big\downarrow \text{16}$$

$$\frac{ben\left(\frac{\log(fx+g)}{df-eg}-\frac{\log(d+ex)}{df-eg}\right)}{f}-\frac{a+b\log\left(c(d+ex)^n\right)}{f(fx+g)}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^2*x^2),x]
```

output
```
-((a + b*Log[c*(d + e*x)^n])/(f*(g + f*x))) + (b*e*n*(-(Log[d + e*x]/(d*f
- e*g)) + Log[g + f*x]/(d*f - e*g)))/f
```

3.307.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.307.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

method	result
parallelrisch	$-\frac{\ln(ex+d)xb\,e^2fn-\ln(fx+g)xb\,e^2fn+\ln(ex+d)b\,e^2gn-\ln(fx+g)b\,e^2gn+\ln(c(ex+d)^n)bdef-\ln(c(ex+d)^n)b\,e^2g+adef-a\,e^{}}{(df-eg)(fx+g)ef}$
risch	$-\frac{b\ln((ex+d)^n)}{f(fx+g)}-\frac{-i\pi\,\mathrm{csgn}(ic(ex+d)^n)^3bdf-i\pi beg\ \mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2+i\pi\ \mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^{}}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(f+g/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(ln(e*x+d)*x*b*e^2*f*n-ln(f*x+g)*x*b*e^2*f*n+ln(e*x+d)*b*e^2*g*n-ln(f*x+g
)*b*e^2*g*n+ln(c*(e*x+d)^n)*b*d*e*f-ln(c*(e*x+d)^n)*b*e^2*g+a*d*e*f-a*e^2*
g)/(d*f-e*g)/(f*x+g)/e/f
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = -\frac{adf-aeg+(befnx+bdfn)\log(ex+d)-(befnx+begn)\log(fx+g)+(bdf-beg)\log(c)}{df^2g-efg^2+(df^3-ef^2g)x}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^2/x^2,x, algorithm="fricas")
```

output
```
-(a*d*f - a*e*g + (b*e*f*n*x + b*d*f*n)*log(e*x + d) - (b*e*f*n*x + b*e*g*
n)*log(f*x + g) + (b*d*f - b*e*g)*log(c))/(d*f^2*g - e*f*g^2 + (d*f^3 - e*
f^2*g)*x)
```

3.307.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(61) = 122.

Time = 34.40 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = \begin{cases} \frac{ax+\frac{bd\log\left(c(d+ex)^n\right)}{e}-bnx+bx\log\left(c(d+ex)^n\right)}{g^2} \\ -\frac{a}{f^2x+fg}-\frac{bn}{f^2x+fg}-\frac{b\log\left(c\left(ex+\frac{eg}{f}\right)^n\right)}{f^2x+fg} \\ -\frac{adf}{df^3x+df^2g-ef^2gx-efg^2}+\frac{aeg}{df^3x+df^2g-ef^2gx-efg^2}-\frac{bdf\log\left(c(d+ex)^n\right)}{df^3x+df^2g-ef^2gx-efg^2}+\frac{befnx\log\left(x+\frac{g}{f}\right)}{df^3x+df^2g-ef^2gx-efg^2}-\frac{befx\log\left(c(d+e\right.}{df^3x+df^2g-ef^2g} \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(f+g/x)**2/x**2,x)
```

output
```
Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)*
*n))/g**2, Eq(f, 0)), (-a/(f**2*x + f*g) - b*n/(f**2*x + f*g) - b*log(c*(e
*x + e*g/f)**n)/(f**2*x + f*g), Eq(d, e*g/f)), (-a*d*f/(d*f**3*x + d*f**2*
g - e*f**2*g*x - e*f*g**2) + a*e*g/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f
*g**2) - b*d*f*log(c*(d + e*x)**n)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f
*g**2) + b*e*f*n*x*log(x + g/f)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g*
*2) - b*e*f*x*log(c*(d + e*x)**n)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*
g**2) + b*e*g*n*log(x + g/f)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g**2)
, True))
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = -ben\left(\frac{\log\left(ex+d\right)}{df^2-efg}-\frac{\log\left(fx+g\right)}{df^2-efg}\right) - \frac{b\log\left((ex+d)^nc\right)}{f^2x+fg}-\frac{a}{f^2x+fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^2/x^2,x, algorithm="maxima")
```

output
```
-b*e*n*(log(e*x + d)/(d*f^2 - e*f*g) - log(f*x + g)/(d*f^2 - e*f*g)) - b*l
og((e*x + d)^n*c)/(f^2*x + f*g) - a/(f^2*x + f*g)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = -\frac{ben\log\left(ex+d\right)}{df^2-efg}+\frac{ben\log\left(fx+g\right)}{df^2-efg} - \frac{bn\log\left(ex+d\right)}{f^2x+fg}-\frac{b\log\left(c\right)+a}{f^2x+fg}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^2/x^2,x, algorithm="giac")
```

output
```
-b*e*n*log(e*x + d)/(d*f^2 - e*f*g) + b*e*n*log(f*x + g)/(d*f^2 - e*f*g) -
 b*n*log(e*x + d)/(f^2*x + f*g) - (b*log(c) + a)/(f^2*x + f*g)
```

3.307.9 Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = -\frac{a}{x\,f^2+g\,f}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{f\,\left(g+f\,x\right)}+\frac{b\,e\,n\,\mathrm{atan}\left(\frac{e\,g\,2{}\mathrm{i}+e\,f\,x\,2{}\mathrm{i}}{d\,f-e\,g}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{f\,\left(d\,f-e\,g\right)}$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g/x)^2),x)
```

output
```
(b*e*n*atan((e*g*2i + e*f*x*2i)/(d*f - e*g) + 1i)*2i)/(f*(d*f - e*g)) - (b
*log(c*(d + e*x)^n))/(f*(g + f*x)) - a/(f*g + f^2*x)
```

3.307. $\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx$

3.308 $\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx$

3.308.1 Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx=-\frac{ben}{2f(df-eg)(g+fx)}+\frac{be^2n\log(d+ex)}{2f(df-eg)^2}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f(g+fx)^2}-\frac{be^2n\log(g+fx)}{2f(df-eg)^2}$$

output
```
-1/2*b*e*n/f/(d*f-e*g)/(f*x+g)+1/2*b*e^2*n*ln(e*x+d)/f/(d*f-e*g)^2+1/2*(-a
-b*ln(c*(e*x+d)^n))/f/(f*x+g)^2-1/2*b*e^2*n*ln(f*x+g)/f/(d*f-e*g)^2
```

3.308.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx=-\frac{a+b\log\left(c(d+ex)^n\right)-\frac{ben(g+fx)(-df+eg+e(g+fx)\log(d+ex)-e(g+fx)\log(g+fx))}{(df-eg)^2}}{2f(g+fx)^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^3*x^3),x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n] - (b*e*n*(g + f*x)*(-(d*f) + e*g + e*(g + f
*x)*Log[d + e*x] - e*(g + f*x)*Log[g + f*x]))/(d*f - e*g)^2)/(f*(g + f*x)^
2)
```

3.308.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2005, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+\frac{g}{x}\right)^3}dx$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(fx+g)^3}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{ben\int \frac{1}{(d+ex)(g+fx)^2}dx}{2f}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f(fx+g)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{ben\int \left(\frac{e^2}{(df-eg)^2(d+ex)}-\frac{fe}{(df-eg)^2(g+fx)}+\frac{f}{(df-eg)(g+fx)^2}\right)dx}{2f}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f(fx+g)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ben\left(-\frac{1}{(fx+g)(df-eg)}+\frac{e\log(d+ex)}{(df-eg)^2}-\frac{e\log(fx+g)}{(df-eg)^2}\right)}{2f}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f(fx+g)^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^3*x^3),x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n])/(f*(g + f*x)^2) + (b*e*n*(-(1/((d*f - e*g)
*(g + f*x))) + (e*Log[d + e*x])/(d*f - e*g)^2 - (e*Log[g + f*x])/(d*f - e*
g)^2))/(2*f)
```

3.308.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.308.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(107) = 214$.

Time = 0.61 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
parallelrisch	$\frac{-a\,d^2e\,f^3-a\,e^3f\,g^2-\ln(c(ex+d)^n)b\,d^2e\,f^3-\ln(c(ex+d)^n)b\,e^3f\,g^2-\ln(fx+g)b\,e^3f\,g^2n-xbd\,e^2f^3n+2\ln(c(ex+d)^n)bd\,e^2f^2g-}{2(f}$
risch	$-\frac{b\ln((ex+d)^n)}{2f(fx+g)^2}-\frac{2\ln(c)b\,d^2f^2+2\ln(c)b\,e^2g^2-4\ln(c)bdefg+2i\pi bdefg\,\mathrm{csgn}(ic(ex+d)^n)^3-i\pi b\,e^2g^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n}{}$

input
```
int((a+b*ln(c*(e*x+d)^n))/(f+g/x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*d^2*e*f^3-a*e^3*f*g^2-ln(c*(e*x+d)^n)*b*d^2*e*f^3-ln(c*(e*x+d)^n)*
b*e^3*f*g^2-ln(f*x+g)*b*e^3*f*g^2*n-x*b*d*e^2*f^3*n+2*ln(c*(e*x+d)^n)*b*d*
e^2*f^2*g-b*d*e^2*f^2*n*g+b*e^3*f^2*g*n*x+2*a*d*e^2*f^2*g+b*e^3*f*g^2*n+2*
ln(e*x+d)*b*e^3*f^2*g*n*x+ln(e*x+d)*x^2*b*e^3*f^3*n-ln(f*x+g)*x^2*b*e^3*f^
3*n-2*ln(f*x+g)*x*b*e^3*f^2*g*n+ln(e*x+d)*b*e^3*f*g^2*n)/(d^2*f^2-2*d*e*f*
g+e^2*g^2)/(f*x+g)^2/e/f^2
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(104) = 208$.

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.43

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx =$$
$$-\frac{ad^2f^2-2\,adefg+ae^2g^2+(bdef^2-be^2fg)nx+(bdefg-be^2g^2)n-(be^2f^2nx^2+2\,be^2fgnx-(bd^2f^2}{2\,(d^2f^3g^2-2\,def^2g^3+e^2fg^4+(d^2f^5-2\,de}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d^2*f^2 - 2*a*d*e*f*g + a*e^2*g^2 + (b*d*e*f^2 - b*e^2*f*g)*n*x +
(b*d*e*f*g - b*e^2*g^2)*n - (b*e^2*f^2*n*x^2 + 2*b*e^2*f*g*n*x - (b*d^2*f^
2 - 2*b*d*e*f*g)*n)*log(e*x + d) + (b*e^2*f^2*n*x^2 + 2*b*e^2*f*g*n*x + b*
e^2*g^2*n)*log(f*x + g) + (b*d^2*f^2 - 2*b*d*e*f*g + b*e^2*g^2)*log(c))/(d
^2*f^3*g^2 - 2*d*e*f^2*g^3 + e^2*f*g^4 + (d^2*f^5 - 2*d*e*f^4*g + e^2*f^3*
g^2)*x^2 + 2*(d^2*f^4*g - 2*d*e*f^3*g^2 + e^2*f^2*g^3)*x)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))/(f+g/x)**3/x**3,x)
```

output
```
Timed out
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.51

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx$$
$$=\frac{1}{2}\,ben\left(\frac{e\log\left(ex+d\right)}{d^2f^3-2\,def^2g+e^2fg^2}-\frac{e\log\left(fx+g\right)}{d^2f^3-2\,def^2g+e^2fg^2}-\frac{1}{df^2g-efg^2+(df^3-ef^2g)x}\right)$$
$$-\frac{b\log\left((ex+d)^nc\right)}{2\,(f^3x^2+2\,f^2gx+fg^2)}-\frac{a}{2\,(f^3x^2+2\,f^2gx+fg^2)}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^3/x^3,x, algorithm="maxima")
```

output
```
1/2*b*e*n*(e*log(e*x + d)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - e*log(f*x
+ g)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - 1/(d*f^2*g - e*f*g^2 + (d*f^3 -
 e*f^2*g)*x)) - 1/2*b*log((e*x + d)^n*c)/(f^3*x^2 + 2*f^2*g*x + f*g^2) - 1
/2*a/(f^3*x^2 + 2*f^2*g*x + f*g^2)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.79

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx = \frac{be^2n\log\left(ex+d\right)}{2\left(d^2f^3-2\,def^2g+e^2fg^2\right)} - \frac{be^2n\log\left(fx+g\right)}{2\left(d^2f^3-2\,def^2g+e^2fg^2\right)} - \frac{bn\log\left(ex+d\right)}{2\left(f^3x^2+2\,f^2gx+fg^2\right)} - \frac{befnx+begn+bdf\log\left(c\right)-beg\log\left(c\right)+adf-aeg}{2\left(df^4x^2-ef^3gx^2+2\,df^3gx-2\,ef^2g^2x+df^2g^2-efg^3\right)}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^3/x^3,x, algorithm="giac")
```

output
```
1/2*b*e^2*n*log(e*x + d)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - 1/2*b*e^2*n
*log(f*x + g)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - 1/2*b*n*log(e*x + d)/(
f^3*x^2 + 2*f^2*g*x + f*g^2) - 1/2*(b*e*f*n*x + b*e*g*n + b*d*f*log(c) - b
*e*g*log(c) + a*d*f - a*e*g)/(d*f^4*x^2 - e*f^3*g*x^2 + 2*d*f^3*g*x - 2*e*
f^2*g^2*x + d*f^2*g^2 - e*f*g^3)
```

3.308.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx = \frac{b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,d^2\,f^3-2\,e^2\,f\,g^2}{2\,f\,{\left(d\,f-e\,g\right)}^2}+\frac{2\,e\,f\,x}{d\,f-e\,g}\right)}{f\,{\left(d\,f-e\,g\right)}^2} - \frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{2\,f\,\left(f^2\,x^2+2\,f\,g\,x+g^2\right)} - \frac{\frac{a\,d\,f-a\,e\,g+b\,e\,g\,n}{d\,f-e\,g}+\frac{b\,e\,f\,n\,x}{d\,f-e\,g}}{2\,f^3\,x^2+4\,f^2\,g\,x+2\,f\,g^2}$$

input
```
int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g/x)^3),x)
```

output
```
(b*e^2*n*atanh((2*d^2*f^3 - 2*e^2*f*g^2)/(2*f*(d*f - e*g)^2) + (2*e*f*x)/(
d*f - e*g)))/(f*(d*f - e*g)^2) - (b*log(c*(d + e*x)^n))/(2*f*(g^2 + f^2*x^
2 + 2*f*g*x)) - ((a*d*f - a*e*g + b*e*g*n)/(d*f - e*g) + (b*e*f*n*x)/(d*f
- e*g))/(2*f*g^2 + 2*f^3*x^2 + 4*f^2*g*x)
```

3.308. $\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx$

3.309 $\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx$

3.309.1 Optimal result

Integrand size = 16, antiderivative size = 247

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = -\frac{x}{c}+\frac{(a+bx)\log(a+bx)}{bc}-\frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}$$

output
```
-x/c+(b*x+a)*ln(b*x+a)/b/c+1/2*ln(b*x+a)*ln(-b*(x*(-c)^(1/2)+d^(1/2))/(a*(
-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/2*ln(b*x+a)*ln(b*(-x*(-c)^(1/2)
+d^(1/2))/(a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/2*polylog(2,(b*x+
a)*(-c)^(1/2)/(a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/2*polylog(2,(
b*x+a)*(-c)^(1/2)/(a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = -\frac{x}{c}+\frac{(a+bx)\log(a+bx)}{bc}-\frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}$$

input
```
Integrate[Log[a + b*x]/(c + d/x^2),x]
```

output
```
-(x/c) + ((a + b*x)*Log[a + b*x])/(b*c) - (Sqrt[d]*Log[a + b*x]*Log[(b*(Sq
rt[d] - Sqrt[-c]*x))/(a*Sqrt[-c] + b*Sqrt[d])]])/(2*(-c)^(3/2)) + (Sqrt[d]*
Log[a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/
(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*
Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sq
rt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2))
```

3.309.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c\,(cx^2+d)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{\sqrt{-c}a+b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{(a+bx)\log(a+bx)}{bc}-\frac{x}{c}$$

input
```
Int[Log[a + b*x]/(c + d/x^2),x]
```

output
```
-(x/c) + ((a + b*x)*Log[a + b*x])/(b*c) - (Sqrt[d]*Log[a + b*x]*Log[(b*(Sq
rt[d] - Sqrt[-c]*x))/(a*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*
Log[a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/
(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*
Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sq
rt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2))
```

3.309.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.309.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{(bx+a)\ln(bx+a)-bx-a}{c}-\frac{d\,b^2\left(-\frac{\ln(bx+a)\left(-\ln\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)+\ln\left(\frac{b\sqrt{-cd}-ca+c(bx+a)}{b\sqrt{-cd}-ca}\right)\right)}{2b\sqrt{-cd}}+\frac{\operatorname{dilog}\left(\frac{b\sqrt{-cd}+ca-c(bx+a}{b\sqrt{-cd}+ca}\right.}{}\right.}{c}}{b}$
default	$\frac{\frac{(bx+a)\ln(bx+a)-bx-a}{c}-\frac{d\,b^2\left(-\frac{\ln(bx+a)\left(-\ln\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)+\ln\left(\frac{b\sqrt{-cd}-ca+c(bx+a)}{b\sqrt{-cd}-ca}\right)\right)}{2b\sqrt{-cd}}+\frac{\operatorname{dilog}\left(\frac{b\sqrt{-cd}+ca-c(bx+a}{b\sqrt{-cd}+ca}\right.}{}\right.}{c}}{b}$
risch	$\frac{\ln(bx+a)x}{c}+\frac{\ln(bx+a)a}{bc}-\frac{x}{c}-\frac{a}{bc}-\frac{d\ln(bx+a)\ln\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)}{2c\sqrt{-cd}}+\frac{d\ln(bx+a)\ln\left(\frac{b\sqrt{-cd}-ca+c(bx+a)}{b\sqrt{-cd}-ca}\right)}{2c\sqrt{-cd}}$

input
```
int(ln(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/c*((b*x+a)*ln(b*x+a)-b*x-a)-d*b^2/c*(-1/2*ln(b*x+a)*(-ln((b*(-c*d)^
(1/2)+c*a-c*(b*x+a))/(b*(-c*d)^(1/2)+c*a))+ln((b*(-c*d)^(1/2)-c*a+c*(b*x+a
))/(b*(-c*d)^(1/2)-c*a)))/b/(-c*d)^(1/2)+1/2*(dilog((b*(-c*d)^(1/2)+c*a-c*
(b*x+a))/(b*(-c*d)^(1/2)+c*a))-dilog((b*(-c*d)^(1/2)-c*a+c*(b*x+a))/(b*(-c
*d)^(1/2)-c*a)))/b/(-c*d)^(1/2)))
```

3.309.5 Fricas [F]

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\log\left(bx+a\right)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(log(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

output
```
integral(x^2*log(b*x + a)/(c*x^2 + d), x)
```

3.309.6 Sympy [F]

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{x^2\log(a+bx)}{cx^2+d}\,dx$$

input
```
integrate(ln(b*x+a)/(c+d/x**2),x)
```

output
```
Integral(x**2*log(a + b*x)/(c*x**2 + d), x)
```

3.309.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.21

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{d\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} - \frac{x}{c}\right)\log(bx+a) - \frac{2\,bcx - 2\,ac\log(bx+a) + \left(b\arctan\left(\frac{(b^2x+ab)\sqrt{c}\sqrt{d}}{a^2c+b^2d}, \frac{abcx+a^2c}{a^2c+b^2d}\right)\log(cx^2+d) - b\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2}{a^2c}\right.\right.}{2\,bc^2}$$

input
```
integrate(log(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

output
```
-(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*log(b*x + a) - 1/2*(2*b*c*x
 - 2*a*c*log(b*x + a) + (b*arctan2((b^2*x + a*b)*sqrt(c)*sqrt(d)/(a^2*c +
b^2*d), (a*b*c*x + a^2*c)/(a^2*c + b^2*d))*log(c*x^2 + d) - b*arctan(sqrt(
c)*x/sqrt(d))*log((b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(a^2*c + b^2*d)) + I*b*d
ilog(-(a*b*c*x + b^2*d + (I*b^2*x - I*a*b)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a
*b*sqrt(c)*sqrt(d) - b^2*d)) - I*b*dilog(-(a*b*c*x + b^2*d - (I*b^2*x - I*
a*b)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*b*sqrt(c)*sqrt(d) - b^2*d)))*sqrt(c)*
sqrt(d))/(b*c^2)
```

3.309.8 Giac [F]

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\log(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(log(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

output
```
integrate(log(b*x + a)/(c + d/x^2), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\ln(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

input
```
int(log(a + b*x)/(c + d/x^2),x)
```

output
```
int(log(a + b*x)/(c + d/x^2), x)
```

3.310 $\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.310.1 Optimal result

Integrand size = 29, antiderivative size = 831

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = -\frac{2abdfnx}{eg^2}+\frac{2b^2dfn^2x}{eg^2}-\frac{2b^2d^3n^2x}{e^3g}-\frac{b^2fn^2(d+ex)^2}{4e^2g^2}+\frac{3b^2d^2n^2(d+ex)^2}{4e^4g}-\frac{2b^2dn^2(d+ex)^3}{9e^4g}+\frac{b^2n^2(d+ex)^4}{32e^4g}+\frac{b^2d^4n^2\log^2(d+ex)}{4e^4g}-\frac{2b^2dfn(d+ex)\log(c(d+ex)^n)}{e^2g^2}+\frac{2bd^3n(d+ex)(a+b\log(c(d+ex)^n))}{e^4g}+\frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g^2}-\frac{3bd^2n(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^4g}+\frac{2bdn(d+ex)^3(a+b\log(c(d+ex)^n))}{3e^4g}-\frac{bn(d+ex)^4(a+b\log(c(d+ex)^n))}{8e^4g}-\frac{bd^4n\log(d+ex)(a+b\log(c(d+ex)^n))}{2e^4g}+\frac{x^4(a+b\log(c(d+ex)^n))^2}{4g}+\frac{df(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g^2}-\frac{f(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g^2}+\frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}+\frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}+\frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}+\frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

output
```
-2*a*b*d*f*n*x/e/g^2+2*b^2*d*f*n^2*x/e/g^2-2*b^2*d^3*n^2*x/e^3/g-1/4*b^2*f
*n^2*(e*x+d)^2/e^2/g^2+3/4*b^2*d^2*n^2*(e*x+d)^2/e^4/g-2/9*b^2*d*n^2*(e*x+
d)^3/e^4/g+1/32*b^2*n^2*(e*x+d)^4/e^4/g+1/4*b^2*d^4*n^2*ln(e*x+d)^2/e^4/g-
2*b^2*d*f*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g^2+2*b*d^3*n*(e*x+d)*(a+b*ln(c*(e
*x+d)^n))/e^4/g+1/2*b*f*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2/g^2-3/2*b*d^
2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^4/g+2/3*b*d*n*(e*x+d)^3*(a+b*ln(c*(e
*x+d)^n))/e^4/g-1/8*b*n*(e*x+d)^4*(a+b*ln(c*(e*x+d)^n))/e^4/g-1/2*b*d^4*n*
ln(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^4/g+1/4*x^4*(a+b*ln(c*(e*x+d)^n))^2/g+d*
f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g^2-1/2*f*(e*x+d)^2*(a+b*ln(c*(e*x+d
)^n))^2/e^2/g^2+1/2*f^2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2)
)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3+1/2*f^2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)
^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+b*f^2*n*(a+b*ln(c*(e*x+d)^
n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+b*f^2*n*(a+b*
ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3-b
^2*f^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3-b^2*f^
2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3
```

3.310.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{-144e^4fgx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+72e^4g^2x^4(a-bn\log(d+ex)+b\log(c(d+ex)^n)}{}$$

input
```
Integrate[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(-144*e^4*f*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 72*e^4
*g^2*x^4*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 144*e^4*f^2*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] - 12*b*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(12*e^2*f*g*(e*x*(2*d - e*x) - 2*
(d^2 - e^2*x^2)*Log[d + e*x]) + g^2*(e*x*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^
2 + 3*e^3*x^3) + 12*(d^4 - e^4*x^4)*Log[d + e*x]) - 24*e^4*f^2*(Log[d + e*
x]*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2,
(Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - 24*e^4*f^2*(Log[d + e
*x]*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (S
qrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])) + b^2*n^2*(-72*e^2*f*g*(e*x
*(-6*d + e*x) + (6*d^2 + 4*d*e*x - 2*e^2*x^2)*Log[d + e*x] - 2*(d^2 - e^2*
x^2)*Log[d + e*x]^2) - g^2*(e*x*(300*d^3 - 78*d^2*e*x + 28*d*e^2*x^2 - 9*e
^3*x^3) - 12*(25*d^4 + 12*d^3*e*x - 6*d^2*e^2*x^2 + 4*d*e^3*x^3 - 3*e^4*x^
4)*Log[d + e*x] + 72*(d^4 - e^4*x^4)*Log[d + e*x]^2) + 144*e^4*f^2*(Log[d
+ e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log
[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2
*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + 144*e^4*f
^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]
+ 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]
 - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(288*...
```

3.310.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{f^2x(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}-\frac{fx(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{x^3(a+b\log(c(d+ex)^n))^2}{g}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2n^2\log^2(d+ex)d^4}{4e^4g}-\frac{bn\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)d^4}{2e^4g}-\frac{2b^2n^2xd^3}{e^3g}+\frac{2bn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)d^3}{e^4g}+\frac{3b^2n^2(d+ex)^2d^2}{4e^4g}-\frac{3bn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)d^2}{2e^4g}-\frac{2b^2n^2(d+ex)^3d}{9e^4g}+\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2d}{e^2g^2}+\frac{2b^2fn^2xd}{eg^2}-\frac{2abfnxd}{eg^2}-\frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)d}{e^2g^2}+\frac{2bn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)d}{3e^4g}+\frac{b^2n^2(d+ex)^4}{32e^4g}-\frac{b^2fn^2(d+ex)^2}{4e^2g^2}+\frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g}-\frac{f(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2}-\frac{bn(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4g}+\frac{bfn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2}+\frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^3}+\frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}+\frac{bf^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}+\frac{bf^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3}$$

input
```
Int[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

3.310. $\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

output
```
(-2*a*b*d*f*n*x)/(e*g^2) + (2*b^2*d*f*n^2*x)/(e*g^2) - (2*b^2*d^3*n^2*x)/(
e^3*g) - (b^2*f*n^2*(d + e*x)^2)/(4*e^2*g^2) + (3*b^2*d^2*n^2*(d + e*x)^2)
/(4*e^4*g) - (2*b^2*d*n^2*(d + e*x)^3)/(9*e^4*g) + (b^2*n^2*(d + e*x)^4)/(
32*e^4*g) + (b^2*d^4*n^2*Log[d + e*x]^2)/(4*e^4*g) - (2*b^2*d*f*n*(d + e*x
)*Log[c*(d + e*x)^n])/(e^2*g^2) + (2*b*d^3*n*(d + e*x)*(a + b*Log[c*(d + e
*x)^n]))/(e^4*g) + (b*f*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g
^2) - (3*b*d^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^4*g) + (2*b*
d*n*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*e^4*g) - (b*n*(d + e*x)^4*(
a + b*Log[c*(d + e*x)^n]))/(8*e^4*g) - (b*d^4*n*Log[d + e*x]*(a + b*Log[c*
(d + e*x)^n]))/(2*e^4*g) + (x^4*(a + b*Log[c*(d + e*x)^n])^2)/(4*g) + (d*f
*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e^2*g^2) - (f*(d + e*x)^2*(a + b
*Log[c*(d + e*x)^n])^2)/(2*e^2*g^2) + (f^2*(a + b*Log[c*(d + e*x)^n])^2*Lo
g[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3) + (f^2*(a
+ b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*S
qrt[g])])/(2*g^3) + (b*f^2*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt
[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 + (b*f^2*n*(a + b*Log[c*(d
+ e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3 -
 (b^2*f^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])
/g^3 - (b^2*f^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g]
)])/g^3
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.310.4 Maple [F]

$$\int \frac{x^5(a + b\ln (c(ex + d)^n))^2}{g\,x^2 + f}dx$$

input
```
int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

output
```
int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

3.310.5 Fricas [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2x^5}{gx^2+f}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*x^5*log((e*x + d)^n*c)^2 + 2*a*b*x^5*log((e*x + d)^n*c) + a^
2*x^5)/(g*x^2 + f), x)
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.310.7 Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2x^5}{gx^2+f}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/4*a^2*(2*f^2*log(g*x^2 + f)/g^3 + (g*x^4 - 2*f*x^2)/g^2) + integrate((b^
2*x^5*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^5*log((e*x + d)^n) + (b^
2*log(c)^2 + 2*a*b*log(c))*x^5)/(g*x^2 + f), x)
```

3.310.8 Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^5}{gx^2+f}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^5/(g*x^2 + f), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x^5\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{g\,x^2+f}\,dx$$

input
```
int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
```

output
```
int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

3.311 $\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.311.1 Optimal result

Integrand size = 29, antiderivative size = 499

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{2abdnx}{eg}-\frac{2b^2dn^2x}{eg}+\frac{b^2n^2(d+ex)^2}{4e^2g}+\frac{2b^2dn(d+ex)\log(c(d+ex)^n)}{e^2g}-\frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g}-\frac{d(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g}+\frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g}-\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}-\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}-\frac{bfn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}-\frac{bfn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}+\frac{b^2fn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}+\frac{b^2fn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$

output
```
2*a*b*d*n*x/e/g-2*b^2*d*n^2*x/e/g+1/4*b^2*n^2*(e*x+d)^2/e^2/g+2*b^2*d*n*(e
*x+d)*ln(c*(e*x+d)^n)/e^2/g-1/2*b*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2/g-
d*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g+1/2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n)
)^2/e^2/g-1/2*f*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f
)^(1/2)+d*g^(1/2)))/g^2-1/2*f*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g
^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-b*f*n*(a+b*ln(c*(e*x+d)^n))*polylog(
2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-b*f*n*(a+b*ln(c*(e*x+d)^n
))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2+b^2*f*n^2*polyl
og(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+b^2*f*n^2*polylog(3,(e
*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2
```

3.311.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.28

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{2e^2gx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 - 2e^2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+$$

input
```
Integrate[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(2*e^2*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*e^2*f*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(e*g*x*(2*d - e*x) - 2*g*(d^2 - e^
2*x^2)*Log[d + e*x] - 2*e^2*f*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/((
-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[
f] + d*Sqrt[g])]) - 2*e^2*f*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/(I*e
*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*S
qrt[g])])) - b^2*n^2*(g*(e*x*(6*d - e*x) + (-6*d^2 - 4*d*e*x + 2*e^2*x^2)*
Log[d + e*x] + 2*(d^2 - e^2*x^2)*Log[d + e*x]^2) + 2*e^2*f*(Log[d + e*x]^2
*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x
]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog
[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + 2*e^2*f*(Log[d +
e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d +
e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog
[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(4*e^2*g^2)
```

3.311.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{x(a+b\log(c(d+ex)^n))^2}{g} - \frac{fx(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g} + \frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g} - \frac{d(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g} - \frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^2} - \frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^2} - \frac{f\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g^2} - \frac{f\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g^2} + \frac{2abdnx}{eg} + \frac{2b^2dn(d+ex)\log(c(d+ex)^n)}{e^2g} + \frac{b^2n^2(d+ex)^2}{4e^2g} + \frac{b^2fn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} + \frac{b^2fn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^2} - \frac{2b^2dn^2x}{eg}$$

input
```
Int[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(2*a*b*d*n*x)/(e*g) - (2*b^2*d*n^2*x)/(e*g) + (b^2*n^2*(d + e*x)^2)/(4*e^2
*g) + (2*b^2*d*n*(d + e*x)*Log[c*(d + e*x)^n])/(e^2*g) - (b*n*(d + e*x)^2*
(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g) - (d*(d + e*x)*(a + b*Log[c*(d + e*x
)^n])^2)/(e^2*g) + ((d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2*g) -
(f*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f]
 + d*Sqrt[g])])/(2*g^2) - (f*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f]
 + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^2) - (b*f*n*(a + b*Log[c*(d
 + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g
^2 - (b*f*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*S
qrt[-f] + d*Sqrt[g])])/g^2 + (b^2*f*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(
e*Sqrt[-f] - d*Sqrt[g]))])/g^2 + (b^2*f*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))
/(e*Sqrt[-f] + d*Sqrt[g])])/g^2
```

3.311. $\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.311.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.311.4 Maple [F]

$$\int \frac{x^3(a+b\ln(c(ex+d)^n))^2}{g\,x^2+f}dx$$

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

output
```
int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

3.311.5 Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^3}{gx^2+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*log((e*x + d)^n*c)^2 + 2*a*b*x^3*log((e*x + d)^n*c) + a^
2*x^3)/(g*x^2 + f), x)
```

3.311.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.311.7 Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^3}{gx^2+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*a^2*(x^2/g - f*log(g*x^2 + f)/g^2) + integrate((b^2*x^3*log((e*x + d)^
n)^2 + 2*(b^2*log(c) + a*b)*x^3*log((e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*l
og(c))*x^3)/(g*x^2 + f), x)
```

3.311.8 Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^3}{gx^2+f}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^3/(g*x^2 + f), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

input
```
int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
```

output
```
int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

3.312 $\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2} dx$

3.312.1 Optimal result

Integrand size = 27, antiderivative size = 317

$$\begin{aligned}\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2} dx &= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} \\ &+ \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} \\ &+ \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} \\ &+ \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g} \\ &- \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} \\ &- \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g}\end{aligned}$$

output
```
1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^
(1/2)))/g+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^
(1/2)-d*g^(1/2)))/g+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(
e*(-f)^(1/2)-d*g^(1/2)))/g+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(
1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g-b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f
)^(1/2)-d*g^(1/2)))/g-b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^
(1/2)))/g
```

3.312.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx^2)+2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\mathrm{lo}\right.}{}$$

input
```
Integrate[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a
 - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*(Log[1 - (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + Log[1 - (Sqrt[g]*(d + e*x))/(
I*e*Sqrt[f] + d*Sqrt[g])]) + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f
] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])
]) + b^2*n^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] +
 d*Sqrt[g])] + Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d
*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f]
 + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f
] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqr
t[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/(2*
g)
```

3.312.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$\Big\downarrow \ 2863$$

$$\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g}+\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g}+\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g}-\frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g}-\frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g}$$

input
```
Int[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/(2*g) + ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqr
t[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g) + (b*n*(a + b*Log[c*(d + e*x)^n]
)*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g + (b*n*(a
 + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sq
rt[g])])/g - (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqr
t[g]))])/g - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[
g])])/g
```

3.312.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.312.4 Maple [F]

$$\int \frac{x(a+b\ln (c(ex+d)^{n}))^{2}}{g\,x^{2}+f}dx$$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

output
```
int(x*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

3.312.5 Fricas [F]

$$\int \frac{x(a+b\log (c(d+ex)^{n}))^{2}}{f+gx^{2}}\,dx=\int \frac{(b\log ((ex+d)^{n}c)+a)^{2}x}{gx^{2}+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*x*log((e*x + d)^n*c)^2 + 2*a*b*x*log((e*x + d)^n*c) + a^2*x)
/(g*x^2 + f), x)
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log (c(d+ex)^{n}))^{2}}{f+gx^{2}}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.312.7 Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x}{gx^2+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*a^2*log(g*x^2 + f)/g + integrate((b^2*x*log((e*x + d)^n)^2 + 2*(b^2*lo
g(c) + a*b)*x*log((e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x)/(g*x^2 +
 f), x)
```

3.312.8 Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x}{gx^2+f}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x/(g*x^2 + f), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

input
```
int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
```

output
```
int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

3.313 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx$

3.313.1 Optimal result

Integrand size = 29, antiderivative size = 397

$$\begin{aligned}
\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx &= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f} \\
&- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} \\
&- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} \\
&- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} \\
&- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} \\
&+ \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f} \\
&+ \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} \\
&+ \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f}
\end{aligned}$$

output
```
ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/f-1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f
)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f-1/2*(a+b*ln(c*(e*x+d)^n))^2
*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f+2*b*n*(a+b*ln(c*(
e*x+d)^n))*polylog(2,1+e*x/d)/f-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+
d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2
,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f-2*b^2*n^2*polylog(3,1+e*x/d)/
f+b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f+b^2*n^2*p
olylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f
```

3.313.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.45

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx =$$
$$-\frac{-2\log(x)\,(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)),x]
```

output
```
-1/2*(-2*Log[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + (a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x
))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x)
)/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt
[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g
])] - 2*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, 1 + (e*x)/d])) + b^2*n^
2*(-2*Log[-((e*x)/d)]*Log[d + e*x]^2 + Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d
+ e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d
 + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(
d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[
g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 4*Log[d + e*x]*PolyLog[2, 1 + (
e*x)/d] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] -
 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 4*PolyLog[3
, 1 + (e*x)/d]))/f
```

3.313.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{fx}-\frac{gx(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{f}-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{f}-\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2f}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2f}+\frac{2bn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{f}+\frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f}+\frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f}+\frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f}-\frac{2b^2n^2\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)}{f}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)),x]
```

output
```
(Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^2)/f - ((a + b*Log[c*(d + e*x)
^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f) - (
(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] -
d*Sqrt[g])])/(2*f) - (b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f - (b*n*(a + b*Log[c*(d + e*x)^n]
)*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f + (2*b*n*(a
+ b*Log[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/f + (b^2*n^2*PolyLog[3, -
((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f + (b^2*n^2*PolyLog[3, (
Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f - (2*b^2*n^2*PolyLog[3, 1
+ (e*x)/d])/f
```

3.313. $\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}dx$

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.313.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x\left(g\,x^{2}+f\right)}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f),x)
```

3.313.5 Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x\left(f+gx^{2}\right)}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}}{(gx^{2}+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^
3 + f*x), x)
```

3.313.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x/(g*x**2+f),x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))**2/(x*(f + g*x**2)), x)
```

3.313.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f),x, algorithm="maxima")
```

output
```
-1/2*a^2*(log(g*x^2 + f)/f - 2*log(x)/f) + integrate((b^2*log((e*x + d)^n)
^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/
(g*x^3 + f*x), x)
```

3.313.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}{x\,\left(g\,x^2+f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)), x)
```

3.314 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx$

3.314.1 Optimal result

Integrand size = 29, antiderivative size = 551

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \frac{b^2e^2n^2\log(x)}{d^2f} - \frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2fx} - \frac{(a+b\log(c(d+ex)^n))^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{be^2n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d^2f} + \frac{b^2e^2n^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2f} + \frac{bgn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} + \frac{bgn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2bgn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2} - \frac{b^2gn^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} - \frac{b^2gn^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} + \frac{2b^2gn^2\,\mathrm{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f^2}$$

output
```
b^2*e^2*n^2*ln(x)/d^2/f-b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/d^2/f/x-1/2*(a
+b*ln(c*(e*x+d)^n))^2/f/x^2-g*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/f^2-b*e^2
*n*(a+b*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^2/f+1/2*g*(a+b*ln(c*(e*x+d)^n))
^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^2+1/2*g*(a+b*ln
(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f^2
+b^2*e^2*n^2*polylog(2,d/(e*x+d))/d^2/f-2*b*g*n*(a+b*ln(c*(e*x+d)^n))*poly
log(2,1+e*x/d)/f^2+b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/
(e*(-f)^(1/2)-d*g^(1/2)))/f^2+b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d
)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2+2*b^2*g*n^2*polylog(3,1+e*x/d)/f^2
-b^2*g*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2-b^2*g*
n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2
```

3.314.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx$$

$$= \frac{-d^2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2-2d^2gx^2\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)),x]
```

output
```
(-(d^2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2) - 2*d^2*g*x^2*Lo
g[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + d^2*g*x^2*(a - b*n*
Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] - 2*b*n*(a - b*n*Log
[d + e*x] + b*Log[c*(d + e*x)^n])*(f*(d*e*x + e^2*x^2*Log[x] + (d^2 - e^2*
x^2)*Log[d + e*x]) - d^2*g*x^2*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x
))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqr
t[f] - I*d*Sqrt[g])]) - d^2*g*x^2*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g
]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqr
t[f] + I*d*Sqrt[g])]) + 2*d^2*g*x^2*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLo
g[2, 1 + (e*x)/d])) + b^2*n^2*(f*(2*e^2*x^2*Log[x] - Log[d + e*x]*(2*e^2*x
^2*Log[-((e*x)/d)] + (d + e*x)*(2*e*x + (d - e*x)*Log[d + e*x])) - 2*e^2*x
^2*PolyLog[2, 1 + (e*x)/d]) + d^2*g*x^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(
d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[
g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e
*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + d^2*g*x^2*(Log[d + e*x]^2*Log[1 - (S
qrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (
Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d +
 e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]) - 2*d^2*g*x^2*(Log[-((e*x)/d)]*Log[d +
e*x]^2 + 2*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] - 2*PolyLog[3, 1 + (e*x)/d
])))/(2*d^2*f^2*x^2)
```

3.314.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{g^2x(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}-\frac{g(a+b\log(c(d+ex)^n))^2}{f^2x}+\frac{(a+b\log(c(d+ex)^n))^2}{fx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{be^2n\log\left(1-\frac{d}{d+ex}\right)(a+b\log(c(d+ex)^n))}{d^2f}-\frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2fx}+\frac{bgn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{f^2}+\frac{bgn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{f^2}-\frac{2bgn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{f^2}-\frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}+\frac{g\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2f^2}+\frac{g\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2f^2}-\frac{(a+b\log(c(d+ex)^n))^2}{2fx^2}+\frac{b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2f}+\frac{b^2e^2n^2\log(x)}{d^2f}-\frac{b^2gn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}-\frac{b^2gn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^2}+\frac{2b^2gn^2\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)}{f^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)),x]
```

output
```
(b^2*e^2*n^2*Log[x])/(d^2*f) - (b*e*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])
)/(d^2*f*x) - (a + b*Log[c*(d + e*x)^n])^2/(2*f*x^2) - (g*Log[-((e*x)/d)]*
(a + b*Log[c*(d + e*x)^n])^2)/f^2 + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e
*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2) + (g*(a + b*Lo
g[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g]
)])/(2*f^2) - (b*e^2*n*(a + b*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/(d
^2*f) + (b^2*e^2*n^2*PolyLog[2, d/(d + e*x)])/(d^2*f) + (b*g*n*(a + b*Log[
c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/f^2 + (b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/
(e*Sqrt[-f] + d*Sqrt[g])])/f^2 - (2*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyL
og[2, 1 + (e*x)/d])/f^2 - (b^2*g*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*S
qrt[-f] - d*Sqrt[g]))])/f^2 - (b^2*g*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e
*Sqrt[-f] + d*Sqrt[g])])/f^2 + (2*b^2*g*n^2*PolyLog[3, 1 + (e*x)/d])/f^2
```

3.314. $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx$

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.314.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{x^3(g\,x^2+f)}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x)
```

3.314.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^
5 + f*x^3), x)
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x**3/(g*x**2+f),x)
```

output
```
Timed out
```

3.314.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*a^2*(g*log(g*x^2 + f)/f^2 - 2*g*log(x)/f^2 - 1/(f*x^2)) + integrate((b
^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)
*log((e*x + d)^n))/(g*x^5 + f*x^3), x)
```

3.314.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x^3), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx=\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}{x^3\,\left(g\,x^2+f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)), x)
```

3.315 $\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.315.1 Optimal result

Integrand size = 29, antiderivative size = 701

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{2abfnx}{g^2} - \frac{2b^2fn^2x}{g^2} + \frac{2b^2d^2n^2x}{e^2g} - \frac{b^2dn^2(d+ex)^2}{2e^3g} + \frac{2b^2n^2(d+ex)^3}{27e^3g}$$
$$- \frac{b^2d^3n^2\log^2(d+ex)}{3e^3g} + \frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{eg^2}$$
$$- \frac{2bd^2n(d+ex)\left(a+b\log(c(d+ex)^n)\right)}{e^3g} + \frac{bdn(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)}{e^3g}$$
$$- \frac{2bn(d+ex)^3\left(a+b\log(c(d+ex)^n)\right)}{9e^3g} + \frac{2bd^3n\log(d+ex)\left(a+b\log(c(d+ex)^n)\right)}{3e^3g}$$
$$+ \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g} - \frac{f(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{eg^2}$$
$$+ \frac{(-f)^{3/2}\left(a+b\log(c(d+ex)^n)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$
$$- \frac{(-f)^{3/2}\left(a+b\log(c(d+ex)^n)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}$$
$$- \frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}$$
$$+ \frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}$$
$$+ \frac{b^2(-f)^{3/2}n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} - \frac{b^2(-f)^{3/2}n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}$$

output
```
2*a*b*f*n*x/g^2-2*b^2*f*n^2*x/g^2+2*b^2*d^2*n^2*x/e^2/g-1/2*b^2*d*n^2*(e*x
+d)^2/e^3/g+2/27*b^2*n^2*(e*x+d)^3/e^3/g-1/3*b^2*d^3*n^2*ln(e*x+d)^2/e^3/g
+2*b^2*f*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2-2*b*d^2*n*(e*x+d)*(a+b*ln(c*(e*x+
d)^n))/e^3/g+b*d*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^3/g-2/9*b*n*(e*x+d)^3
*(a+b*ln(c*(e*x+d)^n))/e^3/g+2/3*b*d^3*n*ln(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e
^3/g+1/3*x^3*(a+b*ln(c*(e*x+d)^n))^2/g-f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e
/g^2+1/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e
*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)-1/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2*ln(
e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)-b*(-f)^(3/2)*n*
(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))
/g^(5/2)+b*(-f)^(3/2)*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e
*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)+b^2*(-f)^(3/2)*n^2*polylog(3,-(e*x+d)*g^(1
/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)-b^2*(-f)^(3/2)*n^2*polylog(3,(e*x+d)
*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)
```

3.315.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{-54e^3f\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+18e^3g^{3/2}x^3(a-bn\log(d+ex)+b\log(c(d+ex)^n)}{}$$

input
```
Integrate[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(-54*e^3*f*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 18*
e^3*g^(3/2)*x^3*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 54*e^3*f
^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*
x)^n])^2 + 6*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(-18*e^2*f*
Sqrt[g]*(d + e*x)*(-1 + Log[d + e*x]) + g^(3/2)*(e*x*(-6*d^2 + 3*d*e*x - 2
*e^2*x^2) + 6*(d^3 + e^3*x^3)*Log[d + e*x]) + (9*I)*e^3*f^(3/2)*(Log[d + e
*x]*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2,
 (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - (9*I)*e^3*f^(3/2)*(L
og[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])) + b^2*n^2*(-54*e^2*
f*Sqrt[g]*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2) +
g^(3/2)*(e*x*(66*d^2 - 15*d*e*x + 4*e^2*x^2) - 6*(11*d^3 + 6*d^2*e*x - 3*d
*e^2*x^2 + 2*e^3*x^3)*Log[d + e*x] + 18*(d^3 + e^3*x^3)*Log[d + e*x]^2) +
(27*I)*e^3*f^(3/2)*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqr
t[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e
*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f]
+ d*Sqrt[g])]) - (27*I)*e^3*f^(3/2)*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d +
e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d +
 e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*
Sqrt[f] + d*Sqrt[g])])))/(54*e^3*g^(5/2))
```

3.315.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{f^2(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{x^2(a+b\log(c(d+ex)^n))^2}{g}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bd^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g}-\frac{2bd^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3g}+\frac{bdn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3g}-\frac{2bn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3g}-\frac{b(-f)^{3/2}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^{5/2}}+\frac{b(-f)^{3/2}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^{5/2}}+\frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^{5/2}}-\frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^{5/2}}-\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}+\frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}+\frac{2abfnx}{g^2}+\frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{b^2d^3n^2\log^2(d+ex)}{3e^3g}+\frac{2b^2d^2n^2x}{e^2g}-\frac{b^2dn^2(d+ex)^2}{2e^3g}+\frac{2b^2n^2(d+ex)^3}{27e^3g}+\frac{b^2(-f)^{3/2}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}-\frac{b^2(-f)^{3/2}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^{5/2}}-\frac{2b^2fn^2x}{g^2}$$

input
```
Int[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(2*a*b*f*n*x)/g^2 - (2*b^2*f*n^2*x)/g^2 + (2*b^2*d^2*n^2*x)/(e^2*g) - (b^2
*d*n^2*(d + e*x)^2)/(2*e^3*g) + (2*b^2*n^2*(d + e*x)^3)/(27*e^3*g) - (b^2*
d^3*n^2*Log[d + e*x]^2)/(3*e^3*g) + (2*b^2*f*n*(d + e*x)*Log[c*(d + e*x)^n
])/(e*g^2) - (2*b*d^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/(e^3*g) + (b
*d*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(e^3*g) - (2*b*n*(d + e*x)^3*
(a + b*Log[c*(d + e*x)^n]))/(9*e^3*g) + (2*b*d^3*n*Log[d + e*x]*(a + b*Log
[c*(d + e*x)^n]))/(3*e^3*g) + (x^3*(a + b*Log[c*(d + e*x)^n])^2)/(3*g) - (
f*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) + ((-f)^(3/2)*(a + b*Log
[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*g^(5/2)) - ((-f)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f]
 + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(5/2)) - (b*(-f)^(3/2)*n*(a
 + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*
Sqrt[g]))])/g^(5/2) + (b*(-f)^(3/2)*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2
, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^(5/2) + (b^2*(-f)^(3/2)
*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^(5/2)
- (b^2*(-f)^(3/2)*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[
g])])/g^(5/2)
```

3.315. $\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.315.4 Maple [F]

$$\int \frac{x^4(a+b\ln(c(ex+d)^n))^2}{g\,x^2+f}dx$$

input
```
int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

output
```
int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

3.315.5 Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^4}{gx^2+f}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*x^4*log((e*x + d)^n*c)^2 + 2*a*b*x^4*log((e*x + d)^n*c) + a^
2*x^4)/(g*x^2 + f), x)
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.315.7 Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^4}{gx^2+f}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/3*a^2*(3*f^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2) + (g*x^3 - 3*f*x)/g^2
) + integrate((b^2*x^4*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^4*log((
e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^4)/(g*x^2 + f), x)
```

3.315.8 Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^4}{gx^2+f}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^4/(g*x^2 + f), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

input
```
int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
```

output
```
int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

3.316 $\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.316.1 Optimal result

Integrand size = 29, antiderivative size = 447

$$\begin{aligned}
&\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx \\
&= -\frac{2abnx}{g}+\frac{2b^2n^2x}{g}-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg} \\
&\quad+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad-\frac{b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}} \\
&\quad+\frac{b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}} \\
&\quad+\frac{b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}}-\frac{b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}}
\end{aligned}$$

output
```
-2*a*b*n*x/g+2*b^2*n^2*x/g-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g+(e*x+d)*(a+
b*ln(c*(e*x+d)^n))^2/e/g+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^
(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)-1/2*(a+b*ln(c*(e*x+d)^
n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(
3/2)-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*
g^(1/2)))*(-f)^(1/2)/g^(3/2)+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g
^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)+b^2*n^2*polylog(3,-(e*
x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(3/2)-b^2*n^2*polylog(
3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)
```

3.316.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.39

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{e\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2-e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+ex}{}$$

input
```
Integrate[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(e*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - e*Sqrt[f]*A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2
 + I*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((-2*I)*Sqrt[g]*(d
+ e*x)*(-1 + Log[d + e*x]) - e*Sqrt[f]*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d +
 e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I
)*e*Sqrt[f] + d*Sqrt[g])]) + e*Sqrt[f]*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d +
 e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sq
rt[f] + d*Sqrt[g])])) + b^2*n^2*(Sqrt[g]*(2*e*x - 2*(d + e*x)*Log[d + e*x]
 + (d + e*x)*Log[d + e*x]^2) - (I/2)*e*Sqrt[f]*(Log[d + e*x]^2*Log[1 - (Sq
rt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2,
 (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]
*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + (I/2)*e*Sqrt[f]*(Log[d + e*x]
^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]
*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3,
(Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(e*g^(3/2))
```

3.316.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{g}-\frac{f(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^{3/2}}+\frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^{3/2}}+\frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g^{3/2}}-\frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g^{3/2}}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}-\frac{2abnx}{g}-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg}+\frac{b^2\sqrt{-f}n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}}-\frac{b^2\sqrt{-f}n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^{3/2}}+\frac{2b^2n^2x}{g}$$

input
```
Int[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

output
```
(-2*a*b*n*x)/g + (2*b^2*n^2*x)/g - (2*b^2*n*(d + e*x)*Log[c*(d + e*x)^n])/
(e*g) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g) + (Sqrt[-f]*(a + b*
Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[
g])])/(2*g^(3/2)) - (Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f
] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(3/2)) - (b*Sqrt[-f]*n*(a
+ b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*S
qrt[g]))])/g^(3/2) + (b*Sqrt[-f]*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (
Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^(3/2) + (b^2*Sqrt[-f]*n^2*
PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^(3/2) - (b^
2*Sqrt[-f]*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g
^(3/2)
```

3.316.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.316.4 Maple [F]

$$\int \frac{x^2(a + b\ln(c(ex + d)^n))^2}{g\,x^2 + f}dx$$

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

output
```
int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

3.316.5 Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^2}{gx^2+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*log((e*x + d)^n*c)^2 + 2*a*b*x^2*log((e*x + d)^n*c) + a^
2*x^2)/(g*x^2 + f), x)
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.316.7 Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^2}{gx^2+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
-a^2*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + integrate((b^2*x^2*lo
g((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log((e*x + d)^n) + (b^2*log(c)
^2 + 2*a*b*log(c))*x^2)/(g*x^2 + f), x)
```

3.316.8 Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^2}{gx^2+f}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^2/(g*x^2 + f), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

input
```
int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
```

output
```
int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

3.317 $\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

3.317.1 Optimal result

Integrand size = 26, antiderivative size = 371

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx =& \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ &- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ &- \frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} \\ &+ \frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} \\ &+ \frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} \\ &- \frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}}\end{aligned}$$

output
```
1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^
(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*
g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-b*n*(a+b*ln(c*(e*x+d
)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1
/2)+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^
(1/2)))/(-f)^(1/2)/g^(1/2)+b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2
)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)
^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+ibn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\mathrm{lo}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])
^2 + I*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*(Lo
g[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - Log[1 - (Sqrt[g]
*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]) + PolyLog[2, (Sqrt[g]*(d + e*x))/(
(-I)*e*Sqrt[f] + d*Sqrt[g])] - PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f]
 + d*Sqrt[g])]) + (I/2)*b^2*n^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x)
)/((-I)*e*Sqrt[f] + d*Sqrt[g])] - Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*
x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d
+ e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I
)*e*Sqrt[f] + d*Sqrt[g])] + 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f]
+ d*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

3.317.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{-f}\sqrt{g}}+\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{-f}\sqrt{g}}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2\sqrt{-f}\sqrt{g}}+\frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}}-\frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{\sqrt{-f}\sqrt{g}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2),x]
```

output
```
((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*
(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) -
(b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] - d*Sqrt[g]))])/(Sqrt[-f]*Sqrt[g]) + (b*n*(a + b*Log[c*(d + e*x)^n])*Po
lyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(Sqrt[-f]*Sqrt[g])
 + (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
Sqrt[-f]*Sqrt[g]) - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/(Sqrt[-f]*Sqrt[g])
```

3.317.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.317.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{g\,x^{2}+f}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

3.317.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^{n}))^{2}}{f+gx^{2}}\,dx=\int \frac{(b\log((ex+d)^{n}c)+a)^{2}}{gx^{2}+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^
2 + f), x)
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.317.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx^2+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
a^2*arctan(g*x/sqrt(f*g))/sqrt(f*g) + integrate((b^2*log((e*x + d)^n)^2 +
b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x^
2 + f), x)
```

3.317.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx^2+f}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x^2 + f), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2), x)
```

3.318 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx$

3.318.1 Optimal result

Integrand size = 29, antiderivative size = 461

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx &= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df}\\
&-\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx}\\
&+\frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}\\
&-\frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}\\
&-\frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}}\\
&+\frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}}\\
&+\frac{2b^2en^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{df}\\
&+\frac{b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}}\\
&-\frac{b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}}\end{aligned}$$

output
```
2*b*e*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/d/f-(e*x+d)*(a+b*ln(c*(e*x+d)^n))
^2/d/f/x+2*b^2*e*n^2*polylog(2,1+e*x/d)/d/f+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln
(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)-1/2
*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/
2)))*g^(1/2)/(-f)^(3/2)-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/
2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)+b*n*(a+b*ln(c*(e*x+d)^n))*
polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)+b^2
*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/
2)-b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f
)^(3/2)
```

3.318.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.45

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx$$
$$= \frac{-2d\sqrt{f}(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2-2d\sqrt{g}x\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)),x]
```

output
```
(-2*d*Sqrt[f]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*d*Sqrt[g
]*x*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^
n])^2 + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(2*Sqrt[f]*(e*
x*Log[x] - (d + e*x)*Log[d + e*x]) + I*d*Sqrt[g]*x*(Log[d + e*x]*Log[(e*(S
qrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[
g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - I*d*Sqrt[g]*x*(Log[d + e*x]*Lo
g[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*S
qrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])) + b^2*n^2*(2*Sqrt[f]*(2*e*x
*Log[-((e*x)/d)]*Log[d + e*x] - (d + e*x)*Log[d + e*x]^2 + 2*e*x*PolyLog[2
, 1 + (e*x)/d]) - I*d*Sqrt[g]*x*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x)
)/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d +
e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-
I)*e*Sqrt[f] + d*Sqrt[g])]) + I*d*Sqrt[g]*x*(Log[d + e*x]^2*Log[1 - (Sqrt[
g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt
[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(2*d*f^(3/2)*x)
```

3.318.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{fx^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{3/2}}+\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{3/2}}+\frac{\sqrt{g}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{3/2}}-\frac{\sqrt{g}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{3/2}}+\frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df}-\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx}+\frac{b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}}-\frac{b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{(-f)^{3/2}}+\frac{2b^2en^2\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{df}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)),x]
```

output
```
(2*b*e*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(d*f) - ((d + e*x)*(a
 + b*Log[c*(d + e*x)^n])^2)/(d*f*x) + (Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^
2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2))
 - (Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e
*Sqrt[-f] - d*Sqrt[g])])/(2*(-f)^(3/2)) - (b*Sqrt[g]*n*(a + b*Log[c*(d + e
*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^
(3/2) + (b*Sqrt[g]*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e
*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(3/2) + (2*b^2*e*n^2*PolyLog[2, 1 + (
e*x)/d])/(d*f) + (b^2*Sqrt[g]*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt
[-f] - d*Sqrt[g]))])/(-f)^(3/2) - (b^2*Sqrt[g]*n^2*PolyLog[3, (Sqrt[g]*(d
+ e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(3/2)
```

3.318.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.318.4 Maple [F]

$$\int \frac{(a+b\ln (c(ex+d)^{n}))^{2}}{x^{2}(g\,x^{2}+f)}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x)
```

3.318.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^
4 + f*x^2), x)
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x**2/(g*x**2+f),x)
```

output
```
Timed out
```

3.318.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
-a^2*(g*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f) + 1/(f*x)) + integrate((b^2*lo
g((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(
(e*x + d)^n))/(g*x^4 + f*x^2), x)
```

3.318.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x^2), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = \int \frac{(a+b\ln(c(d+ex)^n))^2}{x^2(gx^2+f)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)), x)
```

3.319 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx$

3.319.1 Optimal result

Integrand size = 29, antiderivative size = 694

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = -\frac{b^2e^2n^2}{3d^2fx} - \frac{b^2e^3n^2\log(x)}{d^3f} + \frac{b^2e^3n^2\log(d+ex)}{3d^3f} - \frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2} + \frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3d^3fx} - \frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3f} - \frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3f} - \frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{2b^2egn^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{df^2} + \frac{b^2g^{3/2}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{b^2g^{3/2}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}$$

output
```
-1/3*b^2*e^2*n^2/d^2/f/x-b^2*e^3*n^2*ln(x)/d^3/f+1/3*b^2*e^3*n^2*ln(e*x+d)
/d^3/f-1/3*b*e*n*(a+b*ln(c*(e*x+d)^n))/d/f/x^2+2/3*b*e^2*n*(e*x+d)*(a+b*ln
(c*(e*x+d)^n))/d^3/f/x-2*b*e*g*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/d/f^2-1/
3*(a+b*ln(c*(e*x+d)^n))^2/f/x^3+g*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/d/f^2/x+
2/3*b*e^3*n*(a+b*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^3/f+1/2*g^(3/2)*(a+b*l
n(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(-
f)^(5/2)-1/2*g^(3/2)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(
e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(5/2)-2/3*b^2*e^3*n^2*polylog(2,d/(e*x+d))/d
^3/f-2*b^2*e*g*n^2*polylog(2,1+e*x/d)/d/f^2-b*g^(3/2)*n*(a+b*ln(c*(e*x+d)^
n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(5/2)+b*g^(3
/2)*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1
/2)))/(-f)^(5/2)+b^2*g^(3/2)*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-
d*g^(1/2)))/(-f)^(5/2)-b^2*g^(3/2)*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(
1/2)+d*g^(1/2)))/(-f)^(5/2)
```

3.319.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx$$
$$= \frac{-2d^3f^{3/2}(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+6d^3\sqrt{f}gx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^4*(f + g*x^2)),x]
```

output
```
(-2*d^3*f^(3/2)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 6*d^3*Sq
rt[f]*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 6*d^3*g^(3/2
)*x^3*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x
)^n])^2 + (2*I)*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((6*I)*d
^2*Sqrt[f]*g*x^2*(e*x*Log[x] - (d + e*x)*Log[d + e*x]) + I*f^(3/2)*(d*e*x*
(d - 2*e*x) - 2*e^3*x^3*Log[x] + 2*(d^3 + e^3*x^3)*Log[d + e*x]) - 3*d^3*g
^(3/2)*x^3*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*
Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]
) + 3*d^3*g^(3/2)*x^3*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqr
t[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*S
qrt[g])])) + I*b^2*n^2*((6*I)*d^2*Sqrt[f]*g*x^2*(2*e*x*Log[-((e*x)/d)]*Log
[d + e*x] - (d + e*x)*Log[d + e*x]^2 + 2*e*x*PolyLog[2, 1 + (e*x)/d]) + (2
*I)*f^(3/2)*(d*e^2*x^2 + 3*e^3*x^3*Log[x] + d^2*e*x*Log[d + e*x] - 2*d*e^2
*x^2*Log[d + e*x] - 3*e^3*x^3*Log[d + e*x] - 2*e^3*x^3*Log[-((e*x)/d)]*Log
[d + e*x] + d^3*Log[d + e*x]^2 + e^3*x^3*Log[d + e*x]^2 - 2*e^3*x^3*PolyLo
g[2, 1 + (e*x)/d]) + 3*d^3*g^(3/2)*x^3*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d
 + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*
x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - 3*d^3*g^(3/2)*x^3*(Log[d + e*x]^2*Log
[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*Po...
```

3.319.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{g^2(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}-\frac{g(a+b\log(c(d+ex)^n))^2}{f^2x^2}+\frac{(a+b\log(c(d+ex)^n))^2}{fx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2be^3n\log\left(1-\frac{d}{d+ex}\right)(a+b\log(c(d+ex)^n))}{3d^3f}+\frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3d^3fx}-\frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2}+\frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x}-\frac{bg^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{5/2}}+\frac{bg^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{5/2}}+\frac{g^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{5/2}}-\frac{g^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{5/2}}-\frac{(a+b\log(c(d+ex)^n))^2}{3fx^3}-\frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2}-\frac{2b^2e^3n^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3f}-\frac{b^2e^3n^2\log(x)}{d^3f}+\frac{b^2e^3n^2\log(d+ex)}{3d^3f}-\frac{b^2e^2n^2}{3d^2fx}-\frac{2b^2egn^2\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{df^2}+\frac{b^2g^{3/2}n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}-\frac{b^2g^{3/2}n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{(-f)^{5/2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x^4*(f + g*x^2)),x]
```

output
```
-1/3*(b^2*e^2*n^2)/(d^2*f*x) - (b^2*e^3*n^2*Log[x])/(d^3*f) + (b^2*e^3*n^2
*Log[d + e*x])/(3*d^3*f) - (b*e*n*(a + b*Log[c*(d + e*x)^n]))/(3*d*f*x^2)
+ (2*b*e^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/(3*d^3*f*x) - (2*b*e*g*
n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(d*f^2) - (a + b*Log[c*(d +
e*x)^n])^2/(3*f*x^3) + (g*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(d*f^2*x
) + (g^(3/2)*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(
e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(5/2)) - (g^(3/2)*(a + b*Log[c*(d + e*x)
^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*(-f)^(
5/2)) + (2*b*e^3*n*(a + b*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/(3*d^3
*f) - (2*b^2*e^3*n^2*PolyLog[2, d/(d + e*x)])/(3*d^3*f) - (b*g^(3/2)*n*(a
+ b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*S
qrt[g]))])/(-f)^(5/2) + (b*g^(3/2)*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2,
 (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2) - (2*b^2*e*g*n^
2*PolyLog[2, 1 + (e*x)/d])/(d*f^2) + (b^2*g^(3/2)*n^2*PolyLog[3, -((Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(5/2) - (b^2*g^(3/2)*n^2*Pol
yLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2)
```

3.319. $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx$

3.319.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.319.4 Maple [F]

$$\int \frac{\left(a+b\ln \left(c (e x+d)^{n}\right)\right)^{2}}{x^{4} \left(g\,x^{2}+f \right)}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x)
```

3.319.5 Fricas [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{x^4\,(f+gx^2)}\,dx=\int \frac{(b\log{((ex+d)^nc)}+a)^2}{(gx^2+f)x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^
6 + f*x^4), x)
```

3.319.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x**4/(g*x**2+f),x)
```

output
```
Timed out
```

3.319.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/3*a^2*(3*g^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f^2) + (3*g*x^2 - f)/(f^2*
x^3)) + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) +
2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x^6 + f*x^4), x)
```

3.319.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x^4), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \int \frac{\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2}{x^4\,\left(g\,x^2+f\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x^4*(f + g*x^2)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x^4*(f + g*x^2)), x)
```

3.320 $\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.320.1 Optimal result

Integrand size = 29, antiderivative size = 936

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= \frac{2abdnx}{eg^2} - \frac{2b^2dn^2x}{eg^2} + \frac{b^2n^2(d+ex)^2}{4e^2g^2} + \frac{2b^2dn(d+ex)\log(c(d+ex)^n)}{e^2g^2}$$
$$- \frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g^2}$$
$$+ \frac{e^2f^2(a+b\log(c(d+ex)^n))^2}{2g^3(e^2f+d^2g)} - \frac{d(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g^2}$$
$$+ \frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g^2} - \frac{f^2(a+b\log(c(d+ex)^n))^2}{2g^3(f+gx^2)}$$
$$- \frac{bef\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)}$$
$$- \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$- \frac{be(-f)^{3/2}\left(e\sqrt{-f}+d\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)}$$
$$- \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$- \frac{b^2e(-f)^{3/2}\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)}$$
$$- \frac{2bfn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$- \frac{b^2e(-f)^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)}$$
$$- \frac{2bfn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$+ \frac{2b^2fn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} + \frac{2b^2fn^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

output
```
2*a*b*d*n*x/e/g^2-2*b^2*d*n^2*x/e/g^2+1/4*b^2*n^2*(e*x+d)^2/e^2/g^2+2*b^2*
d*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g^2-1/2*b*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n)
)/e^2/g^2+1/2*e^2*f^2*(a+b*ln(c*(e*x+d)^n))^2/g^3/(d^2*g+e^2*f)-d*(e*x+d)*
(a+b*ln(c*(e*x+d)^n))^2/e^2/g^2+1/2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2/
g^2-1/2*f^2*(a+b*ln(c*(e*x+d)^n))^2/g^3/(g*x^2+f)-f*(a+b*ln(c*(e*x+d)^n))^
2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^3-f*(a+b*ln(c*(e
*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^3-2*b*
f*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/
2)))/g^3-2*b*f*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(
1/2)+d*g^(1/2)))/g^3+2*b^2*f*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-
d*g^(1/2)))/g^3+2*b^2*f*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1
/2)))/g^3-1/2*b^2*e*(-f)^(3/2)*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)
+d*g^(1/2)))*(e*(-f)^(1/2)-d*g^(1/2))/g^3/(d^2*g+e^2*f)-1/2*b*e*(-f)^(3/2)
*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/
2)))*(e*(-f)^(1/2)+d*g^(1/2))/g^3/(d^2*g+e^2*f)-1/2*b^2*e*(-f)^(3/2)*n^2*p
olylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(e*(-f)^(1/2)+d*g^(1/2
))/g^3/(d^2*g+e^2*f)-1/2*b*e*f*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*
g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/g^3/(d^2*g+e
^2*f)
```

3.320.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.34

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{2gx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2-\frac{2f^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2}-4f(a-bn\log(d+ex)\cdots}{}$$

input
```
Integrate[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
(2*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - (2*f^2*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 4*f*(a - b*n*Log[d
+ e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[d + e
*x] + b*Log[c*(d + e*x)^n])*((g*(e*x*(2*d - e*x) - 2*(d^2 - e^2*x^2)*Log[d
 + e*x]))/e^2 + (f^(3/2)*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] - e*(Sqrt[f] +
I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt
[f] + I*Sqrt[g]*x)) + (I*f^(3/2)*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*(I
*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x)) - 4*f*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[
g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e
*Sqrt[f] - I*d*Sqrt[g])]) - 4*f*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*
x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[
f] + I*d*Sqrt[g])])) + b^2*n^2*((g*(e*x*(-6*d + e*x) + (6*d^2 + 4*d*e*x -
2*e^2*x^2)*Log[d + e*x] - 2*(d^2 - e^2*x^2)*Log[d + e*x]^2))/e^2 + (I*f^(3
/2)*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log
[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e
*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d
*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - (f^(3/2
)*(Log[d + e*x]*((-I)*Sqrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] + I*Sq
rt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + ...
```

3.320.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{f^2x(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)^2}-\frac{2fx(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}+\frac{x(a+b\log(c(d+ex)^n))^2}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{n^2(d+ex)^2b^2}{4e^2g^2}-\frac{2dn^2xb^2}{eg^2}+\frac{2dn(d+ex)\log\left(c(d+ex)^n\right)b^2}{e^2g^2}-\frac{e(-f)^{3/2}\left(\sqrt{g}d+e\sqrt{-f}\right)n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{2g^3\left(gd^2+e^2f\right)}-\frac{e(-f)^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{2g^3\left(gd^2+e^2f\right)}+\frac{2fn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{g^3}+\frac{2fn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{g^3}+\frac{2adnxb}{eg^2}-\frac{n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)b}{2e^2g^2}-\frac{ef\left(\sqrt{-f}\sqrt{g}d+ef\right)n(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{2g^3\left(gd^2+e^2f\right)}-\frac{e(-f)^{3/2}\left(\sqrt{g}d+e\sqrt{-f}\right)n(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)b}{2g^3\left(gd^2+e^2f\right)}-\frac{2fn(a+b\log\left(c(d+ex)^n\right))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b}{g^3}-\frac{2fn(a+b\log\left(c(d+ex)^n\right))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{g^3}+\frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2}-\frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2}+\frac{e^2f^2(a+b\log\left(c(d+ex)^n\right))^2}{2g^3\left(gd^2+e^2f\right)}-\frac{f^2(a+b\log\left(c(d+ex)^n\right))^2}{2g^3\left(gx^2+f\right)}-\frac{f(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3}-\frac{f(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$

input
```
Int[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

3.320. $\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

output
```
(2*a*b*d*n*x)/(e*g^2) - (2*b^2*d*n^2*x)/(e*g^2) + (b^2*n^2*(d + e*x)^2)/(4
*e^2*g^2) + (2*b^2*d*n*(d + e*x)*Log[c*(d + e*x)^n])/(e^2*g^2) - (b*n*(d +
 e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g^2) + (e^2*f^2*(a + b*Log[c*(d
 + e*x)^n])^2)/(2*g^3*(e^2*f + d^2*g)) - (d*(d + e*x)*(a + b*Log[c*(d + e*
x)^n])^2)/(e^2*g^2) + ((d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2*g^
2) - (f^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*g^3*(f + g*x^2)) - (b*e*f*(e*f
+ d*Sqrt[-f]*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt
[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3*(e^2*f + d^2*g)) - (f*(a + b*Log
[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/g^3 - (b*e*(-f)^(3/2)*(e*Sqrt[-f] + d*Sqrt[g])*n*(a + b*Log[c*(d + e*x)
^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^3*(e^2
*f + d^2*g)) - (f*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*
x))/(e*Sqrt[-f] - d*Sqrt[g])])/g^3 - (b^2*e*(-f)^(3/2)*(e*Sqrt[-f] + d*Sqr
t[g])*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*
g^3*(e^2*f + d^2*g)) - (2*b*f*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((S
qrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 - (b^2*e*(-f)^(3/2)*(e*S
qrt[-f] - d*Sqrt[g])*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sq
rt[g])])/(2*g^3*(e^2*f + d^2*g)) - (2*b*f*n*(a + b*Log[c*(d + e*x)^n])*Pol
yLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3 + (2*b^2*f*n^2*
PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 + (2*b...
```

3.320.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.320.4 Maple [F]

$$\int \frac{x^5(a+b\ln\left(c(ex+d)^n\right))^2}{\left(g\,x^2+f\right)^2}dx$$

input
```
int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

output
```
int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

3.320.5 Fricas [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^5}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^5*log((e*x + d)^n*c)^2 + 2*a*b*x^5*log((e*x + d)^n*c) + a^
2*x^5)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.320.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.320.7 Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^5}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(f^2/(g^4*x^2 + f*g^3) - x^2/g^2 + 2*f*log(g*x^2 + f)/g^3) + inte
grate((b^2*x^5*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^5*log((e*x + d)
^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^5)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.320.8 Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^5}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^5/(g*x^2 + f)^2, x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{x^5\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
```

output
```
int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

3.321 $\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.321.1 Optimal result

Integrand size = 29, antiderivative size = 739

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= -\frac{e^2f(a+b\log(c(d+ex)^n))^2}{2g^2(e^2f+d^2g)}+\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}$$
$$+\frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)}$$
$$+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$
$$+\frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)}$$
$$+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}$$
$$-\frac{b^2e\sqrt{-f}\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)}$$
$$+\frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}$$
$$+\frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)}$$
$$+\frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$
$$-\frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}-\frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$

output
```
-1/2*e^2*f*(a+b*ln(c*(e*x+d)^n))^2/g^2/(d^2*g+e^2*f)+1/2*f*(a+b*ln(c*(e*x+
d)^n))^2/g^2/(g*x^2+f)+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1
/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^2+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^
(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+b*n*(a+b*ln(c*(e*x+d)^n))*p
olylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+b*n*(a+b*ln(c*(e*x
+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2-b^2*n^2*po
lylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-b^2*n^2*polylog(3,(
e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2-1/2*b^2*e*n^2*polylog(2,-(e*x
+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)*(e*(-f)^(1/2)+d*g^(1/2))/
g^2/(d^2*g+e^2*f)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/
2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*f-d*(-f)^(1/2)*g^(1/2))/g^2/(d^2*g+e^2*f)
+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)
+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/g^2/(d^2*g+e^2*f)+1/2*b^2*e*n^2*po
lylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2
))/g^2/(d^2*g+e^2*f)
```

3.321.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 1103, normalized size of antiderivative = 1.49

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2}+2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx^2)+2bn(a-bn\log \cdots}{\cdots}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
((2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) + 2*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Sqrt[f]*((-I)*Sqrt[g]*(d + e*x)*
Log[d + e*x] + e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*
Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*(I*Sqrt[g]*(d +
 e*x)*Log[d + e*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x])
)/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 2*(Log[d + e*x]*Lo
g[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I
)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2*(Log[d + e*x]*Log[(e*
(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g
]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])) + b^2*n^2*(2*Log[d + e*x]^2*Log[
1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]^2*L
og[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(Log[d +
e*x]*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] - I*Sqrt[g]*x)*Log[(
e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) + 2*e*(Sqrt[f] - I*
Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/(
(e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 4*Log[d + e*x]*PolyLo
g[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 4*Log[d + e*x]*Po
lyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(Log[d
+ e*x]*((-I)*Sqrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] + I*Sqrt[g]*...
```

3.321.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{x(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}-\frac{fx(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{ben\left(d\sqrt{-f}\sqrt{g}+ef\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^2(d^2g+e^2f)}+\frac{ben\left(ef-d\sqrt{-f}\sqrt{g}\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^2(d^2g+e^2f)}-\frac{e^2f(a+b\log(c(d+ex)^n))^2}{2g^2(d^2g+e^2f)}+\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g^2}+\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g^2}-\frac{b^2e\sqrt{-f}n^2\left(d\sqrt{g}+e\sqrt{-f}\right)\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(d^2g+e^2f)}+\frac{b^2en^2\left(d\sqrt{-f}\sqrt{g}+ef\right)\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^2(d^2g+e^2f)}-\frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}-\frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^2}$$

input
```
Int[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
-1/2*(e^2*f*(a + b*Log[c*(d + e*x)^n])^2)/(g^2*(e^2*f + d^2*g)) + (f*(a +
b*Log[c*(d + e*x)^n])^2)/(2*g^2*(f + g*x^2)) + (b*e*(e*f + d*Sqrt[-f]*Sqrt
[g])*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-
f] + d*Sqrt[g])])/(2*g^2*(e^2*f + d^2*g)) + ((a + b*Log[c*(d + e*x)^n])^2*
Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2) + (b*e*(
e*f - d*Sqrt[-f]*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] +
Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^2*(e^2*f + d^2*g)) + ((a + b*L
og[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g
])])/(2*g^2) - (b^2*e*Sqrt[-f]*(e*Sqrt[-f] + d*Sqrt[g])*n^2*PolyLog[2, -((
Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^2*(e^2*f + d^2*g)) + (
b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f
] - d*Sqrt[g]))])/g^2 + (b^2*e*(e*f + d*Sqrt[-f]*Sqrt[g])*n^2*PolyLog[2, (
Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2*(e^2*f + d^2*g)) + (b
*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/g^2 - (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f]
- d*Sqrt[g]))])/g^2 - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f]
+ d*Sqrt[g])])/g^2
```

3.321. $\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.321.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.321.4 Maple [F]

$$\int \frac{x^3(a+b\ln(c(ex+d)^n))^2}{(g\,x^2+f)^2}dx$$

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

output
```
int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

3.321.5 Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^2x^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^3*log((e*x + d)^n*c)^2 + 2*a*b*x^3*log((e*x + d)^n*c) + a^
2*x^3)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.321.7 Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(f/(g^3*x^2 + f*g^2) + log(g*x^2 + f)/g^2) + integrate((b^2*x^3*lo
g((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^3*log((e*x + d)^n) + (b^2*log(c)
^2 + 2*a*b*log(c))*x^3)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.321.8 Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^3/(g*x^2 + f)^2, x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
```

output
```
int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

3.322 $\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.322.1 Optimal result

Integrand size = 27, antiderivative size = 430

$$\begin{aligned}\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx &= \frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} \\ &- \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fg(e^2f+d^2g)} \\ &- \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fg(e^2f+d^2g)} \\ &- \frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g(e^2f+d^2g)} \\ &- \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fg(e^2f+d^2g)}\end{aligned}$$

output
```
1/2*e^2*(a+b*ln(c*(e*x+d)^n))^2/g/(d^2*g+e^2*f)-1/2*(a+b*ln(c*(e*x+d)^n))^
2/g/(g*x^2+f)-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(
1/2)))*(e*(-f)^(1/2)+d*g^(1/2))/g/(d^2*g+e^2*f)/(-f)^(1/2)-1/2*b*e*n*(a+b*
ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*
f-d*(-f)^(1/2)*g^(1/2))/f/g/(d^2*g+e^2*f)-1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*
ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^
(1/2))/f/g/(d^2*g+e^2*f)-1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(
1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/f/g/(d^2*g+e^2*f)
```

3.322.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.37

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{-\frac{2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2}+\frac{2bn(-a+bn\log(d+ex)-b\log(c(d+ex)^n))\left(2\sqrt{f}g\left(d^2-e^2x^2\right)\log(d+ex)+e\left(f+gx^2\right)\left(\left(e\sqrt{f}+id\sqrt{g}\right)\log\right.\right.}{\sqrt{f}(e^2f+d^2g)(f+gx^2)}}{}$$

input
```
Integrate[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
((-2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) + (2*b*n
*(-a + b*n*Log[d + e*x] - b*Log[c*(d + e*x)^n])*(2*Sqrt[f]*g*(d^2 - e^2*x^
2)*Log[d + e*x] + e*(f + g*x^2)*((e*Sqrt[f] + I*d*Sqrt[g])*Log[I*Sqrt[f] -
 Sqrt[g]*x] + (e*Sqrt[f] - I*d*Sqrt[g])*Log[I*Sqrt[f] + Sqrt[g]*x])))/(Sqr
t[f]*(e^2*f + d^2*g)*(f + g*x^2)) + (I*b^2*n^2*((-(Sqrt[g]*(d + e*x)*Log[d
 + e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*
Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyL
og[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*
Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + (Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d
 + e*x] + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/
(e*Sqrt[f] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (
Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])/((e*Sqrt[f] - I*d*Sqrt[g])*
(Sqrt[f] + I*Sqrt[g]*x))))/Sqrt[f])/(4*g)
```

3.322.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2860, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}dx$$

$$\Big\downarrow \ 2860$$

$$\frac{ben\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(gx^2+f)}dx}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)}$$

$$\Big\downarrow \ 2865$$

$$\frac{ben\int \left(\frac{e^2(a+b\log(c(d+ex)^n))}{(gd^2+e^2f)(d+ex)}-\frac{g(ex-d)(a+b\log(c(d+ex)^n))}{(gd^2+e^2f)(gx^2+f)}\right)dx}{g}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{ben\left(\frac{e(a+b\log(c(d+ex)^n))^2}{2bn(d^2g+e^2f)}-\frac{(d\sqrt{-f}\sqrt{g}+ef)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2f(d^2g+e^2f)}-\frac{(ef-d\sqrt{-f}\sqrt{g})\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+e \cdots}{2f(d^2g+e^2f)}\cdots\right.}{g}$$

$$\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)}$$

input
```
Int[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x)^n])^2/(g*(f + g*x^2)) + (b*e*n*((e*(a + b*Log[
c*(d + e*x)^n])^2)/(2*b*(e^2*f + d^2*g)*n) - ((e*f + d*Sqrt[-f]*Sqrt[g])*(
a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*f*(e^2*f + d^2*g)) - ((e*f - d*Sqrt[-f]*Sqrt[g])*(a + b*Log[c
*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(
2*f*(e^2*f + d^2*g)) - (b*(e*f - d*Sqrt[-f]*Sqrt[g])*n*PolyLog[2, -((Sqrt[
g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*f*(e^2*f + d^2*g)) - (b*(e*f
+ d*Sqrt[-f]*Sqrt[g])*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f*(e^2*f + d^2*g))))/g
```

3.322.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2860
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Simp[b*e*n*(p/(g*r*(q + 1)))
   Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && N
eQ[q, -1] && IGtQ[p, 0]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.322.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 1231, normalized size of antiderivative = 2.86

method	result	size
risch	Expression too large to display	1231

input
```
int(x*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*ln((e*x+d)^n)^2/g/(g*x^2+f)-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*ln(e*
x+d)^2+b^2/g*n*e^2/(d^2*g+e^2*f)*ln(e*x+d)*ln((e*x+d)^n)+1/2*b^2/g*n^2*e^2
/(d^2*g+e^2*f)*ln(g*(e*x+d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)*ln(e*x+d)-1/2*b^2
/g*n*e^2/(d^2*g+e^2*f)*ln(g*(e*x+d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)*ln((e*x+d
)^n)-b^2*n^2*e/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/
e/(f*g)^(1/2))*ln(e*x+d)+b^2*n*e/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(1/2*(2
*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)-1/2*b^2/g*n^2*e^2/(d^2*g+e^
2*f)*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2
*b^2/g*n^2*e^2/(d^2*g+e^2*f)*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(
e*(-f*g)^(1/2)-d*g))+1/2*b^2*n^2*e/(d^2*g+e^2*f)*ln(e*x+d)/(-f*g)^(1/2)*ln
((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d-1/2*b^2*n^2*e/(d^2
*g+e^2*f)*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*
g)^(1/2)-d*g))*d-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*dilog((e*(-f*g)^(1/2)-g*(
e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*dilog((e
*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b^2*n^2*e/(d^2*g+e^
2*f)/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g
))*d-1/2*b^2*n^2*e/(d^2*g+e^2*f)/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*d+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*cs
gn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)...
```

3.322.5 Fricas [F]

$$\int \frac{x(a+b\log\left(c(d+ex)^n\right))^2}{\left(f+gx^2\right)^2}\,dx=\int \frac{\left(b\log\left((ex+d)^nc\right)+a\right)^2x}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x*log((e*x + d)^n*c)^2 + 2*a*b*x*log((e*x + d)^n*c) + a^2*x)
/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.322.7 Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x}{(gx^2+f)^2}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a*b*e*n*(e*log(g*x^2 + f)/(e^2*f*g + d^2*g^2) - 2*e*log(e*x + d)/(e^2
*f*g + d^2*g^2) - 2*d*arctan(g*x/sqrt(f*g))/((e^2*f + d^2*g)*sqrt(f*g))) -
 1/2*b^2*(log((e*x + d)^n)^2/(g^2*x^2 + f*g) - 2*integrate((e*g*x^2*log(c)
^2 + d*g*x*log(c)^2 + (2*d*g*x*log(c) + e*f*n + (e*g*n + 2*e*g*log(c))*x^2
)*log((e*x + d)^n))/(e*g^3*x^5 + d*g^3*x^4 + 2*e*f*g^2*x^3 + 2*d*f*g^2*x^2
 + e*f^2*g*x + d*f^2*g), x)) - a*b*log((e*x + d)^n*c)/(g^2*x^2 + f*g) - 1/
2*a^2/(g^2*x^2 + f*g)
```

3.322.8 Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x}{(gx^2+f)^2}\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x/(g*x^2 + f)^2, x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{(g\,x^2+f)^2}\,dx$$

input
```
int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
```

output
```
int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

3.323 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$

3.323.1 Optimal result

Integrand size = 29, antiderivative size = 814

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$$
$$= -\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)}$$
$$+ \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}$$
$$+ \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$- \frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}(e^2f+d^2g)}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{2bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2} + \frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2b^2n^2\,\text{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f^2}$$

output
```
-1/2*e^2*(a+b*ln(c*(e*x+d)^n))^2/f/(d^2*g+e^2*f)+1/2*(a+b*ln(c*(e*x+d)^n))
^2/f/(g*x^2+f)+ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/f^2-1/2*(a+b*ln(c*(e*x+d
)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^2-1/2*(a+b
*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/
f^2+2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/f^2-b*n*(a+b*ln(c*(e*x+
d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2-b*n*(a+b*l
n(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2-2*
b^2*n^2*polylog(3,1+e*x/d)/f^2+b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^
(1/2)-d*g^(1/2)))/f^2+b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^
(1/2)))/f^2-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/
2)))*(e*(-f)^(1/2)+d*g^(1/2))/(-f)^(3/2)/(d^2*g+e^2*f)+1/2*b*e*n*(a+b*ln(c
*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*f-d*
(-f)^(1/2)*g^(1/2))/f^2/(d^2*g+e^2*f)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e
*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2
))/f^2/(d^2*g+e^2*f)+1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)
+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/f^2/(d^2*g+e^2*f)
```

3.323.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 1209, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} + 4\log(x)\,(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 - 2(a-bn\log(d+ex)}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)^2),x]
```

output
```
((2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) + 4*Log
[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*(a - b*n*Log[d + e
*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[d + e*x]
 + b*Log[c*(d + e*x)^n])*((Sqrt[f]*((-I)*Sqrt[g]*(d + e*x)*Log[d + e*x] +
e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*S
qrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*(I*Sqrt[g]*(d + e*x)*Log[d + e
*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] +
 I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 2*(Log[d + e*x]*Log[(e*(Sqrt[f] +
 I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d +
e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - 2*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sq
rt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e
*Sqrt[f] + I*d*Sqrt[g])]) + 4*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, 1
 + (e*x)/d])) + b^2*n^2*(4*Log[-((e*x)/d)]*Log[d + e*x]^2 - 2*Log[d + e*x]
^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*Log[d + e
*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(L
og[d + e*x]*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] - I*Sqrt[g]*x
)*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) + 2*e*(Sqrt[
f] - I*Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g
])]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 4*Log[d + e*x]
*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 4*Log[d...
```

3.323.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{gx(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}+\frac{(a+b\log(c(d+ex)^n))^2}{f^2x}-\frac{gx(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2e\left(\sqrt{g}d+e\sqrt{-f}\right)\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2(-f)^{3/2}\left(gd^2+e^2f\right)}+\frac{b^2e\left(\sqrt{-f}\sqrt{g}d+ef\right)\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2f^2\left(gd^2+e^2f\right)}+\frac{b^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{f^2}+\frac{b^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{f^2}-\frac{2b^2\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)n^2}{f^2}+\frac{be\left(\sqrt{-f}\sqrt{g}d+ef\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2f^2\left(gd^2+e^2f\right)}+\frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2f^2\left(gd^2+e^2f\right)}-\frac{b\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{f^2}-\frac{b\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{f^2}+\frac{2b\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)n}{f^2}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^2}-\frac{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f\left(gd^2+e^2f\right)}+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f\left(gx^2+f\right)}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)^2),x]
```

3.323. $\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$

output
```
-1/2*(e^2*(a + b*Log[c*(d + e*x)^n])^2)/(f*(e^2*f + d^2*g)) + (a + b*Log[c
*(d + e*x)^n])^2/(2*f*(f + g*x^2)) + (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*
x)^n])^2)/f^2 + (b*e*(e*f + d*Sqrt[-f]*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n
])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2*(e^2*f
 + d^2*g)) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/
(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2) + (b*e*(e*f - d*Sqrt[-f]*Sqrt[g])*n*(a
+ b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqr
t[g])])/(2*f^2*(e^2*f + d^2*g)) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sq
rt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2) - (b^2*e*(e*Sqrt[-
f] + d*Sqrt[g])*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[
g]))])/(2*(-f)^(3/2)*(e^2*f + d^2*g)) - (b*n*(a + b*Log[c*(d + e*x)^n])*Po
lyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f^2 + (b^2*e*(e
*f + d*Sqrt[-f]*Sqrt[g])*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/(2*f^2*(e^2*f + d^2*g)) - (b*n*(a + b*Log[c*(d + e*x)^n])*Pol
yLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f^2 + (2*b*n*(a + b
*Log[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/f^2 + (b^2*n^2*PolyLog[3, -(
(Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f^2 + (b^2*n^2*PolyLog[3,
(Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f^2 - (2*b^2*n^2*PolyLog[3,
 1 + (e*x)/d])/f^2
```

3.323.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.323.4 Maple [F]

$$\int \frac{(a + b\ln (c(ex + d)^n))^2}{x\,(g\,x^2 + f)^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x)
```

3.323.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*
x^5 + 2*f*g*x^3 + f^2*x), x)
```

3.323.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.323.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + 2*log(x)/f^2) + integrat
e((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) +
a*b)*log((e*x + d)^n))/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

3.323.8 Giac [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{x\left(f+gx^2\right)^2}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^2}{(gx^2+f)^2 x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)^2*x), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{x\left(f+gx^2\right)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x\right)^n\right)\right)^2}{x\left(g\,x^2+f\right)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)^2), x)
```

3.324 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx$

3.324.1 Optimal result

Integrand size = 29, antiderivative size = 970

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx$$
$$= \frac{b^2e^2n^2\log(x)}{d^2f^2} - \frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2f^2x}$$
$$+ \frac{e^2g(a+b\log(c(d+ex)^n))^2}{2f^2(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2}$$
$$- \frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3}$$
$$- \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3(e^2f+d^2g)}$$
$$+ \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}$$
$$- \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^3(e^2f+d^2g)}$$
$$+ \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$- \frac{be^2n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2} + \frac{b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2f^2}$$
$$- \frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)gn^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}(e^2f+d^2g)}$$
$$+ \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$- \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)gn^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3(e^2f+d^2g)}$$
$$+ \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}$$
$$- \frac{4bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3} - \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$- \frac{2b^2gn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{4b^2gn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f^3}$$

output
```
b^2*e^2*n^2*ln(x)/d^2/f^2-b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/d^2/f^2/x+1/
2*e^2*g*(a+b*ln(c*(e*x+d)^n))^2/f^2/(d^2*g+e^2*f)-1/2*(a+b*ln(c*(e*x+d)^n)
)^2/f^2/x^2-1/2*g*(a+b*ln(c*(e*x+d)^n))^2/f^2/(g*x^2+f)-2*g*ln(-e*x/d)*(a+
b*ln(c*(e*x+d)^n))^2/f^3-b*e^2*n*(a+b*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^2
/f^2+g*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d
*g^(1/2)))/f^3+g*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-
f)^(1/2)-d*g^(1/2)))/f^3+b^2*e^2*n^2*polylog(2,d/(e*x+d))/d^2/f^2-4*b*g*n*
(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/f^3+2*b*g*n*(a+b*ln(c*(e*x+d)^n))
*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^3+2*b*g*n*(a+b*ln(
c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^3+4*b^
2*g*n^2*polylog(3,1+e*x/d)/f^3-2*b^2*g*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)-d*g^(1/2)))/f^3-2*b^2*g*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1
/2)+d*g^(1/2)))/f^3-1/2*b^2*e*g*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/
2)-d*g^(1/2)))*(e*(-f)^(1/2)+d*g^(1/2))/(-f)^(5/2)/(d^2*g+e^2*f)-1/2*b*e*g
*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/
2)))*(e*f-d*(-f)^(1/2)*g^(1/2))/f^3/(d^2*g+e^2*f)-1/2*b*e*g*n*(a+b*ln(c*(e
*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f
)^(1/2)*g^(1/2))/f^3/(d^2*g+e^2*f)-1/2*b^2*e*g*n^2*polylog(2,(e*x+d)*g^(1/
2)/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/f^3/(d^2*g+e^2*f)
```

3.324.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 1391, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)^2),x]
```

output
```
((-2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/x^2 - (2*f*g*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 8*g*Log[x]*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 4*g*(a - b*n*Log[d + e*x] + b
*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[d + e*x] + b*Lo
g[c*(d + e*x)^n])*((-2*f*(d*e*x + e^2*x^2*Log[x] + (d^2 - e^2*x^2)*Log[d +
 e*x]))/(d^2*x^2) + (I*Sqrt[f]*g*(Sqrt[g]*(d + e*x)*Log[d + e*x] + I*e*(Sq
rt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g
])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*Sqrt[f]*g*(-(Sqrt[g]*(d + e*x)*Log[d + e*
x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] +
 I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 4*g*(Log[d + e*x]*Log[(e*(Sqrt[f]
 + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d
+ e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 4*g*(Log[d + e*x]*Log[(e*(Sqrt[f] -
I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x)
)/(e*Sqrt[f] + I*d*Sqrt[g])]) - 8*g*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLo
g[2, 1 + (e*x)/d])) + b^2*n^2*((I*Sqrt[f]*g*(-(Sqrt[g]*(d + e*x)*Log[d + e
*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt
[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2
, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqr
t[g])*(Sqrt[f] - I*Sqrt[g]*x)) - (Sqrt[f]*g*(Log[d + e*x]*((-I)*Sqrt[g]*(d
 + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*...
```

3.324.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 970, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{2g^2x(a+b\log(c(d+ex)^n))^2}{f^3(f+gx^2)}-\frac{2g(a+b\log(c(d+ex)^n))^2}{f^3x}+\frac{g^2x(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)^2}+\frac{(a+b\log(c(d-}{f^2x^3}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g(a+b\log(c(d+ex)^n))^2e^2}{2f^2(gd^2+e^2f)}+\frac{b^2n^2\log(x)e^2}{d^2f^2}-\frac{bn(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)e^2}{d^2f^2}+\frac{b^2n^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)e^2}{d^2f^2}-\frac{bn(d+ex)(a+b\log(c(d+ex)^n))e}{d^2f^2x}-\frac{b\left(\sqrt{-f}\sqrt{g}d+ef\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)e}{2f^3(gd^2+e^2f)}-\frac{b\left(ef-d\sqrt{-f}\sqrt{g}\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)e}{2f^3(gd^2+e^2f)}-\frac{b^2\left(\sqrt{g}d+e\sqrt{-f}\right)gn^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)e}{2(-f)^{5/2}(gd^2+e^2f)}-\frac{b^2\left(\sqrt{-f}\sqrt{g}d+ef\right)gn^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)e}{2f^3(gd^2+e^2f)}-\frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3}-\frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(gx^2+f)}-\frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2}+\frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3}+\frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}+\frac{2bgn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}+\frac{2bgn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3}-\frac{4bgn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3}-\frac{2b^2gn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}-\frac{2b^2gn^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3}+\frac{4b^2gn^2\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)}{f^3}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)^2),x]
```

3.324. $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx$

output
```
(b^2*e^2*n^2*Log[x])/(d^2*f^2) - (b*e*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n
]))/(d^2*f^2*x) + (e^2*g*(a + b*Log[c*(d + e*x)^n])^2)/(2*f^2*(e^2*f + d^2
*g)) - (a + b*Log[c*(d + e*x)^n])^2/(2*f^2*x^2) - (g*(a + b*Log[c*(d + e*x
)^n])^2)/(2*f^2*(f + g*x^2)) - (2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)
^n])^2)/f^3 - (b*e*(e*f + d*Sqrt[-f]*Sqrt[g])*g*n*(a + b*Log[c*(d + e*x)^n
])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^3*(e^2*f
 + d^2*g)) + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x)
)/(e*Sqrt[-f] + d*Sqrt[g])])/f^3 - (b*e*(e*f - d*Sqrt[-f]*Sqrt[g])*g*n*(a
+ b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqr
t[g])])/(2*f^3*(e^2*f + d^2*g)) + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(
Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/f^3 - (b*e^2*n*(a + b*Lo
g[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/(d^2*f^2) + (b^2*e^2*n^2*PolyLog[2
, d/(d + e*x)])/(d^2*f^2) - (b^2*e*(e*Sqrt[-f] + d*Sqrt[g])*g*n^2*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*(-f)^(5/2)*(e^2*f
+ d^2*g)) + (2*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d +
 e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f^3 - (b^2*e*(e*f + d*Sqrt[-f]*Sqrt[g])
*g*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^3*(e
^2*f + d^2*g)) + (2*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f^3 - (4*b*g*n*(a + b*Log[c*(d + e*x)
^n])*PolyLog[2, 1 + (e*x)/d])/f^3 - (2*b^2*g*n^2*PolyLog[3, -((Sqrt[g]*...
```

3.324.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.324.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{x^3(gx^2+f)^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x)
```

3.324.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*
x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

3.324.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x**3/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.324.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*((2*g*x^2 + f)/(f^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 4
*g*log(x)/f^3) + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*
log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g^2*x^7 + 2*f*g*x^5 + f^2
*x^3), x)
```

3.324.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2 x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)^2*x^3), x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{x^3\,(g\,x^2+f)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)^2), x)
```

3.325 $\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.325.1 Optimal result

Integrand size = 29, antiderivative size = 897

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= -\frac{2abnx}{g^2}+\frac{2b^2n^2x}{g^2}-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg^2}$$
$$+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2}-\frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$-\frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{befn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$
$$+\frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}$$
$$+\frac{befn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$-\frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}+\frac{b^2efn^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$-\frac{3b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}$$
$$-\frac{b^2efn^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$
$$+\frac{3b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$
$$+\frac{3b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}-\frac{3b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$

output
```
-2*a*b*n*x/g^2+2*b^2*n^2*x/g^2-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+(e*x+
d)*(a+b*ln(c*(e*x+d)^n))^2/e/g^2+3/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1
/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)-3/4*(a+b*ln(c*
(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(
1/2)/g^(5/2)-3/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(5/2)+3/2*b*n*(a+b*ln(c*(e*x+d)^n))*pol
ylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)+3/2*b^
2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(5
/2)-3/2*b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(
1/2)/g^(5/2)+1/2*b*e*f*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))
/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/2)-d*g^(1/2))+1/2*b^2*e*f*n^
2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/
2)-d*g^(1/2))-1/2*b*e*f*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2)
)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/2*b^2*e*f*n
^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/
2)+d*g^(1/2))-1/4*f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/g^2/(e*(-f)^(1/2)+d*g^
(1/2))/((-f)^(1/2)-x*g^(1/2))-1/4*f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/g^2/(e
*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(1/2))
```

3.325.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 1237, normalized size of antiderivative = 1.38

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{4\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+\frac{2f\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2}-6\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(\cdots}{\cdots}$$

input
```
Integrate[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
(4*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + (2*f*Sqrt[g
]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 6*Sqrt[
f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n
])^2 + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((4*Sqrt[g]*(d
+ e*x)*(-1 + Log[d + e*x]))/e + (f*(Sqrt[g]*(d + e*x)*Log[d + e*x] + I*e*(
Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt
[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (f*(Sqrt[g]*(d + e*x)*Log[d + e*x] + e*((-
I)*Sqrt[f] - Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqr
t[g])*(Sqrt[f] - I*Sqrt[g]*x)) + (3*I)*Sqrt[f]*(Log[d + e*x]*Log[(e*(Sqrt[
f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(
d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - (3*I)*Sqrt[f]*(Log[d + e*x]*Log[(e
*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[
g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])) + b^2*n^2*((4*Sqrt[g]*(2*e*x -
2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2))/e - (f*(-(Sqrt[g]*(d
 + e*x)*Log[d + e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*
(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqr
t[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*
Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + (f*(Log[d + e*x]*(Sqrt[g
]*(d + e*x)*Log[d + e*x] + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f]
 + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqr...
```

3.325.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{f^2(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)^2}-\frac{2f(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}+\frac{(a+b\log(c(d+ex)^n))^2}{g^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2n^2xb^2}{g^2}-\frac{2n(d+ex)\log\left(c(d+ex)^n\right)b^2}{eg^2}+\frac{efn^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}-\frac{efn^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{5/2}}+\frac{3\sqrt{-f}n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{2g^{5/2}}-\frac{3\sqrt{-f}n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{2g^{5/2}}-\frac{2anxb}{g^2}-\frac{efn(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{5/2}}+\frac{efn(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)b}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}-\frac{3\sqrt{-f}n(a+b\log\left(c(d+ex)^n\right))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b}{2g^{5/2}}+\frac{3\sqrt{-f}n(a+b\log\left(c(d+ex)^n\right))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{2g^{5/2}}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}-\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(\sqrt{g}d+e\sqrt{-f}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{g}x+\sqrt{-f}\right)}+\frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4g^{5/2}}-\frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{g}x+\sqrt{-f}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}$$

input
```
Int[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

3.325. $\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

output
```
(-2*a*b*n*x)/g^2 + (2*b^2*n^2*x)/g^2 - (2*b^2*n*(d + e*x)*Log[c*(d + e*x)^
n])/(e*g^2) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) - (f*(d + e
*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-f] + d*Sqrt[g])*g^2*(Sqrt[-f
] - Sqrt[g]*x)) - (f*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-f
] - d*Sqrt[g])*g^2*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*f*n*(a + b*Log[c*(d + e*
x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(e*Sqr
t[-f] + d*Sqrt[g])*g^(5/2)) + (3*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])^2*Log
[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2)) + (b*e*
f*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f]
- d*Sqrt[g])])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(5/2)) - (3*Sqrt[-f]*(a + b*L
og[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g
])])/(4*g^(5/2)) + (b^2*e*f*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] - d*Sqrt[g]))])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(5/2)) - (3*b*Sqrt[-f]*n*
(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] -
d*Sqrt[g]))])/(2*g^(5/2)) - (b^2*e*f*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e
*Sqrt[-f] + d*Sqrt[g])])/(2*(e*Sqrt[-f] + d*Sqrt[g])*g^(5/2)) + (3*b*Sqrt[
-f]*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f
] + d*Sqrt[g])])/(2*g^(5/2)) + (3*b^2*Sqrt[-f]*n^2*PolyLog[3, -((Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^(5/2)) - (3*b^2*Sqrt[-f]*n^2*Po
lyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^(5/2))
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.325.4 Maple [F]

$$\int \frac{x^4(a+b\ln(c(ex+d)^n))^2}{(g\,x^2+f)^2}dx$$

input
```
int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

output
```
int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

3.325.5 Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^2x^4}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4*log((e*x + d)^n*c)^2 + 2*a*b*x^4*log((e*x + d)^n*c) + a^
2*x^4)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.325.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.325.7 Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^2x^4}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(f*x/(g^3*x^2 + f*g^2) - 3*f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2)
 + 2*x/g^2) + integrate((b^2*x^4*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)
*x^4*log((e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^4)/(g^2*x^4 + 2*f*
g*x^2 + f^2), x)
```

3.325.8 Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^4}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^4/(g*x^2 + f)^2, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{x^4\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
```

output
```
int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

3.326 $\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2} dx$

3.326.1 Optimal result

Integrand size = 29, antiderivative size = 815

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{b^2en^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} + \frac{b^2en^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{bn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} + \frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} - \frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$

output
```
1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^
(1/2)))/g^(3/2)/(-f)^(1/2)-1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*
g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/2*b*n*(a+b*ln(c*(e
*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f
)^(1/2)+1/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1
/2)+d*g^(1/2)))/g^(3/2)/(-f)^(1/2)+1/2*b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/
(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/2*b^2*n^2*polylog(3,(e*x+d)
*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/2*b*e*n*(a+b*ln(c*
(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/
(e*(-f)^(1/2)-d*g^(1/2))-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^
(1/2)-d*g^(1/2)))/g^(3/2)/(e*(-f)^(1/2)-d*g^(1/2))+1/2*b*e*n*(a+b*ln(c*(e*
x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^(3/2)/(e*
(-f)^(1/2)+d*g^(1/2))+1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2
)+d*g^(1/2)))/g^(3/2)/(e*(-f)^(1/2)+d*g^(1/2))+1/4*(e*x+d)*(a+b*ln(c*(e*x+
d)^n))^2/g/(e*(-f)^(1/2)+d*g^(1/2))/((-f)^(1/2)-x*g^(1/2))+1/4*(e*x+d)*(a+
b*ln(c*(e*x+d)^n))^2/g/(e*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(1/2))
```

3.326.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 1132, normalized size of antiderivative = 1.39

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{-\frac{2\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2}+\frac{2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{\sqrt{f}}+2bn(a-bn\log(d+ex)+b}{}$$

input
```
Integrate[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
((-2*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2
) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*
x)^n])^2)/Sqrt[f] + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((
-(Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*((-I)*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqr
t[f] - Sqrt[g]*x])/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (
-(Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f
] + Sqrt[g]*x])/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - (I*(
Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] +
PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]))/Sqrt[f] +
 (I*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g]
)] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/Sqrt[f]
) + b^2*n^2*((-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[
g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g
])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt
[f] + I*d*Sqrt[g])])/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) -
 (Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d + e*x] + (2*I)*e*(Sqrt[f] + I*Sqrt
[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + (2*I)
*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d
*Sqrt[g])])/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*(Log[
d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + ...
```

3.326.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2e\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}+\frac{b^2e\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{3/2}}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2\sqrt{-f}g^{3/2}}-\frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2\sqrt{-f}g^{3/2}}+\frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{3/2}}-\frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}-\frac{b(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2\sqrt{-f}g^{3/2}}+\frac{b(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2\sqrt{-f}g^{3/2}}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(\sqrt{g}d+e\sqrt{-f}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{g}x+\sqrt{-f}\right)}+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$

input
```
Int[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

output
```
((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-f] + d*Sqrt[g])*g*(Sq
rt[-f] - Sqrt[g]*x)) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt
[-f] - d*Sqrt[g])*g*(Sqrt[-f] + Sqrt[g]*x)) + (b*e*n*(a + b*Log[c*(d + e*x
)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(e*Sqrt
[-f] + d*Sqrt[g])*g^(3/2)) + ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f
] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2)) - (b*e*n*(
a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*S
qrt[g])])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - ((a + b*Log[c*(d + e*x)^n
])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*Sqrt[-f]
*g^(3/2)) - (b^2*e*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sq
rt[g]))])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - (b*n*(a + b*Log[c*(d + e*
x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqr
t[-f]*g^(3/2)) + (b^2*e*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d
*Sqrt[g])])/(2*(e*Sqrt[-f] + d*Sqrt[g])*g^(3/2)) + (b*n*(a + b*Log[c*(d +
e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt
[-f]*g^(3/2)) + (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*
Sqrt[g]))])/(2*Sqrt[-f]*g^(3/2)) - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))
/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*g^(3/2))
```

3.326. $\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.326.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.326.4 Maple [F]

$$\int \frac{x^2(a+b\ln(c(ex+d)^n))^2}{(g\,x^2+f)^2}dx$$

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

output
```
int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

3.326.5 Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^2}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*log((e*x + d)^n*c)^2 + 2*a*b*x^2*log((e*x + d)^n*c) + a^
2*x^2)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.326.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))^2}{\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.326.7 Maxima [F]

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))^2}{\left(f+gx^2\right)^2}\,dx=\int \frac{\left(b\log\left((ex+d)^nc\right)+a\right)^2x^2}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(x/(g^2*x^2 + f*g) - arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g)) + integ
rate((b^2*x^2*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log((e*x + d)^
n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^2)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.326.8 Giac [F]

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))^2}{\left(f+gx^2\right)^2}\,dx=\int \frac{\left(b\log\left((ex+d)^nc\right)+a\right)^2x^2}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^2/(g*x^2 + f)^2, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
```

output
```
int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

3.327 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.327.1 Optimal result

Integrand size = 26, antiderivative size = 821

$$
\begin{aligned}
\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx =& -\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} \\
& - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} \\
& - \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} \\
& - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} \\
& - \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
& + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} \\
& - \frac{b^2en^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
& + \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
& - \frac{b^2en^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} \\
& - \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
& - \frac{b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
& + \frac{b^2n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}}
\end{aligned}
$$

output
```
-1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g
^(1/2)))/(-f)^(3/2)/g^(1/2)+1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x
*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/2*b*n*(a+b*ln(c*(
e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)
/g^(1/2)-1/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(
1/2)+d*g^(1/2)))/(-f)^(3/2)/g^(1/2)-1/2*b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)
/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/2*b^2*n^2*polylog(3,(e*x+d
)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(3/2)/g^(1/2)-1/2*b*e*n*(a+b*ln(c
*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f/g^(1/
2)/(e*(-f)^(1/2)+d*g^(1/2))-1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f
)^(1/2)+d*g^(1/2)))/f/g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/2*b*e*n*(a+b*ln(c
*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(1/2)
/(e*(-f)^(3/2)+d*f*g^(1/2))-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-
f)^(1/2)-d*g^(1/2)))/g^(1/2)/(e*(-f)^(3/2)+d*f*g^(1/2))-1/4*(e*x+d)*(a+b*l
n(c*(e*x+d)^n))^2/f/(e*(-f)^(1/2)+d*g^(1/2))/((-f)^(1/2)-x*g^(1/2))-1/4*(e
*x+d)*(a+b*ln(c*(e*x+d)^n))^2/f/(e*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(
1/2))
```

3.327.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 1143, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2\sqrt{f}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} + \frac{2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{\sqrt{g}} + \frac{2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2)^2,x]
```

output
```
((2*Sqrt[f]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)
 + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x
)^n])^2)/Sqrt[g] + (2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((
Sqrt[f]*(Sqrt[g]*(d + e*x)*Log[d + e*x] + I*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[
I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x
)) + (Sqrt[f]*(Sqrt[g]*(d + e*x)*Log[d + e*x] + e*((-I)*Sqrt[f] - Sqrt[g]*
x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x)) - I*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I
*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g
])]) + I*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sq
rt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])))/S
qrt[g] + (b^2*n^2*(-((Sqrt[f]*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(
I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqr
t[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(
d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f]
 - I*Sqrt[g]*x))) + (Sqrt[f]*(Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d + e*x]
 + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt
[f] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqrt[g]
*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[
f] + I*Sqrt[g]*x)) + I*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I...
```

3.327.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}dx$$

$$\Big\downarrow \text{ 2856}$$

$$\int \left(-\frac{g(a+b\log(c(d+ex)^n))^2}{2f(-fg-g^2x^2)}-\frac{g(a+b\log(c(d+ex)^n))^2}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2 e \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}-\frac{b^2 e \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2f\left(\sqrt{g}d+e\sqrt{-f}\right)\sqrt{g}}-\frac{b^2 \operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2(-f)^{3/2}\sqrt{g}}+\frac{b^2 \operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2(-f)^{3/2}\sqrt{g}}-\frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2f\left(\sqrt{g}d+e\sqrt{-f}\right)\sqrt{g}}-\frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}+\frac{b(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2(-f)^{3/2}\sqrt{g}}-\frac{b(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2(-f)^{3/2}\sqrt{g}}-\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(\sqrt{g}d+e\sqrt{-f}\right)\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{g}x+\sqrt{-f}\right)}-\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{3/2}\sqrt{g}}+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2)^2,x]
```

output
```
-1/4*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(f*(e*Sqrt[-f] + d*Sqrt[g])*
(Sqrt[-f] - Sqrt[g]*x)) - ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*f*(e
*Sqrt[-f] - d*Sqrt[g])*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*n*(a + b*Log[c*(d +
e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*(e
*Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sq
rt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g]) - (
b*e*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f
] - d*Sqrt[g])])/(2*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) + ((a + b*Log[c*
(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/
(4*(-f)^(3/2)*Sqrt[g]) - (b^2*e*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sq
rt[-f] - d*Sqrt[g]))])/(2*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) + (b*n*(a
+ b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*S
qrt[g]))])/(2*(-f)^(3/2)*Sqrt[g]) - (b^2*e*n^2*PolyLog[2, (Sqrt[g]*(d + e*
x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*(e*Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) - (b
*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/(2*(-f)^(3/2)*Sqrt[g]) - (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d +
 e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*(-f)^(3/2)*Sqrt[g]) + (b^2*n^2*PolyL
og[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2)*Sqrt[g]
)
```

3.327. $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

3.327.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.327.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{(g\,x^2+f)^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

3.327.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*
x^4 + 2*f*g*x^2 + f^2), x)
```

3.327.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.327.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(x/(f*g*x^2 + f^2) + arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f)) + integr
ate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c)
+ a*b)*log((e*x + d)^n))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

3.327.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/(g*x^2 + f)^2, x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{(f + gx^2)^2}\,dx = \int \frac{(a + b\,\ln(c\,(d + e\,x)^n))^2}{(g\,x^2 + f)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2)^2,x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2)^2, x)
```

3.328 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx$

3.328.1 Optimal result

Integrand size = 29, antiderivative size = 919

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)} + \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)} + \frac{3b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2b^2en^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{df^2} - \frac{3b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{3b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$

output
```
2*b*e*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/d/f^2-(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^2/d/f^2/x+2*b^2*e*n^2*polylog(2,1+e*x/d)/d/f^2-3/4*(a+b*ln(c*(e*x+d)^n)
)^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(5/
2)+3/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d
*g^(1/2)))*g^(1/2)/(-f)^(5/2)+3/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*
x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(5/2)-3/2*b*n*(a+b*ln(
c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/
(-f)^(5/2)-3/2*b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2))
)*g^(1/2)/(-f)^(5/2)+3/2*b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d
*g^(1/2)))*g^(1/2)/(-f)^(5/2)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(
1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/f^2/(e*(-f)^(1/2)+d*g^(1
/2))+1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(
1/2)/f^2/(e*(-f)^(1/2)+d*g^(1/2))+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-
f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/f/(e*(-f)^(3/2)+d*f*
g^(1/2))+1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2))
)*g^(1/2)/f/(e*(-f)^(3/2)+d*f*g^(1/2))+1/4*g*(e*x+d)*(a+b*ln(c*(e*x+d)^n))
^2/f^2/(e*(-f)^(1/2)+d*g^(1/2))/((-f)^(1/2)-x*g^(1/2))+1/4*g*(e*x+d)*(a+b*
ln(c*(e*x+d)^n))^2/f^2/(e*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(1/2))
```

3.328.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.68 (sec) , antiderivative size = 1304, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx$$

$$= \frac{-\frac{4\sqrt{f}(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x} - \frac{2\sqrt{f}gx(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} - 6\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)^2),x]
```

output
```
((-4*Sqrt[f]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/x - (2*Sqrt[
f]*g*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 6*Sq
rt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x
)^n])^2 + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((4*Sqrt[f]*
(e*x*Log[x] - (d + e*x)*Log[d + e*x]))/(d*x) - (Sqrt[f]*Sqrt[g]*(Sqrt[g]*(
d + e*x)*Log[d + e*x] + I*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g
]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*Sqrt
[g]*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*S
qrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))
+ (3*I)*Sqrt[g]*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] -
 I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt
[g])]) - (3*I)*Sqrt[g]*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sq
rt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*
Sqrt[g])])) + b^2*n^2*((Sqrt[f]*Sqrt[g]*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]^
2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*
x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I
*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x)) - (Sqrt[f]*Sqrt[g]*(Log[d + e*x]*(Sqrt[g]*(d +
e*x)*Log[d + e*x] + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sq
rt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)...
```

3.328.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{g(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}+\frac{(a+b\log(c(d+ex)^n))^2}{f^2x^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2e\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)}+\frac{b^2e\sqrt{g}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2f^2\left(\sqrt{g}d+e\sqrt{-f}\right)}+\frac{2b^2e\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)n^2}{df^2}-\frac{3b^2\sqrt{g}\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2(-f)^{5/2}}+\frac{3b^2\sqrt{g}\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2(-f)^{5/2}}+\frac{2be\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))n}{df^2}+\frac{be\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2f^2\left(\sqrt{g}d+e\sqrt{-f}\right)}+\frac{be\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)}+\frac{3b\sqrt{g}(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2(-f)^{5/2}}-\frac{3b\sqrt{g}(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2(-f)^{5/2}}-\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x}+\frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(\sqrt{g}d+e\sqrt{-f}\right)\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{g}x+\sqrt{-f}\right)}-\frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{5/2}}+\frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)^2),x]
```

3.328. $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx$

output
```
(2*b*e*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(d*f^2) - ((d + e*x)*
(a + b*Log[c*(d + e*x)^n])^2)/(d*f^2*x) + (g*(d + e*x)*(a + b*Log[c*(d + e
*x)^n])^2)/(4*f^2*(e*Sqrt[-f] + d*Sqrt[g])*(Sqrt[-f] - Sqrt[g]*x)) + (g*(d
 + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*f^2*(e*Sqrt[-f] - d*Sqrt[g])*(Sqr
t[-f] + Sqrt[g]*x)) + (b*e*Sqrt[g]*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sq
rt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2*(e*Sqrt[-f] + d*Sqr
t[g])) - (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g
]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(5/2)) + (b*e*Sqrt[g]*n*(a + b*Lo
g[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]
)/(2*f*(e*(-f)^(3/2) + d*f*Sqrt[g])) + (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n
])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*(-f)^(5/
2)) + (b^2*e*Sqrt[g]*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*
Sqrt[g]))])/(2*f*(e*(-f)^(3/2) + d*f*Sqrt[g])) + (3*b*Sqrt[g]*n*(a + b*Log
[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g])
)])/(2*(-f)^(5/2)) + (b^2*e*Sqrt[g]*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*
Sqrt[-f] + d*Sqrt[g])])/(2*f^2*(e*Sqrt[-f] + d*Sqrt[g])) - (3*b*Sqrt[g]*n*
(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(2*(-f)^(5/2)) + (2*b^2*e*n^2*PolyLog[2, 1 + (e*x)/d])/(d*f^2)
- (3*b^2*Sqrt[g]*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt
[g]))])/(2*(-f)^(5/2)) + (3*b^2*Sqrt[g]*n^2*PolyLog[3, (Sqrt[g]*(d + e*...
```

3.328.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.328.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{x^2(g\,x^2+f)^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x)
```

3.328.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*
x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

3.328.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2/x**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

3.328.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*((3*g*x^2 + 2*f)/(f^2*g*x^3 + f^3*x) + 3*g*arctan(g*x/sqrt(f*g))/
(sqrt(f*g)*f^2)) + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*
b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g^2*x^6 + 2*f*g*x^4 + f
^2*x^2), x)
```

3.328.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)^2x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)^2*x^2), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx=\int \frac{(a+b\ln(c\,(d+e\,x)^n))^2}{x^2\,(g\,x^2+f)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)^2),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)^2), x)
```

3.329 $\int \frac{\log^3(c(a+bx)^n)}{d+ex^2} dx$

3.329.1 Optimal result

Integrand size = 22, antiderivative size = 477

$$\begin{aligned}\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx &= \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad + \frac{3n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{3n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{3n^3\operatorname{PolyLog}\left(4,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{3n^3\operatorname{PolyLog}\left(4,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*ln(c*(b*x+a)^n)^3*ln(b*((-d)^(1/2)-x*e^(1/2))/(b*(-d)^(1/2)+a*e^(1/2))
)/(-d)^(1/2)/e^(1/2)-1/2*ln(c*(b*x+a)^n)^3*ln(b*((-d)^(1/2)+x*e^(1/2))/(b*
(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-3/2*n*ln(c*(b*x+a)^n)^2*polylog(
2,-(b*x+a)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+3/2*n*ln(c
*(b*x+a)^n)^2*polylog(2,(b*x+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/
2)/e^(1/2)+3*n^2*ln(c*(b*x+a)^n)*polylog(3,-(b*x+a)*e^(1/2)/(b*(-d)^(1/2)-
a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-3*n^2*ln(c*(b*x+a)^n)*polylog(3,(b*x+a)*e^(
1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-3*n^3*polylog(4,-(b*x+a)
*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+3*n^3*polylog(4,(b*x
+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

3.329.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.58

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex^2}\,dx$$
$$= \frac{-2n^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^3(a+bx)+6n^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^2(a+bx)\log\left(c(a+bx)^n\right)-6n\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^3/(d + e*x^2),x]
```

output
```
(-2*n^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[a + b*x]^3 + 6*n^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]]*Log[a + b*x]^2*Log[c*(a + b*x)^n] - 6*n*ArcTan[(Sqrt[e]*x)/Sq
rt[d]]*Log[a + b*x]*Log[c*(a + b*x)^n]^2 + 2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*L
og[c*(a + b*x)^n]^3 + I*n^3*Log[a + b*x]^3*Log[1 - (Sqrt[e]*(a + b*x))/((-
I)*b*Sqrt[d] + a*Sqrt[e])] - (3*I)*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*L
og[1 - (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + (3*I)*n*Log[a +
 b*x]*Log[c*(a + b*x)^n]^2*Log[1 - (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a
*Sqrt[e])] - I*n^3*Log[a + b*x]^3*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d]
 + a*Sqrt[e])] + (3*I)*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*Log[1 - (Sqrt
[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] - (3*I)*n*Log[a + b*x]*Log[c*(a
+ b*x)^n]^2*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] + (3*I)
*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a
*Sqrt[e])] - (3*I)*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (Sqrt[e]*(a + b*x))/(
I*b*Sqrt[d] + a*Sqrt[e])] - (6*I)*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (Sqrt[
e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + (6*I)*n^2*Log[c*(a + b*x)^n]
*PolyLog[3, (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] + (6*I)*n^3*Pol
yLog[4, (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] - (6*I)*n^3*Poly
Log[4, (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])])/(2*Sqrt[d]*Sqrt[e])
```

3.329.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(\frac{\sqrt{-d}\log^3\left(c(a+bx)^n\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log^3\left(c(a+bx)^n\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3n^2\log\left(c(a+bx)^n\right)\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{3n^2\log\left(c(a+bx)^n\right)\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{3n\log^2\left(c(a+bx)^n\right)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{3n\log^2\left(c(a+bx)^n\right)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log^3\left(c(a+bx)^n\right)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{a\sqrt{e}+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log^3\left(c(a+bx)^n\right)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{3n^3\operatorname{PolyLog}\left(4,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{3n^3\operatorname{PolyLog}\left(4,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

input
```
Int[Log[c*(a + b*x)^n]^3/(d + e*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]^3*Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[
e])])/(2*Sqrt[-d]*Sqrt[e]) - (Log[c*(a + b*x)^n]^3*Log[(b*(Sqrt[-d] + Sqrt
[e]*x))/(b*Sqrt[-d] - a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (3*n*Log[c*(a +
b*x)^n]^2*PolyLog[2, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(2*
Sqrt[-d]*Sqrt[e]) + (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (Sqrt[e]*(a + b*x
))/(b*Sqrt[-d] + a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (3*n^2*Log[c*(a + b*x
)^n]*PolyLog[3, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d
]*Sqrt[e]) - (3*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (Sqrt[e]*(a + b*x))/(b*S
qrt[-d] + a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - (3*n^3*PolyLog[4, -((Sqrt[e]*(
a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d]*Sqrt[e]) + (3*n^3*PolyLog[
4, (Sqrt[e]*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])
```

3.329.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.329.4 Maple [F]

$$\int \frac{\ln\left(c(bx+a)^n\right)^3}{e\,x^2+d}dx$$

input
```
int(ln(c*(b*x+a)^n)^3/(e*x^2+d),x)
```

output
```
int(ln(c*(b*x+a)^n)^3/(e*x^2+d),x)
```

3.329.5 Fricas [F]

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^3/(e*x^2 + d), x)
```

3.329.6 Sympy [F]

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left(c(a+bx)^n\right)^3}{d+ex^2}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)**3/(e*x**2+d),x)
```

output
```
Integral(log(c*(a + b*x)**n)**3/(d + e*x**2), x)
```

3.329.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(b*x+a)^n)^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.329.8 Giac [F]

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^3/(e*x^2 + d), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^3}{e\,x^2+d}\,dx$$

input
```
int(log(c*(a + b*x)^n)^3/(d + e*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)^3/(d + e*x^2), x)
```

3.330 $\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx$

3.330.1 Optimal result

Integrand size = 22, antiderivative size = 347

$$\begin{aligned}\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx =& \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}\\ &+\frac{n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}\\ &+\frac{n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*ln(c*(b*x+a)^n)^2*ln(b*((-d)^(1/2)-x*e^(1/2))/(b*(-d)^(1/2)+a*e^(1/2))
)/(-d)^(1/2)/e^(1/2)-1/2*ln(c*(b*x+a)^n)^2*ln(b*((-d)^(1/2)+x*e^(1/2))/(b*
(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-n*ln(c*(b*x+a)^n)*polylog(2,-(b*
x+a)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+n*ln(c*(b*x+a)^n
)*polylog(2,(b*x+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+n
^2*polylog(3,-(b*x+a)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)
-n^2*polylog(3,(b*x+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2
)
```

3.330.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.41

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx$$
$$= \frac{2n^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^2(a+bx)-4n\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(a+bx)\log(c(a+bx)^n)+2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^2(c(a\ldots}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^2/(d + e*x^2),x]
```

output
```
(2*n^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[a + b*x]^2 - 4*n*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[a + b*x]*Log[c*(a + b*x)^n] + 2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[c*(a + b*x)^n]^2 - I*n^2*Log[a + b*x]^2*Log[1 - (Sqrt[e]*(a + b*x))/((
-I)*b*Sqrt[d] + a*Sqrt[e])] + (2*I)*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[
1 - (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + I*n^2*Log[a + b*x]
^2*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] - (2*I)*n*Log[a
+ b*x]*Log[c*(a + b*x)^n]*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqr
t[e])] + (2*I)*n*Log[c*(a + b*x)^n]*PolyLog[2, (Sqrt[e]*(a + b*x))/((-I)*b
*Sqrt[d] + a*Sqrt[e])] - (2*I)*n*Log[c*(a + b*x)^n]*PolyLog[2, (Sqrt[e]*(a
 + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] - (2*I)*n^2*PolyLog[3, (Sqrt[e]*(a + b
*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + (2*I)*n^2*PolyLog[3, (Sqrt[e]*(a + b*
x))/(I*b*Sqrt[d] + a*Sqrt[e])])/(2*Sqrt[d]*Sqrt[e])
```

3.330.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx$$
$$\Big\downarrow \; 2856$$

$$\int \left(\frac{\sqrt{-d}\log^2(c(a+bx)^n)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log^2(c(a+bx)^n)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{a\sqrt{e}+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

input
```
Int[Log[c*(a + b*x)^n]^2/(d + e*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]^2*Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[
e])])/(2*Sqrt[-d]*Sqrt[e]) - (Log[c*(a + b*x)^n]^2*Log[(b*(Sqrt[-d] + Sqrt
[e]*x))/(b*Sqrt[-d] - a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*Log[c*(a + b*
x)^n]*PolyLog[2, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-
d]*Sqrt[e]) + (n*Log[c*(a + b*x)^n]*PolyLog[2, (Sqrt[e]*(a + b*x))/(b*Sqrt
[-d] + a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + (n^2*PolyLog[3, -((Sqrt[e]*(a + b
*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d]*Sqrt[e]) - (n^2*PolyLog[3, (Sqr
t[e]*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])
```

3.330.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.330.4 Maple [F]

$$\int \frac{\ln \left(c(bx+a)^n\right)^2}{e\,x^2+d}dx$$

input
```
int(ln(c*(b*x+a)^n)^2/(e*x^2+d),x)
```

output
```
int(ln(c*(b*x+a)^n)^2/(e*x^2+d),x)
```

3.330.5 Fricas [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^2/(e*x^2 + d), x)
```

3.330.6 Sympy [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left(c(a+bx)^n\right)^2}{d+ex^2}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)**2/(e*x**2+d),x)
```

output
```
Integral(log(c*(a + b*x)**n)**2/(d + e*x**2), x)
```

3.330.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(b*x+a)^n)^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.330.8 Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^2/(e*x^2 + d), x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{e\,x^2+d}\,dx$$

input
```
int(log(c*(a + b*x)^n)^2/(d + e*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)^2/(d + e*x^2), x)
```

3.331 $\int \frac{\log(c(a+bx)^n)}{d+ex^2} dx$

3.331.1 Optimal result

Integrand size = 20, antiderivative size = 229

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2} dx = \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\text{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

output
```
1/2*ln(c*(b*x+a)^n)*ln(b*((-d)^(1/2)-x*e^(1/2))/(b*(-d)^(1/2)+a*e^(1/2)))/
(-d)^(1/2)/e^(1/2)-1/2*ln(c*(b*x+a)^n)*ln(b*((-d)^(1/2)+x*e^(1/2))/(b*(-d)
^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,-(b*x+a)*e^(1/2)/(b*
(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,(b*x+a)*e^(1/2)/
(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

3.331.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\log (c(a+bx)^n)}{d+ex^2} \, dx$$
$$= \frac{\log (c(a+bx)^n) \left(\log \left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right) - \log \left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)\right) - n \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right) + n \operatorname{PolyLog}\left(2, \ldots\right.}{2\sqrt{-d}\sqrt{e}}$$

input
```
Integrate[Log[c*(a + b*x)^n]/(d + e*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]*(Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[e
])] - Log[(b*(Sqrt[-d] + Sqrt[e]*x))/(b*Sqrt[-d] - a*Sqrt[e])]) - n*PolyLo
g[2, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))] + n*PolyLog[2, (Sqrt
[e]*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

3.331.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(a+bx)^n)}{d+ex^2} \, dx$$
$$\Big\downarrow \; 2856$$
$$\int \left(\frac{\sqrt{-d}\log (c(a+bx)^n)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\log (c(a+bx)^n)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\log (c(a+bx)^n)\log \left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{a\sqrt{e}+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log (c(a+bx)^n)\log \left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n \operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n \operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[Log[c*(a + b*x)^n]/(d + e*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]*Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[e]
)])/(2*Sqrt[-d]*Sqrt[e]) - (Log[c*(a + b*x)^n]*Log[(b*(Sqrt[-d] + Sqrt[e]*
x))/(b*Sqrt[-d] - a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, -((Sqr
t[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(2*Sqrt[-d]*Sqrt[e]) + (n*Poly
Log[2, (Sqrt[e]*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

3.331.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

3.331.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\arctan\left(\frac{2e(bx+a)-2ae}{2\sqrt{de}\,b}\right)n\ln(bx+a)}{\sqrt{de}}+\frac{\arctan\left(\frac{2e(bx+a)-2ae}{2\sqrt{de}\,b}\right)\ln((bx+a)^n)}{\sqrt{de}}+\frac{n\ln(bx+a)\ln\left(\frac{b\sqrt{-de}-e(bx+a)+ae}{b\sqrt{-de}+ae}\right)}{2\sqrt{-de}}-\frac{n\ln(bx+a}{}$

input
```
int(ln(c*(b*x+a)^n)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*e)^(1/2)*arctan(1/2*(2*e*(b*x+a)-2*a*e)/(d*e)^(1/2)/b)*n*ln(b*x+a)+1
/(d*e)^(1/2)*arctan(1/2*(2*e*(b*x+a)-2*a*e)/(d*e)^(1/2)/b)*ln((b*x+a)^n)+1
/2*n*ln(b*x+a)/(-d*e)^(1/2)*ln((b*(-d*e)^(1/2)-e*(b*x+a)+a*e)/(b*(-d*e)^(1
/2)+a*e))-1/2*n*ln(b*x+a)/(-d*e)^(1/2)*ln((b*(-d*e)^(1/2)+e*(b*x+a)-a*e)/(
b*(-d*e)^(1/2)-a*e))+1/2*n/(-d*e)^(1/2)*dilog((b*(-d*e)^(1/2)-e*(b*x+a)+a*
e)/(b*(-d*e)^(1/2)+a*e))-1/2*n/(-d*e)^(1/2)*dilog((b*(-d*e)^(1/2)+e*(b*x+a
)-a*e)/(b*(-d*e)^(1/2)-a*e))+(-1/2*I*Pi*csgn(I*c*(b*x+a)^n)^3+1/2*I*Pi*csg
n(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+1/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I
*c)-1/2*I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+ln(c))/(d*e)^
(1/2)*arctan(x*e/(d*e)^(1/2))
```

3.331.5 Fricas [F]

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log((bx+a)^n c)}{ex^2+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)/(e*x^2 + d), x)
```

3.331.6 Sympy [F]

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log(c(a+bx)^n)}{d+ex^2}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)/(e*x**2+d),x)
```

output
```
Integral(log(c*(a + b*x)**n)/(d + e*x**2), x)
```

3.331.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.331.8 Giac [F]

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)}{ex^2+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)/(e*x^2 + d), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)}{e\,x^2+d}\,dx$$

input
```
int(log(c*(a + b*x)^n)/(d + e*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)/(d + e*x^2), x)
```

3.332 $\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx$

3.332.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = -\frac{\text{Int}\left(\frac{1}{(\sqrt{-d}-\sqrt{e}x)\log(c(a+bx)^n)},x\right)}{2\sqrt{-d}} - \frac{\text{Int}\left(\frac{1}{(\sqrt{-d}+\sqrt{e}x)\log(c(a+bx)^n)},x\right)}{2\sqrt{-d}}$$

output
```
-1/2*Unintegrable(1/ln(c*(b*x+a)^n)/((-d)^(1/2)-x*e^(1/2)),x)/(-d)^(1/2)-1
/2*Unintegrable(1/ln(c*(b*x+a)^n)/((-d)^(1/2)+x*e^(1/2)),x)/(-d)^(1/2)
```

3.332.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx$$

input
```
Integrate[1/((d + e*x^2)*Log[c*(a + b*x)^n]),x]
```

output
```
Integrate[1/((d + e*x^2)*Log[c*(a + b*x)^n]), x]
```

3.332.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{\sqrt{-d}}{2d\left(\sqrt{-d}-\sqrt{e}x\right)\log(c(a+bx)^n)}+\frac{\sqrt{-d}}{2d\left(\sqrt{-d}+\sqrt{e}x\right)\log(c(a+bx)^n)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int \frac{1}{\left(\sqrt{-d}-\sqrt{e}x\right)\log(c(a+bx)^n)}dx}{2\sqrt{-d}}-\frac{\int \frac{1}{\left(\sqrt{e}x+\sqrt{-d}\right)\log(c(a+bx)^n)}dx}{2\sqrt{-d}}$$

input
```
Int[1/((d + e*x^2)*Log[c*(a + b*x)^n]),x]
```

output
```
$Aborted
```

3.332.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.332.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(e\,x^{2}+d\right) \ln \left(c \left(b x +a \right)^{n}\right)}d x$$

input
```
int(1/(e*x^2+d)/ln(c*(b*x+a)^n),x)
```

output
```
int(1/(e*x^2+d)/ln(c*(b*x+a)^n),x)
```

3.332.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+d)\log\left((bx+a)^n c\right)}\,dx$$

input
```
integrate(1/(e*x^2+d)/log(c*(b*x+a)^n),x, algorithm="fricas")
```

output
```
integral(1/((e*x^2 + d)*log((b*x + a)^n*c)), x)
```

3.332.6 Sympy [N/A]

Not integrable

Time = 21.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(d+ex^2)\log\left(c\,(a+bx)^n\right)}\,dx$$

input
```
integrate(1/(e*x**2+d)/ln(c*(b*x+a)**n),x)
```

output
```
Integral(1/((d + e*x**2)*log(c*(a + b*x)**n)), x)
```

3.332.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+d)\log((bx+a)^n c)}\,dx$$

input
```
integrate(1/(e*x^2+d)/log(c*(b*x+a)^n),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*log((b*x + a)^n*c)), x)
```

3.332.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+d)\log((bx+a)^n c)}\,dx$$

input
```
integrate(1/(e*x^2+d)/log(c*(b*x+a)^n),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*log((b*x + a)^n*c)), x)
```

3.332.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{\ln\left(c\,{\left(a+b\,x\right)}^n\right)\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/(log(c*(a + b*x)^n)*(d + e*x^2)),x)
```

output
```
int(1/(log(c*(a + b*x)^n)*(d + e*x^2)), x)
```

3.333 $\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x(a+bx^m)}\,dx$

3.333.1 Optimal result

Integrand size = 32, antiderivative size = 27

$$\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\left(a+bx^m\right)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{(1-c)\left(b+ax^{-m}\right)}{b}\right)}{am}$$

output
```
polylog(2,(1-c)*(b+a/(x^m))/b)/a/m
```

3.333.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\left(a+bx^m\right)}\,dx=\frac{\operatorname{PolyLog}\left(2,-\frac{(-1+c)x^{-m}(a+bx^m)}{b}\right)}{am}$$

input
```
Integrate[Log[c - (a*(1 - c))/(b*x^m)]/(x*(a + b*x^m)),x]
```

output
```
PolyLog[2, -(((-1 + c)*(a + b*x^m))/(b*x^m))]/(a*m)
```

3.333.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2925, 2005, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x (a+bx^m)} \, dx$$
$$\Big\downarrow \ 2925$$
$$-\frac{\int \frac{x^m \log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{bx^m+a}dx^{-m}}{m}$$
$$\Big\downarrow \ 2005$$
$$-\frac{\int \frac{\log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{ax^{-m}+b}dx^{-m}}{m}$$
$$\Big\downarrow \ 2840$$
$$-\frac{\int x^m \log \left(1-\frac{(1-c)(ax^{-m}+b)}{b}\right)d(ax^{-m}+b)}{am}$$
$$\Big\downarrow \ 2838$$
$$\frac{\text{PolyLog}\left(2,\frac{(1-c)(ax^{-m}+b)}{b}\right)}{am}$$

input
```
Int[Log[c - (a*(1 - c))/(b*x^m)]/(x*(a + b*x^m)),x]
```

output
```
PolyLog[2, ((1 - c)*(b + a/x^m))/b]/(a*m)
```

3.333.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

3.333.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$	27
default	$\frac{\operatorname{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$	27
risch	Expression too large to display	1267

input
```
int(ln(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x,method=_RETURNVERBOSE)
```

output
```
1/m/a*dilog(1/b*(a*c-a)/(x^m)+c)
```

3.333. $\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x(a+bx^m)}\,dx$

3.333.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\log\left(c - \frac{a(1-c)x^{-m}}{b}\right)}{x\left(a + bx^m\right)}\,dx = \frac{\mathrm{Li}_2\left(-\frac{bcx^m + ac - a}{bx^m} + 1\right)}{am}$$

input
```
integrate(log(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x, algorithm="fricas")
```

output
```
dilog(-(b*c*x^m + a*c - a)/(b*x^m) + 1)/(a*m)
```

3.333.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c - \frac{a(1-c)x^{-m}}{b}\right)}{x\left(a + bx^m\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c-a*(1-c)/b/(x**m))/x/(a+b*x**m),x)
```

output
```
Timed out
```

3.333.7 Maxima [F]

$$\int \frac{\log\left(c - \frac{a(1-c)x^{-m}}{b}\right)}{x\left(a + bx^m\right)}\,dx = \int \frac{\log\left(c + \frac{a(c-1)}{bx^m}\right)}{(bx^m + a)x}\,dx$$

input
```
integrate(log(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x, algorithm="maxima")
```

output
```
(c*m - m)*integrate(log(x)/(b*c*x*x^m + a*(c - 1)*x), x) + (log(b*c*x^m +
a*c - a)*log(x) - log(b)*log(x) - log(x)*log(x^m))/a + log(b)*log((b*x^m +
 a)/b)/(a*m) + (log(x^m)*log(b*x^m/a + 1) + dilog(-b*x^m/a))/(a*m) - (log(
b*c*x^m + a*c - a)*log((b*c*x^m + a*(c - 1))/a + 1) + dilog(-(b*c*x^m + a*
(c - 1))/a))/(a*m)
```

3.333.8 Giac [F]

$$\int \frac{\log\left(c - \frac{a(1-c)x^{-m}}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\log\left(c + \frac{a(c-1)}{bx^m}\right)}{(bx^m+a)x}\,dx$$

input
```
integrate(log(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x, algorithm="giac")
```

output
```
integrate(log(c + a*(c - 1)/(b*x^m))/((b*x^m + a)*x), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c - \frac{a(1-c)x^{-m}}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\ln\left(c + \frac{a\,(c-1)}{b\,x^m}\right)}{x\,(a+b\,x^m)}\,dx$$

input
```
int(log(c + (a*(c - 1))/(b*x^m))/(x*(a + b*x^m)),x)
```

output
```
int(log(c + (a*(c - 1))/(b*x^m))/(x*(a + b*x^m)), x)
```

3.334 $\int \frac{\log \left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)} dx$

3.334.1 Optimal result

Integrand size = 36, antiderivative size = 27

$$\int \frac{\log \left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)} dx = \frac{\text{PolyLog}\left(2, \frac{(1-c)(b+ax^{-m})}{b}\right)}{am}$$

output
```
polylog(2,(1-c)*(b+a/(x^m))/b)/a/m
```

3.334.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)} dx = \frac{\text{PolyLog}\left(2, -\frac{(-1+c)x^{-m}(a+bx^m)}{b}\right)}{am}$$

input
```
Integrate[Log[(-a + a*c + b*c*x^m)/(b*x^m)]/(x*(a + b*x^m)),x]
```

output
```
PolyLog[2, -(((-1 + c)*(a + b*x^m))/(b*x^m))]/(a*m)
```

3.334.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2930, 2925, 2005, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{x^{-m}(ac-a+bcx^m)}{b}\right)}{x(a+bx^m)}dx$$
$$\Big\downarrow \text{2930}$$
$$\int \frac{\log\left(\frac{(ac-a)x^{-m}}{b}+c\right)}{x(a+bx^m)}dx$$
$$\Big\downarrow \text{2925}$$
$$-\frac{\int \frac{x^m\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{bx^m+a}dx^{-m}}{m}$$
$$\Big\downarrow \text{2005}$$
$$-\frac{\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{ax^{-m}+b}dx^{-m}}{m}$$
$$\Big\downarrow \text{2840}$$
$$-\frac{\int x^m\log\left(1-\frac{(1-c)(ax^{-m}+b)}{b}\right)d(ax^{-m}+b)}{am}$$
$$\Big\downarrow \text{2838}$$
$$\frac{\text{PolyLog}\left(2,\frac{(1-c)(ax^{-m}+b)}{b}\right)}{am}$$

input
```
Int[Log[(-a + a*c + b*c*x^m)/(b*x^m)]/(x*(a + b*x^m)),x]
```

output
```
PolyLog[2, ((1 - c)*(b + a/x^m))/b]/(a*m)
```

3.334. $\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)}dx$

3.334.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 2930
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.)*((h_.)*(x_))^(m_
.), x_Symbol] :> Int[(h*x)^m*ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v
, x]^p])^q, x] /; FreeQ[{a, b, c, h, m, p, q, r}, x] && BinomialQ[{u, v}, x
] &&  !BinomialMatchQ[{u, v}, x]
```

3.334.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$	27
default	$\frac{\operatorname{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$	27
risch	Expression too large to display	1267

3.334. $\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)}\,dx$

input
```
int(ln((-a+c*a+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x,method=_RETURNVERBOSE)
```

output
```
1/m/a*dilog(1/b*(a*c-a)/(x^m)+c)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx=\frac{\operatorname{Li}_2\left(-\frac{bcx^m+ac-a}{bx^m}+1\right)}{am}$$

input
```
integrate(log((-a+a*c+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x, algorithm="fricas")
```

output
```
dilog(-(b*c*x^m + a*c - a)/(b*x^m) + 1)/(a*m)
```

3.334.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx=\text{Timed out}$$

input
```
integrate(ln((-a+a*c+b*c*x**m)/b/(x**m))/x/(a+b*x**m),x)
```

output
```
Timed out
```

3.334.7 Maxima [F]

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx=\int \frac{\log\left(\frac{bcx^m+ac-a}{bx^m}\right)}{(bx^m+a)x}\,dx$$

input
```
integrate(log((-a+a*c+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x, algorithm="maxima")
```

output
```
(c*m - m)*integrate(log(x)/(b*c*x*x^m + a*(c - 1)*x), x) + (log(b*c*x^m +
a*c - a)*log(x) - log(b)*log(x) - log(x)*log(x^m))/a + log(b)*log((b*x^m +
 a)/b)/(a*m) + (log(x^m)*log(b*x^m/a + 1) + dilog(-b*x^m/a))/(a*m) - (log(
b*c*x^m + a*c - a)*log((b*c*x^m + a*(c - 1))/a + 1) + dilog(-(b*c*x^m + a*
(c - 1))/a))/(a*m)
```

3.334.8 Giac [F]

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\log\left(\frac{bcx^m+ac-a}{bx^m}\right)}{(bx^m+a)x}\,dx$$

input
```
integrate(log((-a+a*c+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x, algorithm="giac")
```

output
```
integrate(log((b*c*x^m + a*c - a)/(b*x^m))/((b*x^m + a)*x), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\ln\left(\frac{a\,c-a+b\,c\,x^m}{b\,x^m}\right)}{x\,\left(a+b\,x^m\right)}\,dx$$

input
```
int(log((a*c - a + b*c*x^m)/(b*x^m))/(x*(a + b*x^m)),x)
```

output
```
int(log((a*c - a + b*c*x^m)/(b*x^m))/(x*(a + b*x^m)), x)
```

3.335 $\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x(d+ex^m)}\,dx$

3.335.1 Optimal result

Integrand size = 38, antiderivative size = 28

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\,(d+ex^m)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{(1-ac)(e+dx^{-m})}{e}\right)}{dm}$$

output
```
polylog(2,(-a*c+1)*(e+d/(x^m))/e)/d/m
```

3.335.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\,(d+ex^m)}\,dx=\frac{\operatorname{PolyLog}\left(2,-\frac{(-1+ac)x^{-m}(d+ex^m)}{e}\right)}{dm}$$

input
```
Integrate[Log[c*(a - (d - a*c*d)/(c*e*x^m))]/(x*(d + e*x^m)),x]
```

output
```
PolyLog[2, -(((-1 + a*c)*(d + e*x^m))/(e*x^m))]/(d*m)
```

3.335.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2925, 2005, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c \left(a-\frac{x^{-m} (d-a c d)}{c e}\right)\right)}{x \left(d+e x^m\right)} \, dx$$

$$\Big\downarrow \ 2925$$

$$-\frac{\int \frac{x^m \log \left(\frac{a c e-(1-a c) d x^{-m}}{e}\right)}{e x^m+d}dx^{-m}}{m}$$

$$\Big\downarrow \ 2005$$

$$-\frac{\int \frac{\log \left(\frac{a c e-(1-a c) d x^{-m}}{e}\right)}{d x^{-m}+e}dx^{-m}}{m}$$

$$\Big\downarrow \ 2840$$

$$-\frac{\int x^m \log \left(1-\frac{(1-a c) \left(d x^{-m}+e\right)}{e}\right)d\left(d x^{-m}+e\right)}{d m}$$

$$\Big\downarrow \ 2838$$

$$\frac{\operatorname{PolyLog}\left(2,\frac{(1-a c) \left(d x^{-m}+e\right)}{e}\right)}{d m}$$

input
```
Int[Log[c*(a - (d - a*c*d)/(c*e*x^m))]/(x*(d + e*x^m)),x]
```

output
```
PolyLog[2, ((1 - a*c)*(e + d/x^m))/e]/(d*m)
```

3.335. $\int \frac{\log \left(c \left(a-\frac{(d-a c d) x^{-m}}{c e}\right)\right)}{x (d+e x^m)} \, dx$

3.335.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

3.335.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$	30
default	$\frac{\operatorname{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$	30
risch	Expression too large to display	1200

input
```
int(ln(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x,method=_RETURNVERBOSE)
```

output
```
1/m/d*dilog(1/e*(a*c*d-d)/(x^m)+c*a)
```

3.335. $\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x(d+ex^m)}\,dx$

3.335.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx = \frac{\operatorname{Li}_2\left(-\frac{acex^m+(ac-1)d}{ex^m}+1\right)}{dm}$$

input
```
integrate(log(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x, algorithm="fricas"
)
```

output
```
dilog(-(a*c*e*x^m + (a*c - 1)*d)/(e*x^m) + 1)/(d*m)
```

3.335.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(ln(c*(a+(a*c*d-d)/c/e/(x**m)))/x/(d+e*x**m),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.335.7 Maxima [F]

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx = \int \frac{\log\left(\left(a+\frac{acd-d}{cex^m}\right)c\right)}{(ex^m+d)x}\,dx$$

input
```
integrate(log(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x, algorithm="maxima"
)
```

output
```
(a*c*m - m)*integrate(log(x)/(a*c*e*x*x^m + (a*c*d - d)*x), x) + (log(a*c*
e*x^m + (a*c - 1)*d)*log(x) - log(e)*log(x) - log(x)*log(x^m))/d + log(e)*
log((e*x^m + d)/e)/(d*m) + (log(x^m)*log(e*x^m/d + 1) + dilog(-e*x^m/d))/(
d*m) - (log(a*c*e*x^m + (a*c - 1)*d)*log((a*c*e*x^m + a*c*d - d)/d + 1) +
dilog(-(a*c*e*x^m + a*c*d - d)/d))/(d*m)
```

3.335.8 Giac [F]

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx=\int \frac{\log\left(\left(a+\frac{acd-d}{cex^m}\right)c\right)}{(ex^m+d)x}\,dx$$

input
```
integrate(log(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x, algorithm="giac")
```

output
```
integrate(log((a + (a*c*d - d)/(c*e*x^m))*c)/((e*x^m + d)*x), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx=\int \frac{\ln\left(c\left(a-\frac{d-a\,c\,d}{c\,e\,x^m}\right)\right)}{x\,\left(d+e\,x^m\right)}\,dx$$

input
```
int(log(c*(a - (d - a*c*d)/(c*e*x^m)))/(x*(d + e*x^m)),x)
```

output
```
int(log(c*(a - (d - a*c*d)/(c*e*x^m)))/(x*(d + e*x^m)), x)
```

3.336 $\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x(d+ex^m)}\,dx$

3.336.1 Optimal result

Integrand size = 38, antiderivative size = 28

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx = \frac{\operatorname{PolyLog}\left(2,\frac{(1-ac)(e+dx^{-m})}{e}\right)}{dm}$$

output
```
polylog(2,(-a*c+1)*(e+d/(x^m))/e)/d/m
```

3.336.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx = \frac{\operatorname{PolyLog}\left(2,-\frac{(-1+ac)x^{-m}(d+ex^m)}{e}\right)}{dm}$$

input
```
Integrate[Log[(-d + a*c*d + a*c*e*x^m)/(e*x^m)]/(x*(d + e*x^m)),x]
```

output
```
PolyLog[2, -(((-1 + a*c)*(d + e*x^m))/(e*x^m))]/(d*m)
```

3.336.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2930, 2925, 2005, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{x^{-m}(acd+acex^m-d)}{e}\right)}{x(d+ex^m)}dx$$
$$\Big\downarrow \text{ 2930}$$
$$\int \frac{\log \left(\frac{x^{-m}(acd-d)}{e}+ac\right)}{x(d+ex^m)}dx$$
$$\Big\downarrow \text{ 2925}$$
$$-\frac{\int \frac{x^m \log \left(ac-\frac{(1-ac)dx^{-m}}{e}\right)}{ex^m+d}dx^{-m}}{m}$$
$$\Big\downarrow \text{ 2005}$$
$$-\frac{\int \frac{\log \left(ac-\frac{(1-ac)dx^{-m}}{e}\right)}{dx^{-m}+e}dx^{-m}}{m}$$
$$\Big\downarrow \text{ 2840}$$
$$-\frac{\int x^m \log \left(1-\frac{(1-ac)(dx^{-m}+e)}{e}\right)d(dx^{-m}+e)}{dm}$$
$$\Big\downarrow \text{ 2838}$$
$$\frac{\text{PolyLog}\left(2,\frac{(1-ac)(dx^{-m}+e)}{e}\right)}{dm}$$

input
```
Int[Log[(-d + a*c*d + a*c*e*x^m)/(e*x^m)]/(x*(d + e*x^m)),x]
```

output
```
PolyLog[2, ((1 - a*c)*(e + d/x^m))/e]/(d*m)
```

3.336. $\int \frac{\log \left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x(d+ex^m)}dx$

3.336.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 2930
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.)*((h_.)*(x_))^(m_
.), x_Symbol] :> Int[(h*x)^m*ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v
, x]^p])^q, x] /; FreeQ[{a, b, c, h, m, p, q, r}, x] && BinomialQ[{u, v}, x
] &&  !BinomialMatchQ[{u, v}, x]
```

3.336.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$	30
default	$\frac{\operatorname{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$	30
risch	Expression too large to display	1200

3.336. $\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x(d+ex^m)}\,dx$

input
```
int(ln((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x,method=_RETURNVERBOSE)
```

output
```
1/m/d*dilog(1/e*(a*c*d-d)/(x^m)+c*a)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x(d+ex^m)}\,dx = \frac{\text{Li}_2\left(-\frac{acex^m+(ac-1)d}{ex^m}+1\right)}{dm}$$

input
```
integrate(log((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x, algorithm="fric
as")
```

output
```
dilog(-(a*c*e*x^m + (a*c - 1)*d)/(e*x^m) + 1)/(d*m)
```

3.336.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x(d+ex^m)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(ln((-d+a*c*d+a*c*e*x**m)/e/(x**m))/x/(d+e*x**m),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.336.7 Maxima [F]

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx = \int \frac{\log\left(\frac{acex^m+acd-d}{ex^m}\right)}{(ex^m+d)x}\,dx$$

input
```
integrate(log((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x, algorithm="maxi
ma")
```

output
```
(a*c*m - m)*integrate(log(x)/(a*c*e*x*x^m + (a*c*d - d)*x), x) + (log(a*c*
e*x^m + (a*c - 1)*d)*log(x) - log(e)*log(x) - log(x)*log(x^m))/d + log(e)*
log((e*x^m + d)/e)/(d*m) + (log(x^m)*log(e*x^m/d + 1) + dilog(-e*x^m/d))/(
d*m) - (log(a*c*e*x^m + (a*c - 1)*d)*log((a*c*e*x^m + a*c*d - d)/d + 1) +
dilog(-(a*c*e*x^m + a*c*d - d)/d))/(d*m)
```

3.336.8 Giac [F]

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx = \int \frac{\log\left(\frac{acex^m+acd-d}{ex^m}\right)}{(ex^m+d)x}\,dx$$

input
```
integrate(log((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x, algorithm="giac
")
```

output
```
integrate(log((a*c*e*x^m + a*c*d - d)/(e*x^m))/((e*x^m + d)*x), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx = \int \frac{\ln\left(\frac{a\,c\,d-d+a\,c\,e\,x^m}{e\,x^m}\right)}{x\,\left(d+e\,x^m\right)}\,dx$$

input
```
int(log((a*c*d - d + a*c*e*x^m)/(e*x^m))/(x*(d + e*x^m)),x)
```

output
```
int(log((a*c*d - d + a*c*e*x^m)/(e*x^m))/(x*(d + e*x^m)), x)
```

3.337 $\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx$

3.337.1 Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx=\frac{\text{PolyLog}\left(2,1-\frac{2a}{a+bx}\right)}{2ab}$$

output
```
1/2*polylog(2,1-2*a/(b*x+a))/a/b
```

3.337.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx=\frac{\text{PolyLog}\left(2,\frac{-a+bx}{a+bx}\right)}{2ab}$$

input
```
Integrate[Log[(2*a)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

output
```
PolyLog[2, (-a + b*x)/(a + b*x)]/(2*a*b)
```

3.337.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \text{2849}$$

$$\frac{\int \frac{\log\left(\frac{2a}{a+bx}\right)}{1-\frac{2a}{a+bx}}d\frac{1}{a+bx}}{b}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{2a}{a+bx}\right)}{2ab}$$

input
```
Int[Log[(2*a)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

output
```
PolyLog[2, 1 - (2*a)/(a + b*x)]/(2*a*b)
```

3.337.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.337.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$	20
default	$\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$	20
risch	$\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$	20
parts	$\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{b\left(\frac{\ln(bx+a)^2}{2a\,b^2}+\frac{-\operatorname{dilog}\left(-\frac{-bx-a}{2a}\right)-\ln(-bx+a)\ln\left(-\frac{-bx-a}{2a}\right)}{a\,b^2}\right)}{2}$	120

input
```
int(ln(2*a/(b*x+a))/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/a*dilog(2*a/(b*x+a))
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx=\frac{\operatorname{Li}_2\left(-\frac{2\,a}{bx+a}+1\right)}{2\,ab}$$

input
```
integrate(log(2*a/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*dilog(-2*a/(b*x + a) + 1)/(a*b)
```

3.337.6 Sympy [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx=-\int \frac{\log(2)}{-a^2+b^2x^2}\,dx-\int \frac{\log\left(\frac{a}{a+bx}\right)}{-a^2+b^2x^2}\,dx$$

input
```
integrate(ln(2*a/(b*x+a))/(-b**2*x**2+a**2),x)
```

output
```
-Integral(log(2)/(-a**2 + b**2*x**2), x) - Integral(log(a/(a + b*x))/(-a**
2 + b**2*x**2), x)
```

3.337.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx$$
$$=\frac{1}{4}b\left(\frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{ab^2}+\frac{2\left(\log(bx+a)\log\left(-\frac{bx+a}{2a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2a}\right)\right)}{ab^2}\right)$$
$$+\frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{2a}{bx+a}\right)$$

input
```
integrate(log(2*a/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/4*b*((log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b^2) + 2*(log(b*x
 + a)*log(-1/2*(b*x + a)/a + 1) + dilog(1/2*(b*x + a)/a))/(a*b^2)) + 1/2*(
log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log(2*a/(b*x + a))
```

3.337.8 Giac [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx=\int -\frac{\log\left(\frac{2a}{bx+a}\right)}{b^2x^2-a^2}\,dx$$

input
```
integrate(log(2*a/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
integrate(-log(2*a/(b*x + a))/(b^2*x^2 - a^2), x)
```

3.337.9 Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx=\frac{\mathrm{Li}_2\left(\frac{2a}{a+bx}\right)}{2ab}$$

input
```
int(log((2*a)/(a + b*x))/(a^2 - b^2*x^2),x)
```

output
```
dilog((2*a)/(a + b*x))/(2*a*b)
```

3.338 $\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$

3.338.1 Optimal result

Integrand size = 27, antiderivative size = 24

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\operatorname{PolyLog}\left(2,1-\frac{2a}{a+bx}\right)}{2ab}$$

output
```
1/2*polylog(2,1-2*a/(b*x+a))/a/b
```

3.338.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\operatorname{PolyLog}\left(2,\frac{-a+bx}{a+bx}\right)}{2ab}$$

input
```
Integrate[Log[(2*a)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

output
```
PolyLog[2, (-a + b*x)/(a + b*x)]/(2*a*b)
```

3.338.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2858, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}dx$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 2778}$$

$$-\frac{\int \frac{(a+bx)\log\left(\frac{2a}{a+bx}\right)}{a-bx}d\frac{1}{a+bx}}{b}$$

$$\Big\downarrow \text{ 2005}$$

$$-\frac{\int \frac{\log\left(\frac{2a}{a+bx}\right)}{\frac{2a}{a+bx}-1}d\frac{1}{a+bx}}{b}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{2a}{a+bx}\right)}{2ab}$$

input
```
Int[Log[(2*a)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

output
```
PolyLog[2, 1 - (2*a)/(a + b*x)]/(2*a*b)
```

3.338.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.338.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$	20
default	$\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$	20
risch	$\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$	20
parts	$\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{b\left(\frac{\ln(bx+a)^2}{2a\,b^2}+\frac{-\operatorname{dilog}\left(-\frac{-bx-a}{2a}\right)-\ln(-bx+a)\ln\left(-\frac{-bx-a}{2a}\right)}{a\,b^2}\right)}{2}$	120

input
```
int(ln(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/a*dilog(2*a/(b*x+a))
```

3.338. $\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$

3.338.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\text{Li}_2\left(-\frac{2a}{bx+a}+1\right)}{2ab}$$

input
```
integrate(log(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*dilog(-2*a/(b*x + a) + 1)/(a*b)
```

3.338.6 Sympy [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = -\int \frac{\log(2)}{-a^2+b^2x^2}\,dx - \int \frac{\log\left(\frac{a}{a+bx}\right)}{-a^2+b^2x^2}\,dx$$

input
```
integrate(ln(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x)
```

output
```
-Integral(log(2)/(-a**2 + b**2*x**2), x) - Integral(log(a/(a + b*x))/(-a**
2 + b**2*x**2), x)
```

3.338.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$$
$$= \frac{1}{4}b\left(\frac{\log(bx+a)^2 - 2\log(bx+a)\log(bx-a)}{ab^2} + \frac{2\left(\log(bx+a)\log\left(-\frac{bx+a}{2a}+1\right) + \text{Li}_2\left(\frac{bx+a}{2a}\right)\right)}{ab^2}\right)$$
$$+ \frac{1}{2}\left(\frac{\log(bx+a)}{ab} - \frac{\log(bx-a)}{ab}\right)\log\left(\frac{2a}{bx+a}\right)$$

input
```
integrate(log(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
1/4*b*((log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b^2) + 2*(log(b*x
 + a)*log(-1/2*(b*x + a)/a + 1) + dilog(1/2*(b*x + a)/a))/(a*b^2)) + 1/2*(
log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log(2*a/(b*x + a))
```

3.338.8 Giac [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int -\frac{\log\left(\frac{2\,a}{bx+a}\right)}{(bx+a)(bx-a)}\,dx$$

input
```
integrate(log(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(-log(2*a/(b*x + a))/((b*x + a)*(b*x - a)), x)
```

3.338.9 Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{2\,a}{a+b\,x}\right)}{2\,a\,b}$$

input
```
int(log((2*a)/(a + b*x))/((a + b*x)*(a - b*x)),x)
```

output
```
dilog((2*a)/(a + b*x))/(2*a*b)
```

3.339 $\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx$

3.339.1 Optimal result

Integrand size = 38, antiderivative size = 37

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{2ab}$$

output
```
1/2*polylog(2,1+(-a*(1-c)-b*(1+c)*x)/(b*x+a))/a/b
```

3.339.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(37) = 74.

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 6.81

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\log^2\left(\frac{2ac}{(1+c)(a+bx)}\right)-2\log(a-bx)\log\left(\frac{a+bx}{2a}\right)+2\log(a-bx)\log\left(\frac{a-ac+b(1+c)x}{2a}\right)+2\log\left(\frac{2ac}{(1+c)(a+bx)}\right)\log\left(-\right.}{}$$

input
```
Integrate[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

output
```
(Log[(2*a*c)/((1 + c)*(a + b*x))]^2 - 2*Log[a - b*x]*Log[(a + b*x)/(2*a)]
+ 2*Log[a - b*x]*Log[(a - a*c + b*(1 + c)*x)/(2*a)] + 2*Log[(2*a*c)/((1 +
c)*(a + b*x))]*Log[-1/2*(a - a*c + b*(1 + c)*x)/(a*c)] - 2*Log[a - b*x]*Lo
g[(a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*Log[(2*a*c)/((1 + c)*(a + b*x))]*
Log[(a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*PolyLog[2, (a - b*x)/(2*a)] + 2
*PolyLog[2, ((1 + c)*(a - b*x))/(2*a)] - 2*PolyLog[2, ((1 + c)*(a + b*x))/
(2*a*c)])/(4*a*b)
```

3.339.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{a^2-b^2x^2} \, dx$$

$$\Big\downarrow \text{ 2897}$$

$$\frac{\operatorname{PolyLog}\left(2,1-\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{2ab}$$

input
```
Int[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

output
```
PolyLog[2, 1 - (a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(2*a*b)
```

3.339.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.339.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
default	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
risch	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
parts	$\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a}{}\right.}{2ca}\right.}{}$

input
```
int(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE
)
```

output
```
1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx=\frac{\operatorname{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

input
```
integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="fr
icas")
```

output
```
1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

3.339.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b**2*x**2+a**2),x)
```

output
```
Timed out
```

3.339.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(33) = 66$.

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.65

$$\begin{aligned}\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx &= \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right)\\ &+\frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{4ab}\\ &+\frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2a}+1\right)+\text{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2a}\right)}{2ab}\\ &+\frac{\log(bx+a)\log\left(-\frac{bx+a}{2a}+1\right)+\text{Li}_2\left(\frac{bx+a}{2a}\right)}{2ab}\\ &-\frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2ac}+1\right)+\text{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2ac}\right)}{2ab}\end{aligned}$$

input
```
integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="ma
xima")
```

output
```
1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*(c + 1)*x - a*(c - 1)
)/(b*x + a)) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) +
1/2*(log(b*x - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b
*(c + 1)*x - a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a
 + 1) + dilog(1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c +
 1)*x + a*(c + 1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c))
)/(a*b)
```

3.339.8 Giac [F]

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int -\frac{\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right)}{b^2x^2-a^2}\,dx$$

input
```
integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="gi
ac")
```

output
```
integrate(-log((b*(c + 1)*x - a*(c - 1))/(b*x + a))/(b^2*x^2 - a^2), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int \frac{\ln\left(-\frac{a\,(c-1)-b\,x\,(c+1)}{a+b\,x}\right)}{a^2-b^2\,x^2}\,dx$$

input
```
int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/(a^2 - b^2*x^2),x)
```

output
```
int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/(a^2 - b^2*x^2), x)
```

3.340 $\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$

3.340.1 Optimal result

Integrand size = 39, antiderivative size = 27

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

output

```
1/2*polylog(2,c*(-b*x+a)/(b*x+a))/a/b
```

3.340.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 9.33

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$$
$$=\frac{\log^2\left(\frac{2ac}{(1+c)(a+bx)}\right)-2\log(a-bx)\log\left(\frac{a+bx}{2a}\right)+2\log(a-bx)\log\left(\frac{a-ac+b(1+c)x}{2a}\right)+2\log\left(\frac{2ac}{(1+c)(a+bx)}\right)\log\left(-\right.}{}$$

input

```
Integrate[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/((a - b*x)*(a + b*x)),x
]
```

output
```
(Log[(2*a*c)/((1 + c)*(a + b*x))]^2 - 2*Log[a - b*x]*Log[(a + b*x)/(2*a)]
+ 2*Log[a - b*x]*Log[(a - a*c + b*(1 + c)*x)/(2*a)] + 2*Log[(2*a*c)/((1 +
c)*(a + b*x))]*Log[-1/2*(a - a*c + b*(1 + c)*x)/(a*c)] - 2*Log[a - b*x]*Lo
g[(a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*Log[(2*a*c)/((1 + c)*(a + b*x))]*
Log[(a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*PolyLog[2, (a - b*x)/(2*a)] + 2
*PolyLog[2, ((1 + c)*(a - b*x))/(2*a)] - 2*PolyLog[2, ((1 + c)*(a + b*x))/
(2*a*c)])/(4*a*b)
```

3.340.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2965, 27, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{(a-bx)(a+bx)} \, dx$$

$$\Big\downarrow \text{2965}$$

$$\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{2ab\left(1-\frac{a(1-c)+b(c+1)x}{a+bx}\right)}d\frac{a(1-c)+b(c+1)x}{a+bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{1-\frac{a(1-c)+b(c+1)x}{a+bx}}d\frac{a(1-c)+b(c+1)x}{a+bx}}{2ab}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{2ab}$$

input
```
Int[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

output
```
PolyLog[2, 1 - (a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(2*a*b)
```

3.340.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

3.340.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
default	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
risch	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
parts	$\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a}{}\right.}{2ca}\right.}{}$

input
```
int(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x,method=_RETURNVERBO
SE)
```

output
```
1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

input
```
integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="
fricas")
```

output
```
1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x)
```

output
```
Timed out
```

3.340.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 9.11

$$\begin{aligned}\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx &= \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right) \\ &\quad+\frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{4\,ab} \\ &\quad+\frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2\,a}+1\right)+\mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2\,a}\right)}{2\,ab} \\ &\quad+\frac{\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)}{2\,ab} \\ &\quad-\frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2\,ac}+1\right)+\mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2\,ac}\right)}{2\,ab}\end{aligned}$$

input
```
integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="
maxima")
```

output
```
1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*(c + 1)*x - a*(c - 1)
)/(b*x + a)) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) +
1/2*(log(b*x - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b
*(c + 1)*x - a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a
 + 1) + dilog(1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c +
 1)*x + a*(c + 1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c))
)/(a*b)
```

3.340.8 Giac [F]

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int -\frac{\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right)}{(bx+a)(bx-a)}\,dx$$

input
```
integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="
giac")
```

output
```
integrate(-log((b*(c + 1)*x - a*(c - 1))/(b*x + a))/((b*x + a)*(b*x - a)),
 x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int \frac{\ln\left(-\frac{a\,(c-1)-b\,x\,(c+1)}{a+b\,x}\right)}{(a+b\,x)\,(a-b\,x)}\,dx$$

input
```
int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/((a + b*x)*(a - b*x)),x)
```

output
```
int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/((a + b*x)*(a - b*x)), x)
```

3.341 $\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx$

3.341.1 Optimal result

Integrand size = 34, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

output
```
1/2*polylog(2,c*(-b*x+a)/(b*x+a))/a/b
```

3.341.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(27) = 54.

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 9.59

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx$$
$$=\frac{4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a}{b}+x\right)-\log^2\left(\frac{a}{b}+x\right)-4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a-ac}{b+bc}+x\right)+2\log\left(\frac{a}{b}+x\right)\log\left(\frac{a-bx}{2a}\right)-2\log}{}$$

input
```
Integrate[Log[1 - (c*(a - b*x))/(a + b*x)]/(a^2 - b^2*x^2),x]
```

output
```
(4*ArcTanh[(b*x)/a]*Log[a/b + x] - Log[a/b + x]^2 - 4*ArcTanh[(b*x)/a]*Log
[(a - a*c)/(b + b*c) + x] + 2*Log[a/b + x]*Log[(a - b*x)/(2*a)] - 2*Log[(a
 - a*c)/(b + b*c) + x]*Log[((1 + c)*(a - b*x))/(2*a)] + 2*Log[(a - a*c)/(b
 + b*c) + x]*Log[((1 + c)*(a + b*x))/(2*a*c)] + 4*ArcTanh[(b*x)/a]*Log[(a
- a*c + b*(1 + c)*x)/(a + b*x)] + 2*PolyLog[2, (a + b*x)/(2*a)] - 2*PolyLo
g[2, (a - a*c + b*(1 + c)*x)/(2*a)] + 2*PolyLog[2, -1/2*(a - a*c + b*(1 +
c)*x)/(a*c)])/(4*a*b)
```

3.341.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{2897\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2} \, dx$$
$$\Big\downarrow \; 2897$$
$$\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

input
```
Int[Log[1 - (c*(a - b*x))/(a + b*x)]/(a^2 - b^2*x^2),x]
```

output
```
PolyLog[2, (c*(a - b*x))/(a + b*x)]/(2*a*b)
```

3.341.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.341.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
default	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
risch	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
parts	$\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(bx+a)}{2ab} - \frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(-bx+a)}{2ab} + \frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a)\ln\left(-\right.}{}\right.}{2ca}\right.}{}$

input
```
int(ln(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\operatorname{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

input
```
integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(1-c*(-b*x+a)/(b*x+a))/(-b**2*x**2+a**2),x)
```

output
```
Timed out
```

3.341.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 9.00

$$\begin{aligned}\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx &= \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{(bx-a)c}{bx+a}+1\right)\\ &+ \frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{4\,ab}\\ &+ \frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2\,a}+1\right)+\mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2\,a}\right)}{2\,ab}\\ &+ \frac{\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)}{2\,ab}\\ &- \frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2\,ac}+1\right)+\mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2\,ac}\right)}{2\,ab}\end{aligned}$$

input
```
integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*x - a)*c/(b*x + a) +
1) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) + 1/2*(log(b
*x - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b*(c + 1)*x
 - a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a + 1) + di
log(1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c + 1)*x + a*
(c + 1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c)))/(a*b)
```

3.341.8 Giac [F]

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx=\int -\frac{\log\left(\frac{(bx-a)c}{bx+a}+1\right)}{b^2x^2-a^2}\,dx$$

input
```
integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
integrate(-log((b*x - a)*c/(b*x + a) + 1)/(b^2*x^2 - a^2), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx=\int \frac{\ln\left(1-\frac{c\,(a-b\,x)}{a+b\,x}\right)}{a^2-b^2\,x^2}\,dx$$

input
```
int(log(1 - (c*(a - b*x))/(a + b*x))/(a^2 - b^2*x^2),x)
```

output
```
int(log(1 - (c*(a - b*x))/(a + b*x))/(a^2 - b^2*x^2), x)
```

3.342 $\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$

3.342.1 Optimal result

Integrand size = 35, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

output
```
1/2*polylog(2,c*(-b*x+a)/(b*x+a))/a/b
```

3.342.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(27) = 54.

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 9.59

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$$
$$=\frac{4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a}{b}+x\right)-\log^2\left(\frac{a}{b}+x\right)-4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a-ac}{b+bc}+x\right)+2\log\left(\frac{a}{b}+x\right)\log\left(\frac{a-bx}{2a}\right)-2\log\cdots}{\cdots}$$

input
```
Integrate[Log[1 - (c*(a - b*x))/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

output
```
(4*ArcTanh[(b*x)/a]*Log[a/b + x] - Log[a/b + x]^2 - 4*ArcTanh[(b*x)/a]*Log
[(a - a*c)/(b + b*c) + x] + 2*Log[a/b + x]*Log[(a - b*x)/(2*a)] - 2*Log[(a
 - a*c)/(b + b*c) + x]*Log[((1 + c)*(a - b*x))/(2*a)] + 2*Log[(a - a*c)/(b
 + b*c) + x]*Log[((1 + c)*(a + b*x))/(2*a*c)] + 4*ArcTanh[(b*x)/a]*Log[(a
- a*c + b*(1 + c)*x)/(a + b*x)] + 2*PolyLog[2, (a + b*x)/(2*a)] - 2*PolyLo
g[2, (a - a*c + b*(1 + c)*x)/(2*a)] + 2*PolyLog[2, -1/2*(a - a*c + b*(1 +
c)*x)/(a*c)])/(4*a*b)
```

3.342.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2997, 2965, 27, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)} \, dx$$

$$\Big\downarrow \text{2997}$$

$$\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{(a-bx)(a+bx)}dx$$

$$\Big\downarrow \text{2965}$$

$$\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{2ab\left(1-\frac{a(1-c)+b(c+1)x}{a+bx}\right)}d\frac{a(1-c)+b(c+1)x}{a+bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\log \left(\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{1-\frac{a(1-c)+b(c+1)x}{a+bx}}d\frac{a(1-c)+b(c+1)x}{a+bx}}{2ab}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{2ab}$$

input
```
Int[Log[1 - (c*(a - b*x))/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

3.342. $\int \frac{\log \left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)} \, dx$

output
```
PolyLog[2, 1 - (a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(2*a*b)
```

3.342.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

rule 2997
```
Int[Log[(e_.)*((f_.)*((g_) + (v_.)/(w_)))^(r_.)]^(s_.)*(u_.), x_Symbol] :>
Int[u*Log[e*(f*(ExpandToSum[v + g*w, x]/ExpandToSum[w, x]))^r]^s, x] /; Fre
eQ[{e, f, g, r, s}, x] && LinearQ[w, x] && (FreeQ[v, x] || LinearQ[v, x]) &
& AlgebraicFunctionQ[u, x]
```

3.342.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
default	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
risch	$\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$
parts	$\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a)\ln\left(-\right.}{}\right.}{2ca}\right.}{}$

input
```
int(ln(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\mathrm{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

input
```
integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="fricas"
)
```

output
```
1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\text{Timed out}$$

input
```
integrate(ln(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x)
```

output
```
Timed out
```

3.342.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 9.00

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{(bx-a)c}{bx+a}+1\right) + \frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{4\,ab} + \frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2\,a}+1\right)+\mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2\,a}\right)}{2\,ab} + \frac{\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)}{2\,ab} - \frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2\,ac}+1\right)+\mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2\,ac}\right)}{2\,ab}$$

input
```
integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="maxima"
)
```

output
```
1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*x - a)*c/(b*x + a) +
1) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) + 1/2*(log(b
*x - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b*(c + 1)*x
 - a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a + 1) + di
log(1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c + 1)*x + a*
(c + 1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c)))/(a*b)
```

3.342.8 Giac [F]

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int -\frac{\log\left(\frac{(bx-a)c}{bx+a}+1\right)}{(bx+a)(bx-a)}\,dx$$

input
```
integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(-log((b*x - a)*c/(b*x + a) + 1)/((b*x + a)*(b*x - a)), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int \frac{\ln \left(1-\frac{c\,(a-b\,x)}{a+b\,x}\right)}{(a+b\,x)\,(a-b\,x)}\,dx$$

input
```
int(log(1 - (c*(a - b*x))/(a + b*x))/((a + b*x)*(a - b*x)),x)
```

output
```
int(log(1 - (c*(a - b*x))/(a + b*x))/((a + b*x)*(a - b*x)), x)
```

3.343 $\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx$

3.343.1 Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx &= \frac{\log\left(-\frac{bx}{a}\right)\log^3(c(a+bx)^n)}{d} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} \\ &\quad - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} \\ &\quad + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d} \\ &\quad + \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d} \\ &\quad - \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)}{d} \\ &\quad - \frac{6n^3\operatorname{PolyLog}\left(4,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{6n^3\operatorname{PolyLog}\left(4,1+\frac{bx}{a}\right)}{d}\end{aligned}$$

output
```
ln(-b*x/a)*ln(c*(b*x+a)^n)^3/d-ln(c*(b*x+a)^n)^3*ln(b*(e*x+d)/(-a*e+b*d))/
d-3*n*ln(c*(b*x+a)^n)^2*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d+3*n*ln(c*(b*x+a
)^n)^2*polylog(2,1+b*x/a)/d+6*n^2*ln(c*(b*x+a)^n)*polylog(3,-e*(b*x+a)/(-a
*e+b*d))/d-6*n^2*ln(c*(b*x+a)^n)*polylog(3,1+b*x/a)/d-6*n^3*polylog(4,-e*(
b*x+a)/(-a*e+b*d))/d+6*n^3*polylog(4,1+b*x/a)/d
```

3.343.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 494 vs. 2(238) = 476.

Time = 0.14 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.08

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx$$
$$= \frac{-\log(x)\left(n\log(a+bx)-\log\left(c(a+bx)^n\right)\right)^3+\left(n\log(a+bx)-\log\left(c(a+bx)^n\right)\right)^3\log(d+ex)+3n(-n\log}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^3/(d*x + e*x^2),x]
```

output
```
(-(Log[x]*(n*Log[a + b*x] - Log[c*(a + b*x)^n])^3) + (n*Log[a + b*x] - Log
[c*(a + b*x)^n])^3*Log[d + e*x] + 3*n*(-(n*Log[a + b*x]) + Log[c*(a + b*x)
^n])^2*(Log[x]*(Log[a + b*x] - Log[1 + (b*x)/a]) - Log[a + b*x]*Log[(b*(d
+ e*x))/(b*d - a*e)] - PolyLog[2, -((b*x)/a)] - PolyLog[2, (e*(a + b*x))/(
-(b*d) + a*e)]) - 3*n^2*(n*Log[a + b*x] - Log[c*(a + b*x)^n])*(Log[-((b*x)
/a)]*Log[a + b*x]^2 - Log[a + b*x]^2*Log[(b*(d + e*x))/(b*d - a*e)] - 2*Lo
g[a + b*x]*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)] + 2*Log[a + b*x]*PolyL
og[2, 1 + (b*x)/a] + 2*PolyLog[3, (e*(a + b*x))/(-(b*d) + a*e)] - 2*PolyLo
g[3, 1 + (b*x)/a]) + n^3*(Log[-((b*x)/a)]*Log[a + b*x]^3 - Log[a + b*x]^3*
Log[(b*(d + e*x))/(b*d - a*e)] - 3*Log[a + b*x]^2*PolyLog[2, (e*(a + b*x))
/(-(b*d) + a*e)] + 3*Log[a + b*x]^2*PolyLog[2, 1 + (b*x)/a] + 6*Log[a + b*
x]*PolyLog[3, (e*(a + b*x))/(-(b*d) + a*e)] - 6*Log[a + b*x]*PolyLog[3, 1
+ (b*x)/a] - 6*PolyLog[4, (e*(a + b*x))/(-(b*d) + a*e)] + 6*PolyLog[4, 1 +
 (b*x)/a]))/d
```

3.343.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2026, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx$$
$$\Big\downarrow \text{2026}$$

$$\int \frac{\log^3(c(a+bx)^n)}{x(d+ex)}dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{\log^3(c(a+bx)^n)}{dx}-\frac{e\log^3(c(a+bx)^n)}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d}-\frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}-\frac{\log^3(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}-\frac{6n^2\operatorname{PolyLog}\left(3,\frac{bx}{a}+1\right)\log(c(a+bx)^n)}{d}+\frac{3n\operatorname{PolyLog}\left(2,\frac{bx}{a}+1\right)\log^2(c(a+bx)^n)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log^3(c(a+bx)^n)}{d}-\frac{6n^3\operatorname{PolyLog}\left(4,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{6n^3\operatorname{PolyLog}\left(4,\frac{bx}{a}+1\right)}{d}$$

input
```
Int[Log[c*(a + b*x)^n]^3/(d*x + e*x^2),x]
```

output
```
(Log[-((b*x)/a)]*Log[c*(a + b*x)^n]^3)/d - (Log[c*(a + b*x)^n]^3*Log[(b*(d
 + e*x))/(b*d - a*e)])/d - (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, -((e*(a +
b*x))/(b*d - a*e))])/d + (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, 1 + (b*x)/a]
)/d + (6*n^2*Log[c*(a + b*x)^n]*PolyLog[3, -((e*(a + b*x))/(b*d - a*e))])/
d - (6*n^2*Log[c*(a + b*x)^n]*PolyLog[3, 1 + (b*x)/a])/d - (6*n^3*PolyLog[
4, -((e*(a + b*x))/(b*d - a*e))])/d + (6*n^3*PolyLog[4, 1 + (b*x)/a])/d
```

3.343.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.343.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 1756, normalized size of antiderivative = 7.38

method	result	size
risch	Expression too large to display	1756

input
```
int(ln(c*(b*x+a)^n)^3/(e*x^2+d*x),x,method=_RETURNVERBOSE)
```

output
```
(ln((b*x+a)^n)-n*ln(b*x+a))^3/d*ln(b*x)-(ln((b*x+a)^n)-n*ln(b*x+a))^3/d*ln
(e*(b*x+a)-a*e+b*d)+n^3/d*ln(b*x+a)^3*ln(1-(b*x+a)/a)+3*n^3/d*ln(b*x+a)^2*
polylog(2,(b*x+a)/a)-6*n^3/d*ln(b*x+a)*polylog(3,(b*x+a)/a)+6*n^3/d*polylo
g(4,(b*x+a)/a)-n^3/d*ln(b*x+a)^3*ln(1+e*(b*x+a)/(-a*e+b*d))-3*n^3/d*ln(b*x
+a)^2*polylog(2,-e*(b*x+a)/(-a*e+b*d))+6*n^3/d*ln(b*x+a)*polylog(3,-e*(b*x
+a)/(-a*e+b*d))-6*n^3*polylog(4,-e*(b*x+a)/(-a*e+b*d))/d+3*b*n*(ln((b*x+a)
^n)-n*ln(b*x+a))^2*(1/b/d*(dilog(-x/a*b)+ln(b*x+a)*ln(-x/a*b))-e/b/d*(dilo
g((e*(b*x+a)-a*e+b*d)/(-a*e+b*d))/e+ln(b*x+a)*ln((e*(b*x+a)-a*e+b*d)/(-a*e
+b*d))/e))+3*b*n^2*(ln((b*x+a)^n)-n*ln(b*x+a))*(1/b/d*(ln(b*x+a)^2*ln(1-(b
*x+a)/a)+2*ln(b*x+a)*polylog(2,(b*x+a)/a)-2*polylog(3,(b*x+a)/a))-1/b/d*(l
n(b*x+a)^2*ln(1+e*(b*x+a)/(-a*e+b*d))+2*ln(b*x+a)*polylog(2,-e*(b*x+a)/(-a
*e+b*d))-2*polylog(3,-e*(b*x+a)/(-a*e+b*d))))+1/8*(-I*Pi*csgn(I*c*(b*x+a)^
n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x+a)^n)
^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*ln(c))
^3*(-1/d*ln(e*x+d)+1/d*ln(x))+(-3/2*I*Pi*csgn(I*c*(b*x+a)^n)^3+3/2*I*Pi*cs
gn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+3/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(
I*c)-3/2*I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+3*ln(c))*((l
n((b*x+a)^n)-n*ln(b*x+a))^2/d*ln(b*x)-(ln((b*x+a)^n)-n*ln(b*x+a))^2/d*ln(e
*(b*x+a)-a*e+b*d)+b*n^2*(1/b/d*(ln(b*x+a)^2*ln(1-(b*x+a)/a)+2*ln(b*x+a)*po
lylog(2,(b*x+a)/a)-2*polylog(3,(b*x+a)/a))-1/b/d*(ln(b*x+a)^2*ln(1+e*(b...
```

3.343.5 Fricas [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(e*x^2+d*x),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^3/(e*x^2 + d*x), x)
```

3.343.6 Sympy [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left(c(a+bx)^n\right)^3}{x\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)**3/(e*x**2+d*x),x)
```

output
```
Integral(log(c*(a + b*x)**n)**3/(x*(d + e*x)), x)
```

3.343.7 Maxima [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(e*x^2+d*x),x, algorithm="maxima")
```

output
```
integrate(log((b*x + a)^n*c)^3/(e*x^2 + d*x), x)
```

3.343.8 Giac [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(e*x^2+d*x),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^3/(e*x^2 + d*x), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^3}{e\,x^2+d\,x}\,dx$$

input
```
int(log(c*(a + b*x)^n)^3/(d*x + e*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)^3/(d*x + e*x^2), x)
```

3.344 $\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx$

3.344.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \frac{\log\left(-\frac{bx}{a}\right)\log^2(c(a+bx)^n)}{d} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d} + \frac{2n^2\operatorname{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{2n^2\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)}{d}$$

output
```
ln(-b*x/a)*ln(c*(b*x+a)^n)^2/d-ln(c*(b*x+a)^n)^2*ln(b*(e*x+d)/(-a*e+b*d))/
d-2*n*ln(c*(b*x+a)^n)*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d+2*n*ln(c*(b*x+a)^
n)*polylog(2,1+b*x/a)/d+2*n^2*polylog(3,-e*(b*x+a)/(-a*e+b*d))/d-2*n^2*pol
ylog(3,1+b*x/a)/d
```

3.344.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.74

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{dx+ex^2}\,dx$$
$$= \frac{\log(x)\left(-n\log(a+bx)+\log\left(c(a+bx)^n\right)\right)^2-\left(-n\log(a+bx)+\log\left(c(a+bx)^n\right)\right)^2\log(d+ex)-2n(n\log}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^2/(d*x + e*x^2),x]
```

output
```
(Log[x]*(-(n*Log[a + b*x]) + Log[c*(a + b*x)^n])^2 - (-(n*Log[a + b*x]) +
Log[c*(a + b*x)^n])^2*Log[d + e*x] - 2*n*(n*Log[a + b*x] - Log[c*(a + b*x)
^n])*(Log[x]*(Log[a + b*x] - Log[1 + (b*x)/a]) - Log[a + b*x]*Log[(b*(d +
e*x))/(b*d - a*e)] - PolyLog[2, -((b*x)/a)] - PolyLog[2, (e*(a + b*x))/(-(
b*d) + a*e)]) + n^2*(Log[-((b*x)/a)]*Log[a + b*x]^2 - Log[a + b*x]^2*Log[(
b*(d + e*x))/(b*d - a*e)] - 2*Log[a + b*x]*PolyLog[2, (e*(a + b*x))/(-(b*d
) + a*e)] + 2*Log[a + b*x]*PolyLog[2, 1 + (b*x)/a] + 2*PolyLog[3, (e*(a +
b*x))/(-(b*d) + a*e)] - 2*PolyLog[3, 1 + (b*x)/a]))/d
```

3.344.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2026, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{dx+ex^2}\,dx$$
$$\Big\downarrow \; 2026$$
$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x(d+ex)}dx$$
$$\Big\downarrow \; 2863$$
$$\int \left(\frac{\log^2\left(c(a+bx)^n\right)}{dx}-\frac{e\log^2\left(c(a+bx)^n\right)}{d(d+ex)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{2n\log\left(c(a+bx)^n\right)\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}-\frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{2n\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)\log\left(c(a+bx)^n\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^n\right)}{d}+\frac{2n^2\,\text{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d}-\frac{2n^2\,\text{PolyLog}\left(3,\frac{bx}{a}+1\right)}{d}$$

input
```
Int[Log[c*(a + b*x)^n]^2/(d*x + e*x^2),x]
```

output
```
(Log[-((b*x)/a)]*Log[c*(a + b*x)^n]^2)/d - (Log[c*(a + b*x)^n]^2*Log[(b*(d
 + e*x))/(b*d - a*e)])/d - (2*n*Log[c*(a + b*x)^n]*PolyLog[2, -((e*(a + b*
x))/(b*d - a*e))])/d + (2*n*Log[c*(a + b*x)^n]*PolyLog[2, 1 + (b*x)/a])/d
+ (2*n^2*PolyLog[3, -((e*(a + b*x))/(b*d - a*e))])/d - (2*n^2*PolyLog[3, 1
 + (b*x)/a])/d
```

3.344.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.344.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.53

method	result
risch	$\frac{(\ln((bx+a)^n)-n\ln(bx+a))^2\ln(bx)}{d} - \frac{(\ln((bx+a)^n)-n\ln(bx+a))^2\ln(e(bx+a)-ae+bd)}{d} + \frac{n^2\ln(bx+a)^2\ln\left(1-\frac{bx+a}{a}\right)}{d} + \frac{2n^2\ln(}{}$

input
```
int(ln(c*(b*x+a)^n)^2/(e*x^2+d*x),x,method=_RETURNVERBOSE)
```

output
```
(ln((b*x+a)^n)-n*ln(b*x+a))^2/d*ln(b*x)-(ln((b*x+a)^n)-n*ln(b*x+a))^2/d*ln
(e*(b*x+a)-a*e+b*d)+n^2/d*ln(b*x+a)^2*ln(1-(b*x+a)/a)+2*n^2/d*ln(b*x+a)*po
lylog(2,(b*x+a)/a)-2*n^2/d*polylog(3,(b*x+a)/a)-n^2/d*ln(b*x+a)^2*ln(1+e*(
b*x+a)/(-a*e+b*d))-2*n^2/d*ln(b*x+a)*polylog(2,-e*(b*x+a)/(-a*e+b*d))+2*n^
2*polylog(3,-e*(b*x+a)/(-a*e+b*d))/d+2*b*n*(ln((b*x+a)^n)-n*ln(b*x+a))*(1/
b/d*(dilog(-x/a*b)+ln(b*x+a)*ln(-x/a*b))-e/b/d*(dilog((e*(b*x+a)-a*e+b*d)/
(-a*e+b*d))/e+ln(b*x+a)*ln((e*(b*x+a)-a*e+b*d)/(-a*e+b*d))/e))+(-I*Pi*csgn
(I*c*(b*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I
*c*(b*x+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(
I*c)+2*ln(c))*(-1/d*ln(e*x+d)*ln((b*x+a)^n)+ln((b*x+a)^n)/d*ln(x)-b*n*(1/d
*dilog((b*x+a)/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+a*e-b*d
)/(a*e-b*d))/b-1/d*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b))+1/4*(-I
*Pi*csgn(I*c*(b*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*P
i*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^
n)*csgn(I*c)+2*ln(c))^2*(-1/d*ln(e*x+d)+1/d*ln(x))
```

3.344.5 Fricas [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(e*x^2+d*x),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^2/(e*x^2 + d*x), x)
```

3.344.6 Sympy [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left(c(a+bx)^n\right)^2}{x\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)**2/(e*x**2+d*x),x)
```

output
```
Integral(log(c*(a + b*x)**n)**2/(x*(d + e*x)), x)
```

3.344.7 Maxima [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(e*x^2+d*x),x, algorithm="maxima")
```

output
```
integrate(log((b*x + a)^n*c)^2/(e*x^2 + d*x), x)
```

3.344.8 Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(e*x^2+d*x),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^2/(e*x^2 + d*x), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2} dx = \int \frac{\ln(c(a+bx)^n)^2}{e x^2 + d x} dx$$

input
```
int(log(c*(a + b*x)^n)^2/(d*x + e*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)^2/(d*x + e*x^2), x)
```

3.345 $\int \frac{\log(c(a+bx)^n)}{dx+ex^2} dx$

3.345.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2} dx = \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^n)}{d} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{n\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{n\,\text{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d}$$

output
```
ln(-b*x/a)*ln(c*(b*x+a)^n)/d-ln(c*(b*x+a)^n)*ln(b*(e*x+d)/(-a*e+b*d))/d-n*
polylog(2,-e*(b*x+a)/(-a*e+b*d))/d+n*polylog(2,1+b*x/a)/d
```

3.345.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2} dx = \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^n)}{d} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{n\,\text{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d} - \frac{n\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}$$

input
```
Integrate[Log[c*(a + b*x)^n]/(d*x + e*x^2),x]
```

output
```
(Log[-((b*x)/a)]*Log[c*(a + b*x)^n])/d - (Log[c*(a + b*x)^n]*Log[(b*(d + e
*x))/(b*d - a*e)])/d + (n*PolyLog[2, (a + b*x)/a])/d - (n*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/d
```

3.345.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2026, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx)^n\right)}{dx+ex^2} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{\log \left(c(a+bx)^n\right)}{x(d+ex)}dx$$

$$\Big\downarrow \text{ 2863}$$

$$\int \left(\frac{\log \left(c(a+bx)^n\right)}{dx}-\frac{e\log \left(c(a+bx)^n\right)}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log \left(c(a+bx)^n\right)\log \left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log \left(-\frac{bx}{a}\right)\log \left(c(a+bx)^n\right)}{d}-\frac{n\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{n\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}$$

input
```
Int[Log[c*(a + b*x)^n]/(d*x + e*x^2),x]
```

output
```
(Log[-((b*x)/a)]*Log[c*(a + b*x)^n])/d - (Log[c*(a + b*x)^n]*Log[(b*(d + e
*x))/(b*d - a*e)])/d - (n*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (n
*PolyLog[2, 1 + (b*x)/a])/d
```

3.345.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.345.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

method	result
parts	$-\frac{\ln(c(bx+a)^n)\ln(ex+d)}{d}+\frac{\ln(c(bx+a)^n)\ln(x)}{d}-bn\left(\frac{\mathrm{dilog}\left(\frac{bx+a}{a}\right)}{db}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{db}-\frac{\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{db}-\frac{\ln(ex-$
risch	$-\frac{\ln(ex+d)\ln((bx+a)^n)}{d}+\frac{\ln((bx+a)^n)\ln(x)}{d}-\frac{n\,\mathrm{dilog}\left(\frac{bx+a}{a}\right)}{d}-\frac{n\ln(x)\ln\left(\frac{bx+a}{a}\right)}{d}+\frac{n\,\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d}+\frac{n\ln(ex+$

input
```
int(ln(c*(b*x+a)^n)/(e*x^2+d*x),x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x+a)^n)/d*ln(e*x+d)+ln(c*(b*x+a)^n)/d*ln(x)-b*n*(1/d*dilog((b*x+a
)/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/
b-1/d*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b)
```

3.345.5 Fricas [F]

$$\int \frac{\log \left(c(a+bx)^n\right)}{dx+ex^2}\,dx=\int \frac{\log \left((bx+a)^n c\right)}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)/(e*x^2+d*x),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)/(e*x^2 + d*x), x)
```

3.345.6 Sympy [F]

$$\int \frac{\log \left(c(a+bx)^n\right)}{dx+ex^2}\,dx=\int \frac{\log \left(c(a+bx)^n\right)}{x\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(b*x+a)**n)/(e*x**2+d*x),x)
```

output
```
Integral(log(c*(a + b*x)**n)/(x*(d + e*x)), x)
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{\log \left(c(a+bx)^n\right)}{dx+ex^2}\,dx = -bn\left(\frac{\log \left(\frac{bx}{a}+1\right)\log (x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{bd}-\frac{\log (ex+d)\log \left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{bd}\right) - \left(\frac{\log (ex+d)}{d}-\frac{\log (x)}{d}\right)\log \left((bx+a)^n c\right)$$

input
```
integrate(log(c*(b*x+a)^n)/(e*x^2+d*x),x, algorithm="maxima")
```

output
```
-b*n*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(e*x + d)*log(-(
b*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/(b*d)) -
 (log(e*x + d)/d - log(x)/d)*log((b*x + a)^n*c)
```

3.345.8 Giac [F]

$$\int \frac{\log\left(c(a+bx)^n\right)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)}{ex^2+dx}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)/(e*x^2+d*x),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)/(e*x^2 + d*x), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^n\right)}{dx+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)}{e\,x^2+d\,x}\,dx$$

input
```
int(log(c*(a + b*x)^n)/(d*x + e*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)/(d*x + e*x^2), x)
```

3.346 $\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx$

3.346.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx=\frac{\text{Int}\left(\frac{1}{x\log(c(a+bx)^n)},x\right)}{d}-\frac{e\text{Int}\left(\frac{1}{(d+ex)\log(c(a+bx)^n)},x\right)}{d}$$

output
```
Unintegrable(1/x/ln(c*(b*x+a)^n),x)/d-e*Unintegrable(1/(e*x+d)/ln(c*(b*x+a
)^n),x)/d
```

3.346.2 Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx=\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx$$

input
```
Integrate[1/((d*x + e*x^2)*Log[c*(a + b*x)^n]),x]
```

output
```
Integrate[1/((d*x + e*x^2)*Log[c*(a + b*x)^n]), x]
```

3.346.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2026, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(d+ex)\log(c(a+bx)^n)}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{1}{dx\log(c(a+bx)^n)}-\frac{e}{d(d+ex)\log(c(a+bx)^n)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{1}{x\log(c(a+bx)^n)}dx}{d}-\frac{e\int \frac{1}{(d+ex)\log(c(a+bx)^n)}dx}{d}$$

input
```
Int[1/((d*x + e*x^2)*Log[c*(a + b*x)^n]),x]
```

output
```
$Aborted
```

3.346.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.346.4 Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+dx)\ln\left(c\,(bx+a)^n\right)}dx$$

input
```
int(1/(e*x^2+d*x)/ln(c*(b*x+a)^n),x)
```

output
```
int(1/(e*x^2+d*x)/ln(c*(b*x+a)^n),x)
```

3.346.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(ex^2+dx)\log\left((bx+a)^n c\right)}\,dx$$

input
```
integrate(1/(e*x^2+d*x)/log(c*(b*x+a)^n),x, algorithm="fricas")
```

output
```
integral(1/((e*x^2 + d*x)*log((b*x + a)^n*c)), x)
```

3.346.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{x\left(d+ex\right)\log\left(c\left(a+bx\right)^n\right)}\,dx$$

input
```
integrate(1/(e*x**2+d*x)/ln(c*(b*x+a)**n),x)
```

output
```
Integral(1/(x*(d + e*x)*log(c*(a + b*x)**n)), x)
```

3.346.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(ex^2+dx)\log\left((bx+a)^n c\right)}\,dx$$

input
```
integrate(1/(e*x^2+d*x)/log(c*(b*x+a)^n),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d*x)*log((b*x + a)^n*c)), x)
```

3.346.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(ex^2+dx)\log\left((bx+a)^n c\right)}\,dx$$

input
```
integrate(1/(e*x^2+d*x)/log(c*(b*x+a)^n),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d*x)*log((b*x + a)^n*c)), x)
```

3.346.9 Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{\ln\left(c\,(a+b\,x)^n\right)\,\left(e\,x^2+d\,x\right)}\,dx$$

input
```
int(1/(log(c*(a + b*x)^n)*(d*x + e*x^2)),x)
```

output
```
int(1/(log(c*(a + b*x)^n)*(d*x + e*x^2)), x)
```

3.347 $\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2} dx$

3.347.1 Optimal result

Integrand size = 25, antiderivative size = 500

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^3\operatorname{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^3\operatorname{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

output
```
ln(c*(b*x+a)^n)^3*ln(-b*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(2*a*f-b*(e-(-4*d*f+e
^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-ln(c*(b*x+a)^n)^3*ln(-b*(e+2*f*x+(-4*d*f+e
^2)^(1/2))/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+3*n*ln(c*(
b*x+a)^n)^2*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*
f+e^2)^(1/2)-3*n*ln(c*(b*x+a)^n)^2*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d
*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-6*n^2*ln(c*(b*x+a)^n)*polylog(3,2*f*(b
*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+6*n^2*ln(c*(b*x
+a)^n)*polylog(3,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2
)^(1/2)+6*n^3*polylog(4,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*
d*f+e^2)^(1/2)-6*n^3*polylog(4,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))
))/(-4*d*f+e^2)^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 993, normalized size of antiderivative = 1.99

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx$$

$$= \frac{-2\sqrt{e^2-4df}n^3\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log^3(a+bx)+6\sqrt{e^2-4df}n^2\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log^2(a+bx)\log(c(a}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^3/(d + e*x + f*x^2),x]
```

output
```
(-2*Sqrt[e^2 - 4*d*f]*n^3*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a + b
*x]^3 + 6*Sqrt[e^2 - 4*d*f]*n^2*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log
[a + b*x]^2*Log[c*(a + b*x)^n] - 6*Sqrt[e^2 - 4*d*f]*n*ArcTan[(e + 2*f*x)/
Sqrt[-e^2 + 4*d*f]]*Log[a + b*x]*Log[c*(a + b*x)^n]^2 + 2*Sqrt[e^2 - 4*d*f
]*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[c*(a + b*x)^n]^3 + Sqrt[-e^2
+ 4*d*f]*n^3*Log[a + b*x]^3*Log[1 - (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sq
rt[e^2 - 4*d*f])] - 3*Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]^2*Log[c*(a + b*x
)^n]*Log[1 - (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 3*S
qrt[-e^2 + 4*d*f]*n*Log[a + b*x]*Log[c*(a + b*x)^n]^2*Log[1 - (2*f*(a + b*
x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] - Sqrt[-e^2 + 4*d*f]*n^3*Log[a
 + b*x]^3*Log[1 + (2*f*(a + b*x))/(-2*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] +
3*Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*Log[1 + (2*f*(a
 + b*x))/(-2*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] - 3*Sqrt[-e^2 + 4*d*f]*n*Lo
g[a + b*x]*Log[c*(a + b*x)^n]^2*Log[1 + (2*f*(a + b*x))/(-2*a*f + b*(e + S
qrt[e^2 - 4*d*f]))] + 3*Sqrt[-e^2 + 4*d*f]*n*Log[c*(a + b*x)^n]^2*PolyLog[
2, (2*f*(a + b*x))/(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))] - 3*Sqrt[-e^2 + 4
*d*f]*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sq
rt[e^2 - 4*d*f]))] - 6*Sqrt[-e^2 + 4*d*f]*n^2*Log[c*(a + b*x)^n]*PolyLog[3
, (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 6*Sqrt[-e^2 +
4*d*f]*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e ...
```

3.347.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{2f\log^3(c(a+bx)^n)}{\sqrt{e^2-4df}\left(-\sqrt{e^2-4df}+e+2fx\right)}-\frac{2f\log^3(c(a+bx)^n)}{\sqrt{e^2-4df}\left(\sqrt{e^2-4df}+e+2fx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(-\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{6n^3\operatorname{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{6n^3\operatorname{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

input
```
Int[Log[c*(a + b*x)^n]^3/(d + e*x + f*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]^3*Log[-((b*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b
*(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] - (Log[c*(a + b*x)^n]^3*Log
[-((b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
)])/Sqrt[e^2 - 4*d*f] + (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (2*f*(a + b*x
))/(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (3*n*Log[c*(a
 + b*x)^n]^2*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])
)])/Sqrt[e^2 - 4*d*f] - (6*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (2*f*(a + b*x
))/(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] + (6*n^2*Log[c*
(a + b*x)^n]*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])
)])/Sqrt[e^2 - 4*d*f] + (6*n^3*PolyLog[4, (2*f*(a + b*x))/(2*a*f - b*(e -
Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (6*n^3*PolyLog[4, (2*f*(a + b*x)
)/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

3.347.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.347.4 Maple [F]

$$\int \frac{\ln \left(c(bx+a)^n\right)^3}{f\,x^2+ex+d}dx$$

input
```
int(ln(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x)
```

output
```
int(ln(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x)
```

3.347.5 Fricas [F]

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{fx^2+ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^3/(f*x^2 + e*x + d), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x+a)**n)**3/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.347.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.347.8 Giac [F]

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{fx^2+ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^3/(f*x^2 + e*x + d), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^3}{f\,x^2+e\,x+d}\,dx$$

input
```
int(log(c*(a + b*x)^n)^3/(d + e*x + f*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)^3/(d + e*x + f*x^2), x)
```

3.348 $\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx$

3.348.1 Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx &= \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\ &\quad - \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\ &\quad + \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\ &\quad - \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\ &\quad - \frac{2n^2\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\ &\quad + \frac{2n^2\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}\end{aligned}$$

output
```
ln(c*(b*x+a)^n)^2*ln(-b*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(2*a*f-b*(e-(-4*d*f+e
^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-ln(c*(b*x+a)^n)^2*ln(-b*(e+2*f*x+(-4*d*f+e
^2)^(1/2))/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+2*n*ln(c*(
b*x+a)^n)*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+
e^2)^(1/2)-2*n*ln(c*(b*x+a)^n)*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e
^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-2*n^2*polylog(3,2*f*(b*x+a)/(2*a*f-b*(e-(-
4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+2*n^2*polylog(3,2*f*(b*x+a)/(2*a*f-
b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.76

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx$$
$$= \frac{2\sqrt{e^2-4df}n^2\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log^2(a+bx)-4\sqrt{e^2-4df}n\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log(a+bx)\log\left(c(a+b\right.}{}$$

input
```
Integrate[Log[c*(a + b*x)^n]^2/(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[e^2 - 4*d*f]*n^2*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a + b*
x]^2 - 4*Sqrt[e^2 - 4*d*f]*n*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a
+ b*x]*Log[c*(a + b*x)^n] + 2*Sqrt[e^2 - 4*d*f]*ArcTan[(e + 2*f*x)/Sqrt[-e
^2 + 4*d*f]]*Log[c*(a + b*x)^n]^2 - Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]^2*
Log[1 - (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 2*Sqrt[-
e^2 + 4*d*f]*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[1 - (2*f*(a + b*x))/(-(
b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]
^2*Log[1 + (2*f*(a + b*x))/(-2*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] - 2*Sqrt[
-e^2 + 4*d*f]*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[1 + (2*f*(a + b*x))/(-
2*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] + 2*Sqrt[-e^2 + 4*d*f]*n*Log[c*(a + b*
x)^n]*PolyLog[2, (2*f*(a + b*x))/(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))] - 2
*Sqrt[-e^2 + 4*d*f]*n*Log[c*(a + b*x)^n]*PolyLog[2, (2*f*(a + b*x))/(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))] - 2*Sqrt[-e^2 + 4*d*f]*n^2*PolyLog[3, (2*f*
(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 2*Sqrt[-e^2 + 4*d*f]*
n^2*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[
-(e^2 - 4*d*f)^2]
```

3.348.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2} \, dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{2f\log^2(c(a+bx)^n)}{\sqrt{e^2-4df}\left(-\sqrt{e^2-4df}+e+2fx\right)}-\frac{2f\log^2(c(a+bx)^n)}{\sqrt{e^2-4df}\left(\sqrt{e^2-4df}+e+2fx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2n\log(c(a+bx)^n)\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{2n\log(c(a+bx)^n)\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(-\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}-\frac{2n^2\text{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{2n^2\text{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

input
```
Int[Log[c*(a + b*x)^n]^2/(d + e*x + f*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]^2*Log[-((b*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b
*(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] - (Log[c*(a + b*x)^n]^2*Log
[-((b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
)])/Sqrt[e^2 - 4*d*f] + (2*n*Log[c*(a + b*x)^n]*PolyLog[2, (2*f*(a + b*x))
/(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (2*n*Log[c*(a +
 b*x)^n]*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/
Sqrt[e^2 - 4*d*f] - (2*n^2*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] + (2*n^2*PolyLog[3, (2*f*(a + b*x))/(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

3.348.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.348.4 Maple [F]

$$\int \frac{\ln \left(c(bx+a)^{n}\right)^{2}}{f\,x^{2}+ex+d}dx$$

input
```
int(ln(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x)
```

output
```
int(ln(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x)
```

3.348.5 Fricas [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{fx^2+ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)^2/(f*x^2 + e*x + d), x)
```

3.348.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x+a)**n)**2/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.348.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.348.8 Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{fx^2+ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)^2/(f*x^2 + e*x + d), x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{f\,x^2+e\,x+d}\,dx$$

input
```
int(log(c*(a + b*x)^n)^2/(d + e*x + f*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)^2/(d + e*x + f*x^2), x)
```

3.349 $\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx$

3.349.1 Optimal result

Integrand size = 23, antiderivative size = 243

$$\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{\log(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\mathrm{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\,\mathrm{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

output
```
ln(c*(b*x+a)^n)*ln(-b*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(2*a*f-b*(e-(-4*d*f+e^2
)^(1/2))))/(-4*d*f+e^2)^(1/2)-ln(c*(b*x+a)^n)*ln(-b*(e+2*f*x+(-4*d*f+e^2)^
(1/2))/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*polylog(2,2*
f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-n*polylog(2
,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.80

$$\int \frac{\log \left(c(a+bx)^n\right)}{d+ex+fx^2} \, dx$$
$$= \frac{\log \left(c(a+bx)^n\right) \left(\log \left(\frac{b\left(-e+\sqrt{e^2-4df}-2fx\right)}{-be+2af+b\sqrt{e^2-4df}}\right) - \log \left(\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)\right) + n \operatorname{PolyLog}\left(2, \frac{2f(a+bx)}{2af+b\left(-e+\sqrt{e^2-4\right.}}\right.}{\sqrt{e^2-4df}}$$

input
```
Integrate[Log[c*(a + b*x)^n]/(d + e*x + f*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]*(Log[(b*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(-(b*e) + 2*
a*f + b*Sqrt[e^2 - 4*d*f])] - Log[(b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(-2*
a*f + b*(e + Sqrt[e^2 - 4*d*f]))]) + n*PolyLog[2, (2*f*(a + b*x))/(2*a*f +
 b*(-e + Sqrt[e^2 - 4*d*f]))] - n*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e
 + Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

3.349.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx)^n\right)}{d+ex+fx^2} \, dx$$
$$\Big\downarrow \text{ 2865}$$
$$\int \left(\frac{2f \log \left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(-\sqrt{e^2-4df}+e+2fx\right)} - \frac{2f \log \left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(\sqrt{e^2-4df}+e+2fx\right)}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(c(a+bx)^n)\log\left(-\frac{b\left(-\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{\log(c(a+bx)^n)\log\left(-\frac{b\left(\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

input
```
Int[Log[c*(a + b*x)^n]/(d + e*x + f*x^2),x]
```

output
```
(Log[c*(a + b*x)^n]*Log[-((b*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(
e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] - (Log[c*(a + b*x)^n]*Log[-((
b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))])/
Sqrt[e^2 - 4*d*f] + (n*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e - Sqrt[e^2
 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b
*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

3.349.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.349.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.29 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.53

method	result
risch	$-\frac{2b\arctan\left(\frac{2(bx+a)f-2af+be}{\sqrt{4b^2df-e^2b^2}}\right)n\ln(bx+a)}{\sqrt{4b^2df-e^2b^2}}+\frac{2b\arctan\left(\frac{2(bx+a)f-2af+be}{\sqrt{4b^2df-e^2b^2}}\right)\ln((bx+a)^n)}{\sqrt{4b^2df-e^2b^2}}+\frac{bn\ln(bx+a)\ln\left(\frac{-2(bx+a)f+2af-be}{2af-be+\sqrt{-4}}\right.}{\sqrt{-4b^2df+e^2b^2}}$

input
```
int(ln(c*(b*x+a)^n)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2*b/(4*b^2*d*f-b^2*e^2)^(1/2)*arctan((2*(b*x+a)*f-2*a*f+b*e)/(4*b^2*d*f-b
^2*e^2)^(1/2))*n*ln(b*x+a)+2*b/(4*b^2*d*f-b^2*e^2)^(1/2)*arctan((2*(b*x+a)
*f-2*a*f+b*e)/(4*b^2*d*f-b^2*e^2)^(1/2))*ln((b*x+a)^n)+b*n*ln(b*x+a)/(-4*b
^2*d*f+b^2*e^2)^(1/2)*ln((-2*(b*x+a)*f+2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2
))/(2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2)))-b*n*ln(b*x+a)/(-4*b^2*d*f+b^2*e
^2)^(1/2)*ln((2*(b*x+a)*f-2*a*f+b*e+(-4*b^2*d*f+b^2*e^2)^(1/2))/(-2*a*f+b*
e+(-4*b^2*d*f+b^2*e^2)^(1/2)))+b*n/(-4*b^2*d*f+b^2*e^2)^(1/2)*dilog((-2*(b
*x+a)*f+2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2))/(2*a*f-b*e+(-4*b^2*d*f+b^2*e
^2)^(1/2)))-b*n/(-4*b^2*d*f+b^2*e^2)^(1/2)*dilog((2*(b*x+a)*f-2*a*f+b*e+(-
4*b^2*d*f+b^2*e^2)^(1/2))/(-2*a*f+b*e+(-4*b^2*d*f+b^2*e^2)^(1/2)))+2*(-1/2
*I*Pi*csgn(I*c*(b*x+a)^n)^3+1/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^
n)+1/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-1/2*I*Pi*csgn(I*c*(b*x+a)^n)*c
sgn(I*(b*x+a)^n)*csgn(I*c)+ln(c))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*
f-e^2)^(1/2))
```

3.349.5 Fricas [F]

$$\int \frac{\log (c(a+bx)^n)}{d+ex+fx^2} \, dx = \int \frac{\log \left((bx+a)^n c \right)}{fx^2+ex+d} \, dx$$

input
```
integrate(log(c*(b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^n*c)/(f*x^2 + e*x + d), x)
```

3.349.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log (c(a+bx)^n)}{d+ex+fx^2} \, dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x+a)**n)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.349.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.349.8 Giac [F]

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)}{fx^2+ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^n*c)/(f*x^2 + e*x + d), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)}{f\,x^2+e\,x+d}\,dx$$

input
```
int(log(c*(a + b*x)^n)/(d + e*x + f*x^2),x)
```

output
```
int(log(c*(a + b*x)^n)/(d + e*x + f*x^2), x)
```

3.350 $\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx$

3.350.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \frac{2f\text{Int}\left(\frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\log(c(a+bx)^n)},x\right)}{\sqrt{e^2-4df}} - \frac{2f\text{Int}\left(\frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\log(c(a+bx)^n)},x\right)}{\sqrt{e^2-4df}}$$

output
```
2*f*Unintegrable(1/ln(c*(b*x+a)^n)/(e+2*f*x-(-4*d*f+e^2)^(1/2)),x)/(-4*d*f
+e^2)^(1/2)-2*f*Unintegrable(1/ln(c*(b*x+a)^n)/(e+2*f*x+(-4*d*f+e^2)^(1/2)
),x)/(-4*d*f+e^2)^(1/2)
```

3.350.2 Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx$$

input
```
Integrate[1/((d + e*x + f*x^2)*Log[c*(a + b*x)^n]),x]
```

output
```
Integrate[1/((d + e*x + f*x^2)*Log[c*(a + b*x)^n]), x]
```

3.350.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{2f}{\sqrt{e^2-4df}\left(-\sqrt{e^2-4df}+e+2fx\right)\log(c(a+bx)^n)}-\frac{2f}{\sqrt{e^2-4df}\left(\sqrt{e^2-4df}+e+2fx\right)\log(c(a+bx)^n)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2f\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\log(c(a+bx)^n)}dx}{\sqrt{e^2-4df}}-\frac{2f\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\log(c(a+bx)^n)}dx}{\sqrt{e^2-4df}}$$

input
```
Int[1/((d + e*x + f*x^2)*Log[c*(a + b*x)^n]),x]
```

output
```
$Aborted
```

3.350.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.350.4 Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f\,x^2+ex+d)\ln\left(c\,(bx+a)^n\right)}dx$$

input
```
int(1/(f*x^2+e*x+d)/ln(c*(b*x+a)^n),x)
```

output
```
int(1/(f*x^2+e*x+d)/ln(c*(b*x+a)^n),x)
```

3.350.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(fx^2+ex+d)\log\left((bx+a)^n c\right)}\,dx$$

input
```
integrate(1/(f*x^2+e*x+d)/log(c*(b*x+a)^n),x, algorithm="fricas")
```

output
```
integral(1/((f*x^2 + e*x + d)*log((b*x + a)^n*c)), x)
```

3.350.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x**2+e*x+d)/ln(c*(b*x+a)**n),x)
```

output
```
Timed out
```

3.350.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(fx^2+ex+d)\log((bx+a)^n c)}\,dx$$

input
```
integrate(1/(f*x^2+e*x+d)/log(c*(b*x+a)^n),x, algorithm="maxima")
```

output
```
integrate(1/((f*x^2 + e*x + d)*log((b*x + a)^n*c)), x)
```

3.350.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(fx^2+ex+d)\log((bx+a)^n c)}\,dx$$

input
```
integrate(1/(f*x^2+e*x+d)/log(c*(b*x+a)^n),x, algorithm="giac")
```

output
```
integrate(1/((f*x^2 + e*x + d)*log((b*x + a)^n*c)), x)
```

3.350.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{\ln\left(c\,(a+b\,x)^n\right)\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(log(c*(a + b*x)^n)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(log(c*(a + b*x)^n)*(d + e*x + f*x^2)), x)
```

3.351 $\int \frac{x^3 \log(x)}{a+bx+cx^2} dx$

3.351.1 Optimal result

Integrand size = 18, antiderivative size = 286

$$\begin{aligned}\int \frac{x^3 \log(x)}{a+bx+cx^2} dx &= \frac{bx}{c^2} - \frac{x^2}{4c} - \frac{bx\log(x)}{c^2} + \frac{x^2\log(x)}{2c} \\ &+ \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3} \\ &+ \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3} \\ &+ \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3} \\ &+ \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3}\end{aligned}$$

output
```
b*x/c^2-1/4*x^2/c-b*x*ln(x)/c^2+1/2*x^2*ln(x)/c+1/2*ln(x)*ln(1+2*c*x/(b-(-
4*a*c+b^2)^(1/2)))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^3+1/2*pol
ylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)
^(1/2))/c^3+1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^2-a*c+b*(-3*a*
c+b^2)/(-4*a*c+b^2)^(1/2))/c^3+1/2*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2))
)*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^3
```

3.351.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.62

$$\int \frac{x^3 \log(x)}{a + bx + cx^2} \, dx$$

$$= \frac{4bcx - c^2x^2 - 4bcx\log(x) + 2c^2x^2\log(x) + \frac{4abc\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + 2(b^2-ac)\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log}{}$$

input
```
Integrate[(x^3*Log[x])/(a + b*x + c*x^2),x]
```

output
```
(4*b*c*x - c^2*x^2 - 4*b*c*x*Log[x] + 2*c^2*x^2*Log[x] + (4*a*b*c*Log[x]*L
og[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*
a*c] + 2*(b^2 - a*c)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b - Sqrt[b^2 -
4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - (4*a*b*c*Log[x]*Log[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] + 2*(b^2
- a*c)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/(b + Sqrt[b^2 - 4*a*c])] + (4*a*b*c*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 -
4*a*c])])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*(1 - b/Sqrt[b^2 - 4*a*c])*Poly
Log[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] - (4*a*b*c*PolyLog[2, (-2*c*x)/(b
 + Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*(1 + b/Sqrt[b^2
- 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(4*c^3)
```

3.351.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log(x)}{a + bx + cx^2} \, dx$$

$$\Big\downarrow \text{2804}$$

$$\int \left(\frac{\log(x)\left(x\left(b^2 - ac\right) + ab\right)}{c^2\left(a + bx + cx^2\right)} - \frac{b\log(x)}{c^2} + \frac{x\log(x)}{c}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3}+\frac{\log(x)\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2c^3}+\frac{\log(x)\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2c^3}+\frac{bx}{c^2}-\frac{bx\log(x)}{c^2}-\frac{x^2}{4c}+\frac{x^2\log(x)}{2c}$$

input
```
Int[(x^3*Log[x])/(a + b*x + c*x^2),x]
```

output
```
(b*x)/c^2 - x^2/(4*c) - (b*x*Log[x])/c^2 + (x^2*Log[x])/(2*c) + ((b^2 - a*
c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[
b^2 - 4*a*c])])/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^3) + ((b^2 - a*c
- (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 -
 4*a*c])])/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Po
lyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^3)
```

3.351.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.351.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(258) = 516$.

Time = 1.21 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.68

method	result
default	$\frac{\frac{x^2\ln(x)}{2}-\frac{x^2}{4}}{c}-\frac{b(\ln(x)x-x)}{c^2}+\frac{-\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,ac-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b^2-3\ln\left(\frac{-2xc+\sqrt{}}{-b+\sqrt{}}\right.}{}}{}$
risch	$\frac{x^2\ln(x)}{2c}-\frac{x^2}{4c}-\frac{bx\ln(x)}{c^2}+\frac{bx}{c^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)a}{2c^2}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2c^3}+\frac{3\ln(x)\ln\left(\frac{-2xc+\sqrt{}}{-b+\sqrt{}}\right.}{2c^2\sqrt{-4}}$

input
```
int(x^3*ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*x^2*ln(x)-1/4*x^2)-b/c^2*(ln(x)*x-x)+1/c^2*(-1/2*ln(x)*(ln((-2*x*
c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln
((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)
*b^2-3*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c+ln(
(-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+ln((2*x*c+(-4*a
*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((2*x*c+
(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2+3*ln(
(2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c-ln((2*x*c+(-4*a
*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^3)/c/(-4*a*c+b^2)^(1/2)-1/2*(di
log((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1
/2)*a*c-dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a
*c+b^2)^(1/2)*b^2-3*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(
1/2)))*a*b*c+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*
b^3+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2
)^(1/2)*a*c-dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4
*a*c+b^2)^(1/2)*b^2+3*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(
1/2)))*a*b*c-dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^
3)/c/(-4*a*c+b^2)^(1/2))
```

3.351.5 Fricas [F]

$$\int \frac{x^3\log(x)}{a+bx+cx^2}\,dx=\int \frac{x^3\log(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(x^3*log(x)/(c*x^2 + b*x + a), x)
```

3.351.6 Sympy [F]

$$\int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx = \int \frac{x^3 \log{(x)}}{a+bx+cx^2}\,dx$$

input
```
integrate(x**3*ln(x)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**3*log(x)/(a + b*x + c*x**2), x)
```

3.351.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.351.8 Giac [F]

$$\int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx = \int \frac{x^3 \log{(x)}}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*log(x)/(c*x^2 + b*x + a), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log(x)}{a+bx+cx^2} \, dx = \int \frac{x^3 \ln(x)}{c\,x^2+b\,x+a} \, dx$$

input
```
int((x^3*log(x))/(a + b*x + c*x^2),x)
```

output
```
int((x^3*log(x))/(a + b*x + c*x^2), x)
```

3.352 $\int \frac{x^2 \log(x)}{a+bx+cx^2} \, dx$

3.352.1 Optimal result

Integrand size = 18, antiderivative size = 234

$$\int \frac{x^2 \log(x)}{a+bx+cx^2} \, dx = -\frac{x}{c} + \frac{x \log(x)}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log(x) \log\left(1 + \frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log(x) \log\left(1 + \frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{PolyLog}\left(2, -\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{PolyLog}\left(2, -\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2}$$

output

```
-x/c+x*ln(x)/c-1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b+(2*a*c-b^2)
/(-4*a*c+b^2)^(1/2))/c^2-1/2*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^2-1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1
/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2-1/2*polylog(2,-2*c*x/(b+(-4*
a*c+b^2)^(1/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2
```

3.352.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.85

$$\int \frac{x^2\log(x)}{a+bx+cx^2}\,dx = -\frac{x}{c}+\frac{x\log(x)}{c}-\frac{a\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} + \frac{a\log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{a\,\mathrm{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} + \frac{a\,\mathrm{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2}$$

input
```
Integrate[(x^2*Log[x])/(a + b*x + c*x^2),x]
```

output
```
-(x/c) + (x*Log[x])/c - (a*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b -
 Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]) - (b*(1 - b/Sqrt[b^2 - 4*a*c])
*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*c
^2) + (a*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c]
)])/(c*Sqrt[b^2 - 4*a*c]) - (b*(1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + S
qrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^2) - (a*PolyLog[2
, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]) - (b*(1 - b/Sqr
t[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*c^2) + (a
*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]) - (b*
(1 + b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2
*c^2)
```

3.352.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log(x)}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{2804}$$

$$\int \left(\frac{\log(x)}{c}-\frac{\log(x)(a+bx)}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{\log(x)\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2c^2}-\frac{\log(x)\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2c^2}-\frac{x}{c}+\frac{x\log(x)}{c}$$

input
```
Int[(x^2*Log[x])/(a + b*x + c*x^2),x]
```

output
```
-(x/c) + (x*Log[x])/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[x]*Log[
1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*c^2) - ((b + (b^2 - 2*a*c)/Sqrt[b
^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^2) - ((
b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4
*a*c])])/(2*c^2) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c
*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^2)
```

3.352.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.352.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(212) = 424$.

Time = 1.10 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.32

method	result
default	$\frac{\ln(x)x-x}{c}+\frac{-\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b+2\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)ac-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2+\ln\left(\frac{2xc+\sqrt{-4ca+}}{b+\sqrt{-4ca+}}\right.}{2c\sqrt{-4ca+b^2}}}{}$
risch	$\frac{x\ln(x)}{c}-\frac{x}{c}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b}{2c^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)a}{c\sqrt{-4ca+b^2}}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2c^2\sqrt{-4ca+b^2}}-\ln(x$

input
```
int(x^2*ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*(ln(x)*x-x)+1/c*(-1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*
a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b+2*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(
-b+(-4*a*c+b^2)^(1/2)))*a*c-ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b
^2)^(1/2)))*b^2+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-
4*a*c+b^2)^(1/2)*b-2*ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)
))*a*c+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/c/(-4*
a*c+b^2)^(1/2)-1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(
1/2)))*(-4*a*c+b^2)^(1/2)*b+2*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*
a*c+b^2)^(1/2)))*a*c-dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^
(1/2)))*b^2+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4
*a*c+b^2)^(1/2)*b-2*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/
2)))*a*c+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/c
/(-4*a*c+b^2)^(1/2))
```

3.352.5 Fricas [F]

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} dx = \int \frac{x^2 \log(x)}{cx^2 + bx + a} dx$$

input
```
integrate(x^2*log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*log(x)/(c*x^2 + b*x + a), x)
```

3.352.6 Sympy [F]

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} dx = \int \frac{x^2 \log(x)}{a + bx + cx^2} dx$$

input
```
integrate(x**2*ln(x)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**2*log(x)/(a + b*x + c*x**2), x)
```

3.352.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.352.8 Giac [F]

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^2 \log(x)}{cx^2 + bx + a} \, dx$$

input
```
integrate(x^2*log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*log(x)/(c*x^2 + b*x + a), x)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^2 \ln(x)}{c\,x^2 + b\,x + a} \, dx$$

input
```
int((x^2*log(x))/(a + b*x + c*x^2),x)
```

output
```
int((x^2*log(x))/(a + b*x + c*x^2), x)
```

3.353 $\int \frac{x\log(x)}{a+bx+cx^2}\,dx$

3.353.1 Optimal result

Integrand size = 16, antiderivative size = 193

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c}$$

output
```
1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2)^(1/2))/c+1/
2*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2)^(1/2))/c+1/2*
ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/2))/c+1/2*po
lylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/2))/c
```

3.353.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \frac{\log(x)\left(\left(-b+\sqrt{b^2-4ac}\right)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)+\left(b+\sqrt{b^2-4ac}\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)\right)+\left(-b+\sqrt{b^2-4\right.}{2c\sqrt{b^2-4ac}}$$

input
```
Integrate[(x*Log[x])/(a + b*x + c*x^2),x]
```

output
```
(Log[x]*((-b + Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b -
 Sqrt[b^2 - 4*a*c])] + (b + Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]) + (-b + Sqrt[b^2 - 4*a*c])*PolyLog[2, (
2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b + Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*
c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c*Sqrt[b^2 - 4*a*c])
```

3.353.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 2804$$

$$\int \left(\frac{\log(x)\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\log(x)\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)}{\sqrt{b^2-4ac}+b+2cx}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c}+\frac{\log(x)\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2c}+\frac{\log(x)\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2c}$$

input
```
Int[(x*Log[x])/(a + b*x + c*x^2),x]
```

output
```
((1 - b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]
)/(2*c) + ((1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2
- 4*a*c])])/(2*c) + ((1 - b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sq
rt[b^2 - 4*a*c])])/(2*c) + ((1 + b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/
(b + Sqrt[b^2 - 4*a*c])])/(2*c)
```

3.353.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.353.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(169) = 338$.

Time = 1.05 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.87

method	result
risch	$\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right.}{2c\sqrt{-4ca+b^2}}$
default	$\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right.}{2c\sqrt{-4ca+b^2}}$

input
```
int(x*ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a
*c+b^2)^(1/2)-ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b+
ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)
+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b)/c/(-4*a*c+b^2)
^(1/2)+1/2/c*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))-
1/2/c/(-4*a*c+b^2)^(1/2)*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b
^2)^(1/2)))*b+1/2/c*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/
2)))+1/2/c/(-4*a*c+b^2)^(1/2)*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*
c+b^2)^(1/2)))*b
```

3.353.5 Fricas [F]

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\log(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(x*log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(x*log(x)/(c*x^2 + b*x + a), x)
```

3.353.6 Sympy [F]

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\log(x)}{a+bx+cx^2}\,dx$$

input
```
integrate(x*ln(x)/(c*x**2+b*x+a),x)
```

output
```
Integral(x*log(x)/(a + b*x + c*x**2), x)
```

3.353.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.353.8 Giac [F]

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\log(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(x*log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(x*log(x)/(c*x^2 + b*x + a), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\ln(x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x*log(x))/(a + b*x + c*x^2),x)
```

output
```
int((x*log(x))/(a + b*x + c*x^2), x)
```

3.354 $\int \frac{\log(x)}{a+bx+cx^2}\,dx$

3.354.1 Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \frac{\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} - \frac{\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \frac{\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} - \frac{\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)-ln(x)*ln(1+2*c
*x/(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+polylog(2,-2*c*x/(b-(-4*a*c+
b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)-polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(
-4*a*c+b^2)^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \frac{\log(x)\left(\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)-\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)\right)+\text{PolyLog}\left(2,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)-\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4a}}\right.}{\sqrt{b^2-4ac}}$$

input
```
Integrate[Log[x]/(a + b*x + c*x^2),x]
```

output
```
(Log[x]*(Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - Lo
g[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]) + PolyLog[2, (
2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] - PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a
*c])])/Sqrt[b^2 - 4*a*c]
```

3.354.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2804}$$

$$\int \left(\frac{2c\log(x)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c\log(x)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+\frac{\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{\sqrt{b^2-4ac}}-\frac{\log(x)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[Log[x]/(a + b*x + c*x^2),x]
```

output
```
(Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] - (Log
[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] + PolyLog[
2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[b^2 - 4*a*c] - PolyLog[2, (-2*c*
x)/(b + Sqrt[b^2 - 4*a*c])]/Sqrt[b^2 - 4*a*c]
```

3.354.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.354.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

method	result	siz
default	$-\frac{\ln(x)\left(\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\right)}{\sqrt{-4ca+b^2}}+\frac{\operatorname{dilog}\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)-\operatorname{dilog}\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)}{\sqrt{-4ca+b^2}}$	16
risch	$-\frac{\ln(x)\left(\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\right)}{\sqrt{-4ca+b^2}}+\frac{\operatorname{dilog}\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)}{\sqrt{-4ca+b^2}}-\frac{\operatorname{dilog}\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)}{\sqrt{-4ca+b^2}}$	17

input
```
int(ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)*(ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))-ln((-2*x*c
+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)+(dilog
((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))-dilog((2*x*c+(-4*a
*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

3.354.5 Fricas [F]

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx=\int \frac{\log(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(log(x)/(c*x^2 + b*x + a), x)
```

3.354.6 Sympy [F]

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \int \frac{\log(x)}{a+bx+cx^2}\,dx$$

input
```
integrate(ln(x)/(c*x**2+b*x+a),x)
```

output
```
Integral(log(x)/(a + b*x + c*x**2), x)
```

3.354.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.354.8 Giac [F]

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \int \frac{\log(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(log(x)/(c*x^2 + b*x + a), x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \int \frac{\ln(x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(log(x)/(a + b*x + c*x^2),x)
```

output
```
int(log(x)/(a + b*x + c*x^2), x)
```

3.355 $\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx$

3.355.1 Optimal result

Integrand size = 18, antiderivative size = 204

$$\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx = \frac{\log^2(x)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a}$$

output
```
1/2*ln(x)^2/a-1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^
2)^(1/2))/a-1/2*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2)
^(1/2))/a-1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(
1/2))/a-1/2*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/
2))/a
```

3.355.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx = \frac{\log(x)\left(\sqrt{b^2-4ac}\log(x)-\left(b+\sqrt{b^2-4ac}\right)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)\right)}{2a\sqrt{b^2-4ac}}$$

input
```
Integrate[Log[x]/(x*(a + b*x + c*x^2)),x]
```

output
```
(Log[x]*(Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*L
og[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]) - (b + Sqrt[b
^2 - 4*a*c])*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[b^2 - 4*
a*c])
```

3.355.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \; 2804$$

$$\int \left(\frac{\log(x)(-b-cx)}{a(a+bx+cx^2)}+\frac{\log(x)}{ax}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a}-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a}-\frac{\log(x)\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2a}-\frac{\log(x)\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2a}+\frac{\log^2(x)}{2a}$$

input
```
Int[Log[x]/(x*(a + b*x + c*x^2)),x]
```

output
```
Log[x]^2/(2*a) - ((1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sq
rt[b^2 - 4*a*c])])/(2*a) - ((1 - b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*
x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a) - ((1 + b/Sqrt[b^2 - 4*a*c])*PolyLog[2,
 (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a) - ((1 - b/Sqrt[b^2 - 4*a*c])*Pol
yLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a)
```

3.355.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.355.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(178) = 356$.

Time = 1.07 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.81

method	result
default	$\frac{\ln(x)^2}{2a} + \frac{-\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}+\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}-\ln\left(\frac{2xc+\sqrt{-4ca+}}{b+\sqrt{-4ca+}}\right.}{2\sqrt{-4ca+b^2}}}{}$
risch	$\frac{\ln(x)^2}{2a} - \frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)}{2a} - \frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b}{2a\sqrt{-4ca+b^2}} - \frac{\ln(x)\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)}{2a} + \frac{\ln(x)\ln\left(\frac{2xc+\sqrt{}}{b+\sqrt{}}\right.}{2a\sqrt{-4}}$

input
```
int(ln(x)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*ln(x)^2+1/a*(-1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*
c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)+ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4
*a*c+b^2)^(1/2)))*b+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))
)*(-4*a*c+b^2)^(1/2)-ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)
))*b)/(-4*a*c+b^2)^(1/2)-1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*
a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-
b+(-4*a*c+b^2)^(1/2)))*b+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2
)^(1/2)))*(-4*a*c+b^2)^(1/2)-dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c
+b^2)^(1/2)))*b)/(-4*a*c+b^2)^(1/2))
```

3.355.5 Fricas [F]

$$\int \frac{\log(x)}{x\left(a+bx+cx^2\right)}\,dx=\int \frac{\log\left(x\right)}{(cx^2+bx+a)x}\,dx$$

input
```
integrate(log(x)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(log(x)/(c*x^3 + b*x^2 + a*x), x)
```

3.355.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(x)/x/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.355.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(x)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.355.8 Giac [F]

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \int \frac{\log(x)}{(cx^2+bx+a)x}\,dx$$

input
```
integrate(log(x)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(log(x)/((c*x^2 + b*x + a)*x), x)
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \int \frac{\ln(x)}{x\,(c\,x^2+b\,x+a)}\,dx$$

input
```
int(log(x)/(x*(a + b*x + c*x^2)),x)
```

output
```
int(log(x)/(x*(a + b*x + c*x^2)), x)
```

3.356 $\int \frac{\log(x)}{x^2(a+bx+cx^2)} dx$

3.356.1 Optimal result

Integrand size = 18, antiderivative size = 251

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)} dx = -\frac{1}{ax} - \frac{\log(x)}{ax} - \frac{b\log^2(x)}{2a^2} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2}$$

output
```
-1/a/x-ln(x)/a/x-1/2*b*ln(x)^2/a^2+1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1
/2)))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2+1/2*polylog(2,-2*c*x/(b+(-4*a
*c+b^2)^(1/2)))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2+1/2*ln(x)*ln(1+2*c*
x/(b-(-4*a*c+b^2)^(1/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2+1/2*poly
log(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a
^2
```

3.356.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx$$
$$= \frac{-\frac{2a}{x}-\frac{2a\log(x)}{x}-b\log^2(x)+\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)+\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b+\sqrt{b^2-\cdots}}{b+\sqrt{b^2\cdots}}\right.\cdots}{2a^2}$$

input
```
Integrate[Log[x]/(x^2*(a + b*x + c*x^2)),x]
```

output
```
((-2*a)/x - (2*a*Log[x])/x - b*Log[x]^2 + (b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*
a*c])*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]
+ (b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + (b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b + (-b^2 + 2*a*c)/Sqrt[b
^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2)
```

3.356.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx$$
$$\Big\downarrow \text{2804}$$
$$\int \left(\frac{\log(x)\left(-ac+b^2+bcx\right)}{a^2(a+bx+cx^2)}-\frac{b\log(x)}{a^2x}+\frac{\log(x)}{ax^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2}+\frac{\log(x)\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2a^2}+\frac{\log(x)\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2a^2}-\frac{b\log^2(x)}{2a^2}-\frac{1}{ax}-\frac{\log(x)}{ax}$$

input
```
Int[Log[x]/(x^2*(a + b*x + c*x^2)),x]
```

output
```
-(1/(a*x)) - Log[x]/(a*x) - (b*Log[x]^2)/(2*a^2) + ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2) +
 ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b
^2 - 4*a*c])])/(2*a^2) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*PolyLog[2,
 (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2) + ((b - (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2)
```

3.356.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.356.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(227) = 454$.

Time = 1.18 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.20

method	result
default	$-\frac{b\ln(x)^2}{2a^2}+\frac{\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b-2\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)ac+\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2+\ln\left(\frac{2xc+\sqrt{-4ca+b}}{b+\sqrt{-4ca+b}}\right.}{2\sqrt{-4ca+b^2}}}{}$
risch	$-\frac{b\ln(x)^2}{2a^2}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b}{2a^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)c}{a\sqrt{-4ca+b^2}}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2a^2\sqrt{-4ca+b^2}}+\frac{\ln(x)\ln}{}$

input
```
int(ln(x)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln(x)^2/a^2+1/a^2*(1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+
(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b-2*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-
b)/(-b+(-4*a*c+b^2)^(1/2)))*a*c+ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a
*c+b^2)^(1/2)))*b^2+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))
)*(-4*a*c+b^2)^(1/2)*b+2*ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(
1/2)))*a*c-ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/(-
4*a*c+b^2)^(1/2)+1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)
^(1/2)))*(-4*a*c+b^2)^(1/2)*b-2*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-
4*a*c+b^2)^(1/2)))*a*c+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2
)^(1/2)))*b^2+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(
-4*a*c+b^2)^(1/2)*b+2*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(
1/2)))*a*c-dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)
/(-4*a*c+b^2)^(1/2))+1/a*(-1/x*ln(x)-1/x)
```

3.356.5 Fricas [F]

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = \int \frac{\log(x)}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate(log(x)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(log(x)/(c*x^4 + b*x^3 + a*x^2), x)
```

3.356.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(ln(x)/x**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.356.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(x)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.356.8 Giac [F]

$$\int \frac{\log(x)}{x^2\left(a+bx+cx^2\right)}\,dx = \int \frac{\log\left(x\right)}{\left(cx^2+bx+a\right)x^2}\,dx$$

input
```
integrate(log(x)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(log(x)/((c*x^2 + b*x + a)*x^2), x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^2\left(a+bx+cx^2\right)}\,dx = \int \frac{\ln\left(x\right)}{x^2\,\left(c\,x^2+b\,x+a\right)}\,dx$$

input
```
int(log(x)/(x^2*(a + b*x + c*x^2)),x)
```

output
```
int(log(x)/(x^2*(a + b*x + c*x^2)), x)
```

3.357 $\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx$

3.357.1 Optimal result

Integrand size = 18, antiderivative size = 308

$$\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx = -\frac{1}{4ax^2}+\frac{b}{a^2x}-\frac{\log(x)}{2ax^2}+\frac{b\log(x)}{a^2x}+\frac{(b^2-ac)\log^2(x)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\mathrm{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\mathrm{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3}$$

output
```
-1/4/a/x^2+b/a^2/x-1/2*ln(x)/a/x^2+b*ln(x)/a^2/x+1/2*(-a*c+b^2)*ln(x)^2/a^
3-1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^2-a*c-b*(-3*a*c+b^2)/(-4
*a*c+b^2)^(1/2))/a^3-1/2*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^2-a*c
-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3-1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^
2)^(1/2)))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3-1/2*polylog(2,-
2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/
a^3
```

3.357.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.01

$$\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx = -\frac{\frac{a^2}{x^2}-\frac{4ab}{x}+\frac{2a^2\log(x)}{x^2}-\frac{4ab\log(x)}{x}-2(b^2-ac)\log^2(x)+2\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{}$$

input
```
Integrate[Log[x]/(x^3*(a + b*x + c*x^2)),x]
```

output
```
-1/4*(a^2/x^2 - (4*a*b)/x + (2*a^2*Log[x])/x^2 - (4*a*b*Log[x])/x - 2*(b^2
 - a*c)*Log[x]^2 + 2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log
[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + 2*(b^2
- a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + 2*(b^2 - a*c + (b*(b^2 - 3*a*c))/
Sqrt[b^2 - 4*a*c])*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(b^2 -
 a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[
b^2 - 4*a*c])])/a^3
```

3.357.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{2804}$$

$$\int \left(\frac{\log(x)\left(-cx(b^2-ac)-b(b^2-2ac)\right)}{a^3(a+bx+cx^2)}+\frac{\log(x)(b^2-ac)}{a^3x}-\frac{b\log(x)}{a^2x^2}+\frac{\log(x)}{ax^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3}-\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3}+\frac{\log^2(x)\left(b^2-ac\right)}{2a^3}-\frac{\log(x)\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2a^3}-\frac{\log(x)\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2a^3}+\frac{b}{a^2x}+\frac{b\log(x)}{a^2x}-\frac{1}{4ax^2}-\frac{\log(x)}{2ax^2}$$

input
```
Int[Log[x]/(x^3*(a + b*x + c*x^2)),x]
```

output
```
-1/4*1/(a*x^2) + b/(a^2*x) - Log[x]/(2*a*x^2) + (b*Log[x])/(a^2*x) + ((b^2
 - a*c)*Log[x]^2)/(2*a^3) - ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^3) - ((b^2 - a*
c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[
b^2 - 4*a*c])])/(2*a^3) - ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^3) - ((b^2 - a*c - (
b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*
a*c])])/(2*a^3)
```

3.357.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.357.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(278) = 556$.

Time = 1.16 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.54

method	result
default	$\frac{(-ca+b^2)\ln(x)^2}{2a^3}+\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,ac-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b^2+3\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)ab}{}$
risch	$-\frac{\ln(x)^2c}{2a^2}+\frac{\ln(x)^2b^2}{2a^3}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)c}{2a^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2a^3}+\frac{3\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)}{2a^2\sqrt{-4ca+b^2}}$

input
```
int(ln(x)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*c+b^2)*ln(x)^2/a^3+1/a^3*(1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)
-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((-2*x*c+(-4*a*c+b^2
)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2+3*ln((-2*x*c+(-
4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c-ln((-2*x*c+(-4*a*c+b^2)
^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+
(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((2*x*c+(-4*a*c+b^2)^(1/2)+b
)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2-3*ln((2*x*c+(-4*a*c+b^2)^
(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+
(-4*a*c+b^2)^(1/2)))*b^3)/(-4*a*c+b^2)^(1/2)+1/2*(dilog((-2*x*c+(-4*a*c+b^
2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-dilog((-2*x*c+
(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2+3*di
log((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c-dilog((-2
*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+dilog((2*x*c+(-4*a
*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-dilog((2*x
*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2-3*
dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c+dilog((2*
x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^3)/(-4*a*c+b^2)^(1/2))
+1/a*(-1/2/x^2*ln(x)-1/4/x^2)-1/a^2*b*(-1/x*ln(x)-1/x)
```

3.357.5 Fricas [F]

$$\int \frac{\log(x)}{x^3\,(a+bx+cx^2)}\,dx=\int \frac{\log(x)}{(cx^2+bx+a)x^3}\,dx$$

input
```
integrate(log(x)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(log(x)/(c*x^5 + b*x^4 + a*x^3), x)
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} \, dx = \text{Timed out}$$

input
```
integrate(ln(x)/x**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.357.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(x)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.357.8 Giac [F]

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} \, dx = \int \frac{\log(x)}{(cx^2 + bx + a)x^3} \, dx$$

input
```
integrate(log(x)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(log(x)/((c*x^2 + b*x + a)*x^3), x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} \, dx = \int \frac{\ln(x)}{x^3 \left(c x^2 + b x + a\right)} \, dx$$

input
```
int(log(x)/(x^3*(a + b*x + c*x^2)),x)
```

output
```
int(log(x)/(x^3*(a + b*x + c*x^2)), x)
```

3.358 $\int x^3 \log (f x^m) (a + b \log (c(d + ex)^n)) \, dx$

3.358.1 Optimal result

Integrand size = 24, antiderivative size = 232

$$\begin{aligned}\int x^3 \log (f x^m) (a+b \log (c(d+ex)^n)) \, dx = &-\frac{5bd^3mnx}{16e^3}+\frac{3bd^2mnx^2}{32e^2}-\frac{7bdmnx^3}{144e}+\frac{1}{32}bmnx^4\\ &+\frac{bd^3nx\log (fx^m)}{4e^3}-\frac{bd^2nx^2\log (fx^m)}{8e^2}\\ &+\frac{bdnx^3\log (fx^m)}{12e}-\frac{1}{16}bnx^4\log (fx^m)\\ &+\frac{bd^4mn\log(d+ex)}{16e^4}-\frac{1}{16}(mx^4\\ &-4x^4\log (fx^m)) (a+b\log (c(d+ex)^n))\\ &-\frac{bd^4n\log (fx^m)\log \left(1+\frac{ex}{d}\right)}{4e^4}\\ &-\frac{bd^4mn \operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{4e^4}\end{aligned}$$

output
```
-5/16*b*d^3*m*n*x/e^3+3/32*b*d^2*m*n*x^2/e^2-7/144*b*d*m*n*x^3/e+1/32*b*m*
n*x^4+1/4*b*d^3*n*x*ln(f*x^m)/e^3-1/8*b*d^2*n*x^2*ln(f*x^m)/e^2+1/12*b*d*n
*x^3*ln(f*x^m)/e-1/16*b*n*x^4*ln(f*x^m)+1/16*b*d^4*m*n*ln(e*x+d)/e^4-1/16*
(m*x^4-4*x^4*ln(f*x^m))*(a+b*ln(c*(e*x+d)^n))-1/4*b*d^4*n*ln(f*x^m)*ln(1+e
*x/d)/e^4-1/4*b*d^4*m*n*polylog(2,-e*x/d)/e^4
```

3.358.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.95

$$\int x^3 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$$
$$= \frac{-6\log(fx^m)(-12ae^4x^4+benx(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+12bd^4n\log(d+ex)-12be^4x^4\log(c$$

input
```
Integrate[x^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(-6*Log[f*x^m]*(-12*a*e^4*x^4 + b*e*n*x*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2
 + 3*e^3*x^3) + 12*b*d^4*n*Log[d + e*x] - 12*b*e^4*x^4*Log[c*(d + e*x)^n])
 + m*(-90*b*d^3*e*n*x + 27*b*d^2*e^2*n*x^2 - 14*b*d*e^3*n*x^3 - 18*a*e^4*x
^4 + 9*b*e^4*n*x^4 + 18*b*d^4*n*(1 + 4*Log[x])*Log[d + e*x] - 18*b*e^4*x^4
*Log[c*(d + e*x)^n] - 72*b*d^4*n*Log[x]*Log[1 + (e*x)/d]) - 72*b*d^4*m*n*P
olyLog[2, -((e*x)/d)])/(288*e^4)
```

3.358.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2873, 49, 2009, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$$
$$\Big\downarrow \; 2873$$
$$-\frac{1}{4}ben\int \frac{x^4\log(fx^m)}{d+ex}dx+\frac{1}{16}bemn\int \frac{x^4}{d+ex}dx-\frac{1}{16}(mx^4-4x^4\log(fx^m))(a+b\log(c(d+ex)^n))$$
$$\Big\downarrow \; 49$$
$$\frac{1}{16}bemn\int \left(\frac{d^4}{e^4(d+ex)}-\frac{d^3}{e^4}+\frac{xd^2}{e^3}-\frac{x^2d}{e^2}+\frac{x^3}{e}\right)dx-\frac{1}{4}ben\int \frac{x^4\log(fx^m)}{d+ex}dx-\frac{1}{16}(mx^4-4x^4\log(fx^m))(a+b\log(c(d+ex)^n))$$
$$\Big\downarrow \; 2009$$

$$-\frac{1}{4}ben\int\frac{x^4\log(fx^m)}{d+ex}dx-\frac{1}{16}\left(mx^4-4x^4\log(fx^m)\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(\frac{d^4\log(d+ex)}{e^5}-\frac{d^3x}{e^4}+\frac{d^2x^2}{2e^3}-\frac{dx^3}{3e^2}+\frac{x^4}{4e}\right)$$

$$\Big\downarrow \text{2793}$$

$$-\frac{1}{4}ben\int\left(\frac{\log(fx^m)d^4}{e^4(d+ex)}-\frac{\log(fx^m)d^3}{e^4}+\frac{x\log(fx^m)d^2}{e^3}-\frac{x^2\log(fx^m)d}{e^2}+\frac{x^3\log(fx^m)}{e}\right)dx-\frac{1}{16}\left(mx^4-4x^4\log(fx^m)\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(\frac{d^4\log(d+ex)}{e^5}-\frac{d^3x}{e^4}+\frac{d^2x^2}{2e^3}-\frac{dx^3}{3e^2}+\frac{x^4}{4e}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\left(mx^4-4x^4\log(fx^m)\right)(a+b\log(c(d+ex)^n))-\frac{1}{4}ben\left(\frac{d^4\log\left(\frac{ex}{d}+1\right)\log(fx^m)}{e^5}+\frac{d^4m\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^5}-\frac{d^3x\log(fx^m)}{e^4}+\frac{d^3mx}{e^4}+\frac{d^2x^2\log(fx^m)}{2e^3}-\frac{d^2mx^2}{4e^3}-\right.$$
$$\frac{1}{16}bemn\left(\frac{d^4\log(d+ex)}{e^5}-\frac{d^3x}{e^4}+\frac{d^2x^2}{2e^3}-\frac{dx^3}{3e^2}+\frac{x^4}{4e}\right)$$

input
```
Int[x^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(b*e*m*n*(-((d^3*x)/e^4) + (d^2*x^2)/(2*e^3) - (d*x^3)/(3*e^2) + x^4/(4*e)
 + (d^4*Log[d + e*x])/e^5))/16 - ((m*x^4 - 4*x^4*Log[f*x^m])*(a + b*Log[c*
(d + e*x)^n]))/16 - (b*e*n*((d^3*m*x)/e^4 - (d^2*m*x^2)/(4*e^3) + (d*m*x^3
)/(9*e^2) - (m*x^4)/(16*e) - (d^3*x*Log[f*x^m])/e^4 + (d^2*x^2*Log[f*x^m])
/(2*e^3) - (d*x^3*Log[f*x^m])/(3*e^2) + (x^4*Log[f*x^m])/(4*e) + (d^4*Log[
f*x^m]*Log[1 + (e*x)/d])/e^5 + (d^4*m*PolyLog[2, -((e*x)/d)])/e^5))/4
```

3.358.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.358.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 78.15 (sec) , antiderivative size = 1180, normalized size of antiderivative = 5.09

method	result	size
risch	Expression too large to display	1180

input
```
int(x^3*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-205/576*b*d^4*m*n/e^4-1/32*I*n*b*x^4*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/32*I*
n*b*x^4*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+
d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I
*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^
n)^3+1/2*b*ln(c)+1/2*a)*(1/4*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*
Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*
f*x^m)^3+2*ln(f))*x^4+1/2*x^4*ln(x^m)-1/8*m*x^4)-1/8/e^2*n*b*ln(x^m)*x^2*d
^2+1/4/e^3*n*b*ln(x^m)*x*d^3-1/4/e^4*n*b*ln(x^m)*d^4*ln(e*x+d)+1/12/e*n*b*
d*x^3*ln(f)-1/8/e^2*n*b*d^2*x^2*ln(f)+1/4/e^3*n*b*d^3*x*ln(f)-1/4/e^4*n*b*
d^4*ln(e*x+d)*ln(f)+1/32*I*n*b*x^4*Pi*csgn(I*f*x^m)^3+1/4*m/e^4*b*d^4*n*di
log(-e*x/d)-1/16*n*b*ln(x^m)*x^4-1/16*n*b*x^4*ln(f)-1/16*I/e^2*n*b*d^2*x^2
*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/8*I/e^3*n*b*d^3*x*Pi*csgn(I*f)*csgn(I*f*
x^m)^2+1/8*I/e^3*n*b*d^3*x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/8*I/e^4*n*b*d^
4*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+(1/4*b*x^4*ln(x^m)+1/16*b*x^4*(-2
*I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+2*I*Pi*csgn(I*f)*csgn(I*f*x^m)^2
+2*I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-2*I*Pi*csgn(I*f*x^m)^3+4*ln(f)-m))*ln(
(e*x+d)^n)-1/24*I/e*n*b*d*x^3*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/16*
I/e^2*n*b*d^2*x^2*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/8*I/e^3*n*b*d^3
*x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*c
sgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*csgn(...
```

3.358.5 Fricas [F]

$$\int x^3 \log(fx^m)(a + b\log(c(d+ex)^n))\, dx = \int (b\log((ex+d)^n c) + a)x^3 \log(fx^m)\, dx$$

input
```
integrate(x^3*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(b*x^3*log((e*x + d)^n*c)*log(f*x^m) + a*x^3*log(f*x^m), x)
```

3.358.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right)\, dx = \text{Timed out}$$

input
```
integrate(x**3*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Timed out
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right)\, dx$$
$$= \frac{1}{288}\left(\frac{72\left(\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right) + \text{Li}_2\left(\frac{ex+d}{d}\right)\right)bd^4n}{e^4} - \frac{18\, be^4x^4\log\left((ex+d)^n\right) + 14\, bde^3nx^3 - 27\, bd}{}\right.$$
$$+ \frac{1}{48}\left(12\, bx^4\log\left((ex+d)^n c\right) + 12\, ax^4 - ben\left(\frac{12\, d^4\log(ex+d)}{e^5} + \frac{3\, e^3x^4 - 4\, de^2x^3 + 6\, d^2ex^2 - 12\, d^3x}{e^4}\right)\right.$$

input
```
integrate(x^3*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/288*(72*(log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d^4*
n/e^4 - (18*b*e^4*x^4*log((e*x + d)^n) + 14*b*d*e^3*n*x^3 - 27*b*d^2*e^2*n
*x^2 + 90*b*d^3*e*n*x - 18*b*d^4*n*log(e*x + d) + 9*(2*a*e^4 - (e^4*n - 2*
e^4*log(c))*b)*x^4)/e^4)*m + 1/48*(12*b*x^4*log((e*x + d)^n*c) + 12*a*x^4
- b*e*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2
- 12*d^3*x)/e^4))*log(f*x^m)
```

3.358.8 Giac [F]

$$\int x^3 \log(fx^m)(a + b\log(c(d + ex)^n))\,dx = \int (b\log((ex + d)^n c) + a)x^3 \log(fx^m)\,dx$$

input
```
integrate(x^3*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^3*log(f*x^m), x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \log(fx^m)(a + b\log(c(d + ex)^n))\,dx = \int x^3 \ln(f\,x^m)\,(a + b\,\ln(c\,(d + e\,x)^n))\,dx$$

input
```
int(x^3*log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(x^3*log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

3.359 $\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$

3.359.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \frac{4bd^2mnx}{9e^2} - \frac{5bdmnx^2}{36e} + \frac{2}{27}bmnx^3 - \frac{bd^2nx\log(fx^m)}{3e^2} + \frac{bdnx^2\log(fx^m)}{6e} - \frac{1}{9}bnx^3\log(fx^m) - \frac{bd^3mn\log(d+ex)}{9e^3} - \frac{1}{9}(mx^3-3x^3\log(fx^m))(a+b\log(c(d+ex)^n)) + \frac{bd^3n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{3e^3} + \frac{bd^3mn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{3e^3}$$

output
```
4/9*b*d^2*m*n*x/e^2-5/36*b*d*m*n*x^2/e+2/27*b*m*n*x^3-1/3*b*d^2*n*x*ln(f*x
^m)/e^2+1/6*b*d*n*x^2*ln(f*x^m)/e-1/9*b*n*x^3*ln(f*x^m)-1/9*b*d^3*m*n*ln(e
*x+d)/e^3-1/9*(m*x^3-3*x^3*ln(f*x^m))*(a+b*ln(c*(e*x+d)^n))+1/3*b*d^3*n*ln
(f*x^m)*ln(1+e*x/d)/e^3+1/3*b*d^3*m*n*polylog(2,-e*x/d)/e^3
```

3.359.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.01

$$\int x^2 \log(fx^m)(a + b\log(c(d + ex)^n)) \, dx$$
$$= \frac{6\log(fx^m)(6ae^3x^3 + benx(-6d^2 + 3dex - 2e^2x^2) + 6bd^3n\log(d + ex) + 6be^3x^3\log(c(d + ex)^n)) + m(48}{}$$

input
```
Integrate[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(6*Log[f*x^m]*(6*a*e^3*x^3 + b*e*n*x*(-6*d^2 + 3*d*e*x - 2*e^2*x^2) + 6*b*
d^3*n*Log[d + e*x] + 6*b*e^3*x^3*Log[c*(d + e*x)^n]) + m*(48*b*d^2*e*n*x -
 15*b*d*e^2*n*x^2 - 12*a*e^3*x^3 + 8*b*e^3*n*x^3 - 12*b*d^3*n*(1 + 3*Log[x
])*Log[d + e*x] - 12*b*e^3*x^3*Log[c*(d + e*x)^n] + 36*b*d^3*n*Log[x]*Log[
1 + (e*x)/d]) + 36*b*d^3*m*n*PolyLog[2, -((e*x)/d)])/(108*e^3)
```

3.359.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2873, 49, 2009, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(fx^m)(a + b\log(c(d + ex)^n)) \, dx$$

$$\Big\downarrow \; 2873$$

$$-\frac{1}{3}ben\int \frac{x^3\log(fx^m)}{d + ex}dx + \frac{1}{9}bemn\int \frac{x^3}{d + ex}dx - \frac{1}{9}(mx^3 - 3x^3\log(fx^m))(a + b\log(c(d + ex)^n))$$

$$\Big\downarrow \; 49$$

$$\frac{1}{9}bemn\int \left(-\frac{d^3}{e^3(d + ex)} + \frac{d^2}{e^3} - \frac{xd}{e^2} + \frac{x^2}{e}\right)dx - \frac{1}{3}ben\int \frac{x^3\log(fx^m)}{d + ex}dx - \frac{1}{9}(mx^3 - 3x^3\log(fx^m))(a + b\log(c(d + ex)^n))$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{3}ben\int\frac{x^3\log\left(fx^m\right)}{d+ex}dx-\frac{1}{9}\left(mx^3-3x^3\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+$$
$$\frac{1}{9}bemn\left(-\frac{d^3\log(d+ex)}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{2e^2}+\frac{x^3}{3e}\right)$$

$$\Big\downarrow \text{2793}$$

$$-\frac{1}{3}ben\int\left(-\frac{\log\left(fx^m\right)d^3}{e^3(d+ex)}+\frac{\log\left(fx^m\right)d^2}{e^3}-\frac{x\log\left(fx^m\right)d}{e^2}+\frac{x^2\log\left(fx^m\right)}{e}\right)dx-$$
$$\frac{1}{9}\left(mx^3-3x^3\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{1}{9}bemn\left(-\frac{d^3\log(d+ex)}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{2e^2}+\frac{x^3}{3e}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{9}\left(mx^3-3x^3\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-$$
$$\frac{1}{3}ben\left(-\frac{d^3\log\left(\frac{ex}{d}+1\right)\log\left(fx^m\right)}{e^4}-\frac{d^3m\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}+\frac{d^2x\log\left(fx^m\right)}{e^3}-\frac{d^2mx}{e^3}-\frac{dx^2\log\left(fx^m\right)}{2e^2}+\frac{dmx^2}{4e^2}+\right.$$
$$\frac{1}{9}bemn\left(-\frac{d^3\log(d+ex)}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{2e^2}+\frac{x^3}{3e}\right)$$

input
```
Int[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(b*e*m*n*((d^2*x)/e^3 - (d*x^2)/(2*e^2) + x^3/(3*e) - (d^3*Log[d + e*x])/e
^4))/9 - ((m*x^3 - 3*x^3*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]))/9 - (b*e*
n*(-((d^2*m*x)/e^3) + (d*m*x^2)/(4*e^2) - (m*x^3)/(9*e) + (d^2*x*Log[f*x^m
])/e^3 - (d*x^2*Log[f*x^m])/(2*e^2) + (x^3*Log[f*x^m])/(3*e) - (d^3*Log[f*
x^m]*Log[1 + (e*x)/d])/e^4 - (d^3*m*PolyLog[2, -((e*x)/d)])/e^4))/3
```

3.359.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.359.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.94 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.19

method	result	size
risch	Expression too large to display	1012

input
```
int(x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/e*n*b*d*x^2*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/18*I*n*b*x^3*
Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/18*I*n*b*x^3*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2
+1/6*I/e^2*n*b*d^2*x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/6*I/e^3*n*b*
d^3*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/3*m/e^3*b*d^3*n*dil
og(-e*x/d)-1/12*I/e*n*b*d*x^2*Pi*csgn(I*f*x^m)^3+1/6*I/e^2*n*b*d^2*x*Pi*cs
gn(I*f*x^m)^3+1/18*I*n*b*x^3*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/3/e^
2*n*b*ln(x^m)*x*d^2+49/108/e^3*n*b*m*d^3+(1/3*b*x^3*ln(x^m)+1/18*b*x^3*(-3
*I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+3*I*Pi*csgn(I*f)*csgn(I*f*x^m)^2
+3*I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-3*I*Pi*csgn(I*f*x^m)^3+6*ln(f)-2*m))*l
n((e*x+d)^n)-1/3*m/e^3*b*d^3*n*ln(e*x+d)*ln(-e*x/d)-1/6*I/e^3*n*b*d^3*ln(e
*x+d)*Pi*csgn(I*f*x^m)^3+2/27*b*m*n*x^3+1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*csg
n(I*f)*csgn(I*f*x^m)^2+1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I
*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*
c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(1/3*(-
I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*
Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))*x^3+2/3*x^3*l
n(x^m)-2/9*m*x^3)+1/3/e^3*n*b*d^3*ln(e*x+d)*ln(f)+1/18*I*n*b*x^3*Pi*csgn(I
*f*x^m)^3+1/3/e^3*n*b*ln(x^m)*d^3*ln(e*x+d)+1/6/e*n*b*ln(x^m)*d*x^2+1/6/e*
n*b*d*x^2*ln(f)-1/3/e^2*n*b*d^2*x*ln(f)-1/9*n*b*x^3*ln(f)+1/12*I/e*n*b*...
```

3.359.5 Fricas [F]

$$\int x^2 \log(fx^m)\,(a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)x^2\log(fx^m)\,dx$$

input
```
integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(b*x^2*log((e*x + d)^n*c)*log(f*x^m) + a*x^2*log(f*x^m), x)
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \log (f x^m) (a + b \log (c(d + ex)^n)) \, dx = \text{Timed out}$$

input
```
integrate(x**2*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Timed out
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.06

$$\int x^2 \log (f x^m) (a + b \log (c(d + ex)^n)) \, dx = -\frac{1}{108} \left(\frac{36 \left(\log (ex + d) \log \left(-\frac{ex+d}{d} + 1\right) + \text{Li}_2\left(\frac{ex+d}{d}\right)\right) b d^3 n}{e^3} + \frac{12 \, b e^3 x^3 \log ((ex + d)^n) + 15 \, b d e^2 n x^2 - 48 \right. + \frac{1}{18} \left(6 \, b x^3 \log ((ex + d)^n c) + 6 \, a x^3 + b e n \left(\frac{6 \, d^3 \log (ex + d)}{e^4} - \frac{2 \, e^2 x^3 - 3 \, d e x^2 + 6 \, d^2 x}{e^3} \right) \right) \log (f x^m)$$

input
```
integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
-1/108*(36*(log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d^3
*n/e^3 + (12*b*e^3*x^3*log((e*x + d)^n) + 15*b*d*e^2*n*x^2 - 48*b*d^2*e*n*
x + 12*b*d^3*n*log(e*x + d) + 4*(3*a*e^3 - (2*e^3*n - 3*e^3*log(c))*b)*x^3
)/e^3)*m + 1/18*(6*b*x^3*log((e*x + d)^n*c) + 6*a*x^3 + b*e*n*(6*d^3*log(e
*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3))*log(f*x^m)
```

3.359.8 Giac [F]

$$\int x^2 \log (f x^m) (a + b \log (c(d + ex)^n)) \, dx = \int (b \log ((ex + d)^n c) + a) x^2 \log (f x^m) \, dx$$

input
```
integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x^2*log(f*x^m), x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int x^2\,\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

input
```
int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

3.360 $\int x \log(fx^m)(a + b\log(c(d + ex)^n))\, dx$

3.360.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int x \log(fx^m)(a + b\log(c(d + ex)^n))\, dx = -\frac{3bdmnx}{4e} + \frac{1}{4}bmnx^2 + \frac{bdnx\log(fx^m)}{2e} - \frac{1}{4}bnx^2\log(fx^m) + \frac{bd^2mn\log(d+ex)}{4e^2} - \frac{1}{4}\left(mx^2 - 2x^2\log(fx^m)\right)(a + b\log(c(d+ex)^n)) - \frac{bd^2n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{2e^2} - \frac{bd^2mn\,\mathrm{PolyLog}\left(2, -\frac{ex}{d}\right)}{2e^2}$$

output
```
-3/4*b*d*m*n*x/e+1/4*b*m*n*x^2+1/2*b*d*n*x*ln(f*x^m)/e-1/4*b*n*x^2*ln(f*x^
m)+1/4*b*d^2*m*n*ln(e*x+d)/e^2-1/4*(m*x^2-2*x^2*ln(f*x^m))*(a+b*ln(c*(e*x+
d)^n))-1/2*b*d^2*n*ln(f*x^m)*ln(1+e*x/d)/e^2-1/2*b*d^2*m*n*polylog(2,-e*x/
d)/e^2
```

3.360.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int x \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$$
$$= \frac{\log(fx^m)(-2bd^2n\log(d+ex)+ex(2bdn+2aex-benx+2bex\log(c(d+ex)^n)))+m(-3bdenx-ae^2x\cdots}{\cdots}$$

input
```
Integrate[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(Log[f*x^m]*(-2*b*d^2*n*Log[d + e*x] + e*x*(2*b*d*n + 2*a*e*x - b*e*n*x +
2*b*e*x*Log[c*(d + e*x)^n])) + m*(-3*b*d*e*n*x - a*e^2*x^2 + b*e^2*n*x^2 +
 b*d^2*n*(1 + 2*Log[x])*Log[d + e*x] - b*e^2*x^2*Log[c*(d + e*x)^n] - 2*b*
d^2*n*Log[x]*Log[1 + (e*x)/d]) - 2*b*d^2*m*n*PolyLog[2, -((e*x)/d)])/(4*e^
2)
```

3.360.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2873, 49, 2009, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$$
$$\Big\downarrow \text{2873}$$
$$-\frac{1}{2}ben\int \frac{x^2\log(fx^m)}{d+ex}dx+\frac{1}{4}bemn\int \frac{x^2}{d+ex}dx-\frac{1}{4}(mx^2-2x^2\log(fx^m))(a+b\log(c(d+ex)^n))$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{4}bemn\int\left(\frac{d^2}{e^2(d+ex)}-\frac{d}{e^2}+\frac{x}{e}\right)dx-\frac{1}{2}ben\int \frac{x^2\log(fx^m)}{d+ex}dx-\frac{1}{4}(mx^2-2x^2\log(fx^m))(a+b\log(c(d+ex)^n))$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}ben\int\frac{x^2\log(fx^m)}{d+ex}dx-\frac{1}{4}\left(mx^2-2x^2\log(fx^m)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{1}{4}bemn\left(\frac{d^2\log(d+ex)}{e^3}-\frac{dx}{e^2}+\frac{x^2}{2e}\right)$$

$$\Big\downarrow \text{ 2793}$$

$$-\frac{1}{2}ben\int\left(\frac{\log(fx^m)\,d^2}{e^2(d+ex)}-\frac{\log(fx^m)\,d}{e^2}+\frac{x\log(fx^m)}{e}\right)dx-\frac{1}{4}\left(mx^2-2x^2\log(fx^m)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{1}{4}bemn\left(\frac{d^2\log(d+ex)}{e^3}-\frac{dx}{e^2}+\frac{x^2}{2e}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{4}\left(mx^2-2x^2\log(fx^m)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{1}{2}ben\left(\frac{d^2\log\left(\frac{ex}{d}+1\right)\log(fx^m)}{e^3}+\frac{d^2m\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}-\frac{dx\log(fx^m)}{e^2}+\frac{dmx}{e^2}+\frac{x^2\log(fx^m)}{2e}-\frac{mx^2}{4e}\right)+\frac{1}{4}bemn\left(\frac{d^2\log(d+ex)}{e^3}-\frac{dx}{e^2}+\frac{x^2}{2e}\right)$$

input
```
Int[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(b*e*m*n*(-((d*x)/e^2) + x^2/(2*e) + (d^2*Log[d + e*x])/e^3))/4 - ((m*x^2
- 2*x^2*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]))/4 - (b*e*n*((d*m*x)/e^2 -
(m*x^2)/(4*e) - (d*x*Log[f*x^m])/e^2 + (x^2*Log[f*x^m])/(2*e) + (d^2*Log[f
*x^m]*Log[1 + (e*x)/d])/e^3 + (d^2*m*PolyLog[2, -((e*x)/d)])/e^3))/2
```

3.360.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.360.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.04 (sec) , antiderivative size = 843, normalized size of antiderivative = 5.34

method	result	size
risch	Expression too large to display	843

input
```
int(x*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/e*n*b*d*x*Pi*csgn(I*f*x^m)^3+1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*csgn(I*
f*x^m)^3+1/8*I*n*b*x^2*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2/e^2*n*b*
ln(x^m)*d^2*ln(e*x+d)+1/2/e^2*n*b*m*d^2*dilog(-e*x/d)-1/2/e^2*n*b*d^2*ln(e
*x+d)*ln(f)+1/8*I*n*b*x^2*Pi*csgn(I*f*x^m)^3-1/4*I/e^2*n*b*d^2*ln(e*x+d)*P
i*csgn(I*f)*csgn(I*f*x^m)^2-1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*csgn(I*x^m)*csg
n(I*f*x^m)^2-1/8*I*n*b*x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I/e*n*b*d*x*
Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/e*n*b*d*x*Pi*csgn(I*f)*csgn(I*f*x^m)
^2+1/4*I/e*n*b*d*x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2/e^2*n*b*m*d^2*ln(e*x
+d)*ln(-e*x/d)+1/4*b*m*n*x^2+(1/2*b*x^2*ln(x^m)+1/4*b*x^2*(-I*Pi*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)
*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f)-m))*ln((e*x+d)^n)-1/8*I*n*b*
x^2*Pi*csgn(I*f)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1
/2*b*ln(c)+1/2*a)*(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csg
n(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)
^3+2*ln(f))*x^2+x^2*ln(x^m)-1/2*m*x^2)-1/4*n*b*x^2*ln(f)-1/4*n*b*ln(x^m)*x
^2-5/8/e^2*n*b*m*d^2+1/2/e*n*b*d*x*ln(f)+1/2/e*n*b*ln(x^m)*d*x-3/4*b*d*m*n
*x/e+1/4*b*d^2*m*n*ln(e*x+d)/e^2
```

3.360.5 Fricas [F]

$$\int x \log \left(f x^m\right) \left(a+b \log \left(c(d+e x)^n\right)\right) \, dx=\int (b \log \left((ex+d)^n c\right)+a)x \log \left(f x^m\right) \, dx$$

input
```
integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(b*x*log((e*x + d)^n*c)*log(f*x^m) + a*x*log(f*x^m), x)
```

3.360.6 Sympy [F(-1)]

Timed out.

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Timed out
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.13

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx$$
$$=\frac{1}{4}\left(\frac{2\left(\log\left(ex+d\right)\log\left(-\frac{ex+d}{d}+1\right)+\text{Li}_2\left(\frac{ex+d}{d}\right)\right)bd^2n}{e^2}-\frac{be^2x^2\log\left((ex+d)^n\right)+3\,bdenx-bd^2n\log\left(ex+\right.}{e^2}\right.$$
$$-\frac{1}{4}\left(ben\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)-2\,bx^2\log\left((ex+d)^nc\right)-2\,ax^2\right)\log\left(fx^m\right)$$

input
```
integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
1/4*(2*(log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d^2*n/e
^2 - (b*e^2*x^2*log((e*x + d)^n) + 3*b*d*e*n*x - b*d^2*n*log(e*x + d) + (a
*e^2 - (e^2*n - e^2*log(c))*b)*x^2)/e^2)*m - 1/4*(b*e*n*(2*d^2*log(e*x + d
)/e^3 + (e*x^2 - 2*d*x)/e^2) - 2*b*x^2*log((e*x + d)^n*c) - 2*a*x^2)*log(f
*x^m)
```

3.360.8 Giac [F]

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int\left(b\log\left((ex+d)^nc\right)+a\right)x\log\left(fx^m\right)\,dx$$

input
```
integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*x*log(f*x^m), x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int x\,\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

input
```
int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

3.361 $\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$

3.361.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = 2bmnx - bnx\log(fx^m) - \frac{bdmn\log(d+ex)}{e} - x(m-\log(fx^m))(a+b\log(c(d+ex)^n)) + \frac{bdn\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{e} + \frac{bdmn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}$$

output
```
2*b*m*n*x-b*n*x*ln(f*x^m)-b*d*m*n*ln(e*x+d)/e-x*(m-ln(f*x^m))*(a+b*ln(c*(e
*x+d)^n))+b*d*n*ln(f*x^m)*ln(1+e*x/d)/e+b*d*m*n*polylog(2,-e*x/d)/e
```

3.361.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \frac{\log(fx^m)(bdn\log(d+ex)+ex(a-bn+b\log(c(d+ex)^n)))-m\big(aex-2benx+bdn(1+\log(x))\log(d}{e}$$

input
```
Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
(Log[f*x^m]*(b*d*n*Log[d + e*x] + e*x*(a - b*n + b*Log[c*(d + e*x)^n])) -
m*(a*e*x - 2*b*e*n*x + b*d*n*(1 + Log[x])*Log[d + e*x] + b*e*x*Log[c*(d +
e*x)^n] - b*d*n*Log[x]*Log[1 + (e*x)/d]) + b*d*m*n*PolyLog[2, -((e*x)/d)])
/e
```

3.361.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2869, 49, 2009, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$$

$$\Big\downarrow \text{2869}$$

$$-ben\int \frac{x\log(fx^m)}{d+ex}dx+bemn\int \frac{x}{d+ex}dx-x(m-\log(fx^m))(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \text{49}$$

$$-ben\int \frac{x\log(fx^m)}{d+ex}dx+bemn\int \left(\frac{1}{e}-\frac{d}{e(d+ex)}\right)dx-x(m-\log(fx^m))(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \text{2009}$$

$$-ben\int \frac{x\log(fx^m)}{d+ex}dx-x(m-\log(fx^m))(a+b\log(c(d+ex)^n))+bemn\left(\frac{x}{e}-\frac{d\log(d+ex)}{e^2}\right)$$

$$\Big\downarrow \text{2793}$$

$$-ben\int \left(\frac{\log(fx^m)}{e}-\frac{d\log(fx^m)}{e(d+ex)}\right)dx-x(m-\log(fx^m))(a+b\log(c(d+ex)^n))+bemn\left(\frac{x}{e}-\frac{d\log(d+ex)}{e^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-x(m-\log(fx^m))(a+b\log(c(d+ex)^n))-ben\left(-\frac{d\log\left(\frac{ex}{d}+1\right)\log(fx^m)}{e^2}-\frac{dm\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}+\frac{x\log(fx^m)}{e}-\frac{mx}{e}\right)+bemn\left(\frac{x}{e}-\frac{d\log(d+ex)}{e^2}\right)$$

input
```
Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
b*e*m*n*(x/e - (d*Log[d + e*x])/e^2) - x*(m - Log[f*x^m])*(a + b*Log[c*(d
+ e*x)^n]) - b*e*n*(-((m*x)/e) + (x*Log[f*x^m])/e - (d*Log[f*x^m]*Log[1 +
(e*x)/d])/e^2 - (d*m*PolyLog[2, -((e*x)/d)])/e^2)
```

3.361.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2869
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.)), x_Symbol] :> Simp[(-x)*(m - Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x]
 + (-Simp[b*e*n   Int[(x*Log[f*x^m])/(d + e*x), x], x] + Simp[b*e*m*n   Int
[x/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.361.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.25 (sec) , antiderivative size = 657, normalized size of antiderivative = 6.64

method	result
risch	$\left(bx\ln(x^m) + \frac{xb\left(-i\pi\ \mathrm{csgn}(if)\,\mathrm{csgn}(ix^m)\,\mathrm{csgn}(if\,x^m)+i\pi\ \mathrm{csgn}(if)\,\mathrm{csgn}(if\,x^m)^2+i\pi\ \mathrm{csgn}(ix^m)\,\mathrm{csgn}(if\,x^m)^2-i\pi\ \mathrm{csgn}(if\,x^m\right.}{2}\right.$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
(b*x*ln(x^m)+1/2*x*b*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(
I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3
+2*ln(f)-2*m))*ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b
*ln(c)+1/2*a)*(I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*x+I*Pi*csgn(I*x^m)*csgn(I*f*
x^m)^2*x+2*x*ln(f)-I*Pi*csgn(I*f*x^m)^3*x-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(
I*f*x^m)*x+2*ln(x^m)*x-2*m*x)+1/2*I*n*b*x*Pi*csgn(I*f*x^m)^3-1/2*I*n*b*x*P
i*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I*n*b*x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*
f*x^m)-1/2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*f*x^m)^3-n*b*x*ln(f)+2*b*m*n*x-1/
2*I*n*b*x*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*x
^m)*csgn(I*f*x^m)^2+1/2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1
/2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/e*n*b*d*ln
(e*x+d)*ln(f)-b*d*m*n*ln(e*x+d)/e-n*b*ln(x^m)*x+1/e*n*b*ln(x^m)*d*ln(e*x+d
)+1/e*n*b*m*d-1/e*n*b*m*d*ln(e*x+d)*ln(-e*x/d)-1/e*n*b*m*d*dilog(-e*x/d)
```

3.361.5 Fricas [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)\log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m), x)
```

3.361.6 Sympy [F(-1)]

Timed out.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Timed out
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx =$$
$$-\left(\frac{\left(\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)\right)bdn}{e}+\frac{bdn\log(ex+d)+bex\log((ex+d)^n)-((2\,en-e\right.$$
$$-\left(ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)-bx\log((ex+d)^n c)-ax\right)\log(fx^m)$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
-((log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d*n/e + (b*d
*n*log(e*x + d) + b*e*x*log((e*x + d)^n) - ((2*e*n - e*log(c))*b - a*e)*x)
/e)*m - (b*e*n*(x/e - d*log(e*x + d)/e^2) - b*x*log((e*x + d)^n*c) - a*x)*
log(f*x^m)
```

3.361.8 Giac [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)\log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m), x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int \ln(f\,x^m)\,(a+b\,\ln(c\,(d+e\,x)^n))\,dx$$

input
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

3.362 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx$

3.362.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m} - \frac{bn\log^2(fx^m)\log\left(1+\frac{ex}{d}\right)}{2m} - bn\log(fx^m)\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right) + bmn\,\mathrm{PolyLog}\left(3,-\frac{ex}{d}\right)$$

output
```
1/2*ln(f*x^m)^2*(a+b*ln(c*(e*x+d)^n))/m-1/2*b*n*ln(f*x^m)^2*ln(1+e*x/d)/m-
b*n*ln(f*x^m)*polylog(2,-e*x/d)+b*m*n*polylog(3,-e*x/d)
```

3.362.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \frac{1}{2}\left(\frac{a\log^2(fx^m)}{m} - bm\log^2(x)\log(c(d+ex)^n) + 2b\log(x)\log(fx^m)\log(c(d+ex)^n) + bmn\log^2(x)\log\left(1+\frac{ex}{d}\right) - 2bn\log(x)\log(fx^m)\log\left(1+\frac{ex}{d}\right) - 2bn\log(fx^m)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right) + 2bmn\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)\right)$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x,x]
```

output
```
((a*Log[f*x^m]^2)/m - b*m*Log[x]^2*Log[c*(d + e*x)^n] + 2*b*Log[x]*Log[f*x
^m]*Log[c*(d + e*x)^n] + b*m*n*Log[x]^2*Log[1 + (e*x)/d] - 2*b*n*Log[x]*Lo
g[f*x^m]*Log[1 + (e*x)/d] - 2*b*n*Log[f*x^m]*PolyLog[2, -((e*x)/d)] + 2*b*
m*n*PolyLog[3, -((e*x)/d)])/2
```

3.362.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2872, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx$$

$$\Big\downarrow \text{ 2872}$$

$$\frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m} - \frac{ben\int \frac{\log^2(fx^m)}{d+ex}dx}{2m}$$

$$\Big\downarrow \text{ 2754}$$

$$\frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m}-\frac{ben\left(\frac{\log\left(\frac{ex}{d}+1\right)\log^2(fx^m)}{e}-\frac{2m\int\frac{\log(fx^m)\log\left(\frac{ex}{d}+1\right)}{x}dx}{e}\right)}{2m}$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m}-\frac{ben\left(\frac{\log\left(\frac{ex}{d}+1\right)\log^2(fx^m)}{e}-\frac{2m\left(m\int\frac{\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{ex}{d}\right)\log(fx^m)\right)}{e}\right)}{2m}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m}-\frac{ben\left(\frac{\log\left(\frac{ex}{d}+1\right)\log^2(fx^m)}{e}-\frac{2m\left(m\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)-\text{PolyLog}\left(2,-\frac{ex}{d}\right)\log(fx^m)\right)}{e}\right)}{2m}$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x,x]
```

output
```
(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*m) - (b*e*n*((Log[f*x^m]^2*Lo
g[1 + (e*x)/d])/e - (2*m*(-(Log[f*x^m]*PolyLog[2, -((e*x)/d)]) + m*PolyLog
[3, -((e*x)/d)]))/e))/(2*m)
```

3.362.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

3.362. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx$

rule 2872
```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[Log[f*x^m]^2*((a + b*Log[c*(d + e*x)^n])/(2*m
)), x] - Simp[b*e*(n/(2*m))   Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{
a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.362.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.60 (sec) , antiderivative size = 756, normalized size of antiderivative = 8.59

method	result	size
risch	Expression too large to display	756

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x,x,method=_RETURNVERBOSE)
```

output
```
(b*ln(x)*ln(x^m)-1/2*b*m*ln(x)^2-1/2*I*Pi*ln(x)*b*csgn(I*f)*csgn(I*x^m)*cs
gn(I*f*x^m)+1/2*I*Pi*ln(x)*b*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*Pi*ln(x)*b*cs
gn(I*x^m)*csgn(I*f*x^m)^2-1/2*I*Pi*ln(x)*b*csgn(I*f*x^m)^3+ln(f)*ln(x)*b)*
ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)
+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(
I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*ln(x)+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*ln(x
)+2*ln(x)*ln(f)-I*Pi*csgn(I*f*x^m)^3*ln(x)-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn
(I*f*x^m)*ln(x)+1/m*ln(x^m)^2)-1/2*I*n*b*ln(x)*ln((e*x+d)/d)*Pi*csgn(I*x^m
)*csgn(I*f*x^m)^2-1/2*I*n*b*dilog((e*x+d)/d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-
1/2*I*n*b*dilog((e*x+d)/d)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I*n*b*dilog(
(e*x+d)/d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I*n*b*ln(x)*ln((e*x+
d)/d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I*n*b*ln(x)*ln((e*x+d)/d)
*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*n*b*dilog((e*x+d)/d)*Pi*csgn(I*f*x^m)^
3+1/2*I*n*b*ln(x)*ln((e*x+d)/d)*Pi*csgn(I*f*x^m)^3-1/2*n*b*m*ln(x)^2*ln(1+
e*x/d)+n*b*ln(x)^2*ln((e*x+d)/d)*m-n*b*m*ln(x)*polylog(2,-e*x/d)+n*b*dilog
((e*x+d)/d)*m*ln(x)-n*b*ln(x)*ln((e*x+d)/d)*ln(x^m)-n*b*ln(x)*ln((e*x+d)/d
)*ln(f)+b*m*n*polylog(3,-e*x/d)-n*b*dilog((e*x+d)/d)*ln(x^m)-n*b*dilog((e*
x+d)/d)*ln(f)
```

3.362.5 Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x, x)
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x,x)
```

output
```
Timed out
```

3.362.7 Maxima [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x,x, algorithm="maxima")
```

output
```
-1/2*(b*m*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^m))*log((e*x + d
)^n) - integrate(-1/2*(b*e*m*n*x*log(x)^2 - 2*b*e*n*x*log(f)*log(x) + 2*b*
d*log(c)*log(f) + 2*a*d*log(f) + 2*(b*e*log(c)*log(f) + a*e*log(f))*x - 2*
(b*e*n*x*log(x) - b*d*log(c) - a*d - (b*e*log(c) + a*e)*x)*log(x^m))/(e*x^
2 + d*x), x)
```

3.362.8 Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x, x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)}{x}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x, x)
```

3.363 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx$

3.363.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = \frac{bemn\log(x)}{d} - \frac{ben\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{d} - \frac{bemn\log(d+ex)}{d} - \left(\frac{m}{x}+\frac{\log(fx^m)}{x}\right)(a+b\log(c(d+ex)^n)) + \frac{bemn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}$$

output
```
b*e*m*n*ln(x)/d-b*e*n*ln(1+d/e/x)*ln(f*x^m)/d-b*e*m*n*ln(e*x+d)/d-(m/x+ln(
f*x^m)/x)*(a+b*ln(c*(e*x+d)^n))+b*e*m*n*polylog(2,-d/e/x)/d
```

3.363.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = -\frac{bemnx\log^2(x)+2(m+\log(fx^m))(ad+benx\log(d+ex)+bd\log(c(d+ex)^n))-2benx\log(x)(m+}{2dx}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^2,x]
```

output
```
-1/2*(b*e*m*n*x*Log[x]^2 + 2*(m + Log[f*x^m])*(a*d + b*e*n*x*Log[d + e*x]
+ b*d*Log[c*(d + e*x)^n]) - 2*b*e*n*x*Log[x]*(m + Log[f*x^m] + m*Log[d + e
*x] - m*Log[1 + (e*x)/d]) + 2*b*e*m*n*x*PolyLog[2, -((e*x)/d)])/(d*x)
```

3.363.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2873, 47, 14, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (f x^m) (a+b \log (c (d+e x)^n))}{x^2} \, dx$$

$$\Big\downarrow \text{2873}$$

$$b e n \int \frac{\log (f x^m)}{x (d+e x)}dx+b e m n \int \frac{1}{x (d+e x)}dx-\left(\left(\frac{\log (f x^m)}{x}+\frac{m}{x}\right) (a+b \log (c (d+e x)^n))\right)$$

$$\Big\downarrow \text{47}$$

$$b e n \int \frac{\log (f x^m)}{x (d+e x)}dx+b e m n \left(\frac{\int \frac{1}{x}dx}{d}-\frac{e \int \frac{1}{d+e x}dx}{d}\right)-\left(\left(\frac{\log (f x^m)}{x}+\frac{m}{x}\right) (a+b \log (c (d+e x)^n))\right)$$

$$\Big\downarrow \text{14}$$

$$b e n \int \frac{\log (f x^m)}{x (d+e x)}dx+b e m n \left(\frac{\log (x)}{d}-\frac{e \int \frac{1}{d+e x}dx}{d}\right)-\left(\left(\frac{\log (f x^m)}{x}+\frac{m}{x}\right) (a+b \log (c (d+e x)^n))\right)$$

$$\Big\downarrow \text{16}$$

$$b e n \int \frac{\log (f x^m)}{x (d+e x)}dx-\left(\frac{\log (f x^m)}{x}+\frac{m}{x}\right) (a+b \log (c (d+e x)^n))+b e m n \left(\frac{\log (x)}{d}-\frac{\log (d+e x)}{d}\right)$$

$$\Big\downarrow \text{2779}$$

$$ben\left(\frac{m\int \frac{\log\left(\frac{d}{ex}+1\right)}{x}dx}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log\left(fx^m\right)}{d}\right)-\left(\frac{\log\left(fx^m\right)}{x}+\frac{m}{x}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+bemn\left(\frac{\log(x)}{d}-\frac{\log(d+ex)}{d}\right)$$

$$\Big\downarrow \text{2838}$$

$$-\left(\frac{\log\left(fx^m\right)}{x}+\frac{m}{x}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+ben\left(\frac{m\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log\left(fx^m\right)}{d}\right)+bemn\left(\frac{\log(x)}{d}-\frac{\log(d+ex)}{d}\right)$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^2,x]
```

output
```
b*e*m*n*(Log[x]/d - Log[d + e*x]/d) - (m/x + Log[f*x^m]/x)*(a + b*Log[c*(d
 + e*x)^n]) + b*e*n*(-((Log[1 + d/(e*x)]*Log[f*x^m])/d) + (m*PolyLog[2, -(
d/(e*x))])/d)
```

3.363.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.363.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.36 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.95

method	result
risch	$\left(-\frac{b\ln(x^m)}{x} - \frac{-i\pi b\ \mathrm{csgn}(if)\ \mathrm{csgn}(ix^m)\ \mathrm{csgn}(if\,x^m)+i\pi b\ \mathrm{csgn}(if)\ \mathrm{csgn}(if\,x^m)^2+i\pi b\ \mathrm{csgn}(ix^m)\ \mathrm{csgn}(if\,x^m)^2-i\pi b\,\mathrm{csgn}(if\,x^m)^3}{2x}\right.$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-b/x*ln(x^m)-1/2*(-I*Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*b*csgn
(I*f)*csgn(I*f*x^m)^2+I*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*b*csgn(I*f*x
^m)^3+2*b*ln(f)+2*b*m)/x)*ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*
I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)
^n)^3+1/2*b*ln(c)+1/2*a)*(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi
*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*
x^m)^3+2*ln(f))/x-2*ln(x^m)/x-2*m/x)-e*n*b*ln(x^m)/d*ln(e*x+d)+e*n*b*ln(x^
m)/d*ln(x)-1/2*e*n*b*m/d*ln(x)^2+e*n*b*m/d*ln(e*x+d)*ln(-e*x/d)+e*n*b*m/d*
dilog(-e*x/d)-1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*I
*e*n*b/d*ln(x)*Pi*csgn(I*f*x^m)^3-1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(I*f)*csg
n(I*f*x^m)^2-1/2*I*e*n*b/d*ln(x)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*
n*b/d*ln(e*x+d)*ln(f)-b*e*m*n*ln(e*x+d)/d+1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(
I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I*e*n*b/d*ln(x)*Pi*csgn(I*f)*csgn(I*f*x
^m)^2+1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(I*f*x^m)^3+1/2*I*e*n*b/d*ln(x)*Pi*cs
gn(I*x^m)*csgn(I*f*x^m)^2+e*n*b/d*ln(x)*ln(f)+b*e*m*n*ln(x)/d
```

3.363.5 Fricas [F]

$$\int \frac{\log(fx^m)\,(a+b\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x^2}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^2, x)
```

3.363.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\,(a+b\log(c(d+ex)^n))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**2,x)
```

output
```
Timed out
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.59

$$\int \frac{\log(fx^m)\,(a+b\log(c(d+ex)^n))}{x^2}\,dx = -\frac{1}{2}\left(\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\text{Li}_2\left(-\frac{ex}{d}\right)\right)ben}{d}+\frac{2\,ben\log(ex+d)}{d}-\frac{2\,benx\log(ex+d)\log(x)-benx\,\text{lo}}{}\right. - \left(ben\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)+\frac{b\log((ex+d)^n c)}{x}+\frac{a}{x}\right)\log(fx^m)$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e*n/d + 2*b*e*n*log(e*x
+ d)/d - (2*b*e*n*x*x*log(e*x + d)*log(x) - b*e*n*x*x*log(x)^2 + 2*b*e*n*x*x*log
(x) - 2*b*d*log((e*x + d)^n) - 2*b*d*log(c) - 2*a*d)/(d*x))*m - (b*e*n*(lo
g(e*x + d)/d - log(x)/d) + b*log((e*x + d)^n*c)/x + a/x)*log(f*x^m)
```

3.363.8 Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^2}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^2, x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}{x^2}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^2,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^2, x)
```

3.364 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3} dx$

3.364.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3} dx = -\frac{3bemn}{4dx} - \frac{be^2mn\log(x)}{4d^2} - \frac{ben\log(fx^m)}{2dx} + \frac{be^2n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{2d^2} + \frac{be^2mn\log(d+ex)}{4d^2} - \frac{1}{4}\left(\frac{m}{x^2}+\frac{2\log(fx^m)}{x^2}\right)(a+b\log(c(d+ex)^n)) - \frac{be^2mn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{2d^2}$$

output
```
-3/4*b*e*m*n/d/x-1/4*b*e^2*m*n*ln(x)/d^2-1/2*b*e*n*ln(f*x^m)/d/x+1/2*b*e^2
*n*ln(1+d/e/x)*ln(f*x^m)/d^2+1/4*b*e^2*m*n*ln(e*x+d)/d^2-1/4*(m/x^2+2*ln(f
*x^m)/x^2)*(a+b*ln(c*(e*x+d)^n))-1/2*b*e^2*m*n*polylog(2,-d/e/x)/d^2
```

3.364.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.31

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx =$$
$$-\frac{ad^2m+3bdemnx-be^2mnx^2\log^2(x)+2ad^2\log(fx^m)+2bdenx\log(fx^m)-be^2mnx^2\log(d+ex)-2}{}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^3,x]
```

output
```
-1/4*(a*d^2*m + 3*b*d*e*m*n*x - b*e^2*m*n*x^2*Log[x]^2 + 2*a*d^2*Log[f*x^m
] + 2*b*d*e*n*x*Log[f*x^m] - b*e^2*m*n*x^2*Log[d + e*x] - 2*b*e^2*n*x^2*Lo
g[f*x^m]*Log[d + e*x] + b*d^2*m*Log[c*(d + e*x)^n] + 2*b*d^2*Log[f*x^m]*Lo
g[c*(d + e*x)^n] + b*e^2*n*x^2*Log[x]*(m + 2*Log[f*x^m] + 2*m*Log[d + e*x]
 - 2*m*Log[1 + (e*x)/d]) - 2*b*e^2*m*n*x^2*PolyLog[2, -((e*x)/d)])/(d^2*x^
2)
```

3.364.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2873, 54, 2009, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}dx$$

$$\Big\downarrow \text{2873}$$

$$\frac{1}{2}ben\int \frac{\log(fx^m)}{x^2(d+ex)}dx+\frac{1}{4}bemn\int \frac{1}{x^2(d+ex)}dx-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}bemn\int \left(\frac{e^2}{d^2(d+ex)}-\frac{e}{d^2x}+\frac{1}{dx^2}\right)dx+\frac{1}{2}ben\int \frac{\log(fx^m)}{x^2(d+ex)}dx-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ben\int \frac{\log(fx^m)}{x^2(d+ex)}dx-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))+\frac{1}{4}bemn\left(-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}-\frac{1}{dx}\right)$$

$\Big\downarrow$ 2780

$$\frac{1}{2}ben\left(\frac{\int \frac{\log(fx^m)}{x^2}dx}{d}-\frac{e\int \frac{\log(fx^m)}{x(d+ex)}dx}{d}\right)-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))+\frac{1}{4}bemn\left(-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}-\frac{1}{dx}\right)$$

$\Big\downarrow$ 2741

$$\frac{1}{2}ben\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\int \frac{\log(fx^m)}{x(d+ex)}dx}{d}\right)-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))+\frac{1}{4}bemn\left(-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}-\frac{1}{dx}\right)$$

$\Big\downarrow$ 2779

$$\frac{1}{2}ben\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\left(\frac{m\int \frac{\log\left(\frac{d}{ex}+1\right)}{x}dx}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d}\right)}{d}\right)-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))+\frac{1}{4}bemn\left(-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}-\frac{1}{dx}\right)$$

$\Big\downarrow$ 2838

$$-\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n))+\frac{1}{4}bemn\left(-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}-\frac{1}{dx}\right)+\frac{1}{2}ben\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\left(\frac{m\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d}\right)}{d}\right)$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^3,x]
```

output
```
(b*e*m*n*(-(1/(d*x)) - (e*Log[x])/d^2 + (e*Log[d + e*x])/d^2))/4 - ((m/x^2
 + (2*Log[f*x^m])/x^2)*(a + b*Log[c*(d + e*x)^n]))/4 + (b*e*n*((-(m/x) - L
og[f*x^m]/x)/d - (e*(-((Log[1 + d/(e*x)]*Log[f*x^m])/d) + (m*PolyLog[2, -(
d/(e*x))])/d))/d))/2
```

3.364. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx$

3.364.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.364. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx$

3.364.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.39 (sec) , antiderivative size = 901, normalized size of antiderivative = 5.78

method	result	size
risch	Expression too large to display	901

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*e^2*n*b*m/d^2*ln(e*x+d)*ln(-e*x/d)+1/2*e^2*n*b*ln(x^m)/d^2*ln(e*x+d)-
1/2*e^2*n*b*ln(x^m)/d^2*ln(x)-1/2*e^2*n*b*m/d^2*dilog(-e*x/d)+1/4*e^2*n*b*
m/d^2*ln(x)^2+1/2*e^2*n*b/d^2*ln(e*x+d)*ln(f)-1/2*e^2*n*b/d^2*ln(x)*ln(f)+
1/4*I*e*n*b/d/x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e^2*n*b/d^2*l
n(x)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I*e^2*n*b/d^2*ln(e*x+d)*Pi
*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I*e^2*n*b/d^2*ln(e*x+d)*Pi*csgn(I
*f*x^m)^3+1/4*I*e^2*n*b/d^2*ln(x)*Pi*csgn(I*f*x^m)^3+1/4*I*e*n*b/d/x*Pi*cs
gn(I*f*x^m)^3-1/2*e*n*b*ln(x^m)/d/x-1/4*I*e*n*b/d/x*Pi*csgn(I*f)*csgn(I*f*
x^m)^2-1/4*I*e*n*b/d/x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/4*I*e^2*n*b/d^2*ln
(e*x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I*e^2*n*b/d^2*ln(e*x+d)*Pi*csgn(I
*x^m)*csgn(I*f*x^m)^2-1/4*I*e^2*n*b/d^2*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2
-1/4*I*e^2*n*b/d^2*ln(x)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(
I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(
e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*
csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(-ln(x^m)/x^2-1/2*m/x^2-1/2*(-I*P
i*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*
csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))/x^2)+(-1/2*b/x^2
*ln(x^m)-1/4*(-I*Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*b*csgn(I*f)
*csgn(I*f*x^m)^2+I*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*b*csgn(I*f*x^m)^3
+2*b*ln(f)+b*m)/x^2)*ln((e*x+d)^n)-1/2*e*n*b/d/x*ln(f)-3/4*b*e*m*n/d/x-...
```

3.364.5 Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x^3}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^3,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^3, x)
```

3.364.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^3} dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**3,x)
```

output
```
Timed out
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.27

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^3} dx$$
$$= \frac{1}{4}\left(\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\text{Li}_2\left(-\frac{ex}{d}\right)\right)be^2n}{d^2}+\frac{be^2n\log(ex+d)}{d^2}-\frac{2\,be^2nx^2\log(ex+d)\log(x)-be^2nx^2\,\mathrm{l}}{}\right.$$
$$+\frac{1}{2}\left(ben\left(\frac{e\log(ex+d)}{d^2}-\frac{e\log(x)}{d^2}-\frac{1}{dx}\right)-\frac{b\log((ex+d)^nc)}{x^2}-\frac{a}{x^2}\right)\log(fx^m)$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^3,x, algorithm="maxima")
```

output
```
1/4*(2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e^2*n/d^2 + b*e^2*n*log(e
*x + d)/d^2 - (2*b*e^2*n*x^2*log(e*x + d)*log(x) - b*e^2*n*x^2*log(x)^2 +
b*e^2*n*x^2*log(x) + 3*b*d*e*n*x + b*d^2*log((e*x + d)^n) + b*d^2*log(c) +
 a*d^2)/(d^2*x^2))*m + 1/2*(b*e*n*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(
d*x)) - b*log((e*x + d)^n*c)/x^2 - a/x^2)*log(f*x^m)
```

3.364.8 Giac [F]

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^3} dx = \int \frac{(b\log ((ex + d)^nc) + a)\log (fx^m)}{x^3} dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^3, x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)}{x^3}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^3,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^3, x)
```

3.365 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx$

3.365.1 Optimal result

Integrand size = 24, antiderivative size = 193

$$\begin{aligned}\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = & -\frac{5bemn}{36dx^2}+\frac{4be^2mn}{9d^2x}+\frac{be^3mn\log(x)}{9d^3} \\ & -\frac{ben\log(fx^m)}{6dx^2}+\frac{be^2n\log(fx^m)}{3d^2x} \\ & -\frac{be^3n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{3d^3} \\ & -\frac{be^3mn\log(d+ex)}{9d^3} \\ & -\frac{1}{9}\left(\frac{m}{x^3}+\frac{3\log(fx^m)}{x^3}\right)(a+b\log(c(d+ex)^n)) \\ & +\frac{be^3mn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^3}\end{aligned}$$

output
```
-5/36*b*e*m*n/d/x^2+4/9*b*e^2*m*n/d^2/x+1/9*b*e^3*m*n*ln(x)/d^3-1/6*b*e*n*
ln(f*x^m)/d/x^2+1/3*b*e^2*n*ln(f*x^m)/d^2/x-1/3*b*e^3*n*ln(1+d/e/x)*ln(f*x
^m)/d^3-1/9*b*e^3*m*n*ln(e*x+d)/d^3-1/9*(m/x^3+3*ln(f*x^m)/x^3)*(a+b*ln(c*
(e*x+d)^n))+1/3*b*e^3*m*n*polylog(2,-d/e/x)/d^3
```

3.365.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx =$$
$$-\frac{4ad^3m+5bd^2emnx-16bde^2mnx^2+6be^3mnx^3\log^2(x)+12ad^3\log(fx^m)+6bd^2enx\log(fx^m)-12bd}{}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^4,x]
```

output
```
-1/36*(4*a*d^3*m + 5*b*d^2*e*m*n*x - 16*b*d*e^2*m*n*x^2 + 6*b*e^3*m*n*x^3*
Log[x]^2 + 12*a*d^3*Log[f*x^m] + 6*b*d^2*e*n*x*Log[f*x^m] - 12*b*d*e^2*n*x
^2*Log[f*x^m] + 4*b*e^3*m*n*x^3*Log[d + e*x] + 12*b*e^3*n*x^3*Log[f*x^m]*L
og[d + e*x] + 4*b*d^3*m*Log[c*(d + e*x)^n] + 12*b*d^3*Log[f*x^m]*Log[c*(d
+ e*x)^n] - 4*b*e^3*n*x^3*Log[x]*(m + 3*Log[f*x^m] + 3*m*Log[d + e*x] - 3*
m*Log[1 + (e*x)/d]) + 12*b*e^3*m*n*x^3*PolyLog[2, -((e*x)/d)])/(d^3*x^3)
```

3.365.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2873, 54, 2009, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx$$

$$\Big\downarrow \; 2873$$

$$\frac{1}{3}ben\int \frac{\log(fx^m)}{x^3(d+ex)}dx+\frac{1}{9}bemn\int \frac{1}{x^3(d+ex)}dx-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \; 54$$

$$\frac{1}{9}bemn\int \left(-\frac{e^3}{d^3(d+ex)}+\frac{e^2}{d^3x}-\frac{e}{d^2x^2}+\frac{1}{dx^3}\right)dx+\frac{1}{3}ben\int \frac{\log(fx^m)}{x^3(d+ex)}dx-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}ben\int \frac{\log(fx^m)}{x^3(d+ex)}dx-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)$$

$$\Big\downarrow \text{2780}$$

$$\frac{1}{3}ben\left(\frac{\int \frac{\log(fx^m)}{x^3}dx}{d}-\frac{e\int \frac{\log(fx^m)}{x^2(d+ex)}dx}{d}\right)-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{3}ben\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\int \frac{\log(fx^m)}{x^2(d+ex)}dx}{d}\right)-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)$$

$$\Big\downarrow \text{2780}$$

$$\frac{1}{3}ben\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{\int \frac{\log(fx^m)}{x^2}dx}{d}-\frac{e\int \frac{\log(fx^m)}{x(d+ex)}dx}{d}\right)}{d}\right)-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{3}ben\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\int \frac{\log(fx^m)}{x(d+ex)}dx}{d}\right)}{d}\right)-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)$$

$$\Big\downarrow \text{2779}$$

3.365. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx$

$$\frac{1}{3}ben\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\left(\frac{m\int\frac{\log\left(\frac{d}{ex}+1\right)}{x}dx}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d}\right)}{d}\right)}{d}\right)-$$
$$\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+$$
$$\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n))+$$
$$\frac{1}{9}bemn\left(\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}+\frac{e}{d^2x}-\frac{1}{2dx^2}\right)+$$
$$\frac{1}{3}ben\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\left(\frac{m\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d}\right)}{d}\right)}{d}\right)$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^4,x]
```

output
```
(b*e*m*n*(-1/2*1/(d*x^2) + e/(d^2*x) + (e^2*Log[x])/d^3 - (e^2*Log[d + e*x
])/d^3))/9 - ((m/x^3 + (3*Log[f*x^m])/x^3)*(a + b*Log[c*(d + e*x)^n]))/9 +
 (b*e*n*((-1/4*m/x^2 - Log[f*x^m]/(2*x^2))/d - (e*((-(m/x) - Log[f*x^m]/x)
/d - (e*(-((Log[1 + d/(e*x)]*Log[f*x^m])/d) + (m*PolyLog[2, -(d/(e*x))])/d
))/d))/d))/3
```

3.365. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx$

3.365.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.365. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx$

3.365.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.38 (sec) , antiderivative size = 1070, normalized size of antiderivative = 5.54

method	result	size
risch	Expression too large to display	1070

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*b/x^3*ln(x^m)-1/18*(-3*I*Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+3*
I*Pi*b*csgn(I*f)*csgn(I*f*x^m)^2+3*I*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-3*I*
Pi*b*csgn(I*f*x^m)^3+6*b*ln(f)+2*b*m)/x^3)*ln((e*x+d)^n)-1/6*I*e^3*n*b/d^3
*ln(x)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/6*I*e^2*n*b/d^2/x*Pi*csgn(
I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/3*e^3*n*b*ln(x^m)/d^3*ln(e*x+d)+1/3*e^3*n
*b*ln(x^m)/d^3*ln(x)-1/6*e^3*n*b*m/d^3*ln(x)^2+1/3*e^3*n*b*m/d^3*dilog(-e*
x/d)-1/3*e^3*n*b/d^3*ln(e*x+d)*ln(f)+1/3*e^3*n*b/d^3*ln(x)*ln(f)-1/6*e*n*b
/d/x^2*ln(f)+1/3*e^3*n*b*m/d^3*ln(e*x+d)*ln(-e*x/d)+1/6*I*e^3*n*b/d^3*ln(e
*x+d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/12*I*e*n*b/d/x^2*Pi*csgn(I*
f)*csgn(I*x^m)*csgn(I*f*x^m)-1/12*I*e*n*b/d/x^2*Pi*csgn(I*f)*csgn(I*f*x^m)
^2-1/12*I*e*n*b/d/x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/6*I*e^3*n*b/d^3*ln(
e*x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/6*I*e^3*n*b/d^3*ln(e*x+d)*Pi*csgn(I*
x^m)*csgn(I*f*x^m)^2+1/6*I*e^3*n*b/d^3*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+
1/6*I*e^3*n*b/d^3*ln(x)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/6*I*e^3*n*b/d^3*l
n(e*x+d)*Pi*csgn(I*f*x^m)^3-1/6*I*e^3*n*b/d^3*ln(x)*Pi*csgn(I*f*x^m)^3-1/6
*I*e^2*n*b/d^2/x*Pi*csgn(I*f*x^m)^3+1/12*I*e*n*b/d/x^2*Pi*csgn(I*f*x^m)^3+
1/6*I*e^2*n*b/d^2/x*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/6*I*e^2*n*b/d^2/x*Pi*cs
gn(I*x^m)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*
c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*
(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b...
```

3.365.5 Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^4}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^4,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^4, x)
```

3.365.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**4,x)
```

output
```
Timed out
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.19

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx =$$
$$-\frac{1}{36}\left(\frac{12\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)be^3n}{d^3}+\frac{4\,be^3n\log(ex+d)}{d^3}-\frac{12\,be^3nx^3\log(ex+d)\log(x)-}{}\right.$$
$$-\frac{1}{6}\left(ben\left(\frac{2\,e^2\log(ex+d)}{d^3}-\frac{2\,e^2\log(x)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)+\frac{2\,b\log((ex+d)^nc)}{x^3}+\frac{2\,a}{x^3}\right)\log(fx^m)$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^4,x, algorithm="maxima")
```

output
```
-1/36*(12*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e^3*n/d^3 + 4*b*e^3*n*
log(e*x + d)/d^3 - (12*b*e^3*n*x^3*log(e*x + d)*log(x) - 6*b*e^3*n*x^3*log
(x)^2 + 4*b*e^3*n*x^3*log(x) + 16*b*d*e^2*n*x^2 - 5*b*d^2*e*n*x - 4*b*d^3*
log((e*x + d)^n) - 4*b*d^3*log(c) - 4*a*d^3)/(d^3*x^3))*m - 1/6*(b*e*n*(2*
e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2)) + 2*b*log
((e*x + d)^n*c)/x^3 + 2*a/x^3)*log(f*x^m)
```

3.365.8 Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^4}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^4,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^4, x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}{x^4}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^4,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^4, x)
```

3.366 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$

3.366.1 Optimal result

Integrand size = 24, antiderivative size = 230

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = -\frac{7bemn}{144dx^3}+\frac{3be^2mn}{32d^2x^2}-\frac{5be^3mn}{16d^3x}-\frac{be^4mn\log(x)}{16d^4} -\frac{ben\log(fx^m)}{12dx^3}+\frac{be^2n\log(fx^m)}{8d^2x^2} -\frac{be^3n\log(fx^m)}{4d^3x}+\frac{be^4n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{4d^4} +\frac{be^4mn\log(d+ex)}{16d^4} -\frac{1}{16}\left(\frac{m}{x^4}+\frac{4\log(fx^m)}{x^4}\right)(a+b\log(c(d+ex)^n)) -\frac{be^4mn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{4d^4}$$

output
```
-7/144*b*e*m*n/d/x^3+3/32*b*e^2*m*n/d^2/x^2-5/16*b*e^3*m*n/d^3/x-1/16*b*e^
4*m*n*ln(x)/d^4-1/12*b*e*n*ln(f*x^m)/d/x^3+1/8*b*e^2*n*ln(f*x^m)/d^2/x^2-1
/4*b*e^3*n*ln(f*x^m)/d^3/x+1/4*b*e^4*n*ln(1+d/e/x)*ln(f*x^m)/d^4+1/16*b*e^
4*m*n*ln(e*x+d)/d^4-1/16*(m/x^4+4*ln(f*x^m)/x^4)*(a+b*ln(c*(e*x+d)^n))-1/4
*b*e^4*m*n*polylog(2,-d/e/x)/d^4
```

3.366.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.19

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = -\frac{18ad^4m+14bd^3emnx-27bd^2e^2mnx^2+90bde^3mnx^3-36be^4mnx^4\log^2(x)+72ad^4\log(fx^m)+24bd^3}{}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^5,x]
```

output
```
-1/288*(18*a*d^4*m + 14*b*d^3*e*m*n*x - 27*b*d^2*e^2*m*n*x^2 + 90*b*d*e^3*
m*n*x^3 - 36*b*e^4*m*n*x^4*Log[x]^2 + 72*a*d^4*Log[f*x^m] + 24*b*d^3*e*n*x
*Log[f*x^m] - 36*b*d^2*e^2*n*x^2*Log[f*x^m] + 72*b*d*e^3*n*x^3*Log[f*x^m]
- 18*b*e^4*m*n*x^4*Log[d + e*x] - 72*b*e^4*n*x^4*Log[f*x^m]*Log[d + e*x] +
 18*b*d^4*m*Log[c*(d + e*x)^n] + 72*b*d^4*Log[f*x^m]*Log[c*(d + e*x)^n] +
18*b*e^4*n*x^4*Log[x]*(m + 4*Log[f*x^m] + 4*m*Log[d + e*x] - 4*m*Log[1 + (
e*x)/d]) - 72*b*e^4*m*n*x^4*PolyLog[2, -((e*x)/d)])/(d^4*x^4)
```

3.366.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2873, 54, 2009, 2780, 2741, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}dx$$

$$\Big\downarrow \text{2873}$$

$$\frac{1}{4}ben\int \frac{\log(fx^m)}{x^4(d+ex)}dx+\frac{1}{16}bemn\int \frac{1}{x^4(d+ex)}dx-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{16}bemn\int \left(\frac{e^4}{d^4(d+ex)}-\frac{e^3}{d^4x}+\frac{e^2}{d^3x^2}-\frac{e}{d^2x^3}+\frac{1}{dx^4}\right)dx+\frac{1}{4}ben\int \frac{\log(fx^m)}{x^4(d+ex)}dx-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}ben\int \frac{\log(fx^m)}{x^4(d+ex)}dx-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{2780}$$

$$\frac{1}{4}ben\left(\frac{\int \frac{\log(fx^m)}{x^4}dx}{d}-\frac{e\int \frac{\log(fx^m)}{x^3(d+ex)}dx}{d}\right)-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\int \frac{\log(fx^m)}{x^3(d+ex)}dx}{d}\right)-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{2780}$$

$$\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\left(\frac{\int \frac{\log(fx^m)}{x^3}dx}{d}-\frac{e\int \frac{\log(fx^m)}{x^2(d+ex)}dx}{d}\right)}{d}\right)-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\int \frac{\log(fx^m)}{x^2(d+ex)}dx}{d}\right)}{d}\right)-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{2780}$$

3.366. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$

$$\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{\int\frac{\log(fx^m)}{x^2}dx}{d}-\frac{e\int\frac{\log(fx^m)}{x(d+ex)}dx}{d}\right)}{d}\right)}{d}\right)-$$
$$\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+$$
$$\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\int\frac{\log(fx^m)}{x(d+ex)}dx}{d}\right)}{d}\right)}{d}\right)-$$
$$\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+$$
$$\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$$\Big\downarrow \text{ 2779}$$

3.366. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$

$$\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\left(\frac{m\int\frac{\log\left(\frac{d}{ex}+1\right)}{x}dx}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d}\right)}{d}\right)}{d}\right)}{d}\right)-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)$$

$\downarrow$ 2838

$$-\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n))+\frac{1}{16}bemn\left(-\frac{e^3\log(x)}{d^4}+\frac{e^3\log(d+ex)}{d^4}-\frac{e^2}{d^3x}+\frac{e}{2d^2x^2}-\frac{1}{3dx^3}\right)+\frac{1}{4}ben\left(\frac{-\frac{\log(fx^m)}{3x^3}-\frac{m}{9x^3}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{2x^2}-\frac{m}{4x^2}}{d}-\frac{e\left(\frac{-\frac{\log(fx^m)}{x}-\frac{m}{x}}{d}-\frac{e\left(\frac{m\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}-\frac{\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d}\right)}{d}\right)}{d}\right)}{d}\right)$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^5,x]
```

3.366. $\int\frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$

output
```
(b*e*m*n*(-1/3*1/(d*x^3) + e/(2*d^2*x^2) - e^2/(d^3*x) - (e^3*Log[x])/d^4
+ (e^3*Log[d + e*x])/d^4))/16 - ((m/x^4 + (4*Log[f*x^m])/x^4)*(a + b*Log[c
*(d + e*x)^n]))/16 + (b*e*n*((-1/9*m/x^3 - Log[f*x^m]/(3*x^3))/d - (e*((-1
/4*m/x^2 - Log[f*x^m]/(2*x^2))/d - (e*((-(m/x) - Log[f*x^m]/x)/d - (e*(-((
Log[1 + d/(e*x)]*Log[f*x^m])/d) + (m*PolyLog[2, -(d/(e*x))])/d))/d))/d))/d
))/4
```

3.366.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.366. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$

rule 2873
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Simp[b*e*(n/(g*(q + 1)))   Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x],
x] + Simp[b*e*m*(n/(g*(q + 1)^2))   Int[(g*x)^(q + 1)/(d + e*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

3.366.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.96 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.38

method	result	size
risch	Expression too large to display	1237

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*I*e*n*b/d/x^3*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/8*I*e^4*n*b/d^4*ln(e*
x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/8*I*e^4*n*b/d^4*ln(e*x+d)*Pi*csgn(I*x^
m)*csgn(I*f*x^m)^2-1/8*I*e^3*n*b/d^3/x*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/8*I*
e^3*n*b/d^3/x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/16*I*e^2*n*b/d^2/x^2*Pi*csg
n(I*f)*csgn(I*f*x^m)^2+1/16*I*e^2*n*b/d^2/x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)
^2-1/8*I*e^4*n*b/d^4*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/8*I*e^4*n*b/d^4*
ln(x)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/24*I*e*n*b/d/x^3*Pi*csgn(I*f)*csgn(
I*f*x^m)^2+1/24*I*e*n*b/d/x^3*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/24*
I*e*n*b/d/x^3*Pi*csgn(I*f*x^m)^3+1/8*I*e^3*n*b/d^3/x*Pi*csgn(I*f*x^m)^3-1/
16*I*e^2*n*b/d^2/x^2*Pi*csgn(I*f*x^m)^3+1/8*I*e^4*n*b/d^4*ln(x)*Pi*csgn(I*
f*x^m)^3-1/8*I*e^4*n*b/d^4*ln(e*x+d)*Pi*csgn(I*f*x^m)^3-1/16*I*e^2*n*b/d^2
/x^2*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/8*I*e^4*n*b/d^4*ln(x)*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/8*I*e^4*n*b/d^4*ln(e*x+d)*Pi*csgn(I*f)*
csgn(I*x^m)*csgn(I*f*x^m)+1/8*I*e^3*n*b/d^3/x*Pi*csgn(I*f)*csgn(I*x^m)*csg
n(I*f*x^m)-1/4*e^3*n*b/d^3/x*ln(f)-1/12*e*n*b/d/x^3*ln(f)+1/8*e^2*n*b/d^2/
x^2*ln(f)-1/4*e^4*n*b*m/d^4*ln(e*x+d)*ln(-e*x/d)+(-1/4*b/x^4*ln(x^m)-1/16*
(-2*I*Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+2*I*Pi*b*csgn(I*f)*csgn(I*f
*x^m)^2+2*I*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-2*I*Pi*b*csgn(I*f*x^m)^3+4*b*
ln(f)+b*m)/x^4)*ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*...
```

3.366.5 Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x^5}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^5,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^5, x)
```

3.366.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**5,x)
```

output
```
Timed out
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.10

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$$
$$= \frac{1}{288}\left(\frac{72\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\operatorname{Li}_2\left(-\frac{ex}{d}\right)\right)be^4n}{d^4}+\frac{18\,be^4n\log(ex+d)}{d^4}-\frac{72\,be^4nx^4\log(ex+d)\log(x)-}{}\right.$$
$$+\frac{1}{24}\left(ben\left(\frac{6\,e^3\log(ex+d)}{d^4}-\frac{6\,e^3\log(x)}{d^4}-\frac{6\,e^2x^2-3\,dex+2\,d^2}{d^3x^3}\right)-\frac{6\,b\log((ex+d)^n c)}{x^4}-\frac{6\,a}{x^4}\right)\log(f$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^5,x, algorithm="maxima")
```

output
```
1/288*(72*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e^4*n/d^4 + 18*b*e^4*n
*log(e*x + d)/d^4 - (72*b*e^4*n*x^4*log(e*x + d)*log(x) - 36*b*e^4*n*x^4*l
og(x)^2 + 18*b*e^4*n*x^4*log(x) + 90*b*d*e^3*n*x^3 - 27*b*d^2*e^2*n*x^2 +
14*b*d^3*e*n*x + 18*b*d^4*log((e*x + d)^n) + 18*b*d^4*log(c) + 18*a*d^4)/(
d^4*x^4))*m + 1/24*(b*e*n*(6*e^3*log(e*x + d)/d^4 - 6*e^3*log(x)/d^4 - (6*
e^2*x^2 - 3*d*e*x + 2*d^2)/(d^3*x^3)) - 6*b*log((e*x + d)^n*c)/x^4 - 6*a/x
^4)*log(f*x^m)
```

3.366.8 Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx=\int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^5}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^5,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^5, x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx=\int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}{x^5}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^5,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^5, x)
```

3.367 $\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$

3.367.1 Optimal result

Integrand size = 26, antiderivative size = 705

$$\begin{aligned}
&\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx \\
&= \frac{2abd^2mnx}{9e^2} - \frac{71b^2d^2mn^2x}{54e^2} + \frac{bd^2mn(6a-11bn)x}{9e^2} + \frac{19b^2dmn^2x^2}{54e} - \frac{2}{27}b^2mn^2x^3 \\
&\quad - \frac{2abd^2nx\log(fx^m)}{3e^2} + \frac{11b^2d^2n^2x\log(fx^m)}{9e^2} - \frac{5b^2dn^2x^2\log(fx^m)}{18e} \\
&\quad + \frac{2}{27}b^2n^2x^3\log(fx^m) + \frac{23b^2d^3mn^2\log(d+ex)}{54e^3} + \frac{5b^2d^3mn^2\log\left(-\frac{ex}{d}\right)\log(d+ex)}{9e^3} \\
&\quad - \frac{5b^2d^3n^2\log(fx^m)\log(d+ex)}{9e^3} + \frac{8b^2d^2mn(d+ex)\log(c(d+ex)^n)}{9e^3} \\
&\quad + \frac{2b^2d^3mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{3e^3} - \frac{2b^2d^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{3e^3} \\
&\quad - \frac{5bdmnx^2(a+b\log(c(d+ex)^n))}{18e} + \frac{4}{27}bmnx^3(a+b\log(c(d+ex)^n)) \\
&\quad + \frac{bdnx^2\log(fx^m)(a+b\log(c(d+ex)^n))}{3e} - \frac{2}{9}bnx^3\log(fx^m)(a+b\log(c(d+ex)^n)) \\
&\quad - \frac{d^3m(a+b\log(c(d+ex)^n))^2}{9e^3} - \frac{1}{9}mx^3(a+b\log(c(d+ex)^n))^2 \\
&\quad - \frac{d^3m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{3e^3} + \frac{d^3\log(fx^m)(a+b\log(c(d+ex)^n))^2}{3e^3} \\
&\quad + \frac{1}{3}x^3\log(fx^m)(a+b\log(c(d+ex)^n))^2 + \frac{11b^2d^3mn^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{9e^3} \\
&\quad - \frac{2bd^3mn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{3e^3} + \frac{2b^2d^3mn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{3e^3}
\end{aligned}$$

output
```
-2/9*b*n*x^3*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))-1/3*d^3*m*ln(-e*x/d)*(a+b*ln(
c*(e*x+d)^n))^2/e^3+4/27*b*m*n*x^3*(a+b*ln(c*(e*x+d)^n))+2/9*a*b*d^2*m*n*x
/e^2+1/9*b*d^2*m*n*(-11*b*n+6*a)*x/e^2-2/3*a*b*d^2*n*x*ln(f*x^m)/e^2+5/9*b
^2*d^3*m*n^2*ln(-e*x/d)*ln(e*x+d)/e^3+8/9*b^2*d^2*m*n*(e*x+d)*ln(c*(e*x+d)
^n)/e^3+2/3*b^2*d^3*m*n*ln(-e*x/d)*ln(c*(e*x+d)^n)/e^3-2/3*b^2*d^2*n*(e*x+
d)*ln(f*x^m)*ln(c*(e*x+d)^n)/e^3-5/18*b*d*m*n*x^2*(a+b*ln(c*(e*x+d)^n))/e+
1/3*b*d*n*x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/e-2/3*b*d^3*m*n*(a+b*ln(c*(e
*x+d)^n))*polylog(2,1+e*x/d)/e^3-1/9*m*x^3*(a+b*ln(c*(e*x+d)^n))^2+1/3*x^3
*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2-2/27*b^2*m*n^2*x^3+2/27*b^2*n^2*x^3*ln(
f*x^m)-1/9*d^3*m*(a+b*ln(c*(e*x+d)^n))^2/e^3+1/3*d^3*ln(f*x^m)*(a+b*ln(c*(
e*x+d)^n))^2/e^3-71/54*b^2*d^2*m*n^2*x/e^2+19/54*b^2*d*m*n^2*x^2/e+11/9*b^
2*d^3*m*n^2*polylog(2,1+e*x/d)/e^3+2/3*b^2*d^3*m*n^2*polylog(3,1+e*x/d)/e^
3+11/9*b^2*d^2*n^2*x*ln(f*x^m)/e^2-5/18*b^2*d*n^2*x^2*ln(f*x^m)/e+23/54*b^
2*d^3*m*n^2*ln(e*x+d)/e^3-5/9*b^2*d^3*n^2*ln(f*x^m)*ln(e*x+d)/e^3
```

3.367.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.38

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{48abd^2emnx - 137b^2d^2emn^2x - 15abde^2mnx^2 + 19b^2de^2mn^2x^2 - 6a^2e^3mx^3 + 8abe^3mnx^3 - 4b^2e^3mn^2x^3}{}$$

input
```
Integrate[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(48*a*b*d^2*e*m*n*x - 137*b^2*d^2*e*m*n^2*x - 15*a*b*d*e^2*m*n*x^2 + 19*b^
2*d*e^2*m*n^2*x^2 - 6*a^2*e^3*m*x^3 + 8*a*b*e^3*m*n*x^3 - 4*b^2*e^3*m*n^2*
x^3 - 36*a*b*d^2*e*n*x*Log[f*x^m] + 66*b^2*d^2*e*n^2*x*Log[f*x^m] + 18*a*b
*d*e^2*n*x^2*Log[f*x^m] - 15*b^2*d*e^2*n^2*x^2*Log[f*x^m] + 18*a^2*e^3*x^3
*Log[f*x^m] - 12*a*b*e^3*n*x^3*Log[f*x^m] + 4*b^2*e^3*n^2*x^3*Log[f*x^m] -
 12*a*b*d^3*m*n*Log[d + e*x] + 71*b^2*d^3*m*n^2*Log[d + e*x] - 36*a*b*d^3*
m*n*Log[x]*Log[d + e*x] + 66*b^2*d^3*m*n^2*Log[x]*Log[d + e*x] + 36*a*b*d^
3*n*Log[f*x^m]*Log[d + e*x] - 66*b^2*d^3*n^2*Log[f*x^m]*Log[d + e*x] + 6*b
^2*d^3*m*n^2*Log[d + e*x]^2 + 36*b^2*d^3*m*n^2*Log[x]*Log[d + e*x]^2 - 18*
b^2*d^3*m*n^2*Log[-((e*x)/d)]*Log[d + e*x]^2 - 18*b^2*d^3*n^2*Log[f*x^m]*L
og[d + e*x]^2 + 48*b^2*d^2*e*m*n*x*Log[c*(d + e*x)^n] - 15*b^2*d*e^2*m*n*x
^2*Log[c*(d + e*x)^n] - 12*a*b*e^3*m*x^3*Log[c*(d + e*x)^n] + 8*b^2*e^3*m*
n*x^3*Log[c*(d + e*x)^n] - 36*b^2*d^2*e*n*x*Log[f*x^m]*Log[c*(d + e*x)^n]
+ 18*b^2*d*e^2*n*x^2*Log[f*x^m]*Log[c*(d + e*x)^n] + 36*a*b*e^3*x^3*Log[f*
x^m]*Log[c*(d + e*x)^n] - 12*b^2*e^3*n*x^3*Log[f*x^m]*Log[c*(d + e*x)^n] -
 12*b^2*d^3*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] - 36*b^2*d^3*m*n*Log[x]*Lo
g[d + e*x]*Log[c*(d + e*x)^n] + 36*b^2*d^3*n*Log[f*x^m]*Log[d + e*x]*Log[c
*(d + e*x)^n] - 6*b^2*e^3*m*x^3*Log[c*(d + e*x)^n]^2 + 18*b^2*e^3*x^3*Log[
f*x^m]*Log[c*(d + e*x)^n]^2 + 36*a*b*d^3*m*n*Log[x]*Log[1 + (e*x)/d] - 66*
b^2*d^3*m*n^2*Log[x]*Log[1 + (e*x)/d] - 36*b^2*d^3*m*n^2*Log[x]*Log[d +...
```

3.367.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2875, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$

$$\Big\downarrow \; 2875$$

$$-m\int \left(-\frac{b^2n^2\log^2(d+ex)d^3}{3e^3x}+\frac{2bn\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)d^3}{3e^3x}+\frac{2b^2n^2d^2}{e^2}-\frac{2bn(d+ex)\left(a+b\log\left(c\right.\right.}{e^3x}\right.$$

$$\frac{2bd^3n\log(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3}-$$

$$\frac{2bd^2n(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3}+$$

$$\frac{bdn(d+ex)^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3}-\frac{2bn(d+ex)^3\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3}+$$

$$\frac{1}{3}x^3\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2-\frac{b^2d^3n^2\log^2(d+ex)\log\left(fx^m\right)}{3e^3}+\frac{2b^2d^2n^2x\log\left(fx^m\right)}{e^2}-$$

$$\frac{b^2dn^2(d+ex)^2\log\left(fx^m\right)}{2e^3}+\frac{2b^2n^2(d+ex)^3\log\left(fx^m\right)}{27e^3}$$

$\downarrow$ 2009

$$-\frac{b^2n^2\log\left(fx^m\right)\log^2(d+ex)d^3}{3e^3}+\frac{2bn\log\left(fx^m\right)\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)d^3}{3e^3}+$$

$$\frac{2b^2n^2x\log\left(fx^m\right)d^2}{e^2}-\frac{2bn(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)d^2}{e^3}-$$

$$\frac{b^2n^2(d+ex)^2\log\left(fx^m\right)d}{2e^3}+\frac{bn(d+ex)^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)d}{e^3}+$$

$$\frac{1}{3}x^3\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2+\frac{2b^2n^2(d+ex)^3\log\left(fx^m\right)}{27e^3}-$$

$$\frac{2bn(d+ex)^3\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3}-$$

$$m\left(-\frac{b^2n^2\log^2(d+ex)d^3}{9e^3}-\frac{b^2n^2\log(x)\log^2(d+ex)d^3}{3e^3}-\frac{\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)^2d^3}{3e^3}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\right.}{3e}\right.$$

input
```
Int[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

3.367. $\int x^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

output
```
(2*b^2*d^2*n^2*x*Log[f*x^m])/e^2 - (b^2*d*n^2*(d + e*x)^2*Log[f*x^m])/(2*e
^3) + (2*b^2*n^2*(d + e*x)^3*Log[f*x^m])/(27*e^3) - (b^2*d^3*n^2*Log[f*x^m
]*Log[d + e*x]^2)/(3*e^3) - (2*b*d^2*n*(d + e*x)*Log[f*x^m]*(a + b*Log[c*(
d + e*x)^n]))/e^3 + (b*d*n*(d + e*x)^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n
]))/e^3 - (2*b*n*(d + e*x)^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/(9*e^3
) + (2*b*d^3*n*Log[f*x^m]*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n]))/(3*e^3)
 + (x^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/3 - m*((-11*a*b*d^2*n*x)/
(9*e^2) + (28*b^2*d^2*n^2*x)/(9*e^2) - (5*b^2*d*n^2*x^2)/(36*e) + (2*b^2*n
^2*x^3)/81 - (13*b^2*d*n^2*(d + e*x)^2)/(36*e^3) + (4*b^2*n^2*(d + e*x)^3)
/(81*e^3) - (23*b^2*d^3*n^2*Log[x])/(54*e^3) - (b^2*d^3*n^2*Log[d + e*x]^2
)/(9*e^3) - (b^2*d^3*n^2*Log[x]*Log[d + e*x]^2)/(3*e^3) - (11*b^2*d^2*n*(d
 + e*x)*Log[c*(d + e*x)^n])/(9*e^3) - (2*b*d^2*n*(d + e*x)*(a + b*Log[c*(d
 + e*x)^n]))/(3*e^3) + (13*b*d*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(
18*e^3) - (4*b*n*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(27*e^3) - (11*b*
d^3*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(9*e^3) + (2*b*d^3*n*Log
[d + e*x]*(a + b*Log[c*(d + e*x)^n]))/(9*e^3) + (2*b*d^3*n*Log[x]*Log[d +
e*x]*(a + b*Log[c*(d + e*x)^n]))/(3*e^3) + (x^3*(a + b*Log[c*(d + e*x)^n])
^2)/9 - (d^3*Log[x]*(a + b*Log[c*(d + e*x)^n])^2)/(3*e^3) + (d^3*Log[-((e*
x)/d)]*(a + b*Log[c*(d + e*x)^n])^2)/(3*e^3) - (11*b^2*d^3*n^2*PolyLog[2,
1 + (e*x)/d])/(9*e^3) + (2*b*d^3*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2...
```

3.367.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2875
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*(a + b*
Log[c*(d + e*x)^n])^p, x]}, Simp[Log[f*x^m]   u, x] - Simp[m   Int[1/x   u,
 x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && IGtQ[p, 1] && IGtQ[
q, 0]
```

3.367.4 Maple [F]

$$\int x^2 \ln (f\, x^m) \left(a + b \ln \left(c(ex + d)^n\right)\right)^2 dx$$

input
```
int(x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.367.5 Fricas [F]

$$\int x^2 \log (fx^m) (a + b \log (c(d + ex)^n))^2 \, dx = \int \left(b \log \left((ex + d)^n c\right) + a\right)^2 x^2 \log (fx^m) \, dx$$

input
```
integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*x^2*log((e*x + d)
^n*c)*log(f*x^m) + a^2*x^2*log(f*x^m), x)
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \log (fx^m) (a + b \log (c(d + ex)^n))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Timed out
```

3.367.7 Maxima [F]

$$\int x^2 \log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int \left(b\log\left((ex+d)^n c\right)+a\right)^2 x^2\log\left(fx^m\right)\,dx$$

input
```
integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-1/9*(b^2*(m - 3*log(f))*x^3 - 3*b^2*x^3*log(x^m))*log((e*x + d)^n)^2 + in
tegrate(1/9*(9*(b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(
f))*x^3 + 9*(b^2*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f))
*x^2 + 2*((9*a*b*e*log(f) + (9*e*log(c)*log(f) + (m*n - 3*n*log(f))*e)*b^2
)*x^3 + 9*(b^2*d*log(c)*log(f) + a*b*d*log(f))*x^2 - 3*(((e*n - 3*e*log(c)
)*b^2 - 3*a*b*e)*x^3 - 3*(b^2*d*log(c) + a*b*d)*x^2)*log(x^m))*log((e*x +
d)^n) + 9*((b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^3 + (b^2*d*log(c)^2
 + 2*a*b*d*log(c) + a^2*d)*x^2)*log(x^m))/(e*x + d), x)
```

3.367.8 Giac [F]

$$\int x^2 \log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int \left(b\log\left((ex+d)^n c\right)+a\right)^2 x^2\log\left(fx^m\right)\,dx$$

input
```
integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x^2*log(f*x^m), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int x^2\,\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2\,dx$$

input
```
int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2, x)
```

3.368 $\int x\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

3.368.1 Optimal result

Integrand size = 24, antiderivative size = 602

$$
\begin{aligned}
&\int x\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx\\
&=-\frac{abdmnx}{2e}+\frac{2b^2dmn^2x}{e}-\frac{2bdmn(a-bn)x}{e}-\frac{1}{8}b^2mn^2x^2-\frac{b^2mn^2(d+ex)^2}{4e^2}\\
&\quad-\frac{b^2d^2mn^2\log(x)}{4e^2}+\frac{2abdnx\log(fx^m)}{e}-\frac{2b^2dn^2x\log(fx^m)}{e}\\
&\quad+\frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2}-\frac{5b^2dmn(d+ex)\log\left(c(d+ex)^n\right)}{2e^2}\\
&\quad-\frac{2b^2d^2mn\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^n\right)}{e^2}+\frac{2b^2dn(d+ex)\log(fx^m)\log\left(c(d+ex)^n\right)}{e^2}\\
&\quad+\frac{bmn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}+\frac{bd^2mn\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}\\
&\quad-\frac{bn(d+ex)^2\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}+\frac{dm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}\\
&\quad-\frac{m(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2}+\frac{d^2m\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}\\
&\quad-\frac{d(d+ex)\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}\\
&\quad+\frac{(d+ex)^2\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}-\frac{3b^2d^2mn^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{2e^2}\\
&\quad+\frac{bd^2mn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}-\frac{b^2d^2mn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e^2}
\end{aligned}
$$

output
```
-1/2*a*b*d*m*n*x/e+2*b^2*d*m*n^2*x/e-2*b*d*m*n*(-b*n+a)*x/e-1/8*b^2*m*n^2*
x^2-1/4*b^2*m*n^2*(e*x+d)^2/e^2-1/4*b^2*d^2*m*n^2*ln(x)/e^2+2*a*b*d*n*x*ln
(f*x^m)/e-2*b^2*d*n^2*x*ln(f*x^m)/e+1/4*b^2*n^2*(e*x+d)^2*ln(f*x^m)/e^2-5/
2*b^2*d*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2-2*b^2*d^2*m*n*ln(-e*x/d)*ln(c*(e*x
+d)^n)/e^2+2*b^2*d*n*(e*x+d)*ln(f*x^m)*ln(c*(e*x+d)^n)/e^2+1/2*b*m*n*(e*x+
d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*b*d^2*m*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^
n))/e^2-1/2*b*n*(e*x+d)^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*d*m*(e*x
+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2-1/4*m*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^
2+1/2*d^2*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/e^2-d*(e*x+d)*ln(f*x^m)*(a+
b*ln(c*(e*x+d)^n))^2/e^2+1/2*(e*x+d)^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/e
^2-3/2*b^2*d^2*m*n^2*polylog(2,1+e*x/d)/e^2+b*d^2*m*n*(a+b*ln(c*(e*x+d)^n)
)*polylog(2,1+e*x/d)/e^2-b^2*d^2*m*n^2*polylog(3,1+e*x/d)/e^2
```

3.368. $\int x \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$

3.368.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.37

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx = b^2n^2(-m\log(x)+\log(fx^m))\left(\frac{1}{2}x^2\log^2(d+ex)-e\left(\frac{3dx}{2e^2}-\frac{x^2}{4e}-\frac{3d^2\log(d+ex)}{2e^3}-\frac{dx\log(d+ex)}{e^2}+\frac{x^2\log(d+ex)}{2e}+\frac{d^2\log^2(d+ex)}{2e^3}\right)\right)+2bn(-m\log(x)+\log(fx^m))\left(\frac{1}{2}x^2\log(d+ex)-\frac{1}{2}e\left(-\frac{dx}{e^2}+\frac{x^2}{2e}+\frac{d^2\log(d+ex)}{e^3}\right)\right)(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))+\frac{1}{2}mx^2\log(x)(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))^2+\frac{1}{4}x^2(-m+2(-m\log(x)+\log(fx^m)))(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))^2+bmn(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))\left(-\frac{1}{2}x^2\log(d+ex)+x^2\log(x)\log(d+ex)+\frac{1}{2}e\left(-\frac{dx}{e^2}+\frac{x^2}{2e}+\frac{d^2\log(d+ex)}{e^3}\right)-e\left(-\frac{dx(-1+\log(x))}{e^2}+\frac{-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)}{e}+\frac{d^2\left(\frac{\log(x)\log\left(\frac{d+ex}{d}\right)}{e}+\frac{\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}\right)}{e^2}\right)\right)+\frac{1}{2}b^2mn^2\left(-\frac{1}{2}x^2\log^2(d+ex)+x^2\log(x)\log^2(d+ex)+e\left(\frac{3dx}{2e^2}-\frac{x^2}{4e}-\frac{3d^2\log(d+ex)}{2e^3}-\frac{dx\log(d+ex)}{e^2}+\frac{x^2\log(d+ex)}{2e}+\frac{d^2\log^2(d+ex)}{2e^3}\right)-2e\left(-\frac{d\left(2ex-d\log(d+ex)-ex\log(d+ex)+\log(x)\left(-ex+ex\log(d+ex)+d\log\left(1+\frac{ex}{d}\right)\right)+d\,\mathrm{Po}\right.}{e^3}+\frac{-3dex+e^2x^2+d^2\log(d+ex)-e^2x^2\log(d+ex)+\log(x)\left(ex(2d-ex)+2e^2x^2\log(d+ex)-2d^2\log\right.}{4e^3}+\frac{d^2\left(\frac{1}{2}\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\log^2(d+ex)-\log(d+ex)\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)+\mathrm{PolyLog}\left(3,\frac{d+ex}{d}\right)\right)}{e^3}\right)\right)$$

input
```
Integrate[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
b^2*n^2*(-(m*Log[x]) + Log[f*x^m])*((x^2*Log[d + e*x]^2)/2 - e*((3*d*x)/(2
*e^2) - x^2/(4*e) - (3*d^2*Log[d + e*x])/(2*e^3) - (d*x*Log[d + e*x])/e^2
+ (x^2*Log[d + e*x])/(2*e) + (d^2*Log[d + e*x]^2)/(2*e^3))) + 2*b*n*(-(m*L
og[x]) + Log[f*x^m])*((x^2*Log[d + e*x])/2 - (e*(-((d*x)/e^2) + x^2/(2*e)
+ (d^2*Log[d + e*x])/e^3))/2)*(a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^
n])) + (m*x^2*Log[x]*(a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n]))^2)/2
 + (x^2*(-m + 2*(-(m*Log[x]) + Log[f*x^m]))*(a + b*(-(n*Log[d + e*x]) + Lo
g[c*(d + e*x)^n]))^2)/4 + b*m*n*(a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x
)^n]))*(-1/2*(x^2*Log[d + e*x]) + x^2*Log[x]*Log[d + e*x] + (e*(-((d*x)/e^
2) + x^2/(2*e) + (d^2*Log[d + e*x])/e^3))/2 - e*(-((d*x*(-1 + Log[x]))/e^2
) + (-1/4*x^2 + (x^2*Log[x])/2)/e + (d^2*((Log[x]*Log[(d + e*x)/d])/e + Po
lyLog[2, -((e*x)/d)]/e))/e^2)) + (b^2*m*n^2*(-1/2*(x^2*Log[d + e*x]^2) + x
^2*Log[x]*Log[d + e*x]^2 + e*((3*d*x)/(2*e^2) - x^2/(4*e) - (3*d^2*Log[d +
 e*x])/(2*e^3) - (d*x*Log[d + e*x])/e^2 + (x^2*Log[d + e*x])/(2*e) + (d^2*
Log[d + e*x]^2)/(2*e^3)) - 2*e*(-((d*(2*e*x - d*Log[d + e*x] - e*x*Log[d +
 e*x] + Log[x]*(-(e*x) + e*x*Log[d + e*x] + d*Log[1 + (e*x)/d]) + d*PolyLo
g[2, -((e*x)/d)]))/e^3) + (-3*d*e*x + e^2*x^2 + d^2*Log[d + e*x] - e^2*x^2
*Log[d + e*x] + Log[x]*(e*x*(2*d - e*x) + 2*e^2*x^2*Log[d + e*x] - 2*d^2*L
og[1 + (e*x)/d]) - 2*d^2*PolyLog[2, -((e*x)/d)])/(4*e^3) + (d^2*(((Log[x]
- Log[-((e*x)/d)])*Log[d + e*x]^2)/2 - Log[d + e*x]*PolyLog[2, (d + e*x...
```

3.368.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2875, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2875}$$

$$-m\int\left(\frac{n^2(d+ex)^2b^2}{4e^2x}+\frac{2dn(d+ex)\log\left(c(d+ex)^n\right)b^2}{e^2x}+\frac{2dn(a-bn)b}{e}-\frac{n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)b}{2e^2x}\right.$$
$$\frac{bn(d+ex)^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}+\frac{(d+ex)^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}-$$
$$\frac{d(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}+\frac{2abdnx\log\left(fx^m\right)}{e}+$$
$$\frac{2b^2dn(d+ex)\log\left(fx^m\right)\log\left(c(d+ex)^n\right)}{e^2}+\frac{b^2n^2(d+ex)^2\log\left(fx^m\right)}{4e^2}-\frac{2b^2dn^2x\log\left(fx^m\right)}{e}$$

$$\Big\downarrow \text{2009}$$

$$-m\left(-\frac{bd^2n\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2}-\frac{bd^2n\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}-\frac{d^2\log\left(-\frac{ex}{d}\right)}{}\right.$$
$$\frac{bn(d+ex)^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}+\frac{(d+ex)^2\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}-$$
$$\frac{d(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}+\frac{2abdnx\log\left(fx^m\right)}{e}+$$
$$\frac{2b^2dn(d+ex)\log\left(fx^m\right)\log\left(c(d+ex)^n\right)}{e^2}+\frac{b^2n^2(d+ex)^2\log\left(fx^m\right)}{4e^2}-\frac{2b^2dn^2x\log\left(fx^m\right)}{e}$$

input
```
Int[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(2*a*b*d*n*x*Log[f*x^m])/e - (2*b^2*d*n^2*x*Log[f*x^m])/e + (b^2*n^2*(d +
e*x)^2*Log[f*x^m])/(4*e^2) + (2*b^2*d*n*(d + e*x)*Log[f*x^m]*Log[c*(d + e*
x)^n])/e^2 - (b*n*(d + e*x)^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/(2*e^
2) - (d*(d + e*x)*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/e^2 + ((d + e*x
)^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2) - m*((a*b*d*n*x)/(2*e
) - (2*b^2*d*n^2*x)/e + (2*b*d*n*(a - b*n)*x)/e + (b^2*n^2*x^2)/8 + (b^2*n
^2*(d + e*x)^2)/(4*e^2) + (b^2*d^2*n^2*Log[x])/(4*e^2) + (5*b^2*d*n*(d + e
*x)*Log[c*(d + e*x)^n])/(2*e^2) + (2*b^2*d^2*n*Log[-((e*x)/d)]*Log[c*(d +
e*x)^n])/e^2 - (b*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2) - (b*d
^2*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(2*e^2) - (d*(d + e*x)*(a
 + b*Log[c*(d + e*x)^n])^2)/(2*e^2) + ((d + e*x)^2*(a + b*Log[c*(d + e*x)^
n])^2)/(4*e^2) - (d^2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2
) + (3*b^2*d^2*n^2*PolyLog[2, 1 + (e*x)/d])/(2*e^2) - (b*d^2*n*(a + b*Log[
c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/e^2 + (b^2*d^2*n^2*PolyLog[3, 1 +
 (e*x)/d])/e^2)
```

3.368.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2875
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*(a + b*
Log[c*(d + e*x)^n])^p, x]}, Simp[Log[f*x^m]   u, x] - Simp[m   Int[1/x   u,
 x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && IGtQ[p, 1] && IGtQ[
q, 0]
```

3.368.4 Maple [F]

$$\int x \ln \left(f\,x^{m}\right) \left(a+b \ln \left(c (e x +d)^{n}\right)\right)^{2}d x$$

input
```
int(x*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(x*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.368.5 Fricas [F]

$$\int x \log \left(f x^{m}\right) \left(a+b \log \left(c(d+e x)^{n}\right)\right)^{2}\, dx=\int \left(b \log \left((e x+d)^{n} c\right)+a\right)^{2} x \log \left(f x^{m}\right)\, dx$$

input
```
integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*x*log((e*x + d)^n*c
)*log(f*x^m) + a^2*x*log(f*x^m), x)
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int x \log \left(f x^{m}\right) \left(a+b \log \left(c(d+e x)^{n}\right)\right)^{2}\, dx=\text{Timed out}$$

input
```
integrate(x*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Timed out
```

3.368.7 Maxima [F]

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx=\int(b\log((ex+d)^nc)+a)^2x\log(fx^m)\,dx$$

input
```
integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(m - 2*log(f))*x^2 - 2*b^2*x^2*log(x^m))*log((e*x + d)^n)^2 + in
tegrate(1/2*(2*(b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(
f))*x^2 + 2*(b^2*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f))
*x + ((4*a*b*e*log(f) + (4*e*log(c)*log(f) + (m*n - 2*n*log(f))*e)*b^2)*x^
2 + 4*(b^2*d*log(c)*log(f) + a*b*d*log(f))*x - 2*(((e*n - 2*e*log(c))*b^2
- 2*a*b*e)*x^2 - 2*(b^2*d*log(c) + a*b*d)*x)*log(x^m))*log((e*x + d)^n) +
2*((b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^2 + (b^2*d*log(c)^2 + 2*a*b
*d*log(c) + a^2*d)*x)*log(x^m))/(e*x + d), x)
```

3.368.8 Giac [F]

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx=\int(b\log((ex+d)^nc)+a)^2x\log(fx^m)\,dx$$

input
```
integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*x*log(f*x^m), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx=\int x\,\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2\,dx$$

input
```
int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2, x)
```

3.369 $\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$

3.369.1 Optimal result

Integrand size = 23, antiderivative size = 309

$$\begin{aligned}
&\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx\\
&= 2abmnx - 4b^2mn^2x + 2bmn(a-bn)x - 2abnx\log(fx^m)\\
&\quad + 2b^2n^2x\log(fx^m) + \frac{4b^2mn(d+ex)\log(c(d+ex)^n)}{e}\\
&\quad + \frac{2b^2dmn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e}\\
&\quad - \frac{m(d+ex)(a+b\log(c(d+ex)^n))^2}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e}\\
&\quad + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} + \frac{2b^2dmn^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}\\
&\quad - \frac{2bdmn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e} + \frac{2b^2dmn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e}
\end{aligned}$$

output
```
2*a*b*m*n*x-4*b^2*m*n^2*x+2*b*m*n*(-b*n+a)*x-2*a*b*n*x*ln(f*x^m)+2*b^2*n^2
*x*ln(f*x^m)+4*b^2*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e+2*b^2*d*m*n*ln(-e*x/d)*ln
(c*(e*x+d)^n)/e-2*b^2*n*(e*x+d)*ln(f*x^m)*ln(c*(e*x+d)^n)/e-m*(e*x+d)*(a+b
*ln(c*(e*x+d)^n))^2/e-d*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/e+(e*x+d)*ln(
f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/e+2*b^2*d*m*n^2*polylog(2,1+e*x/d)/e-2*b*d*
m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/e+2*b^2*d*m*n^2*polylog(3,1+e
*x/d)/e
```

3.369.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.48

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{-b^2n^2(m\log(x)-\log(fx^m))\left(2ex-2(d+ex)\log(d+ex)+(d+ex)\log^2(d+ex)\right)+2bn(m\log(x)-\log}{}$$

input
```
Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
(-(b^2*n^2*(m*Log[x] - Log[f*x^m])*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d
+ e*x)*Log[d + e*x]^2)) + 2*b*n*(m*Log[x] - Log[f*x^m])*(e*x - (d + e*x)*L
og[d + e*x])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) + e*m*x*Log[x]*
(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - e*x*(m + m*Log[x] - Log[
f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 2*b*m*n*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(d + e*x - e*x*(-1 + Log[x]) - (d +
e*x)*Log[d + e*x] + e*x*Log[x]*Log[d + e*x] + d*(Log[x]*Log[1 + (e*x)/d] +
 PolyLog[2, -((e*x)/d)])) + b^2*m*n^2*(-6*e*x + 2*e*x*Log[x] + 4*d*Log[d +
 e*x] + 4*e*x*Log[d + e*x] - 2*e*x*Log[x]*Log[d + e*x] - d*Log[d + e*x]^2
- e*x*Log[d + e*x]^2 + d*Log[x]*Log[d + e*x]^2 + e*x*Log[x]*Log[d + e*x]^2
 - d*Log[-((e*x)/d)]*Log[d + e*x]^2 - 2*d*Log[x]*Log[1 + (e*x)/d] - 2*d*Po
lyLog[2, -((e*x)/d)] - 2*d*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] + 2*d*Poly
Log[3, 1 + (e*x)/d]))/e
```

3.369.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2870, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$$
$$\Big\downarrow \; 2870$$

$$-m\int\left(-\frac{2n(d+ex)\log\left(c(d+ex)^n\right)b^2}{ex}-2n(a-bn)b+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{ex}\right)dx+\frac{(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}-2abnx\log\left(fx^m\right)-\frac{2b^2n(d+ex)\log\left(fx^m\right)\log\left(c(d+ex)^n\right)}{e}+2b^2n^2x\log\left(fx^m\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-m\left(\frac{2bdn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{d\log\left(-\frac{ex}{d}\right)\left(a+b\log\right.}{e}\right.$$
$$+\frac{(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}-2abnx\log\left(fx^m\right)-\frac{2b^2n(d+ex)\log\left(fx^m\right)\log\left(c(d+ex)^n\right)}{e}+2b^2n^2x\log\left(fx^m\right)$$

input
```
Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
-2*a*b*n*x*Log[f*x^m] + 2*b^2*n^2*x*Log[f*x^m] - (2*b^2*n*(d + e*x)*Log[f*
x^m]*Log[c*(d + e*x)^n])/e + ((d + e*x)*Log[f*x^m]*(a + b*Log[c*(d + e*x)^
n])^2)/e - m*(-2*a*b*n*x + 4*b^2*n^2*x - 2*b*n*(a - b*n)*x - (4*b^2*n*(d +
 e*x)*Log[c*(d + e*x)^n])/e - (2*b^2*d*n*Log[-((e*x)/d)]*Log[c*(d + e*x)^n
])/e + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e + (d*Log[-((e*x)/d)]*(a
+ b*Log[c*(d + e*x)^n])^2)/e - (2*b^2*d*n^2*PolyLog[2, 1 + (e*x)/d])/e + (
2*b*d*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/e - (2*b^2*d*n
^2*PolyLog[3, 1 + (e*x)/d])/e)
```

3.369.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2870
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*(d + e*x)^n])^p, x]},
 Simp[Log[f*x^m]   u, x] - Simp[m   Int[1/x   u, x], x]] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && IGtQ[p, 1]
```

3.369.4 Maple [F]

$$\int \ln \left(f\,x^{m}\right) \left(a+b \ln \left(c \left(e x +d \right)^{n}\right)\right)^{2}d x$$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.369.5 Fricas [F]

$$\int \log \left(fx^{m}\right) \left(a+b\log \left(c(d+ex)^{n}\right)\right)^{2}\,dx=\int \left(b\log \left((ex+d)^{n}c\right)+a\right)^{2}\log \left(fx^{m}\right)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*lo
g(f*x^m) + a^2*log(f*x^m), x)
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \log \left(fx^{m}\right) \left(a+b\log \left(c(d+ex)^{n}\right)\right)^{2}\,dx=\text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Timed out
```

3.369.7 Maxima [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx = \int (b\log((ex+d)^n c)+a)^2\log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(b^2*(m - log(f))*x - b^2*x*log(x^m))*log((e*x + d)^n)^2 + integrate((b^2
*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f) + (b^2*e*log(c)^
2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f))*x + 2*(b^2*d*log(c)*log(f
) + a*b*d*log(f) + (a*b*e*log(f) + (e*log(c)*log(f) + (m*n - n*log(f))*e)*
b^2)*x + (b^2*d*log(c) + a*b*d - ((e*n - e*log(c))*b^2 - a*b*e)*x)*log(x^m
))*log((e*x + d)^n) + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*lo
g(c)^2 + 2*a*b*e*log(c) + a^2*e)*x)*log(x^m))/(e*x + d), x)
```

3.369.8 Giac [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx = \int (b\log((ex+d)^n c)+a)^2\log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m), x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx = \int \ln(f\,x^m)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2\,dx$$

input
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2, x)
```

3.370 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx$

3.370.1 Optimal result

Integrand size = 26, antiderivative size = 823

$$
\begin{aligned}
&\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx \\
&= \frac{1}{2}m\log^2(x)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 \\
&\quad+\log(x)\left(-m\log(x)+\log\left(fx^m\right)\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 \\
&\quad+2bn\left(-m\log(x)+\log\left(fx^m\right)\right)\left(a-bn\log(d+ex)\right. \\
&\qquad\left.+b\log\left(c(d+ex)^n\right)\right)\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\right) \\
&\quad+2bmn\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)\left(\frac{1}{2}\log^2(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)\right. \\
&\qquad\left.-\log(x)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)+\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)\right) \\
&\quad-b^2n^2\left(m\log(x)-\log\left(fx^m\right)\right)\left(\log\left(-\frac{ex}{d}\right)\log^2(d+ex)+2\log(d+ex)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)\right. \\
&\qquad\left.-2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)\right)+\frac{1}{12}b^2mn^2\left(\log^4\left(-\frac{ex}{d}\right)+6\log^2\left(-\frac{ex}{d}\right)\log^2\left(-\frac{ex}{d+ex}\right)\right. \\
&\qquad-4\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{d}{d+ex}\right)\right)\log^3\left(-\frac{ex}{d+ex}\right)+\log^4\left(-\frac{ex}{d+ex}\right) \\
&\qquad+6\log^2(x)\log^2(d+ex)+4\left(2\log^3\left(-\frac{ex}{d}\right)-3\log^2(x)\log(d+ex)\right)\log\left(1+\frac{ex}{d}\right) \\
&\qquad+6\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\left(\log(x)+3\log\left(-\frac{ex}{d}\right)\right)\log^2\left(1+\frac{ex}{d}\right) \\
&\qquad-4\log^2\left(-\frac{ex}{d}\right)\log\left(-\frac{ex}{d+ex}\right)\left(\log\left(-\frac{ex}{d}\right)+3\log\left(1+\frac{ex}{d}\right)\right) \\
&\qquad+12\left(\log^2\left(-\frac{ex}{d}\right)-2\log\left(-\frac{ex}{d}\right)\left(\log\left(-\frac{ex}{d+ex}\right)+\log\left(1+\frac{ex}{d}\right)\right)\right. \\
&\qquad\left.+2\log(x)\left(-\log(d+ex)+\log\left(1+\frac{ex}{d}\right)\right)\right)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right) \\
&\qquad-12\log^2\left(-\frac{ex}{d+ex}\right)\operatorname{PolyLog}\left(2,\frac{ex}{d+ex}\right) \\
&\qquad+12\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{ex}{d+ex}\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&\qquad+24\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\log\left(1+\frac{ex}{d}\right)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&\qquad+24\left(\log\left(-\frac{ex}{d+ex}\right)+\log(d+ex)\right)\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right) \\
&\qquad+24\log\left(-\frac{ex}{d+ex}\right)\operatorname{PolyLog}\left(3,\frac{ex}{d+ex}\right) \\
&\qquad+24\left(-\log(x)+\log\left(-\frac{ex}{d+ex}\right)\right)\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right) \\
&\qquad\left.-24\left(\operatorname{PolyLog}\left(4,-\frac{ex}{d}\right)+\operatorname{PolyLog}\left(4,\frac{ex}{d+ex}\right)-\operatorname{PolyLog}\left(4,1+\frac{ex}{d}\right)\right)\right)
\end{aligned}
$$

output
```
1/2*m*ln(x)^2*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2+ln(x)*(-m*ln(x)+ln(f*x
^m))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2+2*b*n*(-m*ln(x)+ln(f*x^m))*(a-b
*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(ln(x)*(ln(e*x+d)-ln(1+e*x/d))-polylog(2,-
e*x/d))+2*b*m*n*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(1/2*ln(x)^2*(ln(e*x+d
)-ln(1+e*x/d))-ln(x)*polylog(2,-e*x/d)+polylog(3,-e*x/d))-b^2*n^2*(m*ln(x)
-ln(f*x^m))*(ln(-e*x/d)*ln(e*x+d)^2+2*ln(e*x+d)*polylog(2,1+e*x/d)-2*polyl
og(3,1+e*x/d))+1/12*b^2*m*n^2*(ln(-e*x/d)^4+6*ln(-e*x/d)^2*ln(-e*x/(e*x+d)
)^2-4*(ln(-e*x/d)+ln(d/(e*x+d)))*ln(-e*x/(e*x+d))^3+ln(-e*x/(e*x+d))^4+6*l
n(x)^2*ln(e*x+d)^2+4*(2*ln(-e*x/d)^3-3*ln(x)^2*ln(e*x+d))*ln(1+e*x/d)+6*(l
n(x)-ln(-e*x/d))*(ln(x)+3*ln(-e*x/d))*ln(1+e*x/d)^2-4*ln(-e*x/d)^2*ln(-e*x
/(e*x+d))*(ln(-e*x/d)+3*ln(1+e*x/d))+12*(ln(-e*x/d)^2-2*ln(-e*x/d)*(ln(-e*
x/(e*x+d))+ln(1+e*x/d))+2*ln(x)*(-ln(e*x+d)+ln(1+e*x/d)))*polylog(2,-e*x/d
)-12*ln(-e*x/(e*x+d))^2*polylog(2,e*x/(e*x+d))+12*(ln(-e*x/d)-ln(-e*x/(e*x
+d)))^2*polylog(2,1+e*x/d)+24*(ln(x)-ln(-e*x/d))*ln(1+e*x/d)*polylog(2,1+e
*x/d)+24*(ln(-e*x/(e*x+d))+ln(e*x+d))*polylog(3,-e*x/d)+24*ln(-e*x/(e*x+d)
)*polylog(3,e*x/(e*x+d))+24*(-ln(x)+ln(-e*x/(e*x+d)))*polylog(3,1+e*x/d)-2
4*polylog(4,-e*x/d)-24*polylog(4,e*x/(e*x+d))+24*polylog(4,1+e*x/d))
```

3.370. $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx$

3.370.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.00

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx$$

$$\begin{aligned}
&= \frac{1}{2}m\log^2(x)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 \\
&+\log(x)\left(-m\log(x)+\log\left(fx^m\right)\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 \\
&+2bn\left(-m\log(x)+\log\left(fx^m\right)\right)\left(a-bn\log(d+ex)\right. \\
&\left.+b\log\left(c(d+ex)^n\right)\right)\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\right) \\
&+2bmn\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)\left(\frac{1}{2}\log^2(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)\right. \\
&\left.-\log(x)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)+\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)\right) \\
&-b^2n^2\left(m\log(x)-\log\left(fx^m\right)\right)\left(\log\left(-\frac{ex}{d}\right)\log^2(d+ex)+2\log(d+ex)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)\right. \\
&\left.-2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)\right)+\frac{1}{12}b^2mn^2\left(\log^4\left(-\frac{ex}{d}\right)+6\log^2\left(-\frac{ex}{d}\right)\log^2\left(-\frac{ex}{d+ex}\right)\right. \\
&-4\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{d}{d+ex}\right)\right)\log^3\left(-\frac{ex}{d+ex}\right)+\log^4\left(-\frac{ex}{d+ex}\right) \\
&+6\log^2(x)\log^2(d+ex)+4\left(2\log^3\left(-\frac{ex}{d}\right)-3\log^2(x)\log(d+ex)\right)\log\left(1+\frac{ex}{d}\right) \\
&+6\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\left(\log(x)+3\log\left(-\frac{ex}{d}\right)\right)\log^2\left(1+\frac{ex}{d}\right) \\
&-4\log^2\left(-\frac{ex}{d}\right)\log\left(-\frac{ex}{d+ex}\right)\left(\log\left(-\frac{ex}{d}\right)+3\log\left(1+\frac{ex}{d}\right)\right) \\
&+12\left(\log^2\left(-\frac{ex}{d}\right)-2\log\left(-\frac{ex}{d}\right)\left(\log\left(-\frac{ex}{d+ex}\right)+\log\left(1+\frac{ex}{d}\right)\right)\right. \\
&\left.+2\log(x)\left(-\log(d+ex)+\log\left(1+\frac{ex}{d}\right)\right)\right)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right) \\
&-12\log^2\left(-\frac{ex}{d+ex}\right)\operatorname{PolyLog}\left(2,\frac{ex}{d+ex}\right) \\
&+12\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{ex}{d+ex}\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&+24\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\log\left(1+\frac{ex}{d}\right)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&+24\left(\log\left(-\frac{ex}{d+ex}\right)+\log(d+ex)\right)\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right) \\
&+24\log\left(-\frac{ex}{d+ex}\right)\operatorname{PolyLog}\left(3,\frac{ex}{d+ex}\right) \\
&+24\left(-\log(x)+\log\left(-\frac{ex}{d+ex}\right)\right)\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right) \\
&\left.-24\left(\operatorname{PolyLog}\left(4,-\frac{ex}{d}\right)+\operatorname{PolyLog}\left(4,\frac{ex}{d+ex}\right)-\operatorname{PolyLog}\left(4,1+\frac{ex}{d}\right)\right)\right)
\end{aligned}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x,x]
```

output
```
(m*Log[x]^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/2 + Log[x]*(-
(m*Log[x]) + Log[f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 +
 2*b*n*(-(m*Log[x]) + Log[f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x
)^n])*(Log[x]*(Log[d + e*x] - Log[1 + (e*x)/d]) - PolyLog[2, -((e*x)/d)])
+ 2*b*m*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[x]^2*(Log[d
+ e*x] - Log[1 + (e*x)/d]))/2 - Log[x]*PolyLog[2, -((e*x)/d)] + PolyLog[3,
 -((e*x)/d)]) - b^2*n^2*(m*Log[x] - Log[f*x^m])*(Log[-((e*x)/d)]*Log[d + e
*x]^2 + 2*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] - 2*PolyLog[3, 1 + (e*x)/d]
) + (b^2*m*n^2*(Log[-((e*x)/d)]^4 + 6*Log[-((e*x)/d)]^2*Log[-((e*x)/(d + e
*x))]^2 - 4*(Log[-((e*x)/d)] + Log[d/(d + e*x)])*Log[-((e*x)/(d + e*x))]^3
 + Log[-((e*x)/(d + e*x))]^4 + 6*Log[x]^2*Log[d + e*x]^2 + 4*(2*Log[-((e*x
)/d)]^3 - 3*Log[x]^2*Log[d + e*x])*Log[1 + (e*x)/d] + 6*(Log[x] - Log[-((e
*x)/d)])*(Log[x] + 3*Log[-((e*x)/d)])*Log[1 + (e*x)/d]^2 - 4*Log[-((e*x)/d
)]^2*Log[-((e*x)/(d + e*x))]*(Log[-((e*x)/d)] + 3*Log[1 + (e*x)/d]) + 12*(
Log[-((e*x)/d)]^2 - 2*Log[-((e*x)/d)]*(Log[-((e*x)/(d + e*x))] + Log[1 + (
e*x)/d]) + 2*Log[x]*(-Log[d + e*x] + Log[1 + (e*x)/d]))*PolyLog[2, -((e*x)
/d)] - 12*Log[-((e*x)/(d + e*x))]^2*PolyLog[2, (e*x)/(d + e*x)] + 12*(Log[
-((e*x)/d)] - Log[-((e*x)/(d + e*x))])^2*PolyLog[2, 1 + (e*x)/d] + 24*(Log
[x] - Log[-((e*x)/d)])*Log[1 + (e*x)/d]*PolyLog[2, 1 + (e*x)/d] + 24*(Log[
-((e*x)/(d + e*x))] + Log[d + e*x])*PolyLog[3, -((e*x)/d)] + 24*Log[-((...
```

3.370.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (fx^m) (a+b\log (c(d+ex)^n))^2}{x} \, dx$$

$$\Big\downarrow \; 2874$$

$$\frac{\log^2 (fx^m) (a+b\log (c(d+ex)^n))^2}{2m} - \frac{ben \int \frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{d+ex}dx}{m}$$

$$\Big\downarrow \; 2891$$

$$\frac{\log^2 (fx^m) (a+b\log (c(d+ex)^n))^2}{2m} - \frac{ben \int \frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{d+ex}dx}{m}$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x,x]
```

output
```
$Aborted
```

3.370.3.1 Defintions of rubi rules used

rule 2874
```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[f*x^m]^2*((a + b*Log[c*(d + e*x)^n]
)^p/(2*m)), x] - Simp[b*e*n*(p/(2*m))   Int[Log[f*x^m]^2*((a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
&& IGtQ[p, 0]
```

rule 2891
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))^(q_.)*((k_.) + (l_.)*(x_))^(r_.),
 x_Symbol] :> Unintegrable[(k + l*x)^r*(a + b*Log[c*(d + e*x)^n])^p*(f + g*
Log[h*(i + j*x)^m])^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, p, q, r}, x]
```

3.370.4 Maple [F]

$$\int \frac{\ln\left(f\,x^{m}\right)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x}dx$$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x,x)
```

output
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x,x)
```

3.370.5 Fricas [F]

$$\int \frac{\log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}\log\left(fx^{m}\right)}{x}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*l
og(f*x^m) + a^2*log(f*x^m))/x, x)
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2/x,x)
```

output
```
Timed out
```

3.370.7 Maxima [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2\log(fx^m)}{x}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x,x, algorithm="maxima")
```

output
```
-1/2*(b^2*m*log(x)^2 - 2*b^2*log(f)*log(x) - 2*b^2*log(x)*log(x^m))*log((e
*x + d)^n)^2 - integrate(-(b^2*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) +
 a^2*d*log(f) + (b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log
(f))*x + (b^2*e*m*n*x*log(x)^2 - 2*b^2*e*n*x*log(f)*log(x) + 2*b^2*d*log(c
)*log(f) + 2*a*b*d*log(f) + 2*(b^2*e*log(c)*log(f) + a*b*e*log(f))*x - 2*(
b^2*e*n*x*log(x) - b^2*d*log(c) - a*b*d - (b^2*e*log(c) + a*b*e)*x)*log(x^
m))*log((e*x + d)^n) + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*l
og(c)^2 + 2*a*b*e*log(c) + a^2*e)*x)*log(x^m))/(e*x^2 + d*x), x)
```

3.370.8 Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2\log(fx^m)}{x}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m)/x, x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx=\int \frac{\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,\left(d+e\,x\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x, x)
```

3.371 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx$

3.371.1 Optimal result

Integrand size = 26, antiderivative size = 607

$$
\begin{aligned}
\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx = &-\frac{b^2emn^2\log^2(x)\log(d+ex)}{d} \\
&+\frac{2b^2emn^2\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{2b^2en^2\log(x)\log(fx^m)\log(d+ex)}{d} \\
&-\frac{b^2emn^2\log^2(d+ex)}{d}-\frac{b^2mn^2\log^2(d+ex)}{x}+\frac{b^2emn^2\log\left(-\frac{ex}{d}\right)\log^2(d+ex)}{d} \\
&-\frac{b^2en^2\log(fx^m)\log^2(d+ex)}{d}-\frac{b^2n^2\log(fx^m)\log^2(d+ex)}{x} \\
&-\frac{2bn(m\log(x)-\log(fx^m))\left(ex\log\left(-\frac{ex}{d}\right)-(d+ex)\log(d+ex)\right)(a-bn\log(d+ex)+b\log(c(d+ex)}{dx} \\
&-\frac{m\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x} \\
&-\frac{(m-m\log(x)+\log(fx^m))(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x} \\
&+\frac{b^2emn^2\log^2(x)\log\left(1+\frac{ex}{d}\right)}{d}-\frac{2b^2en^2\log(x)\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{d} \\
&-\frac{2b^2en^2\log(fx^m)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d} \\
&+\frac{bmn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(2ex\log\left(-\frac{ex}{d}\right)-2(d+ex)\log(d+ex)-2d\log(x)\log(d}{dx} \\
&+\frac{2b^2emn^2(1+\log(d+ex))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d} \\
&+\frac{2b^2emn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d}-\frac{2b^2emn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{d}
\end{aligned}
$$

output
```
-b^2*e*m*n^2*ln(x)^2*ln(e*x+d)/d+2*b^2*e*m*n^2*ln(-e*x/d)*ln(e*x+d)/d+2*b^
2*e*n^2*ln(x)*ln(f*x^m)*ln(e*x+d)/d-b^2*e*m*n^2*ln(e*x+d)^2/d-b^2*m*n^2*ln
(e*x+d)^2/x+b^2*e*m*n^2*ln(-e*x/d)*ln(e*x+d)^2/d-b^2*e*n^2*ln(f*x^m)*ln(e*
x+d)^2/d-b^2*n^2*ln(f*x^m)*ln(e*x+d)^2/x-2*b*n*(m*ln(x)-ln(f*x^m))*(e*x*ln
(-e*x/d)-(e*x+d)*ln(e*x+d))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))/d/x-m*ln(x
)*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2/x-(m-m*ln(x)+ln(f*x^m))*(a-b*n*ln(
e*x+d)+b*ln(c*(e*x+d)^n))^2/x+b^2*e*m*n^2*ln(x)^2*ln(1+e*x/d)/d-2*b^2*e*n^
2*ln(x)*ln(f*x^m)*ln(1+e*x/d)/d-2*b^2*e*n^2*ln(f*x^m)*polylog(2,-e*x/d)/d+
b*m*n*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(2*e*x*ln(-e*x/d)-2*(e*x+d)*ln(e
*x+d)-2*d*ln(x)*ln(e*x+d)+e*x*(ln(x)^2-2*ln(x)*ln(1+e*x/d)-2*polylog(2,-e*
x/d)))/d/x+2*b^2*e*m*n^2*(1+ln(e*x+d))*polylog(2,1+e*x/d)/d+2*b^2*e*m*n^2*
polylog(3,-e*x/d)/d-2*b^2*e*m*n^2*polylog(3,1+e*x/d)/d
```

3.371.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.85

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx$$
$$= \frac{2bn(m\log(x)-\log(fx^m))\left(-ex\log\left(-\frac{ex}{d}\right)+(d+ex)\log(d+ex)\right)(a-bn\log(d+ex)+b\log(c(d+ex)}{}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^2,x]
```

output
```
(2*b*n*(m*Log[x] - Log[f*x^m])*(-(e*x*Log[-((e*x)/d)]) + (d + e*x)*Log[d +
 e*x])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) - d*m*Log[x]*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + d*(-m + m*Log[x] - Log[f*x^m])*(
a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - b*m*n*(a - b*n*Log[d + e*
x] + b*Log[c*(d + e*x)^n])*(-2*e*x*Log[-((e*x)/d)] + 2*(d + e*x)*Log[d + e
*x] + 2*d*Log[x]*Log[d + e*x] - e*x*(Log[x]^2 - 2*(Log[x]*Log[1 + (e*x)/d]
 + PolyLog[2, -((e*x)/d)]))) + b^2*n^2*(e*m*x*Log[x]^2*Log[d + e*x] + 2*e*
m*x*Log[-((e*x)/d)]*Log[d + e*x] - 2*e*m*x*Log[x]*Log[-((e*x)/d)]*Log[d +
e*x] + 2*e*x*Log[-((e*x)/d)]*Log[f*x^m]*Log[d + e*x] - d*m*Log[d + e*x]^2
- e*m*x*Log[d + e*x]^2 + e*m*x*Log[-((e*x)/d)]*Log[d + e*x]^2 - d*Log[f*x^
m]*Log[d + e*x]^2 - e*x*Log[f*x^m]*Log[d + e*x]^2 - e*m*x*Log[x]^2*Log[1 +
 (e*x)/d] - 2*e*m*x*Log[x]*PolyLog[2, -((e*x)/d)] + 2*e*x*(m - m*Log[x] +
Log[f*x^m] + m*Log[d + e*x])*PolyLog[2, 1 + (e*x)/d] + 2*e*m*x*PolyLog[3,
-((e*x)/d)] - 2*e*m*x*PolyLog[3, 1 + (e*x)/d]))/(d*x)
```

3.371.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx$$

$$\Big\downarrow \text{ 2876}$$

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}dx$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^2,x]
```

output
```
$Aborted
```

3.371.3.1 Defintions of rubi rules used

rule 2876
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*Log[f*x^m]*
(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}
, x]
```

3.371.4 Maple [F]

$$\int \frac{\ln(f\,x^m)(a+b\ln(c(ex+d)^n))^2}{x^2}dx$$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^2,x)
```

output
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^2,x)
```

3.371.5 Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2\log(fx^m)}{x^2}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*l
og(f*x^m) + a^2*log(f*x^m))/x^2, x)
```

3.371.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2/x**2,x)
```

output
```
Timed out
```

3.371.7 Maxima [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2\log(fx^m)}{x^2}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^2,x, algorithm="maxima")
```

output
```
-(b^2*(m + log(f)) + b^2*log(x^m))*log((e*x + d)^n)^2/x + integrate((b^2*d
*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f) + (b^2*e*log(c)^2*
log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f))*x + 2*(b^2*d*log(c)*log(f)
+ a*b*d*log(f) + (a*b*e*log(f) + (e*log(c)*log(f) + (m*n + n*log(f))*e)*b^
2)*x + (b^2*d*log(c) + a*b*d + ((e*n + e*log(c))*b^2 + a*b*e)*x)*log(x^m))
*log((e*x + d)^n) + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(
c)^2 + 2*a*b*e*log(c) + a^2*e)*x)*log(x^m))/(e*x^3 + d*x^2), x)
```

3.371.8 Giac [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left((ex+d)^nc\right)+a\right)^2\log(fx^m)}{x^2}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m)/x^2, x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx=\int \frac{\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{x^2}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^2,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^2, x)
```

3.372 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx$

3.372.1 Optimal result

Integrand size = 26, antiderivative size = 939

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3}\,dx = \frac{b^2e^2mn^2\log(x)}{d^2} - \frac{b^2e^2mn^2\log^2(x)}{2d^2}$$
$$+ \frac{b^2e^2mn^2\log\left(-\frac{ex}{d}\right)}{2d^2} + \frac{b^2e^2n^2\log(x)\log(fx^m)}{d^2} - \frac{3b^2e^2mn^2\log(d+ex)}{2d^2}$$
$$- \frac{3b^2emn^2\log(d+ex)}{2dx} + \frac{b^2e^2mn^2\log(x)\log(d+ex)}{d^2} + \frac{b^2e^2mn^2\log^2(x)\log(d+ex)}{2d^2}$$
$$- \frac{b^2e^2mn^2\log\left(-\frac{ex}{d}\right)\log(d+ex)}{2d^2} - \frac{b^2e^2n^2\log(fx^m)\log(d+ex)}{d^2}$$
$$- \frac{b^2en^2\log(fx^m)\log(d+ex)}{dx} - \frac{b^2e^2n^2\log(x)\log(fx^m)\log(d+ex)}{d^2}$$
$$+ \frac{b^2e^2mn^2\log^2(d+ex)}{4d^2} - \frac{b^2mn^2\log^2(d+ex)}{4x^2} - \frac{b^2e^2mn^2\log\left(-\frac{ex}{d}\right)\log^2(d+ex)}{2d^2}$$
$$+ \frac{b^2e^2n^2\log(fx^m)\log^2(d+ex)}{2d^2} - \frac{b^2n^2\log(fx^m)\log^2(d+ex)}{2x^2}$$
$$+ \frac{bn(m\log(x)-\log(fx^m))\left(e^2x^2\log\left(-\frac{ex}{d}\right)+(d+ex)(ex+(d-ex)\log(d+ex))\right)(a-bn\log(d+ex)+}{d^2x^2}$$
$$- \frac{m\log(x)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2}{2x^2}$$
$$- \frac{(m-2m\log(x)+2\log(fx^m))\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2}{4x^2}$$
$$- \frac{b^2e^2mn^2\log(x)\log\left(1+\frac{ex}{d}\right)}{d^2} - \frac{b^2e^2mn^2\log^2(x)\log\left(1+\frac{ex}{d}\right)}{2d^2}$$
$$+ \frac{b^2e^2n^2\log(x)\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{d^2} - \frac{b^2e^2n^2(m-\log(fx^m))\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2}$$
$$- \frac{bmn\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)\left(ex(d+ex)+e^2x^2\log\left(-\frac{ex}{d}\right)+(d^2-e^2x^2)\log(d+ex)+\right.}{2}$$
$$- \frac{b^2e^2mn^2(1+2\log(d+ex))\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{2d^2}$$
$$- \frac{b^2e^2mn^2\,\mathrm{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^2} + \frac{b^2e^2mn^2\,\mathrm{PolyLog}\left(3,1+\frac{ex}{d}\right)}{d^2}$$

output
```
b^2*e^2*m*n^2*ln(x)*ln(e*x+d)/d^2-b^2*e^2*n^2*ln(x)*ln(f*x^m)*ln(e*x+d)/d^
2-b^2*e^2*m*n^2*ln(x)*ln(1+e*x/d)/d^2+b^2*e^2*n^2*ln(x)*ln(f*x^m)*ln(1+e*x
/d)/d^2-b^2*e*n^2*ln(f*x^m)*ln(e*x+d)/d/x+b*n*(m*ln(x)-ln(f*x^m))*(e^2*x^2
*ln(-e*x/d)+(e*x+d)*(e*x+(-e*x+d)*ln(e*x+d)))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x
+d)^n))/d^2/x^2-1/4*(m-2*m*ln(x)+2*ln(f*x^m))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x
+d)^n))^2/x^2-3/2*b^2*e*m*n^2*ln(e*x+d)/d/x+1/2*b^2*e^2*m*n^2*ln(x)^2*ln(e
*x+d)/d^2-1/2*b^2*e^2*m*n^2*ln(-e*x/d)*ln(e*x+d)/d^2-1/2*b^2*e^2*m*n^2*ln(
-e*x/d)*ln(e*x+d)^2/d^2-1/2*b^2*e^2*m*n^2*ln(x)^2*ln(1+e*x/d)/d^2-1/2*b*m*
n*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(e*x*(e*x+d)+e^2*x^2*ln(-e*x/d)+(-e^
2*x^2+d^2)*ln(e*x+d)+2*d^2*ln(x)*ln(e*x+d)+e*x*(e*x*ln(x)^2+2*d*(1+ln(x))-
2*e*x*(ln(x)*ln(1+e*x/d)+polylog(2,-e*x/d))))/d^2/x^2-1/2*b^2*e^2*m*n^2*(1
+2*ln(e*x+d))*polylog(2,1+e*x/d)/d^2-1/4*b^2*m*n^2*ln(e*x+d)^2/x^2-1/2*b^2
*n^2*ln(f*x^m)*ln(e*x+d)^2/x^2-1/2*m*ln(x)*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)
^n))^2/x^2-1/2*b^2*e^2*m*n^2*ln(x)^2/d^2+1/2*b^2*e^2*m*n^2*ln(-e*x/d)/d^2-
3/2*b^2*e^2*m*n^2*ln(e*x+d)/d^2+1/4*b^2*e^2*m*n^2*ln(e*x+d)^2/d^2+1/2*b^2*
e^2*n^2*ln(f*x^m)*ln(e*x+d)^2/d^2-b^2*e^2*m*n^2*polylog(3,-e*x/d)/d^2+b^2*
e^2*m*n^2*polylog(3,1+e*x/d)/d^2-b^2*e^2*n^2*ln(f*x^m)*ln(e*x+d)/d^2+b^2*e
^2*m*n^2*ln(x)/d^2+b^2*e^2*n^2*ln(x)*ln(f*x^m)/d^2-b^2*e^2*n^2*(m-ln(f*x^m
))*polylog(2,-e*x/d)/d^2
```

3.372.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 781, normalized size of antiderivative = 0.83

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx$$
$$= \frac{4bn(m\log(x)-\log(fx^m))\left(e^2x^2\log\left(-\frac{ex}{d}\right)+(d+ex)(ex+(d-ex)\log(d+ex))\right)(a-bn\log(d+ex)+}{}$$

input
```
Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^3,x]
```

output
```
(4*b*n*(m*Log[x] - Log[f*x^m])*(e^2*x^2*Log[-((e*x)/d)] + (d + e*x)*(e*x +
 (d - e*x)*Log[d + e*x]))*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) -
2*d^2*m*Log[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + d^2*(-m +
 2*m*Log[x] - 2*Log[f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^
2 - 2*b*m*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(e*x*(d + e*x) +
 e^2*x^2*Log[-((e*x)/d)] + (d^2 - e^2*x^2)*Log[d + e*x] + 2*d^2*Log[x]*Log
[d + e*x] + e*x*(e*x*Log[x]^2 + 2*d*(1 + Log[x]) - 2*e*x*(Log[x]*Log[1 + (
e*x)/d] + PolyLog[2, -((e*x)/d)]))) + b^2*n^2*(4*e^2*m*x^2*Log[x] - 2*e^2*
m*x^2*Log[x]^2 + 2*e^2*m*x^2*Log[-((e*x)/d)] + 4*e^2*x^2*Log[x]*Log[f*x^m]
 - 6*d*e*m*x*Log[d + e*x] - 6*e^2*m*x^2*Log[d + e*x] + 4*e^2*m*x^2*Log[x]*
Log[d + e*x] - 2*e^2*m*x^2*Log[x]^2*Log[d + e*x] - 2*e^2*m*x^2*Log[-((e*x)
/d)]*Log[d + e*x] + 4*e^2*m*x^2*Log[x]*Log[-((e*x)/d)]*Log[d + e*x] - 4*d*
e*x*Log[f*x^m]*Log[d + e*x] - 4*e^2*x^2*Log[f*x^m]*Log[d + e*x] - 4*e^2*x^
2*Log[-((e*x)/d)]*Log[f*x^m]*Log[d + e*x] - d^2*m*Log[d + e*x]^2 + e^2*m*x
^2*Log[d + e*x]^2 - 2*e^2*m*x^2*Log[-((e*x)/d)]*Log[d + e*x]^2 - 2*d^2*Log
[f*x^m]*Log[d + e*x]^2 + 2*e^2*x^2*Log[f*x^m]*Log[d + e*x]^2 - 4*e^2*m*x^2
*Log[x]*Log[1 + (e*x)/d] + 2*e^2*m*x^2*Log[x]^2*Log[1 + (e*x)/d] + 4*e^2*m
*x^2*(-1 + Log[x])*PolyLog[2, -((e*x)/d)] - 2*e^2*x^2*(m - 2*m*Log[x] + 2*
Log[f*x^m] + 2*m*Log[d + e*x])*PolyLog[2, 1 + (e*x)/d] - 4*e^2*m*x^2*PolyL
og[3, -((e*x)/d)] + 4*e^2*m*x^2*PolyLog[3, 1 + (e*x)/d]))/(4*d^2*x^2)
```

3.372.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (fx^m) (a + b\log (c(d+ex)^n))^2}{x^3}\,dx$$

$$\Big\downarrow \text{2876}$$

$$\int \frac{\log (fx^m) (a + b\log (c(d+ex)^n))^2}{x^3}dx$$

input
```
Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^3,x]
```

output
```
$Aborted
```

3.372.3.1 Defintions of rubi rules used

rule 2876
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*Log[f*x^m]*
(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}
, x]
```

3.372.4 Maple [F]

$$\int \frac{\ln(f\,x^{m})\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^3,x)
```

output
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^3,x)
```

3.372.5 Fricas [F]

$$\int \frac{\log(fx^{m})\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x^{3}}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}\log(fx^{m})}{x^{3}}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*l
og(f*x^m) + a^2*log(f*x^m))/x^3, x)
```

3.372.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^{m})\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x^{3}}\,dx=\text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2/x**3,x)
```

output
```
Timed out
```

3.372.7 Maxima [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^2\log(fx^m)}{x^3}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(m + 2*log(f)) + 2*b^2*log(x^m))*log((e*x + d)^n)^2/x^2 + integr
ate(1/2*(2*b^2*d*log(c)^2*log(f) + 4*a*b*d*log(c)*log(f) + 2*a^2*d*log(f)
+ 2*(b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f))*x + (4*
b^2*d*log(c)*log(f) + 4*a*b*d*log(f) + (4*a*b*e*log(f) + (4*e*log(c)*log(f
) + (m*n + 2*n*log(f))*e)*b^2)*x + 2*(2*b^2*d*log(c) + 2*a*b*d + ((e*n + 2
*e*log(c))*b^2 + 2*a*b*e)*x)*log(x^m))*log((e*x + d)^n) + 2*(b^2*d*log(c)^
2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x)*
log(x^m))/(e*x^4 + d*x^3), x)
```

3.372.8 Giac [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^2\log(fx^m)}{x^3}\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m)/x^3, x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3}\,dx = \int \frac{\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{x^3}\,dx$$

input
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^3,x)
```

output
```
int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^3, x)
```

3.373 $\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx$

3.373.1 Optimal result

Integrand size = 23, antiderivative size = 522

$$\begin{aligned}
&\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx \\
&= -12ab^2mn^2x+18b^3mn^3x-6b^2mn^2(a-bn)x+6ab^2n^2x\log(fx^m) \\
&\quad -6b^3n^3x\log(fx^m)-\frac{18b^3mn^2(d+ex)\log(c(d+ex)^n)}{e} \\
&\quad -\frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e}+\frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&\quad +\frac{6bmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{3bdmn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e} \\
&\quad -\frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e}-\frac{m(d+ex)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad -\frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^3}{e}+\frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad -\frac{6b^3dmn^3\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}+\frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e} \\
&\quad -\frac{3bdmn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}-\frac{6b^3dmn^3\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e} \\
&\quad +\frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e}-\frac{6b^3dmn^3\operatorname{PolyLog}\left(4,1+\frac{ex}{d}\right)}{e}
\end{aligned}$$

output
```
-12*a*b^2*m*n^2*x+18*b^3*m*n^3*x-6*b^2*m*n^2*(-b*n+a)*x+6*a*b^2*n^2*x*ln(f
*x^m)-6*b^3*n^3*x*ln(f*x^m)-18*b^3*m*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-6*b^3*d
*m*n^2*ln(-e*x/d)*ln(c*(e*x+d)^n)/e+6*b^3*n^2*(e*x+d)*ln(f*x^m)*ln(c*(e*x+
d)^n)/e+6*b*m*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e+3*b*d*m*n*ln(-e*x/d)*(a+
b*ln(c*(e*x+d)^n))^2/e-3*b*n*(e*x+d)*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/e-m
*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e-d*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^3/
e+(e*x+d)*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^3/e-6*b^3*d*m*n^3*polylog(2,1+e*
x/d)/e+6*b^2*d*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/e-3*b*d*m*n*
(a+b*ln(c*(e*x+d)^n))^2*polylog(2,1+e*x/d)/e-6*b^3*d*m*n^3*polylog(3,1+e*x
/d)/e+6*b^2*d*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,1+e*x/d)/e-6*b^3*d*m*n
^3*polylog(4,1+e*x/d)/e
```

3.373.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1163 vs. 2(522) = 1044.

Time = 0.46 (sec) , antiderivative size = 1163, normalized size of antiderivative = 2.23

$$\int \log (fx^m) (a + b\log (c(d + ex)^n))^3 \, dx = \text{Too large to display}$$

input
```
Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
(-(b^3*n^3*(d + e*x)*(m*Log[x] - Log[f*x^m])*(-6 + 6*Log[d + e*x] - 3*Log[
d + e*x]^2 + Log[d + e*x]^3)) - 3*b^2*n^2*(m*Log[x] - Log[f*x^m])*(2*e*x -
 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2)*(a - b*n*Log[d + e*x
] + b*Log[c*(d + e*x)^n]) - 3*b*e*n*x*(m - Log[f*x^m])*Log[d + e*x]*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 3*b*d*n*(m + m*Log[x] - Log[f*
x^m])*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + e*x*(
a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(3*b*m*n + 3*b*n*(m*Log[x]
- Log[f*x^m]) + a*(-(m*Log[x]) + Log[f*x^m]) + b*(-(m*Log[x]) + Log[f*x^m]
)*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])) + a*d*m*(a - b*n*Log[d + e*x]
+ b*Log[c*(d + e*x)^n])^2*(Log[x]*Log[1 + (e*x)/d] + PolyLog[2, -((e*x)/d)
]) - b*d*m*(n*Log[d + e*x] - Log[c*(d + e*x)^n])*(a - b*n*Log[d + e*x] + b
*Log[c*(d + e*x)^n])^2*(Log[x]*Log[1 + (e*x)/d] + PolyLog[2, -((e*x)/d)])
- a*m*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(e*x + Log[x]*(-(e*x
) + d*Log[1 + (e*x)/d]) + d*PolyLog[2, -((e*x)/d)]) + 3*b*m*n*(a - b*n*Log
[d + e*x] + b*Log[c*(d + e*x)^n])^2*(e*x + Log[x]*(-(e*x) + d*Log[1 + (e*x
)/d]) + d*PolyLog[2, -((e*x)/d)]) + b*m*(n*Log[d + e*x] - Log[c*(d + e*x)^
n])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(e*x + Log[x]*(-(e*x)
+ d*Log[1 + (e*x)/d]) + d*PolyLog[2, -((e*x)/d)]) - 3*b^2*m*n^2*(-a + b*n*
Log[d + e*x] - b*Log[c*(d + e*x)^n])*(-6*e*x + 2*e*x*Log[x] + 4*d*Log[d +
e*x] + 4*e*x*Log[d + e*x] - 2*e*x*Log[x]*Log[d + e*x] - d*Log[d + e*x]^...
```

3.373.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2870, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(f x^m\right) \left(a+b \log \left(c (d+e x)^n\right)\right)^3 \, dx$$

$$\Big\downarrow \text{2870}$$

$$-m \int \Bigg(\frac{6n^2(d+ex)\log\left(c(d+ex)^n\right)b^3}{ex}+6n^2(a-bn)b^2-\frac{3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2 b}{ex}+\frac{(d+ex)(a+$$

$$6ab^2n^2x\log\left(fx^m\right)-\frac{3bn(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+$$

$$\frac{(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{6b^3n^2(d+ex)\log\left(fx^m\right)\log\left(c(d+ex)^n\right)}{e}-$$

$$6b^3n^3x\log\left(fx^m\right)$$

$$\Big\downarrow \text{2009}$$

$$6ab^2n^2x\log(fx^m) - m\left(-\frac{6b^2dn^2\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{e} - \frac{6b^2dn^2\,\mathrm{PolyLog}\left(3,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{e} + \right.$$
$$\frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} +$$
$$\frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} - 6b^3n^3x\log(fx^m)$$

input
```
Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^3,x]
```

output
```
6*a*b^2*n^2*x*Log[f*x^m] - 6*b^3*n^3*x*Log[f*x^m] + (6*b^3*n^2*(d + e*x)*L
og[f*x^m]*Log[c*(d + e*x)^n])/e - (3*b*n*(d + e*x)*Log[f*x^m]*(a + b*Log[c
*(d + e*x)^n])^2)/e + ((d + e*x)*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^3)/
e - m*(12*a*b^2*n^2*x - 18*b^3*n^3*x + 6*b^2*n^2*(a - b*n)*x + (18*b^3*n^2
*(d + e*x)*Log[c*(d + e*x)^n])/e + (6*b^3*d*n^2*Log[-((e*x)/d)]*Log[c*(d +
 e*x)^n])/e - (6*b*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - (3*b*d*n*
Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^2)/e + ((d + e*x)*(a + b*Log[c*
(d + e*x)^n])^3)/e + (d*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^3)/e +
(6*b^3*d*n^3*PolyLog[2, 1 + (e*x)/d])/e - (6*b^2*d*n^2*(a + b*Log[c*(d + e
*x)^n])*PolyLog[2, 1 + (e*x)/d])/e + (3*b*d*n*(a + b*Log[c*(d + e*x)^n])^2
*PolyLog[2, 1 + (e*x)/d])/e + (6*b^3*d*n^3*PolyLog[3, 1 + (e*x)/d])/e - (6
*b^2*d*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, 1 + (e*x)/d])/e + (6*b^3*
d*n^3*PolyLog[4, 1 + (e*x)/d])/e)
```

3.373.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2870
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*(d + e*x)^n])^p, x]},
 Simp[Log[f*x^m]   u, x] - Simp[m   Int[1/x   u, x], x]] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && IGtQ[p, 1]
```

3.373. $\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx$

3.373.4 Maple [F]

$$\int \ln\left(f\,x^{m}\right)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{3}dx$$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^3,x)
```

output
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^3,x)
```

3.373.5 Fricas [F]

$$\int \log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{3}\,dx=\int \left(b\log\left((ex+d)^{n}c\right)+a\right)^{3}\log\left(fx^{m}\right)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*log((e*x + d)^n*c)^3*log(f*x^m) + 3*a*b^2*log((e*x + d)^n*c)^
2*log(f*x^m) + 3*a^2*b*log((e*x + d)^n*c)*log(f*x^m) + a^3*log(f*x^m), x)
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{3}\,dx=\text{Timed out}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**3,x)
```

output
```
Timed out
```

3.373.7 Maxima [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx = \int (b\log((ex+d)^nc)+a)^3\log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

output
```
-(b^3*(m - log(f))*x - b^3*x*log(x^m))*log((e*x + d)^n)^3 + integrate((b^3
*d*log(c)^3*log(f) + 3*a*b^2*d*log(c)^2*log(f) + 3*a^2*b*d*log(c)*log(f) +
 a^3*d*log(f) + 3*(b^3*d*log(c)*log(f) + a*b^2*d*log(f) + (a*b^2*e*log(f)
+ (e*log(c)*log(f) + (m*n - n*log(f))*e)*b^3)*x + (b^3*d*log(c) + a*b^2*d
- ((e*n - e*log(c))*b^3 - a*b^2*e)*x)*log(x^m))*log((e*x + d)^n)^2 + (b^3*
e*log(c)^3*log(f) + 3*a*b^2*e*log(c)^2*log(f) + 3*a^2*b*e*log(c)*log(f) +
a^3*e*log(f))*x + 3*(b^3*d*log(c)^2*log(f) + 2*a*b^2*d*log(c)*log(f) + a^2
*b*d*log(f) + (b^3*e*log(c)^2*log(f) + 2*a*b^2*e*log(c)*log(f) + a^2*b*e*l
og(f))*x + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d + (b^3*e*log(c)^2
+ 2*a*b^2*e*log(c) + a^2*b*e)*x)*log(x^m))*log((e*x + d)^n) + (b^3*d*log(c
)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d + (b^3*e*log(c)^3 + 3*
a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x)*log(x^m))/(e*x + d), x)
```

3.373.8 Giac [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx = \int (b\log((ex+d)^nc)+a)^3\log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3*log(f*x^m), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx=\int \ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^3\,dx$$

input
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^3,x)
```

output
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^3, x)
```

3.374 $\int \frac{\log(x)\log^2(a+bx)}{x}\,dx$

3.374.1 Optimal result

Integrand size = 14, antiderivative size = 519

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \frac{1}{12}\left(\log^4\left(-\frac{bx}{a}\right)+6\log^2\left(-\frac{bx}{a}\right)\log^2\left(-\frac{bx}{a+bx}\right)\right.$$
$$-4\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{a}{a+bx}\right)\right)\log^3\left(-\frac{bx}{a+bx}\right)$$
$$+\log^4\left(-\frac{bx}{a+bx}\right)+6\log^2(x)\log^2(a+bx)$$
$$+4\left(2\log^3\left(-\frac{bx}{a}\right)-3\log^2(x)\log(a+bx)\right)\log\left(1+\frac{bx}{a}\right)$$
$$+6\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\left(\log(x)+3\log\left(-\frac{bx}{a}\right)\right)\log^2\left(1\right.$$
$$\left.+\frac{bx}{a}\right)-4\log^2\left(-\frac{bx}{a}\right)\log\left(-\frac{bx}{a+bx}\right)\left(\log\left(-\frac{bx}{a}\right)\right.$$
$$\left.+3\log\left(1+\frac{bx}{a}\right)\right)+12\left(\log^2\left(-\frac{bx}{a}\right)\right.$$
$$\left.-2\log\left(-\frac{bx}{a}\right)\left(\log\left(-\frac{bx}{a+bx}\right)+\log\left(1+\frac{bx}{a}\right)\right)\right)$$
$$\left.+2\log(x)\left(-\log(a+bx)+\log\left(1+\frac{bx}{a}\right)\right)\right)\mathrm{PolyLog}\left(2,-\frac{bx}{a}\right)$$
$$-12\log^2\left(-\frac{bx}{a+bx}\right)\mathrm{PolyLog}\left(2,\frac{bx}{a+bx}\right)$$
$$+12\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{bx}{a+bx}\right)\right)^2\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)$$
$$+24\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\log\left(1+\frac{bx}{a}\right)\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)$$
$$+24\left(\log\left(-\frac{bx}{a+bx}\right)+\log(a+bx)\right)\mathrm{PolyLog}\left(3,-\frac{bx}{a}\right)$$
$$+24\log\left(-\frac{bx}{a+bx}\right)\mathrm{PolyLog}\left(3,\frac{bx}{a+bx}\right)$$
$$+24\left(-\log(x)+\log\left(-\frac{bx}{a+bx}\right)\right)\mathrm{PolyLog}\left(3,1+\frac{bx}{a}\right)$$
$$-24\left(\mathrm{PolyLog}\left(4,-\frac{bx}{a}\right)+\mathrm{PolyLog}\left(4,\frac{bx}{a+bx}\right)\right.$$
$$\left.\left.-\mathrm{PolyLog}\left(4,1+\frac{bx}{a}\right)\right)\right)$$

output
```
1/12*ln(-b*x/a)^4+1/2*ln(-b*x/a)^2*ln(-b*x/(b*x+a))^2-1/3*(ln(-b*x/a)+ln(a
/(b*x+a)))*ln(-b*x/(b*x+a))^3+1/12*ln(-b*x/(b*x+a))^4+1/2*ln(x)^2*ln(b*x+a
)^2+1/3*(2*ln(-b*x/a)^3-3*ln(x)^2*ln(b*x+a))*ln(1+b*x/a)+1/2*(ln(x)-ln(-b*
x/a))*(ln(x)+3*ln(-b*x/a))*ln(1+b*x/a)^2-1/3*ln(-b*x/a)^2*ln(-b*x/(b*x+a))
*(ln(-b*x/a)+3*ln(1+b*x/a))+(ln(-b*x/a)^2-2*ln(-b*x/a)*(ln(-b*x/(b*x+a))+l
n(1+b*x/a))+2*ln(x)*(-ln(b*x+a)+ln(1+b*x/a)))*polylog(2,-b*x/a)-ln(-b*x/(b
*x+a))^2*polylog(2,b*x/(b*x+a))+(ln(-b*x/a)-ln(-b*x/(b*x+a)))^2*polylog(2,
1+b*x/a)+2*(ln(x)-ln(-b*x/a))*ln(1+b*x/a)*polylog(2,1+b*x/a)+2*(ln(-b*x/(b
*x+a))+ln(b*x+a))*polylog(3,-b*x/a)+2*ln(-b*x/(b*x+a))*polylog(3,b*x/(b*x+
a))+2*(-ln(x)+ln(-b*x/(b*x+a)))*polylog(3,1+b*x/a)-2*polylog(4,-b*x/a)-2*p
olylog(4,b*x/(b*x+a))+2*polylog(4,1+b*x/a)
```

3.374. $\int \frac{\log(x)\log^2(a+bx)}{x}\,dx$

3.374.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \frac{1}{12}\left(\log^4\left(-\frac{bx}{a}\right)+6\log^2\left(-\frac{bx}{a}\right)\log^2\left(-\frac{bx}{a+bx}\right) - 4\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{a}{a+bx}\right)\right)\log^3\left(-\frac{bx}{a+bx}\right) + \log^4\left(-\frac{bx}{a+bx}\right)+6\log^2(x)\log^2(a+bx) + 4\left(2\log^3\left(-\frac{bx}{a}\right)-3\log^2(x)\log(a+bx)\right)\log\left(1+\frac{bx}{a}\right) + 6\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\left(\log(x)+3\log\left(-\frac{bx}{a}\right)\right)\log^2\left(1+\frac{bx}{a}\right) - 4\log^2\left(-\frac{bx}{a}\right)\log\left(-\frac{bx}{a+bx}\right)\left(\log\left(-\frac{bx}{a}\right) + 3\log\left(1+\frac{bx}{a}\right)\right) + 12\left(\log^2\left(-\frac{bx}{a}\right) - 2\log\left(-\frac{bx}{a}\right)\left(\log\left(-\frac{bx}{a+bx}\right)+\log\left(1+\frac{bx}{a}\right)\right) + 2\log(x)\left(-\log(a+bx)+\log\left(1+\frac{bx}{a}\right)\right)\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right) - 12\log^2\left(-\frac{bx}{a+bx}\right)\text{PolyLog}\left(2,\frac{bx}{a+bx}\right) + 12\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{bx}{a+bx}\right)\right)^2\text{PolyLog}\left(2,1+\frac{bx}{a}\right) + 24\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\log\left(1+\frac{bx}{a}\right)\text{PolyLog}\left(2,1+\frac{bx}{a}\right) + 24\left(\log\left(-\frac{bx}{a+bx}\right)+\log(a+bx)\right)\text{PolyLog}\left(3,-\frac{bx}{a}\right) + 24\log\left(-\frac{bx}{a+bx}\right)\text{PolyLog}\left(3,\frac{bx}{a+bx}\right) + 24\left(-\log(x)+\log\left(-\frac{bx}{a+bx}\right)\right)\text{PolyLog}\left(3,1+\frac{bx}{a}\right) - 24\left(\text{PolyLog}\left(4,-\frac{bx}{a}\right)+\text{PolyLog}\left(4,\frac{bx}{a+bx}\right) - \text{PolyLog}\left(4,1+\frac{bx}{a}\right)\right)\right)$$

input
```
Integrate[(Log[x]*Log[a + b*x]^2)/x,x]
```

output
```
(Log[-((b*x)/a)]^4 + 6*Log[-((b*x)/a)]^2*Log[-((b*x)/(a + b*x))]^2 - 4*(Lo
g[-((b*x)/a)] + Log[a/(a + b*x)])*Log[-((b*x)/(a + b*x))]^3 + Log[-((b*x)/
(a + b*x))]^4 + 6*Log[x]^2*Log[a + b*x]^2 + 4*(2*Log[-((b*x)/a)]^3 - 3*Log
[x]^2*Log[a + b*x])*Log[1 + (b*x)/a] + 6*(Log[x] - Log[-((b*x)/a)])*(Log[x
] + 3*Log[-((b*x)/a)])*Log[1 + (b*x)/a]^2 - 4*Log[-((b*x)/a)]^2*Log[-((b*x
)/(a + b*x))]*(Log[-((b*x)/a)] + 3*Log[1 + (b*x)/a]) + 12*(Log[-((b*x)/a)]
^2 - 2*Log[-((b*x)/a)]*(Log[-((b*x)/(a + b*x))] + Log[1 + (b*x)/a]) + 2*Lo
g[x]*(-Log[a + b*x] + Log[1 + (b*x)/a]))*PolyLog[2, -((b*x)/a)] - 12*Log[-
((b*x)/(a + b*x))]^2*PolyLog[2, (b*x)/(a + b*x)] + 12*(Log[-((b*x)/a)] - L
og[-((b*x)/(a + b*x))])^2*PolyLog[2, 1 + (b*x)/a] + 24*(Log[x] - Log[-((b*
x)/a)])*Log[1 + (b*x)/a]*PolyLog[2, 1 + (b*x)/a] + 24*(Log[-((b*x)/(a + b*
x))] + Log[a + b*x])*PolyLog[3, -((b*x)/a)] + 24*Log[-((b*x)/(a + b*x))]*P
olyLog[3, (b*x)/(a + b*x)] + 24*(-Log[x] + Log[-((b*x)/(a + b*x))])*PolyLo
g[3, 1 + (b*x)/a] - 24*(PolyLog[4, -((b*x)/a)] + PolyLog[4, (b*x)/(a + b*x
)] - PolyLog[4, 1 + (b*x)/a]))/12
```

3.374.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{ 2874}$$

$$\frac{1}{2}\log^2(x)\log^2(a+bx)-b\int\frac{\log^2(x)\log(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{ 2891}$$

$$\frac{1}{2}\log^2(x)\log^2(a+bx)-b\int\frac{\log^2(x)\log(a+bx)}{a+bx}dx$$

input
```
Int[(Log[x]*Log[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.374.3.1 Defintions of rubi rules used

rule 2874
```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[f*x^m]^2*((a + b*Log[c*(d + e*x)^n]
)^p/(2*m)), x] - Simp[b*e*n*(p/(2*m))   Int[Log[f*x^m]^2*((a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
&& IGtQ[p, 0]
```

rule 2891
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))^(q_.)*((k_.) + (l_.)*(x_))^(r_.),
 x_Symbol] :> Unintegrable[(k + l*x)^r*(a + b*Log[c*(d + e*x)^n])^p*(f + g*
Log[h*(i + j*x)^m])^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, p, q, r}, x]
```

3.374.4 Maple [F]

$$\int \frac{\ln(x)\ln(bx+a)^{2}}{x}dx$$

input
```
int(ln(x)/x*ln(b*x+a)^2,x)
```

output
```
int(ln(x)/x*ln(b*x+a)^2,x)
```

3.374.5 Fricas [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log(bx+a)^2\log(x)}{x}\,dx$$

input
```
integrate(log(x)*log(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(log(b*x + a)^2*log(x)/x, x)
```

3.374.6 Sympy [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = -b\int \frac{\log(x)^2\log(a+bx)}{a+bx}\,dx + \frac{\log(x)^2\log(a+bx)^2}{2}$$

input
```
integrate(ln(x)*ln(b*x+a)**2/x,x)
```

output
```
-b*Integral(log(x)**2*log(a + b*x)/(a + b*x), x) + log(x)**2*log(a + b*x)*
*2/2
```

3.374.7 Maxima [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log(bx+a)^2\log(x)}{x}\,dx$$

input
```
integrate(log(x)*log(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)^2*log(x)^2 - b*integrate(log(b*x + a)*log(x)^2/(b*x + a),
 x)
```

3.374.8 Giac [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log(bx+a)^2\log(x)}{x}\,dx$$

input
```
integrate(log(x)*log(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(log(b*x + a)^2*log(x)/x, x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\ln(a+b\,x)^2\,\ln(x)}{x}\,dx$$

input
```
int((log(a + b*x)^2*log(x))/x,x)
```

output
```
int((log(a + b*x)^2*log(x))/x, x)
```

3.375 $\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx$

3.375.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \text{Int}\left(\frac{\log(fx^m)}{a+b\log(c(d+ex)^n)},x\right)$$

output
```
Unintegrable(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.375.2 Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx$$

input
```
Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]), x]
```

3.375.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2871}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}dx$$
$$\Big\downarrow \; 2871$$
$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}dx$$

input
```
Int[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]),x]
```

output
```
$Aborted
```

3.375.3.1 Defintions of rubi rules used

rule 2871
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_.), x_Symbol] :> Unintegrable[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^
p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.375.4 Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\ln(f\,x^m)}{a+b\ln(c\,(ex+d)^n)}dx$$

input
```
int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n)),x)
```

output
```
int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n)),x)
```

3.375.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{b\log((ex+d)^n c)+a}\,dx$$

input
```
integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

output
```
integral(log(f*x^m)/(b*log((e*x + d)^n*c) + a), x)
```

3.375.6 Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

input
```
integrate(ln(f*x**m)/(a+b*ln(c*(e*x+d)**n)),x)
```

output
```
Integral(log(f*x**m)/(a + b*log(c*(d + e*x)**n)), x)
```

3.375.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{b\log((ex+d)^n c)+a}\,dx$$

input
```
integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

output
```
integrate(log(f*x^m)/(b*log((e*x + d)^n*c) + a), x)
```

3.375.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a + b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{b\log((ex+d)^n c) + a}\,dx$$

input
```
integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

output
```
integrate(log(f*x^m)/(b*log((e*x + d)^n*c) + a), x)
```

3.375.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a + b\log(c(d+ex)^n)}\,dx = \int \frac{\ln(f\,x^m)}{a + b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

input
```
int(log(f*x^m)/(a + b*log(c*(d + e*x)^n)),x)
```

output
```
int(log(f*x^m)/(a + b*log(c*(d + e*x)^n)), x)
```

3.376 $\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$

3.376.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Int}\left(\frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2},x\right)$$

output
```
Unintegrable(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.376.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$$

input
```
Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2, x]
```

3.376.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2871}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}dx$$
$$\Big\downarrow \text{2871}$$
$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}dx$$

input
```
Int[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2,x]
```

output
```
$Aborted
```

3.376.3.1 Defintions of rubi rules used

rule 2871
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_.), x_Symbol] :> Unintegrable[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^
p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.376.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\ln(f\,x^m)}{(a+b\ln(c\,(ex+d)^n))^2}dx$$

input
```
int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

output
```
int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

3.376.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

output
```
integral(log(f*x^m)/(b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) +
 a^2), x)
```

3.376.6 Sympy [N/A]

Not integrable

Time = 31.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$$

input
```
integrate(ln(f*x**m)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

output
```
Integral(log(f*x**m)/(a + b*log(c*(d + e*x)**n))**2, x)
```

3.376.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.70

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

output
```
-(e*x*log(f) + d*log(f) + (e*x + d)*log(x^m))/(b^2*e*n*log((e*x + d)^n) +
b^2*e*n*log(c) + a*b*e*n) + integrate((e*(m + log(f))*x + e*x*log(x^m) + d
*m)/(b^2*e*n*x*log((e*x + d)^n) + (b^2*e*n*log(c) + a*b*e*n)*x), x)
```

3.376.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(b\log((ex+d)^n c)+a)^2}\,dx$$

input
```
integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

output
```
integrate(log(f*x^m)/(b*log((e*x + d)^n*c) + a)^2, x)
```

3.376.9 Mupad [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\ln(f\,x^m)}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

input
```
int(log(f*x^m)/(a + b*log(c*(d + e*x)^n))^2,x)
```

output
```
int(log(f*x^m)/(a + b*log(c*(d + e*x)^n))^2, x)
```

3.377 $\int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx$

3.377.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx = \text{Int}(\log(fx^m)(a+b\log(c(d+ex)^n))^p,x)$$

output
```
Unintegrable(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^p,x)
```

3.377.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx = \int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx$$

input
```
Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p,x]
```

output
```
Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p, x]
```

3.377.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2871}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log (fx^m) (a+b\log (c(d+ex)^n))^p \, dx$$

$$\Big\downarrow \ 2871$$

$$\int \log (fx^m) (a+b\log (c(d+ex)^n))^p dx$$

input
```
Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p,x]
```

output
```
$Aborted
```

3.377.3.1 Defintions of rubi rules used

rule 2871
```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_.), x_Symbol] :> Unintegrable[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^
p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.377.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \ln \left(f\,x^{m}\right) \left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{p}dx$$

input
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^p,x)
```

output
```
int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^p,x)
```

3.377.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx = \int (b\log((ex+d)^n c)+a)^p \log(fx^m)\,dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^p,x, algorithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c) + a)^p*log(f*x^m), x)
```

3.377.6 Sympy [F(-2)]

Exception generated.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.377.7 Maxima [F(-2)]

Exception generated.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.377.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p \, dx = \int (b\log((ex+d)^n c)+a)^p \log(fx^m) \, dx$$

input
```
integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^p*log(f*x^m), x)
```

3.377.9 Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^p \, dx = \int \ln(f\,x^m)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^p dx$$

input
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^p,x)
```

output
```
int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^p, x)
```

3.378 $\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx$

3.378.1 Optimal result

Integrand size = 16, antiderivative size = 364

$$\begin{aligned}\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx &= \log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx) \\ &\quad+\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{bc-ad}{b(c+dx)}\right)\right. \\ &\quad\left.-\log\left(-\frac{(bc-ad)x}{a(c+dx)}\right)\right)\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right)-\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)\right. \\ &\quad\left.-\log\left(-\frac{dx}{c}\right)\right)\left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)^2 \\ &\quad+\left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right) \\ &\quad+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{c(a+bx)}{a(c+dx)}\right) \\ &\quad-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right) \\ &\quad+\left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right) \\ &\quad-\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)+\operatorname{PolyLog}\left(3,\frac{c(a+bx)}{a(c+dx)}\right) \\ &\quad-\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\operatorname{PolyLog}\left(3,1+\frac{dx}{c}\right)\end{aligned}$$

output
```
ln(-b*x/a)*ln(b*x+a)*ln(d*x+c)+1/2*(ln(-b*x/a)+ln((-a*d+b*c)/b/(d*x+c))-ln
(-(-a*d+b*c)*x/a/(d*x+c)))*ln(a*(d*x+c)/c/(b*x+a))^2-1/2*(ln(-b*x/a)-ln(-d
*x/c))*(ln(b*x+a)+ln(a*(d*x+c)/c/(b*x+a)))^2+(ln(d*x+c)-ln(a*(d*x+c)/c/(b*
x+a)))*polylog(2,1+b*x/a)+ln(a*(d*x+c)/c/(b*x+a))*polylog(2,c*(b*x+a)/a/(d
*x+c))-ln(a*(d*x+c)/c/(b*x+a))*polylog(2,d*(b*x+a)/b/(d*x+c))+(ln(b*x+a)+l
n(a*(d*x+c)/c/(b*x+a)))*polylog(2,1+d*x/c)-polylog(3,1+b*x/a)+polylog(3,c*
(b*x+a)/a/(d*x+c))-polylog(3,d*(b*x+a)/b/(d*x+c))-polylog(3,1+d*x/c)
```

3.378.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx &= \log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx)\\ &\quad+\frac{1}{2}\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right)\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{-bc+ad}{d(a+bx)}\right)\right.\\ &\quad\left.-\log\left(\frac{bcx-adx}{ac+bcx}\right)\right)\\ &\quad+\left(-\log\left(-\frac{bx}{a}\right)+\log\left(-\frac{dx}{c}\right)\right)\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\log\left(1\right.\\ &\quad\left.+\frac{dx}{c}\right)+\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{dx}{c}\right)\right)\log\left(1\right.\\ &\quad\left.+\frac{dx}{c}\right)\left(-2\log(a+bx)+\log\left(1+\frac{dx}{c}\right)\right)\\ &\quad+\left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)\\ &\quad+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\left(-\operatorname{PolyLog}\left(2,\frac{a(c+dx)}{c(a+bx)}\right)\right.\\ &\quad\left.+\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\right)\\ &\quad+\left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)\\ &\quad-\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)+\operatorname{PolyLog}\left(3,\frac{a(c+dx)}{c(a+bx)}\right)\\ &\quad-\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\operatorname{PolyLog}\left(3,1+\frac{dx}{c}\right)\end{aligned}$$

input
```
Integrate[(Log[a + b*x]*Log[c + d*x])/x,x]
```

output
```
Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x] + (Log[(a*(c + d*x))/(c*(a + b*x
))]^2*(Log[-((b*x)/a)] + Log[(-(b*c) + a*d)/(d*(a + b*x))] - Log[(b*c*x -
a*d*x)/(a*c + b*c*x)]))/2 + (-Log[-((b*x)/a)] + Log[-((d*x)/c)])*Log[(a*(c
 + d*x))/(c*(a + b*x))]*Log[1 + (d*x)/c] + ((Log[-((b*x)/a)] - Log[-((d*x)
/c)])*Log[1 + (d*x)/c]*(-2*Log[a + b*x] + Log[1 + (d*x)/c]))/2 + (Log[c +
d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a] + Log[(a*
(c + d*x))/(c*(a + b*x))]*(-PolyLog[2, (a*(c + d*x))/(c*(a + b*x))] + Poly
Log[2, (b*(c + d*x))/(d*(a + b*x))]) + (Log[a + b*x] + Log[(a*(c + d*x))/(
c*(a + b*x))])*PolyLog[2, 1 + (d*x)/c] - PolyLog[3, 1 + (b*x)/a] + PolyLog
[3, (a*(c + d*x))/(c*(a + b*x))] - PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]
 - PolyLog[3, 1 + (d*x)/c]
```

3.378.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx$$

$$\Big\downarrow \; 2885$$

$$\text{PolyLog}\left(3,\frac{c(a+bx)}{a(c+dx)}\right)-\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)+\text{PolyLog}\left(2,\frac{c(a+bx)}{a(c+dx)}\right)\log\left(\frac{a(c+dx)}{c(a+bx)}\right)-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(\frac{a(c+dx)}{c(a+bx)}\right)+\text{PolyLog}\left(2,\frac{bx}{a}+1\right)\left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)+\text{PolyLog}\left(2,\frac{dx}{c}+1\right)\left(\log\left(\frac{a(c+dx)}{c(a+bx)}\right)+\log(a+bx)\right)+\frac{1}{2}\left(\log\left(\frac{bc-ad}{b(c+dx)}\right)-\log\left(-\frac{x(bc-ad)}{a(c+dx)}\right)+\log\left(-\frac{bx}{a}\right)\right)\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right)-\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{dx}{c}\right)\right)\left(\log\left(\frac{a(c+dx)}{c(a+bx)}\right)+\log(a+bx)\right)^2+\log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx)-\text{PolyLog}\left(3,\frac{bx}{a}+1\right)-\text{PolyLog}\left(3,\frac{dx}{c}+1\right)$$

input
```
Int[(Log[a + b*x]*Log[c + d*x])/x,x]
```

output
```
Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x] + ((Log[-((b*x)/a)] + Log[(b*c -
 a*d)/(b*(c + d*x))] - Log[-(((b*c - a*d)*x)/(a*(c + d*x)))])*Log[(a*(c +
d*x))/(c*(a + b*x))]^2)/2 - ((Log[-((b*x)/a)] - Log[-((d*x)/c)])*(Log[a +
b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2 + (Log[c + d*x] - Log[(a*(c
+ d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a] + Log[(a*(c + d*x))/(c*(a
+ b*x))]*PolyLog[2, (c*(a + b*x))/(a*(c + d*x))] - Log[(a*(c + d*x))/(c*(a
 + b*x))]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] + (Log[a + b*x] + Log[(a
*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (d*x)/c] - PolyLog[3, 1 + (b*x)
/a] + PolyLog[3, (c*(a + b*x))/(a*(c + d*x))] - PolyLog[3, (d*(a + b*x))/(
b*(c + d*x))] - PolyLog[3, 1 + (d*x)/c]
```

3.378.3.1 Defintions of rubi rules used

rule 2885
```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

3.378.4 Maple [F]

$$\int \frac{\ln(bx+a)\ln(dx+c)}{x}dx$$

input
```
int(ln(b*x+a)*ln(d*x+c)/x,x)
```

output
```
int(ln(b*x+a)*ln(d*x+c)/x,x)
```

3.378.5 Fricas [F]

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \int \frac{\log(bx+a)\log(dx+c)}{x}\,dx$$

input
```
integrate(log(b*x+a)*log(d*x+c)/x,x, algorithm="fricas")
```

output
```
integral(log(b*x + a)*log(d*x + c)/x, x)
```

3.378.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(b*x+a)*ln(d*x+c)/x,x)
```

output
```
Timed out
```

3.378.7 Maxima [F]

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \int \frac{\log(bx+a)\log(dx+c)}{x}\,dx$$

input
```
integrate(log(b*x+a)*log(d*x+c)/x,x, algorithm="maxima")
```

output
```
integrate(log(b*x + a)*log(d*x + c)/x, x)
```

3.378.8 Giac [F]

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx=\int \frac{\log{(bx+a)}\log{(dx+c)}}{x}\,dx$$

input
```
integrate(log(b*x+a)*log(d*x+c)/x,x, algorithm="giac")
```

output
```
integrate(log(b*x + a)*log(d*x + c)/x, x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx=\int \frac{\ln{(a+b\,x)}\,\ln{(c+d\,x)}}{x}\,dx$$

input
```
int((log(a + b*x)*log(c + d*x))/x,x)
```

output
```
int((log(a + b*x)*log(c + d*x))/x, x)
```

3.379 $\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))$

3.379.1 Optimal result

Integrand size = 32, antiderivative size = 258

$$\begin{aligned}
&\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx\\
&=\frac{2bd^2gn^2x}{e^2}-\frac{bdgn^2(d+ex)^2}{2e^3}+\frac{2bgn^2(d+ex)^3}{27e^3}-\frac{bd^3gn^2\log^2(d+ex)}{3e^3}\\
&\quad+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\\
&\quad-\frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3}\\
&\quad+\frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3}\\
&\quad-\frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3}\\
&\quad+\frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3}
\end{aligned}$$

output
```
2*b*d^2*g*n^2*x/e^2-1/2*b*d*g*n^2*(e*x+d)^2/e^3+2/27*b*g*n^2*(e*x+d)^3/e^3
-1/3*b*d^3*g*n^2*ln(e*x+d)^2/e^3+1/3*x^3*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(
e*x+d)^n))-d^2*n*(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^3+1/2*d*n*(e*x+
d)^2*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^3-1/9*n*(e*x+d)^3*(b*f+a*g+2*b*g*ln
(c*(e*x+d)^n))/e^3+1/3*d^3*n*ln(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^3
```

3.379.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{ex(3a(-6d^2gn+3degnx+2e^2(3f-gn)x^2)+bn(d^2(-18f+66gn)+3de(3f-5gn)x+2e^2(-3f+2gn}{}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

output
```
(e*x*(3*a*(-6*d^2*g*n + 3*d*e*g*n*x + 2*e^2*(3*f - g*n)*x^2) + b*n*(d^2*(-
18*f + 66*g*n) + 3*d*e*(3*f - 5*g*n)*x + 2*e^2*(-3*f + 2*g*n)*x^2)) + 18*d
^3*(b*f + a*g)*n*Log[d + e*x] - 6*(-3*a*e^3*g*x^3 + b*(11*d^3*g*n + 6*d^2*
e*g*n*x - 3*d*e^2*g*n*x^2 + e^3*(-3*f + 2*g*n)*x^3))*Log[c*(d + e*x)^n] +
18*b*g*(d^3 + e^3*x^3)*Log[c*(d + e*x)^n]^2)/(54*e^3)
```

3.379.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2883, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)\,dx$$

$$\Big\downarrow \text{2883}$$

$$\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-\frac{1}{3}en\int\frac{x^3(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}dx$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-\frac{1}{3}n\int\frac{x^3(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}n\int -\frac{x^3(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$\downarrow$ 27

$$\frac{n\int -\frac{e^3x^3(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)}{3e^3}+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$\downarrow$ 2772

$$\frac{n\left(-2bgn\int\left(\frac{\log(d+ex)d^3}{d+ex}-3d^2+\frac{3}{2}(d+ex)d-\frac{1}{3}(d+ex)^2\right)d(d+ex)+d^3\log(d+ex)(ag+2bg\log(c(d+ex)^n)-\right.}{}$$
$$\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$\downarrow$ 2009

$$\frac{n\left(d^3\log(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)-3d^2(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)+\frac{3}{2}d(d+ex)^2(a\right.}{}$$
$$\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

output
```
(x^3*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/3 + (n*(-2*b*g
*n*(-3*d^2*(d + e*x) + (3*d*(d + e*x)^2)/4 - (d + e*x)^3/9 + (d^3*Log[d +
e*x]^2)/2) - 3*d^2*(d + e*x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]) + (3*d
*(d + e*x)^2*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/2 - ((d + e*x)^3*(b*f
 + a*g + 2*b*g*Log[c*(d + e*x)^n]))/3 + d^3*Log[d + e*x]*(b*f + a*g + 2*b*
g*Log[c*(d + e*x)^n])))/(3*e^3)
```

3.379.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.379. $\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

3.379.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.37

method	result
parallelrisch	$\frac{18a\,e^3 f\,x^3-66b\,d^3 g\,n^2-12x^3\ln(c(ex+d)^n)b\,e^3 gn-102\ln(ex+d)b\,d^3 g\,n^2+18\ln(ex+d)a\,d^3 gn+18\ln(ex+d)b\,d^3 fn-15bd\,e^2 g\,n}{}$
risch	Expression too large to display

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOS
E)
```

output
```
1/54*(18*a*e^3*f*x^3-66*b*d^3*g*n^2-12*x^3*ln(c*(e*x+d)^n)*b*e^3*g*n-102*l
n(e*x+d)*b*d^3*g*n^2+18*ln(e*x+d)*a*d^3*g*n+18*ln(e*x+d)*b*d^3*f*n-15*b*d*
e^2*g*n^2*x^2+66*b*d^2*e*g*n^2*x+18*a*d^3*g*n+18*b*d^3*f*n+4*b*e^3*g*n^2*x
^3-6*n*a*e^3*g*x^3-6*n*b*e^3*f*x^3-36*x*ln(c*(e*x+d)^n)*b*d^2*e*g*n+18*x^2
*ln(c*(e*x+d)^n)*b*d*e^2*g*n+18*x^3*ln(c*(e*x+d)^n)^2*b*e^3*g+18*x^3*ln(c*
(e*x+d)^n)*a*e^3*g+18*x^3*ln(c*(e*x+d)^n)*b*e^3*f+36*ln(c*(e*x+d)^n)*b*d^3
*g*n+18*ln(c*(e*x+d)^n)^2*b*d^3*g-18*a*d^2*e*g*n*x-18*b*d^2*e*f*n*x+9*a*d*
e^2*g*n*x^2+9*b*d*e^2*f*n*x^2)/e^3
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.28

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{18\,be^3gx^3\log(c)^2+2\,(2\,be^3gn^2+9\,ae^3f-3\,(be^3f+ae^3g)n)x^3-3\,(5\,bde^2gn^2-3\,(bde^2f+ade^2g)n)x^2+}{}$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="
fricas")
```

output
```
1/54*(18*b*e^3*g*x^3*log(c)^2 + 2*(2*b*e^3*g*n^2 + 9*a*e^3*f - 3*(b*e^3*f
+ a*e^3*g)*n)*x^3 - 3*(5*b*d*e^2*g*n^2 - 3*(b*d*e^2*f + a*d*e^2*g)*n)*x^2
+ 18*(b*e^3*g*n^2*x^3 + b*d^3*g*n^2)*log(e*x + d)^2 + 6*(11*b*d^2*e*g*n^2
- 3*(b*d^2*e*f + a*d^2*e*g)*n)*x + 6*(3*b*d*e^2*g*n^2*x^2 - 6*b*d^2*e*g*n^
2*x - 11*b*d^3*g*n^2 - (2*b*e^3*g*n^2 - 3*(b*e^3*f + a*e^3*g)*n)*x^3 + 3*(
b*d^3*f + a*d^3*g)*n + 6*(b*e^3*g*n*x^3 + b*d^3*g*n)*log(c))*log(e*x + d)
+ 6*(3*b*d*e^2*g*n*x^2 - 6*b*d^2*e*g*n*x - (2*b*e^3*g*n - 3*b*e^3*f - 3*a*
e^3*g)*x^3)*log(c))/e^3
```

3.379.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.49

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\begin{cases}\frac{ad^3g\log(c(d+ex)^n)}{3e^3}-\frac{ad^2gnx}{3e^2}+\frac{adgnx^2}{6e}+\frac{afx^3}{3}-\frac{agnx^3}{9}+\frac{agx^3\log(c(d+ex)^n)}{3}+\frac{bd^3f\log(c(d+ex)^n)}{3e^3}-\frac{11bd^3gn\log(c(d+ex)^n)}{9e^3} \\ \frac{x^3(a+b\log(cd^n))(f+g\log(cd^n))}{3}\end{cases}$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*d**3*g*log(c*(d + e*x)**n)/(3*e**3) - a*d**2*g*n*x/(3*e**2) +
 a*d*g*n*x**2/(6*e) + a*f*x**3/3 - a*g*n*x**3/9 + a*g*x**3*log(c*(d + e*x)
**n)/3 + b*d**3*f*log(c*(d + e*x)**n)/(3*e**3) - 11*b*d**3*g*n*log(c*(d +
e*x)**n)/(9*e**3) + b*d**3*g*log(c*(d + e*x)**n)**2/(3*e**3) - b*d**2*f*n*
x/(3*e**2) + 11*b*d**2*g*n**2*x/(9*e**2) - 2*b*d**2*g*n*x*log(c*(d + e*x)*
*n)/(3*e**2) + b*d*f*n*x**2/(6*e) - 5*b*d*g*n**2*x**2/(18*e) + b*d*g*n*x**
2*log(c*(d + e*x)**n)/(3*e) - b*f*n*x**3/9 + b*f*x**3*log(c*(d + e*x)**n)/
3 + 2*b*g*n**2*x**3/27 - 2*b*g*n*x**3*log(c*(d + e*x)**n)/9 + b*g*x**3*log
(c*(d + e*x)**n)**2/3, Ne(e, 0)), (x**3*(a + b*log(c*d**n))*(f + g*log(c*d
**n))/3, True))
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.06

$$\int x^2(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{1}{3}\,bgx^3\log\left((ex+d)^nc\right)^2+\frac{1}{3}\,bfx^3\log\left((ex+d)^nc\right)+\frac{1}{3}\,agx^3\log\left((ex+d)^nc\right)$$
$$+\frac{1}{3}\,afx^3+\frac{1}{18}\,befn\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)$$
$$+\frac{1}{18}\,aegn\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)$$
$$+\frac{1}{54}\left(6\,en\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)\log\left((ex+d)^nc\right)+\frac{(4\,e^3x^3-15\,de^2x^2-18\,d^3\log\right.$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="
maxima")
```

output
```
1/3*b*g*x^3*log((e*x + d)^n*c)^2 + 1/3*b*f*x^3*log((e*x + d)^n*c) + 1/3*a*
g*x^3*log((e*x + d)^n*c) + 1/3*a*f*x^3 + 1/18*b*e*f*n*(6*d^3*log(e*x + d)/
e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 1/18*a*e*g*n*(6*d^3*log(e*x
 + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 1/54*(6*e*n*(6*d^3*lo
g(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c)
 + (4*e^3*x^3 - 15*d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3
*log(e*x + d))*n^2/e^3)*b*g
```

3.379.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(244) = 488$.

Time = 0.31 (sec) , antiderivative size = 741, normalized size of antiderivative = 2.87

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{(ex+d)^3bgn^2\log(ex+d)^2}{3e^3} - \frac{(ex+d)^2bdgn^2\log(ex+d)^2}{e^3} + \frac{(ex+d)bd^2gn^2\log(ex+d)^2}{e^3} - \frac{2(ex+d)^3bgn^2\log(ex+d)}{9e^3} + \frac{(ex+d)^2bdgn^2\log(ex+d)}{e^3} - \frac{2(ex+d)bd^2gn^2\log(ex+d)}{e^3} + \frac{2(ex+d)^3bgn\log(ex+d)\log(c)}{3e^3} - \frac{2(ex+d)^2bdgn\log(ex+d)\log(c)}{e^3} + \frac{2(ex+d)bd^2gn\log(ex+d)\log(c)}{e^3} + \frac{2(ex+d)^3bgn^2}{27e^3} - \frac{(ex+d)^2bdgn^2}{2e^3} + \frac{2(ex+d)bd^2gn^2}{e^3} + \frac{(ex+d)^3bfn\log(ex+d)}{3e^3} - \frac{(ex+d)^2bdfn\log(ex+d)}{e^3} + \frac{(ex+d)bd^2fn\log(ex+d)}{e^3} + \frac{(ex+d)^3agn\log(ex+d)}{3e^3} - \frac{(ex+d)^2adgn\log(ex+d)}{e^3} + \frac{(ex+d)ad^2gn\log(ex+d)}{e^3} - \frac{2(ex+d)^3bgn\log(c)}{9e^3} + \frac{(ex+d)^2bdgn\log(c)}{e^3} - \frac{2(ex+d)bd^2gn\log(c)}{e^3} + \frac{(ex+d)^3bg\log(c)^2}{3e^3} - \frac{(ex+d)^2bdg\log(c)^2}{e^3} + \frac{(ex+d)bd^2g\log(c)^2}{e^3} - \frac{(ex+d)^3bfn}{9e^3} + \frac{(ex+d)^2bdfn}{2e^3} - \frac{(ex+d)bd^2fn}{e^3} - \frac{(ex+d)^3agn}{9e^3} + \frac{(ex+d)^2adgn}{2e^3} - \frac{(ex+d)ad^2gn}{e^3} + \frac{(ex+d)^3bf\log(c)}{3e^3} - \frac{(ex+d)^2bdf\log(c)}{e^3} + \frac{(ex+d)bd^2f\log(c)}{e^3} + \frac{(ex+d)^3ag\log(c)}{3e^3} - \frac{(ex+d)^2adg\log(c)}{e^3} + \frac{(ex+d)ad^2g\log(c)}{e^3} + \frac{(ex+d)^3af}{3e^3} - \frac{(ex+d)^2adf}{e^3} + \frac{(ex+d)ad^2f}{e^3}$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="
giac")
```

output

```
1/3*(e*x + d)^3*b*g*n^2*log(e*x + d)^2/e^3 - (e*x + d)^2*b*d*g*n^2*log(e*x
 + d)^2/e^3 + (e*x + d)*b*d^2*g*n^2*log(e*x + d)^2/e^3 - 2/9*(e*x + d)^3*b
*g*n^2*log(e*x + d)/e^3 + (e*x + d)^2*b*d*g*n^2*log(e*x + d)/e^3 - 2*(e*x
+ d)*b*d^2*g*n^2*log(e*x + d)/e^3 + 2/3*(e*x + d)^3*b*g*n*log(e*x + d)*log
(c)/e^3 - 2*(e*x + d)^2*b*d*g*n*log(e*x + d)*log(c)/e^3 + 2*(e*x + d)*b*d^
2*g*n*log(e*x + d)*log(c)/e^3 + 2/27*(e*x + d)^3*b*g*n^2/e^3 - 1/2*(e*x +
d)^2*b*d*g*n^2/e^3 + 2*(e*x + d)*b*d^2*g*n^2/e^3 + 1/3*(e*x + d)^3*b*f*n*l
og(e*x + d)/e^3 - (e*x + d)^2*b*d*f*n*log(e*x + d)/e^3 + (e*x + d)*b*d^2*f
*n*log(e*x + d)/e^3 + 1/3*(e*x + d)^3*a*g*n*log(e*x + d)/e^3 - (e*x + d)^2
*a*d*g*n*log(e*x + d)/e^3 + (e*x + d)*a*d^2*g*n*log(e*x + d)/e^3 - 2/9*(e*
x + d)^3*b*g*n*log(c)/e^3 + (e*x + d)^2*b*d*g*n*log(c)/e^3 - 2*(e*x + d)*b
*d^2*g*n*log(c)/e^3 + 1/3*(e*x + d)^3*b*g*log(c)^2/e^3 - (e*x + d)^2*b*d*g
*log(c)^2/e^3 + (e*x + d)*b*d^2*g*log(c)^2/e^3 - 1/9*(e*x + d)^3*b*f*n/e^3
 + 1/2*(e*x + d)^2*b*d*f*n/e^3 - (e*x + d)*b*d^2*f*n/e^3 - 1/9*(e*x + d)^3
*a*g*n/e^3 + 1/2*(e*x + d)^2*a*d*g*n/e^3 - (e*x + d)*a*d^2*g*n/e^3 + 1/3*(
e*x + d)^3*b*f*log(c)/e^3 - (e*x + d)^2*b*d*f*log(c)/e^3 + (e*x + d)*b*d^2
*f*log(c)/e^3 + 1/3*(e*x + d)^3*a*g*log(c)/e^3 - (e*x + d)^2*a*d*g*log(c)/
e^3 + (e*x + d)*a*d^2*g*log(c)/e^3 + 1/3*(e*x + d)^3*a*f/e^3 - (e*x + d)^2
*a*d*f/e^3 + (e*x + d)*a*d^2*f/e^3
```

3.379. $\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$

3.379.9 Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.25

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \ln(c(d+e\,x)^n)\left(\frac{x^3\left(a\,g+b\,f-\frac{2\,b\,g\,n}{3}\right)}{3}+\frac{x^2\left(\frac{3\,d\,(a\,g+b\,f)}{2\,e}-\frac{d\,(9\,a\,g+9\,b\,f-6\,b\,g\,n)}{6\,e}\right)}{3}-\frac{d\,x\left(\frac{9\,d\,(a\,g+b\,f)}{e}-\frac{d\,(9\,a\,g+9\,b\,f-6\,b\,g\,n)}{e}\right)}{9\,e}\right)$$
$$+x^2\left(\frac{d\,(3\,a\,f-b\,g\,n^2)}{6\,e}-\frac{d\left(a\,f-\frac{a\,g\,n}{3}-\frac{b\,f\,n}{3}+\frac{2\,b\,g\,n^2}{9}\right)}{2\,e}\right)$$
$$+\ln(c(d+e\,x)^n)^2\left(\frac{b\,g\,x^3}{3}+\frac{b\,d^3\,g}{3\,e^3}\right)$$
$$-x\left(\frac{d\left(\frac{d\,(3\,a\,f-b\,g\,n^2)}{3\,e}-\frac{d\left(a\,f-\frac{a\,g\,n}{3}-\frac{b\,f\,n}{3}+\frac{2\,b\,g\,n^2}{9}\right)}{e}\right)}{e}-\frac{2\,b\,d^2\,g\,n^2}{3\,e^2}\right)$$
$$+x^3\left(\frac{a\,f}{3}-\frac{a\,g\,n}{9}-\frac{b\,f\,n}{9}+\frac{2\,b\,g\,n^2}{27}\right)+\frac{\ln(d+e\,x)\,(3\,a\,d^3\,g\,n+3\,b\,d^3\,f\,n-11\,b\,d^3\,g\,n^2)}{9\,e^3}$$

input
```
int(x^2*(a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)),x)
```

output
```
log(c*(d + e*x)^n)*((x^3*(a*g + b*f - (2*b*g*n)/3))/3 + (x^2*((3*d*(a*g +
b*f))/(2*e) - (d*(9*a*g + 9*b*f - 6*b*g*n))/(6*e)))/3 - (d*x*((9*d*(a*g +
b*f))/e - (d*(9*a*g + 9*b*f - 6*b*g*n))/e))/(9*e)) + x^2*((d*(3*a*f - b*g*
n^2))/(6*e) - (d*(a*f - (a*g*n)/3 - (b*f*n)/3 + (2*b*g*n^2)/9))/(2*e)) + l
og(c*(d + e*x)^n)^2*((b*g*x^3)/3 + (b*d^3*g)/(3*e^3)) - x*((d*((d*(3*a*f -
 b*g*n^2))/(3*e) - (d*(a*f - (a*g*n)/3 - (b*f*n)/3 + (2*b*g*n^2)/9))/e))/e
 - (2*b*d^2*g*n^2)/(3*e^2)) + x^3*((a*f)/3 - (a*g*n)/9 - (b*f*n)/9 + (2*b*
g*n^2)/27) + (log(d + e*x)*(3*a*d^3*g*n + 3*b*d^3*f*n - 11*b*d^3*g*n^2))/(
9*e^3)
```

3.380 $\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$

3.380.1 Optimal result

Integrand size = 30, antiderivative size = 196

$$\begin{aligned}\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx &= -\frac{2bdgn^2x}{e}+\frac{bgn^2(d+ex)^2}{4e^2}+\frac{bd^2gn^2\log^2(d+ex)}{2e^2}\\ &+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\\ &+\frac{dn(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^2}\\ &-\frac{n(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{4e^2}\\ &-\frac{d^2n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2e^2}\end{aligned}$$

output
```
-2*b*d*g*n^2*x/e+1/4*b*g*n^2*(e*x+d)^2/e^2+1/2*b*d^2*g*n^2*ln(e*x+d)^2/e^2
+1/2*x^2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))+d*n*(e*x+d)*(b*f+a*g+
2*b*g*ln(c*(e*x+d)^n))/e^2-1/4*n*(e*x+d)^2*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))
/e^2-1/2*d^2*n*ln(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^2
```

3.380.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.78

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{ex(2adgn+2bdn(f-3gn)+ae(2f-gn)x+ben(-f+gn)x)-2d^2(bf+ag)n\log(d+ex)+2(ae^2gx^2}{4e^2}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

output
```
(e*x*(2*a*d*g*n + 2*b*d*n*(f - 3*g*n) + a*e*(2*f - g*n)*x + b*e*n*(-f + g*
n)*x) - 2*d^2*(b*f + a*g)*n*Log[d + e*x] + 2*(a*e^2*g*x^2 + b*(3*d^2*g*n +
 2*d*e*g*n*x + e^2*(f - g*n)*x^2))*Log[c*(d + e*x)^n] - 2*b*g*(d^2 - e^2*x
^2)*Log[c*(d + e*x)^n]^2)/(4*e^2)
```

3.380.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2883, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)\,dx$$
$$\Big\downarrow \text{2883}$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-\frac{1}{2}en\int\frac{x^2(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}dx$$
$$\Big\downarrow \text{2858}$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-\frac{1}{2}n\int\frac{x^2(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-\frac{n\int\frac{e^2x^2(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)}{2e^2}$$

$\big\downarrow$ 2772

$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-$$
$$\frac{n\left(-2bgn\int\left(\frac{\log(d+ex)d^2}{d+ex}-2d+\frac{1}{2}(d+ex)\right)d(d+ex)+d^2\log(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)-2d(d+e\right.}{2e^2}$$

$\big\downarrow$ 2009

$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-$$
$$\frac{n\left(d^2\log(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)-2d(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)+\frac{1}{2}(d+ex)^2(ag\right.}{2e^2}$$

input
```
Int[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

output
```
(x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/2 - (n*(-2*b*g
*n*(-2*d*(d + e*x) + (d + e*x)^2/4 + (d^2*Log[d + e*x]^2)/2) - 2*d*(d + e*
x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]) + ((d + e*x)^2*(b*f + a*g + 2*b*
g*Log[c*(d + e*x)^n]))/2 + d^2*Log[d + e*x]*(b*f + a*g + 2*b*g*Log[c*(d +
e*x)^n])))/(2*e^2)
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

3.380.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.45

method	result
parallelrisch	$-\frac{-2x^2 \ln(c(ex+d)^n)^2 b e^2 g+2x^2 \ln(c(ex+d)^n) b e^2 g n-x^2 e^2 b g n^2-10 \ln(ex+d) b d^2 g n^2-2x^2 \ln(c(ex+d)^n) a e^2 g-2x^2 \ln(c(ex-}{}$
risch	Expression too large to display

input
```
int(x*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*x^2*ln(c*(e*x+d)^n)^2*b*e^2*g+2*x^2*ln(c*(e*x+d)^n)*b*e^2*g*n-x^2
*e^2*b*g*n^2-10*ln(e*x+d)*b*d^2*g*n^2-2*x^2*ln(c*(e*x+d)^n)*a*e^2*g-2*x^2*
ln(c*(e*x+d)^n)*b*e^2*f+x^2*e^2*n*a*g+b*e^2*f*n*x^2-4*x*ln(c*(e*x+d)^n)*b*
d*e*g*n+6*x*e*b*d*g*n^2+2*ln(e*x+d)*a*d^2*g*n+2*ln(e*x+d)*b*d^2*f*n-2*a*e^
2*f*x^2-2*a*d*e*g*n*x-2*b*d*e*f*n*x+2*ln(c*(e*x+d)^n)^2*b*d^2*g+4*ln(c*(e*
x+d)^n)*b*d^2*g*n-6*b*d^2*g*n^2+2*a*d^2*g*n+2*d^2*b*f*n)/e^2
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.31

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{2\,be^2gx^2\log(c)^2+(be^2gn^2+2\,ae^2f-(be^2f+ae^2g)n)x^2+2\,(be^2gn^2x^2-bd^2gn^2)\log(ex+d)^2-2\,(3\,bde}{}$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="fr
icas")
```

output
```
1/4*(2*b*e^2*g*x^2*log(c)^2 + (b*e^2*g*n^2 + 2*a*e^2*f - (b*e^2*f + a*e^2*
g)*n)*x^2 + 2*(b*e^2*g*n^2*x^2 - b*d^2*g*n^2)*log(e*x + d)^2 - 2*(3*b*d*e*
g*n^2 - (b*d*e*f + a*d*e*g)*n)*x + 2*(2*b*d*e*g*n^2*x + 3*b*d^2*g*n^2 - (b
*e^2*g*n^2 - (b*e^2*f + a*e^2*g)*n)*x^2 - (b*d^2*f + a*d^2*g)*n + 2*(b*e^2
*g*n*x^2 - b*d^2*g*n)*log(c))*log(e*x + d) + 2*(2*b*d*e*g*n*x - (b*e^2*g*n
 - b*e^2*f - a*e^2*g)*x^2)*log(c))/e^2
```

3.380.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.51

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \begin{cases} -\frac{ad^2g\log(c(d+ex)^n)}{2e^2}+\frac{adgnx}{2e}+\frac{afx^2}{2}-\frac{agnx^2}{4}+\frac{agx^2\log(c(d+ex)^n)}{2}-\frac{bd^2f\log(c(d+ex)^n)}{2e^2}+\frac{3bd^2gn\log(c(d+ex)^n)}{2e^2}-\frac{bd^2g}{} \\ \frac{x^2(a+b\log(cd^n))(f+g\log(cd^n))}{2} \end{cases}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((-a*d**2*g*log(c*(d + e*x)**n)/(2*e**2) + a*d*g*n*x/(2*e) + a*f*
x**2/2 - a*g*n*x**2/4 + a*g*x**2*log(c*(d + e*x)**n)/2 - b*d**2*f*log(c*(d
 + e*x)**n)/(2*e**2) + 3*b*d**2*g*n*log(c*(d + e*x)**n)/(2*e**2) - b*d**2*
g*log(c*(d + e*x)**n)**2/(2*e**2) + b*d*f*n*x/(2*e) - 3*b*d*g*n**2*x/(2*e)
 + b*d*g*n*x*log(c*(d + e*x)**n)/e - b*f*n*x**2/4 + b*f*x**2*log(c*(d + e*
x)**n)/2 + b*g*n**2*x**2/4 - b*g*n*x**2*log(c*(d + e*x)**n)/2 + b*g*x**2*l
og(c*(d + e*x)**n)**2/2, Ne(e, 0)), (x**2*(a + b*log(c*d**n))*(f + g*log(c
*d**n))/2, True))
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.14

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx = \frac{1}{2}bgx^2\log((ex+d)^nc)^2 - \frac{1}{4}befn\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right) - \frac{1}{4}aegn\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right) + \frac{1}{2}bfx^2\log((ex+d)^nc) + \frac{1}{2}agx^2\log((ex+d)^nc) + \frac{1}{2}afx^2 - \frac{1}{4}\left(2en\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right)\log((ex+d)^nc) - \frac{(e^2x^2+2d^2\log(ex+d)^2-6dex+6d^2\mathrm{l}}{e^2}\right.$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="ma
xima")
```

output
```
1/2*b*g*x^2*log((e*x + d)^n*c)^2 - 1/4*b*e*f*n*(2*d^2*log(e*x + d)/e^3 + (
e*x^2 - 2*d*x)/e^2) - 1/4*a*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x
)/e^2) + 1/2*b*f*x^2*log((e*x + d)^n*c) + 1/2*a*g*x^2*log((e*x + d)^n*c) +
 1/2*a*f*x^2 - 1/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*l
og((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(
e*x + d))*n^2/e^2)*b*g
```

3.380.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(186) = 372$.

Time = 0.32 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.38

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{(ex+d)^2bgn^2\log(ex+d)^2}{2e^2}-\frac{(ex+d)bdgn^2\log(ex+d)^2}{e^2}-\frac{(ex+d)^2bgn^2\log(ex+d)}{2e^2}$$
$$+\frac{2(ex+d)bdgn^2\log(ex+d)}{e^2}+\frac{(ex+d)^2bgn\log(ex+d)\log(c)}{e^2}$$
$$-\frac{2(ex+d)bdgn\log(ex+d)\log(c)}{e^2}+\frac{(ex+d)^2bgn^2}{4e^2}-\frac{2(ex+d)bdgn^2}{e^2}$$
$$+\frac{(ex+d)^2bfn\log(ex+d)}{2e^2}-\frac{(ex+d)bdfn\log(ex+d)}{e^2}+\frac{(ex+d)^2agn\log(ex+d)}{2e^2}$$
$$-\frac{(ex+d)adgn\log(ex+d)}{e^2}-\frac{(ex+d)^2bgn\log(c)}{2e^2}+\frac{2(ex+d)bdgn\log(c)}{e^2}$$
$$+\frac{(ex+d)^2bg\log(c)^2}{2e^2}-\frac{(ex+d)bdg\log(c)^2}{e^2}-\frac{(ex+d)^2bfn}{4e^2}+\frac{(ex+d)bdfn}{e^2}$$
$$-\frac{(ex+d)^2agn}{4e^2}+\frac{(ex+d)adgn}{e^2}+\frac{(ex+d)^2bf\log(c)}{2e^2}-\frac{(ex+d)bdf\log(c)}{e^2}$$
$$+\frac{(ex+d)^2ag\log(c)}{2e^2}-\frac{(ex+d)adg\log(c)}{e^2}+\frac{(ex+d)^2af}{2e^2}-\frac{(ex+d)adf}{e^2}$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="gi
ac")
```

output
```
1/2*(e*x + d)^2*b*g*n^2*log(e*x + d)^2/e^2 - (e*x + d)*b*d*g*n^2*log(e*x +
 d)^2/e^2 - 1/2*(e*x + d)^2*b*g*n^2*log(e*x + d)/e^2 + 2*(e*x + d)*b*d*g*n
^2*log(e*x + d)/e^2 + (e*x + d)^2*b*g*n*log(e*x + d)*log(c)/e^2 - 2*(e*x +
 d)*b*d*g*n*log(e*x + d)*log(c)/e^2 + 1/4*(e*x + d)^2*b*g*n^2/e^2 - 2*(e*x
 + d)*b*d*g*n^2/e^2 + 1/2*(e*x + d)^2*b*f*n*log(e*x + d)/e^2 - (e*x + d)*b
*d*f*n*log(e*x + d)/e^2 + 1/2*(e*x + d)^2*a*g*n*log(e*x + d)/e^2 - (e*x +
d)*a*d*g*n*log(e*x + d)/e^2 - 1/2*(e*x + d)^2*b*g*n*log(c)/e^2 + 2*(e*x +
d)*b*d*g*n*log(c)/e^2 + 1/2*(e*x + d)^2*b*g*log(c)^2/e^2 - (e*x + d)*b*d*g
*log(c)^2/e^2 - 1/4*(e*x + d)^2*b*f*n/e^2 + (e*x + d)*b*d*f*n/e^2 - 1/4*(e
*x + d)^2*a*g*n/e^2 + (e*x + d)*a*d*g*n/e^2 + 1/2*(e*x + d)^2*b*f*log(c)/e
^2 - (e*x + d)*b*d*f*log(c)/e^2 + 1/2*(e*x + d)^2*a*g*log(c)/e^2 - (e*x +
d)*a*d*g*log(c)/e^2 + 1/2*(e*x + d)^2*a*f/e^2 - (e*x + d)*a*d*f/e^2
```

3.380. $\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$

3.380.9 Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.04

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=x\left(\frac{d(af-bgn^2)}{e}-\frac{d\left(af-\frac{agn}{2}-\frac{bfn}{2}+\frac{bgn^2}{2}\right)}{e}\right)$$
$$+\ln(c(d+ex)^n)\left(\left(\frac{ag}{2}+\frac{bf}{2}-\frac{bgn}{2}\right)x^2+\left(\frac{d(ag+bf)}{e}-\frac{d(ag+bf-bgn)}{e}\right)x\right)$$
$$+\ln(c(d+ex)^n)^2\left(\frac{bgx^2}{2}-\frac{bd^2g}{2e^2}\right)+x^2\left(\frac{af}{2}-\frac{agn}{4}-\frac{bfn}{4}+\frac{bgn^2}{4}\right)$$
$$-\frac{\ln(d+ex)\,(ad^2gn+bd^2fn-3bd^2gn^2)}{2e^2}$$

input
```
int(x*(a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)),x)
```

output
```
x*((d*(a*f - b*g*n^2))/e - (d*(a*f - (a*g*n)/2 - (b*f*n)/2 + (b*g*n^2)/2))
/e) + log(c*(d + e*x)^n)*(x*((d*(a*g + b*f))/e - (d*(a*g + b*f - b*g*n))/e
) + x^2*((a*g)/2 + (b*f)/2 - (b*g*n)/2)) + log(c*(d + e*x)^n)^2*((b*g*x^2)
/2 - (b*d^2*g)/(2*e^2)) + x^2*((a*f)/2 - (a*g*n)/4 - (b*f*n)/4 + (b*g*n^2)
/4) - (log(d + e*x)*(a*d^2*g*n + b*d^2*f*n - 3*b*d^2*g*n^2))/(2*e^2)
```

3.381 $\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$

3.381.1 Optimal result

Integrand size = 29, antiderivative size = 110

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= -((bf+ag)nx)+2bgn^2x-\frac{2bgn(d+ex)\log(c(d+ex)^n)}{e}$$
$$+x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))+\frac{d(bf+ag+2bg\log(c(d+ex)^n))^2}{4beg}$$

output
```
-(a*g+b*f)*n*x+2*b*g*n^2*x-2*b*g*n*(e*x+d)*ln(c*(e*x+d)^n)/e+x*(a+b*ln(c*(
e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))+1/4*d*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))^2/b
/e/g
```

3.381.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.69

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{e(a(f-gn)+bn(-f+2gn))x+(ag+b(f-2gn))(d+ex)\log(c(d+ex)^n)+bg(d+ex)\log^2(c(d+ex}{e}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

output
```
(e*(a*(f - g*n) + b*n*(-f + 2*g*n))*x + (a*g + b*(f - 2*g*n))*(d + e*x)*Lo
g[c*(d + e*x)^n] + b*g*(d + e*x)*Log[c*(d + e*x)^n]^2)/e
```

3.381.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2878, 2858, 25, 27, 2788, 2009, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)\,dx$$

$$\Big\downarrow \text{2878}$$

$$x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-en\int \frac{x(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}dx$$

$$\Big\downarrow \text{2858}$$

$$x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-n\int \frac{x(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)$$

$$\Big\downarrow \text{25}$$

$$n\int -\frac{x(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)+x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$$\Big\downarrow \text{27}$$

$$\frac{n\int -\frac{ex(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}d(d+ex)}{e}+x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$$\Big\downarrow \text{2788}$$

$$\frac{n\left(d\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{d+ex}d(d+ex)-\int (bf+ag+2bg\log(c(d+ex)^n))d(d+ex)\right)}{e}+x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$$\Big\downarrow \text{2009}$$

$$\frac{n\left(d\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{d+ex}d(d+ex)-((d+ex)(ag+bf))-2bg(d+ex)\log(c(d+ex)^n)+2bgn(d+ex)\right)}{e}+x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)$$

$$\Big\downarrow \text{2738}$$

$$\frac{x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)+n\left(\frac{d(ag+2bg\log(c(d+ex)^n)+bf)^2}{4bgn}-(d+ex)(ag+bf)-2bg(d+ex)\log(c(d+ex)^n)+2bgn(d+ex)\right)}{e}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

output
```
x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]) + (n*(-((b*f + a*g
)*(d + e*x)) + 2*b*g*n*(d + e*x) - 2*b*g*(d + e*x)*Log[c*(d + e*x)^n] + (d
*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n])^2)/(4*b*g*n)))/e
```

3.381.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2878
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c*(d + e
*x)^n])*(f + g*Log[c*(d + e*x)^n]), x] - Simp[e*n   Int[(x*(b*f + a*g + 2*b
*g*Log[c*(d + e*x)^n]))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}
, x]
```

3.381.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
norman	$(2bg\,n^2-agn-bfn+af)\,x+(-2bgn+ag+bf)\,x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+bgx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^2$
parts	$xaf+(ag+bf)\left(x\ln\left(c(ex+d)^n\right)-en\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^2}\right)\right)+bgx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^2+\frac{dgb\ln(c\,\mathrm{e}^{n}}{}$
default	$xaf+xag\ln\left(c(ex+d)^n\right)-agnx+\frac{agnd\ln(ex+d)}{e}+xb\ln\left(c(ex+d)^n\right)f-bfnx+\frac{bfnd\ln(ex+d}{e}$
parallelrisch	$\frac{x\ln(c(ex+d)^n)^2begn-2x\ln(c(ex+d)^n)beg\,n^2+2xbeg\,n^3+x\ln(c(ex+d)^n)aegn+x\ln(c(ex+d)^n)befn-xaeg\,n^2-xbef\,n^2+\ln(c}{en}$
risch	Expression too large to display

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

output
```
(2*b*g*n^2-a*g*n-b*f*n+a*f)*x+(-2*b*g*n+a*g+b*f)*x*ln(c*exp(n*ln(e*x+d)))+
b*g*x*ln(c*exp(n*ln(e*x+d)))^2+n*(-2*b*d*g*n+a*d*g+b*d*f)/e*ln(e*x+d)+d*g*
b/e*ln(c*exp(n*ln(e*x+d)))^2
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int (a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{begx\log(c)^2+(begn^2x+bdgn^2)\log(ex+d)^2-(2\,begn-bef-aeg)x\log(c)+(2\,begn^2+aef-(bef+}{}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="fric
as")
```

output
```
(b*e*g*x*log(c)^2 + (b*e*g*n^2*x + b*d*g*n^2)*log(e*x + d)^2 - (2*b*e*g*n
- b*e*f - a*e*g)*x*log(c) + (2*b*e*g*n^2 + a*e*f - (b*e*f + a*e*g)*n)*x -
(2*b*d*g*n^2 - (b*d*f + a*d*g)*n + (2*b*e*g*n^2 - (b*e*f + a*e*g)*n)*x - 2
*(b*e*g*n*x + b*d*g*n)*log(c))*log(e*x + d))/e
```

3.381.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.72

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \begin{cases} \frac{adg\log(c(d+ex)^n)}{e} + afx - agnx + agx\log(c(d+ex)^n) + \frac{bdf\log(c(d+ex)^n)}{e} - \frac{2bdgn\log(c(d+ex)^n)}{e} + \frac{bdg\log(c(d+ex)}{e} \\ x(a+b\log(cd^n))(f+g\log(cd^n)) \end{cases}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n)),x)
```

output
```
Piecewise((a*d*g*log(c*(d + e*x)**n)/e + a*f*x - a*g*n*x + a*g*x*log(c*(d
+ e*x)**n) + b*d*f*log(c*(d + e*x)**n)/e - 2*b*d*g*n*log(c*(d + e*x)**n)/e
 + b*d*g*log(c*(d + e*x)**n)**2/e - b*f*n*x + b*f*x*log(c*(d + e*x)**n) +
2*b*g*n**2*x - 2*b*g*n*x*log(c*(d + e*x)**n) + b*g*x*log(c*(d + e*x)**n)**
2, Ne(e, 0)), (x*(a + b*log(c*d**n))*(f + g*log(c*d**n)), True))
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= -befn\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) - aegn\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + bgx\log\left((ex+d)^n c\right)^2 + bfx\log\left((ex+d)^n c\right) + agx\log\left((ex+d)^n c\right) - \left(2\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^n c\right) + \frac{\left(d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d)\right)n^2}{e}\right)bg + afx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="maxi
ma")
```

output
```
-b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a*e*g*n*(x/e - d*log(e*x + d)/e^2) +
 b*g*x*log((e*x + d)^n*c)^2 + b*f*x*log((e*x + d)^n*c) + a*g*x*log((e*x +
d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*
x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b*g + a*f*x
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int (a+b\log (c(d+ex)^n))\,(f+g\log (c(d+ex)^n))\;dx = \frac{(ex+d)bgn^2\log (ex+d)^2}{e} - \frac{2\,(ex+d)bgn^2\log (ex+d)}{e} + \frac{2\,(ex+d)bgn\log (ex+d)\log (c)}{e} + \frac{2\,(ex+d)bgn^2}{e} + \frac{(ex+d)bfn\log (ex+d)}{e} + \frac{(ex+d)agn\log (ex+d)}{e} - \frac{2\,(ex+d)bgn\log (c)}{e} + \frac{(ex+d)bg\log (c)^2}{e} - \frac{(ex+d)bfn}{e} - \frac{(ex+d)agn}{e} + \frac{(ex+d)bf\log (c)}{e} + \frac{(ex+d)ag\log (c)}{e} + \frac{(ex+d)af}{e}$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="giac
")
```

output
```
(e*x + d)*b*g*n^2*log(e*x + d)^2/e - 2*(e*x + d)*b*g*n^2*log(e*x + d)/e +
2*(e*x + d)*b*g*n*log(e*x + d)*log(c)/e + 2*(e*x + d)*b*g*n^2/e + (e*x + d
)*b*f*n*log(e*x + d)/e + (e*x + d)*a*g*n*log(e*x + d)/e - 2*(e*x + d)*b*g*
n*log(c)/e + (e*x + d)*b*g*log(c)^2/e - (e*x + d)*b*f*n/e - (e*x + d)*a*g*
n/e + (e*x + d)*b*f*log(c)/e + (e*x + d)*a*g*log(c)/e + (e*x + d)*a*f/e
```

3.381.9 Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\ln\left(c\,(d+e\,x)^n\right)^2\left(b\,g\,x+\frac{b\,d\,g}{e}\right)+x\,\left(a\,f-a\,g\,n-b\,f\,n+2\,b\,g\,n^2\right)$$
$$+x\,\ln\left(c\,(d+e\,x)^n\right)\,\left(a\,g+b\,f-2\,b\,g\,n\right)+\frac{\ln\left(d+e\,x\right)\,\left(a\,d\,g\,n-2\,b\,d\,g\,n^2+b\,d\,f\,n\right)}{e}$$

input
```
int((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)),x)
```

output
```
log(c*(d + e*x)^n)^2*(b*g*x + (b*d*g)/e) + x*(a*f - a*g*n - b*f*n + 2*b*g*
n^2) + x*log(c*(d + e*x)^n)*(a*g + b*f - 2*b*g*n) + (log(d + e*x)*(a*d*g*n
 - 2*b*d*g*n^2 + b*d*f*n))/e
```

3.382 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$

3.382.1 Optimal result

Integrand size = 32, antiderivative size = 158

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx &= \log(x)\,(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n)) \\ &\quad - \frac{\log(x)\,(bf+ag+2bg\log(c(d+ex)^n))^2}{4bg} + \frac{\log\left(-\frac{ex}{d}\right)(bf+ag+2bg\log(c(d+ex)^n))^2}{4bg} \\ &\quad + n(bf+ag+2bg\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right) - 2bgn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)\end{aligned}$$

output
```
ln(x)*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))-1/4*ln(x)*(b*f+a*g+2*b*g
*ln(c*(e*x+d)^n))^2/b/g+1/4*ln(-e*x/d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))^2/b
/g+n*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)-2*b*g*n^2*polylog(
3,1+e*x/d)
```

3.382.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$$
$$= af\log(x)+bf\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)+ag\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)$$
$$+bg\log(x)(-n\log(d+ex)+\log(c(d+ex)^n))^2+2bgn(-n\log(d+ex)$$
$$+\log(c(d+ex)^n))\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\right)$$
$$+bfn\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)+agn\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)$$
$$+2bgn^2\left(\frac{1}{2}\log^2(d+ex)\log\left(1-\frac{d+ex}{d}\right)+\log(d+ex)\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)\right.$$
$$\left.-\operatorname{PolyLog}\left(3,\frac{d+ex}{d}\right)\right)$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x,x]
```

output
```
a*f*Log[x] + b*f*Log[-((e*x)/d)]*Log[c*(d + e*x)^n] + a*g*Log[-((e*x)/d)]*
Log[c*(d + e*x)^n] + b*g*Log[x]*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])^2
 + 2*b*g*n*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])*(Log[x]*(Log[d + e*x]
- Log[1 + (e*x)/d]) - PolyLog[2, -((e*x)/d)]) + b*f*n*PolyLog[2, (d + e*x)
/d] + a*g*n*PolyLog[2, (d + e*x)/d] + 2*b*g*n^2*((Log[d + e*x]^2*Log[1 - (
d + e*x)/d])/2 + Log[d + e*x]*PolyLog[2, (d + e*x)/d] - PolyLog[3, (d + e*
x)/d])
```

3.382.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2882, 2881, 2822, 25, 27, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x}\,dx$$
$$\Big\downarrow \text{2882}$$

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-en\int\frac{\log(x)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)}{d+ex}dx$$

$\Big\downarrow$ 2881

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\int\frac{\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)\log\left(\frac{d+ex}{e}-\frac{d}{e}\right)}{d+ex}d(d+ex)$$

$\Big\downarrow$ 2822

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\left(\frac{\log\left(\frac{d+ex}{e}-\frac{d}{e}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)^2}{4bgn}-\frac{\int\frac{\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{x}d(d+ex)}{4begn}\right)$$

$\Big\downarrow$ 25

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\left(\frac{\int-\frac{\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{x}d(d+ex)}{4begn}+\frac{\log\left(\frac{d+ex}{e}-\frac{d}{e}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)^2}{4bgn}\right)$$

$\Big\downarrow$ 27

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\left(\frac{\int-\frac{\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{ex}d(d+ex)}{4bgn}+\frac{\log\left(\frac{d+ex}{e}-\frac{d}{e}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)^2}{4bgn}\right)$$

$\Big\downarrow$ 2754

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\left(\frac{4bgn\int\frac{\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d+ex}{d}\right)}{d+ex}d(d+ex)-\log\left(1-\frac{d+ex}{d}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)^2}{4bgn}+\log(\ldots\right.$$

$\Big\downarrow$ 2821

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\left(\frac{4bgn\left(2bgn\int\frac{\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,\frac{d+ex}{d}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)\right)-\log\left(1-\frac{d+ex}{d}\right)\ldots}{4bgn}\right.$$

$\Big\downarrow$ 7143

$$\log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)-n\left(\frac{4bgn\left(2bgn\,\text{PolyLog}\left(3,\frac{d+ex}{d}\right)-\text{PolyLog}\left(2,\frac{d+ex}{d}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)\right)-\log\left(1-\frac{d+ex}{d}\right)\left(ag+2b\ldots\right.}{4bgn}\right.$$

3.382. $\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)}{x}\,dx$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x,x]
```

output
```
Log[x]*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]) - n*(((b*f +
a*g + 2*b*g*Log[c*(d + e*x)^n])^2*Log[-(d/e) + (d + e*x)/e])/(4*b*g*n) + (
-((b*f + a*g + 2*b*g*Log[c*(d + e*x)^n])^2*Log[1 - (d + e*x)/d]) + 4*b*g*n
*(-((b*f + a*g + 2*b*g*Log[c*(d + e*x)^n])*PolyLog[2, (d + e*x)/d]) + 2*b*
g*n*PolyLog[3, (d + e*x)/d]))/(4*b*g*n))
```

3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

3.382. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2882
```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.
)*((d_) + (e_.)*(x_))^(n_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b*
Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]), x] - Simp[e*n   Int[(Log[x]
*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d + e*x), x], x] /; FreeQ[{a, b,
c, d, e, f, g, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.382.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.96

method	result
risch	$\left(-i\pi bg \operatorname{csgn}(ic) \operatorname{csgn}(i(ex+d)^n) \operatorname{csgn}(ic(ex+d)^n) + i\pi bg \operatorname{csgn}(ic) \operatorname{csgn}(ic(ex+d)^n)^2 + i\pi bg\right.$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x,x,method=_RETURNVERBOSE)
```

output
```
(-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I
*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
-I*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(ln((e*x+d)^n)*ln(x)-
e*n*(dilog((e*x+d)/d)/e+ln(x)*ln((e*x+d)/d)/e))+ln(e*x+d)^2*ln(e*x)*b*g*n^
2+ln(e*x+d)^2*ln(1-(e*x+d)/d)*b*g*n^2-2*ln(e*x+d)^2*ln(-e*x/d)*b*g*n^2-2*l
n(e*x+d)*ln(e*x)*ln((e*x+d)^n)*b*g*n+2*ln(e*x+d)*polylog(2,(e*x+d)/d)*b*g*
n^2-2*ln(e*x+d)*dilog(-e*x/d)*b*g*n^2+2*ln(e*x+d)*ln(-e*x/d)*ln((e*x+d)^n)
*b*g*n+ln(e*x)*ln((e*x+d)^n)^2*b*g-2*polylog(3,(e*x+d)/d)*b*g*n^2+2*dilog(
-e*x/d)*ln((e*x+d)^n)*b*g*n+1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csg
n(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^
n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*(-I
*g*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*g*Pi*csgn(I*c)*csg
n(I*c*(e*x+d)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*g*Pi*c
sgn(I*c*(e*x+d)^n)^3+2*g*ln(c)+2*f)*ln(x)
```

3.382.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x,x, algorithm="fr
icas")
```

output
```
integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))
/x, x)
```

3.382.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x, x)
```

3.382.7 Maxima [F]

$$\int \frac{(a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c) + a)(g\log((ex+d)^n c) + f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x,x, algorithm="ma
xima")
```

output
```
a*f*log(x) + integrate((b*g*log((e*x + d)^n)^2 + a*g*log(c) + (g*log(c)^2
+ f*log(c))*b + ((2*g*log(c) + f)*b + a*g)*log((e*x + d)^n))/x, x)
```

3.382.8 Giac [F]

$$\int \frac{(a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c) + a)(g\log((ex+d)^n c) + f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x,x, algorithm="gi
ac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$$
$$=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(c\,(d+e\,x)^n))}{x}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x, x)
```

3.383 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx$

3.383.1 Optimal result

Integrand size = 32, antiderivative size = 96

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x} + \frac{en(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d} - \frac{2begn^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d}$$

output
```
-(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x+e*n*(b*f+a*g+2*b*g*ln(c*(e*
x+d)^n))*ln(1-d/(e*x+d))/d-2*b*e*g*n^2*polylog(2,d/(e*x+d))/d
```

3.383.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = -\frac{af}{x} + \frac{befn\log(x)}{d} + \frac{aegn\log(x)}{d} - \frac{befn\log(d+ex)}{d} - \frac{aegn\log(d+ex)}{d} - \frac{bf\log(c(d+ex)^n)}{x} - \frac{ag\log(c(d+ex)^n)}{x} + \frac{2begn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{d} - \frac{beg\log^2(c(d+ex)^n)}{d} - \frac{bg\log^2(c(d+ex)^n)}{x} + \frac{2begn^2\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d}$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^2,x]
```

output
```
-((a*f)/x) + (b*e*f*n*Log[x])/d + (a*e*g*n*Log[x])/d - (b*e*f*n*Log[d + e*
x])/d - (a*e*g*n*Log[d + e*x])/d - (b*f*Log[c*(d + e*x)^n])/x - (a*g*Log[c
*(d + e*x)^n])/x + (2*b*e*g*n*Log[-((e*x)/d)]*Log[c*(d + e*x)^n])/d - (b*e
*g*Log[c*(d + e*x)^n]^2)/d - (b*g*Log[c*(d + e*x)^n]^2)/x + (2*b*e*g*n^2*P
olyLog[2, (d + e*x)/d])/d
```

3.383.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2883, 2858, 25, 27, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x^2}dx$$

$$\Big\downarrow \text{ 2883}$$

$$en\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{x(d+ex)}dx-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x}$$

$$\Big\downarrow \text{ 2858}$$

$$n\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{x(d+ex)}d(d+ex)-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-n\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{x(d+ex)}d(d+ex)-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-en\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x}$$

$$\Big\downarrow \text{ 2779}$$

$$-en\left(\frac{2bgn\int \frac{\log\left(1-\frac{d}{d+ex}\right)}{d+ex}d(d+ex)}{d}-\frac{\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{d}\right)-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x}$$

$$\Big\downarrow \text{2838}$$

$$-en\left(\frac{2bgn\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d}-\frac{\log\left(1-\frac{d}{d+ex}\right)\left(ag+2bg\log\left(c(d+ex)^n\right)+bf\right)}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{x}$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^2,x]
```

output
```
-(((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x) - e*n*(-(((b*
f + a*g + 2*b*g*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/d) + (2*b*g*n*Po
lyLog[2, d/(d + e*x)])/d)
```

3.383.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.383. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx$

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

3.383.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.39

method	result
risch	$\left(-i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i(ex+d)^{n}\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)+i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)^{2}+i\pi bg\right.$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^2,x,method=_RETURNVERBOS
E)
```

output
```
(-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I
*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
-I*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(-ln((e*x+d)^n)/x+e*n
*(-1/d*ln(e*x+d)+1/d*ln(x)))-1/x*ln((e*x+d)^n)^2*b*g-2*b*g*e*n*ln((e*x+d)^
n)/d*ln(e*x+d)+2*b*g*e*n*ln((e*x+d)^n)/d*ln(x)-2*b*g*e*n^2/d*dilog((e*x+d)
/d)-2*b*g*e*n^2/d*ln(x)*ln((e*x+d)/d)+b*g*e*n^2/d*ln(e*x+d)^2-1/4*(-I*b*Pi
*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(
e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c
*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*(-I*g*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)+I*g*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*g*Pi*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)^2-I*g*Pi*csgn(I*c*(e*x+d)^n)^3+2*g*ln(c)+2*f)/x
```

3.383.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^2,x, algorithm="
fricas")
```

output
```
integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))
/x^2, x)
```

3.383.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x**2, x)
```

3.383.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^2,x, algorithm="
maxima")
```

output
```
-b*e*f*n*(log(e*x + d)/d - log(x)/d) - a*e*g*n*(log(e*x + d)/d - log(x)/d)
 - b*g*(log((e*x + d)^n)^2/x - integrate((e*x*log(c)^2 + d*log(c)^2 + 2*((
e*n + e*log(c))*x + d*log(c))*log((e*x + d)^n))/(e*x^3 + d*x^2), x)) - b*f
*log((e*x + d)^n*c)/x - a*g*log((e*x + d)^n*c)/x - a*f/x
```

3.383.8 Giac [F]

$$\int \frac{(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex + d)^n c) + a)(g\log((ex + d)^n c) + f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^2,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x^2, x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n))}{x^2}\,dx = \int \frac{(a + b\ln(c\,(d + e\,x)^n))\,(f + g\ln(c\,(d + e\,x)^n))}{x^2}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^2,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^2, x)
```

3.384 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3} dx$

3.384.1 Optimal result

Integrand size = 32, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3} dx &= \frac{be^2gn^2\log(x)}{d^2} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} \\ &\quad - \frac{en(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2d^2x} \\ &\quad - \frac{e^2n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{2d^2} + \frac{be^2gn^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2}\end{aligned}$$

output
```
b*e^2*g*n^2*ln(x)/d^2-1/2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^2-
1/2*e*n*(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/d^2/x-1/2*e^2*n*(b*f+a*g+2
*b*g*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^2+b*e^2*g*n^2*polylog(2,d/(e*x+d))
/d^2
```

3.384.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.61

$$\begin{aligned}&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx\\&=-\frac{af}{2x^2}+\frac{1}{2}befn\left(-\frac{1}{dx}-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}\right)\\&\quad+\frac{1}{2}aegn\left(-\frac{1}{dx}-\frac{e\log(x)}{d^2}+\frac{e\log(d+ex)}{d^2}\right)-\frac{bf\log(c(d+ex)^n)}{2x^2}-\frac{ag\log(c(d+ex)^n)}{2x^2}\\&\quad-\frac{bg\log^2(c(d+ex)^n)}{2x^2}+begn\left(\frac{en\log(x)}{d^2}-\frac{en\log(d+ex)}{d^2}-\frac{\log(c(d+ex)^n)}{dx}\right.\\&\qquad\left.-\frac{e\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{d^2}+\frac{e\log^2(c(d+ex)^n)}{2d^2n}-\frac{en\,\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^2}\right)\end{aligned}$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^3,x]
```

output
```
-1/2*(a*f)/x^2 + (b*e*f*n*(-(1/(d*x)) - (e*Log[x])/d^2 + (e*Log[d + e*x])/
d^2))/2 + (a*e*g*n*(-(1/(d*x)) - (e*Log[x])/d^2 + (e*Log[d + e*x])/d^2))/2
 - (b*f*Log[c*(d + e*x)^n])/(2*x^2) - (a*g*Log[c*(d + e*x)^n])/(2*x^2) - (
b*g*Log[c*(d + e*x)^n]^2)/(2*x^2) + b*e*g*n*((e*n*Log[x])/d^2 - (e*n*Log[d
 + e*x])/d^2 - Log[c*(d + e*x)^n]/(d*x) - (e*Log[-((e*x)/d)]*Log[c*(d + e*
x)^n])/d^2 + (e*Log[c*(d + e*x)^n]^2)/(2*d^2*n) - (e*n*PolyLog[2, (d + e*x
)/d])/d^2)
```

3.384.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2883, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x^3}\,dx$$

$$\Big\downarrow \ 2883$$

$$\frac{1}{2}en\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{x^2(d+ex)}dx-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{2x^2}$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}n\int \frac{bf+ag+2bg\log\left(c(d+ex)^n\right)}{x^2(d+ex)}d(d+ex)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}e^2n\int \frac{bf+ag+2bg\log\left(c(d+ex)^n\right)}{e^2x^2(d+ex)}d(d+ex)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}e^2n\left(\frac{\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{e^2x^2}d(d+ex)}{d}+\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

$$\Big\downarrow \text{2751}$$

$$\frac{1}{2}e^2n\left(\frac{-\frac{2bgn\int -\frac{1}{ex}d(d+ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}+\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}e^2n\left(\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)}{d}+\frac{\frac{2bgn\log(-ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

$$\Big\downarrow \text{2779}$$

$$\frac{1}{2}e^2n\left(\frac{\frac{2bgn\int \frac{\log\left(1-\frac{d}{d+ex}\right)}{d+ex}d(d+ex)}{d}-\frac{\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{d}}{d}+\frac{\frac{2bgn\log(-ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}\right.$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}e^2n\left(\frac{\frac{2bgn\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d}-\frac{\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{d}}{d}+\frac{\frac{2bgn\log(-ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{2x^2}$$

3.384. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^3,x]
```

output
```
-1/2*((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^2 + (e^2*n*
(((2*b*g*n*Log[-(e*x)])/d - ((d + e*x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^
n]))/(d*e*x))/d + (-(((b*f + a*g + 2*b*g*Log[c*(d + e*x)^n])*Log[1 - d/(d
+ e*x)])/d) + (2*b*g*n*PolyLog[2, d/(d + e*x)])/d)/d))/2
```

3.384.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

3.384.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.78

method	result
risch	$\left(-i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i(ex+d)^{n}\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)+i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)^{2}+i\pi b g\right.$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^3,x,method=_RETURNVERBOS
E)
```

output
```
(-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I
*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
-I*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(-1/2*ln((e*x+d)^n)/x
^2+1/2*e*n*(e/d^2*ln(e*x+d)-1/d/x-e/d^2*ln(x)))-1/2/x^2*ln((e*x+d)^n)^2*b*
g+b*g*e^2*n*ln((e*x+d)^n)/d^2*ln(e*x+d)-b*g*e*n*ln((e*x+d)^n)/d/x-b*g*e^2*
n*ln((e*x+d)^n)/d^2*ln(x)-1/2*b*g*e^2*n^2/d^2*ln(e*x+d)^2-b*g*e^2*n^2/d^2*
ln(e*x+d)+b*e^2*g*n^2*ln(x)/d^2+b*g*e^2*n^2/d^2*dilog((e*x+d)/d)+b*g*e^2*n
^2/d^2*ln(x)*ln((e*x+d)/d)-1/8*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn
(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n
)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*(-I*
g*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*g*Pi*csgn(I*c)*csgn
(I*c*(e*x+d)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*g*Pi*cs
gn(I*c*(e*x+d)^n)^3+2*g*ln(c)+2*f)/x^2
```

3.384.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^3,x, algorithm="
fricas")
```

output
```
integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))
/x^3, x)
```

3.384.6 Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x**3, x)
```

3.384.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^3,x, algorithm="
maxima")
```

output
```
1/2*b*e*f*n*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*a*e*g*n*(e
*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/2*b*g*(log((e*x + d)^n)^2/
x^2 - 2*integrate((e*x*log(c)^2 + d*log(c)^2 + ((e*n + 2*e*log(c))*x + 2*d
*log(c))*log((e*x + d)^n))/(e*x^4 + d*x^3), x)) - 1/2*b*f*log((e*x + d)^n*
c)/x^2 - 1/2*a*g*log((e*x + d)^n*c)/x^2 - 1/2*a*f/x^2
```

3.384.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((ex+d)^nc)+f)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^3,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x^3, x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(c\,(d+e\,x)^n))}{x^3}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^3,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^3, x)
```

3.385 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$

3.385.1 Optimal result

Integrand size = 32, antiderivative size = 234

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx \\
&= -\frac{be^2gn^2}{3d^2x}-\frac{be^3gn^2\log(x)}{d^3}+\frac{be^3gn^2\log(d+ex)}{3d^3} \\
&\quad -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} \\
&\quad -\frac{en(bf+ag+2bg\log(c(d+ex)^n))}{6dx^2}+\frac{e^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3d^3x} \\
&\quad +\frac{e^3n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3}-\frac{2be^3gn^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3}
\end{aligned}$$

output
```
-1/3*b*e^2*g*n^2/d^2/x-b*e^3*g*n^2*ln(x)/d^3+1/3*b*e^3*g*n^2*ln(e*x+d)/d^3
-1/3*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^3-1/6*e*n*(b*f+a*g+2*b*
g*ln(c*(e*x+d)^n))/d/x^2+1/3*e^2*n*(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))
/d^3/x+1/3*e^3*n*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^3-2/3*b
*e^3*g*n^2*polylog(2,d/(e*x+d))/d^3
```

3.385.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$$
$$= -\frac{af}{3x^3}+\frac{1}{3}befn\left(-\frac{1}{2dx^2}+\frac{e}{d^2x}+\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}\right)$$
$$+\frac{1}{3}aegn\left(-\frac{1}{2dx^2}+\frac{e}{d^2x}+\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex)}{d^3}\right)$$
$$-\frac{bf\log(c(d+ex)^n)}{3x^3}-\frac{ag\log(c(d+ex)^n)}{3x^3}-\frac{bg\log^2(c(d+ex)^n)}{3x^3}$$
$$+\frac{2}{3}begn\left(-\frac{en}{2d^2x}-\frac{3e^2n\log(x)}{2d^3}+\frac{3e^2n\log(d+ex)}{2d^3}-\frac{\log(c(d+ex)^n)}{2dx^2}+\frac{e\log(c(d+ex)^n)}{d^2x}\right.$$
$$\left.+\frac{e^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{d^3}-\frac{e^2\log^2(c(d+ex)^n)}{2d^3n}+\frac{e^2n\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^3}\right)$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^4,x]
```

output
```
-1/3*(a*f)/x^3 + (b*e*f*n*(-1/2*1/(d*x^2) + e/(d^2*x) + (e^2*Log[x]])/d^3 -
 (e^2*Log[d + e*x]])/d^3))/3 + (a*e*g*n*(-1/2*1/(d*x^2) + e/(d^2*x) + (e^2*
Log[x]])/d^3 - (e^2*Log[d + e*x]])/d^3))/3 - (b*f*Log[c*(d + e*x)^n]])/(3*x^3
) - (a*g*Log[c*(d + e*x)^n]])/(3*x^3) - (b*g*Log[c*(d + e*x)^n]^2)/(3*x^3)
+ (2*b*e*g*n*(-1/2*(e*n)/(d^2*x) - (3*e^2*n*Log[x]])/(2*d^3) + (3*e^2*n*Log
[d + e*x]])/(2*d^3) - Log[c*(d + e*x)^n]/(2*d*x^2) + (e*Log[c*(d + e*x)^n]])
/(d^2*x) + (e^2*Log[-((e*x)/d)]*Log[c*(d + e*x)^n]])/d^3 - (e^2*Log[c*(d +
e*x)^n]^2)/(2*d^3*n) + (e^2*n*PolyLog[2, (d + e*x)/d]])/d^3))/3
```

3.385.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2883, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{x^4}\,dx$$

$$\Big\downarrow \text{2883}$$

$$\frac{1}{3}en\int \frac{bf+ag+2bg\log\left(c(d+ex)^n\right)}{x^3(d+ex)}dx-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{3}n\int \frac{bf+ag+2bg\log\left(c(d+ex)^n\right)}{x^3(d+ex)}d(d+ex)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}n\int -\frac{bf+ag+2bg\log\left(c(d+ex)^n\right)}{x^3(d+ex)}d(d+ex)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}e^3n\int -\frac{bf+ag+2bg\log\left(c(d+ex)^n\right)}{e^3x^3(d+ex)}d(d+ex)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{2789}$$

$$-\frac{1}{3}e^3n\left(\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{e^3x^3}d(d+ex)}{d}+\frac{\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{e^2x^2(d+ex)}d(d+ex)}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{2756}$$

$$-\frac{1}{3}e^3n\left(\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bgn\int \frac{1}{e^2x^2(d+ex)}d(d+ex)}{d}+\frac{\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{e^2x^2(d+ex)}d(d+ex)}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{3}e^3n\left(\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bgn\int \left(\frac{1}{e^2x^2d}-\frac{1}{exd^2}+\frac{1}{(d+ex)d^2}\right)d(d+ex)}{d}+\frac{\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{e^2x^2(d+ex)}d(d+ex)}{d}\right)-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(g\log\left(c(d+ex)^n\right)+f\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

3.385. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$

$$-\frac{1}{3}e^3n\left(\frac{\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{e^2x^2(d+ex)}d(d+ex)}{d}+\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bgn\left(-\frac{\log(-ex)}{d^2}+\frac{\log(d+ex)}{d^2}-\frac{1}{dex}\right)}{d}\right)-\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3}$$

$\downarrow$ 2789

$$-\frac{1}{3}e^3n\left(\frac{\frac{\int \frac{bf+ag+2bg\log(c(d+ex)^n)}{e^2x^2}d(d+ex)}{d}+\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)}{d}}{d}+\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bgn\left(-\frac{\log(-ex)}{d^2}+\right.}{d}\right.$$
$$\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3}$$

$\downarrow$ 2751

$$-\frac{1}{3}e^3n\left(\frac{\frac{-\frac{2bgn\int -\frac{1}{ex}d(d+ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}+\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)}{d}}{d}+\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bg}{}\right.$$
$$\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3}$$

$\downarrow$ 16

$$-\frac{1}{3}e^3n\left(\frac{\frac{\int -\frac{bf+ag+2bg\log(c(d+ex)^n)}{ex(d+ex)}d(d+ex)}{d}+\frac{\frac{2bgn\log(-ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}}{d}+\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bgn\left(-\right.}{d}\right.$$
$$\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3}$$

$\downarrow$ 2779

$$-\frac{1}{3}e^3n\left(\frac{\frac{\frac{2bgn\int \frac{\log\left(1-\frac{d}{d+ex}\right)}{d+ex}d(d+ex)}{d}-\frac{\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{d}}{d}+\frac{\frac{2bgn\log(-ex)}{d}-\frac{(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{dex}}{d}}{d}+\frac{ag+2bg}{}\right.$$
$$\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3}$$

$\downarrow$ 2838

3.385. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$

$$-\frac{1}{3}e^3n\left(\frac{\frac{ag+2bg\log(c(d+ex)^n)+bf}{2e^2x^2}-bgn\left(-\frac{\log(-ex)}{d^2}+\frac{\log(d+ex)}{d^2}-\frac{1}{dex}\right)}{d}+\frac{\frac{2bgn\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d}-\frac{\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)}{d}}{d}\right.$$

$$\frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3}$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^4,x]
```

output
```
-1/3*((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^3 - (e^3*n*
((-(b*g*n*(-(1/(d*e*x)) - Log[-(e*x)]/d^2 + Log[d + e*x]/d^2)) + (b*f + a*
g + 2*b*g*Log[c*(d + e*x)^n])/(2*e^2*x^2))/d + (((2*b*g*n*Log[-(e*x)])/d -
 ((d + e*x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d*e*x))/d + (-(((b*f
+ a*g + 2*b*g*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/d) + (2*b*g*n*Poly
Log[2, d/(d + e*x)])/d)/d)/d))/3
```

3.385.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

3.385. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$

3.385.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.74

method	result
risch	$\left(-i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i(ex+d)^{n}\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)+i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)^{2}+i\pi bg\right.$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^4,x,method=_RETURNVERBOS
E)
```

output
```
(-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I
*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
-I*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(-1/3*ln((e*x+d)^n)/x
^3+1/3*e*n*(-e^2/d^3*ln(e*x+d)-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x))-1/3/x^3*l
n((e*x+d)^n)^2*b*g-2/3*b*g*e^3*n*ln((e*x+d)^n)/d^3*ln(e*x+d)-1/3*b*g*e*n*l
n((e*x+d)^n)/d/x^2+2/3*b*g*e^3*n*ln((e*x+d)^n)/d^3*ln(x)+2/3*b*g*e^2*n*ln(
(e*x+d)^n)/d^2/x+b*e^3*g*n^2*ln(e*x+d)/d^3-1/3*b*e^2*g*n^2/d^2/x-b*e^3*g*n
^2*ln(x)/d^3-2/3*b*g*e^3*n^2/d^3*dilog((e*x+d)/d)-2/3*b*g*e^3*n^2/d^3*ln(x
)*ln((e*x+d)/d)+1/3*b*g*e^3*n^2/d^3*ln(e*x+d)^2-1/12*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b
+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3
*b+2*b*ln(c)+2*a)*(-I*g*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)
+I*g*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*
(e*x+d)^n)^2-I*g*Pi*csgn(I*c*(e*x+d)^n)^3+2*g*ln(c)+2*f)/x^3
```

3.385.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((ex+d)^nc)+f)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^4,x, algorithm="
fricas")
```

output
```
integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))
/x^4, x)
```

3.385.6 Sympy [F]

$$\int \frac{(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n))}{x^4}\,dx$$
$$= \int \frac{(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n))}{x^4}\,dx$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x**4,x)
```

output
```
Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x**4, x)
```

3.385.7 Maxima [F]

$$\int \frac{(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n))}{x^4}\,dx$$
$$= \int \frac{(b\log((ex + d)^n c) + a)(g\log((ex + d)^n c) + f)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^4,x, algorithm="
maxima")
```

output
```
-1/6*b*e*f*n*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2
*x^2)) - 1/6*a*e*g*n*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x -
 d)/(d^2*x^2)) - 1/3*b*g*(log((e*x + d)^n)^2/x^3 - 3*integrate(1/3*(3*e*x*
log(c)^2 + 3*d*log(c)^2 + 2*((e*n + 3*e*log(c))*x + 3*d*log(c))*log((e*x +
 d)^n))/(e*x^5 + d*x^4), x)) - 1/3*b*f*log((e*x + d)^n*c)/x^3 - 1/3*a*g*lo
g((e*x + d)^n*c)/x^3 - 1/3*a*f/x^3
```

3.385.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^4,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x^4, x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(c\,(d+e\,x)^n))}{x^4}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^4,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^4, x)
```

3.386 $\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$

3.386.1 Optimal result

Integrand size = 32, antiderivative size = 742

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$= \frac{agi^3mx}{4j^3}+\frac{bd^3fnx}{4e^3}-\frac{5bd^3gmnx}{16e^3}-\frac{5bgi^3mnx}{16j^3}-\frac{5bdgi^2mnx}{24ej^2}-\frac{5bd^2gimnx}{24e^2j}$$
$$+\frac{3bd^2gmnx^2}{32e^2}+\frac{3bgi^2mnx^2}{32j^2}+\frac{bdgimnx^2}{12ej}-\frac{7bdgmnx^3}{144e}-\frac{7bgimnx^3}{144j}+\frac{1}{32}bgmnx^4$$
$$+\frac{bd^4gmn\log(d+ex)}{16e^4}+\frac{bd^2gi^2mn\log(d+ex)}{8e^2j^2}+\frac{bd^3gimn\log(d+ex)}{12e^3j}$$
$$+\frac{bgi^3m(d+ex)\log(c(d+ex)^n)}{4ej^3}-\frac{gi^2mx^2(a+b\log(c(d+ex)^n))}{8j^2}$$
$$+\frac{gimx^3(a+b\log(c(d+ex)^n))}{12j}-\frac{1}{16}gmx^4(a+b\log(c(d+ex)^n))$$
$$+\frac{bgi^4mn\log(i+jx)}{16j^4}+\frac{bdgi^3mn\log(i+jx)}{12ej^3}+\frac{bd^2gi^2mn\log(i+jx)}{8e^2j^2}$$
$$-\frac{gi^4m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^4}+\frac{bd^3gn(i+jx)\log(h(i+jx)^m)}{4e^3j}$$
$$-\frac{bd^2nx^2(f+g\log(h(i+jx)^m))}{8e^2}+\frac{bdnx^3(f+g\log(h(i+jx)^m))}{12e}$$
$$-\frac{1}{16}bnx^4(f+g\log(h(i+jx)^m))-\frac{bd^4n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{4e^4}$$
$$+\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$
$$-\frac{bgi^4mn\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{4j^4}-\frac{bd^4gmn\,\mathrm{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{4e^4}$$

output
```
1/4*x^4*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))-5/16*b*d^3*g*m*n*x/e^3
-5/16*b*g*i^3*m*n*x/j^3+3/32*b*d^2*g*m*n*x^2/e^2+3/32*b*g*i^2*m*n*x^2/j^2-
7/144*b*d*g*m*n*x^3/e-7/144*b*g*i*m*n*x^3/j-1/4*b*g*i^4*m*n*polylog(2,-j*(
e*x+d)/(-d*j+e*i))/j^4-1/4*b*d^4*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^4
-5/24*b*d*g*i^2*m*n*x/e/j^2+1/16*b*d^4*g*m*n*ln(e*x+d)/e^4+1/16*b*g*i^4*m*
n*ln(j*x+i)/j^4+1/4*b*g*i^3*m*(e*x+d)*ln(c*(e*x+d)^n)/e/j^3+1/4*b*d^3*g*n*
(j*x+i)*ln(h*(j*x+i)^m)/e^3/j+1/32*b*g*m*n*x^4-1/16*g*m*x^4*(a+b*ln(c*(e*x
+d)^n))-1/16*b*n*x^4*(f+g*ln(h*(j*x+i)^m))-5/24*b*d^2*g*i*m*n*x/e^2/j+1/12
*b*d*g*i*m*n*x^2/e/j+1/8*b*d^2*g*i^2*m*n*ln(e*x+d)/e^2/j^2+1/12*b*d^3*g*i*
m*n*ln(e*x+d)/e^3/j+1/12*b*d*g*i^3*m*n*ln(j*x+i)/e/j^3+1/8*b*d^2*g*i^2*m*n
*ln(j*x+i)/e^2/j^2+1/4*a*g*i^3*m*x/j^3+1/4*b*d^3*f*n*x/e^3-1/8*g*i^2*m*x^2
*(a+b*ln(c*(e*x+d)^n))/j^2+1/12*g*i*m*x^3*(a+b*ln(c*(e*x+d)^n))/j-1/4*g*i^
4*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^4-1/8*b*d^2*n*x^2*(f+
g*ln(h*(j*x+i)^m))/e^2+1/12*b*d*n*x^3*(f+g*ln(h*(j*x+i)^m))/e-1/4*b*d^4*n*
ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e^4
```

3.386.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.82

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$

$$= \frac{6bn\log(d+ex)\left(12e^4gi^4m\log(i+jx)-12g(e^4i^4-d^4j^4)\,m\log\left(\frac{e(i+jx)}{ei-dj}\right)+dj(12e^3gi^3m+6de^2gi^2jm+\cdots\right.}{}$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(6*b*n*Log[d + e*x]*(12*e^4*g*i^4*m*Log[i + j*x] - 12*g*(e^4*i^4 - d^4*j^4
)*m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(12*e^3*g*i^3*m + 6*d*e^2*g*i^2*j
*m + 4*d^2*e*g*i*j^2*m + 3*d^3*j^3*(-4*f + g*m) - 12*d^3*g*j^3*Log[h*(i +
j*x)^m])) + e*(6*g*i*m*(-12*a*e^3*i^3 + b*(3*e^3*i^3 + 4*d*e^2*i^2*j + 6*d
^2*e*i*j^2 + 12*d^3*j^3)*n)*Log[i + j*x] - 6*b*e^3*Log[c*(d + e*x)^n]*(-12
*f*j^4*x^4 + g*j*m*x*(-12*i^3 + 6*i^2*j*x - 4*i*j^2*x^2 + 3*j^3*x^3) + 12*
g*i^4*m*Log[i + j*x] - 12*g*j^4*x^4*Log[h*(i + j*x)^m]) + j*(6*a*e^3*x*(12
*f*j^3*x^3 + g*m*(12*i^3 - 6*i^2*j*x + 4*i*j^2*x^2 - 3*j^3*x^3)) - b*n*(18
*d^3*j^3*(-4*f + 5*g*m)*x + 3*d^2*e*j^2*x*(12*f*j*x + g*m*(20*i - 9*j*x))
+ e^3*x*(18*f*j^3*x^3 + g*m*(90*i^3 - 27*i^2*j*x + 14*i*j^2*x^2 - 9*j^3*x^
3)) + 2*d*e^2*(-12*f*j^3*x^3 + g*m*(36*i^3 + 30*i^2*j*x - 12*i*j^2*x^2 + 7
*j^3*x^3))) - 6*g*j^3*x*(-12*a*e^3*x^3 + b*n*(-12*d^3 + 6*d^2*e*x - 4*d*e^
2*x^2 + 3*e^3*x^3))*Log[h*(i + j*x)^m])) - 72*b*g*(e^4*i^4 - d^4*j^4)*m*n*
PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)])/(288*e^4*j^4)
```

3.386.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2889, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2889}$$

$$-\frac{1}{4}gjm\int\frac{x^4(a+b\log(c(d+ex)^n))}{i+jx}dx-\frac{1}{4}ben\int\frac{x^4(f+g\log(h(i+jx)^m))}{d+ex}dx+\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-\frac{1}{4}gjm\int\left(\frac{(a+b\log(c(d+ex)^n))i^4}{j^4(i+jx)}-\frac{(a+b\log(c(d+ex)^n))i^3}{j^4}+\frac{x(a+b\log(c(d+ex)^n))i^2}{j^3}-\frac{x^2(a+b\log(\right.$$
$$\frac{1}{4}ben\int\left(\frac{(f+g\log(h(i+jx)^m))d^4}{e^4(d+ex)}-\frac{(f+g\log(h(i+jx)^m))d^3}{e^4}+\frac{x(f+g\log(h(i+jx)^m))d^2}{e^3}-\frac{x^2(f+g\log\right.$$
$$\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}(a+b\log\left(c(d+ex)^n\right))\left(f+g\log\left(h(i+jx)^m\right)\right)x^4-$$
$$\frac{1}{4}gjm\left(-\frac{bn\log(d+ex)d^4}{4e^4j}+\frac{bnxd^3}{4e^3j}-\frac{bin\log(d+ex)d^3}{3e^3j^2}-\frac{bnx^2d^2}{8e^2j}+\frac{binxd^2}{3e^2j^2}-\frac{bi^2n\log(d+ex)d^2}{2e^2j^3}+\frac{bnx^3d}{12ej}-\frac{bi}{6}\right.$$
$$\frac{1}{4}ben\left(\frac{\log\left(-\frac{j(d+ex)}{ei-dj}\right)\left(f+g\log\left(h(i+jx)^m\right)\right)d^4}{e^5}+\frac{gm\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)d^4}{e^5}-\frac{fxd^3}{e^4}+\frac{gmxd^3}{e^4}-\frac{g(i+jx)\log}{}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(x^4*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/4 - (g*j*m*(-(
(a*i^3*x)/j^4) + (b*i^3*n*x)/j^4 + (b*d*i^2*n*x)/(2*e*j^3) + (b*d^2*i*n*x)
/(3*e^2*j^2) + (b*d^3*n*x)/(4*e^3*j) - (b*i^2*n*x^2)/(4*j^3) - (b*d*i*n*x^
2)/(6*e*j^2) - (b*d^2*n*x^2)/(8*e^2*j) + (b*i*n*x^3)/(9*j^2) + (b*d*n*x^3)
/(12*e*j) - (b*n*x^4)/(16*j) - (b*d^2*i^2*n*Log[d + e*x])/(2*e^2*j^3) - (b
*d^3*i*n*Log[d + e*x])/(3*e^3*j^2) - (b*d^4*n*Log[d + e*x])/(4*e^4*j) - (b
*i^3*(d + e*x)*Log[c*(d + e*x)^n])/(e*j^4) + (i^2*x^2*(a + b*Log[c*(d + e*
x)^n]))/(2*j^3) - (i*x^3*(a + b*Log[c*(d + e*x)^n]))/(3*j^2) + (x^4*(a + b
*Log[c*(d + e*x)^n]))/(4*j) + (i^4*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i +
j*x))/(e*i - d*j)])/j^5 + (b*i^4*n*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))
])/j^5))/4 - (b*e*n*(-((d^3*f*x)/e^4) + (d^3*g*m*x)/e^4 + (g*i^3*m*x)/(4*e
*j^3) + (d*g*i^2*m*x)/(3*e^2*j^2) + (d^2*g*i*m*x)/(2*e^3*j) - (d^2*g*m*x^2
)/(4*e^3) - (g*i^2*m*x^2)/(8*e*j^2) - (d*g*i*m*x^2)/(6*e^2*j) + (d*g*m*x^3
)/(9*e^2) + (g*i*m*x^3)/(12*e*j) - (g*m*x^4)/(16*e) - (g*i^4*m*Log[i + j*x
])/(4*e*j^4) - (d*g*i^3*m*Log[i + j*x])/(3*e^2*j^3) - (d^2*g*i^2*m*Log[i +
 j*x])/(2*e^3*j^2) - (d^3*g*(i + j*x)*Log[h*(i + j*x)^m])/(e^4*j) + (d^2*x
^2*(f + g*Log[h*(i + j*x)^m]))/(2*e^3) - (d*x^3*(f + g*Log[h*(i + j*x)^m])
)/(3*e^2) + (x^4*(f + g*Log[h*(i + j*x)^m]))/(4*e) + (d^4*Log[-((j*(d + e*
x))/(e*i - d*j))]*(f + g*Log[h*(i + j*x)^m]))/e^5 + (d^4*g*m*PolyLog[2, (e
*(i + j*x))/(e*i - d*j)])/e^5))/4
```

3.386. $\int x^3(a+b\log\left(c(d+ex)^n\right))\left(f+g\log\left(h(i+jx)^m\right)\right)\,dx$

3.386.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

3.386.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.06 (sec) , antiderivative size = 2094, normalized size of antiderivative = 2.82

Expression too large to display

input
```
int(x^3*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x)
```

output
```
1/4/j^4*b*g*i^4*m*n*ln(j*x+i)*ln(((j*x+i)*e+d*j-e*i)/(d*j-e*i))-3/16/e^3/j
*b*d^3*g*i*m*n-11/96/e^2/j^2*b*d^2*g*i^2*m*n+1/4/e/j^3*ln(e*x+d)*b*d*g*i^3
*m*n+1/4/e^3/j*g*i*m*ln((e*x+d)*j-d*j+e*i)*b*d^3*n+1/8/e^2/j^2*g*i^2*m*ln(
(e*x+d)*j-d*j+e*i)*b*d^2*n+1/12/e/j^3*g*i^3*m*ln((e*x+d)*j-d*j+e*i)*b*d*n+
1/16/j^4*g*i^4*m*ln((e*x+d)*j-d*j+e*i)*b*n+1/4/e^4*b*d^4*g*m*n*dilog(((e*x
+d)*j-d*j+e*i)/(-d*j+e*i))-1/16/e/j^3*b*d*g*i^3*m*n-205/576/e^4*b*d^4*g*m*
n+1/4/e^4*b*d^4*g*m*n*ln(e*x+d)*ln(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/16*ln
(h)*x^4*b*g*n-1/16*n*b*g*ln((j*x+i)^m)*x^4+(1/4*x^4*b*g*ln((j*x+i)^m)-1/48
*b*(-6*I*Pi*g*j^4*x^4*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+6*I*Pi*g*j^4
*x^4*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+6*I*Pi*g*j^4*x^4*csgn
(I*h*(j*x+i)^m)^3-6*I*Pi*g*j^4*x^4*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-12*ln(h
)*g*j^4*x^4+3*g*j^4*m*x^4-12*f*j^4*x^4-4*g*i*j^3*m*x^3+6*g*i^2*j^2*m*x^2+1
2*g*i^4*m*ln(j*x+i)-12*g*i^3*j*m*x)/j^4)*ln((e*x+d)^n)+1/32*b*g*m*n*x^4+1/
16*I/e^2*n*b*Pi*x^2*d^2*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-
1/24*I/e*n*b*Pi*x^3*d*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/
4/j^4*b*g*i^4*m*n*dilog(((j*x+i)*e+d*j-e*i)/(d*j-e*i))+3/32*b*g*i^2*m*n*x^
2/j^2-7/144*b*d*g*m*n*x^3/e-7/144*b*g*i*m*n*x^3/j+1/16*b*d^4*g*m*n*ln(e*x+
d)/e^4+1/12/e*ln(h)*x^3*b*d*g*n-1/8/e^2*ln(h)*x^2*b*d^2*g*n+1/4/e^3*ln(h)*
b*d^3*g*n*x+1/12/e*x^3*b*d*f*n-1/8/e^2*x^2*b*d^2*f*n-1/4/e^4*ln(e*x+d)*b*d
^4*f*n+1/4*b*d^3*f*n*x/e^3-1/16*x^4*b*f*n-1/8*I/e^4*n*b*d^4*ln(e*x+d)*P...
```

3.386.5 Fricas [F]

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="
fricas")
```

output
```
integral(b*f*x^3*log((e*x + d)^n*c) + a*f*x^3 + (b*g*x^3*log((e*x + d)^n*c
) + a*g*x^3)*log((j*x + i)^m*h), x)
```

3.386.6 Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.386.7 Maxima [F]

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$= \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="
maxima")
```

output
```
1/4*b*f*x^4*log((e*x + d)^n*c) + 1/4*a*g*x^4*log((j*x + i)^m*h) + 1/4*a*f*
x^4 - 1/48*b*e*f*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6
*d^2*e*x^2 - 12*d^3*x)/e^4) - 1/48*a*g*j*m*(12*i^4*log(j*x + i)/j^5 + (3*j
^3*x^4 - 4*i*j^2*x^3 + 6*i^2*j*x^2 - 12*i^3*x)/j^4) + 1/48*b*g*((12*e^4*i^
4*m*n*log(e*x + d)*log(j*x + i) + (4*e^4*i*j^3*m*x^3 - 6*e^4*i^2*j^2*m*x^2
 + 12*e^4*i^3*j*m*x - 12*e^4*i^4*m*log(j*x + i) - 3*(j^4*m - 4*j^4*log(h))
*e^4*x^4)*log((e*x + d)^n) + (12*e^4*j^4*x^4*log((e*x + d)^n) + 4*d*e^3*j^
4*n*x^3 - 6*d^2*e^2*j^4*n*x^2 + 12*d^3*e*j^4*n*x - 12*d^4*j^4*n*log(e*x +
d) - 3*(e^4*j^4*n - 4*e^4*j^4*log(c))*x^4)*log((j*x + i)^m))/(e^4*j^4) + 4
8*integrate(-1/48*(6*(2*(j^4*m - 4*j^4*log(h))*e^5*log(c) - (j^4*m*n - 2*j
^4*n*log(h))*e^5)*x^5 + (d*e^4*j^4*m*n + (i*j^3*m*n + 12*i*j^3*n*log(h))*e
^5 - 12*(4*e^5*i*j^3*log(h) - (j^4*m - 4*j^4*log(h))*d*e^4)*log(c))*x^4 -
2*(e^5*i^2*j^2*m*n + d^2*e^3*j^4*m*n + 24*d*e^4*i*j^3*log(c)*log(h))*x^3 +
 6*(e^5*i^3*j*m*n + d^3*e^2*j^4*m*n)*x^2 + 12*(e^5*i^4*m*n + d^4*e*j^4*m*n
)*x + 12*(d*e^4*i^4*m*n - d^5*j^4*m*n + (e^5*i^4*m*n - d^4*e*j^4*m*n)*x)*l
og(e*x + d))/(e^5*j^4*x^2 + d*e^4*i*j^3 + (e^5*i*j^3 + d*e^4*j^4)*x), x))
```

3.386.8 Giac [F]

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)*x^3, x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int x^3\,(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

3.387 $\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$

3.387.1 Optimal result

Integrand size = 32, antiderivative size = 558

$$\begin{aligned}
&\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx\\
&=-\frac{agi^2mx}{3j^2}-\frac{bd^2fnx}{3e^2}+\frac{4bd^2gmnx}{9e^2}+\frac{4bgi^2mnx}{9j^2}+\frac{bdgimnx}{3ej}-\frac{5bdgmnx^2}{36e}\\
&-\frac{5bgimnx^2}{36j}+\frac{2}{27}bgmnx^3-\frac{bd^3gmn\log(d+ex)}{9e^3}-\frac{bd^2gimn\log(d+ex)}{6e^2j}\\
&-\frac{bgi^2m(d+ex)\log(c(d+ex)^n)}{3ej^2}+\frac{gimx^2(a+b\log(c(d+ex)^n))}{6j}\\
&-\frac{1}{9}gmx^3(a+b\log(c(d+ex)^n))-\frac{bgi^3mn\log(i+jx)}{9j^3}\\
&-\frac{bdgi^2mn\log(i+jx)}{6ej^2}+\frac{gi^3m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{3j^3}\\
&-\frac{bd^2gn(i+jx)\log(h(i+jx)^m)}{3e^2j}+\frac{bdnx^2(f+g\log(h(i+jx)^m))}{6e}\\
&-\frac{1}{9}bnx^3(f+g\log(h(i+jx)^m))+\frac{bd^3n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{3e^3}\\
&+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\
&+\frac{bgi^3mn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{3j^3}+\frac{bd^3gmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{3e^3}
\end{aligned}$$

output
```
-1/3*a*g*i^2*m*x/j^2-1/3*b*d^2*f*n*x/e^2+4/9*b*d^2*g*m*n*x/e^2+4/9*b*g*i^2
*m*n*x/j^2+1/3*b*d*g*i*m*n*x/e/j-5/36*b*d*g*m*n*x^2/e-5/36*b*g*i*m*n*x^2/j
+2/27*b*g*m*n*x^3-1/9*b*d^3*g*m*n*ln(e*x+d)/e^3-1/6*b*d^2*g*i*m*n*ln(e*x+d
)/e^2/j-1/3*b*g*i^2*m*(e*x+d)*ln(c*(e*x+d)^n)/e/j^2+1/6*g*i*m*x^2*(a+b*ln(
c*(e*x+d)^n))/j-1/9*g*m*x^3*(a+b*ln(c*(e*x+d)^n))-1/9*b*g*i^3*m*n*ln(j*x+i
)/j^3-1/6*b*d*g*i^2*m*n*ln(j*x+i)/e/j^2+1/3*g*i^3*m*(a+b*ln(c*(e*x+d)^n))*
ln(e*(j*x+i)/(-d*j+e*i))/j^3-1/3*b*d^2*g*n*(j*x+i)*ln(h*(j*x+i)^m)/e^2/j+1
/6*b*d*n*x^2*(f+g*ln(h*(j*x+i)^m))/e-1/9*b*n*x^3*(f+g*ln(h*(j*x+i)^m))+1/3
*b*d^3*n*ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e^3+1/3*x^3*(a+b*
ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))+1/3*b*g*i^3*m*n*polylog(2,-j*(e*x+d
)/(-d*j+e*i))/j^3+1/3*b*d^3*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^3
```

3.387.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.88

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$

$$= \frac{6bn\log(d+ex)\left(-6e^3gi^3m\log(i+jx)+6g(e^3i^3-d^3j^3)m\log\left(\frac{e(i+jx)}{ei-dj}\right)+dj(-6e^2gi^2m-3degijm+2d\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(6*b*n*Log[d + e*x]*(-6*e^3*g*i^3*m*Log[i + j*x] + 6*g*(e^3*i^3 - d^3*j^3)
*m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(-6*e^2*g*i^2*m - 3*d*e*g*i*j*m +
2*d^2*j^2*(3*f - g*m) + 6*d^2*g*j^2*Log[h*(i + j*x)^m])) + e*(6*g*i*m*(6*a
*e^2*i^2 - b*(2*e^2*i^2 + 3*d*e*i*j + 6*d^2*j^2)*n)*Log[i + j*x] + 6*b*e^2
*Log[c*(d + e*x)^n]*(6*f*j^3*x^3 + g*j*m*x*(-6*i^2 + 3*i*j*x - 2*j^2*x^2)
+ 6*g*i^3*m*Log[i + j*x] + 6*g*j^3*x^3*Log[h*(i + j*x)^m]) + j*(6*a*e^2*x*
(6*f*j^2*x^2 + g*m*(-6*i^2 + 3*i*j*x - 2*j^2*x^2)) + b*n*(12*d^2*j^2*(-3*f
 + 4*g*m)*x + 3*d*e*(6*f*j^2*x^2 + g*m*(12*i^2 + 12*i*j*x - 5*j^2*x^2)) +
e^2*x*(-12*f*j^2*x^2 + g*m*(48*i^2 - 15*i*j*x + 8*j^2*x^2))) - 6*g*j^2*x*(
-6*a*e^2*x^2 + b*n*(6*d^2 - 3*d*e*x + 2*e^2*x^2))*Log[h*(i + j*x)^m])) + 3
6*b*g*(e^3*i^3 - d^3*j^3)*m*n*PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)])/(1
08*e^3*j^3)
```

3.387.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2889, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2889}$$

$$-\frac{1}{3}gjm\int\frac{x^3(a+b\log(c(d+ex)^n))}{i+jx}dx-\frac{1}{3}ben\int\frac{x^3(f+g\log(h(i+jx)^m))}{d+ex}dx+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-\frac{1}{3}gjm\int\left(-\frac{(a+b\log(c(d+ex)^n))i^3}{j^3(i+jx)}+\frac{(a+b\log(c(d+ex)^n))i^2}{j^3}-\frac{x(a+b\log(c(d+ex)^n))i}{j^2}+\frac{x^2(a+b\log\right.$$
$$\frac{1}{3}ben\int\left(-\frac{(f+g\log(h(i+jx)^m))d^3}{e^3(d+ex)}+\frac{(f+g\log(h(i+jx)^m))d^2}{e^3}-\frac{x(f+g\log(h(i+jx)^m))d}{e^2}+\frac{x^2(f+g\log\right.$$
$$\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}gjm\left(-\frac{i^3\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^4}-\frac{ix^2(a+b\log(c(d+ex)^n))}{2j^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3j}+\right.$$
$$\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))-$$
$$\frac{1}{3}ben\left(-\frac{d^3\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e^4}-\frac{d^3gm\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e^4}+\frac{d^2fx}{e^3}+\frac{d^2g(i+jx)\log(h(i\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(x^3*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/3 - (g*j*m*((a
*i^2*x)/j^3 - (b*i^2*n*x)/j^3 - (b*d*i*n*x)/(2*e*j^2) - (b*d^2*n*x)/(3*e^2
*j) + (b*i*n*x^2)/(4*j^2) + (b*d*n*x^2)/(6*e*j) - (b*n*x^3)/(9*j) + (b*d^2
*i*n*Log[d + e*x])/(2*e^2*j^2) + (b*d^3*n*Log[d + e*x])/(3*e^3*j) + (b*i^2
*(d + e*x)*Log[c*(d + e*x)^n])/(e*j^3) - (i*x^2*(a + b*Log[c*(d + e*x)^n])
)/(2*j^2) + (x^3*(a + b*Log[c*(d + e*x)^n]))/(3*j) - (i^3*(a + b*Log[c*(d
+ e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/j^4 - (b*i^3*n*PolyLog[2, -((j*
(d + e*x))/(e*i - d*j))])/j^4))/3 - (b*e*n*((d^2*f*x)/e^3 - (d^2*g*m*x)/e^
3 - (g*i^2*m*x)/(3*e*j^2) - (d*g*i*m*x)/(2*e^2*j) + (d*g*m*x^2)/(4*e^2) +
(g*i*m*x^2)/(6*e*j) - (g*m*x^3)/(9*e) + (g*i^3*m*Log[i + j*x])/(3*e*j^3) +
 (d*g*i^2*m*Log[i + j*x])/(2*e^2*j^2) + (d^2*g*(i + j*x)*Log[h*(i + j*x)^m
])/(e^3*j) - (d*x^2*(f + g*Log[h*(i + j*x)^m]))/(2*e^2) + (x^3*(f + g*Log[
h*(i + j*x)^m]))/(3*e) - (d^3*Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log
[h*(i + j*x)^m]))/e^4 - (d^3*g*m*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/e^
4))/3
```

3.387.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

3.387.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 234.84 (sec) , antiderivative size = 1724, normalized size of antiderivative = 3.09

method	result	size
risch	Expression too large to display	1724

input
```
int(x^2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x,method=_RETURNVERBOS
E)
```

output
```
-1/3/j^3*b*g*i^3*m*n*dilog(((j*x+i)*e+d*j-e*i)/(d*j-e*i))+49/108*b*d^3*g*m
*n/e^3-1/9*n*b*f*x^3+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*
x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+
d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1
/2*a)*(1/3*(I*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-I*g*Pi*csgn(I*(
j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-I*g*Pi*csgn(I*h*(j*x+i)^m)^3+I*g*P
i*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+2*g*ln(h)+2*f)*x^3+2/3*g*ln((j*x+i)^m)*x
^3-2/9*g*m*x^3+1/3*g*m/j*x^2*i-2/3*g*m/j^2*x*i^2+2/3*g*m/j^3*i^3*ln(j*x+i)
)-1/3/e^3*b*d^3*n*g*m*dilog(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/9*n*b*g*ln((
j*x+i)^m)*x^3-1/3/j^3*b*g*i^3*m*n*ln(j*x+i)*ln(((j*x+i)*e+d*j-e*i)/(d*j-e*
i))+1/9*b*d*g*i^2*m*n/e/j^2+2/9*b*d^2*g*i*m*n/e^2/j-1/9/j^3*g*i^3*m*ln((e*
x+d)*j-d*j+e*i)*b*n-1/18*I*n*b*Pi*x^3*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)
^m)^2-1/18*I*n*b*Pi*x^3*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+(1/3*x^3*b*g*ln(
(j*x+i)^m)+1/18*b*(3*I*Pi*g*j^3*x^3*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^
2-3*I*Pi*g*j^3*x^3*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-3*I*Pi*
g*j^3*x^3*csgn(I*h*(j*x+i)^m)^3+3*I*Pi*g*j^3*x^3*csgn(I*h*(j*x+i)^m)^2*csg
n(I*h)+6*j^3*x^3*ln(h)*g-2*g*j^3*m*x^3+6*f*j^3*x^3+3*g*i*j^2*m*x^2+6*g*i^3
*m*ln(j*x+i)-6*g*i^2*j*m*x)/j^3)*ln((e*x+d)^n)+1/6/e*ln(h)*x^2*b*d*g*n-1/3
/e^2*ln(h)*x*b*d^2*g*n-1/3/e^3*b*d^3*n*g*m*ln(e*x+d)*ln(((e*x+d)*j-d*j+e*i
)/(-d*j+e*i))-1/9*ln(h)*x^3*b*g*n+2/27*b*g*m*n*x^3+1/6/e*n*b*g*ln((j*x+...
```

3.387.5 Fricas [F]

$$\int x^2(a+b\log(c(d+ex)^n))\left(f+g\log\left(h(i+jx)^m\right)\right)\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="
fricas")
```

output
```
integral(b*f*x^2*log((e*x + d)^n*c) + a*f*x^2 + (b*g*x^2*log((e*x + d)^n*c
) + a*g*x^2)*log((j*x + i)^m*h), x)
```

3.387.6 Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.387.7 Maxima [F]

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="
maxima")
```

output
```
1/3*b*f*x^3*log((e*x + d)^n*c) + 1/3*a*g*x^3*log((j*x + i)^m*h) + 1/3*a*f*
x^3 + 1/18*b*e*f*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^
2*x)/e^3) + 1/18*a*g*j*m*(6*i^3*log(j*x + i)/j^4 - (2*j^2*x^3 - 3*i*j*x^2
+ 6*i^2*x)/j^3) - 1/18*b*g*((6*e^3*i^3*m*n*log(e*x + d)*log(j*x + i) - (3*
e^3*i*j^2*m*x^2 - 6*e^3*i^2*j*m*x + 6*e^3*i^3*m*log(j*x + i) - 2*(j^3*m -
3*j^3*log(h))*e^3*x^3)*log((e*x + d)^n) - (6*e^3*j^3*x^3*log((e*x + d)^n)
+ 3*d*e^2*j^3*n*x^2 - 6*d^2*e*j^3*n*x + 6*d^3*j^3*n*log(e*x + d) - 2*(e^3*
j^3*n - 3*e^3*j^3*log(c))*x^3)*log((j*x + i)^m))/(e^3*j^3) + 18*integrate(
1/18*(2*(3*(j^3*m - 3*j^3*log(h))*e^4*log(c) - (2*j^3*m*n - 3*j^3*n*log(h)
)*e^4)*x^4 + (d*e^3*j^3*m*n + (i*j^2*m*n + 6*i*j^2*n*log(h))*e^4 - 6*(3*e^
4*i*j^2*log(h) - (j^3*m - 3*j^3*log(h))*d*e^3)*log(c))*x^3 - 3*(e^4*i^2*j*
m*n + d^2*e^2*j^3*m*n + 6*d*e^3*i*j^2*log(c)*log(h))*x^2 - 6*(e^4*i^3*m*n
+ d^3*e*j^3*m*n)*x - 6*(d*e^3*i^3*m*n - d^4*j^3*m*n + (e^4*i^3*m*n - d^3*e
*j^3*m*n)*x)*log(e*x + d))/(e^4*j^3*x^2 + d*e^3*i*j^2 + (e^4*i*j^2 + d*e^3
*j^3)*x), x))
```

3.387.8 Giac [F]

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)*x^2, x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int x^2\,(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

3.387. $\int x^2(a+b\log(c(d+ex)^n))\,(f+g\log(h(i+jx)^m))\,dx$

3.388 $\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$

3.388.1 Optimal result

Integrand size = 30, antiderivative size = 397

$$\begin{aligned}
&\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx\\
&=\frac{agimx}{2j}+\frac{bdfnx}{2e}-\frac{3bdgmnx}{4e}-\frac{3bgimnx}{4j}+\frac{1}{4}bgmnx^2+\frac{bd^2gmn\log(d+ex)}{4e^2}\\
&\quad+\frac{bgim(d+ex)\log(c(d+ex)^n)}{2ej}-\frac{1}{4}gmx^2(a+b\log(c(d+ex)^n))+\frac{bgi^2mn\log(i+jx)}{4j^2}\\
&\quad-\frac{gi^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}+\frac{bdgn(i+jx)\log(h(i+jx)^m)}{2ej}\\
&\quad-\frac{1}{4}bnx^2(f+g\log(h(i+jx)^m))-\frac{bd^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2e^2}\\
&\quad+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\
&\quad-\frac{bgi^2mn\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2j^2}-\frac{bd^2gmn\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{2e^2}
\end{aligned}$$

output
```
1/2*a*g*i*m*x/j+1/2*b*d*f*n*x/e-3/4*b*d*g*m*n*x/e-3/4*b*g*i*m*n*x/j+1/4*b*
g*m*n*x^2+1/4*b*d^2*g*m*n*ln(e*x+d)/e^2+1/2*b*g*i*m*(e*x+d)*ln(c*(e*x+d)^n
)/e/j-1/4*g*m*x^2*(a+b*ln(c*(e*x+d)^n))+1/4*b*g*i^2*m*n*ln(j*x+i)/j^2-1/2*
g*i^2*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^2+1/2*b*d*g*n*(j*
x+i)*ln(h*(j*x+i)^m)/e/j-1/4*b*n*x^2*(f+g*ln(h*(j*x+i)^m))-1/2*b*d^2*n*ln(
-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e^2+1/2*x^2*(a+b*ln(c*(e*x+d)
^n))*(f+g*ln(h*(j*x+i)^m))-1/2*b*g*i^2*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i)
)/j^2-1/2*b*d^2*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^2
```

3.388.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.86

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$= \frac{bn\log(d+ex)\left(2e^2gi^2m\log(i+jx)+2g(-e^2i^2+d^2j^2)\,m\log\left(\frac{e(i+jx)}{ei-dj}\right)+dj(-2dfj+2egim+dgjm-2\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(b*n*Log[d + e*x]*(2*e^2*g*i^2*m*Log[i + j*x] + 2*g*(-(e^2*i^2) + d^2*j^2)
*m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(-2*d*f*j + 2*e*g*i*m + d*g*j*m -
2*d*g*j*Log[h*(i + j*x)^m])) + e*(g*i*m*(-2*a*e*i + b*(e*i + 2*d*j)*n)*Log
[i + j*x] + j*(a*e*x*(2*f*j*x + g*m*(2*i - j*x)) - b*n*(e*x*(3*g*i*m + f*j
*x - g*j*m*x) + d*(2*g*i*m - 2*f*j*x + 3*g*j*m*x)) + g*j*x*(2*a*e*x + b*n*
(2*d - e*x))*Log[h*(i + j*x)^m]) + b*e*Log[c*(d + e*x)^n]*(-2*g*i^2*m*Log[
i + j*x] + j*x*(2*g*i*m + 2*f*j*x - g*j*m*x + 2*g*j*x*Log[h*(i + j*x)^m]))
) + 2*b*g*(-(e^2*i^2) + d^2*j^2)*m*n*PolyLog[2, (j*(d + e*x))/(-(e*i) + d*
j)])/(4*e^2*j^2)
```

3.388.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2889, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2889}$$

$$-\frac{1}{2}gjm\int\frac{x^2(a+b\log(c(d+ex)^n))}{i+jx}dx-\frac{1}{2}ben\int\frac{x^2(f+g\log(h(i+jx)^m))}{d+ex}dx+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-\frac{1}{2}gjm\int\left(\frac{(a+b\log(c(d+ex)^n))\,i^2}{j^2(i+jx)}-\frac{(a+b\log(c(d+ex)^n))\,i}{j^2}+\frac{x(a+b\log(c(d+ex)^n))}{j}\right)dx-\frac{1}{2}ben\int\left(\frac{(f+g\log(h(i+jx)^m))\,d^2}{e^2(d+ex)}-\frac{(f+g\log(h(i+jx)^m))\,d}{e^2}+\frac{x(f+g\log(h(i+jx)^m))}{e}\right)dx+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))\,(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}gjm\left(\frac{i^2\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^3}+\frac{x^2(a+b\log(c(d+ex)^n))}{2j}-\frac{aix}{j^2}-\frac{bi(d+ex)\log(c(d+ex)^n}{ej^2}\right.$$

$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))\,(f+g\log(h(i+jx)^m))-$$

$$\frac{1}{2}ben\left(\frac{d^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e^3}+\frac{d^2gm\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e^3}-\frac{dfx}{e^2}-\frac{dg(i+jx)\log(h(i+jx)}{e^2j}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/2 - (g*j*m*(-(
(a*i*x)/j^2) + (b*i*n*x)/j^2 + (b*d*n*x)/(2*e*j) - (b*n*x^2)/(4*j) - (b*d^
2*n*Log[d + e*x])/(2*e^2*j) - (b*i*(d + e*x)*Log[c*(d + e*x)^n])/(e*j^2) +
 (x^2*(a + b*Log[c*(d + e*x)^n]))/(2*j) + (i^2*(a + b*Log[c*(d + e*x)^n])*
Log[(e*(i + j*x))/(e*i - d*j)])/j^3 + (b*i^2*n*PolyLog[2, -((j*(d + e*x))/
(e*i - d*j))])/j^3))/2 - (b*e*n*(-((d*f*x)/e^2) + (d*g*m*x)/e^2 + (g*i*m*x
)/(2*e*j) - (g*m*x^2)/(4*e) - (g*i^2*m*Log[i + j*x])/(2*e*j^2) - (d*g*(i +
 j*x)*Log[h*(i + j*x)^m])/(e^2*j) + (x^2*(f + g*Log[h*(i + j*x)^m]))/(2*e)
 + (d^2*Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log[h*(i + j*x)^m]))/e^3
+ (d^2*g*m*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/e^3))/2
```

3.388.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

3.388.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 111.26 (sec) , antiderivative size = 1372, normalized size of antiderivative = 3.46

method	result	size
risch	Expression too large to display	1372

input
```
int(x*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x,method=_RETURNVERBOSE)
```

output
```
1/2/e*n*b*g*ln((j*x+i)^m)*x*d-1/2*n*b*d^2*f/e^2*ln(e*x+d)-1/4*b*f*n*x^2-1/
4*ln(h)*x^2*b*g*n+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^
n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*
a)*(1/2*(I*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-I*g*Pi*csgn(I*(j*x
+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-I*g*Pi*csgn(I*h*(j*x+i)^m)^3+I*g*Pi*c
sgn(I*h*(j*x+i)^m)^2*csgn(I*h)+2*g*ln(h)+2*f)*x^2+g*ln((j*x+i)^m)*x^2-1/2*
g*m*x^2+g*m/j*x*i-g*m/j^2*i^2*ln(j*x+i))+1/2/e*n*b/j*d*ln(e*x+d)*g*i*m+1/2
/e*n*b*g*m/j*i*ln((e*x+d)*j-d*j+e*i)*d-1/4*I/e*n*b*x*Pi*d*g*csgn(I*h*(j*x+
i)^m)^3+1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*g*csgn(I*h*(j*x+i)^m)^3+1/8*I*n*b*P
i*x^2*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+(1/2*x^2*b*g*ln((j
*x+i)^m)-1/4*b*(-I*Pi*g*j^2*x^2*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+I*
Pi*g*j^2*x^2*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+I*Pi*g*j^2*x^
2*csgn(I*h*(j*x+i)^m)^3-I*Pi*g*j^2*x^2*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-2*j
^2*x^2*ln(h)*g+g*j^2*m*x^2+2*g*i^2*m*ln(j*x+i)-2*f*j^2*x^2-2*g*i*j*m*x)/j^
2)*ln((e*x+d)^n)-1/4*b*d*g*i*m*n/e/j+1/2/e^2*n*b*g*m*d^2*ln(e*x+d)*ln(((e*
x+d)*j-d*j+e*i)/(-d*j+e*i))+1/2*n*b*g*i^2*m/j^2*ln(j*x+i)*ln(((j*x+i)*e+d*
j-e*i)/(d*j-e*i))-1/8*I*n*b*Pi*x^2*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-1/8*I
*n*b*Pi*x^2*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-5/8*b*d^2*g*m*n/e^2-
1/4*I/e*n*b*x*Pi*d*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/...
```

3.388.5 Fricas [F]

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="fr
icas")
```

output
```
integral(b*f*x*log((e*x + d)^n*c) + a*f*x + (b*g*x*log((e*x + d)^n*c) + a*
g*x)*log((j*x + i)^m*h), x)
```

3.388.6 Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.388.7 Maxima [F]

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="ma
xima")
```

output
```
-1/4*b*e*f*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) - 1/4*a*g*j*m*
(2*i^2*log(j*x + i)/j^3 + (j*x^2 - 2*i*x)/j^2) + 1/2*b*f*x^2*log((e*x + d)
^n*c) + 1/2*a*g*x^2*log((j*x + i)^m*h) + 1/2*a*f*x^2 + 1/4*b*g*((2*e^2*i^2
*m*n*log(e*x + d)*log(j*x + i) + (2*e^2*i*j*m*x - 2*e^2*i^2*m*log(j*x + i)
 - (j^2*m - 2*j^2*log(h))*e^2*x^2)*log((e*x + d)^n) + (2*e^2*j^2*x^2*log((
e*x + d)^n) + 2*d*e*j^2*n*x - 2*d^2*j^2*n*log(e*x + d) - (e^2*j^2*n - 2*e^
2*j^2*log(c))*x^2)*log((j*x + i)^m))/(e^2*j^2) + 4*integrate(-1/4*(2*((j^2
*m - 2*j^2*log(h))*e^3*log(c) - (j^2*m*n - j^2*n*log(h))*e^3)*x^3 + (d*e^2
*j^2*m*n + (i*j*m*n + 2*i*j*n*log(h))*e^3 - 2*(2*e^3*i*j*log(h) - (j^2*m -
 2*j^2*log(h))*d*e^2)*log(c))*x^2 + 2*(e^3*i^2*m*n + d^2*e*j^2*m*n - 2*d*e
^2*i*j*log(c)*log(h))*x + 2*(d*e^2*i^2*m*n - d^3*j^2*m*n + (e^3*i^2*m*n -
d^2*e*j^2*m*n)*x)*log(e*x + d))/(e^3*j^2*x^2 + d*e^2*i*j + (e^3*i*j + d*e^
2*j^2)*x), x))
```

3.388.8 Giac [F]

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="gi
ac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)*x, x)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int x\,(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int(x*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

3.389 $\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$

3.389.1 Optimal result

Integrand size = 29, antiderivative size = 232

$$\begin{aligned}\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx &= -agmx-bfnx+2bgmnx-\frac{bgm(d+ex)\log(c(d+ex)^n)}{e}\\ &\quad+\frac{gim(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}-\frac{bgn(i+jx)\log(h(i+jx)^m)}{j}\\ &\quad+\frac{bdn\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e}\\ &\quad+x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\ &\quad+\frac{bgimn\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}+\frac{bdgmn\,\mathrm{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}\end{aligned}$$

output
```
-a*g*m*x-b*f*n*x+2*b*g*m*n*x-b*g*m*(e*x+d)*ln(c*(e*x+d)^n)/e+g*i*m*(a+b*ln
(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j-b*g*n*(j*x+i)*ln(h*(j*x+i)^m)/j+
b*d*n*ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e+x*(a+b*ln(c*(e*x+d
)^n))*(f+g*ln(h*(j*x+i)^m))+b*g*i*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j+b
*d*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e
```

3.389.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.42

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$= \frac{-bdfjn+bdgjmn+aefjx-aegjmx-befjnx+2begjmnx+befjx\log(c(d+ex)^n)-begjmx\log(c(}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(-(b*d*f*j*n) + b*d*g*j*m*n + a*e*f*j*x - a*e*g*j*m*x - b*e*f*j*n*x + 2*b*
e*g*j*m*n*x + b*e*f*j*x*Log[c*(d + e*x)^n] - b*e*g*j*m*x*Log[c*(d + e*x)^n
] + a*e*g*i*m*Log[i + j*x] - b*e*g*i*m*n*Log[i + j*x] + b*d*g*j*m*n*Log[i
+ j*x] + b*e*g*i*m*Log[c*(d + e*x)^n]*Log[i + j*x] - b*d*g*j*n*Log[h*(i +
j*x)^m] + a*e*g*j*x*Log[h*(i + j*x)^m] - b*e*g*j*n*x*Log[h*(i + j*x)^m] +
b*e*g*j*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + b*n*Log[d + e*x]*(-(e*g*
i*m*Log[i + j*x]) + g*(e*i - d*j)*m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(
f - g*m + g*Log[h*(i + j*x)^m])) + b*g*(e*i - d*j)*m*n*PolyLog[2, (j*(d +
e*x))/(-(e*i) + d*j)])/(e*j)
```

3.389.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2879, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2879}$$

$$-gjm\int \frac{x(a+b\log(c(d+ex)^n))}{i+jx}dx-ben\int \frac{x(f+g\log(h(i+jx)^m))}{d+ex}dx+ x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-gjm\int\left(\frac{a+b\log\left(c(d+ex)^n\right)}{j}-\frac{i(a+b\log\left(c(d+ex)^n\right))}{j(i+jx)}\right)dx-$$
$$ben\int\left(\frac{f+g\log\left(h(i+jx)^m\right)}{e}-\frac{d(f+g\log\left(h(i+jx)^m\right))}{e(d+ex)}\right)dx+$$
$$x(a+b\log\left(c(d+ex)^n\right))\left(f+g\log\left(h(i+jx)^m\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$x(a+b\log\left(c(d+ex)^n\right))\left(f+g\log\left(h(i+jx)^m\right)\right)-$$
$$gjm\left(-\frac{i\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log\left(c(d+ex)^n\right))}{j^2}+\frac{ax}{j}+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{ej}-\frac{bin\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j^2}-\right.$$
$$ben\left(-\frac{d\log\left(-\frac{j(d+ex)}{ei-dj}\right)\left(f+g\log\left(h(i+jx)^m\right)\right)}{e^2}-\frac{dgm\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e^2}+\frac{fx}{e}+\frac{g(i+jx)\log\left(h(i+jx)^m\right)}{ej}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]) - g*j*m*((a*x)/j -
 (b*n*x)/j + (b*(d + e*x)*Log[c*(d + e*x)^n])/(e*j) - (i*(a + b*Log[c*(d +
 e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/j^2 - (b*i*n*PolyLog[2, -((j*(d
+ e*x))/(e*i - d*j))])/j^2) - b*e*n*((f*x)/e - (g*m*x)/e + (g*(i + j*x)*Lo
g[h*(i + j*x)^m])/(e*j) - (d*Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log[
h*(i + j*x)^m]))/e^2 - (d*g*m*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/e^2)
```

3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2879
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Simp[g*j*m   Int[x*((a
+ b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Simp[b*e*n*p   Int[x*(a + b*
Log[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

3.389.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 49.44 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.36

method	result	size
risch	Expression too large to display	1012

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x,method=_RETURNVERBOSE)
```

output
```
(x*b*g*ln((j*x+i)^m)+1/2*b*(I*Pi*g*j*x*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^
m)^2-I*Pi*g*j*x*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-I*Pi*g*j*x
*csgn(I*h*(j*x+i)^m)^3+I*Pi*g*j*x*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+2*j*x*ln
(h)*g+2*g*i*m*ln(j*x+i)-2*x*g*m*j+2*f*j*x)/j)*ln((e*x+d)^n)+(-1/4*I*b*Pi*c
sgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I
*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b
*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(I*Pi*g*x*csgn(I*(j*x+i)^m)*c
sgn(I*h*(j*x+i)^m)^2+I*Pi*g*x*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+2*f*x+2*ln(h
)*g*x+2*g*ln((j*x+i)^m)*x-2*g*m*x+2*g*m/j*i*ln(j*x+i)-I*Pi*g*x*csgn(I*h*(j
*x+i)^m)^3-I*Pi*g*x*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h))+1/2*I
*n*b*x*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/2*I*n*b*x*g*
Pi*csgn(I*h*(j*x+i)^m)^3-1/2*I*n*b*x*g*Pi*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+
1/2*I/e*n*b*d*ln(e*x+d)*g*Pi*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-ln(h)*x*b*g*n
+2*b*g*m*n*x-b*f*n*x+1/2*I/e*n*b*d*ln(e*x+d)*g*Pi*csgn(I*(j*x+i)^m)*csgn(I
*h*(j*x+i)^m)^2-1/2*I*n*b*x*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-1
/2*I/e*n*b*d*ln(e*x+d)*g*Pi*csgn(I*h*(j*x+i)^m)^3-1/2*I/e*n*b*d*ln(e*x+d)*
g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/e*n*b*d*ln(e*x+d)*g
*ln(h)-1/e*n*b*d*ln(e*x+d)*g*m+b*f/e*n*d*ln(e*x+d)-n*b*g*ln((j*x+i)^m)*x+1
/e*n*b*g*ln((j*x+i)^m)*d*ln(e*x+d)+b*d*g*m*n/e-n*b*g*m/j*i*ln((e*x+d)*j-d*
j+e*i)-1/e*n*b*g*m*d*dilog(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/e*n*b*g*m*...
```

3.389.5 Fricas [F]

$$\int (a + b\log (c(d + ex)^n)) (f + g\log (h(i + jx)^m)) \, dx = \int (b\log ((ex + d)^n c) + a)(g\log ((jx + i)^m h) + f) \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="fric
as")
```

output
```
integral(b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*log
((j*x + i)^m*h), x)
```

3.389.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\log (c(d + ex)^n)) (f + g\log (h(i + jx)^m)) \, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.389.7 Maxima [F]

$$\int (a + b\log (c(d + ex)^n)) (f + g\log (h(i + jx)^m)) \, dx = \int (b\log ((ex + d)^n c) + a)(g\log ((jx + i)^m h) + f) \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="maxi
ma")
```

output
```
-b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a*g*j*m*(x/j - i*log(j*x + i)/j^2) +
 b*f*x*log((e*x + d)^n*c) + a*g*x*log((j*x + i)^m*h) + a*f*x - b*g*((e*i*m
*n*log(e*x + d)*log(j*x + i) - (e*i*m*log(j*x + i) - (j*m - j*log(h))*e*x)
*log((e*x + d)^n) - (d*j*n*log(e*x + d) + e*j*x*log((e*x + d)^n) - (e*j*n
- e*j*log(c))*x)*log((j*x + i)^m))/(e*j) + integrate(-(d*e*i*log(c)*log(h)
 - ((j*m - j*log(h))*e^2*log(c) - (2*j*m*n - j*n*log(h))*e^2)*x^2 + (d*e*j
*m*n + (i*m*n - i*n*log(h))*e^2 + (e^2*i*log(h) - (j*m - j*log(h))*d*e)*lo
g(c))*x + (d*e*i*m*n - d^2*j*m*n + (e^2*i*m*n - d*e*j*m*n)*x)*log(e*x + d)
)/(e^2*j*x^2 + d*e*i + (e^2*i + d*e*j)*x), x))
```

3.389.8 Giac [F]

$$\int (a+b\log (c(d+ex)^n)) (f+g\log (h(i+jx)^m)) \, dx = \int (b\log ((ex+d)^n c)+a)(g\log ((jx+i)^m h)+f) \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="giac
")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f), x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n)) (f+g\log (h(i+jx)^m)) \, dx = \int (a+b\,\ln (c\,(d+e\,x)^n))\,(f+g\,\ln (h\,(i+j\,x)^m)) \, dx$$

input
```
int((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

3.390 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$

3.390.1 Optimal result

Integrand size = 32, antiderivative size = 637

$$
\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx \\
&= f\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n)) + bgmn\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx) \\
&\quad - bgm\log\left(-\frac{jx}{i}\right)(n\log(d+ex)-\log(c(d+ex)^n))\log(i+jx) \\
&\quad + \frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{ei-dj}{e(i+jx)}\right)-\log\left(-\frac{(ei-dj)x}{d(i+jx)}\right)\right)\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right) \\
&\quad - \frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)^2 \\
&\quad - bg\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)(m\log(i+jx)-\log(h(i+jx)^m)) \\
&\quad + ag\log\left(-\frac{jx}{i}\right)\log(h(i+jx)^m) + bfn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&\quad + bgmn\left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\text{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&\quad - bgn(m\log(i+jx)-\log(h(i+jx)^m))\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&\quad + bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\text{PolyLog}\left(2,\frac{i(d+ex)}{d(i+jx)}\right) \\
&\quad - bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\text{PolyLog}\left(2,\frac{j(d+ex)}{e(i+jx)}\right) + agm\,\text{PolyLog}\left(2,1+\frac{jx}{i}\right) \\
&\quad - bgm(n\log(d+ex)-\log(c(d+ex)^n))\,\text{PolyLog}\left(2,1+\frac{jx}{i}\right) \\
&\quad + bgmn\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\text{PolyLog}\left(2,1+\frac{jx}{i}\right) \\
&\quad - bgmn\,\text{PolyLog}\left(3,1+\frac{ex}{d}\right) + bgmn\,\text{PolyLog}\left(3,\frac{i(d+ex)}{d(i+jx)}\right) \\
&\quad - bgmn\,\text{PolyLog}\left(3,\frac{j(d+ex)}{e(i+jx)}\right) - bgmn\,\text{PolyLog}\left(3,1+\frac{jx}{i}\right)
\end{aligned}
$$

output
```
f*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))+b*g*m*n*ln(-e*x/d)*ln(e*x+d)*ln(j*x+i)-
b*g*m*ln(-j*x/i)*(n*ln(e*x+d)-ln(c*(e*x+d)^n))*ln(j*x+i)+1/2*b*g*m*n*(ln(-
e*x/d)+ln((-d*j+e*i)/e/(j*x+i))-ln(-(-d*j+e*i)*x/d/(j*x+i)))*ln(d*(j*x+i)/
i/(e*x+d))^2-1/2*b*g*m*n*(ln(-e*x/d)-ln(-j*x/i))*(ln(e*x+d)+ln(d*(j*x+i)/i
/(e*x+d)))^2-b*g*ln(-e*x/d)*ln(c*(e*x+d)^n)*(m*ln(j*x+i)-ln(h*(j*x+i)^m))+
a*g*ln(-j*x/i)*ln(h*(j*x+i)^m)+b*f*n*polylog(2,1+e*x/d)+b*g*m*n*(ln(j*x+i)
-ln(d*(j*x+i)/i/(e*x+d)))*polylog(2,1+e*x/d)-b*g*n*(m*ln(j*x+i)-ln(h*(j*x+
i)^m))*polylog(2,1+e*x/d)+b*g*m*n*ln(d*(j*x+i)/i/(e*x+d))*polylog(2,i*(e*x
+d)/d/(j*x+i))-b*g*m*n*ln(d*(j*x+i)/i/(e*x+d))*polylog(2,j*(e*x+d)/e/(j*x+
i))+a*g*m*polylog(2,1+j*x/i)-b*g*m*(n*ln(e*x+d)-ln(c*(e*x+d)^n))*polylog(2
,1+j*x/i)+b*g*m*n*(ln(e*x+d)+ln(d*(j*x+i)/i/(e*x+d)))*polylog(2,1+j*x/i)-b
*g*m*n*polylog(3,1+e*x/d)+b*g*m*n*polylog(3,i*(e*x+d)/d/(j*x+i))-b*g*m*n*p
olylog(3,j*(e*x+d)/e/(j*x+i))-b*g*m*n*polylog(3,1+j*x/i)
```

3.390. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$

3.390.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx \\
&= \log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))(f-gm\log(i+jx)+g\log(h(i+jx)^m)) \\
&\quad + bn(f-gm\log(i+jx)+g\log(h(i+jx)^m))\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)\right. \\
&\qquad \left. -\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\right) \\
&\quad + agm\left(\log(x)\left(\log(i+jx)-\log\left(1+\frac{jx}{i}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{jx}{i}\right)\right) \\
&\quad + bgm(-n\log(d+ex)+\log(c(d+ex)^n))\left(\log(x)\left(\log(i+jx)-\log\left(1+\frac{jx}{i}\right)\right)\right. \\
&\qquad \left. -\operatorname{PolyLog}\left(2,-\frac{jx}{i}\right)\right)+bgmn\left(\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx)\right. \\
&\qquad +\frac{1}{2}\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right)\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{-ei+dj}{j(d+ex)}\right)-\log\left(\frac{eix-djx}{di+eix}\right)\right) \\
&\qquad +\left(-\log\left(-\frac{ex}{d}\right)+\log\left(-\frac{jx}{i}\right)\right)\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\log\left(1+\frac{jx}{i}\right) \\
&\qquad +\frac{1}{2}\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\log\left(1+\frac{jx}{i}\right)\left(-2\log(d+ex)+\log\left(1+\frac{jx}{i}\right)\right) \\
&\qquad +\left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right) \\
&\qquad +\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\left(-\operatorname{PolyLog}\left(2,\frac{d(i+jx)}{i(d+ex)}\right)+\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{j(d+ex)}\right)\right) \\
&\qquad +\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{jx}{i}\right)-\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right) \\
&\qquad \left.+\operatorname{PolyLog}\left(3,\frac{d(i+jx)}{i(d+ex)}\right)-\operatorname{PolyLog}\left(3,\frac{e(i+jx)}{j(d+ex)}\right)-\operatorname{PolyLog}\left(3,1+\frac{jx}{i}\right)\right)
\end{aligned}
$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

output
```
Log[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(f - g*m*Log[i + j*x]
 + g*Log[h*(i + j*x)^m]) + b*n*(f - g*m*Log[i + j*x] + g*Log[h*(i + j*x)^m
])*(Log[x]*(Log[d + e*x] - Log[1 + (e*x)/d]) - PolyLog[2, -((e*x)/d)]) + a
*g*m*(Log[x]*(Log[i + j*x] - Log[1 + (j*x)/i]) - PolyLog[2, -((j*x)/i)]) +
 b*g*m*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])*(Log[x]*(Log[i + j*x] - Lo
g[1 + (j*x)/i]) - PolyLog[2, -((j*x)/i)]) + b*g*m*n*(Log[-((e*x)/d)]*Log[d
 + e*x]*Log[i + j*x] + (Log[(d*(i + j*x))/(i*(d + e*x))]^2*(Log[-((e*x)/d)
] + Log[(-(e*i) + d*j)/(j*(d + e*x))] - Log[(e*i*x - d*j*x)/(d*i + e*i*x)]
))/2 + (-Log[-((e*x)/d)] + Log[-((j*x)/i)])*Log[(d*(i + j*x))/(i*(d + e*x)
)]*Log[1 + (j*x)/i] + ((Log[-((e*x)/d)] - Log[-((j*x)/i)])*Log[1 + (j*x)/i
]*(-2*Log[d + e*x] + Log[1 + (j*x)/i]))/2 + (Log[i + j*x] - Log[(d*(i + j*
x))/(i*(d + e*x))])*PolyLog[2, 1 + (e*x)/d] + Log[(d*(i + j*x))/(i*(d + e*
x))]*(-PolyLog[2, (d*(i + j*x))/(i*(d + e*x))] + PolyLog[2, (e*(i + j*x))/
(j*(d + e*x))]) + (Log[d + e*x] + Log[(d*(i + j*x))/(i*(d + e*x))])*PolyLo
g[2, 1 + (j*x)/i] - PolyLog[3, 1 + (e*x)/d] + PolyLog[3, (d*(i + j*x))/(i*
(d + e*x))] - PolyLog[3, (e*(i + j*x))/(j*(d + e*x))] - PolyLog[3, 1 + (j*
x)/i])
```

3.390.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 553, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2888, 2841, 2752, 2888, 2841, 2752, 2887, 2841, 2752, 2887, 2841, 2752, 2885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$$

$$\Big\downarrow \text{2888}$$

$$f\int \frac{a+b\log(c(d+ex)^n)}{x}dx+g\int \frac{(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{x}dx$$

$$\Big\downarrow \text{2841}$$

$$f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))-ben\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx\right)+g\int \frac{(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{x}dx$$

$$\Big\downarrow \text{2752}$$

$$g\int \frac{(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{x}dx+ f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2888

$$g\left(a\int \frac{\log(h(i+jx)^m)}{x}dx+b\int \frac{\log(c(d+ex)^n)\log(h(i+jx)^m)}{x}dx\right)+ f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2841

$$g\left(a\left(\log\left(-\frac{jx}{i}\right)\log(h(i+jx)^m)-jm\int \frac{\log\left(-\frac{jx}{i}\right)}{i+jx}dx\right)+b\int \frac{\log(c(d+ex)^n)\log(h(i+jx)^m)}{x}dx\right)+ f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2752

$$g\left(b\int \frac{\log(c(d+ex)^n)\log(h(i+jx)^m)}{x}dx+a\left(\log\left(-\frac{jx}{i}\right)\log(h(i+jx)^m)+m\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right)\right)\right)+ f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2887

$$g\left(b\left(m\int \frac{\log(c(d+ex)^n)\log(i+jx)}{x}dx-(m\log(i+jx)-\log(h(i+jx)^m))\int \frac{\log(c(d+ex)^n)}{x}dx\right)+a\left(\log\left(\right.\right.\right.$$
$$f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2841

$$g\left(b\left(m\int \frac{\log(c(d+ex)^n)\log(i+jx)}{x}dx-(m\log(i+jx)-\log(h(i+jx)^m))\left(\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)-en\right.\right.\right.$$
$$f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2752

$$g\left(b\left(m\int \frac{\log(c(d+ex)^n)\log(i+jx)}{x}dx-(m\log(i+jx)-\log(h(i+jx)^m))\left(\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)+n\,\text{P}\right.\right.\right.$$
$$f\left(\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2887

3.390. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$

$$g\left(b\left(m\left(n\int \frac{\log(d+ex)\log(i+jx)}{x}dx-(n\log(d+ex)-\log\left(c(d+ex)^n\right))\int \frac{\log(i+jx)}{x}dx\right)-(m\log(i+jx)\right.\right.$$
$$f\left(\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2841

$$g\left(b\left(m\left(n\int \frac{\log(d+ex)\log(i+jx)}{x}dx-(n\log(d+ex)-\log\left(c(d+ex)^n\right))\left(\log\left(-\frac{jx}{i}\right)\log(i+jx)-j\int \frac{\log}{i}\right.\right.\right.\right.$$
$$f\left(\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2752

$$g\left(b\left(m\left(n\int \frac{\log(d+ex)\log(i+jx)}{x}dx-\left(\text{PolyLog}\left(2,\frac{jx}{i}+1\right)+\log\left(-\frac{jx}{i}\right)\log(i+jx)\right)(n\log(d+ex)-\log\right.\right.\right.$$
$$f\left(\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)$$

$\downarrow$ 2885

$$f\left(\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\right)+$$
$$g\left(a\left(\log\left(-\frac{jx}{i}\right)\log\left(h(i+jx)^m\right)+m\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right)\right)+b\left(m\left(n\left(\text{PolyLog}\left(3,\frac{i(d+ex)}{d(i+jx)}\right)-\text{PolyLog}\left(\right.\right.\right.\right.\right.$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

output
```
f*(Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + b*n*PolyLog[2, 1 + (e*x)/d
]) + g*(a*(Log[-((j*x)/i)]*Log[h*(i + j*x)^m] + m*PolyLog[2, 1 + (j*x)/i])
 + b*(-((m*Log[i + j*x] - Log[h*(i + j*x)^m])*(Log[-((e*x)/d)]*Log[c*(d +
e*x)^n] + n*PolyLog[2, 1 + (e*x)/d])) + m*(-((n*Log[d + e*x] - Log[c*(d +
e*x)^n])*(Log[-((j*x)/i)]*Log[i + j*x] + PolyLog[2, 1 + (j*x)/i])) + n*(Lo
g[-((e*x)/d)]*Log[d + e*x]*Log[i + j*x] + ((Log[-((e*x)/d)] + Log[(e*i - d
*j)/(e*(i + j*x))] - Log[-(((e*i - d*j)*x)/(d*(i + j*x)))])*Log[(d*(i + j*
x))/(i*(d + e*x))]^2)/2 - ((Log[-((e*x)/d)] - Log[-((j*x)/i)])*(Log[d + e*
x] + Log[(d*(i + j*x))/(i*(d + e*x))])^2)/2 + (Log[i + j*x] - Log[(d*(i +
j*x))/(i*(d + e*x))])*PolyLog[2, 1 + (e*x)/d] + Log[(d*(i + j*x))/(i*(d +
e*x))]*PolyLog[2, (i*(d + e*x))/(d*(i + j*x))] - Log[(d*(i + j*x))/(i*(d +
 e*x))]*PolyLog[2, (j*(d + e*x))/(e*(i + j*x))] + (Log[d + e*x] + Log[(d*(
i + j*x))/(i*(d + e*x))])*PolyLog[2, 1 + (j*x)/i] - PolyLog[3, 1 + (e*x)/d
] + PolyLog[3, (i*(d + e*x))/(d*(i + j*x))] - PolyLog[3, (j*(d + e*x))/(e*
(i + j*x))] - PolyLog[3, 1 + (j*x)/i]))))
```

3.390. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$

3.390.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2885
```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

rule 2887
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Simp[m   Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x),
x], x] - Simp[(m*Log[i + j*x] - Log[h*(i + j*x)^m])   Int[Log[c*(d + e*x)^n
]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && N
eQ[i + j*x, h*(i + j*x)^m]
```

rule 2888
```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Simp[f   Int[(a + b
*Log[c*(d + e*x)^n])/x, x], x] + Simp[g   Int[Log[h*(i + j*x)^m]*((a + b*Lo
g[c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n},
 x] && NeQ[e*i - d*j, 0]
```

3.390. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$

3.390.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))(f+g\ln(h(jx+i)^m))}{x}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x,x)
```

3.390.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="fr
icas")
```

output
```
integral((b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*lo
g((j*x + i)^m*h))/x, x)
```

3.390.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m))/x,x)
```

output
```
Timed out
```

3.390.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="ma
xima")
```

output
```
a*f*log(x) + integrate(((g*log(h) + f)*b*log((e*x + d)^n) + (g*log(h) + f)
*b*log(c) + a*g*log(h) + (b*g*log((e*x + d)^n) + b*g*log(c) + a*g)*log((j*
x + i)^m))/x, x)
```

3.390.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="gi
ac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)/x, x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\ln(c(d+e\,x)^n))\,(f+g\ln(h\,(i+j\,x)^m))}{x}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x, x)
```

3.391 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx$

3.391.1 Optimal result

Integrand size = 32, antiderivative size = 270

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx \\
&= \frac{gjm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{i} - \frac{gjm(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{i} \\
&\quad + \frac{ben\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{ben\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{d} \\
&\quad - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} \\
&\quad - \frac{bgjmn\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{i} + \frac{bgjmn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{i} \\
&\quad - \frac{begmn\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{d} + \frac{begmn\operatorname{PolyLog}\left(2,1+\frac{jx}{i}\right)}{d}
\end{aligned}$$

output
```
g*j*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/i-g*j*m*(a+b*ln(c*(e*x+d)^n))*ln(e*
(j*x+i)/(-d*j+e*i))/i+b*e*n*ln(-j*x/i)*(f+g*ln(h*(j*x+i)^m))/d-b*e*n*ln(-j
*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/d-(a+b*ln(c*(e*x+d)^n))*(f+g*ln
(h*(j*x+i)^m))/x-b*g*j*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/i+b*g*j*m*n*po
lylog(2,1+e*x/d)/i-b*e*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/d+b*e*g*m*n*p
olylog(2,1+j*x/i)/d
```

3.391.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx =$$

$$-\frac{adfi - befinx\log(x) - adgjmx\log\left(-\frac{jx}{i}\right) + befinx\log(d+ex) - bdgjmnx\log\left(-\frac{ex}{d}\right)\log(d+ex) + b\cdots}{}$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

output
```
-((a*d*f*i - b*e*f*i*n*x*Log[x] - a*d*g*j*m*x*Log[-((j*x)/i)] + b*e*f*i*n*
x*Log[d + e*x] - b*d*g*j*m*n*x*Log[-((e*x)/d)]*Log[d + e*x] + b*d*g*j*m*n*
x*Log[-((j*x)/i)]*Log[d + e*x] + b*d*f*i*Log[c*(d + e*x)^n] - b*d*g*j*m*x*
Log[-((j*x)/i)]*Log[c*(d + e*x)^n] + a*d*g*j*m*x*Log[i + j*x] - b*e*g*i*m*
n*x*Log[d + e*x]*Log[i + j*x] - b*d*g*j*m*n*x*Log[d + e*x]*Log[i + j*x] +
b*e*g*i*m*n*x*Log[(j*(d + e*x))/(-(e*i) + d*j)]*Log[i + j*x] + b*d*g*j*m*x
*Log[c*(d + e*x)^n]*Log[i + j*x] + b*d*g*j*m*n*x*Log[d + e*x]*Log[(e*(i +
j*x))/(e*i - d*j)] + a*d*g*i*Log[h*(i + j*x)^m] - b*e*g*i*n*x*Log[x]*Log[h
*(i + j*x)^m] + b*e*g*i*n*x*Log[d + e*x]*Log[h*(i + j*x)^m] + b*d*g*i*Log[
c*(d + e*x)^n]*Log[h*(i + j*x)^m] + b*e*g*i*m*n*x*Log[x]*Log[1 + (j*x)/i]
+ b*e*g*i*m*n*x*PolyLog[2, -((j*x)/i)] + b*d*g*j*m*n*x*PolyLog[2, (j*(d +
e*x))/(-(e*i) + d*j)] - b*d*g*j*m*n*x*PolyLog[2, 1 + (e*x)/d] + b*e*g*i*m*
n*x*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/(d*i*x))
```

3.391.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2889, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

$$\Big\downarrow \ 2889$$

$$gjm\int \frac{a+b\log \left(c(d+ex)^n\right)}{x(i+jx)}dx+ben\int \frac{f+g\log \left(h(i+jx)^m\right)}{x(d+ex)}dx-\frac{\left(a+b\log \left(c(d+ex)^n\right)\right)\left(f+g\log \left(h(i+jx)^m\right)\right)}{x}$$

$$\Big\downarrow \ \textbf{2863}$$

$$gjm\int \left(\frac{a+b\log \left(c(d+ex)^n\right)}{ix}-\frac{j\left(a+b\log \left(c(d+ex)^n\right)\right)}{i(i+jx)}\right)dx+ben\int \left(\frac{f+g\log \left(h(i+jx)^m\right)}{dx}-\frac{e\left(f+g\log \left(h(i+jx)^m\right)\right)}{d(d+ex)}\right)dx-\frac{\left(a+b\log \left(c(d+ex)^n\right)\right)\left(f+g\log \left(h(i+jx)^m\right)\right)}{x}$$

$$\Big\downarrow \ \textbf{2009}$$

$$-\frac{\left(a+b\log \left(c(d+ex)^n\right)\right)\left(f+g\log \left(h(i+jx)^m\right)\right)}{x}+gjm\left(-\frac{\log \left(\frac{e(i+jx)}{ei-dj}\right)\left(a+b\log \left(c(d+ex)^n\right)\right)}{i}+\frac{\log \left(-\frac{ex}{d}\right)\left(a+b\log \left(c(d+ex)^n\right)\right)}{i}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{i}+\right.$$
$$ben\left(-\frac{\log \left(-\frac{j(d+ex)}{ei-dj}\right)\left(f+g\log \left(h(i+jx)^m\right)\right)}{d}-\frac{gm\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{d}+\frac{\log \left(-\frac{jx}{i}\right)\left(f+g\log \left(h(i+jx)^m\right)\right)}{d}\right.$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

output
```
-(((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x) + g*j*m*((Log
[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/i - ((a + b*Log[c*(d + e*x)^n])*L
og[(e*(i + j*x))/(e*i - d*j)])/i - (b*n*PolyLog[2, -((j*(d + e*x))/(e*i -
d*j))])/i + (b*n*PolyLog[2, 1 + (e*x)/d])/i) + b*e*n*((Log[-((j*x)/i)]*(f
+ g*Log[h*(i + j*x)^m]))/d - (Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log
[h*(i + j*x)^m]))/d - (g*m*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/d + (g*m
*PolyLog[2, 1 + (j*x)/i])/d)
```

3.391. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx$

3.391.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

3.391.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))(f+g\ln(h(jx+i)^m))}{x^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

3.391.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)}{x^2}dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm="
fricas")
```

output
```
integral((b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*lo
g((j*x + i)^m*h))/x^2, x)
```

3.391.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m))/x**2,x)
```

output
```
Timed out
```

3.391.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx$$
$$= \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm="
maxima")
```

output
```
-b*e*f*n*(log(e*x + d)/d - log(x)/d) - a*g*j*m*(log(j*x + i)/i - log(x)/i)
 + b*g*integrate(((log((e*x + d)^n) + log(c))*log((j*x + i)^m) + log((e*x
+ d)^n)*log(h) + log(c)*log(h))/x^2, x) - b*f*log((e*x + d)^n*c)/x - a*g*l
og((j*x + i)^m*h)/x - a*f/x
```

3.391.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)/x^2, x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))}{x^2}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^2,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^2, x)
```

3.392 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx$

3.392.1 Optimal result

Integrand size = 32, antiderivative size = 421

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx \\
&= \frac{begjmn\log(x)}{di} - \frac{begjmn\log(d+ex)}{2di} - \frac{gjm(a+b\log(c(d+ex)^n))}{2ix} \\
&\quad - \frac{gj^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2i^2} - \frac{begjmn\log(i+jx)}{2di} \\
&\quad + \frac{gj^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2i^2} - \frac{ben(f+g\log(h(i+jx)^m))}{2dx} \\
&\quad - \frac{be^2n\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{2d^2} + \frac{be^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2d^2} \\
&\quad - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} \\
&\quad + \frac{bgj^2mn\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2i^2} - \frac{bgj^2mn\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{2i^2} \\
&\quad + \frac{be^2gmn\,\mathrm{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{2d^2} - \frac{be^2gmn\,\mathrm{PolyLog}\left(2,1+\frac{jx}{i}\right)}{2d^2}
\end{aligned}$$

output

```
b*e*g*j*m*n*ln(x)/d/i-1/2*b*e*g*j*m*n*ln(e*x+d)/d/i-1/2*g*j*m*(a+b*ln(c*(e
*x+d)^n))/i/x-1/2*g*j^2*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/i^2-1/2*b*e*g*j
*m*n*ln(j*x+i)/d/i+1/2*g*j^2*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*
i))/i^2-1/2*b*e*n*(f+g*ln(h*(j*x+i)^m))/d/x-1/2*b*e^2*n*ln(-j*x/i)*(f+g*ln
(h*(j*x+i)^m))/d^2+1/2*b*e^2*n*ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)
^m))/d^2-1/2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^2+1/2*b*g*j^2*m
*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/i^2-1/2*b*g*j^2*m*n*polylog(2,1+e*x/d)
/i^2+1/2*b*e^2*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/d^2-1/2*b*e^2*g*m*n*p
olylog(2,1+j*x/i)/d^2
```

3.392. $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx$

3.392.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.82

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx$$
$$= -\frac{be^2n\log(x)(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2d^2} + \frac{be^2n\log(d+ex)(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2d^2} - \frac{bn\log(d+ex)(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2x^2} - \frac{(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2x^2} - \frac{e(bfn+bgn(-m\log(i+jx)+\log(h(i+jx)^m)))}{2dx} + \frac{1}{2}agm\left(\frac{j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)} - \left(\frac{j^2(i+jx)^2}{i^4\left(1-\frac{i+jx}{i}\right)^2} + \frac{2j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)}\right)\log(i+jx) - \frac{j^2\log\left(1-\frac{i+jx}{i}\right)}{i^2}\right) + \frac{1}{2}bgm(-n\log(d+ex)+\log(c(d+ex)^n))\left(\frac{j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)} - \left(\frac{j^2(i+jx)^2}{i^4\left(1-\frac{i+jx}{i}\right)^2} + \frac{2j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)}\right)\log(i+jx) - \frac{j^2\log\left(1-\frac{i+jx}{i}\right)}{i^2}\right) + \frac{1}{2}bgmn\left(-\frac{\log(d+ex)\log(i+jx)}{x^2} + j\left(\frac{\frac{e\log(x)}{d}-\frac{e\log(d+ex)}{d}-\frac{\log(d+ex)}{x}}{i} - \frac{j\left(\log\left(-\frac{ex}{d}\right)\log(d+ex)+\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)\right)}{i^2} + \frac{j^2\left(\frac{\log(d+ex)\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + \frac{\operatorname{PolyLog}\left(2,\frac{j(d+ex)}{-ei+dj}\right)}{j}\right)}{i^2}\right) + e\left(\frac{\frac{j\log(x)}{i}-\frac{j\log(i+jx)}{i}-\frac{\log(i+jx)}{x}}{d} - \frac{e\left(\log(x)\left(\log(i+jx)-\log\left(1+\frac{jx}{i}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{jx}{i}\right)\right)}{d^2} + \frac{e^2\left(\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)\log(i+jx)}{e} + \frac{\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}\right)}{d^2}\right)\right)$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^3,x]
```

output
```
-1/2*(b*e^2*n*Log[x]*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/d^2
 + (b*e^2*n*Log[d + e*x]*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))
/(2*d^2) - (b*n*Log[d + e*x]*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m
])))/(2*x^2) - ((a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n]))*(f + g*(-
(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/(2*x^2) - (e*(b*f*n + b*g*n*(-(m*
Log[i + j*x]) + Log[h*(i + j*x)^m])))/(2*d*x) + (a*g*m*((j^2*(i + j*x))/(i
^3*(1 - (i + j*x)/i)) - ((j^2*(i + j*x)^2)/(i^4*(1 - (i + j*x)/i)^2) + (2*
j^2*(i + j*x))/(i^3*(1 - (i + j*x)/i)))*Log[i + j*x] - (j^2*Log[1 - (i + j
*x)/i])/i^2))/2 + (b*g*m*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])*((j^2*(i
 + j*x))/(i^3*(1 - (i + j*x)/i)) - ((j^2*(i + j*x)^2)/(i^4*(1 - (i + j*x)/
i)^2) + (2*j^2*(i + j*x))/(i^3*(1 - (i + j*x)/i)))*Log[i + j*x] - (j^2*Log
[1 - (i + j*x)/i])/i^2))/2 + (b*g*m*n*(-((Log[d + e*x]*Log[i + j*x])/x^2)
+ j*(((e*Log[x])/d - (e*Log[d + e*x])/d - Log[d + e*x]/x)/i - (j*(Log[-((e
*x)/d)]*Log[d + e*x] + PolyLog[2, (d + e*x)/d]))/i^2 + (j^2*((Log[d + e*x]
*Log[(e*(i + j*x))/(e*i - d*j)])/j + PolyLog[2, (j*(d + e*x))/(-(e*i) + d*
j)]/j))/i^2) + e*(((j*Log[x])/i - (j*Log[i + j*x])/i - Log[i + j*x]/x)/d -
 (e*(Log[x]*(Log[i + j*x] - Log[1 + (j*x)/i]) - PolyLog[2, -((j*x)/i)]))/d
^2 + (e^2*((Log[(j*(d + e*x))/(-(e*i) + d*j)]*Log[i + j*x])/e + PolyLog[2,
 (e*(i + j*x))/(e*i - d*j)]/e))/d^2)))/2
```

3.392.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2889, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}dx$$

$$\Big\downarrow \text{2889}$$

$$\frac{1}{2}gjm\int \frac{a+b\log(c(d+ex)^n)}{x^2(i+jx)}dx+\frac{1}{2}ben\int \frac{f+g\log(h(i+jx)^m)}{x^2(d+ex)}dx-\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2}$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{2}gjm\int\left(\frac{(a+b\log(c(d+ex)^n))\,j^2}{i^2(i+jx)}-\frac{(a+b\log(c(d+ex)^n))\,j}{i^2x}+\frac{a+b\log(c(d+ex)^n)}{ix^2}\right)dx+$$
$$\frac{1}{2}ben\int\left(\frac{(f+g\log(h(i+jx)^m))\,e^2}{d^2(d+ex)}-\frac{(f+g\log(h(i+jx)^m))\,e}{d^2x}+\frac{f+g\log(h(i+jx)^m)}{dx^2}\right)dx-$$
$$\frac{(a+b\log(c(d+ex)^n))\,(f+g\log(h(i+jx)^m))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\log(c(d+ex)^n))\,(f+g\log(h(i+jx)^m))}{2x^2}+$$
$$\frac{1}{2}gjm\left(-\frac{j\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{i^2}+\frac{j\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{i^2}-\frac{a+b\log(c(d+ex)^n)}{ix}+\right.$$
$$\frac{1}{2}ben\left(-\frac{e\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{d^2}+\frac{e\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{d^2}+\frac{egm\,\text{PolyLog}\left(2,\frac{e(i-}{ei-}\right.}{d^2}\right.$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^3,x]
```

output
```
-1/2*((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^2 + (g*j*m*
((b*e*n*Log[x])/(d*i) - (b*e*n*Log[d + e*x])/(d*i) - (a + b*Log[c*(d + e*x
)^n])/(i*x) - (j*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/i^2 + (j*(a +
 b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/i^2 + (b*j*n*PolyLo
g[2, -((j*(d + e*x))/(e*i - d*j))])/i^2 - (b*j*n*PolyLog[2, 1 + (e*x)/d])/
i^2))/2 + (b*e*n*((g*j*m*Log[x])/(d*i) - (g*j*m*Log[i + j*x])/(d*i) - (f +
 g*Log[h*(i + j*x)^m])/(d*x) - (e*Log[-((j*x)/i)]*(f + g*Log[h*(i + j*x)^m
]))/d^2 + (e*Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log[h*(i + j*x)^m]))
/d^2 + (e*g*m*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/d^2 - (e*g*m*PolyLog[
2, 1 + (j*x)/i])/d^2))/2
```

3.392.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

3.392.4 Maple [F]

$$\int \frac{(a+b\ln (c(ex+d)^n)) (f+g\ln (h(jx+i)^m))}{x^3}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^3,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^3,x)
```

3.392.5 Fricas [F]

$$\int \frac{(a+b\log (c(d+ex)^n)) (f+g\log (h(i+jx)^m))}{x^3} dx = \int \frac{(b\log ((ex+d)^n c)+a)(g\log ((jx+i)^m h)+f)}{x^3} dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^3,x, algorithm="
fricas")
```

output
```
integral((b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*lo
g((j*x + i)^m*h))/x^3, x)
```

3.392.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m))/x**3,x)
```

output
```
Timed out
```

3.392.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^3,x, algorithm="
maxima")
```

output
```
1/2*b*e*f*n*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*a*g*j*m*(j
*log(j*x + i)/i^2 - j*log(x)/i^2 - 1/(i*x)) + b*g*integrate(((log((e*x + d
)^n) + log(c))*log((j*x + i)^m) + log((e*x + d)^n)*log(h) + log(c)*log(h))
/x^3, x) - 1/2*b*f*log((e*x + d)^n*c)/x^2 - 1/2*a*g*log((j*x + i)^m*h)/x^2
 - 1/2*a*f/x^2
```

3.392.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^3,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)/x^3, x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,\left(f+g\,\ln\left(h\,{\left(i+j\,x\right)}^m\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^3,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^3, x)
```

3.393 $\int x(a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))$

3.393.1 Optimal result

Integrand size = 32, antiderivative size = 1210

$$
\begin{aligned}
&\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx \\
&= -\frac{2abdgmnx}{e}-\frac{3abgimnx}{2j}-\frac{2b^2dfn^2x}{e}+\frac{15b^2dgmn^2x}{4e}+\frac{7b^2gimn^2x}{4j}-\frac{1}{4}b^2gmn^2x^2 \\
&\quad+\frac{b^2fn^2(d+ex)^2}{4e^2}-\frac{b^2gmn^2(d+ex)^2}{8e^2}-\frac{b^2d^2gmn^2\log(d+ex)}{4e^2}+\frac{b^2d^2fn^2\log^2(d+ex)}{2e^2} \\
&\quad-\frac{2b^2dgmn(d+ex)\log(c(d+ex)^n)}{e^2}-\frac{3b^2gimn(d+ex)\log(c(d+ex)^n)}{2ej} \\
&\quad+\frac{1}{4}bgmnx^2(a+b\log(c(d+ex)^n))+\frac{2bdfn(d+ex)(a+b\log(c(d+ex)^n))}{e^2} \\
&\quad-\frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2}+\frac{bgmn(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2} \\
&\quad-\frac{bd^2fn\log(d+ex)(a+b\log(c(d+ex)^n))}{e^2}+\frac{dgm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} \\
&\quad+\frac{gim(d+ex)(a+b\log(c(d+ex)^n))^2}{2ej}-\frac{gm(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} \\
&\quad-\frac{b^2gi^2mn^2\log(i+jx)}{4j^2}+\frac{bgi^2mn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} \\
&\quad+\frac{bdgimn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{ej} \\
&\quad+\frac{d^2gm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2e^2} \\
&\quad-\frac{gi^2m(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}+\frac{1}{4}b^2gn^2x^2\log(h(i+jx)^m) \\
&\quad-\frac{3b^2dgn^2(i+jx)\log(h(i+jx)^m)}{2ej}+\frac{3b^2d^2gn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{2e^2} \\
&\quad+\frac{bdgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e} \\
&\quad-\frac{1}{2}bgnx^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m) \\
&\quad-\frac{d^2g(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{2e^2} \\
&\quad+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) \\
&\quad+\frac{b^2gi^2mn^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2j^2}+\frac{b^2dgimn^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{ej} \\
&\quad+\frac{bd^2gmn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e^2} \\
&\quad-\frac{bgi^2mn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j^2}
\end{aligned}
$$

output
```
b*d*g*i*m*n*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/e/j+1/2*x^2*(a+
b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))+1/4*b*g*m*n*x^2*(a+b*ln(c*(e*x+
d)^n))-1/2*b*f*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*d*g*m*(e*x+d)*(a+
b*ln(c*(e*x+d)^n))^2/e^2+1/2*d^2*g*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/
(-d*j+e*i))/e^2-1/2*g*i^2*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i
))/j^2-1/2*b*g*n*x^2*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)-1/4*b^2*d^2*g*m
*n^2*ln(e*x+d)/e^2+2*b*d*f*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^2+1/4*b*g*m*n
*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*g*i*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^2/e/j-1/4*b^2*g*i^2*m*n^2*ln(j*x+i)/j^2+3/2*b^2*d^2*g*n^2*ln(-j*(e*x+d)
/(-d*j+e*i))*ln(h*(j*x+i)^m)/e^2+15/4*b^2*d*g*m*n^2*x/e+7/4*b^2*g*i*m*n^2*
x/j+1/2*b^2*g*i^2*m*n^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2+3/2*b^2*d^2*g
*m*n^2*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^2+b^2*d*g*i*m*n^2*polylog(2,-j*(e
*x+d)/(-d*j+e*i))/e/j-2*a*b*d*g*m*n*x/e-3/2*a*b*g*i*m*n*x/j-2*b^2*d*g*m*n*
(e*x+d)*ln(c*(e*x+d)^n)/e^2+1/2*b*g*i^2*m*n*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*
x+i)/(-d*j+e*i))/j^2-3/2*b^2*d*g*n^2*(j*x+i)*ln(h*(j*x+i)^m)/e/j-1/2*d^2*g
*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e^2-1/4*g*m*(e*x+d)^2*(a+b*ln(c*(
e*x+d)^n))^2/e^2+1/4*b^2*g*n^2*x^2*ln(h*(j*x+i)^m)-1/4*b^2*g*m*n^2*x^2+1/4
*b^2*f*n^2*(e*x+d)^2/e^2+b*d*g*n*x*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)/e
+b*d^2*g*m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e^2-3/
2*b^2*g*i*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e/j-2*b^2*d*f*n^2*x/e-1/8*b^2*g*m...
```

3.393.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 2067, normalized size of antiderivative = 1.71

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx = \text{Result too large to show}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(-8*a*b*d*e*g*i*j*m*n + 4*b^2*d*e*g*i*j*m*n^2 + 8*b^2*d^2*g*j^2*m*n^2 + 4*
a^2*e^2*g*i*j*m*x + 8*a*b*d*e*f*j^2*n*x - 12*a*b*e^2*g*i*j*m*n*x - 12*a*b*
d*e*g*j^2*m*n*x - 12*b^2*d*e*f*j^2*n^2*x + 14*b^2*e^2*g*i*j*m*n^2*x + 28*b
^2*d*e*g*j^2*m*n^2*x + 4*a^2*e^2*f*j^2*x^2 - 2*a^2*e^2*g*j^2*m*x^2 - 4*a*b
*e^2*f*j^2*n*x^2 + 4*a*b*e^2*g*j^2*m*n*x^2 + 2*b^2*e^2*f*j^2*n^2*x^2 - 3*b
^2*e^2*g*j^2*m*n^2*x^2 - 8*a*b*d^2*f*j^2*n*Log[d + e*x] + 8*a*b*d*e*g*i*j*
m*n*Log[d + e*x] + 4*a*b*d^2*g*j^2*m*n*Log[d + e*x] + 12*b^2*d^2*f*j^2*n^2
*Log[d + e*x] - 4*b^2*d*e*g*i*j*m*n^2*Log[d + e*x] - 16*b^2*d^2*g*j^2*m*n^
2*Log[d + e*x] + 4*b^2*d^2*f*j^2*n^2*Log[d + e*x]^2 - 4*b^2*d*e*g*i*j*m*n^
2*Log[d + e*x]^2 - 2*b^2*d^2*g*j^2*m*n^2*Log[d + e*x]^2 - 8*b^2*d*e*g*i*j*
m*n*Log[c*(d + e*x)^n] + 8*a*b*e^2*g*i*j*m*x*Log[c*(d + e*x)^n] + 8*b^2*d*
e*f*j^2*n*x*Log[c*(d + e*x)^n] - 12*b^2*e^2*g*i*j*m*n*x*Log[c*(d + e*x)^n]
 - 12*b^2*d*e*g*j^2*m*n*x*Log[c*(d + e*x)^n] + 8*a*b*e^2*f*j^2*x^2*Log[c*(
d + e*x)^n] - 4*a*b*e^2*g*j^2*m*x^2*Log[c*(d + e*x)^n] - 4*b^2*e^2*f*j^2*n
*x^2*Log[c*(d + e*x)^n] + 4*b^2*e^2*g*j^2*m*n*x^2*Log[c*(d + e*x)^n] - 8*b
^2*d^2*f*j^2*n*Log[d + e*x]*Log[c*(d + e*x)^n] + 8*b^2*d*e*g*i*j*m*n*Log[d
 + e*x]*Log[c*(d + e*x)^n] + 4*b^2*d^2*g*j^2*m*n*Log[d + e*x]*Log[c*(d + e
*x)^n] + 4*b^2*e^2*g*i*j*m*x*Log[c*(d + e*x)^n]^2 + 4*b^2*e^2*f*j^2*x^2*Lo
g[c*(d + e*x)^n]^2 - 2*b^2*e^2*g*j^2*m*x^2*Log[c*(d + e*x)^n]^2 - 4*a^2*e^
2*g*i^2*m*Log[i + j*x] + 4*a*b*e^2*g*i^2*m*n*Log[i + j*x] + 8*a*b*d*e*g...
```

3.393.3 Rubi [A] (verified)

Time = 3.51 (sec) , antiderivative size = 1273, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2889, 2863, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2889}$$

$$-ben\int\frac{x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}dx-\frac{1}{2}gjm\int\frac{x^2(a+b\log(c(d+ex)^n))^2}{i+jx}dx+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-ben\int \frac{x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}dx-$$
$$\frac{1}{2}gjm\int\left(\frac{x(a+b\log(c(d+ex)^n))^2}{j}+\frac{i^2(a+b\log(c(d+ex)^n))^2}{j^2(i+jx)}-\frac{i(a+b\log(c(d+ex)^n))^2}{j^2}\right)dx+$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$\downarrow$ 2009

$$-ben\int \frac{x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}dx-$$
$$\frac{1}{2}gjm\left(-\frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2j}+\frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2j}-\frac{d(d+ex)(a+b\log(c(d+e$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$\downarrow$ 7293

$$-ben\int\left(\frac{f(a+b\log(c(d+ex)^n))x^2}{d+ex}+\frac{g(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)x^2}{d+ex}\right)dx-$$
$$\frac{1}{2}gjm\left(-\frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2j}+\frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2j}-\frac{d(d+ex)(a+b\log(c(d+e$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$\downarrow$ 2009

$$\frac{1}{2}x^2(f+g\log(h(i+jx)^m))(a+b\log(c(d+ex)^n))^2-$$
$$ben\left(-\frac{bfn\log^2(d+ex)d^2}{2e^3}+\frac{bgmn\log(d+ex)d^2}{4e^3}+\frac{f\log(d+ex)(a+b\log(c(d+ex)^n))d^2}{e^3}-\frac{gm(a+b\log(c(d$$
$$\frac{1}{2}gjm\left(\frac{n^2(d+ex)^2b^2}{4e^2j}-\frac{2dn^2xb^2}{ej}-\frac{2in^2xb^2}{j^2}+\frac{2dn(d+ex)\log(c(d+ex)^n)b^2}{e^2j}+\frac{2in(d+ex)\log(c(d+ex)^n)b^2}{ej^2}$$

input
```
Int[x*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

3.393. $\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$

output
```
(x^2*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/2 - b*e*n*((
a*d*g*m*x)/e^2 + (a*g*i*m*x)/(2*e*j) + (2*b*d*f*n*x)/e^2 - (11*b*d*g*m*n*x
)/(4*e^2) - (3*b*g*i*m*n*x)/(4*e*j) + (b*g*m*n*x^2)/(4*e) - (b*f*n*(d + e*
x)^2)/(4*e^3) + (b*d^2*g*m*n*Log[d + e*x])/(4*e^3) - (b*d^2*f*n*Log[d + e*
x]^2)/(2*e^3) + (b*d*g*m*(d + e*x)*Log[c*(d + e*x)^n])/e^3 + (b*g*i*m*(d +
 e*x)*Log[c*(d + e*x)^n])/(2*e^2*j) - (g*m*x^2*(a + b*Log[c*(d + e*x)^n]))
/(4*e) - (2*d*f*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/e^3 + (f*(d + e*x)^2
*(a + b*Log[c*(d + e*x)^n]))/(2*e^3) + (d^2*f*Log[d + e*x]*(a + b*Log[c*(d
 + e*x)^n]))/e^3 + (b*g*i^2*m*n*Log[i + j*x])/(4*e*j^2) - (g*i^2*m*(a + b*
Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(2*e*j^2) - (d*g*i*m*(
a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(e^2*j) - (d^2*g
*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(2*b*e^3*n
) - (b*g*n*x^2*Log[h*(i + j*x)^m])/(4*e) + (3*b*d*g*n*(i + j*x)*Log[h*(i +
 j*x)^m])/(2*e^2*j) - (3*b*d^2*g*n*Log[-((j*(d + e*x))/(e*i - d*j))]*Log[h
*(i + j*x)^m])/(2*e^3) - (d*g*x*(a + b*Log[c*(d + e*x)^n])*Log[h*(i + j*x)
^m])/e^2 + (g*x^2*(a + b*Log[c*(d + e*x)^n])*Log[h*(i + j*x)^m])/(2*e) + (
d^2*g*(a + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j*x)^m])/(2*b*e^3*n) - (b*g*
i^2*m*n*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(2*e*j^2) - (b*d*g*i*m*n
*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(e^2*j) - (d^2*g*m*(a + b*Log[c
*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e^3 - (3*b*d^2...
```

3.393.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

3.393. $\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.393.4 Maple [F]

$$\int x(a+b\ln(c(ex+d)^n))^2(f+g\ln(h(jx+i)^m))\,dx$$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
```

output
```
int(x*(a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
```

3.393.5 Fricas [F]

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$
$$=\int (b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="
fricas")
```

output
```
integral(b^2*f*x*log((e*x + d)^n*c)^2 + 2*a*b*f*x*log((e*x + d)^n*c) + a^2
*f*x + (b^2*g*x*log((e*x + d)^n*c)^2 + 2*a*b*g*x*log((e*x + d)^n*c) + a^2*
g*x)*log((j*x + i)^m*h), x)
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.393.7 Maxima [F]

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="
maxima")
```

output
```
1/2*b^2*f*x^2*log((e*x + d)^n*c)^2 - 1/2*a*b*e*f*n*(2*d^2*log(e*x + d)/e^3
 + (e*x^2 - 2*d*x)/e^2) - 1/4*a^2*g*j*m*(2*i^2*log(j*x + i)/j^3 + (j*x^2 -
 2*i*x)/j^2) + a*b*f*x^2*log((e*x + d)^n*c) + 1/2*a^2*g*x^2*log((j*x + i)^
m*h) + 1/2*a^2*f*x^2 - 1/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x
)/e^2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*
d^2*log(e*x + d))*n^2/e^2)*b^2*f + 1/4*((2*b^2*e^2*g*i*j*m*x - 2*b^2*e^2*g
*i^2*m*log(j*x + i) - (j^2*m - 2*j^2*log(h))*b^2*e^2*g*x^2)*log((e*x + d)^
n)^2 + (2*b^2*d^2*g*j^2*n^2*log(e*x + d)^2 + 2*b^2*e^2*g*j^2*x^2*log((e*x
+ d)^n)^2 - (2*(e^2*g*j^2*n - 2*e^2*g*j^2*log(c))*a*b - (e^2*g*j^2*n^2 - 2
*e^2*g*j^2*n*log(c) + 2*e^2*g*j^2*log(c)^2)*b^2)*x^2 + 2*(2*a*b*d*e*g*j^2*
n - (3*d*e*g*j^2*n^2 - 2*d*e*g*j^2*n*log(c))*b^2)*x - 2*(2*a*b*d^2*g*j^2*n
 - (3*d^2*g*j^2*n^2 - 2*d^2*g*j^2*n*log(c))*b^2)*log(e*x + d) + 2*(2*b^2*d
*e*g*j^2*n*x - 2*b^2*d^2*g*j^2*n*log(e*x + d) + (2*a*b*e^2*g*j^2 - (e^2*g*
j^2*n - 2*e^2*g*j^2*log(c))*b^2)*x^2)*log((e*x + d)^n))*log((j*x + i)^m))/
(e^2*j^2) + integrate(1/4*((2*(e^3*g*j^3*m*n - 2*(j^3*m - 2*j^3*log(h))*e^
3*g*log(c))*a*b - (e^3*g*j^3*m*n^2 - 2*e^3*g*j^3*m*n*log(c) + 2*(j^3*m - 2
*j^3*log(h))*e^3*g*log(c)^2)*b^2)*x^3 - (2*(d*e^2*g*j^3*m*n - 2*(2*e^3*g*i
*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*log(c))*a*b - (5*d*e^2*g*j^3
*m*n^2 - 2*d*e^2*g*j^3*m*n*log(c) + 2*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j
^3*log(h))*d*e^2*g)*log(c)^2)*b^2)*x^2 - 2*(b^2*d^2*e*g*j^3*m*n^2*x + b...
```

3.393.8 Giac [F]

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f)*x, x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$
$$=\int x\,(a+b\,\ln(c\,(d+e\,x)^n))^2\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int(x*(a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)), x)
```

3.394 $\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))\, dx$

3.394.1 Optimal result

Integrand size = 31, antiderivative size = 649

$$\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))\,dx$$
$$= -2abfnx+4abgmnx+2b^2fn^2x-6b^2gmn^2x-\frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{e}$$
$$+\frac{4b^2gmn(d+ex)\log(c(d+ex)^n)}{e}+\frac{df(a+b\log(c(d+ex)^n))^2}{e}$$
$$-\frac{gm(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-\frac{2bgimn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$-\frac{dgm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e}+\frac{gim(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$+\frac{2b^2gn^2(i+jx)\log(h(i+jx)^m)}{j}-\frac{2b^2dgn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{e}$$
$$-2bgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)$$
$$+\frac{dg(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e}$$
$$+x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))-\frac{2b^2gimn^2\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$
$$-\frac{2bdgmn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e}$$
$$+\frac{2bgimn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$
$$-\frac{2b^2dgmn^2\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}+\frac{2b^2dgmn^2\,\text{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{e}$$
$$-\frac{2b^2gimn^2\,\text{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$

output
```
-2*a*b*f*n*x+4*a*b*g*m*n*x+2*b^2*f*n^2*x-6*b^2*g*m*n^2*x-2*b^2*f*n*(e*x+d)
*ln(c*(e*x+d)^n)/e+4*b^2*g*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e+d*f*(a+b*ln(c*(e*
x+d)^n))^2/e-g*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e-2*b*g*i*m*n*(a+b*ln(c*(
e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j-d*g*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(
j*x+i)/(-d*j+e*i))/e+g*i*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i)
)/j+2*b^2*g*n^2*(j*x+i)*ln(h*(j*x+i)^m)/j-2*b^2*d*g*n^2*ln(-j*(e*x+d)/(-d*
j+e*i))*ln(h*(j*x+i)^m)/e-2*b*g*n*x*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)+
d*g*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e+x*(a+b*ln(c*(e*x+d)^n))^2*(f
+g*ln(h*(j*x+i)^m))-2*b^2*g*i*m*n^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j-2*b
*d*g*m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e+2*b*g*i*
m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j-2*b^2*d*g*m*n
^2*polylog(2,e*(j*x+i)/(-d*j+e*i))/e+2*b^2*d*g*m*n^2*polylog(3,-j*(e*x+d)/
(-d*j+e*i))/e-2*b^2*g*i*m*n^2*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j
```

3.394.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1355 vs. $2(649) = 1298$.

Time = 0.32 (sec) , antiderivative size = 1355, normalized size of antiderivative = 2.09

$$\int (a+b\log (c(d+ex)^n))^2 (f+g\log (h(i+jx)^m))\, dx$$
$$= \frac{-2abdfjn + 2abdgjmn - 2b^2dgjmn^2 + a^2efjx - a^2egjmx - 2abefjnx + 4abegjmnx + 2b^2efjn^2x - 6b\cdots}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(-2*a*b*d*f*j*n + 2*a*b*d*g*j*m*n - 2*b^2*d*g*j*m*n^2 + a^2*e*f*j*x - a^2*
e*g*j*m*x - 2*a*b*e*f*j*n*x + 4*a*b*e*g*j*m*n*x + 2*b^2*e*f*j*n^2*x - 6*b^
2*e*g*j*m*n^2*x + 2*a*b*d*f*j*n*Log[d + e*x] - 2*a*b*d*g*j*m*n*Log[d + e*x
] + 2*b^2*d*g*j*m*n^2*Log[d + e*x] - b^2*d*f*j*n^2*Log[d + e*x]^2 + b^2*d*
g*j*m*n^2*Log[d + e*x]^2 - 2*b^2*d*f*j*n*Log[c*(d + e*x)^n] + 2*b^2*d*g*j*
m*n*Log[c*(d + e*x)^n] + 2*a*b*e*f*j*x*Log[c*(d + e*x)^n] - 2*a*b*e*g*j*m*
x*Log[c*(d + e*x)^n] - 2*b^2*e*f*j*n*x*Log[c*(d + e*x)^n] + 4*b^2*e*g*j*m*
n*x*Log[c*(d + e*x)^n] + 2*b^2*d*f*j*n*Log[d + e*x]*Log[c*(d + e*x)^n] - 2
*b^2*d*g*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] + b^2*e*f*j*x*Log[c*(d + e*
x)^n]^2 - b^2*e*g*j*m*x*Log[c*(d + e*x)^n]^2 + a^2*e*g*i*m*Log[i + j*x] -
2*a*b*e*g*i*m*n*Log[i + j*x] + 2*a*b*d*g*j*m*n*Log[i + j*x] + 2*b^2*e*g*i*
m*n^2*Log[i + j*x] - 2*a*b*e*g*i*m*n*Log[d + e*x]*Log[i + j*x] + 2*b^2*e*g
*i*m*n^2*Log[d + e*x]*Log[i + j*x] - 2*b^2*d*g*j*m*n^2*Log[d + e*x]*Log[i
+ j*x] + b^2*e*g*i*m*n^2*Log[d + e*x]^2*Log[i + j*x] + 2*a*b*e*g*i*m*Log[c
*(d + e*x)^n]*Log[i + j*x] - 2*b^2*e*g*i*m*n*Log[c*(d + e*x)^n]*Log[i + j*
x] + 2*b^2*d*g*j*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] - 2*b^2*e*g*i*m*n*Log
[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x] + b^2*e*g*i*m*Log[c*(d + e*x)^n]
^2*Log[i + j*x] + 2*a*b*e*g*i*m*n*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*
j)] - 2*a*b*d*g*j*m*n*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] - 2*b^2*
e*g*i*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] + 2*b^2*d*g*j*m...
```

3.394.3 Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2879, 2863, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2879}$$

$$-2ben\int \frac{x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}dx-gjm\int \frac{x(a+b\log(c(d+ex)^n))^2}{i+jx}dx+x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-2ben\int \frac{x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}dx-gjm\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{j}-\frac{i(a+b\log(c(d+ex)^n))^2}{j(i+jx)}\right)dx+x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$\big\downarrow$ 2009

$$-2ben\int \frac{x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}dx-gjm\left(-\frac{2bin\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^2}-\frac{i\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^2}{j^2}+\frac{(d+ex)}{}\right.$$
$$x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$\big\downarrow$ 7293

$$-2ben\int \left(\frac{gx\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))}{d+ex}+\frac{fx(a+b\log(c(d+ex)^n))}{d+ex}\right)dx-gjm\left(-\frac{2bin\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^2}-\frac{i\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^2}{j^2}+\frac{(d+ex)}{}\right.$$
$$x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

$\big\downarrow$ 2009

$$-gjm\left(-\frac{2bin\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^2}-\frac{i\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^2}{j^2}+\frac{(d+e}{}\right.$$
$$2ben\left(-\frac{df(a+b\log(c(d+ex)^n))^2}{2be^2n}-\frac{dg\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))^2}{2be^2n}+\frac{dgm\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e^2}\right.$$
$$x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

3.394. $\int (a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$

output
```
x*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]) - 2*b*e*n*((a*f*
x)/e - (a*g*m*x)/e - (b*f*n*x)/e + (2*b*g*m*n*x)/e + (b*f*(d + e*x)*Log[c*
(d + e*x)^n])/e^2 - (b*g*m*(d + e*x)*Log[c*(d + e*x)^n])/e^2 - (d*f*(a + b
*Log[c*(d + e*x)^n])^2)/(2*b*e^2*n) + (g*i*m*(a + b*Log[c*(d + e*x)^n])*Lo
g[(e*(i + j*x))/(e*i - d*j)])/(e*j) + (d*g*m*(a + b*Log[c*(d + e*x)^n])^2*
Log[(e*(i + j*x))/(e*i - d*j)])/(2*b*e^2*n) - (b*g*n*(i + j*x)*Log[h*(i +
j*x)^m])/(e*j) + (b*d*g*n*Log[-((j*(d + e*x))/(e*i - d*j))]*Log[h*(i + j*x
)^m])/e^2 + (g*x*(a + b*Log[c*(d + e*x)^n])*Log[h*(i + j*x)^m])/e - (d*g*(
a + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j*x)^m])/(2*b*e^2*n) + (b*g*i*m*n*P
olyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(e*j) + (d*g*m*(a + b*Log[c*(d +
e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e^2 + (b*d*g*m*n*PolyLo
g[2, (e*(i + j*x))/(e*i - d*j)])/e^2 - (b*d*g*m*n*PolyLog[3, -((j*(d + e*x
))/(e*i - d*j))])/e^2) - g*j*m*((-2*a*b*n*x)/j + (2*b^2*n^2*x)/j - (2*b^2*
n*(d + e*x)*Log[c*(d + e*x)^n])/(e*j) + ((d + e*x)*(a + b*Log[c*(d + e*x)^
n])^2)/(e*j) - (i*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*
j)])/j^2 - (2*b*i*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/
(e*i - d*j))])/j^2 + (2*b^2*i*n^2*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))]
)/j^2)
```

3.394.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2879
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Simp[g*j*m   Int[x*((a
+ b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Simp[b*e*n*p   Int[x*(a + b*
Log[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.394. $\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))\,dx$

3.394.4 Maple [F]

$$\int (a+b\ln (c(ex+d)^n))^2(f+g\ln (h(jx+i)^m))\,dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
```

3.394.5 Fricas [F]

$$\int (a+b\log (c(d+ex)^n))^2 (f+g\log (h(i+jx)^m))\,dx$$
$$= \int (b\log ((ex+d)^n c)+a)^2(g\log ((jx+i)^m h)+f)\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="fr
icas")
```

output
```
integral(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f +
 (b^2*g*log((e*x + d)^n*c)^2 + 2*a*b*g*log((e*x + d)^n*c) + a^2*g)*log((j*
x + i)^m*h), x)
```

3.394.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^2 (f+g\log (h(i+jx)^m))\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.394.7 Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m)) \, dx = \int (b\log((ex+d)^n c)+a)^2 (g\log((jx+i)^m h)+f) \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="ma
xima")
```

output
```
-2*a*b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a^2*g*j*m*(x/j - i*log(j*x + i)/
j^2) + b^2*f*x*log((e*x + d)^n*c)^2 + 2*a*b*f*x*log((e*x + d)^n*c) + a^2*g
*x*log((j*x + i)^m*h) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*
c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2*f + a^2*f*x
+ ((b^2*e*g*i*m*log(j*x + i) - (j*m - j*log(h))*b^2*e*g*x)*log((e*x + d)^n
)^2 - (b^2*d*g*j*n^2*log(e*x + d)^2 - b^2*e*g*j*x*log((e*x + d)^n)^2 + (2*
(e*g*j*n - e*g*j*log(c))*a*b - (2*e*g*j*n^2 - 2*e*g*j*n*log(c) + e*g*j*log
(c)^2)*b^2)*x - 2*(a*b*d*g*j*n - (d*g*j*n^2 - d*g*j*n*log(c))*b^2)*log(e*x
 + d) - 2*(b^2*d*g*j*n*log(e*x + d) + (a*b*e*g*j - (e*g*j*n - e*g*j*log(c)
)*b^2)*x)*log((e*x + d)^n))*log((j*x + i)^m))/(e*j) - integrate(-(b^2*d*e*
g*i*j*log(c)^2*log(h) + 2*a*b*d*e*g*i*j*log(c)*log(h) + (2*(e^2*g*j^2*m*n
- (j^2*m - j^2*log(h))*e^2*g*log(c))*a*b - (2*e^2*g*j^2*m*n^2 - 2*e^2*g*j^
2*m*n*log(c) + (j^2*m - j^2*log(h))*e^2*g*log(c)^2)*b^2)*x^2 + (b^2*d*e*g*
j^2*m*n^2*x + b^2*d^2*g*j^2*m*n^2)*log(e*x + d)^2 + (2*(d*e*g*j^2*m*n + (e
^2*g*i*j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*log(c))*a*b - (2*d*e*g*j^2*m
*n^2 - 2*d*e*g*j^2*m*n*log(c) - (e^2*g*i*j*log(h) - (j^2*m - j^2*log(h))*d
*e*g)*log(c)^2)*b^2)*x - 2*(a*b*d^2*g*j^2*m*n - (d^2*g*j^2*m*n^2 - d^2*g*j
^2*m*n*log(c))*b^2 + (a*b*d*e*g*j^2*m*n - (d*e*g*j^2*m*n^2 - d*e*g*j^2*m*n
*log(c))*b^2)*x)*log(e*x + d) + 2*(b^2*d*e*g*i*j*log(c)*log(h) + a*b*d*e*g
*i*j*log(h) - ((j^2*m - j^2*log(h))*a*b*e^2*g + ((j^2*m - j^2*log(h))*e...
```

3.394.8 Giac [F]

$$\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m)) \, dx = \int (b\log((ex+d)^n c)+a)^2 (g\log((jx+i)^m h)+f) \, dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="gi
ac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^2 (f+g\log (h(i+jx)^m))\,dx = \int \left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2\,\left(f+g\,\ln\left(h\,{\left(i+j\,x\right)}^m\right)\right)\,dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)), x)
```

3.395 $\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$

3.395.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x,x)
```

3.395.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

output
```
Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x, x]
```

3.395.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2891}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}dx$$

$$\Big\downarrow \text{ 2891}$$

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}dx$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

output
```
$Aborted
```

3.395.3.1 Defintions of rubi rules used

rule 2891
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))^(q_.)*((k_.) + (l_.)*(x_))^(r_.),
 x_Symbol] :> Unintegrable[(k + l*x)^r*(a + b*Log[c*(d + e*x)^n])^p*(f + g*
Log[h*(i + j*x)^m])^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, p, q, r}, x]
```

3.395.4 Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^2(f+g\ln(h(jx+i)^m))}{x}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x,x)
```

3.395.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.74

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="
fricas")
```

output
```
integral((b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f
+ (b^2*g*log((e*x + d)^n*c)^2 + 2*a*b*g*log((e*x + d)^n*c) + a^2*g)*log((j
*x + i)^m*h))/x, x)
```

3.395.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m))/x,x)
```

output
```
Timed out
```

3.395.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.94

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="
maxima")
```

output
```
a^2*f*log(x) + integrate(((g*log(h) + f)*b^2*log((e*x + d)^n)^2 + (g*log(h
) + f)*b^2*log(c)^2 + 2*(g*log(h) + f)*a*b*log(c) + a^2*g*log(h) + 2*((g*l
og(h) + f)*b^2*log(c) + (g*log(h) + f)*a*b)*log((e*x + d)^n) + (b^2*g*log(
(e*x + d)^n)^2 + b^2*g*log(c)^2 + 2*a*b*g*log(c) + a^2*g + 2*(b^2*g*log(c)
 + a*b*g)*log((e*x + d)^n))*log((j*x + i)^m))/x, x)
```

3.395.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f)/x, x)
```

3.395.9 Mupad [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2\,(f+g\,\ln(h\,(i+j\,x)^m))}{x}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x, x)
```

3.396 $\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$

3.396.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

3.396.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

output
```
Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2, x
]
```

3.396.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2891}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}dx$$
$$\downarrow \text{2891}$$
$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}dx$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

output
```
$Aborted
```

3.396.3.1 Defintions of rubi rules used

rule 2891
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))^(q_.)*((k_.) + (l_.)*(x_))^(r_.),
 x_Symbol] :> Unintegrable[(k + l*x)^r*(a + b*Log[c*(d + e*x)^n])^p*(f + g*
Log[h*(i + j*x)^m])^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, p, q, r}, x]
```

3.396.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^2(f+g\ln(h(jx+i)^m))}{x^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

3.396.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.74

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm
="fricas")
```

output
```
integral((b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f
+ (b^2*g*log((e*x + d)^n*c)^2 + 2*a*b*g*log((e*x + d)^n*c) + a^2*g)*log((j
*x + i)^m*h))/x^2, x)
```

3.396.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m))/x**2,x)
```

output
```
Timed out
```

3.396.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 6.09

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm
="maxima")
```

output
```
-2*a*b*e*f*n*(log(e*x + d)/d - log(x)/d) - 2*a*b*f*log((e*x + d)^n*c)/x -
a^2*f/x + integrate(((g*log(h) + f)*b^2*log((e*x + d)^n)^2 + (g*log(h) + f
)*b^2*log(c)^2 + 2*a*b*g*log(c)*log(h) + a^2*g*log(h) + 2*((g*log(h) + f)*
b^2*log(c) + a*b*g*log(h))*log((e*x + d)^n) + (b^2*g*log((e*x + d)^n)^2 +
b^2*g*log(c)^2 + 2*a*b*g*log(c) + a^2*g + 2*(b^2*g*log(c) + a*b*g)*log((e*
x + d)^n))*log((j*x + i)^m))/x^2, x)
```

3.396.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm
="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f)/x^2, x)
```

3.396.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2\,(f+g\,\ln(h\,(i+j\,x)^m))}{x^2}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x^2,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x^2, x)
```

3.396. $\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$

3.397 $\int x(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))$

3.397.1 Optimal result

Integrand size = 32, antiderivative size = 2050

$$\int x(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx = \text{Too large to display}$$

output
```
1/2*x^2*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))+6*b^3*d*f*n^3*x/e+3/
8*b^3*g*m*n^3*(e*x+d)^2/e^2+1/2*d*g*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^2+
1/2*d^2*g*m*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/e^2-1/2*g*i^2
*m*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/j^2+3/4*b^2*g*n^2*x^2*
(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)-3/4*b*g*n*x^2*(a+b*ln(c*(e*x+d)^n))^
2*ln(h*(j*x+i)^m)-3/8*b^2*g*m*n^2*x^2*(a+b*ln(c*(e*x+d)^n))+3/4*b^2*f*n^2*
(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2-3/4*b*f*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^
n))^2/e^2-3/4*b^3*g*i^2*m*n^3*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2-21/4*b^
3*d^2*g*m*n^3*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^2+9/2*b^3*d^2*g*m*n^3*poly
log(3,-j*(e*x+d)/(-d*j+e*i))/e^2-3/2*b^3*g*i^2*m*n^3*polylog(3,-j*(e*x+d)/
(-d*j+e*i))/j^2+3*b^3*d^2*g*m*n^3*polylog(4,-j*(e*x+d)/(-d*j+e*i))/e^2-3*b
^3*g*i^2*m*n^3*polylog(4,-j*(e*x+d)/(-d*j+e*i))/j^2+3*b*d*f*n*(e*x+d)*(a+b
*ln(c*(e*x+d)^n))^2/e^2+3/4*b*g*m*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+
1/2*g*i*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e/j+3/8*b^3*g*i^2*m*n^3*ln(j*x+i
)/j^2-21/4*b^3*d^2*g*n^3*ln(-j*(e*x+d)/(-d*j+e*i))*ln(h*(j*x+i)^m)/e^2+9/4
*b*d^2*g*n*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e^2+12*a*b^2*d*g*m*n^2*
x/e+21/4*a*b^2*g*i*m*n^2*x/j+12*b^3*d*g*m*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2-
15/4*b*d*g*m*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2-3/4*b^2*g*i^2*m*n^2*(a+
b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^2-9/4*b*d^2*g*m*n*(a+b*ln(c*
(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/e^2+3/4*b*g*i^2*m*n*(a+b*ln(c*(e...
```

3.397.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4971 vs. 2(2050) = 4100.

Time = 1.32 (sec) , antiderivative size = 4971, normalized size of antiderivative = 2.42

$$\int x(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\, dx = \text{Result too large to show}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(-12*a^2*b*d*e*g*i*j*m*n + 12*a*b^2*d*e*g*i*j*m*n^2 + 24*a*b^2*d^2*g*j^2*m
*n^2 - 6*b^3*d*e*g*i*j*m*n^3 - 36*b^3*d^2*g*j^2*m*n^3 + 4*a^3*e^2*g*i*j*m*
x + 12*a^2*b*d*e*f*j^2*n*x - 18*a^2*b*e^2*g*i*j*m*n*x - 18*a^2*b*d*e*g*j^2
*m*n*x - 36*a*b^2*d*e*f*j^2*n^2*x + 42*a*b^2*e^2*g*i*j*m*n^2*x + 84*a*b^2*
d*e*g*j^2*m*n^2*x + 42*b^3*d*e*f*j^2*n^3*x - 45*b^3*e^2*g*i*j*m*n^3*x - 13
5*b^3*d*e*g*j^2*m*n^3*x + 4*a^3*e^2*f*j^2*x^2 - 2*a^3*e^2*g*j^2*m*x^2 - 6*
a^2*b*e^2*f*j^2*n*x^2 + 6*a^2*b*e^2*g*j^2*m*n*x^2 + 6*a*b^2*e^2*f*j^2*n^2*
x^2 - 9*a*b^2*e^2*g*j^2*m*n^2*x^2 - 3*b^3*e^2*f*j^2*n^3*x^2 + 6*b^3*e^2*g*
j^2*m*n^3*x^2 - 12*a^2*b*d^2*f*j^2*n*Log[d + e*x] + 12*a^2*b*d*e*g*i*j*m*n
*Log[d + e*x] + 6*a^2*b*d^2*g*j^2*m*n*Log[d + e*x] + 36*a*b^2*d^2*f*j^2*n^
2*Log[d + e*x] - 12*a*b^2*d*e*g*i*j*m*n^2*Log[d + e*x] - 48*a*b^2*d^2*g*j^
2*m*n^2*Log[d + e*x] - 42*b^3*d^2*f*j^2*n^3*Log[d + e*x] + 30*b^3*d*e*g*i*
j*m*n^3*Log[d + e*x] + 69*b^3*d^2*g*j^2*m*n^3*Log[d + e*x] + 12*a*b^2*d^2*
f*j^2*n^2*Log[d + e*x]^2 - 12*a*b^2*d*e*g*i*j*m*n^2*Log[d + e*x]^2 - 6*a*b
^2*d^2*g*j^2*m*n^2*Log[d + e*x]^2 - 18*b^3*d^2*f*j^2*n^3*Log[d + e*x]^2 +
6*b^3*d*e*g*i*j*m*n^3*Log[d + e*x]^2 + 24*b^3*d^2*g*j^2*m*n^3*Log[d + e*x]
^2 - 4*b^3*d^2*f*j^2*n^3*Log[d + e*x]^3 + 4*b^3*d*e*g*i*j*m*n^3*Log[d + e*
x]^3 + 2*b^3*d^2*g*j^2*m*n^3*Log[d + e*x]^3 - 24*a*b^2*d*e*g*i*j*m*n*Log[c
*(d + e*x)^n] + 12*b^3*d*e*g*i*j*m*n^2*Log[c*(d + e*x)^n] + 24*b^3*d^2*g*j
^2*m*n^2*Log[c*(d + e*x)^n] + 12*a^2*b*e^2*g*i*j*m*x*Log[c*(d + e*x)^n]...
```

3.397.3 Rubi [A] (verified)

Time = 7.34 (sec) , antiderivative size = 2278, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2889, 2863, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2889}$$

$$-\frac{3}{2}ben\int\frac{x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}dx-\frac{1}{2}gjm\int\frac{x^2(a+b\log(c(d+ex)^n))^3}{i+jx}dx+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-\frac{3}{2}ben\int\frac{x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}dx-\frac{1}{2}gjm\int\left(\frac{x(a+b\log(c(d+ex)^n))^3}{j}+\frac{i^2(a+b\log(c(d+ex)^n))^3}{j^2(i+jx)}-\frac{i(a+b\log(c(d+ex)^n))^3}{j^2}\right)dx+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}ben\int\frac{x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}dx-\frac{1}{2}gjm\left(\frac{3b^2n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2j}-\frac{6b^2i^2n^2\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^3}-\frac{6ab^2d}{ej}\right.$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{3}{2}ben\int\left(\frac{gx^2\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))^2}{d+ex}+\frac{fx^2(a+b\log(c(d+ex)^n))^2}{d+ex}\right)dx-\frac{1}{2}gjm\left(\frac{3b^2n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2j}-\frac{6b^2i^2n^2\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^3}-\frac{6ab^2d}{ej}\right.$$
$$\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(f+g\log(h(i+jx)^m))(a+b\log(c(d+ex)^n))^3-\frac{3}{2}ben\left(-\frac{d^2gm\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^3}{3be^3n}+\frac{d^2g\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))^3}{3be^3n}+\frac{d^2f(a+b\right.$$
$$\frac{1}{2}gjm\left(-\frac{3n^3(d+ex)^2b^3}{8e^2j}+\frac{6dn^3xb^3}{ej}+\frac{6in^3xb^3}{j^2}-\frac{6dn^2(d+ex)\log(c(d+ex)^n)b^3}{e^2j}-\frac{6in^2(d+ex)\log(c(d+ex)}{ej^2}\right.$$

3.397. $\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$

input
```
Int[x*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(x^2*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/2 - (3*b*e*n
*((4*a*b*d*f*n*x)/e^2 - (6*a*b*d*g*m*n*x)/e^2 - (3*a*b*g*i*m*n*x)/(2*e*j)
- (4*b^2*d*f*n^2*x)/e^2 + (39*b^2*d*g*m*n^2*x)/(4*e^2) + (7*b^2*g*i*m*n^2*
x)/(4*e*j) - (b^2*g*m*n^2*x^2)/(4*e) + (b^2*f*n^2*(d + e*x)^2)/(4*e^3) - (
b^2*g*m*n^2*(d + e*x)^2)/(8*e^3) - (b^2*d^2*g*m*n^2*Log[d + e*x])/(4*e^3)
+ (4*b^2*d*f*n*(d + e*x)*Log[c*(d + e*x)^n])/e^3 - (6*b^2*d*g*m*n*(d + e*x
)*Log[c*(d + e*x)^n])/e^3 - (3*b^2*g*i*m*n*(d + e*x)*Log[c*(d + e*x)^n])/(
2*e^2*j) + (b*g*m*n*x^2*(a + b*Log[c*(d + e*x)^n]))/(4*e) - (b*f*n*(d + e*
x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^3) + (b*g*m*n*(d + e*x)^2*(a + b*Log
[c*(d + e*x)^n]))/(4*e^3) - (2*d*f*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)
/e^3 + (3*d*g*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^3) + (g*i*m*(
d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2*j) + (f*(d + e*x)^2*(a + b*L
og[c*(d + e*x)^n])^2)/(2*e^3) - (g*m*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]
)^2)/(4*e^3) + (d^2*f*(a + b*Log[c*(d + e*x)^n])^3)/(3*b*e^3*n) - (b^2*g*i
^2*m*n^2*Log[i + j*x])/(4*e*j^2) + (b*g*i^2*m*n*(a + b*Log[c*(d + e*x)^n])
*Log[(e*(i + j*x))/(e*i - d*j)])/(2*e*j^2) + (3*b*d*g*i*m*n*(a + b*Log[c*(
d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(e^2*j) + (3*d^2*g*m*(a + b*L
og[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(2*e^3) - (g*i^2*m*(a
 + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(2*e*j^2) - (d*
g*i*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(e^2...
```

3.397.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.397.4 Maple [F]

$$\int x(a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))\,dx$$

input
```
int(x*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)
```

output
```
int(x*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)
```

3.397.5 Fricas [F]

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$=\int (b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="
fricas")
```

output
```
integral(b^3*f*x*log((e*x + d)^n*c)^3 + 3*a*b^2*f*x*log((e*x + d)^n*c)^2 +
 3*a^2*b*f*x*log((e*x + d)^n*c) + a^3*f*x + (b^3*g*x*log((e*x + d)^n*c)^3
+ 3*a*b^2*g*x*log((e*x + d)^n*c)^2 + 3*a^2*b*g*x*log((e*x + d)^n*c) + a^3*
g*x)*log((j*x + i)^m*h), x)
```

3.397.6 Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.397.7 Maxima [F]

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$= \int (b\log((ex+d)^n c)+a)^3(g\log((jx+i)^m h)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="
maxima")
```

output
```
1/2*b^3*f*x^2*log((e*x + d)^n*c)^3 + 3/2*a*b^2*f*x^2*log((e*x + d)^n*c)^2
- 3/4*a^2*b*e*f*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) - 1/4*a^3
*g*j*m*(2*i^2*log(j*x + i)/j^3 + (j*x^2 - 2*i*x)/j^2) + 3/2*a^2*b*f*x^2*lo
g((e*x + d)^n*c) + 1/2*a^3*g*x^2*log((j*x + i)^m*h) + 1/2*a^3*f*x^2 - 3/4*
(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) -
 (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*
a*b^2*f - 1/8*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e
*x + d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*x +
 d)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*log(e
*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n*log((e*x + d)^n*c)/e^3))*b^3*f
 + 1/8*(2*(2*b^3*e^2*g*i*j*m*x - 2*b^3*e^2*g*i^2*m*log(j*x + i) - (j^2*m -
 2*j^2*log(h))*b^3*e^2*g*x^2)*log((e*x + d)^n)^3 - (4*b^3*d^2*g*j^2*n^3*lo
g(e*x + d)^3 - 4*b^3*e^2*g*j^2*x^2*log((e*x + d)^n)^3 + (6*(e^2*g*j^2*n -
2*e^2*g*j^2*log(c))*a^2*b - 6*(e^2*g*j^2*n^2 - 2*e^2*g*j^2*n*log(c) + 2*e^
2*g*j^2*log(c)^2)*a*b^2 + (3*e^2*g*j^2*n^3 - 6*e^2*g*j^2*n^2*log(c) + 6*e^
2*g*j^2*n*log(c)^2 - 4*e^2*g*j^2*log(c)^3)*b^3)*x^2 - 6*(2*a*b^2*d^2*g*j^2
*n^2 - (3*d^2*g*j^2*n^3 - 2*d^2*g*j^2*n^2*log(c))*b^3)*log(e*x + d)^2 - 6*
(2*b^3*d*e*g*j^2*n*x - 2*b^3*d^2*g*j^2*n*log(e*x + d) + (2*a*b^2*e^2*g*j^2
 - (e^2*g*j^2*n - 2*e^2*g*j^2*log(c))*b^3)*x^2)*log((e*x + d)^n)^2 - 6*(2*
a^2*b*d*e*g*j^2*n - 2*(3*d*e*g*j^2*n^2 - 2*d*e*g*j^2*n*log(c))*a*b^2 + ...
```

3.397.8 Giac [F]

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)x\,dx$$

input
```
integrate(x*(a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f)*x, x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$=\int x\,(a+b\,\ln(c\,(d+e\,x)^n))^3\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int(x*(a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)), x)
```

3.398 $\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,d$

3.398.1 Optimal result

Integrand size = 31, antiderivative size = 1147

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx = \text{Too large to display}$$

output
```
-g*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e+d*g*(a+b*ln(c*(e*x+d)^n))^3*ln(h*(j
*x+i)^m)/e-d*g*m*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/e+g*i*m*
(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/j-18*a*b^2*g*m*n^2*x+6*b^
3*f*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-3*b*f*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/
e-6*b^3*g*n^3*(j*x+i)*ln(h*(j*x+i)^m)/j+6*b^2*g*n^2*x*(a+b*ln(c*(e*x+d)^n)
)*ln(h*(j*x+i)^m)-3*b*g*n*x*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)+6*a*b^
2*f*n^2*x+24*b^3*g*m*n^3*x-6*b^3*f*n^3*x+6*b^2*g*i*m*n^2*(a+b*ln(c*(e*x+d)
^n))*ln(e*(j*x+i)/(-d*j+e*i))/j+3*b*d*g*m*n*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(
j*x+i)/(-d*j+e*i))/e-3*b*g*i*m*n*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*
j+e*i))/j+6*b^2*d*g*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j
+e*i))/e-6*b^2*g*i*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+
e*i))/j-3*b*d*g*m*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-j*(e*x+d)/(-d*j+e*i
))/e+3*b*g*i*m*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/
j+6*b^2*d*g*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e
-6*b^2*g*i*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j+
6*b^3*g*i*m*n^3*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j+6*b^3*d*g*m*n^3*polylog
(2,e*(j*x+i)/(-d*j+e*i))/e-6*b^3*d*g*m*n^3*polylog(3,-j*(e*x+d)/(-d*j+e*i)
)/e+6*b^3*g*i*m*n^3*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j-6*b^3*d*g*m*n^3*pol
ylog(4,-j*(e*x+d)/(-d*j+e*i))/e+6*b^3*g*i*m*n^3*polylog(4,-j*(e*x+d)/(-d*j
+e*i))/j+6*b^3*d*g*n^3*ln(-j*(e*x+d)/(-d*j+e*i))*ln(h*(j*x+i)^m)/e-3*b*...
```

3.398.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3163 vs. 2(1147) = 2294.

Time = 0.65 (sec) , antiderivative size = 3163, normalized size of antiderivative = 2.76

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m)) \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

output
```
(-3*a^2*b*d*f*j*n + 3*a^2*b*d*g*j*m*n - 6*a*b^2*d*g*j*m*n^2 + 6*b^3*d*g*j*
m*n^3 + a^3*e*f*j*x - a^3*e*g*j*m*x - 3*a^2*b*e*f*j*n*x + 6*a^2*b*e*g*j*m*
n*x + 6*a*b^2*e*f*j*n^2*x - 18*a*b^2*e*g*j*m*n^2*x - 6*b^3*e*f*j*n^3*x + 2
4*b^3*e*g*j*m*n^3*x + 3*a^2*b*d*f*j*n*Log[d + e*x] - 3*a^2*b*d*g*j*m*n*Log
[d + e*x] + 6*a*b^2*d*g*j*m*n^2*Log[d + e*x] + 6*b^3*d*f*j*n^3*Log[d + e*x
] - 12*b^3*d*g*j*m*n^3*Log[d + e*x] - 3*a*b^2*d*f*j*n^2*Log[d + e*x]^2 + 3
*a*b^2*d*g*j*m*n^2*Log[d + e*x]^2 - 3*b^3*d*g*j*m*n^3*Log[d + e*x]^2 + b^3
*d*f*j*n^3*Log[d + e*x]^3 - b^3*d*g*j*m*n^3*Log[d + e*x]^3 - 6*a*b^2*d*f*j
*n*Log[c*(d + e*x)^n] + 6*a*b^2*d*g*j*m*n*Log[c*(d + e*x)^n] - 6*b^3*d*g*j
*m*n^2*Log[c*(d + e*x)^n] + 3*a^2*b*e*f*j*x*Log[c*(d + e*x)^n] - 3*a^2*b*e
*g*j*m*x*Log[c*(d + e*x)^n] - 6*a*b^2*e*f*j*n*x*Log[c*(d + e*x)^n] + 12*a*
b^2*e*g*j*m*n*x*Log[c*(d + e*x)^n] + 6*b^3*e*f*j*n^2*x*Log[c*(d + e*x)^n]
- 18*b^3*e*g*j*m*n^2*x*Log[c*(d + e*x)^n] + 6*a*b^2*d*f*j*n*Log[d + e*x]*L
og[c*(d + e*x)^n] - 6*a*b^2*d*g*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] + 6*
b^3*d*g*j*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n] - 3*b^3*d*f*j*n^2*Log[d +
e*x]^2*Log[c*(d + e*x)^n] + 3*b^3*d*g*j*m*n^2*Log[d + e*x]^2*Log[c*(d + e*
x)^n] - 3*b^3*d*f*j*n*Log[c*(d + e*x)^n]^2 + 3*b^3*d*g*j*m*n*Log[c*(d + e*
x)^n]^2 + 3*a*b^2*e*f*j*x*Log[c*(d + e*x)^n]^2 - 3*a*b^2*e*g*j*m*x*Log[c*(
d + e*x)^n]^2 - 3*b^3*e*f*j*n*x*Log[c*(d + e*x)^n]^2 + 6*b^3*e*g*j*m*n*x*L
og[c*(d + e*x)^n]^2 + 3*b^3*d*f*j*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2 -...
```

3.398.3 Rubi [A] (verified)

Time = 3.71 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2879, 2863, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx$$

$$\Big\downarrow \text{2879}$$

$$-3ben\int \frac{x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}dx-gjm\int \frac{x(a+b\log(c(d+ex)^n))^3}{i+jx}dx+x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2863}$$

$$-3ben\int \frac{x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}dx-gjm\int \left(\frac{(a+b\log(c(d+ex)^n))^3}{j}-\frac{i(a+b\log(c(d+ex)^n))^3}{j(i+jx)}\right)dx+x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$-3ben\int \frac{x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}dx-gjm\left(\frac{6b^2in^2\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^2}+\frac{6ab^2n^2x}{j}-\frac{3bin\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(\right.$$
$$x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{7293}$$

$$-3ben\int \left(\frac{gx\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))^2}{d+ex}+\frac{fx(a+b\log(c(d+ex)^n))^2}{d+ex}\right)dx-gjm\left(\frac{6b^2in^2\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j^2}+\frac{6ab^2n^2x}{j}-\frac{3bin\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(\right.$$
$$x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$

$$\Big\downarrow \text{2009}$$

$$x(f+g\log(h(i+jx)^m))(a+b\log(c(d+ex)^n))^3-3ben\left(\frac{dgm\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^3}{3be^2n}-\frac{dg\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))^3}{3be^2n}-\frac{df(a+b\log(c\right.}{3be}$$
$$gjm\left(-\frac{6n^3xb^3}{j}+\frac{6n^2(d+ex)\log(c(d+ex)^n)b^3}{ej}-\frac{6in^3\operatorname{PolyLog}\left(4,-\frac{j(d+ex)}{ei-dj}\right)b^3}{j^2}+\frac{6an^2xb^2}{j}+\frac{6in^2(a+b\log(c\right.}{}$$

input
```
Int[(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

3.398. $\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx$

output
```
x*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]) - 3*b*e*n*((-2*a
*b*f*n*x)/e + (4*a*b*g*m*n*x)/e + (2*b^2*f*n^2*x)/e - (6*b^2*g*m*n^2*x)/e
- (2*b^2*f*n*(d + e*x)*Log[c*(d + e*x)^n])/e^2 + (4*b^2*g*m*n*(d + e*x)*Lo
g[c*(d + e*x)^n])/e^2 + (f*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 - (
g*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 - (d*f*(a + b*Log[c*(d + e
*x)^n])^3)/(3*b*e^2*n) - (2*b*g*i*m*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i
 + j*x))/(e*i - d*j)])/(e*j) - (d*g*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*
(i + j*x))/(e*i - d*j)])/e^2 + (g*i*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*
(i + j*x))/(e*i - d*j)])/(e*j) + (d*g*m*(a + b*Log[c*(d + e*x)^n])^3*Log[(
e*(i + j*x))/(e*i - d*j)])/(3*b*e^2*n) + (2*b^2*g*n^2*(i + j*x)*Log[h*(i +
 j*x)^m])/(e*j) - (2*b^2*d*g*n^2*Log[-((j*(d + e*x))/(e*i - d*j))]*Log[h*(
i + j*x)^m])/e^2 - (2*b*g*n*x*(a + b*Log[c*(d + e*x)^n])*Log[h*(i + j*x)^m
])/e + (d*g*(a + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j*x)^m])/e^2 + (g*x*(a
 + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j*x)^m])/e - (d*g*(a + b*Log[c*(d +
e*x)^n])^3*Log[h*(i + j*x)^m])/(3*b*e^2*n) - (2*b^2*g*i*m*n^2*PolyLog[2, -
((j*(d + e*x))/(e*i - d*j))])/(e*j) - (2*b*d*g*m*n*(a + b*Log[c*(d + e*x)^
n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e^2 + (2*b*g*i*m*n*(a + b*Lo
g[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(e*j) + (d*g*m
*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e^
2 - (2*b^2*d*g*m*n^2*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/e^2 + (2*b^...
```

3.398.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2879
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Simp[g*j*m   Int[x*((a
+ b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Simp[b*e*n*p   Int[x*(a + b*
Log[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

3.398. $\int (a+b\log(c(d+ex)^n))^3\,(f+g\log(h(i+jx)^m))\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.398.4 Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))\,dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)
```

3.398.5 Fricas [F]

$$\int (a+b\log(c(d+ex)^n))^3\,(f+g\log(h(i+jx)^m))\,dx$$
$$=\int (b\log((ex+d)^n c)+a)^3(g\log((jx+i)^m h)+f)\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="fr
icas")
```

output
```
integral(b^3*f*log((e*x + d)^n*c)^3 + 3*a*b^2*f*log((e*x + d)^n*c)^2 + 3*a
^2*b*f*log((e*x + d)^n*c) + a^3*f + (b^3*g*log((e*x + d)^n*c)^3 + 3*a*b^2*
g*log((e*x + d)^n*c)^2 + 3*a^2*b*g*log((e*x + d)^n*c) + a^3*g)*log((j*x +
i)^m*h), x)
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^3\,(f+g\log(h(i+jx)^m))\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m)),x)
```

output
```
Timed out
```

3.398.7 Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)^3 (g\log((jx+i)^m h)+f)\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="ma
xima")
```

output
```
b^3*f*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*f*n*(x/e - d*log(e*x + d)/e^2) -
a^3*g*j*m*(x/j - i*log(j*x + i)/j^2) + 3*a*b^2*f*x*log((e*x + d)^n*c)^2 +
3*a^2*b*f*x*log((e*x + d)^n*c) + a^3*g*x*log((j*x + i)^m*h) - 3*(2*e*n*(x/
e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2
*d*log(e*x + d))*n^2/e)*a*b^2*f - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e
*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d
*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n
*log((e*x + d)^n*c)/e^2))*b^3*f + a^3*f*x + ((b^3*e*g*i*m*log(j*x + i) - (
j*m - j*log(h))*b^3*e*g*x)*log((e*x + d)^n)^3 + (b^3*d*g*j*n^3*log(e*x + d
)^3 + b^3*e*g*j*x*log((e*x + d)^n)^3 - 3*(a*b^2*d*g*j*n^2 - (d*g*j*n^3 - d
*g*j*n^2*log(c))*b^3)*log(e*x + d)^2 + 3*(b^3*d*g*j*n*log(e*x + d) + (a*b^
2*e*g*j - (e*g*j*n - e*g*j*log(c))*b^3)*x)*log((e*x + d)^n)^2 - (3*(e*g*j*
n - e*g*j*log(c))*a^2*b - 3*(2*e*g*j*n^2 - 2*e*g*j*n*log(c) + e*g*j*log(c)
^2)*a*b^2 + (6*e*g*j*n^3 - 6*e*g*j*n^2*log(c) + 3*e*g*j*n*log(c)^2 - e*g*j
*log(c)^3)*b^3)*x + 3*(a^2*b*d*g*j*n - 2*(d*g*j*n^2 - d*g*j*n*log(c))*a*b^
2 + (2*d*g*j*n^3 - 2*d*g*j*n^2*log(c) + d*g*j*n*log(c)^2)*b^3)*log(e*x + d
) - 3*(b^3*d*g*j*n^2*log(e*x + d)^2 - (a^2*b*e*g*j - 2*(e*g*j*n - e*g*j*lo
g(c))*a*b^2 + (2*e*g*j*n^2 - 2*e*g*j*n*log(c) + e*g*j*log(c)^2)*b^3)*x - 2
*(a*b^2*d*g*j*n - (d*g*j*n^2 - d*g*j*n*log(c))*b^3)*log(e*x + d))*log((e*x
 + d)^n))*log((j*x + i)^m))/(e*j) - integrate(-(b^3*d*e*g*i*j*log(c)^3*...
```

3.398.8 Giac [F]

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)^3 (g\log((jx+i)^m h)+f)\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="gi
ac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f), x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^3 (f+g\log (h(i+jx)^m))\ dx$$
$$=\int (a+b\ \ln (c\,(d+e\,x)^n))^3\,(f+g\ \ln (h\,(i+j\,x)^m))\ dx$$

input
```
int((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)),x)
```

output
```
int((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)), x)
```

3.399 $\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$

3.399.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x,x)
```

3.399.2 Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

output
```
Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x, x]
```

3.399.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2891}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}dx$$
$$\Big\downarrow \text{2891}$$
$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}dx$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

output
```
$Aborted
```

3.399.3.1 Defintions of rubi rules used

rule 2891
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))^(q_.)*((k_.) + (l_.)*(x_))^(r_.),
 x_Symbol] :> Unintegrable[(k + l*x)^r*(a + b*Log[c*(d + e*x)^n])^p*(f + g*
Log[h*(i + j*x)^m])^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, p, q, r}, x]
```

3.399.4 Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))}{x}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x,x)
```

3.399.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.97

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$$
$$= \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="
fricas")
```

output
```
integral((b^3*f*log((e*x + d)^n*c)^3 + 3*a*b^2*f*log((e*x + d)^n*c)^2 + 3*
a^2*b*f*log((e*x + d)^n*c) + a^3*f + (b^3*g*log((e*x + d)^n*c)^3 + 3*a*b^2
*g*log((e*x + d)^n*c)^2 + 3*a^2*b*g*log((e*x + d)^n*c) + a^3*g)*log((j*x +
 i)^m*h))/x, x)
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m))/x,x)
```

output
```
Timed out
```

3.399.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 292, normalized size of antiderivative = 8.59

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$$
$$= \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="
maxima")
```

output
```
a^3*f*log(x) + integrate(((g*log(h) + f)*b^3*log((e*x + d)^n)^3 + (g*log(h
) + f)*b^3*log(c)^3 + 3*(g*log(h) + f)*a*b^2*log(c)^2 + 3*(g*log(h) + f)*a
^2*b*log(c) + a^3*g*log(h) + 3*((g*log(h) + f)*b^3*log(c) + (g*log(h) + f)
*a*b^2)*log((e*x + d)^n)^2 + 3*((g*log(h) + f)*b^3*log(c)^2 + 2*(g*log(h)
+ f)*a*b^2*log(c) + (g*log(h) + f)*a^2*b)*log((e*x + d)^n) + (b^3*g*log((e
*x + d)^n)^3 + b^3*g*log(c)^3 + 3*a*b^2*g*log(c)^2 + 3*a^2*b*g*log(c) + a^
3*g + 3*(b^3*g*log(c) + a*b^2*g)*log((e*x + d)^n)^2 + 3*(b^3*g*log(c)^2 +
2*a*b^2*g*log(c) + a^2*b*g)*log((e*x + d)^n))*log((j*x + i)^m))/x, x)
```

3.399.8 Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="
giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f)/x, x)
```

3.399.9 Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3\,(f+g\,\ln(h\,(i+j\,x)^m))}{x}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x, x)
```

3.399. $\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$

3.400 $\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$

3.400.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2},x\right)$$

output
```
Unintegrable((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

3.400.2 Mathematica [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

output
```
Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2, x
]
```

3.400.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2891}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}dx$$
$$\Big\downarrow \text{2891}$$
$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}dx$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

output
```
$Aborted
```

3.400.3.1 Defintions of rubi rules used

rule 2891
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))^(q_.)*((k_.) + (l_.)*(x_))^(r_.),
 x_Symbol] :> Unintegrable[(k + l*x)^r*(a + b*Log[c*(d + e*x)^n])^p*(f + g*
Log[h*(i + j*x)^m])^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, p, q, r}, x]
```

3.400.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))}{x^2}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

3.400.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.97

$$\int \frac{(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3(g\log((jx+i)^m h)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm
="fricas")
```

output
```
integral((b^3*f*log((e*x + d)^n*c)^3 + 3*a*b^2*f*log((e*x + d)^n*c)^2 + 3*
a^2*b*f*log((e*x + d)^n*c) + a^3*f + (b^3*g*log((e*x + d)^n*c)^3 + 3*a*b^2
*g*log((e*x + d)^n*c)^2 + 3*a^2*b*g*log((e*x + d)^n*c) + a^3*g)*log((j*x +
 i)^m*h))/x^2, x)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m))/x**2,x)
```

output
```
Timed out
```

3.400.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 335, normalized size of antiderivative = 9.85

$$\int \frac{(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3(g\log((jx+i)^m h)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm
="maxima")
```

output
```
-3*a^2*b*e*f*n*(log(e*x + d)/d - log(x)/d) - 3*a^2*b*f*log((e*x + d)^n*c)/
x - a^3*f/x + integrate(((g*log(h) + f)*b^3*log((e*x + d)^n)^3 + (g*log(h)
 + f)*b^3*log(c)^3 + 3*(g*log(h) + f)*a*b^2*log(c)^2 + 3*a^2*b*g*log(c)*lo
g(h) + a^3*g*log(h) + 3*((g*log(h) + f)*b^3*log(c) + (g*log(h) + f)*a*b^2)
*log((e*x + d)^n)^2 + 3*((g*log(h) + f)*b^3*log(c)^2 + 2*(g*log(h) + f)*a*
b^2*log(c) + a^2*b*g*log(h))*log((e*x + d)^n) + (b^3*g*log((e*x + d)^n)^3
+ b^3*g*log(c)^3 + 3*a*b^2*g*log(c)^2 + 3*a^2*b*g*log(c) + a^3*g + 3*(b^3*
g*log(c) + a*b^2*g)*log((e*x + d)^n)^2 + 3*(b^3*g*log(c)^2 + 2*a*b^2*g*log
(c) + a^2*b*g)*log((e*x + d)^n))*log((j*x + i)^m))/x^2, x)
```

3.400.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm
="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f)/x^2, x)
```

3.400.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3\,(f+g\,\ln(h\,(i+j\,x)^m))}{x^2}\,dx$$

input
```
int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x^2,x)
```

output
```
int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x^2, x)
```

3.400. $\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$

3.401 $\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx$

3.401.1 Optimal result

Integrand size = 40, antiderivative size = 66

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = -\frac{(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{e}+\frac{bn\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{e}$$

output
```
-(a+b*ln(c*(e*x+d)^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/e+b*n*polylog(3,-g
*(e*x+d)/(-d*g+e*f))/e
```

3.401.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \frac{-\left((a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)\right)+bn\,\text{PolyLog}\left(3,\frac{g(d+ex)}{-ef+dg}\right)}{e}$$

input
```
Integrate[((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(d +
 e*x),x]
```

output
```
(-((a + b*Log[c*(d + e*x)^n])*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) +
b*n*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)])/e
```

3.401.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{e(f+gx)}{ef-dg}\right) (a+b\log (c(d+ex)^n))}{d+ex} \, dx$$

$$\Big\downarrow \; 2881$$

$$\frac{\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(f-\frac{dg}{e}\right)+g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)}{e}$$

$$\Big\downarrow \; 2821$$

$$\frac{bn\int \frac{\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{d+ex}d(d+ex)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log (c(d+ex)^n))}{e}$$

$$\Big\downarrow \; 7143$$

$$\frac{bn\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)-\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log (c(d+ex)^n))}{e}$$

input
```
Int[((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(d + e*x),
x]
```

output
```
(-((a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))]) +
b*n*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/e
```

3.401.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.401.4 Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))\ln\left(\frac{e(gx+f)}{-dg+ef}\right)}{ex+d}dx$$

input
```
int((a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x)
```

output
```
int((a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x)
```

3.401.5 Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log\left(\frac{(gx+f)e}{ef-dg}\right)}{ex+d}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*log(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x, algo
rithm="fricas")
```

output
```
integral((b*log((e*x + d)^n*c)*log((e*g*x + e*f)/(e*f - d*g)) + a*log((e*g
*x + e*f)/(e*f - d*g)))/(e*x + d), x)
```

3.401.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*(e*x+d)**n))*ln(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.401.7 Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log\left(\frac{(gx+f)e}{ef-dg}\right)}{ex+d}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*log(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x, algo
rithm="maxima")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log((g*x + f)*e/(e*f - d*g))/(e*x + d
), x)
```

3.401.8 Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log\left(\frac{(gx+f)e}{ef-dg}\right)}{ex+d}\,dx$$

input
```
integrate((a+b*log(c*(e*x+d)^n))*log(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x, algo
rithm="giac")
```

output
```
integrate((b*log((e*x + d)^n*c) + a)*log((g*x + f)*e/(e*f - d*g))/(e*x + d
), x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx=\int \frac{\ln\left(-\frac{e\,(f+g\,x)}{d\,g-e\,f}\right)\,(a+b\,\ln\left(c\,(d+e\,x)^n\right))}{d+e\,x}\,dx$$

input
```
int((log(-(e*(f + g*x))/(d*g - e*f))*(a + b*log(c*(d + e*x)^n)))/(d + e*x)
,x)
```

output
```
int((log(-(e*(f + g*x))/(d*g - e*f))*(a + b*log(c*(d + e*x)^n)))/(d + e*x)
, x)
```

3.402 $\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2} dx$

3.402.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2} dx = -\frac{b}{e(d+ex)} - \frac{b\log(c(d+ex))}{e(d+ex)} - \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{e(d+ex)} - \frac{a+b+b\log(c(d+ex))}{e(d+ex)}$$

output
```
-b/e/(e*x+d)-b*ln(c*(e*x+d))/e/(e*x+d)-ln(c*(e*x+d))*(a+b*ln(c*(e*x+d)))/e
/(e*x+d)+(-a-b-b*ln(c*(e*x+d)))/e/(e*x+d)
```

3.402.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2} dx = -\frac{a+2b+(a+2b)\log(c(d+ex))+b\log^2(c(d+ex))}{e(d+ex)}$$

input
```
Integrate[(Log[c*(d + e*x)]*(a + b*Log[c*(d + e*x)]))/(d + e*x)^2,x]
```

output
```
-((a + 2*b + (a + 2*b)*Log[c*(d + e*x)] + b*Log[c*(d + e*x)]^2)/(e*(d + e*
x)))
```

3.402.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2816, 27, 2813, 25, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}dx$$

$$\Big\downarrow \text{2816}$$

$$\frac{\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}d(c(d+ex))}{ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{c^2(d+ex)^2}d(c(d+ex))}{e}$$

$$\Big\downarrow \text{2813}$$

$$\frac{c\left(-\int -\frac{a+b+b\log(c(d+ex))}{c^2(d+ex)^2}d(c(d+ex))-\frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{c(d+ex)}-\frac{b\log(c(d+ex))}{c(d+ex)}\right)}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(\int \frac{a+b+b\log(c(d+ex))}{c^2(d+ex)^2}d(c(d+ex))-\frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{c(d+ex)}-\frac{b\log(c(d+ex))}{c(d+ex)}\right)}{e}$$

$$\Big\downarrow \text{2741}$$

$$\frac{c\left(-\frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{c(d+ex)}-\frac{a+b\log(c(d+ex))+b}{c(d+ex)}-\frac{b}{c(d+ex)}-\frac{b\log(c(d+ex))}{c(d+ex)}\right)}{e}$$

input
```
Int[(Log[c*(d + e*x)]*(a + b*Log[c*(d + e*x)]))/(d + e*x)^2,x]
```

output
```
(c*(-(b/(c*(d + e*x))) - (b*Log[c*(d + e*x)])/(c*(d + e*x)) - (Log[c*(d +
e*x)]*(a + b*Log[c*(d + e*x)]))/(c*(d + e*x)) - (a + b + b*Log[c*(d + e*x)
])/(c*(d + e*x))))/e
```

3.402.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 2816
```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.)*(u_)^(m_.),
 x_Symbol] :> With[{e = Coeff[u, x, 0], f = Coeff[u, x, 1], g = Coeff[v, x,
 0], h = Coeff[v, x, 1]}, Simp[1/h   Subst[Int[(f*(x/h))^m*(a + b*Log[x])^p
*(c + d*Log[x])^q, x], x, v], x] /; EqQ[f*g - e*h, 0] && NeQ[g, 0]] /; Free
Q[{a, b, c, d, m, p, q}, x] && LinearQ[{u, v}, x]
```

3.402. $\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx$

3.402.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

method	result	size
norman	$\frac{-\frac{a+2b}{e}-\frac{b\ln(c(ex+d))^2}{e}-\frac{(a+2b)\ln(c(ex+d))}{e}}{ex+d}$	54
parallelrisch	$\frac{-\ln(c(ex+d))^2 b\,e^2-\ln(c(ex+d))a\,e^2-2\ln(c(ex+d))b\,e^2-a\,e^2-2e^2b}{(ex+d)e^3}$	69
risch	$-\frac{b\ln(c(ex+d))^2}{e(ex+d)}-\frac{(a+2b)\ln(c(ex+d))}{e(ex+d)}-\frac{a}{e(ex+d)}-\frac{2b}{e(ex+d)}$	76
parts	$\frac{ac\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)}{e}+\frac{bc\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{e}$	105
derivativedivides	$\frac{c^2a\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2b\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$	110
default	$\frac{c^2a\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2b\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$	110

input
```
int(ln(c*(e*x+d))*(a+b*ln(c*(e*x+d)))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(a+2*b)/e-b/e*ln(c*(e*x+d))^2-(a+2*b)/e*ln(c*(e*x+d)))/(e*x+d)
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{b\log{(cex+cd)}^2+(a+2\,b)\log{(cex+cd)}+a+2\,b}{e^2x+de}$$

input
```
integrate(log(c*(e*x+d))*(a+b*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm="fric
as")
```

output
```
-(b*log(c*e*x + c*d)^2 + (a + 2*b)*log(c*e*x + c*d) + a + 2*b)/(e^2*x + d*
e)
```

3.402.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{b\log\left(c(d+ex)\right)^2}{de+e^2x} + \frac{(-a-2b)\log\left(c(d+ex)\right)}{de+e^2x} - \frac{a+2b}{de+e^2x}$$

input
```
integrate(ln(c*(e*x+d))*(a+b*ln(c*(e*x+d)))/(e*x+d)**2,x)
```

output
```
-b*log(c*(d + e*x))**2/(d*e + e**2*x) + (-a - 2*b)*log(c*(d + e*x))/(d*e +
 e**2*x) - (a + 2*b)/(d*e + e**2*x)
```

3.402.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\left(b\left(\frac{ce}{ce^3x+cde^2} + \frac{\log\left(cex+cd\right)}{e^2x+de}\right) + \frac{a}{e^2x+de}\right)\log\left((ex+d)c\right) - \frac{(b(\log\left(c\right)+2) + b\log\left(ex+d\right)+a)e}{e^3x+de^2}$$

input
```
integrate(log(c*(e*x+d))*(a+b*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm="maxi
ma")
```

output
```
-(b*(c*e/(c*e^3*x + c*d*e^2) + log(c*e*x + c*d)/(e^2*x + d*e)) + a/(e^2*x
+ d*e))*log((e*x + d)*c) - (b*(log(c) + 2) + b*log(e*x + d) + a)*e/(e^3*x
+ d*e^2)
```

3.402.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{e\left(\frac{bc\log((ex+d)c)^2}{(ex+d)e^2} + \frac{(ac^2+2\,bc^2)\log((ex+d)c)}{(ex+d)ce^2} + \frac{ac^2+2\,bc^2}{(ex+d)ce^2}\right)}{c}$$

input
```
integrate(log(c*(e*x+d))*(a+b*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm="giac
")
```

output
```
-e*(b*c*log((e*x + d)*c)^2/((e*x + d)*e^2) + (a*c^2 + 2*b*c^2)*log((e*x +
d)*c)/((e*x + d)*c*e^2) + (a*c^2 + 2*b*c^2)/((e*x + d)*c*e^2))/c
```

3.402.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{d\left(b\ln\left(c\left(d+e\,x\right)\right)^2 + a\,\ln\left(c\left(d+e\,x\right)\right) + 2\,b\,\ln\left(c\left(d+e\,x\right)\right)\right) - e\left(a\,x+2\,b\,x\right)}{d\,e\,\left(d+e\,x\right)}$$

input
```
int((log(c*(d + e*x))*(a + b*log(c*(d + e*x))))/(d + e*x)^2,x)
```

output
```
-(d*(b*log(c*(d + e*x))^2 + a*log(c*(d + e*x)) + 2*b*log(c*(d + e*x))) - e
*(a*x + 2*b*x))/(d*e*(d + e*x))
```

3.403 $\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx$

3.403.1 Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{bg}{e(d+ex)}-\frac{g(a+b+b\log(c(d+ex)))}{e(d+ex)}-\frac{b(f+g\log(c(d+ex)))}{e(d+ex)} - \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{e(d+ex)}$$

output
```
-b*g/e/(e*x+d)-g*(a+b+b*ln(c*(e*x+d)))/e/(e*x+d)-b*(f+g*ln(c*(e*x+d)))/e/(
e*x+d)-(a+b*ln(c*(e*x+d)))*(f+g*ln(c*(e*x+d)))/e/(e*x+d)
```

3.403.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{a(f+g)+b(f+2g)+(ag+b(f+2g))\log(c(d+ex))+bg\log^2(c(d+ex))}{e(d+ex)}$$

input
```
Integrate[((a + b*Log[c*(d + e*x)])*(f + g*Log[c*(d + e*x)]))/(d + e*x)^2,
x]
```

output
```
-((a*(f + g) + b*(f + 2*g) + (a*g + b*(f + 2*g))*Log[c*(d + e*x)] + b*g*Lo
g[c*(d + e*x)]^2)/(e*(d + e*x)))
```

3.403.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2816, 27, 2813, 25, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d+ex)))(g\log(c(d+ex))+f)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{2816}$$

$$\frac{\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}d(c(d+ex))}{ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{c^2(d+ex)^2}d(c(d+ex))}{e}$$

$$\Big\downarrow \text{2813}$$

$$\frac{c\left(-g\int -\frac{a+b+b\log(c(d+ex))}{c^2(d+ex)^2}d(c(d+ex))-\frac{(a+b\log(c(d+ex)))(g\log(c(d+ex))+f)}{c(d+ex)}-\frac{b(g\log(c(d+ex))+f)}{c(d+ex)}\right)}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(g\int \frac{a+b+b\log(c(d+ex))}{c^2(d+ex)^2}d(c(d+ex))-\frac{(a+b\log(c(d+ex)))(g\log(c(d+ex))+f)}{c(d+ex)}-\frac{b(g\log(c(d+ex))+f)}{c(d+ex)}\right)}{e}$$

$$\Big\downarrow \text{2741}$$

$$\frac{c\left(-\frac{(a+b\log(c(d+ex)))(g\log(c(d+ex))+f)}{c(d+ex)}+g\left(-\frac{a+b\log(c(d+ex))+b}{c(d+ex)}-\frac{b}{c(d+ex)}\right)-\frac{b(g\log(c(d+ex))+f)}{c(d+ex)}\right)}{e}$$

input
```
Int[((a + b*Log[c*(d + e*x)])*(f + g*Log[c*(d + e*x)]))/(d + e*x)^2,x]
```

output
```
(c*(-((b*(f + g*Log[c*(d + e*x)]))/(c*(d + e*x))) - ((a + b*Log[c*(d + e*x
)])*(f + g*Log[c*(d + e*x)]))/(c*(d + e*x)) + g*(-(b/(c*(d + e*x))) - (a +
 b + b*Log[c*(d + e*x)])/(c*(d + e*x)))))/e
```

3.403.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 2816
```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.)*(u_)^(m_.),
 x_Symbol] :> With[{e = Coeff[u, x, 0], f = Coeff[u, x, 1], g = Coeff[v, x,
 0], h = Coeff[v, x, 1]}, Simp[1/h   Subst[Int[(f*(x/h))^m*(a + b*Log[x])^p
*(c + d*Log[x])^q, x], x, v], x] /; EqQ[f*g - e*h, 0] && NeQ[g, 0]] /; Free
Q[{a, b, c, d, m, p, q}, x] && LinearQ[{u, v}, x]
```

3.403. $\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx$

3.403.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

method	result
norman	$\frac{-\frac{af+ag+bf+2bg}{e}-\frac{(ag+bf+2bg)\ln(c(ex+d))}{e}-\frac{bg\ln(c(ex+d))^2}{e}}{ex+d}$
parallelrisch	$\frac{-\ln(c(ex+d))^2 b\,e^2 g-\ln(c(ex+d))a\,e^2 g-\ln(c(ex+d))b\,e^2 f-2\ln(c(ex+d))b\,e^2 g-a\,e^2 f-a\,e^2 g-b\,e^2 f-2b\,e^2 g}{(ex+d)e^3}$
risch	$-\frac{bg\ln(c(ex+d))^2}{e(ex+d)}-\frac{(ag+bf+2bg)\ln(c(ex+d))}{e(ex+d)}-\frac{af}{e(ex+d)}-\frac{ag}{e(ex+d)}-\frac{bf}{e(ex+d)}-\frac{2bg}{e(ex+d)}$
parts	$-\frac{af}{e(ex+d)}+\frac{(ag+bf)c\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)}{e}+\frac{bgc\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{e}$
derivativedivides	$\frac{-\frac{c^2af}{cex+cd}+c^2ag\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bf\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bg\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$
default	$\frac{-\frac{c^2af}{cex+cd}+c^2ag\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bf\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bg\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$

input
```
int((a+b*ln(c*(e*x+d)))*(f+g*ln(c*(e*x+d)))/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
(-(a*f+a*g+b*f+2*b*g)/e-(a*g+b*f+2*b*g)/e*ln(c*(e*x+d))-b*g/e*ln(c*(e*x+d)
)^2)/(e*x+d)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{bg\log(cex+cd)^2+(a+b)f+(a+2b)g+(bf+(a+2b)g)\log(cex+cd)}{e^2x+de}$$

input
```
integrate((a+b*log(c*(e*x+d)))*(f+g*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm
="fricas")
```

output
```
-(b*g*log(c*e*x + c*d)^2 + (a + b)*f + (a + 2*b)*g + (b*f + (a + 2*b)*g)*l
og(c*e*x + c*d))/(e^2*x + d*e)
```

3.403.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx$$
$$= -\frac{bg\log\left(c(d+ex)\right)^2}{de+e^2x} + \frac{(-ag-bf-2bg)\log\left(c(d+ex)\right)}{de+e^2x} - \frac{af+ag+bf+2bg}{de+e^2x}$$

input
```
integrate((a+b*ln(c*(e*x+d)))*(f+g*ln(c*(e*x+d)))/(e*x+d)**2,x)
```

output
```
-b*g*log(c*(d + e*x))**2/(d*e + e**2*x) + (-a*g - b*f - 2*b*g)*log(c*(d +
e*x))/(d*e + e**2*x) - (a*f + a*g + b*f + 2*b*g)/(d*e + e**2*x)
```

3.403.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx$$
$$= -b\left(\frac{ce}{ce^3x+cde^2} + \frac{\log\left(cex+cd\right)}{e^2x+de}\right)f - a\left(\frac{ce}{ce^3x+cde^2} + \frac{\log\left(cex+cd\right)}{e^2x+de}\right)g$$
$$- \frac{af}{e^2x+de} - \frac{\left(c^2\log\left(cex+cd\right)^2 + 2\,c^2\log\left(cex+cd\right) + 2\,c^2\right)bg}{(cex+cd)ce}$$

input
```
integrate((a+b*log(c*(e*x+d)))*(f+g*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm
="maxima")
```

output
```
-b*(c*e/(c*e^3*x + c*d*e^2) + log(c*e*x + c*d)/(e^2*x + d*e))*f - a*(c*e/(
c*e^3*x + c*d*e^2) + log(c*e*x + c*d)/(e^2*x + d*e))*g - a*f/(e^2*x + d*e)
 - (c^2*log(c*e*x + c*d)^2 + 2*c^2*log(c*e*x + c*d) + 2*c^2)*b*g/((c*e*x +
 c*d)*c*e)
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx$$
$$= -\frac{\left(\frac{bcg\log((ex+d)c)^2}{(ex+d)e^2}+\frac{(bc^2f+ac^2g+2\,bc^2g)\log((ex+d)c)}{(ex+d)ce^2}+\frac{ac^2f+bc^2f+ac^2g+2\,bc^2g}{(ex+d)ce^2}\right)e}{c}$$

input
```
integrate((a+b*log(c*(e*x+d)))*(f+g*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm
="giac")
```

output
```
-(b*c*g*log((e*x + d)*c)^2/((e*x + d)*e^2) + (b*c^2*f + a*c^2*g + 2*b*c^2*
g)*log((e*x + d)*c)/((e*x + d)*c*e^2) + (a*c^2*f + b*c^2*f + a*c^2*g + 2*b
*c^2*g)/((e*x + d)*c*e^2))*e/c
```

3.403.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx =$$
$$-\frac{d\left(b\,g\ln\left(c\,d+c\,e\,x\right)^2+a\,g\,\ln\left(c\,d+c\,e\,x\right)+b\,f\,\ln\left(c\,d+c\,e\,x\right)+2\,b\,g\,\ln\left(c\,d+c\,e\,x\right)\right)-e\left(a\,f\,x+a\,g\,x\right.}{d^2\,e+x\,d\,e^2}$$

input
```
int(((a + b*log(c*(d + e*x)))*(f + g*log(c*(d + e*x))))/(d + e*x)^2,x)
```

output
```
-(d*(b*g*log(c*d + c*e*x)^2 + a*g*log(c*d + c*e*x) + b*f*log(c*d + c*e*x)
+ 2*b*g*log(c*d + c*e*x)) - e*(a*f*x + a*g*x + b*f*x + 2*b*g*x))/(d^2*e +
d*e^2*x)
```

3.404 $\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx$

3.404.1 Optimal result

Integrand size = 20, antiderivative size = 160

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx = -24ab^3m^3n^3x+24b^4m^4n^4x - \frac{24b^4m^3n^3(e+fx)\log(c(d(e+fx)^m)^n)}{f} + \frac{12b^2m^2n^2(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f} - \frac{4bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^4}{f}$$

output
```
-24*a*b^3*m^3*n^3*x+24*b^4*m^4*n^4*x-24*b^4*m^3*n^3*(f*x+e)*ln(c*(d*(f*x+e
)^m)^n)/f+12*b^2*m^2*n^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^2/f-4*b*m*n*(
f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^3/f+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))
^4/f
```

3.404.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int (a+b\log (c(d(e+fx)^m)^n))^4\,dx$$
$$=\frac{(e+fx)\,(a+b\log (c(d(e+fx)^m)^n))^4-4bmn\Big((e+fx)\,(a+b\log (c(d(e+fx)^m)^n))^3-3bmn\Big((e+fx)}{f}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^4,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^4 - 4*b*m*n*((e + f*x)*(a + b*
Log[c*(d*(e + f*x)^m)^n])^3 - 3*b*m*n*((e + f*x)*(a + b*Log[c*(d*(e + f*x)
^m)^n])^2 - 2*b*m*n*(f*(a - b*m*n)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n
]))))/f
```

3.404.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2895, 2836, 2733, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d(e+fx)^m)^n))^4\,dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int (a+b\log (c(d(e+fx)^m)^n))^4dx$$
$$\Big\downarrow \text{ 2836}$$
$$\frac{\int (a+b\log (cd^n(e+fx)^{mn}))^4d(e+fx)}{f}$$
$$\Big\downarrow \text{ 2733}$$
$$\frac{(e+fx)\,(a+b\log (cd^n(e+fx)^{mn}))^4-4bmn\int (a+b\log (cd^n(e+fx)^{mn}))^3d(e+fx)}{f}$$
$$\Big\downarrow \text{ 2733}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^4-4bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^3-3bmn\int\left(a+b\log\left(cd^n(e+\right.\right.}{f}$$

$\Big\downarrow$ 2733

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^4-4bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^3-3bmn\Big((e+fx)\left(a+b\log\right.}{f}$$

$\Big\downarrow$ 2009

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^4-4bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^3-3bmn\Big((e+fx)\left(a+b\log\right.}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^4,x]
```

output
```
((e + f*x)*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^4 - 4*b*m*n*((e + f*x)*(a +
b*Log[c*d^n*(e + f*x)^(m*n)])^3 - 3*b*m*n*((e + f*x)*(a + b*Log[c*d^n*(e +
 f*x)^(m*n)])^2 - 2*b*m*n*(a*(e + f*x) - b*m*n*(e + f*x) + b*(e + f*x)*Log
[c*d^n*(e + f*x)^(m*n)]))))/f
```

3.404.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.404.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(160) = 320$.

Time = 5.58 (sec) , antiderivative size = 642, normalized size of antiderivative = 4.01

method	result
parallelrisch	$\frac{-12x\ln(c(d(fx+e)^m)^n)^2 a b^3 efmn-12x\ln(c(d(fx+e)^m)^n)a^2b^2 efmn+24x\ln(c(d(fx+e)^m)^n)a b^3 ef m^2n^2+4\ln(c(d(fx+e)^m}{}$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^4,x,method=_RETURNVERBOSE)
```

output
```
(-12*x*ln(c*(d*(f*x+e)^m)^n)^2*a*b^3*e*f*m*n-12*x*ln(c*(d*(f*x+e)^m)^n)*a^
2*b^2*e*f*m*n+24*x*ln(c*(d*(f*x+e)^m)^n)*a*b^3*e*f*m^2*n^2+4*ln(c*(d*(f*x+
e)^m)^n)^3*a*b^3*e^2+6*ln(c*(d*(f*x+e)^m)^n)^2*a^2*b^2*e^2+x*a^4*e*f-24*x*
ln(c*(d*(f*x+e)^m)^n)*b^4*e*f*m^3*n^3+12*x*ln(c*(d*(f*x+e)^m)^n)^2*b^4*e*f
*m^2*n^2+12*ln(c*(d*(f*x+e)^m)^n)^2*b^4*e^2*m^2*n^2-4*ln(c*(d*(f*x+e)^m)^n
)^3*b^4*e^2*m*n-24*ln(f*x+e)*b^4*e^2*m^4*n^4+x*ln(c*(d*(f*x+e)^m)^n)^4*b^4
*e*f+24*x*b^4*e*f*m^4*n^4-12*ln(c*(d*(f*x+e)^m)^n)^2*a*b^3*e^2*m*n+24*ln(f
*x+e)*a*b^3*e^2*m^3*n^3-12*ln(f*x+e)*a^2*b^2*e^2*m^2*n^2-24*x*a*b^3*e*f*m^
3*n^3-4*x*ln(c*(d*(f*x+e)^m)^n)^3*b^4*e*f*m*n+12*x*a^2*b^2*e*f*m^2*n^2-4*x
*a^3*b*e*f*m*n+24*a*b^3*e^2*m^3*n^3-12*a^2*b^2*e^2*m^2*n^2+4*a^3*b*e^2*m*n
+4*ln(f*x+e)*a^3*b*e^2*m*n+4*x*ln(c*(d*(f*x+e)^m)^n)*a^3*b*e*f+4*x*ln(c*(d
*(f*x+e)^m)^n)^3*a*b^3*e*f+6*x*ln(c*(d*(f*x+e)^m)^n)^2*a^2*b^2*e*f-a^4*e^2
-24*b^4*e^2*m^4*n^4+ln(c*(d*(f*x+e)^m)^n)^4*b^4*e^2)/e/f
```

3.404.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(160) = 320$.

Time = 0.32 (sec) , antiderivative size = 1409, normalized size of antiderivative = 8.81

$$\int (a + b\log (c(d(e + fx)^m)^n))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^4,x, algorithm="fricas")
```

output
```
(b^4*f*n^4*x*log(d)^4 + b^4*f*x*log(c)^4 + (b^4*f*m^4*n^4*x + b^4*e*m^4*n^
4)*log(f*x + e)^4 - 4*(b^4*f*m*n - a*b^3*f)*x*log(c)^3 - 4*(b^4*e*m^4*n^4
- a*b^3*e*m^3*n^3 + (b^4*f*m^4*n^4 - a*b^3*f*m^3*n^3)*x - (b^4*f*m^3*n^3*x
 + b^4*e*m^3*n^3)*log(c) - (b^4*f*m^3*n^4*x + b^4*e*m^3*n^4)*log(d))*log(f
*x + e)^3 + 6*(2*b^4*f*m^2*n^2 - 2*a*b^3*f*m*n + a^2*b^2*f)*x*log(c)^2 + 4
*(b^4*f*n^3*x*log(c) - (b^4*f*m*n^4 - a*b^3*f*n^3)*x)*log(d)^3 + 6*(2*b^4*
e*m^4*n^4 - 2*a*b^3*e*m^3*n^3 + a^2*b^2*e*m^2*n^2 + (b^4*f*m^2*n^2*x + b^4
*e*m^2*n^2)*log(c)^2 + (b^4*f*m^2*n^4*x + b^4*e*m^2*n^4)*log(d)^2 + (2*b^4
*f*m^4*n^4 - 2*a*b^3*f*m^3*n^3 + a^2*b^2*f*m^2*n^2)*x - 2*(b^4*e*m^3*n^3 -
 a*b^3*e*m^2*n^2 + (b^4*f*m^3*n^3 - a*b^3*f*m^2*n^2)*x)*log(c) - 2*(b^4*e*
m^3*n^4 - a*b^3*e*m^2*n^3 + (b^4*f*m^3*n^4 - a*b^3*f*m^2*n^3)*x - (b^4*f*m
^2*n^3*x + b^4*e*m^2*n^3)*log(c))*log(d))*log(f*x + e)^2 - 4*(6*b^4*f*m^3*
n^3 - 6*a*b^3*f*m^2*n^2 + 3*a^2*b^2*f*m*n - a^3*b*f)*x*log(c) + 6*(b^4*f*n
^2*x*log(c)^2 - 2*(b^4*f*m*n^3 - a*b^3*f*n^2)*x*log(c) + (2*b^4*f*m^2*n^4
- 2*a*b^3*f*m*n^3 + a^2*b^2*f*n^2)*x)*log(d)^2 + (24*b^4*f*m^4*n^4 - 24*a*
b^3*f*m^3*n^3 + 12*a^2*b^2*f*m^2*n^2 - 4*a^3*b*f*m*n + a^4*f)*x - 4*(6*b^4
*e*m^4*n^4 - 6*a*b^3*e*m^3*n^3 + 3*a^2*b^2*e*m^2*n^2 - a^3*b*e*m*n - (b^4*
f*m*n*x + b^4*e*m*n)*log(c)^3 - (b^4*f*m*n^4*x + b^4*e*m*n^4)*log(d)^3 + 3
*(b^4*e*m^2*n^2 - a*b^3*e*m*n + (b^4*f*m^2*n^2 - a*b^3*f*m*n)*x)*log(c)^2
+ 3*(b^4*e*m^2*n^4 - a*b^3*e*m*n^3 + (b^4*f*m^2*n^4 - a*b^3*f*m*n^3)*x ...
```

3.404.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(155) = 310$.

Time = 2.56 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.81

$$\int (a + b\log (c(d(e + fx)^m)^n))^4 \, dx$$

$$= \begin{cases} a^4x + \frac{4a^3be\log (c(d(e+fx)^m)^n)}{f} - 4a^3bmnx + 4a^3bx\log (c(d(e+fx)^m)^n) - \frac{12a^2b^2emn\log (c(d(e+fx)^m)^n)}{f} + \frac{6a^2b^2e\ldots}{} \\ x(a + b\log (c(de^m)^n))^4 \end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*e*log(c*(d*(e + f*x)**m)**n)/f - 4*a**3*b*m*n
*x + 4*a**3*b*x*log(c*(d*(e + f*x)**m)**n) - 12*a**2*b**2*e*m*n*log(c*(d*(
e + f*x)**m)**n)/f + 6*a**2*b**2*e*log(c*(d*(e + f*x)**m)**n)**2/f + 12*a*
*2*b**2*m**2*n**2*x - 12*a**2*b**2*m*n*x*log(c*(d*(e + f*x)**m)**n) + 6*a*
*2*b**2*x*log(c*(d*(e + f*x)**m)**n)**2 + 24*a*b**3*e*m**2*n**2*log(c*(d*(
e + f*x)**m)**n)/f - 12*a*b**3*e*m*n*log(c*(d*(e + f*x)**m)**n)**2/f + 4*a
*b**3*e*log(c*(d*(e + f*x)**m)**n)**3/f - 24*a*b**3*m**3*n**3*x + 24*a*b**
3*m**2*n**2*x*log(c*(d*(e + f*x)**m)**n) - 12*a*b**3*m*n*x*log(c*(d*(e + f
*x)**m)**n)**2 + 4*a*b**3*x*log(c*(d*(e + f*x)**m)**n)**3 - 24*b**4*e*m**3
*n**3*log(c*(d*(e + f*x)**m)**n)/f + 12*b**4*e*m**2*n**2*log(c*(d*(e + f*x
)**m)**n)**2/f - 4*b**4*e*m*n*log(c*(d*(e + f*x)**m)**n)**3/f + b**4*e*log
(c*(d*(e + f*x)**m)**n)**4/f + 24*b**4*m**4*n**4*x - 24*b**4*m**3*n**3*x*l
og(c*(d*(e + f*x)**m)**n) + 12*b**4*m**2*n**2*x*log(c*(d*(e + f*x)**m)**n)
**2 - 4*b**4*m*n*x*log(c*(d*(e + f*x)**m)**n)**3 + b**4*x*log(c*(d*(e + f*
x)**m)**n)**4, Ne(f, 0)), (x*(a + b*log(c*(d*e**m)**n))**4, True))
```

3.404.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(160) = 320$.

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.49

$$\int (a + b\log(c(d(e+fx)^m)^n))^4\,dx = b^4x\log(((fx+e)^m d)^n c)^4$$
$$- 4a^3bfmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right) + 4ab^3x\log(((fx+e)^m d)^n c)^3$$
$$+ 6a^2b^2x\log(((fx+e)^m d)^n c)^2 + 4a^3bx\log(((fx+e)^m d)^n c)$$
$$-6\left(2fmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^m d)^n c) + \frac{(e\log(fx+e)^2 - 2fx + 2e\log(fx+e))m^2n\right.$$
$$-4\left(3fmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^m d)^n c)^2 - \left(\frac{(e\log(fx+e)^3 + 3e\log(fx+e)^2 - 6fx\right.\right.$$
$$-\left(4fmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^m d)^n c)^3 + \left(\left(\frac{(e\log(fx+e)^4 + 4e\log(fx+e)^3 + 12e\log\right.\right.\right.$$
$$+ a^4x$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(((f*x + e)^m*d)^n*c)^4 - 4*a^3*b*f*m*n*(x/f - e*log(f*x + e)/f^2
) + 4*a*b^3*x*log(((f*x + e)^m*d)^n*c)^3 + 6*a^2*b^2*x*log(((f*x + e)^m*d)
^n*c)^2 + 4*a^3*b*x*log(((f*x + e)^m*d)^n*c) - 6*(2*f*m*n*(x/f - e*log(f*x
 + e)/f^2)*log(((f*x + e)^m*d)^n*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(
f*x + e))*m^2*n^2/f)*a^2*b^2 - 4*(3*f*m*n*(x/f - e*log(f*x + e)/f^2)*log((
(f*x + e)^m*d)^n*c)^2 - ((e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x +
6*e*log(f*x + e))*m^2*n^2/f^2 - 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x
+ e))*m*n*log(((f*x + e)^m*d)^n*c)/f^2)*f*m*n)*a*b^3 - (4*f*m*n*(x/f - e*l
og(f*x + e)/f^2)*log(((f*x + e)^m*d)^n*c)^3 + (((e*log(f*x + e)^4 + 4*e*lo
g(f*x + e)^3 + 12*e*log(f*x + e)^2 - 24*f*x + 24*e*log(f*x + e))*m^2*n^2/f
^3 - 4*(e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*
m*n*log(((f*x + e)^m*d)^n*c)/f^3)*f*m*n + 6*(e*log(f*x + e)^2 - 2*f*x + 2*
e*log(f*x + e))*m*n*log(((f*x + e)^m*d)^n*c)^2/f^2)*f*m*n)*b^4 + a^4*x
```

3.404.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(160) = 320$.

Time = 0.35 (sec) , antiderivative size = 1697, normalized size of antiderivative = 10.61

$$\int (a + b\log(c(d(e + fx)^m)^n))^4\, dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^4,x, algorithm="giac")
```

output
```
(f*x + e)*b^4*m^4*n^4*log(f*x + e)^4/f - 4*(f*x + e)*b^4*m^4*n^4*log(f*x +
 e)^3/f + 4*(f*x + e)*b^4*m^3*n^4*log(f*x + e)^3*log(d)/f + 12*(f*x + e)*b
^4*m^4*n^4*log(f*x + e)^2/f + 4*(f*x + e)*b^4*m^3*n^3*log(f*x + e)^3*log(c
)/f - 12*(f*x + e)*b^4*m^3*n^4*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^4*m
^2*n^4*log(f*x + e)^2*log(d)^2/f - 24*(f*x + e)*b^4*m^4*n^4*log(f*x + e)/f
 + 4*(f*x + e)*a*b^3*m^3*n^3*log(f*x + e)^3/f - 12*(f*x + e)*b^4*m^3*n^3*l
og(f*x + e)^2*log(c)/f + 24*(f*x + e)*b^4*m^3*n^4*log(f*x + e)*log(d)/f +
12*(f*x + e)*b^4*m^2*n^3*log(f*x + e)^2*log(c)*log(d)/f - 12*(f*x + e)*b^4
*m^2*n^4*log(f*x + e)*log(d)^2/f + 4*(f*x + e)*b^4*m*n^4*log(f*x + e)*log(
d)^3/f + 24*(f*x + e)*b^4*m^4*n^4/f - 12*(f*x + e)*a*b^3*m^3*n^3*log(f*x +
 e)^2/f + 24*(f*x + e)*b^4*m^3*n^3*log(f*x + e)*log(c)/f + 6*(f*x + e)*b^4
*m^2*n^2*log(f*x + e)^2*log(c)^2/f - 24*(f*x + e)*b^4*m^3*n^4*log(d)/f + 1
2*(f*x + e)*a*b^3*m^2*n^3*log(f*x + e)^2*log(d)/f - 24*(f*x + e)*b^4*m^2*n
^3*log(f*x + e)*log(c)*log(d)/f + 12*(f*x + e)*b^4*m^2*n^4*log(d)^2/f + 12
*(f*x + e)*b^4*m*n^3*log(f*x + e)*log(c)*log(d)^2/f - 4*(f*x + e)*b^4*m*n^
4*log(d)^3/f + (f*x + e)*b^4*n^4*log(d)^4/f + 24*(f*x + e)*a*b^3*m^3*n^3*l
og(f*x + e)/f - 24*(f*x + e)*b^4*m^3*n^3*log(c)/f + 12*(f*x + e)*a*b^3*m^2
*n^2*log(f*x + e)^2*log(c)/f - 12*(f*x + e)*b^4*m^2*n^2*log(f*x + e)*log(c
)^2/f - 24*(f*x + e)*a*b^3*m^2*n^3*log(f*x + e)*log(d)/f + 24*(f*x + e)*b^
4*m^2*n^3*log(c)*log(d)/f + 12*(f*x + e)*b^4*m*n^2*log(f*x + e)*log(c)^...
```

3.404.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.38

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx$$
$$= \ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)^3\left(\frac{4\left(a\,b^3\,e-b^4\,e\,m\,n\right)}{f}+4\,b^3\,x\left(a-b\,m\,n\right)\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)^4\left(b^4\,x+\frac{b^4\,e}{f}\right)$$
$$+x\left(a^4-4\,a^3\,b\,m\,n+12\,a^2\,b^2\,m^2\,n^2-24\,a\,b^3\,m^3\,n^3+24\,b^4\,m^4\,n^4\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)^2\left(\frac{6\left(e\,a^2\,b^2-2\,e\,a\,b^3\,m\,n+2\,e\,b^4\,m^2\,n^2\right)}{f}+6\,b^2\,x\left(a^2-2\,a\,b\,m\,n+2\,b^2\,m^2\,n^2\right)\right)$$
$$-\frac{\ln\left(e+f\,x\right)\left(-4\,e\,a^3\,b\,m\,n+12\,e\,a^2\,b^2\,m^2\,n^2-24\,e\,a\,b^3\,m^3\,n^3+24\,e\,b^4\,m^4\,n^4\right)}{f}$$
$$+\frac{\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\left(4\,b\,f\left(a^3-3\,a^2\,b\,m\,n+6\,a\,b^2\,m^2\,n^2-6\,b^3\,m^3\,n^3\right)x^2+4\,b\,e\left(a^3-3\,a^2\,b\,m\,n+6\,a\right.\right.}{e+f\,x}$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^4,x)
```

output
```
log(c*(d*(e + f*x)^m)^n)^3*((4*(a*b^3*e - b^4*e*m*n))/f + 4*b^3*x*(a - b*m
*n)) + log(c*(d*(e + f*x)^m)^n)^4*(b^4*x + (b^4*e)/f) + x*(a^4 + 24*b^4*m^
4*n^4 - 24*a*b^3*m^3*n^3 - 4*a^3*b*m*n + 12*a^2*b^2*m^2*n^2) + log(c*(d*(e
 + f*x)^m)^n)^2*((6*(a^2*b^2*e + 2*b^4*e*m^2*n^2 - 2*a*b^3*e*m*n))/f + 6*b
^2*x*(a^2 + 2*b^2*m^2*n^2 - 2*a*b*m*n)) - (log(e + f*x)*(24*b^4*e*m^4*n^4
- 24*a*b^3*e*m^3*n^3 - 4*a^3*b*e*m*n + 12*a^2*b^2*e*m^2*n^2))/f + (log(c*(
d*(e + f*x)^m)^n)*(4*b*f*x^2*(a^3 - 6*b^3*m^3*n^3 + 6*a*b^2*m^2*n^2 - 3*a^
2*b*m*n) + 4*b*e*x*(a^3 - 6*b^3*m^3*n^3 + 6*a*b^2*m^2*n^2 - 3*a^2*b*m*n)))
/(e + f*x)
```

3.404. $\int (a+b\log (c(d(e+fx)^m)^n))^4\,dx$

3.405 $\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx$

3.405.1 Optimal result

Integrand size = 20, antiderivative size = 121

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx = 6ab^2m^2n^2x-6b^3m^3n^3x + \frac{6b^3m^2n^2(e+fx)\log(c(d(e+fx)^m)^n)}{f} - \frac{3bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f}$$

output
```
6*a*b^2*m^2*n^2*x-6*b^3*m^3*n^3*x+6*b^3*m^2*n^2*(f*x+e)*ln(c*(d*(f*x+e)^m)
^n)/f-3*b*m*n*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^2/f+(f*x+e)*(a+b*ln(c*(d
*(f*x+e)^m)^n))^3/f
```

3.405.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx = \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3-3bmn\Big((e+fx)(a+b\log(c(d(e+fx)^m)^n))^2-2bmn(f(a-bm$$
$$f$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^3,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^3 - 3*b*m*n*((e + f*x)*(a + b*
Log[c*(d*(e + f*x)^m)^n])^2 - 2*b*m*n*(f*(a - b*m*n)*x + b*(e + f*x)*Log[c
*(d*(e + f*x)^m)^n])))/f
```

3.405.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2895, 2836, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log(c(d(e+fx)^m)^n))^3dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(cd^n(e+fx)^{mn}))^3d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)(a+b\log(cd^n(e+fx)^{mn}))^3-3bmn\int (a+b\log(cd^n(e+fx)^{mn}))^2d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)(a+b\log(cd^n(e+fx)^{mn}))^3-3bmn\Big((e+fx)(a+b\log(cd^n(e+fx)^{mn}))^2-2bmn\int (a+b\log(cd^n(e+$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)(a+b\log(cd^n(e+fx)^{mn}))^3-3bmn\Big((e+fx)(a+b\log(cd^n(e+fx)^{mn}))^2-2bmn(a(e+fx)+b(e+$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^3,x]
```

output
```
((e + f*x)*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^3 - 3*b*m*n*((e + f*x)*(a +
b*Log[c*d^n*(e + f*x)^(m*n)])^2 - 2*b*m*n*(a*(e + f*x) - b*m*n*(e + f*x) +
 b*(e + f*x)*Log[c*d^n*(e + f*x)^(m*n)])))/f
```

3.405.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.405.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(121) = 242$.

Time = 1.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
parallelrisch	$\frac{6 \ln(fx+e) b^3 e^2 m^3 n^3 - 6x\, b^3 ef\, m^3 n^3 + 6x \ln(c(d(fx+e)^m)^n) b^3 ef\, m^2 n^2 + 6 b^3 e^2 m^3 n^3 - 6 \ln(fx+e) a\, b^2 e^2 m^2 n^2 - 3x \ln(c(d(fx+e}{}$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
(6*ln(f*x+e)*b^3*e^2*m^3*n^3-6*x*b^3*e*f*m^3*n^3+6*x*ln(c*(d*(f*x+e)^m)^n)
*b^3*e*f*m^2*n^2+6*b^3*e^2*m^3*n^3-6*ln(f*x+e)*a*b^2*e^2*m^2*n^2-3*x*ln(c*
(d*(f*x+e)^m)^n)^2*b^3*e*f*m*n+6*x*a*b^2*e*f*m^2*n^2+x*ln(c*(d*(f*x+e)^m)^
n)^3*b^3*e*f-6*x*ln(c*(d*(f*x+e)^m)^n)*a*b^2*e*f*m*n-3*ln(c*(d*(f*x+e)^m)^
n)^2*b^3*e^2*m*n-6*a*b^2*e^2*m^2*n^2+3*ln(f*x+e)*a^2*b*e^2*m*n+3*x*ln(c*(d
*(f*x+e)^m)^n)^2*a*b^2*e*f-3*x*a^2*b*e*f*m*n+ln(c*(d*(f*x+e)^m)^n)^3*b^3*e
^2+3*x*ln(c*(d*(f*x+e)^m)^n)*a^2*b*e*f+3*ln(c*(d*(f*x+e)^m)^n)^2*a*b^2*e^2
+3*a^2*b*e^2*m*n+x*a^3*e*f-a^3*e^2)/e/f
```

3.405.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 639, normalized size of antiderivative = 5.28

$$\int (a+b\log (c(d(e+fx)^m)^n))^3\,dx$$
$$= \frac{b^3fn^3x\log (d)^3 + b^3fx\log (c)^3 + (b^3fm^3n^3x + b^3em^3n^3)\log (fx+e)^3 - 3\,(b^3fmn - ab^2f)x\log (c)^2 - 3\,(}{}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="fricas")
```

output
```
(b^3*f*n^3*x*log(d)^3 + b^3*f*x*log(c)^3 + (b^3*f*m^3*n^3*x + b^3*e*m^3*n^
3)*log(f*x + e)^3 - 3*(b^3*f*m*n - a*b^2*f)*x*log(c)^2 - 3*(b^3*e*m^3*n^3
- a*b^2*e*m^2*n^2 + (b^3*f*m^3*n^3 - a*b^2*f*m^2*n^2)*x - (b^3*f*m^2*n^2*x
 + b^3*e*m^2*n^2)*log(c) - (b^3*f*m^2*n^3*x + b^3*e*m^2*n^3)*log(d))*log(f
*x + e)^2 + 3*(2*b^3*f*m^2*n^2 - 2*a*b^2*f*m*n + a^2*b*f)*x*log(c) + 3*(b^
3*f*n^2*x*log(c) - (b^3*f*m*n^3 - a*b^2*f*n^2)*x)*log(d)^2 - (6*b^3*f*m^3*
n^3 - 6*a*b^2*f*m^2*n^2 + 3*a^2*b*f*m*n - a^3*f)*x + 3*(2*b^3*e*m^3*n^3 -
2*a*b^2*e*m^2*n^2 + a^2*b*e*m*n + (b^3*f*m*n*x + b^3*e*m*n)*log(c)^2 + (b^
3*f*m*n^3*x + b^3*e*m*n^3)*log(d)^2 + (2*b^3*f*m^3*n^3 - 2*a*b^2*f*m^2*n^2
 + a^2*b*f*m*n)*x - 2*(b^3*e*m^2*n^2 - a*b^2*e*m*n + (b^3*f*m^2*n^2 - a*b^
2*f*m*n)*x)*log(c) - 2*(b^3*e*m^2*n^3 - a*b^2*e*m*n^2 + (b^3*f*m^2*n^3 - a
*b^2*f*m*n^2)*x - (b^3*f*m*n^2*x + b^3*e*m*n^2)*log(c))*log(d))*log(f*x +
e) + 3*(b^3*f*n*x*log(c)^2 - 2*(b^3*f*m*n^2 - a*b^2*f*n)*x*log(c) + (2*b^3
*f*m^2*n^3 - 2*a*b^2*f*m*n^2 + a^2*b*f*n)*x)*log(d))/f
```

3.405.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(117) = 234$.

Time = 1.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.98

$$\int (a + b\log(c(d(e+fx)^m)^n))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2be\log(c(d(e+fx)^m)^n)}{f} - 3a^2bmnx + 3a^2bx\log(c(d(e+fx)^m)^n) - \frac{6ab^2emn\log(c(d(e+fx)^m)^n)}{f} + \frac{3ab^2e\log(\cdots}{} \\ x(a+b\log(c(de^m)^n))^3 \end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*e*log(c*(d*(e + f*x)**m)**n)/f - 3*a**2*b*m*n
*x + 3*a**2*b*x*log(c*(d*(e + f*x)**m)**n) - 6*a*b**2*e*m*n*log(c*(d*(e +
f*x)**m)**n)/f + 3*a*b**2*e*log(c*(d*(e + f*x)**m)**n)**2/f + 6*a*b**2*m**
2*n**2*x - 6*a*b**2*m*n*x*log(c*(d*(e + f*x)**m)**n) + 3*a*b**2*x*log(c*(d
*(e + f*x)**m)**n)**2 + 6*b**3*e*m**2*n**2*log(c*(d*(e + f*x)**m)**n)/f -
3*b**3*e*m*n*log(c*(d*(e + f*x)**m)**n)**2/f + b**3*e*log(c*(d*(e + f*x)**
m)**n)**3/f - 6*b**3*m**3*n**3*x + 6*b**3*m**2*n**2*x*log(c*(d*(e + f*x)**
m)**n) - 3*b**3*m*n*x*log(c*(d*(e + f*x)**m)**n)**2 + b**3*x*log(c*(d*(e +
 f*x)**m)**n)**3, Ne(f, 0)), (x*(a + b*log(c*(d*e**m)**n))**3, True))
```

3.405.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(121) = 242$.

Time = 0.24 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.62

$$\int (a + b\log(c(d(e+fx)^m)^n))^3\,dx = -3\,a^2bfmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)$$
$$+ b^3x\log(((fx+e)^m d)^n c)^3 + 3\,ab^2x\log(((fx+e)^m d)^n c)^2 + 3\,a^2bx\log(((fx+e)^m d)^n c)$$
$$-3\left(2\,fmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^m d)^n c) + \frac{(e\log(fx+e)^2 - 2\,fx + 2\,e\log(fx+e))m^2n\cdots}{f}\right.$$
$$-\left(3\,fmn\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^m d)^n c)^2 - \left(\frac{(e\log(fx+e)^3 + 3\,e\log(fx+e)^2 - 6\,fx + \cdots}{f^2}\right.\right.$$
$$+ a^3x$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="maxima")
```

output
```
-3*a^2*b*f*m*n*(x/f - e*log(f*x + e)/f^2) + b^3*x*log(((f*x + e)^m*d)^n*c)
^3 + 3*a*b^2*x*log(((f*x + e)^m*d)^n*c)^2 + 3*a^2*b*x*log(((f*x + e)^m*d)^
n*c) - 3*(2*f*m*n*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^m*d)^n*c) + (e
*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*m^2*n^2/f)*a*b^2 - (3*f*m*n*(x
/f - e*log(f*x + e)/f^2)*log(((f*x + e)^m*d)^n*c)^2 - ((e*log(f*x + e)^3 +
 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*m^2*n^2/f^2 - 3*(e*log(f*x
 + e)^2 - 2*f*x + 2*e*log(f*x + e))*m*n*log(((f*x + e)^m*d)^n*c)/f^2)*f*m*
n)*b^3 + a^3*x
```

3.405.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 772, normalized size of antiderivative = 6.38

$$
\begin{aligned}
\int (a+b\log (c(d(e+fx)^m)^n))^3\, dx = {} & \frac{(fx+e)b^3m^3n^3\log (fx+e)^3}{f} \\
& - \frac{3\,(fx+e)b^3m^3n^3\log (fx+e)^2}{f} \\
& + \frac{3\,(fx+e)b^3m^2n^3\log (fx+e)^2\log (d)}{f} \\
& + \frac{6\,(fx+e)b^3m^3n^3\log (fx+e)}{f} \\
& + \frac{3\,(fx+e)b^3m^2n^2\log (fx+e)^2\log (c)}{f} \\
& - \frac{6\,(fx+e)b^3m^2n^3\log (fx+e)\log (d)}{f} \\
& + \frac{3\,(fx+e)b^3mn^3\log (fx+e)\log (d)^2}{f} \\
& - \frac{6\,(fx+e)b^3m^3n^3}{f} \\
& + \frac{3\,(fx+e)ab^2m^2n^2\log (fx+e)^2}{f} \\
& - \frac{6\,(fx+e)b^3m^2n^2\log (fx+e)\log (c)}{f} \\
& + \frac{6\,(fx+e)b^3m^2n^3\log (d)}{f} \\
& + \frac{6\,(fx+e)b^3mn^2\log (fx+e)\log (c)\log (d)}{f} \\
& - \frac{3\,(fx+e)b^3mn^3\log (d)^2}{f} + \frac{(fx+e)b^3n^3\log (d)^3}{f} \\
& - \frac{6\,(fx+e)ab^2m^2n^2\log (fx+e)}{f} \\
& + \frac{6\,(fx+e)b^3m^2n^2\log (c)}{f} \\
& + \frac{3\,(fx+e)b^3mn\log (fx+e)\log (c)^2}{f} \\
& + \frac{6\,(fx+e)ab^2mn^2\log (fx+e)\log (d)}{f} \\
& - \frac{6\,(fx+e)b^3mn^2\log (c)\log (d)}{f} \\
& + \frac{3\,(fx+e)b^3n^2\log (c)\log (d)^2}{f} + \frac{6\,(fx+e)ab^2m^2n^2}{f} \\
& + \frac{6\,(fx+e)ab^2mn\log (fx+e)\log (c)}{f} \\
& - \frac{3\,(fx+e)b^3mn\log (c)^2}{f} - \frac{6\,(fx+e)ab^2mn^2\log (d)}{f} \\
& + \frac{3\,(fx+e)b^3n\log (c)^2\log (d)}{f}
\end{aligned}
$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="giac")
```

output
```
(f*x + e)*b^3*m^3*n^3*log(f*x + e)^3/f - 3*(f*x + e)*b^3*m^3*n^3*log(f*x +
 e)^2/f + 3*(f*x + e)*b^3*m^2*n^3*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^
3*m^3*n^3*log(f*x + e)/f + 3*(f*x + e)*b^3*m^2*n^2*log(f*x + e)^2*log(c)/f
 - 6*(f*x + e)*b^3*m^2*n^3*log(f*x + e)*log(d)/f + 3*(f*x + e)*b^3*m*n^3*l
og(f*x + e)*log(d)^2/f - 6*(f*x + e)*b^3*m^3*n^3/f + 3*(f*x + e)*a*b^2*m^2
*n^2*log(f*x + e)^2/f - 6*(f*x + e)*b^3*m^2*n^2*log(f*x + e)*log(c)/f + 6*
(f*x + e)*b^3*m^2*n^3*log(d)/f + 6*(f*x + e)*b^3*m*n^2*log(f*x + e)*log(c)
*log(d)/f - 3*(f*x + e)*b^3*m*n^3*log(d)^2/f + (f*x + e)*b^3*n^3*log(d)^3/
f - 6*(f*x + e)*a*b^2*m^2*n^2*log(f*x + e)/f + 6*(f*x + e)*b^3*m^2*n^2*log
(c)/f + 3*(f*x + e)*b^3*m*n*log(f*x + e)*log(c)^2/f + 6*(f*x + e)*a*b^2*m*
n^2*log(f*x + e)*log(d)/f - 6*(f*x + e)*b^3*m*n^2*log(c)*log(d)/f + 3*(f*x
 + e)*b^3*n^2*log(c)*log(d)^2/f + 6*(f*x + e)*a*b^2*m^2*n^2/f + 6*(f*x + e
)*a*b^2*m*n*log(f*x + e)*log(c)/f - 3*(f*x + e)*b^3*m*n*log(c)^2/f - 6*(f*
x + e)*a*b^2*m*n^2*log(d)/f + 3*(f*x + e)*b^3*n*log(c)^2*log(d)/f + 3*(f*x
 + e)*a*b^2*n^2*log(d)^2/f + 3*(f*x + e)*a^2*b*m*n*log(f*x + e)/f - 6*(f*x
 + e)*a*b^2*m*n*log(c)/f + (f*x + e)*b^3*log(c)^3/f + 6*(f*x + e)*a*b^2*n*
log(c)*log(d)/f - 3*(f*x + e)*a^2*b*m*n/f + 3*(f*x + e)*a*b^2*log(c)^2/f +
 3*(f*x + e)*a^2*b*n*log(d)/f + 3*(f*x + e)*a^2*b*log(c)/f + (f*x + e)*a^3
/f
```

3.405.9 Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.00

$$
\begin{aligned}
\int (a+b\log (c(d(e+fx)^m)^n))^3 \, dx &= x\,\left(a^3-3\,a^2\,b\,m\,n+6\,a\,b^2\,m^2\,n^2-6\,b^3\,m^3\,n^3\right) \\
&+\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^m\right)}^n\right)^2\,\left(\frac{3\,\left(a\,b^2\,e-b^3\,e\,m\,n\right)}{f}+3\,b^2\,x\,\left(a-b\,m\,n\right)\right) \\
&+\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^m\right)}^n\right)^3\,\left(b^3\,x+\frac{b^3\,e}{f}\right) \\
&+\frac{\ln\left(e+f\,x\right)\,\left(3\,e\,a^2\,b\,m\,n-6\,e\,a\,b^2\,m^2\,n^2+6\,e\,b^3\,m^3\,n^3\right)}{f} \\
&+\frac{\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^m\right)}^n\right)\,\left(3\,b\,f\,\left(a^2-2\,a\,b\,m\,n+2\,b^2\,m^2\,n^2\right)\,x^2+3\,b\,e\,\left(a^2-2\,a\,b\,m\,n+2\,b^2\,m^2\,n^2\right)\,x\right)}{e+f\,x}
\end{aligned}
$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^3,x)
```

output
```
x*(a^3 - 6*b^3*m^3*n^3 + 6*a*b^2*m^2*n^2 - 3*a^2*b*m*n) + log(c*(d*(e + f*
x)^m)^n)^2*((3*(a*b^2*e - b^3*e*m*n))/f + 3*b^2*x*(a - b*m*n)) + log(c*(d*
(e + f*x)^m)^n)^3*(b^3*x + (b^3*e)/f) + (log(e + f*x)*(6*b^3*e*m^3*n^3 - 6
*a*b^2*e*m^2*n^2 + 3*a^2*b*e*m*n))/f + (log(c*(d*(e + f*x)^m)^n)*(3*b*e*x*
(a^2 + 2*b^2*m^2*n^2 - 2*a*b*m*n) + 3*b*f*x^2*(a^2 + 2*b^2*m^2*n^2 - 2*a*b
*m*n)))/(e + f*x)
```

3.405. $\int (a + b\log (c(d(e+fx)^m)^n))^3 \, dx$

3.406 $\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx$

3.406.1 Optimal result

Integrand size = 20, antiderivative size = 78

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx = -2abmnx+2b^2m^2n^2x - \frac{2b^2mn(e+fx)\log(c(d(e+fx)^m)^n)}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f}$$

output
```
-2*a*b*m*n*x+2*b^2*m^2*n^2*x-2*b^2*m*n*(f*x+e)*ln(c*(d*(f*x+e)^m)^n)/f+(f*
x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^2/f
```

3.406.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx = \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f} - 2bmn\left(ax - bmnx + \frac{b(e+fx)\log(c(d(e+fx)^m)^n)}{f}\right)$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^2,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^2)/f - 2*b*m*n*(a*x - b*m*n*x
+ (b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n])/f)
```

3.406.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2895, 2836, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d(e+fx)^m)^n))^2\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log (c(d(e+fx)^m)^n))^2dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log (cd^n(e+fx)^{mn}))^2d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)(a+b\log (cd^n(e+fx)^{mn}))^2-2bmn\int (a+b\log (cd^n(e+fx)^{mn}))d(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)(a+b\log (cd^n(e+fx)^{mn}))^2-2bmn(a(e+fx)+b(e+fx)\log (cd^n(e+fx)^{mn})-bmn(e+fx))}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^2,x]
```

output
```
((e + f*x)*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^2 - 2*b*m*n*(a*(e + f*x) - b
*m*n*(e + f*x) + b*(e + f*x)*Log[c*d^n*(e + f*x)^(m*n)]))/f
```

3.406.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.406.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(78) = 156.

Time = 0.42 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.12

method	result
parallelrisch	$\frac{-2\ln(fx+e)b^2e^2m^2n^2+2x b^2ef m^2n^2-2x\ln(c(d(fx+e)^m)^n)b^2efmn+2\ln(fx+e)ab e^2mn+x\ln(c(d(fx+e)^m)^n)^2b^2ef-2xab}{ef}$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*ln(f*x+e)*b^2*e^2*m^2*n^2+2*x*b^2*e*f*m^2*n^2-2*x*ln(c*(d*(f*x+e)^m)^n
)*b^2*e*f*m*n+2*ln(f*x+e)*a*b*e^2*m*n+x*ln(c*(d*(f*x+e)^m)^n)^2*b^2*e*f-2*
x*a*b*e*f*m*n+2*x*ln(c*(d*(f*x+e)^m)^n)*a*b*e*f+ln(c*(d*(f*x+e)^m)^n)^2*b^
2*e^2+e*a^2*f*x)/e/f
```

3.406.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

Time = 0.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

$$\int (a + b\log (c(d(e + fx)^m)^n))^2 \, dx$$
$$= \frac{b^2fn^2x\log (d)^2 + b^2fx\log (c)^2 + (b^2fm^2n^2x + b^2em^2n^2)\log (fx + e)^2 - 2\,(b^2fmn - abf)x\log (c) + (2\,b^2}{}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="fricas")
```

output
```
(b^2*f*n^2*x*log(d)^2 + b^2*f*x*log(c)^2 + (b^2*f*m^2*n^2*x + b^2*e*m^2*n^
2)*log(f*x + e)^2 - 2*(b^2*f*m*n - a*b*f)*x*log(c) + (2*b^2*f*m^2*n^2 - 2*
a*b*f*m*n + a^2*f)*x - 2*(b^2*e*m^2*n^2 - a*b*e*m*n + (b^2*f*m^2*n^2 - a*b
*f*m*n)*x - (b^2*f*m*n*x + b^2*e*m*n)*log(c) - (b^2*f*m*n^2*x + b^2*e*m*n^
2)*log(d))*log(f*x + e) + 2*(b^2*f*n*x*log(c) - (b^2*f*m*n^2 - a*b*f*n)*x)
*log(d))/f
```

3.406.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.28

$$\int (a + b\log (c(d(e + fx)^m)^n))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{2abe\log (c(d(e+fx)^m)^n)}{f} - 2abmnx + 2abx\log (c(d(e + fx)^m)^n) - \frac{2b^2emn\log (c(d(e+fx)^m)^n)}{f} + \frac{b^2e\log (c(d(e+}{f} \\ x(a + b\log (c(de^m)^n))^2 \end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*e*log(c*(d*(e + f*x)**m)**n)/f - 2*a*b*m*n*x + 2
*a*b*x*log(c*(d*(e + f*x)**m)**n) - 2*b**2*e*m*n*log(c*(d*(e + f*x)**m)**n
)/f + b**2*e*log(c*(d*(e + f*x)**m)**n)**2/f + 2*b**2*m**2*n**2*x - 2*b**2
*m*n*x*log(c*(d*(e + f*x)**m)**n) + b**2*x*log(c*(d*(e + f*x)**m)**n)**2,
Ne(f, 0)), (x*(a + b*log(c*(d*e**m)**n))**2, True))
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx$$
$$= -2\,abfmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)+b^2x\log\left(\left(\left((fx+e)^m d\right)^n c\right)^2+2\,abx\log\left(\left((fx+e)^m d\right)^n c\right) - \left(2\,fmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(\left((fx+e)^m d\right)^n c\right)+\frac{\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)m^2n^2}{f}\right. + a^2x$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="maxima")
```

output
```
-2*a*b*f*m*n*(x/f - e*log(f*x + e)/f^2) + b^2*x*log(((f*x + e)^m*d)^n*c)^2
 + 2*a*b*x*log(((f*x + e)^m*d)^n*c) - (2*f*m*n*(x/f - e*log(f*x + e)/f^2)*
log(((f*x + e)^m*d)^n*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*m
^2*n^2/f)*b^2 + a^2*x
```

3.406.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(78) = 156$.

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.63

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx = \frac{(fx+e)b^2m^2n^2\log(fx+e)^2}{f} - \frac{2(fx+e)b^2m^2n^2\log(fx+e)}{f} + \frac{2(fx+e)b^2mn^2\log(fx+e)\log(d)}{f} + \frac{2(fx+e)b^2m^2n^2}{f} + \frac{2(fx+e)b^2mn\log(fx+e)\log(c)}{f} - \frac{2(fx+e)b^2mn^2\log(d)}{f} + \frac{(fx+e)b^2n^2\log(d)^2}{f} + \frac{2(fx+e)abmn\log(fx+e)}{f} - \frac{2(fx+e)b^2mn\log(c)}{f} + \frac{2(fx+e)b^2n\log(c)\log(d)}{f} - \frac{2(fx+e)abmn}{f} + \frac{(fx+e)b^2\log(c)^2}{f} + \frac{2(fx+e)abn\log(d)}{f} + \frac{2(fx+e)ab\log(c)}{f} + \frac{(fx+e)a^2}{f}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="giac")
```

output
```
(f*x + e)*b^2*m^2*n^2*log(f*x + e)^2/f - 2*(f*x + e)*b^2*m^2*n^2*log(f*x +
 e)/f + 2*(f*x + e)*b^2*m*n^2*log(f*x + e)*log(d)/f + 2*(f*x + e)*b^2*m^2*
n^2/f + 2*(f*x + e)*b^2*m*n*log(f*x + e)*log(c)/f - 2*(f*x + e)*b^2*m*n^2*
log(d)/f + (f*x + e)*b^2*n^2*log(d)^2/f + 2*(f*x + e)*a*b*m*n*log(f*x + e)
/f - 2*(f*x + e)*b^2*m*n*log(c)/f + 2*(f*x + e)*b^2*n*log(c)*log(d)/f - 2*
(f*x + e)*a*b*m*n/f + (f*x + e)*b^2*log(c)^2/f + 2*(f*x + e)*a*b*n*log(d)/
f + 2*(f*x + e)*a*b*log(c)/f + (f*x + e)*a^2/f
```

3.406. $\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx$

3.406.9 Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx = \ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)^2\left(b^2\,x+\frac{b^2\,e}{f}\right) + x\left(a^2-2\,a\,b\,m\,n+2\,b^2\,m^2\,n^2\right) - \frac{\ln\left(e+f\,x\right)\,\left(2\,b^2\,e\,m^2\,n^2-2\,a\,b\,e\,m\,n\right)}{f} + 2\,b\,x\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\,\left(a-b\,m\,n\right)$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^2,x)
```

output
```
log(c*(d*(e + f*x)^m)^n)^2*(b^2*x + (b^2*e)/f) + x*(a^2 + 2*b^2*m^2*n^2 -
2*a*b*m*n) - (log(e + f*x)*(2*b^2*e*m^2*n^2 - 2*a*b*e*m*n))/f + 2*b*x*log(
c*(d*(e + f*x)^m)^n)*(a - b*m*n)
```

3.407 $\int (a+b\log(c(d(e+fx)^m)^n))\,dx$

3.407.1 Optimal result

Integrand size = 18, antiderivative size = 34

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = ax-bmnx+\frac{b(e+fx)\log(c(d(e+fx)^m)^n)}{f}$$

output
```
a*x-b*m*n*x+b*(f*x+e)*ln(c*(d*(f*x+e)^m)^n)/f
```

3.407.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = ax-bmnx+\frac{b(e+fx)\log(c(d(e+fx)^m)^n)}{f}$$

input
```
Integrate[a + b*Log[c*(d*(e + f*x)^m)^n],x]
```

output
```
a*x - b*m*n*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n])/f
```

3.407.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b(e+fx)\log(c(d(e+fx)^m)^n)}{f}-bmnx$$

input
```
Int[a + b*Log[c*(d*(e + f*x)^m)^n],x]
```

output
```
a*x - b*m*n*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n])/f
```

3.407.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.407.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result	size
default	$ax+b\ln(c(d(fx+e)^m)^n)x-bmnx+\frac{bnme\ln(fx+e)}{f}$	42
parts	$ax+b\ln(c(d(fx+e)^m)^n)x-bmnx+\frac{bnme\ln(fx+e)}{f}$	42
parallelrisch	$\frac{b(2\ln(fx+e)e^2mn-xefmn+x\ln(c(d(fx+e)^m)^n)ef-\ln(c(d(fx+e)^m)^n)e^2)}{ef}+ax$	71

input
```
int(a+b*ln(c*(d*(f*x+e)^m)^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*ln(c*(d*(f*x+e)^m)^n)*x-b*m*n*x+b*n*m/f*e*ln(f*x+e)
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int (a + b\log (c(d(e + fx)^m)^n))\, dx = \frac{bfnx\log (d) + bfx\log (c) - (bfmn - af)x + (bfmnx + bemn)\log (fx + e)}{f}$$

input
```
integrate(a+b*log(c*(d*(f*x+e)^m)^n),x, algorithm="fricas")
```

output
```
(b*f*n*x*log(d) + b*f*x*log(c) - (b*f*m*n - a*f)*x + (b*f*m*n*x + b*e*m*n)
*log(f*x + e))/f
```

3.407.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a + b\log (c(d(e + fx)^m)^n))\, dx = ax + b\left(\begin{cases} \frac{e\log (c(d(e+fx)^m)^n)}{f} - mnx + x\log (c(d(e + fx)^m)^n) & \text{for } f \neq 0 \\ x\log (c(de^m)^n) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*ln(c*(d*(f*x+e)**m)**n),x)
```

output
```
a*x + b*Piecewise((e*log(c*(d*(e + f*x)**m)**n)/f - m*n*x + x*log(c*(d*(e
+ f*x)**m)**n), Ne(f, 0)), (x*log(c*(d*e**m)**n), True))
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int (a + b\log (c(d(e + fx)^m)^n))\, dx = -bfmn\left(\frac{x}{f} - \frac{e\log (fx + e)}{f^2}\right) + bx\log \left(\left(\left(fx + e\right)^m d\right)^n c\right) + ax$$

input
```
integrate(a+b*log(c*(d*(f*x+e)^m)^n),x, algorithm="maxima")
```

output
```
-b*f*m*n*(x/f - e*log(f*x + e)/f^2) + b*x*log(((f*x + e)^m*d)^n*c) + a*x
```

3.407.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int (a + b\log(c(d(e+fx)^m)^n))\,dx$$
$$= \left(\frac{(fx+e)mn\log(fx+e)}{f} - \frac{(fx+e)mn}{f} + \frac{(fx+e)n\log(d)}{f} + \frac{(fx+e)\log(c)}{f}\right)b + ax$$

input
```
integrate(a+b*log(c*(d*(f*x+e)^m)^n),x, algorithm="giac")
```

output
```
((f*x + e)*m*n*log(f*x + e)/f - (f*x + e)*m*n/f + (f*x + e)*n*log(d)/f + (
f*x + e)*log(c)/f)*b + a*x
```

3.407.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (a + b\log(c(d(e+fx)^m)^n))\,dx = x\,(a - b\,m\,n) + b\,x\,\ln\left(c\left(d\,(e+f\,x)^m\right)^n\right) + \frac{b\,e\,m\,n\,\ln(e+f\,x)}{f}$$

input
```
int(a + b*log(c*(d*(e + f*x)^m)^n),x)
```

output
```
x*(a - b*m*n) + b*x*log(c*(d*(e + f*x)^m)^n) + (b*e*m*n*log(e + f*x))/f
```

3.408 $\int \frac{1}{a+b\log(c(d(e+fx)^m)^n)}\,dx$

3.408.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{bfmn}$$

output
```
(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)/b/exp(a/b/m/n)/f/m/n/((c*(d*
(f*x+e)^m)^n)^(1/m/n))
```

3.408.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{bfmn}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-1),x]
```

output
```
((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)])/(b*E^(
a/(b*m*n))*f*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*n)))
```

3.408.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2895, 2836, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\log \left(c\left(d(e+fx)^m\right)^n\right)} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{a+b\log \left(c\left(d(e+fx)^m\right)^n\right)}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{a+b\log\left(cd^n(e+fx)^{mn}\right)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \frac{\left(cd^n(e+fx)^{mn}\right)^{\frac{1}{mn}}}{a+b\log\left(cd^n(e+fx)^{mn}\right)}d\log\left(cd^n(e+fx)^{mn}\right)}{fmn}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(e+fx)e^{-\frac{a}{bmn}}\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log\left(cd^n(e+fx)^{mn}\right)}{bmn}\right)}{bfmn}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-1),x]
```

output
```
((e + f*x)*ExpIntegralEi[(a + b*Log[c*d^n*(e + f*x)^(m*n)])/(b*m*n)])/(b*E
^(a/(b*m*n))*f*m*n*(c*d^n*(e + f*x)^(m*n))^(1/(m*n)))
```

3.408.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.408.4 Maple [F]

$$\int \frac{1}{a + b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n)),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n)),x)
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{e^{\left(-\frac{bn\log(d)+b\log(c)+a}{bmn}\right)}\log\_integral\left((fx+e)e^{\left(\frac{bn\log(d)+b\log(c)+a}{bmn}\right)}\right)}{bfmn}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n)),x, algorithm="fricas")
```

output
```
e^(-(b*n*log(d) + b*log(c) + a)/(b*m*n))*log_integral((f*x + e)*e^((b*n*lo
g(d) + b*log(c) + a)/(b*m*n)))/(b*f*m*n)
```

3.408.6 Sympy [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \frac{1}{a+b\log\left(c\left(d\left(e+fx\right)^m\right)^n\right)}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n)),x)
```

output
```
Integral(1/(a + b*log(c*(d*(e + f*x)**m)**n)), x)
```

3.408.7 Maxima [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \frac{1}{b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n)),x, algorithm="maxima")
```

output
```
integrate(1/(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{\text{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bmn}\right)}}{bc^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}fmn}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n)),x, algorithm="giac")
```

output
```
Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))/(b*c
^(1/(m*n))*d^(1/m)*f*m*n)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \frac{1}{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n)),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n)), x)
```

3.409 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx$

3.409.1 Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{b^2fm^2n^2} - \frac{e+fx}{bfmn\,(a+b\log(c(d(e+fx)^m)^n))}$$

output
```
(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)/b^2/exp(a/b/m/n)/f/m^2/n^2/(
(c*(d*(f*x+e)^m)^n)^(1/m/n))+(-f*x-e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))
```

3.409.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = -\frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(be^{\frac{a}{bmn}}mn\left(c(d(e+fx)^m)^n\right)^{\frac{1}{mn}} - \text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)}{bmn}\right.\right.}{b^2fm^2n^2\,(a+b\log(c(d(e+fx)^m)^n))}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-2),x]
```

output
```
-(((e + f*x)*(b*E^(a/(b*m*n))*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*n)) - ExpInt
egralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)]*(a + b*Log[c*(d*(e + f*x
)^m)^n])))/(b^2*E^(a/(b*m*n))*f*m^2*n^2*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(a
 + b*Log[c*(d*(e + f*x)^m)^n])))
```

3.409.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2895, 2836, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^2}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{a+b\log(cd^n(e+fx)^{mn})}d(e+fx)}{bmn}-\frac{e+fx}{bmn(a+b\log(cd^n(e+fx)^{mn}))}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\int \frac{(cd^n(e+fx)^{mn})^{\frac{1}{mn}}}{a+b\log(cd^n(e+fx)^{mn})}d\log(cd^n(e+fx)^{mn})}{bm^2n^2}-\frac{e+fx}{bmn(a+b\log(cd^n(e+fx)^{mn}))}}{f}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{(e+fx)e^{-\frac{a}{bmn}}(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^n(e+fx)^{mn})}{bmn}\right)}{b^2m^2n^2}-\frac{e+fx}{bmn(a+b\log(cd^n(e+fx)^{mn}))}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-2),x]
```

output
```
(((e + f*x)*ExpIntegralEi[(a + b*Log[c*d^n*(e + f*x)^(m*n)])/(b*m*n)])/(b^
2*E^(a/(b*m*n))*m^2*n^2*(c*d^n*(e + f*x)^(m*n))^(1/(m*n))) - (e + f*x)/(b*
m*n*(a + b*Log[c*d^n*(e + f*x)^(m*n)])))/f
```

3.409.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.409. $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx$

3.409.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^2,x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^2,x)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = -\frac{\left((bfmnx+bemn)e^{\left(\frac{bn\log(d)+b\log(c)+a}{bmn}\right)}-(bmn\log(fx+e)+bn\log(d)+b\log(c)+a)\,\text{log\_integral}\left((f\ldots\right.\right.}{b^3fm^3n^3\log(fx+e)+b^3fm^2n^3\log(d)+b^3fm^2n^2\log(c)+ab^2\ldots}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="fricas")
```

output
```
-((b*f*m*n*x + b*e*m*n)*e^((b*n*log(d) + b*log(c) + a)/(b*m*n)) - (b*m*n*l
og(f*x + e) + b*n*log(d) + b*log(c) + a)*log_integral((f*x + e)*e^((b*n*lo
g(d) + b*log(c) + a)/(b*m*n))))*e^(-(b*n*log(d) + b*log(c) + a)/(b*m*n))/(
b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log(c) +
 a*b^2*f*m^2*n^2)
```

3.409.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-2), x)
```

3.409.7 Maxima [F]

$$\int \frac{1}{(a+b\log (c\,(d(e+fx)^m)^n))^2}\,dx = \int \frac{1}{(b\log (((fx+e)^m d)^n\, c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="maxima")
```

output
```
-(f*x + e)/(b^2*f*m*n*log(((f*x + e)^m)^n) + a*b*f*m*n + (f*m*n^2*log(d) +
 f*m*n*log(c))*b^2) + integrate(1/(b^2*m*n*log(((f*x + e)^m)^n) + a*b*m*n
+ (m*n^2*log(d) + m*n*log(c))*b^2), x)
```

3.409.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(123) = 246$.

Time = 0.34 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.73

$$\int \frac{1}{(a+b\log (c\,(d(e+fx)^m)^n))^2}\,dx$$
$$= -\frac{(fx+e)bmn}{b^3fm^3n^3\log (fx+e)+b^3fm^2n^3\log (d)+b^3fm^2n^2\log (c)+ab^2fm^2n^2}$$
$$+\frac{bmn\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log (fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}\log (fx+e)}{(b^3fm^3n^3\log (fx+e)+b^3fm^2n^3\log (d)+b^3fm^2n^2\log (c)+ab^2fm^2n^2)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}}$$
$$+\frac{bn\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log (fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}\log (d)}{(b^3fm^3n^3\log (fx+e)+b^3fm^2n^3\log (d)+b^3fm^2n^2\log (c)+ab^2fm^2n^2)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}}$$
$$+\frac{b\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log (fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}\log (c)}{(b^3fm^3n^3\log (fx+e)+b^3fm^2n^3\log (d)+b^3fm^2n^2\log (c)+ab^2fm^2n^2)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}}$$
$$+\frac{a\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log (fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}}{(b^3fm^3n^3\log (fx+e)+b^3fm^2n^3\log (d)+b^3fm^2n^2\log (c)+ab^2fm^2n^2)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="giac")
```

output
```
-(f*x + e)*b*m*n/(b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d) + b^3*
f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2) + b*m*n*Ei(log(d)/m + log(c)/(m*n) + a
/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)/((b^3*f*m^3*n^3*log(f
*x + e) + b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)*c
^(1/(m*n))*d^(1/m)) + b*n*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x
 + e))*e^(-a/(b*m*n))*log(d)/((b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*
log(d) + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)*c^(1/(m*n))*d^(1/m)) + b*
Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(
c)/((b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log
(c) + a*b^2*f*m^2*n^2)*c^(1/(m*n))*d^(1/m)) + a*Ei(log(d)/m + log(c)/(m*n)
 + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))/((b^3*f*m^3*n^3*log(f*x + e) +
 b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)*c^(1/(m*n)
)*d^(1/m))
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = \int \frac{1}{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^2}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^2,x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^2, x)
```

3.410 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx$

3.410.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{2b^3fm^3n^3} - \frac{e+fx}{2bfmn\,(a+b\log(c(d(e+fx)^m)^n))^2} - \frac{e+fx}{2b^2fm^2n^2\,(a+b\log(c(d(e+fx)^m)^n))}$$

output
```
1/2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)/b^3/exp(a/b/m/n)/f/m^3/n
^3/((c*(d*(f*x+e)^m)^n)^(1/m/n))+1/2*(-f*x-e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)
^m)^n))^2+1/2*(-f*x-e)/b^2/f/m^2/n^2/(a+b*ln(c*(d*(f*x+e)^m)^n))
```

3.410.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = -\frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(-\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log(c(d(e+fx)^m)^n\right.}{2b^3fm^3n^3\,(a+b\log(c(d(e+fx)^m}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3),x]
```

output
```
-1/2*((e + f*x)*(-(ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)]
*(a + b*Log[c*(d*(e + f*x)^m)^n])^2) + b*E^(a/(b*m*n))*m*n*(c*(d*(e + f*x)
^m)^n)^(1/(m*n))*(a + b*m*n + b*Log[c*(d*(e + f*x)^m)^n])))/(b^3*E^(a/(b*m
*n))*f*m^3*n^3*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(a + b*Log[c*(d*(e + f*x)^m
)^n])^2)
```

3.410.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2895, 2836, 2734, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log (c (d(e+fx)^m)^n))^3} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{(a+b\log (c (d(e+fx)^m)^n))^3}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^3}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^2}d(e+fx)}{2bmn}-\frac{e+fx}{2bmn(a+b\log(cd^n(e+fx)^{mn}))^2}}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\frac{\int \frac{1}{a+b\log(cd^n(e+fx)^{mn})}d(e+fx)}{bmn}-\frac{e+fx}{bmn(a+b\log(cd^n(e+fx)^{mn}))}}{2bmn}-\frac{e+fx}{2bmn(a+b\log(cd^n(e+fx)^{mn}))^2}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{\frac{(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\int \frac{(cd^n(e+fx)^{mn})^{\frac{1}{mn}}}{a+b\log(cd^n(e+fx)^{mn})}d\log(cd^n(e+fx)^{mn})}{bm^2n^2}-\frac{e+fx}{bmn(a+b\log(cd^n(e+fx)^{mn}))}}{2bmn}-\frac{e+fx}{2bmn(a+b\log(cd^n(e+fx)^{mn}))^2}}{f}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{\frac{(e+fx)e^{-\frac{a}{bmn}}(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^n(e+fx)^{mn})}{bmn}\right)}{b^2m^2n^2}-\frac{e+fx}{bmn(a+b\log(cd^n(e+fx)^{mn}))}}{2bmn}-\frac{e+fx}{2bmn(a+b\log(cd^n(e+fx)^{mn}))^2}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3),x]
```

output
```
(-1/2*(e + f*x)/(b*m*n*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^2) + (((e + f*x)
*ExpIntegralEi[(a + b*Log[c*d^n*(e + f*x)^(m*n)])/(b*m*n)])/(b^2*E^(a/(b*m
*n))*m^2*n^2*(c*d^n*(e + f*x)^(m*n))^(1/(m*n))) - (e + f*x)/(b*m*n*(a + b*
Log[c*d^n*(e + f*x)^(m*n)])))/(2*b*m*n))/f
```

3.410.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.410. $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx$

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.410.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{3}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^3,x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^3,x)
```

3.410.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(163) = 326.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.63

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = -\frac{\left((b^2em^2n^2+abemn+(b^2fm^2n^2+abfmn)x+(b^2fm^2n^2x+b^2em^2n^2)\log(fx+e)+(b^2fmnx+b^2e\right.}{2\left(b^5fm^5n^5\log\right.}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*e*m^2*n^2 + a*b*e*m*n + (b^2*f*m^2*n^2 + a*b*f*m*n)*x + (b^2*f*
m^2*n^2*x + b^2*e*m^2*n^2)*log(f*x + e) + (b^2*f*m*n*x + b^2*e*m*n)*log(c)
 + (b^2*f*m*n^2*x + b^2*e*m*n^2)*log(d))*e^((b*n*log(d) + b*log(c) + a)/(b
*m*n)) - (b^2*m^2*n^2*log(f*x + e)^2 + b^2*n^2*log(d)^2 + b^2*log(c)^2 + 2
*a*b*log(c) + a^2 + 2*(b^2*m*n^2*log(d) + b^2*m*n*log(c) + a*b*m*n)*log(f*
x + e) + 2*(b^2*n*log(c) + a*b*n)*log(d))*log_integral((f*x + e)*e^((b*n*l
og(d) + b*log(c) + a)/(b*m*n))))*e^(-(b*n*log(d) + b*log(c) + a)/(b*m*n))/
(b^5*f*m^5*n^5*log(f*x + e)^2 + b^5*f*m^3*n^5*log(d)^2 + b^5*f*m^3*n^3*log
(c)^2 + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3 + 2*(b^5*f*m^4*n^5*lo
g(d) + b^5*f*m^4*n^4*log(c) + a*b^4*f*m^4*n^4)*log(f*x + e) + 2*(b^5*f*m^3
*n^4*log(c) + a*b^4*f*m^3*n^4)*log(d))
```

3.410.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**3,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-3), x)
```

3.410.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="maxima")
```

output
```
-1/2*((e*m*n + e*n*log(d) + e*log(c))*b + a*e + ((f*m*n + f*n*log(d) + f*l
og(c))*b + a*f)*x + (b*f*x + b*e)*log(((f*x + e)^m)^n))/(b^4*f*m^2*n^2*log
(((f*x + e)^m)^n)^2 + a^2*b^2*f*m^2*n^2 + 2*(f*m^2*n^3*log(d) + f*m^2*n^2*
log(c))*a*b^3 + (f*m^2*n^4*log(d)^2 + 2*f*m^2*n^3*log(c)*log(d) + f*m^2*n^
2*log(c)^2)*b^4 + 2*(a*b^3*f*m^2*n^2 + (f*m^2*n^3*log(d) + f*m^2*n^2*log(c
))*b^4)*log(((f*x + e)^m)^n)) + integrate(1/2/(b^3*m^2*n^2*log(((f*x + e)^
m)^n) + a*b^2*m^2*n^2 + (m^2*n^3*log(d) + m^2*n^2*log(c))*b^3), x)
```

3.410.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3401 vs. $2(163) = 326$.

Time = 0.38 (sec) , antiderivative size = 3401, normalized size of antiderivative = 20.12

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="giac")
```

output
```
-1/2*(f*x + e)*b^2*m^2*n^2*log(f*x + e)/(b^5*f*m^5*n^5*log(f*x + e)^2 + 2*
b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) +
b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*
log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^
4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3) + 1/2*b^2*m^2*n^2*Ei(log(d)/m + lo
g(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)^2/((b^5
*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*
m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*l
og(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a
*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1
/(m*n))*d^(1/m)) - 1/2*(f*x + e)*b^2*m^2*n^2/(b^5*f*m^5*n^5*log(f*x + e)^2
 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(
c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3
*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2
*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3) - 1/2*(f*x + e)*b^2*m*n^2*log
(d)/(b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) +
2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m
^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)
^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n
^3) + b^2*m*n^2*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*...
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^m)^n))^3}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^3,x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^3, x)
```

3.411 $\int (a+b\log (c(d(e+fx)^m)^n))^{5/2} \, dx$

3.411.1 Optimal result

Integrand size = 22, antiderivative size = 219

$$\int (a+b\log (c(d(e+fx)^m)^n))^{5/2} \, dx = -\frac{15b^{5/2}e^{-\frac{a}{bmn}}m^{5/2}n^{5/2}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{8f} + \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{4f} - \frac{5bmn(e+fx)\left(a+b\log(c(d(e+fx)^m)^n)\right)^{3/2}}{2f} + \frac{(e+fx)\left(a+b\log(c(d(e+fx)^m)^n)\right)^{5/2}}{f}$$

output
```
-5/2*b*m*n*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)/f+(f*x+e)*(a+b*ln(c*(
d*(f*x+e)^m)^n))^(5/2)/f-15/8*b^(5/2)*m^(5/2)*n^(5/2)*(f*x+e)*erfi((a+b*ln
(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/2)/exp(a/b/m/n)/
f/((c*(d*(f*x+e)^m)^n)^(1/m/n))+15/4*b^2*m^2*n^2*(f*x+e)*(a+b*ln(c*(d*(f*x
+e)^m)^n))^(1/2)/f
```

3.411.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.87

$$\int (a + b \log (c(d(e+fx)^m)^n))^{5/2} \, dx = \frac{(e+fx) \left(8(a+b\log (c(d(e+fx)^m)^n))^{5/2} - 5bmn\left(3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2),x]
```

output
```
((e + f*x)*(8*(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2) - 5*b*m*n*((3*b^(3/2)
*m^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[
b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*(d*(e + f*x)^m)^n)^(1/(m*n))) + 2*
Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]*(2*a - 3*b*m*n + 2*b*Log[c*(d*(e + f*
x)^m)^n]))))/(8*f)
```

3.411.3 Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2895, 2836, 2733, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b \log (c(d(e+fx)^m)^n))^{5/2} \, dx$$

$$\Big\downarrow \ 2895$$

$$\int (a + b \log (c(d(e+fx)^m)^n))^{5/2} dx$$

$$\Big\downarrow \ 2836$$

$$\frac{\int (a + b \log (cd^n(e+fx)^{mn}))^{5/2} d(e+fx)}{f}$$

$$\Big\downarrow \ 2733$$

$$\frac{(e+fx)(a+b\log (cd^n(e+fx)^{mn}))^{5/2} - \frac{5}{2}bmn\int (a+b\log (cd^n(e+fx)^{mn}))^{3/2} d(e+fx)}{f}$$

$\downarrow$ 2733

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{5/2}-\frac{5}{2}bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\int\sqrt{a+b\log\left(c\right.}}{f}$$

$\downarrow$ 2733

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{5/2}-\frac{5}{2}bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\Big((e+fx)\sqrt{a+}}{f}$$

$\downarrow$ 2737

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{5/2}-\frac{5}{2}bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\Big((e+fx)\sqrt{a+}}{f}$$

$\downarrow$ 2611

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{5/2}-\frac{5}{2}bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\Big((e+fx)\sqrt{a+}}{}$$

$\downarrow$ 2633

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{5/2}-\frac{5}{2}bmn\Big((e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\Big((e+fx)\sqrt{a+}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2),x]
```

output
```
((e + f*x)*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^(5/2) - (5*b*m*n*((e + f*x)*
(a + b*Log[c*d^n*(e + f*x)^(m*n)])^(3/2) - (3*b*m*n*(-1/2*(Sqrt[b]*Sqrt[m]
*Sqrt[n]*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]/(S
qrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*d^n*(e + f*x)^(m*n))^(1/(m*n))
) + (e + f*x)*Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]))/2))/2)/f
```

3.411. $\int \left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}\,dx$

3.411.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.411.4 Maple [F]

$$\int (a+b\ln (c(d(fx+e)^m)^n))^{\frac{5}{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

3.411.5 Fricas [F(-2)]

Exception generated.

$$\int (a + b\log (c(d(e + fx)^m)^n))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\log (c(d(e + fx)^m)^n))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**(5/2),x)
```

output
```
Timed out
```

3.411.7 Maxima [F]

$$\int (a + b\log (c(d(e + fx)^m)^n))^{5/2}\,dx = \int (b\log (((fx + e)^m d)^n c) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(5/2), x)
```

3.411.8 Giac [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx=\int (b\log(((fx+e)^m d)^n c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(5/2), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx=\int (a+b\,\ln(c\,(d\,(e+f\,x)^m)^n))^{5/2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^(5/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^(5/2), x)
```

3.412 $\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx$

3.412.1 Optimal result

Integrand size = 22, antiderivative size = 176

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \frac{3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{4f} - \frac{3bmn(e+fx)\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{2f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{f}$$

output
```
(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)/f+3/4*b^(3/2)*m^(3/2)*n^(3/2)*(f
*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(
1/2)/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m/n))-3/2*b*m*n*(f*x+e)*(a+b*l
n(c*(d*(f*x+e)^m)^n))^(1/2)/f
```

3.412.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \frac{(e+fx)\left(3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx}{\sqrt{b}\sqrt{m}\sqrt{n}}\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(3/2),x]
```

output
```
((e + f*x)*((3*b^(3/2)*m^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d*(
e + f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*(d*(e + f*x)
^m)^n)^(1/(m*n))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]*(2*a - 3*b*m*n
+ 2*b*Log[c*(d*(e + f*x)^m)^n])))/(4*f)
```

3.412.3 Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2895, 2836, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\int \sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\left((e+fx)\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}-\frac{1}{2}bmn\int \frac{1}{\sqrt{a+b\log(cd^n(e+fx$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\left((e+fx)\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}-\frac{1}{2}b(e+fx)\left(cd^n(e+fx)\right.}{f}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\Big((e+fx)\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}-(e+fx)\left(cd^n(e+fx)^{mn}\right.}{f}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}-\frac{3}{2}bmn\Big((e+fx)\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{m}\sqrt{n}(e+fx)e}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(3/2),x]
```

output
```
((e + f*x)*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^(3/2) - (3*b*m*n*(-1/2*(Sqrt
[b]*Sqrt[m]*Sqrt[n]*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^n*(e + f*x)
^(m*n)]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*d^n*(e + f*x)^(m*n)
)^(1/(m*n))) + (e + f*x)*Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]))/2)/f
```

3.412.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.412.4 Maple [F]

$$\int (a+b\ln (c(d(fx+e)^{m})^{n}))^{\frac{3}{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

3.412.5 Fricas [F(-2)]

Exception generated.

$$\int (a+b\log (c(d(e+fx)^{m})^{n}))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.412.6 Sympy [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx=\int (a+b\log(c(d(e+fx)^m)^n))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(3/2), x)
```

3.412.7 Maxima [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx=\int (b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(3/2), x)
```

3.412.8 Giac [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx=\int (b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(3/2), x)
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c(d(e+fx)^m)^n\right)\right)^{3/2}\,dx=\int \left(a+b\,\ln \left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^{3/2}dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^(3/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^(3/2), x)
```

3.413 $\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx$

3.413.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bmn}}\sqrt{m}\sqrt{n}\sqrt{\pi}(e+fx)\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{2f} + \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f}$$

output
```
-1/2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2
))*b^(1/2)*m^(1/2)*n^(1/2)*Pi^(1/2)/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1
/m/n))+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/f
```

3.413.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{(e+fx)\left(-\sqrt{b}e^{-\frac{a}{bmn}}\sqrt{m}\sqrt{n}\sqrt{\pi}\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)+2\sqrt{a+b\log\left(c\left(d(e+\right.\right.}\right.}{2f}$$

input
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
```

output
```
((e + f*x)*(-((Sqrt[b]*Sqrt[m]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d*(
e + f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*(d*(e + f*x)
^m)^n)^(1/(m*n)))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]))/(2*f)
```

3.413.3 Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2895, 2836, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\log \left(c \left(d(e+fx)^m\right)^n\right)} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \sqrt{a+b\log \left(c \left(d(e+fx)^m\right)^n\right)}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \sqrt{a+b\log \left(cd^n(e+fx)^{mn}\right)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)\sqrt{a+b\log \left(cd^n(e+fx)^{mn}\right)}-\frac{1}{2}bmn\int \frac{1}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\sqrt{a+b\log \left(cd^n(e+fx)^{mn}\right)}-\frac{1}{2}b(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \frac{\left(cd^n(e+fx)^{mn}\right)^{\frac{1}{mn}}}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d\log \left(cd^n(e+fx)^{\right.}}{f}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(e+fx)\sqrt{a+b\log \left(cd^n(e+fx)^{mn}\right)}-(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \exp \left(\frac{a+b\log(cd^n(e+fx)^{mn})}{bmn}-\frac{a}{bmn}\right)d\sqrt{a+b}}{f}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(e+fx)\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{m}\sqrt{n}(e+fx)e^{-\frac{a}{bmn}}\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^n(e+fx)^{r}}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right.}{f}$$

input
```
Int[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
```

output
```
(-1/2*(Sqrt[b]*Sqrt[m]*Sqrt[n]*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^
n*(e + f*x)^(m*n)]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*d^n*(e +
 f*x)^(m*n))^(1/(m*n))) + (e + f*x)*Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]
)/f
```

3.413.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.413.4 Maple [F]

$$\int \sqrt{a+b\ln \left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
```

3.413.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log \left(c\left(d(e+fx)^{m}\right)^{n}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.413.6 Sympy [F]

$$\int \sqrt{a+b\log \left(c\left(d(e+fx)^{m}\right)^{n}\right)}\,dx = \int \sqrt{a+b\log \left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**m)**n)), x)
```

3.413.7 Maxima [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{b\log\left(\left(\left((fx+e)^m d\right)^n c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

3.413.8 Giac [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{b\log\left(\left(\left((fx+e)^m d\right)^n c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{a+b\,\ln\left(c\left(d\,(e+f\,x)^m\right)^n\right)}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^(1/2), x)
```

3.414 $\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}\,dx$

3.414.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \frac{e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

output
```
(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi
^(1/2)/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m/n))/b^(1/2)/m^(1/2)/n^(1/2
)
```

3.414.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \frac{e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

input
```
Integrate[1/Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
```

output
```
(Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]*Sqr
t[m]*Sqrt[n])])/(Sqrt[b]*E^(a/(b*m*n))*f*Sqrt[m]*Sqrt[n]*(c*(d*(e + f*x)^m
)^n)^(1/(m*n)))
```

3.414.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2895, 2836, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \frac{\left(cd^n(e+fx)^{mn}\right)^{\frac{1}{mn}}}{\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}d\log\left(cd^n(e+fx)^{mn}\right)}{fmn}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \exp\left(\frac{a+b\log\left(cd^n(e+fx)^{mn}\right)}{bmn}-\frac{a}{bmn}\right)d\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}{bfmn}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

input
```
Int[1/Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
```

output
```
(Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]/(Sqrt[b]*S
qrt[m]*Sqrt[n])])/(Sqrt[b]*E^(a/(b*m*n))*f*Sqrt[m]*Sqrt[n]*(c*d^n*(e + f*x
)^(m*n))^(1/(m*n)))
```

3.414.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.414. $\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}\,dx$

3.414.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
```

3.414.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.414.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*log(c*(d*(e + f*x)**m)**n)), x)
```

3.414.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

3.414.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(1/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(1/2), x)
```

3.415 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx$

3.415.1 Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}fm^{3/2}n^{3/2}} - \frac{2(e+fx)}{bfmn\sqrt{a+b\log(c(d(e+fx)^m)^n)}}$$

output
```
2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*
Pi^(1/2)/b^(3/2)/exp(a/b/m/n)/f/m^(3/2)/n^(3/2)/((c*(d*(f*x+e)^m)^n)^(1/m/
n))-2*(f*x+e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = -\frac{2e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(e^{\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{\frac{1}{mn}} - \Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\sqrt{-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}}\right)}{bfmn\sqrt{a+b\log(c(d(e+fx)^m)^n)}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3/2),x]
```

output
```
(-2*(e + f*x)*(E^(a/(b*m*n))*(c*(d*(e + f*x)^m)^n)^(1/(m*n)) - Gamma[1/2,
-((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*Sqrt[-((a + b*Log[c*(d*(e + f
*x)^m)^n])/(b*m*n))]))/(b*E^(a/(b*m*n))*f*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*
n))*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]])
```

3.415.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2895, 2836, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d(e+fx)}{bmn}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{2(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\int \frac{(cd^n(e+fx)^{mn})^{\frac{1}{mn}}}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d\log(cd^n(e+fx)^{mn})}{bm^2n^2}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}}{f}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{4(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\int \exp\left(\frac{a+b\log(cd^n(e+fx)^{mn})}{bmn}-\frac{a}{bmn}\right)d\sqrt{a+b\log(cd^n(e+fx)^{mn})}}{b^2m^2n^2}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}}{f}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}m^{3/2}n^{3/2}}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3/2),x]
```

output
```
((2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]/(Sqrt[b
]*Sqrt[m]*Sqrt[n])])/(b^(3/2)*E^(a/(b*m*n))*m^(3/2)*n^(3/2)*(c*d^n*(e + f*
x)^(m*n))^(1/(m*n))) - (2*(e + f*x))/(b*m*n*Sqrt[a + b*Log[c*d^n*(e + f*x)
^(m*n)]]))/f
```

3.415.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.415. $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx$

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.415.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

3.415.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.415.6 Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-3/2), x)
```

3.415.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-3/2), x)
```

3.415.8 Giac [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-3/2), x)
```

3.415.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^m)^n))^{3/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(3/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(3/2), x)
```

3.416 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx$

3.416.1 Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{3b^{5/2}fm^{5/2}n^{5/2}} - \frac{2(e+fx)}{3bfmn\,(a+b\log(c(d(e+fx)^m)^n))^{3/2}} - \frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log(c(d(e+fx)^m)^n)}}$$

output
```
-2/3*(f*x+e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)+4/3*(f*x+e)*erfi((a
+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/
exp(a/b/m/n)/f/m^(5/2)/n^(5/2)/((c*(d*(f*x+e)^m)^n)^(1/m/n))-4/3*(f*x+e)/b
^2/f/m^2/n^2/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = -\frac{2e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(2bmn\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\left(-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)^{3/2}+e^{\frac{a}{bmn}}\right.}{3b^2fm^2n^2\,(a+b\log(c(d(e+fx)^m)^n))^{3/2}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-5/2),x]
```

output
```
(-2*(e + f*x)*(2*b*m*n*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*
n))]*(-((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)))^(3/2) + E^(a/(b*m*n))*(
c*(d*(e + f*x)^m)^n)^(1/(m*n))*(2*a + b*m*n + 2*b*Log[c*(d*(e + f*x)^m)^n]
)))/(3*b^2*E^(a/(b*m*n))*f*m^2*n^2*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(a + b*
Log[c*(d*(e + f*x)^m)^n])^(3/2))
```

3.416.3 Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2895, 2836, 2734, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx$$

$$\Big\downarrow \ 2895$$

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}dx$$

$$\Big\downarrow \ 2836$$

$$\frac{\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{5/2}}d(e+fx)}{f}$$

$$\Big\downarrow \ 2734$$

$$\frac{\frac{2\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}d(e+fx)}{3bmn}-\frac{2(e+fx)}{3bmn(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}}{f}$$

$$\Big\downarrow \ 2734$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d(e+fx)}{bmn}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}\right)}{3bmn}-\frac{2(e+fx)}{3bmn(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}}{f}$$

$$\Big\downarrow \ 2737$$

$$\frac{\frac{2\left(\frac{2(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \frac{\left(cd^n(e+fx)^{mn}\right)^{\frac{1}{mn}}}{\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}d\log\left(cd^n(e+fx)^{mn}\right)}{bm^2n^2}-\frac{2(e+fx)}{bmn\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}\right)}{3bmn}-\frac{2(e+fx)}{3bmn\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/}}}{f}$$

$\Big\downarrow$ 2611

$$\frac{\frac{2\left(\frac{4(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \exp\left(\frac{a+b\log\left(cd^n(e+fx)^{mn}\right)}{bmn}-\frac{a}{bmn}\right)d\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}{b^2m^2n^2}-\frac{2(e+fx)}{bmn\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}\right)}{3bmn}-\frac{2(e+}{3bmn(a+b\log(cd}}{f}$$

$\Big\downarrow$ 2633

$$\frac{\frac{2\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}m^{3/2}n^{3/2}}-\frac{2(e+fx)}{bmn\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}\right)}{3bmn}-\frac{2(e+fx)}{3bmn\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-5/2),x]
```

output
```
((-2*(e + f*x))/(3*b*m*n*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^(3/2)) + (2*((
2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]/(Sqrt[b]*
Sqrt[m]*Sqrt[n])])/(b^(3/2)*E^(a/(b*m*n))*m^(3/2)*n^(3/2)*(c*d^n*(e + f*x)
^(m*n))^(1/(m*n))) - (2*(e + f*x))/(b*m*n*Sqrt[a + b*Log[c*d^n*(e + f*x)^(
m*n)]])))/(3*b*m*n))/f
```

3.416.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

3.416. $\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}}\,dx$

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.416.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

3.416.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.416.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-5/2), x)
```

3.416.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-5/2), x)
```

3.416.8 Giac [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-5/2), x)
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\ln(c(d(e+fx)^m)^n))^{5/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(5/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(5/2), x)
```

3.417 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx$

3.417.1 Optimal result

Integrand size = 22, antiderivative size = 237

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx = \frac{8e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{15b^{7/2}fm^{7/2}n^{7/2}} - \frac{2(e+fx)}{5bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

output
```
-2/5*(f*x+e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2)-4/15*(f*x+e)/b^2/f/
m^2/n^2/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)+8/15*(f*x+e)*erfi((a+b*ln(c*(d*(
f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/2)/b^(7/2)/exp(a/b/m/n)
/f/m^(7/2)/n^(7/2)/((c*(d*(f*x+e)^m)^n)^(1/m/n))-8/15*(f*x+e)/b^3/f/m^3/n^
3/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)
```

3.417.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx =$$
$$-\frac{2e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(-4\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log\left(c(d(e+fx)^m)^n\right))^2\sqrt{-\frac{a-}{}}\right.}{15b^3}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-7/2),x]
```

output
```
(-2*(e + f*x)*(-4*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*
(a + b*Log[c*(d*(e + f*x)^m)^n])^2*Sqrt[-((a + b*Log[c*(d*(e + f*x)^m)^n])
/(b*m*n))] + E^(a/(b*m*n))*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(4*a^2 + 2*a*b*
m*n + 3*b^2*m^2*n^2 + 2*b*(4*a + b*m*n)*Log[c*(d*(e + f*x)^m)^n] + 4*b^2*L
og[c*(d*(e + f*x)^m)^n]^2)))/(15*b^3*E^(a/(b*m*n))*f*m^3*n^3*(c*(d*(e + f*
x)^m)^n)^(1/(m*n))*(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2))
```

3.417.3 Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2895, 2836, 2734, 2734, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}dx$$
$$\Big\downarrow \text{ 2836}$$
$$\frac{\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{7/2}}d(e+fx)}{f}$$
$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{2\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{5/2}}d(e+fx)}{5bmn}-\frac{2(e+fx)}{5bmn(a+b\log(cd^n(e+fx)^{mn}))^{5/2}}}{f}$$

$$\Big\downarrow \ 2734$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}d(e+fx)}{3bmn}-\frac{2(e+fx)}{3bmn(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}\right)}{5bmn}-\frac{2(e+fx)}{5bmn(a+b\log(cd^n(e+fx)^{mn}))^{5/2}}}{f}$$

$$\Big\downarrow \ 2734$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d(e+fx)}{bmn}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}\right)}{3bmn}-\frac{2(e+fx)}{3bmn(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}\right)}{5bmn}-\frac{2(e+fx)}{5bmn(a+b\log(cd^n(e+fx)^{mn}))^{5/2}}}{f}$$

$$\Big\downarrow \ 2737$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\int \frac{(cd^n(e+fx)^{mn})^{\frac{1}{mn}}}{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}d\log(cd^n(e+fx)^{mn})}{bm^2n^2}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}\right)}{3bmn}-\frac{2(e+fx)}{3bmn(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}\right)}{5bmn}}{f}$$

$$\Big\downarrow \ 2611$$

$$\frac{\frac{2\left(\frac{2\left(\frac{4(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\int \exp\left(\frac{a+b\log(cd^n(e+fx)^{mn})}{bmn}-\frac{a}{bmn}\right)d\sqrt{a+b\log(cd^n(e+fx)^{mn})}}{b^2m^2n^2}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}\right)}{3bmn}-\frac{2(e+f}{3bmn(a+b\log(cd^n}\right.}{5bmn}}{f}$$

$$\Big\downarrow \ 2633$$

3.417. $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^n(e+fx)^{mn})}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}m^{3/2}n^{3/2}}-\frac{2(e+fx)}{bmn\sqrt{a+b\log(cd^n(e+fx)^{mn})}}\right)}{3bmn}-\frac{2(e+fx)}{3bmn(a+b\log(cd^n(e+fx)^{mn}))^{3/2}}\right)}{5bmn}-}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-7/2),x]
```

output
```
((-2*(e + f*x))/(5*b*m*n*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^(5/2)) + (2*((
-2*(e + f*x))/(3*b*m*n*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^(3/2)) + (2*((2*
Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*n)]]/(Sqrt[b]*Sq
rt[m]*Sqrt[n])])/(b^(3/2)*E^(a/(b*m*n))*m^(3/2)*n^(3/2)*(c*d^n*(e + f*x)^(
m*n))^(1/(m*n))) - (2*(e + f*x))/(b*m*n*Sqrt[a + b*Log[c*d^n*(e + f*x)^(m*
n)]])))/(3*b*m*n)))/(5*b*m*n))/f
```

3.417.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

3.417. $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx$

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.417.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(7/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(7/2),x)
```

3.417.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(7/2),x)
```

output
```
Timed out
```

3.417.7 Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-7/2), x)
```

3.417.8 Giac [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-7/2), x)
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^m)^n))^{7/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(7/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(7/2), x)
```

3.418 $\int (a+b\log(c(d(e+fx)^m)^n))^p\,dx$

3.418.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\int (a+b\log(c(d(e+fx)^m)^n))^p\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log(c(d(e+fx)^m)^n))^p\left(-\frac{a+b\log\right.}{f}$$

output
```
(f*x+e)*GAMMA(p+1,(-a-b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)*(a+b*ln(c*(d*(f*x+e)
^m)^n))^p/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m/n))/(((-a-b*ln(c*(d*(f*
x+e)^m)^n))/b/m/n)^p)
```

3.418.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d(e+fx)^m)^n))^p\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log(c(d(e+fx)^m)^n))^p\left(-\frac{a+b\log\right.}{f}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^p,x]
```

output
```
((e + f*x)*Gamma[1 + p, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*(a +
b*Log[c*(d*(e + f*x)^m)^n])^p)/(E^(a/(b*m*n))*f*(c*(d*(e + f*x)^m)^n)^(1/(
m*n))*(-((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)))^p)
```

3.418.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2895, 2836, 2737, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d(e+fx)^m)^n))^p\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log (c(d(e+fx)^m)^n))^p\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log (cd^n(e+fx)^{mn}))^p\,d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\int \left(cd^n(e+fx)^{mn}\right)^{\frac{1}{mn}}(a+b\log (cd^n(e+fx)^{mn}))^p\,d\log \left(cd^n(e+fx)^{mn}\right)}{fmn}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(e+fx)e^{-\frac{a}{bmn}}\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}(a+b\log (cd^n(e+fx)^{mn}))^p\left(-\frac{a+b\log(cd^n(e+fx)^{mn})}{bmn}\right)^{-p}\Gamma\Big(p+1,-\frac{a+b\log(cd^n(e}{bmn}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^p,x]
```

output
```
((e + f*x)*Gamma[1 + p, -((a + b*Log[c*d^n*(e + f*x)^(m*n)])/(b*m*n))]*(a
+ b*Log[c*d^n*(e + f*x)^(m*n)])^p)/(E^(a/(b*m*n))*f*(c*d^n*(e + f*x)^(m*n)
)^(1/(m*n))*(-((a + b*Log[c*d^n*(e + f*x)^(m*n)])/(b*m*n)))^p)
```

3.418.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.418.4 Maple [F]

$$\int (a+b\ln(c(d(fx+e)^m)^n))^p dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^p,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^m)^n))^p,x)
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \frac{e^{\left(-\frac{bmnp\log\left(-\frac{1}{bmn}\right)+bn\log(d)+b\log(c)+a}{bmn}\right)}\Gamma\left(p+1,-\frac{bmn\log(fx+e)+bn\log(d)+b\log(c)+a}{bmn}\right)}{f}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^p,x, algorithm="fricas")
```

output
```
e^(-(b*m*n*p*log(-1/(b*m*n)) + b*n*log(d) + b*log(c) + a)/(b*m*n))*gamma(p
 + 1, -(b*m*n*log(f*x + e) + b*n*log(d) + b*log(c) + a)/(b*m*n))/f
```

3.418.6 Sympy [F]

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \int (a+b\log (c(d(e+fx)^m)^n))^p \, dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**p,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**m)**n))**p, x)
```

3.418.7 Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.418.8 Giac [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^p\,dx=\int (b\log(((fx+e)^m d)^n c)+a)^p\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^m)^n))^p,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^m*d)^n*c) + a)^p, x)
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d(e+fx)^m)^n))^p\,dx=\int \left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^p\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^p,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^m)^n))^p, x)
```

3.419 $\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx$

3.419.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx = \frac{4^{-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)}{b}\right)^{-p}}{c^4d^2f}$$

output
```
GAMMA(p+1,-4*(a+b*ln(c*(d*(f*x+e)^(1/2))^(1/2)))/b)*(a+b*ln(c*(d*(f*x+e)^(
1/2))^(1/2)))^p/(4^p)/c^4/d^2/exp(4*a/b)/f/(((-a-b*ln(c*(d*(f*x+e)^(1/2))^
(1/2)))/b)^p)
```

3.419.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx = \frac{2^{-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)}{b}\right)^{-p}}{c^4d^2f}$$

input
```
Integrate[(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]])^p,x]
```

output
```
(Gamma[1 + p, (-4*(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]]))/b]*(a + b*Log[c*Sq
rt[d*Sqrt[e + f*x]]])^p)/(2^(2*p)*c^4*d^2*E^((4*a)/b)*f*(-((a + b*Log[c*Sq
rt[d*Sqrt[e + f*x]]])/b))^p)
```

3.419.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2895, 2836, 2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \left(a+b\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)\right)^p d(e+fx)}{f}$$

$$\Big\downarrow \text{ 2736}$$

$$\frac{4\int c^4d^2(e+fx)\left(a+b\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)\right)^p d\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)}{c^4d^2f}$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{4^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)\right)}{b}\right)}{c^4d^2f}$$

input
```
Int[(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]])^p,x]
```

output
```
(Gamma[1 + p, (-4*(a + b*Log[c*Sqrt[d]*(e + f*x)^(1/4)]))/b]*(a + b*Log[c*
Sqrt[d]*(e + f*x)^(1/4)])^p)/(4^p*c^4*d^2*E^((4*a)/b)*f*(-((a + b*Log[c*Sq
rt[d]*(e + f*x)^(1/4)])/b))^p)
```

3.419. $\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx$

3.419.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.419.4 Maple [F]

$$\int \left(a+b\ln\left(c\sqrt{d\sqrt{fx+e}}\right)\right)^p dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x)
```

3.419.5 Fricas [F]

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx=\int \left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x, algorithm="fricas")
```

output
```
integral((b*log(sqrt(sqrt(f*x + e)*d)*c) + a)^p, x)
```

3.419.6 Sympy [F]

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx=\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**(1/2))**(1/2)))**p,x)
```

output
```
Integral((a + b*log(c*sqrt(d*sqrt(e + f*x))))**p, x)
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx = -\frac{4\left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)^{p+1}e^{\left(-\frac{4a}{b}\right)}E_{-p}\left(-\frac{4\left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)}{b}\right)}{bc^4d^2f}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x, algorithm="maxima")
```

output
```
-4*(b*log(sqrt(sqrt(f*x + e)*d)*c) + a)^(p + 1)*e^(-4*a/b)*exp_integral_e(
-p, -4*(b*log(sqrt(sqrt(f*x + e)*d)*c) + a)/b)/(b*c^4*d^2*f)
```

3.419.8 Giac [F]

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx = \int \left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x, algorithm="giac")
```

output
```
integrate((b*log(sqrt(sqrt(f*x + e)*d)*c) + a)^p, x)
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx = \int \left(a+b\,\ln\left(c\,\sqrt{d\,\sqrt{e+f\,x}}\right)\right)^p dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^(1/2))^(1/2)))^p,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^(1/2))^(1/2)))^p, x)
```

3.420 $\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx$

3.420.1 Optimal result

Integrand size = 26, antiderivative size = 158

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{b(fg-eh)^3pqx}{4f^3} - \frac{b(fg-eh)^2pq(g+hx)^2}{8f^2h} - \frac{b(fg-eh)pq(g+hx)^3}{12fh} - \frac{bpq(g+hx)^4}{16h} - \frac{b(fg-eh)^4pq\log(e+fx)}{4f^4h} + \frac{(g+hx)^4(a+b\log(c(d(e+fx)^p)^q))}{4h}$$

output
```
-1/4*b*(-e*h+f*g)^3*p*q*x/f^3-1/8*b*(-e*h+f*g)^2*p*q*(h*x+g)^2/f^2/h-1/12*
b*(-e*h+f*g)*p*q*(h*x+g)^3/f/h-1/16*b*p*q*(h*x+g)^4/h-1/4*b*(-e*h+f*g)^4*p
*q*ln(f*x+e)/f^4/h+1/4*(h*x+g)^4*(a+b*ln(c*(d*(f*x+e)^p)^q))/h
```

3.420.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.47

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx$$
$$= \frac{fx(12af^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3) - bpq(-12e^3h^3+6e^2fh^2(8g+hx) - 4ef^2h(18g^2+6ghx+h^2x}{}$$

input
```
Integrate[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
(f*x*(12*a*f^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) - b*p*q*(-12*e^
3*h^3 + 6*e^2*f*h^2*(8*g + h*x) - 4*e*f^2*h*(18*g^2 + 6*g*h*x + h^2*x^2) +
 f^3*(48*g^3 + 36*g^2*h*x + 16*g*h^2*x^2 + 3*h^3*x^3))) - 12*b*e^2*h*(6*f^
2*g^2 - 4*e*f*g*h + e^2*h^2)*p*q*Log[e + f*x] + 12*b*f^3*(4*e*g^3 + f*x*(4
*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3))*Log[c*(d*(e + f*x)^p)^q])/(48*f
^4)
```

3.420.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2895, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(g+hx)^4 (a+b\log(c(d(e+fx)^p)^q))}{4h}-\frac{bfpq\int \frac{(g+hx)^4}{e+fx}dx}{4h}$$

$$\Big\downarrow \text{49}$$

$$\frac{(g+hx)^4 (a+b\log(c(d(e+fx)^p)^q))}{4h}-\frac{bfpq\int \left(\frac{(fg-eh)^4}{f^4(e+fx)}+\frac{h(fg-eh)^3}{f^4}+\frac{h(g+hx)(fg-eh)^2}{f^3}+\frac{h(g+hx)^2(fg-eh)}{f^2}+\frac{h(g+hx)^3}{f}\right)dx}{4h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^4 (a+b\log(c(d(e+fx)^p)^q))}{4h}-\frac{bfpq\left(\frac{(fg-eh)^4\log(e+fx)}{f^5}+\frac{hx(fg-eh)^3}{f^4}+\frac{(g+hx)^2(fg-eh)^2}{2f^3}+\frac{(g+hx)^3(fg-eh)}{3f^2}+\frac{(g+hx)^4}{4f}\right)}{4h}$$

input
```
Int[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
-1/4*(b*f*p*q*((h*(f*g - e*h)^3*x)/f^4 + ((f*g - e*h)^2*(g + h*x)^2)/(2*f^
3) + ((f*g - e*h)*(g + h*x)^3)/(3*f^2) + (g + h*x)^4/(4*f) + ((f*g - e*h)^
4*Log[e + f*x])/f^5))/h + ((g + h*x)^4*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(
4*h)
```

3.420.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.420.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(146) = 292$.

Time = 5.98 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.95

method	result
parallelrisch	$-\frac{-48\ln(fx+e)b\,e^3 fg\,h^2pq+72\ln(fx+e)b\,e^2f^2g^2hpq-24x^2be\,f^3g\,h^2pq+48xb\,e^2f^2g\,h^2pq-48x^3a\,f^4g\,h^2-72x^2a\,f^4g^2h-48x}{}$

input
```
int((h*x+g)^3*(a+b*ln(c*(d*(f*x+e)^p)^q)),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-48*ln(f*x+e)*b*e^3*f*g*h^2*p*q+72*ln(f*x+e)*b*e^2*f^2*g^2*h*p*q-24
*x^2*b*e*f^3*g*h^2*p*q+48*x*b*e^2*f^2*g*h^2*p*q-48*x^3*a*f^4*g*h^2-72*x^2*
a*f^4*g^2*h-48*x*ln(c*(d*(f*x+e)^p)^q)*b*f^4*g^3+48*ln(c*(d*(f*x+e)^p)^q)*
b*e*f^3*g^3-48*b*e*f^3*g^3*p*q+12*b*e^4*h^3*p*q-72*x*b*e*f^3*g^2*h*p*q+72*
b*e^2*g^2*h*p*q*f^2-48*b*e^3*f*g*h^2*p*q-12*x^4*ln(c*(d*(f*x+e)^p)^q)*b*f^
4*h^3+48*a*e*g^3*f^3-72*x^2*ln(c*(d*(f*x+e)^p)^q)*b*f^4*g^2*h+48*x*b*f^4*g
^3*p*q+12*ln(f*x+e)*b*e^4*h^3*p*q+3*x^4*b*f^4*h^3*p*q-48*x^3*ln(c*(d*(f*x+
e)^p)^q)*b*f^4*g*h^2-12*x^4*a*f^4*h^3-48*x*a*f^4*g^3+36*x^2*b*f^4*g^2*h*p*
q-12*x*b*e^3*f*h^3*p*q-96*ln(f*x+e)*b*e*f^3*g^3*p*q-4*x^3*b*e*f^3*h^3*p*q+
16*x^3*b*f^4*g*h^2*p*q+6*x^2*b*e^2*f^2*h^3*p*q)/f^4
```

3.420.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(146) = 292$.

Time = 0.36 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.56

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)\,dx = -\frac{3\,(bf^4h^3pq-4\,af^4h^3)x^4-4\,(12\,af^4gh^2-(4\,bf^4gh^2-bef^3h^3)pq)x^3-6\,(12\,af^4g^2h-(6\,bf^4g^2h-4\,be}{}$$

input
```
integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
-1/48*(3*(b*f^4*h^3*p*q - 4*a*f^4*h^3)*x^4 - 4*(12*a*f^4*g*h^2 - (4*b*f^4*
g*h^2 - b*e*f^3*h^3)*p*q)*x^3 - 6*(12*a*f^4*g^2*h - (6*b*f^4*g^2*h - 4*b*e
*f^3*g*h^2 + b*e^2*f^2*h^3)*p*q)*x^2 - 12*(4*a*f^4*g^3 - (4*b*f^4*g^3 - 6*
b*e*f^3*g^2*h + 4*b*e^2*f^2*g*h^2 - b*e^3*f*h^3)*p*q)*x - 12*(b*f^4*h^3*p*
q*x^4 + 4*b*f^4*g*h^2*p*q*x^3 + 6*b*f^4*g^2*h*p*q*x^2 + 4*b*f^4*g^3*p*q*x
+ (4*b*e*f^3*g^3 - 6*b*e^2*f^2*g^2*h + 4*b*e^3*f*g*h^2 - b*e^4*h^3)*p*q)*l
og(f*x + e) - 12*(b*f^4*h^3*x^4 + 4*b*f^4*g*h^2*x^3 + 6*b*f^4*g^2*h*x^2 +
4*b*f^4*g^3*x)*log(c) - 12*(b*f^4*h^3*q*x^4 + 4*b*f^4*g*h^2*q*x^3 + 6*b*f^
4*g^2*h*q*x^2 + 4*b*f^4*g^3*q*x)*log(d))/f^4
```

3.420.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(139) = 278$.

Time = 2.50 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.89

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx$$
$$= \begin{cases} ag^3x + \frac{3ag^2hx^2}{2} + agh^2x^3 + \frac{ah^3x^4}{4} - \frac{be^4h^3\log(c(d(e+fx)^p)^q)}{4f^4} + \frac{be^3gh^2\log(c(d(e+fx)^p)^q)}{f^3} + \frac{be^3h^3pqx}{4f^3} - \frac{3be^2g^2h\log(c(d(e}{2f^2} \\ (a+b\log(c(de^p)^q))\left(g^3x + \frac{3g^2hx^2}{2} + gh^2x^3 + \frac{h^3x^4}{4}\right) \end{cases}$$

input
```
integrate((h*x+g)**3*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Piecewise((a*g**3*x + 3*a*g**2*h*x**2/2 + a*g*h**2*x**3 + a*h**3*x**4/4 -
b*e**4*h**3*log(c*(d*(e + f*x)**p)**q)/(4*f**4) + b*e**3*g*h**2*log(c*(d*(
e + f*x)**p)**q)/f**3 + b*e**3*h**3*p*q*x/(4*f**3) - 3*b*e**2*g**2*h*log(c
*(d*(e + f*x)**p)**q)/(2*f**2) - b*e**2*g*h**2*p*q*x/f**2 - b*e**2*h**3*p*
q*x**2/(8*f**2) + b*e*g**3*log(c*(d*(e + f*x)**p)**q)/f + 3*b*e*g**2*h*p*q
*x/(2*f) + b*e*g*h**2*p*q*x**2/(2*f) + b*e*h**3*p*q*x**3/(12*f) - b*g**3*p
*q*x + b*g**3*x*log(c*(d*(e + f*x)**p)**q) - 3*b*g**2*h*p*q*x**2/4 + 3*b*g
**2*h*x**2*log(c*(d*(e + f*x)**p)**q)/2 - b*g*h**2*p*q*x**3/3 + b*g*h**2*x
**3*log(c*(d*(e + f*x)**p)**q) - b*h**3*p*q*x**4/16 + b*h**3*x**4*log(c*(d
*(e + f*x)**p)**q)/4, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))*(g**3*x + 3*g
**2*h*x**2/2 + g*h**2*x**3 + h**3*x**4/4), True))
```

3.420.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(146) = 292$.

Time = 0.20 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q)) \, dx$$
$$= \frac{1}{4}bh^3x^4\log(((fx+e)^pd)^q c) + \frac{1}{4}ah^3x^4 - bfg^3pq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)$$
$$- \frac{1}{48}bfh^3pq\left(\frac{12e^4\log(fx+e)}{f^5} + \frac{3f^3x^4 - 4ef^2x^3 + 6e^2fx^2 - 12e^3x}{f^4}\right)$$
$$+ \frac{1}{6}bfgh^2pq\left(\frac{6e^3\log(fx+e)}{f^4} - \frac{2f^2x^3 - 3efx^2 + 6e^2x}{f^3}\right)$$
$$- \frac{3}{4}bfg^2hpq\left(\frac{2e^2\log(fx+e)}{f^3} + \frac{fx^2 - 2ex}{f^2}\right) + bgh^2x^3\log(((fx+e)^pd)^q c) + agh^2x^3$$
$$+ \frac{3}{2}bg^2hx^2\log(((fx+e)^pd)^q c) + \frac{3}{2}ag^2hx^2 + bg^3x\log(((fx+e)^pd)^q c) + ag^3x$$

input
```
integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
1/4*b*h^3*x^4*log(((f*x + e)^p*d)^q*c) + 1/4*a*h^3*x^4 - b*f*g^3*p*q*(x/f
- e*log(f*x + e)/f^2) - 1/48*b*f*h^3*p*q*(12*e^4*log(f*x + e)/f^5 + (3*f^3
*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/f^4) + 1/6*b*f*g*h^2*p*q*(6*e
^3*log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3) - 3/4*b*f*g^2
*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2) + b*g*h^2*x^3*log(((
f*x + e)^p*d)^q*c) + a*g*h^2*x^3 + 3/2*b*g^2*h*x^2*log(((f*x + e)^p*d)^q*c
) + 3/2*a*g^2*h*x^2 + b*g^3*x*log(((f*x + e)^p*d)^q*c) + a*g^3*x
```

3.420.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(146) = 292$.

Time = 0.34 (sec) , antiderivative size = 987, normalized size of antiderivative = 6.25

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q)) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
(f*x + e)*b*g^3*p*q*log(f*x + e)/f + 3/2*(f*x + e)^2*b*g^2*h*p*q*log(f*x +
 e)/f^2 - 3*(f*x + e)*b*e*g^2*h*p*q*log(f*x + e)/f^2 + (f*x + e)^3*b*g*h^2
*p*q*log(f*x + e)/f^3 - 3*(f*x + e)^2*b*e*g*h^2*p*q*log(f*x + e)/f^3 + 3*(
f*x + e)*b*e^2*g*h^2*p*q*log(f*x + e)/f^3 + 1/4*(f*x + e)^4*b*h^3*p*q*log(
f*x + e)/f^4 - (f*x + e)^3*b*e*h^3*p*q*log(f*x + e)/f^4 + 3/2*(f*x + e)^2*
b*e^2*h^3*p*q*log(f*x + e)/f^4 - (f*x + e)*b*e^3*h^3*p*q*log(f*x + e)/f^4
- (f*x + e)*b*g^3*p*q/f - 3/4*(f*x + e)^2*b*g^2*h*p*q/f^2 + 3*(f*x + e)*b*
e*g^2*h*p*q/f^2 - 1/3*(f*x + e)^3*b*g*h^2*p*q/f^3 + 3/2*(f*x + e)^2*b*e*g*
h^2*p*q/f^3 - 3*(f*x + e)*b*e^2*g*h^2*p*q/f^3 - 1/16*(f*x + e)^4*b*h^3*p*q
/f^4 + 1/3*(f*x + e)^3*b*e*h^3*p*q/f^4 - 3/4*(f*x + e)^2*b*e^2*h^3*p*q/f^4
 + (f*x + e)*b*e^3*h^3*p*q/f^4 + (f*x + e)*b*g^3*q*log(d)/f + 3/2*(f*x + e
)^2*b*g^2*h*q*log(d)/f^2 - 3*(f*x + e)*b*e*g^2*h*q*log(d)/f^2 + (f*x + e)^
3*b*g*h^2*q*log(d)/f^3 - 3*(f*x + e)^2*b*e*g*h^2*q*log(d)/f^3 + 3*(f*x + e
)*b*e^2*g*h^2*q*log(d)/f^3 + 1/4*(f*x + e)^4*b*h^3*q*log(d)/f^4 - (f*x + e
)^3*b*e*h^3*q*log(d)/f^4 + 3/2*(f*x + e)^2*b*e^2*h^3*q*log(d)/f^4 - (f*x +
 e)*b*e^3*h^3*q*log(d)/f^4 + (f*x + e)*b*g^3*log(c)/f + 3/2*(f*x + e)^2*b*
g^2*h*log(c)/f^2 - 3*(f*x + e)*b*e*g^2*h*log(c)/f^2 + (f*x + e)^3*b*g*h^2*
log(c)/f^3 - 3*(f*x + e)^2*b*e*g*h^2*log(c)/f^3 + 3*(f*x + e)*b*e^2*g*h^2*
log(c)/f^3 + 1/4*(f*x + e)^4*b*h^3*log(c)/f^4 - (f*x + e)^3*b*e*h^3*log(c)
/f^4 + 3/2*(f*x + e)^2*b*e^2*h^3*log(c)/f^4 - (f*x + e)*b*e^3*h^3*log(c...
```

3.420. $\int (g+hx)^3 (a+b\log (c(d(e+fx)^p)^q))\,dx$

3.420.9 Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.34

$$\int (g+hx)^3 (a+b\log (c(d(e+fx)^p)^q))\,dx$$

$$= \ln \left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\,\left(b\,g^3\,x+\frac{3\,b\,g^2\,h\,x^2}{2}+b\,g\,h^2\,x^3+\frac{b\,h^3\,x^4}{4}\right) - x^2\left(\frac{e\left(\frac{h^2\,(a\,e\,h+3\,a\,f\,g-b\,f\,g\,p\,q)}{f}-\frac{e\,h^3\,(4\,a-b\,p\,q)}{4\,f}\right)}{2\,f}-\frac{3\,g\,h\,(2\,a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{4\,f}\right) + x\left(\frac{4\,a\,f\,g^3+12\,a\,e\,g^2\,h-4\,b\,f\,g^3\,p\,q}{4\,f} + \frac{e\left(\frac{e\left(\frac{h^2\,(a\,e\,h+3\,a\,f\,g-b\,f\,g\,p\,q)}{f}-\frac{e\,h^3\,(4\,a-b\,p\,q)}{4\,f}\right)}{f}-\frac{3\,g\,h\,(2\,a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{2\,f}\right)}{f}\right) + x^3\left(\frac{h^2\,(a\,e\,h+3\,a\,f\,g-b\,f\,g\,p\,q)}{3\,f}-\frac{e\,h^3\,(4\,a-b\,p\,q)}{12\,f}\right) - \frac{\ln\left(e+f\,x\right)\,\left(b\,p\,q\,e^4\,h^3-4\,b\,p\,q\,e^3\,f\,g\,h^2+6\,b\,p\,q\,e^2\,f^2\,g^2\,h-4\,b\,p\,q\,e\,f^3\,g^3\right)}{4\,f^4} + \frac{h^3\,x^4\,(4\,a-b\,p\,q)}{16}$$

input
```
int((g + h*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
log(c*(d*(e + f*x)^p)^q)*((b*h^3*x^4)/4 + b*g^3*x + (3*b*g^2*h*x^2)/2 + b*
g*h^2*x^3) - x^2*((e*((h^2*(a*e*h + 3*a*f*g - b*f*g*p*q))/f - (e*h^3*(4*a
- b*p*q))/(4*f)))/(2*f) - (3*g*h*(2*a*e*h + 2*a*f*g - b*f*g*p*q))/(4*f)) +
 x*((4*a*f*g^3 + 12*a*e*g^2*h - 4*b*f*g^3*p*q)/(4*f) + (e*((e*((h^2*(a*e*h
 + 3*a*f*g - b*f*g*p*q))/f - (e*h^3*(4*a - b*p*q))/(4*f)))/f - (3*g*h*(2*a
*e*h + 2*a*f*g - b*f*g*p*q))/(2*f)))/f) + x^3*((h^2*(a*e*h + 3*a*f*g - b*f
*g*p*q))/(3*f) - (e*h^3*(4*a - b*p*q))/(12*f)) - (log(e + f*x)*(b*e^4*h^3*
p*q - 4*b*e*f^3*g^3*p*q + 6*b*e^2*f^2*g^2*h*p*q - 4*b*e^3*f*g*h^2*p*q))/(4
*f^4) + (h^3*x^4*(4*a - b*p*q))/16
```

3.421 $\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx$

3.421.1 Optimal result

Integrand size = 26, antiderivative size = 128

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{b(fg-eh)^2pqx}{3f^2} - \frac{b(fg-eh)pq(g+hx)^2}{6fh} - \frac{bpq(g+hx)^3}{9h} - \frac{b(fg-eh)^3pq\log(e+fx)}{3f^3h} + \frac{(g+hx)^3(a+b\log(c(d(e+fx)^p)^q))}{3h}$$

output
```
-1/3*b*(-e*h+f*g)^2*p*q*x/f^2-1/6*b*(-e*h+f*g)*p*q*(h*x+g)^2/f/h-1/9*b*p*q
*(h*x+g)^3/h-1/3*b*(-e*h+f*g)^3*p*q*ln(f*x+e)/f^3/h+1/3*(h*x+g)^3*(a+b*ln(
c*(d*(f*x+e)^p)^q))/h
```

3.421.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.22

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{6be^2h(-3fg+eh)pq\log(e+fx)+f(x(6af^2(3g^2+3ghx+h^2x^2)-bpq(6e^2h^2-3efh(6g+hx)+f^2(18}{18f^3}$$

input
```
Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
(6*b*e^2*h*(-3*f*g + e*h)*p*q*Log[e + f*x] + f*(x*(6*a*f^2*(3*g^2 + 3*g*h*
x + h^2*x^2) - b*p*q*(6*e^2*h^2 - 3*e*f*h*(6*g + h*x) + f^2*(18*g^2 + 9*g*
h*x + 2*h^2*x^2))) + 6*b*f*(3*e*g^2 + f*x*(3*g^2 + 3*g*h*x + h^2*x^2))*Log
[c*(d*(e + f*x)^p)^q]))/(18*f^3)
```

3.421.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2895, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(g+hx)^3 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{3h}-\frac{bfpq\int \frac{(g+hx)^3}{e+fx}dx}{3h}$$

$$\Big\downarrow \text{49}$$

$$\frac{(g+hx)^3 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{3h}-\frac{bfpq\int \left(\frac{(fg-eh)^3}{f^3(e+fx)}+\frac{h(fg-eh)^2}{f^3}+\frac{h(g+hx)(fg-eh)}{f^2}+\frac{h(g+hx)^2}{f}\right)dx}{3h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^3 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{3h}-\frac{bfpq\left(\frac{(fg-eh)^3\log(e+fx)}{f^4}+\frac{hx(fg-eh)^2}{f^3}+\frac{(g+hx)^2(fg-eh)}{2f^2}+\frac{(g+hx)^3}{3f}\right)}{3h}$$

input
```
Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
-1/3*(b*f*p*q*((h*(f*g - e*h)^2*x)/f^3 + ((f*g - e*h)*(g + h*x)^2)/(2*f^2)
 + (g + h*x)^3/(3*f) + ((f*g - e*h)^3*Log[e + f*x])/f^4))/h + ((g + h*x)^3
*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(3*h)
```

3.421.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.421.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(118) = 236$.

Time = 1.97 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.27

method	result
parallelrisch	$\frac{-2x^3 be\, f^3 h^2 pq+6x^3 \ln(c(d(fx+e)^p)^q) be\, f^3 h^2+3x^2 b\, e^2 f^2 h^2 pq-9x^2 be\, f^3 ghpq+6 \ln(fx+e) b\, e^4 h^2 pq-18 \ln(fx+e) b\, e^3 fghpq+}{}$

input
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q)),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-2*x^3*b*e*f^3*h^2*p*q+6*x^3*ln(c*(d*(f*x+e)^p)^q)*b*e*f^3*h^2+3*x^2
*b*e^2*f^2*h^2*p*q-9*x^2*b*e*f^3*g*h*p*q+6*ln(f*x+e)*b*e^4*h^2*p*q-18*ln(f
*x+e)*b*e^3*f*g*h*p*q+36*ln(f*x+e)*b*e^2*f^2*g^2*p*q+6*x^3*a*e*f^3*h^2+18*
x^2*ln(c*(d*(f*x+e)^p)^q)*b*e*f^3*g*h-6*x*b*e^3*f*h^2*p*q+18*x*b*e^2*f^2*g
*h*p*q-18*x*b*e*f^3*g^2*p*q+18*x^2*a*e*f^3*g*h+18*x*ln(c*(d*(f*x+e)^p)^q)*
b*e*f^3*g^2+18*x*a*e*f^3*g^2-18*ln(c*(d*(f*x+e)^p)^q)*b*e^2*f^2*g^2)/f^3/e
```

3.421.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.09

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right) dx = -\frac{2\,(bf^3h^2pq-3\,af^3h^2)x^3-3\,(6\,af^3gh-(3\,bf^3gh-bef^2h^2)pq)x^2-6\,(3\,af^3g^2-(3\,bf^3g^2-3\,bef^2gh+}{}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
-1/18*(2*(b*f^3*h^2*p*q - 3*a*f^3*h^2)*x^3 - 3*(6*a*f^3*g*h - (3*b*f^3*g*h
 - b*e*f^2*h^2)*p*q)*x^2 - 6*(3*a*f^3*g^2 - (3*b*f^3*g^2 - 3*b*e*f^2*g*h +
 b*e^2*f*h^2)*p*q)*x - 6*(b*f^3*h^2*p*q*x^3 + 3*b*f^3*g*h*p*q*x^2 + 3*b*f^
3*g^2*p*q*x + (3*b*e*f^2*g^2 - 3*b*e^2*f*g*h + b*e^3*h^2)*p*q)*log(f*x + e
) - 6*(b*f^3*h^2*x^3 + 3*b*f^3*g*h*x^2 + 3*b*f^3*g^2*x)*log(c) - 6*(b*f^3*
h^2*q*x^3 + 3*b*f^3*g*h*q*x^2 + 3*b*f^3*g^2*q*x)*log(d))/f^3
```

3.421.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(112) = 224$.

Time = 1.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.23

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right) dx = \begin{cases} ag^2x + aghx^2 + \frac{ah^2x^3}{3} + \frac{be^3h^2\log\left(c(d(e+fx)^p)^q\right)}{3f^3} - \frac{be^2gh\log\left(c(d(e+fx)^p)^q\right)}{f^2} - \frac{be^2h^2pqx}{3f^2} + \frac{beg^2\log\left(c(d(e+fx)^p)^q\right)}{f} + \frac{beghp}{f} \\ \left(a+b\log\left(c(de^p)^q\right)\right)\left(g^2x + ghx^2 + \frac{h^2x^3}{3}\right) \end{cases}$$

input
```
integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Piecewise((a*g**2*x + a*g*h*x**2 + a*h**2*x**3/3 + b*e**3*h**2*log(c*(d*(e
 + f*x)**p)**q)/(3*f**3) - b*e**2*g*h*log(c*(d*(e + f*x)**p)**q)/f**2 - b*
e**2*h**2*p*q*x/(3*f**2) + b*e*g**2*log(c*(d*(e + f*x)**p)**q)/f + b*e*g*h
*p*q*x/f + b*e*h**2*p*q*x**2/(6*f) - b*g**2*p*q*x + b*g**2*x*log(c*(d*(e +
 f*x)**p)**q) - b*g*h*p*q*x**2/2 + b*g*h*x**2*log(c*(d*(e + f*x)**p)**q) -
 b*h**2*p*q*x**3/9 + b*h**2*x**3*log(c*(d*(e + f*x)**p)**q)/3, Ne(f, 0)),
((a + b*log(c*(d*e**p)**q))*(g**2*x + g*h*x**2 + h**2*x**3/3), True))
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.58

$$\begin{aligned}\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx &= -bfg^2pq\left(\frac{x}{f}-\frac{e\log\left(fx+e\right)}{f^2}\right) \\ &\quad+\frac{1}{18}\,bfh^2pq\left(\frac{6\,e^3\log\left(fx+e\right)}{f^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{f^3}\right) \\ &\quad-\frac{1}{2}\,bfghpq\left(\frac{2\,e^2\log\left(fx+e\right)}{f^3}+\frac{fx^2-2\,ex}{f^2}\right)+\frac{1}{3}\,bh^2x^3\log\left(\left(\left(fx+e\right)^pd\right)^q c\right) \\ &\quad+\frac{1}{3}\,ah^2x^3+bghx^2\log\left(\left(\left(fx+e\right)^pd\right)^q c\right)+aghx^2+bg^2x\log\left(\left(\left(fx+e\right)^pd\right)^q c\right)+ag^2x\end{aligned}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
-b*f*g^2*p*q*(x/f - e*log(f*x + e)/f^2) + 1/18*b*f*h^2*p*q*(6*e^3*log(f*x
+ e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3) - 1/2*b*f*g*h*p*q*(2*e^2
*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2) + 1/3*b*h^2*x^3*log(((f*x + e)^p*
d)^q*c) + 1/3*a*h^2*x^3 + b*g*h*x^2*log(((f*x + e)^p*d)^q*c) + a*g*h*x^2 +
 b*g^2*x*log(((f*x + e)^p*d)^q*c) + a*g^2*x
```

3.421.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(118) = 236$.

Time = 0.36 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.25

$$
\begin{aligned}
&\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx \\
&= \frac{(fx+e)bg^2pq\log(fx+e)}{f} + \frac{(fx+e)^2bghpq\log(fx+e)}{f^2} \\
&\quad - \frac{2\,(fx+e)beghpq\log(fx+e)}{f^2} + \frac{(fx+e)^3bh^2pq\log(fx+e)}{3\,f^3} \\
&\quad - \frac{(fx+e)^2beh^2pq\log(fx+e)}{f^3} + \frac{(fx+e)be^2h^2pq\log(fx+e)}{f^3} \\
&\quad - \frac{(fx+e)bg^2pq}{f} - \frac{(fx+e)^2bghpq}{2\,f^2} + \frac{2\,(fx+e)beghpq}{f^2} - \frac{(fx+e)^3bh^2pq}{9\,f^3} \\
&\quad + \frac{(fx+e)^2beh^2pq}{2\,f^3} - \frac{(fx+e)be^2h^2pq}{f^3} + \frac{(fx+e)bg^2q\log(d)}{f} \\
&\quad + \frac{(fx+e)^2bghq\log(d)}{f^2} - \frac{2\,(fx+e)beghq\log(d)}{f^2} + \frac{(fx+e)^3bh^2q\log(d)}{3\,f^3} \\
&\quad - \frac{(fx+e)^2beh^2q\log(d)}{f^3} + \frac{(fx+e)be^2h^2q\log(d)}{f^3} + \frac{(fx+e)bg^2\log(c)}{f} \\
&\quad + \frac{(fx+e)^2bgh\log(c)}{f^2} - \frac{2\,(fx+e)begh\log(c)}{f^2} + \frac{(fx+e)^3bh^2\log(c)}{3\,f^3} \\
&\quad - \frac{(fx+e)^2beh^2\log(c)}{f^3} + \frac{(fx+e)be^2h^2\log(c)}{f^3} + \frac{(fx+e)ag^2}{f} + \frac{(fx+e)^2agh}{f^2} \\
&\quad - \frac{2\,(fx+e)aegh}{f^2} + \frac{(fx+e)^3ah^2}{3\,f^3} - \frac{(fx+e)^2aeh^2}{f^3} + \frac{(fx+e)ae^2h^2}{f^3}
\end{aligned}
$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
(f*x + e)*b*g^2*p*q*log(f*x + e)/f + (f*x + e)^2*b*g*h*p*q*log(f*x + e)/f^
2 - 2*(f*x + e)*b*e*g*h*p*q*log(f*x + e)/f^2 + 1/3*(f*x + e)^3*b*h^2*p*q*l
og(f*x + e)/f^3 - (f*x + e)^2*b*e*h^2*p*q*log(f*x + e)/f^3 + (f*x + e)*b*e
^2*h^2*p*q*log(f*x + e)/f^3 - (f*x + e)*b*g^2*p*q/f - 1/2*(f*x + e)^2*b*g*
h*p*q/f^2 + 2*(f*x + e)*b*e*g*h*p*q/f^2 - 1/9*(f*x + e)^3*b*h^2*p*q/f^3 +
1/2*(f*x + e)^2*b*e*h^2*p*q/f^3 - (f*x + e)*b*e^2*h^2*p*q/f^3 + (f*x + e)*
b*g^2*q*log(d)/f + (f*x + e)^2*b*g*h*q*log(d)/f^2 - 2*(f*x + e)*b*e*g*h*q*
log(d)/f^2 + 1/3*(f*x + e)^3*b*h^2*q*log(d)/f^3 - (f*x + e)^2*b*e*h^2*q*lo
g(d)/f^3 + (f*x + e)*b*e^2*h^2*q*log(d)/f^3 + (f*x + e)*b*g^2*log(c)/f + (
f*x + e)^2*b*g*h*log(c)/f^2 - 2*(f*x + e)*b*e*g*h*log(c)/f^2 + 1/3*(f*x +
e)^3*b*h^2*log(c)/f^3 - (f*x + e)^2*b*e*h^2*log(c)/f^3 + (f*x + e)*b*e^2*h
^2*log(c)/f^3 + (f*x + e)*a*g^2/f + (f*x + e)^2*a*g*h/f^2 - 2*(f*x + e)*a*
e*g*h/f^2 + 1/3*(f*x + e)^3*a*h^2/f^3 - (f*x + e)^2*a*e*h^2/f^3 + (f*x + e
)*a*e^2*h^2/f^3
```

3.421.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.76

$$
\begin{aligned}
&\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx \\
&= \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(b\,g^2\,x+b\,g\,h\,x^2+\frac{b\,h^2\,x^3}{3}\right) \\
&\quad + x^2\left(\frac{h\,(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{2\,f}-\frac{e\,h^2\,(3\,a-b\,p\,q)}{6\,f}\right) \\
&\quad + x\left(\frac{3\,a\,f\,g^2+6\,a\,e\,g\,h-3\,b\,f\,g^2\,p\,q}{3\,f}-\frac{e\left(\frac{h\,(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{f}-\frac{e\,h^2\,(3\,a-b\,p\,q)}{3\,f}\right)}{f}\right) \\
&\quad + \frac{\ln\left(e+f\,x\right)\,\left(b\,p\,q\,e^3\,h^2-3\,b\,p\,q\,e^2\,f\,g\,h+3\,b\,p\,q\,e\,f^2\,g^2\right)}{3\,f^3}+\frac{h^2\,x^3\,(3\,a-b\,p\,q)}{9}
\end{aligned}
$$

input
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
log(c*(d*(e + f*x)^p)^q)*((b*h^2*x^3)/3 + b*g^2*x + b*g*h*x^2) + x^2*((h*(
a*e*h + 2*a*f*g - b*f*g*p*q))/(2*f) - (e*h^2*(3*a - b*p*q))/(6*f)) + x*((3
*a*f*g^2 + 6*a*e*g*h - 3*b*f*g^2*p*q)/(3*f) - (e*((h*(a*e*h + 2*a*f*g - b*
f*g*p*q))/f - (e*h^2*(3*a - b*p*q))/(3*f)))/f) + (log(e + f*x)*(b*e^3*h^2*
p*q + 3*b*e*f^2*g^2*p*q - 3*b*e^2*f*g*h*p*q))/(3*f^3) + (h^2*x^3*(3*a - b*
p*q))/9
```

3.422 $\int (g+hx) (a+b\log (c(d(e+fx)^p)^q)) \, dx$

3.422.1 Optimal result

Integrand size = 24, antiderivative size = 98

$$\int (g+hx) (a+b\log (c(d(e+fx)^p)^q)) \, dx = -\frac{b(fg-eh)pqx}{2f} - \frac{bpq(g+hx)^2}{4h} - \frac{b(fg-eh)^2pq\log(e+fx)}{2f^2h} + \frac{(g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))}{2h}$$

output
```
-1/2*b*(-e*h+f*g)*p*q*x/f-1/4*b*p*q*(h*x+g)^2/h-1/2*b*(-e*h+f*g)^2*p*q*ln(
f*x+e)/f^2/h+1/2*(h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/h
```

3.422.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int (g+hx) (a+b\log (c(d(e+fx)^p)^q)) \, dx = agx - bgpqx + \frac{behpqx}{2f} + \frac{1}{2}ahx^2 - \frac{1}{4}bhpqx^2 - \frac{be^2hpq\log(e+fx)}{2f^2} + \frac{1}{2}bhx^2 \log (c(d(e+fx)^p)^q) + \frac{bg(e+fx)\log (c(d(e+fx)^p)^q)}{f}$$

input
```
Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
a*g*x - b*g*p*q*x + (b*e*h*p*q*x)/(2*f) + (a*h*x^2)/2 - (b*h*p*q*x^2)/4 -
(b*e^2*h*p*q*Log[e + f*x])/(2*f^2) + (b*h*x^2*Log[c*(d*(e + f*x)^p)^q])/2
+ (b*g*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

3.422.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2895, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}-\frac{bfpq\int \frac{(g+hx)^2}{e+fx}dx}{2h}$$

$$\Big\downarrow \text{49}$$

$$\frac{(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}-\frac{bfpq\int \left(\frac{(fg-eh)^2}{f^2(e+fx)}+\frac{h(fg-eh)}{f^2}+\frac{h(g+hx)}{f}\right)dx}{2h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}-\frac{bfpq\left(\frac{(fg-eh)^2\log(e+fx)}{f^3}+\frac{hx(fg-eh)}{f^2}+\frac{(g+hx)^2}{2f}\right)}{2h}$$

input
```
Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
-1/2*(b*f*p*q*((h*(f*g - e*h)*x)/f^2 + (g + h*x)^2/(2*f) + ((f*g - e*h)^2*
Log[e + f*x])/f^3))/h + ((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(2*
h)
```

3.422.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.422.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.77

method	result
parallelrisch	$-\frac{x^{2} b\,f^{2} h p q+2 \ln \left(f x+e\right) b\,e^{2} h p q-8 \ln \left(f x+e\right) b e f g p q-2 x^{2} \ln \left(c \left(d \left(f x+e\right)^{p}\right)^{q}\right) b\,f^{2} h-2 x b e f h p q+4 x b\,f^{2} g p q-2 x^{2} a\,f^{2} h-4 x \ln (c(\ldots}{4 f^{2}}$

input
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x^2*b*f^2*h*p*q+2*ln(f*x+e)*b*e^2*h*p*q-8*ln(f*x+e)*b*e*f*g*p*q-2*x^
2*ln(c*(d*(f*x+e)^p)^q)*b*f^2*h-2*x*b*e*f*h*p*q+4*x*b*f^2*g*p*q-2*x^2*a*f^
2*h-4*x*ln(c*(d*(f*x+e)^p)^q)*b*f^2*g+2*b*e^2*h*p*q-4*b*e*f*g*p*q-4*x*a*f^
2*g+4*ln(c*(d*(f*x+e)^p)^q)*b*e*f*g+4*a*e*f*g)/f^2
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx =$$
$$-\frac{(bf^2hpq-2\,af^2h)x^2-2\,(2\,af^2g-(2\,bf^2g-befh)pq)x-2\,(bf^2hpqx^2+2\,bf^2gpqx+(2\,befg-be^2h)p}{4\,f^2}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
-1/4*((b*f^2*h*p*q - 2*a*f^2*h)*x^2 - 2*(2*a*f^2*g - (2*b*f^2*g - b*e*f*h)
*p*q)*x - 2*(b*f^2*h*p*q*x^2 + 2*b*f^2*g*p*q*x + (2*b*e*f*g - b*e^2*h)*p*q
)*log(f*x + e) - 2*(b*f^2*h*x^2 + 2*b*f^2*g*x)*log(c) - 2*(b*f^2*h*q*x^2 +
 2*b*f^2*g*q*x)*log(d))/f^2
```

3.422.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx$$
$$=\begin{cases} agx+\frac{ahx^2}{2}-\frac{be^2h\log\left(c(d(e+fx)^p)^q\right)}{2f^2}+\frac{beg\log\left(c(d(e+fx)^p)^q\right)}{f}+\frac{behpqx}{2f}-bgpqx+bgx\log\left(c(d(e+fx)^p)^q\right)-\frac{bhpqx}{4} \\ \left(a+b\log\left(c(de^p)^q\right)\right)\left(gx+\frac{hx^2}{2}\right) \end{cases}$$

input
```
integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Piecewise((a*g*x + a*h*x**2/2 - b*e**2*h*log(c*(d*(e + f*x)**p)**q)/(2*f**
2) + b*e*g*log(c*(d*(e + f*x)**p)**q)/f + b*e*h*p*q*x/(2*f) - b*g*p*q*x +
b*g*x*log(c*(d*(e + f*x)**p)**q) - b*h*p*q*x**2/4 + b*h*x**2*log(c*(d*(e +
 f*x)**p)**q)/2, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))*(g*x + h*x**2/2),
True))
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = -bfgpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right) - \frac{1}{4}bfhpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right) + \frac{1}{2}bhx^2\log\left(\left(\left((fx+e)^pd\right)^q c\right)\right) + \frac{1}{2}ahx^2 + bgx\log\left(\left(\left((fx+e)^pd\right)^q c\right)\right) + agx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
-b*f*g*p*q*(x/f - e*log(f*x + e)/f^2) - 1/4*b*f*h*p*q*(2*e^2*log(f*x + e)/
f^3 + (f*x^2 - 2*e*x)/f^2) + 1/2*b*h*x^2*log(((f*x + e)^p*d)^q*c) + 1/2*a*
h*x^2 + b*g*x*log(((f*x + e)^p*d)^q*c) + a*g*x
```

3.422.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(90) = 180.

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.41

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \frac{(fx+e)bgpq\log(fx+e)}{f} + \frac{(fx+e)^2bhpq\log(fx+e)}{2f^2} - \frac{(fx+e)behpq\log(fx+e)}{f^2} - \frac{(fx+e)bgpq}{f} - \frac{(fx+e)^2bhpq}{4f^2} + \frac{(fx+e)behpq}{f^2} + \frac{(fx+e)bgq\log(d)}{f} + \frac{(fx+e)^2bhq\log(d)}{2f^2} - \frac{(fx+e)behq\log(d)}{f^2} + \frac{(fx+e)bg\log(c)}{f} + \frac{(fx+e)^2bh\log(c)}{2f^2} - \frac{(fx+e)beh\log(c)}{f^2} + \frac{(fx+e)ag}{f} + \frac{(fx+e)^2ah}{2f^2} - \frac{(fx+e)aeh}{f^2}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
(f*x + e)*b*g*p*q*log(f*x + e)/f + 1/2*(f*x + e)^2*b*h*p*q*log(f*x + e)/f^
2 - (f*x + e)*b*e*h*p*q*log(f*x + e)/f^2 - (f*x + e)*b*g*p*q/f - 1/4*(f*x
+ e)^2*b*h*p*q/f^2 + (f*x + e)*b*e*h*p*q/f^2 + (f*x + e)*b*g*q*log(d)/f +
1/2*(f*x + e)^2*b*h*q*log(d)/f^2 - (f*x + e)*b*e*h*q*log(d)/f^2 + (f*x + e
)*b*g*log(c)/f + 1/2*(f*x + e)^2*b*h*log(c)/f^2 - (f*x + e)*b*e*h*log(c)/f
^2 + (f*x + e)*a*g/f + 1/2*(f*x + e)^2*a*h/f^2 - (f*x + e)*a*e*h/f^2
```

3.422.9 Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(\frac{b\,h\,x^2}{2}+b\,g\,x\right) + x\left(\frac{2\,a\,e\,h+2\,a\,f\,g-2\,b\,f\,g\,p\,q}{2\,f} - \frac{e\,h\,(2\,a-b\,p\,q)}{2\,f}\right) + \frac{h\,x^2\,(2\,a-b\,p\,q)}{4} - \frac{\ln\left(e+f\,x\right)\,\left(b\,e^2\,h\,p\,q-2\,b\,e\,f\,g\,p\,q\right)}{2\,f^2}$$

input
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
log(c*(d*(e + f*x)^p)^q)*((b*h*x^2)/2 + b*g*x) + x*((2*a*e*h + 2*a*f*g - 2
*b*f*g*p*q)/(2*f) - (e*h*(2*a - b*p*q))/(2*f)) + (h*x^2*(2*a - b*p*q))/4 -
 (log(e + f*x)*(b*e^2*h*p*q - 2*b*e*f*g*p*q))/(2*f^2)
```

3.423 $\int (a+b\log(c(d(e+fx)^p)^q))\,dx$

3.423.1 Optimal result

Integrand size = 18, antiderivative size = 34

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = ax - bpqx + \frac{b(e+fx)\log(c(d(e+fx)^p)^q)}{f}$$

output
```
a*x-b*p*q*x+b*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f
```

3.423.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = ax - bpqx + \frac{b(e+fx)\log(c(d(e+fx)^p)^q)}{f}$$

input
```
Integrate[a + b*Log[c*(d*(e + f*x)^p)^q],x]
```

output
```
a*x - b*p*q*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

3.423.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx$$
$$\Big\downarrow \text{2009}$$
$$ax+\frac{b(e+fx)\log(c(d(e+fx)^p)^q)}{f}-bpqx$$

input
```
Int[a + b*Log[c*(d*(e + f*x)^p)^q],x]
```

output
```
a*x - b*p*q*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

3.423.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.423.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result	size
default	$ax+b\ln(c(d(fx+e)^p)^q)x-bpqx+\frac{bqpe\ln(fx+e)}{f}$	42
parts	$ax+b\ln(c(d(fx+e)^p)^q)x-bpqx+\frac{bqpe\ln(fx+e)}{f}$	42
parallelrisch	$\frac{b(2\ln(fx+e)e^2pq-xefpq+x\ln(c(d(fx+e)^p)^q)ef-\ln(c(d(fx+e)^p)^q)e^2)}{ef}+ax$	71

input
```
int(a+b*ln(c*(d*(f*x+e)^p)^q),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*ln(c*(d*(f*x+e)^p)^q)*x-b*p*q*x+b*q*p/f*e*ln(f*x+e)
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{bfqx\log(d)+bfx\log(c)-(bfpq-af)x+(bfpqx+bepq)\log(fx+e)}{f}$$

input
```
integrate(a+b*log(c*(d*(f*x+e)^p)^q),x, algorithm="fricas")
```

output
```
(b*f*q*x*log(d) + b*f*x*log(c) - (b*f*p*q - a*f)*x + (b*f*p*q*x + b*e*p*q)
*log(f*x + e))/f
```

3.423.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = ax+b\left(\begin{cases}\frac{e\log(c(d(e+fx)^p)^q)}{f}-pqx+x\log(c(d(e+fx)^p)^q) & \text{for } f\neq 0\\ x\log(c(de^p)^q) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*ln(c*(d*(f*x+e)**p)**q),x)
```

output
```
a*x + b*Piecewise((e*log(c*(d*(e + f*x)**p)**q)/f - p*q*x + x*log(c*(d*(e
+ f*x)**p)**q), Ne(f, 0)), (x*log(c*(d*e**p)**q), True))
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = -bfpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)+bx\log(((fx+e)^pd)^q\,c)+ax$$

input
```
integrate(a+b*log(c*(d*(f*x+e)^p)^q),x, algorithm="maxima")
```

output
```
-b*f*p*q*(x/f - e*log(f*x + e)/f^2) + b*x*log(((f*x + e)^p*d)^q*c) + a*x
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int (a + b\log(c(d(e + fx)^p)^q))\,dx$$
$$= \left(\frac{(fx + e)pq\log(fx + e)}{f} - \frac{(fx + e)pq}{f} + \frac{(fx + e)q\log(d)}{f} + \frac{(fx + e)\log(c)}{f}\right)b + ax$$

input
```
integrate(a+b*log(c*(d*(f*x+e)^p)^q),x, algorithm="giac")
```

output
```
((f*x + e)*p*q*log(f*x + e)/f - (f*x + e)*p*q/f + (f*x + e)*q*log(d)/f + (
f*x + e)*log(c)/f)*b + a*x
```

3.423.9 Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = x\,(a-b\,p\,q)+b\,x\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)+\frac{b\,e\,p\,q\,\ln(e+f\,x)}{f}$$

input
```
int(a + b*log(c*(d*(e + f*x)^p)^q),x)
```

output
```
x*(a - b*p*q) + b*x*log(c*(d*(e + f*x)^p)^q) + (b*e*p*q*log(e + f*x))/f
```

3.424 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}\,dx$

3.424.1 Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{g+hx}\,dx=\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h+b*p*q*polylog(2,-h*
(f*x+e)/(-e*h+f*g))/h
```

3.424.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{g+hx}\,dx=\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\,\mathrm{PolyLog}\left(2,\frac{h(e+fx)}{-fg+eh}\right)}{h}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p
*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

3.424.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2895, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}dx$$

$$\Big\downarrow \text{2841}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2840}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bpq\int\frac{\log\left(\frac{h(e+fx)}{fg-eh}+1\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}+\frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p
*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h
```

3.424.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.424.4 Maple [F]

$$\int \frac{a + b\ln\left(c(d(fx+e)^{p})^{q}\right)}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

3.424.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.424.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x), x)
```

3.424.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxima")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*x + g), x) + a*l
og(h*x + g)/h
```

3.424.8 Giac [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log \left(\left(\left((fx+e)^pd\right)^q c\right)+a}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x), x)
```

3.425 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^2}\,dx$

3.425.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx=\frac{bfpq\log(e+fx)}{h(fg-eh)}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)}-\frac{bfpq\log(g+hx)}{h(fg-eh)}$$

output
```
b*f*p*q*ln(f*x+e)/h/(-e*h+f*g)+(-a-b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)-b*f*
p*q*ln(h*x+g)/h/(-e*h+f*g)
```

3.425.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx=\frac{-\frac{a}{g+hx}-\frac{b\log(c(d(e+fx)^p)^q)}{g+hx}+\frac{bfpq(\log(e+fx)-\log(g+hx))}{fg-eh}}{h}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^2,x]
```

output
```
(-(a/(g + h*x)) - (b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x) + (b*f*p*q*(Log[e
 + f*x] - Log[g + h*x]))/(f*g - e*h))/h
```

3.425.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2895, 2842, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{bfpq\int \frac{1}{(e+fx)(g+hx)}dx}{h}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)}$$

$$\Big\downarrow \text{47}$$

$$\frac{bfpq\left(\frac{f\int \frac{1}{e+fx}dx}{fg-eh}-\frac{h\int \frac{1}{g+hx}dx}{fg-eh}\right)}{h}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{bfpq\left(\frac{\log(e+fx)}{fg-eh}-\frac{\log(g+hx)}{fg-eh}\right)}{h}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^2,x]
```

output
```
-((a + b*Log[c*(d*(e + f*x)^p)^q])/(h*(g + h*x))) + (b*f*p*q*(Log[e + f*x]
/(f*g - e*h) - Log[g + h*x]/(f*g - e*h)))/h
```

3.425.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.425.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.74

method	result
parallelrisch	$-\frac{\ln(fx+e)xb\,f^2hpq-\ln(hx+g)xb\,f^2hpq+\ln(fx+e)b\,f^2gpq-\ln(hx+g)b\,f^2gpq+\ln(c(d(fx+e)^p)^q)befh-\ln(c(d(fx+e)^p)^q)b\,f^2}{(eh-fg)(hx+g)fh}$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
-(ln(f*x+e)*x*b*f^2*h*p*q-ln(h*x+g)*x*b*f^2*h*p*q+ln(f*x+e)*b*f^2*g*p*q-ln
(h*x+g)*b*f^2*g*p*q+ln(c*(d*(f*x+e)^p)^q)*b*e*f*h-ln(c*(d*(f*x+e)^p)^q)*b*
f^2*g+a*e*f*h-a*f^2*g)/(e*h-f*g)/(h*x+g)/f/h
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = -\frac{afg-aeh+(bfg-beh)q\log(d)-(bfhpqx+behpq)\log(fx+e)+(bfhpqx+bfgpq)\log(hx+g)+}{fg^2h-egh^2+(fgh^2-eh^3)x}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x, algorithm="fricas")
```

output
```
-(a*f*g - a*e*h + (b*f*g - b*e*h)*q*log(d) - (b*f*h*p*q*x + b*e*h*p*q)*log
(f*x + e) + (b*f*h*p*q*x + b*f*g*p*q)*log(h*x + g) + (b*f*g - b*e*h)*log(c
))/(f*g^2*h - e*g*h^2 + (f*g*h^2 - e*h^3)*x)
```

3.425.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(68) = 136.

Time = 3.56 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.46

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = \begin{cases} \frac{ax+\frac{be\log\left(c(d(e+fx)^p)^q\right)}{f}-bpqx+bx\log\left(c(d(e+fx)^p)^q\right)}{g^2} \\ -\frac{a}{gh+h^2x}-\frac{bpq}{gh+h^2x}-\frac{b\log\left(c\left(d\left(\frac{fg}{h}+fx\right)^p\right)^q\right)}{gh+h^2x} \\ -\frac{aeh}{egh^2+eh^3x-fg^2h-fgh^2x}+\frac{afg}{egh^2+eh^3x-fg^2h-fgh^2x}-\frac{beh\log\left(c(d(e+fx)^p)^q\right)}{egh^2+eh^3x-fg^2h-fgh^2x}+\frac{bfgpq\log\left(\frac{g}{h}+x\right)}{egh^2+eh^3x-fg^2h-fgh^2x}+\frac{bfhpqx\log}{egh^2+eh^3x-}\end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**2,x)
```

output
```
Piecewise(((a*x + b*e*log(c*(d*(e + f*x)**p)**q)/f - b*p*q*x + b*x*log(c*(
d*(e + f*x)**p)**q))/g**2, Eq(h, 0)), (-a/(g*h + h**2*x) - b*p*q/(g*h + h*
*2*x) - b*log(c*(d*(f*g/h + f*x)**p)**q)/(g*h + h**2*x), Eq(e, f*g/h)), (-
a*e*h/(e*g*h**2 + e*h**3*x - f*g**2*h - f*g*h**2*x) + a*f*g/(e*g*h**2 + e*
h**3*x - f*g**2*h - f*g*h**2*x) - b*e*h*log(c*(d*(e + f*x)**p)**q)/(e*g*h*
*2 + e*h**3*x - f*g**2*h - f*g*h**2*x) + b*f*g*p*q*log(g/h + x)/(e*g*h**2
+ e*h**3*x - f*g**2*h - f*g*h**2*x) + b*f*h*p*q*x*log(g/h + x)/(e*g*h**2 +
 e*h**3*x - f*g**2*h - f*g*h**2*x) - b*f*h*x*log(c*(d*(e + f*x)**p)**q)/(e
*g*h**2 + e*h**3*x - f*g**2*h - f*g*h**2*x), True))
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = bfpq\left(\frac{\log(fx+e)}{fgh-eh^2}-\frac{\log(hx+g)}{fgh-eh^2}\right) - \frac{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)\right)}{h^2x+gh}-\frac{a}{h^2x+gh}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x, algorithm="maxima")
```

output
```
b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2)) - b*
log(((f*x + e)^p*d)^q*c)/(h^2*x + g*h) - a/(h^2*x + g*h)
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = \frac{bfpq\log(fx+e)}{fgh-eh^2}-\frac{bfpq\log(hx+g)}{fgh-eh^2} - \frac{bpq\log(fx+e)}{h^2x+gh}-\frac{bq\log(d)+b\log(c)+a}{h^2x+gh}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x, algorithm="giac")
```

output
```
b*f*p*q*log(f*x + e)/(f*g*h - e*h^2) - b*f*p*q*log(h*x + g)/(f*g*h - e*h^2
) - b*p*q*log(f*x + e)/(h^2*x + g*h) - (b*q*log(d) + b*log(c) + a)/(h^2*x
+ g*h)
```

3.425.9 Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = -\frac{a}{x\,h^2+g\,h}-\frac{b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)}{h\,\left(g+h\,x\right)} + \frac{b\,f\,p\,q\,\mathrm{atan}\left(\frac{f\,g\,2{}\mathrm{i}+f\,h\,x\,2{}\mathrm{i}}{e\,h-f\,g}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{h\,\left(e\,h-f\,g\right)}$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^2,x)
```

output
```
(b*f*p*q*atan((f*g*2i + f*h*x*2i)/(e*h - f*g) + 1i)*2i)/(h*(e*h - f*g)) -
(b*log(c*(d*(e + f*x)^p)^q))/(h*(g + h*x)) - a/(g*h + h^2*x)
```

3.425. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^2}\,dx$

3.426 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^3} dx$

3.426.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3} dx = \frac{bfpq}{2h(fg-eh)(g+hx)} + \frac{bf^2pq\log(e+fx)}{2h(fg-eh)^2} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2} - \frac{bf^2pq\log(g+hx)}{2h(fg-eh)^2}$$

output
```
1/2*b*f*p*q/h/(-e*h+f*g)/(h*x+g)+1/2*b*f^2*p*q*ln(f*x+e)/h/(-e*h+f*g)^2+1/
2*(-a-b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^2-1/2*b*f^2*p*q*ln(h*x+g)/h/(-e*h
+f*g)^2
```

3.426.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3} dx = -\frac{a+b\log\left(c(d(e+fx)^p)^q\right) - \frac{bfpq(g+hx)(fg-eh+f(g+hx)\log(e+fx)-f(g+hx)\log(g+hx))}{(fg-eh)^2}}{2h(g+hx)^2}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d*(e + f*x)^p)^q] - (b*f*p*q*(g + h*x)*(f*g - e*h + f*(
g + h*x)*Log[e + f*x] - f*(g + h*x)*Log[g + h*x]))/(f*g - e*h)^2)/(h*(g +
h*x)^2)
```

3.426.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2895, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{bfpq\int \frac{1}{(e+fx)(g+hx)^2}dx}{2h}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{bfpq\int \left(\frac{f^2}{(fg-eh)^2(e+fx)}-\frac{hf}{(fg-eh)^2(g+hx)}-\frac{h}{(fg-eh)(g+hx)^2}\right)dx}{2h}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bfpq\left(\frac{1}{(g+hx)(fg-eh)}+\frac{f\log(e+fx)}{(fg-eh)^2}-\frac{f\log(g+hx)}{(fg-eh)^2}\right)}{2h}-\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d*(e + f*x)^p)^q])/(h*(g + h*x)^2) + (b*f*p*q*(1/((f*g
- e*h)*(g + h*x)) + (f*Log[e + f*x])/(f*g - e*h)^2 - (f*Log[g + h*x])/(f*g
 - e*h)^2))/(2*h)
```

3.426.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.426.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(114) = 228$.

Time = 3.76 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.56

method	result
parallelrisch	$\frac{2 \ln(fx+e)xb f^3 g h^2 pq - a e^2 f h^3 - a f^3 g^2 h + \ln(fx+e)x^2 b f^3 h^3 pq - \ln(hx+g)x^2 b f^3 h^3 pq + \ln(fx+e) b f^3 g^2 hpq - \ln(hx+g) b f^3 g^2}{}$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*ln(f*x+e)*x*b*f^3*g*h^2*p*q-a*e^2*f*h^3-a*f^3*g^2*h+ln(f*x+e)*x^2*b
*f^3*h^3*p*q-ln(h*x+g)*x^2*b*f^3*h^3*p*q+ln(f*x+e)*b*f^3*g^2*h*p*q-ln(h*x+
g)*b*f^3*g^2*h*p*q-x*b*e*f^2*h^3*p*q+x*b*f^3*g*h^2*p*q-2*ln(h*x+g)*x*b*f^3
*g*h^2*p*q-b*e*f^2*g*h^2*p*q+b*f^3*g^2*h*p*q+2*a*e*f^2*g*h^2+2*ln(c*(d*(f*
x+e)^p)^q)*b*e*f^2*g*h^2-ln(c*(d*(f*x+e)^p)^q)*b*e^2*f*h^3-ln(c*(d*(f*x+e)
^p)^q)*b*f^3*g^2*h)/(e^2*h^2-2*e*f*g*h+f^2*g^2)/(h*x+g)^2/f/h^2
```

3.426.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(111) = 222.

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.61

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = -\frac{af^2g^2-2\,aefgh+ae^2h^2-(bf^2gh-befh^2)pqx-(bf^2g^2-befgh)pq+(bf^2g^2-2\,befgh+be^2h^2)q\,\mathrm{lo}}{2\,(f^2g^4h-2\,efg^3h^2}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*f^2*g^2 - 2*a*e*f*g*h + a*e^2*h^2 - (b*f^2*g*h - b*e*f*h^2)*p*q*x
- (b*f^2*g^2 - b*e*f*g*h)*p*q + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*q*lo
g(d) - (b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + (2*b*e*f*g*h - b*e^2*h^2)*
p*q)*log(f*x + e) + (b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + b*f^2*g^2*p*q
)*log(h*x + g) + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))/(f^2*g^4*h
- 2*e*f*g^3*h^2 + e^2*g^2*h^3 + (f^2*g^2*h^3 - 2*e*f*g*h^4 + e^2*h^5)*x^2
+ 2*(f^2*g^3*h^2 - 2*e*f*g^2*h^3 + e^2*g*h^4)*x)
```

3.426.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1984 vs. 2(105) = 210.

Time = 15.25 (sec) , antiderivative size = 1984, normalized size of antiderivative = 16.67

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**3,x)
```

output
```
Piecewise(((a*x + b*e*log(c*(d*(e + f*x)**p)**q)/f - b*p*q*x + b*x*log(c*(
d*(e + f*x)**p)**q))/g**3, Eq(h, 0)), (-2*a/(4*g**2*h + 8*g*h**2*x + 4*h**
3*x**2) - b*p*q/(4*g**2*h + 8*g*h**2*x + 4*h**3*x**2) - 2*b*log(c*(d*(f*g/
h + f*x)**p)**q)/(4*g**2*h + 8*g*h**2*x + 4*h**3*x**2), Eq(e, f*g/h)), (-a
*e**2*h**2/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*
g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2
*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) + 2*a*e*f*g*h/(2*e**2*g**2*h**3 + 4*
e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4
*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x
**2) - a*f**2*g**2/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2
- 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h
+ 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) - b*e**2*h**2*log(c*(d*(e +
f*x)**p)**q)/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*
f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f*
*2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) - b*e*f*g*h*p*q/(2*e**2*g**2*h**3
+ 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x
 - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h*
*3*x**2) + 2*b*e*f*g*h*log(c*(d*(e + f*x)**p)**q)/(2*e**2*g**2*h**3 + 4*e*
*2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e
*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*...
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.45

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx$$
$$=\frac{1}{2}bfpq\left(\frac{f\log\left(fx+e\right)}{f^2g^2h-2\,efgh^2+e^2h^3}-\frac{f\log\left(hx+g\right)}{f^2g^2h-2\,efgh^2+e^2h^3}+\frac{1}{fg^2h-egh^2+(fgh^2-eh^3)x}\right)$$
$$-\frac{b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}-\frac{a}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x, algorithm="maxima")
```

output
```
1/2*b*f*p*q*(f*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - f*log(h*
x + g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) + 1/(f*g^2*h - e*g*h^2 + (f*g*h
^2 - e*h^3)*x)) - 1/2*b*log(((f*x + e)^p*d)^q*c)/(h^3*x^2 + 2*g*h^2*x + g^
2*h) - 1/2*a/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \frac{bf^2pq\log\left(fx+e\right)}{2\left(f^2g^2h-2\,efgh^2+e^2h^3\right)} - \frac{bf^2pq\log\left(hx+g\right)}{2\left(f^2g^2h-2\,efgh^2+e^2h^3\right)} - \frac{bpq\log\left(fx+e\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} + \frac{bfhpqx+bfgpq-bfgq\log\left(d\right)+behq\log\left(d\right)-bfg\log\left(c\right)+beh\log\left(c\right)-afg+aeh}{2\left(fgh^3x^2-eh^4x^2+2\,fg^2h^2x-2\,egh^3x+fg^3h-eg^2h^2\right)}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/2*b*f^2*p*q*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - 1/2*b*f^2
*p*q*log(h*x + g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - 1/2*b*p*q*log(f*x
+ e)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*(b*f*h*p*q*x + b*f*g*p*q - b*f*g*
q*log(d) + b*e*h*q*log(d) - b*f*g*log(c) + b*e*h*log(c) - a*f*g + a*e*h)/(
f*g*h^3*x^2 - e*h^4*x^2 + 2*f*g^2*h^2*x - 2*e*g*h^3*x + f*g^3*h - e*g^2*h^
2)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.51

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \frac{b\,f^2\,p\,q\,\mathrm{atanh}\left(\frac{2\,e^2\,h^3-2\,f^2\,g^2\,h}{2\,h\,{\left(e\,h-f\,g\right)}^2}+\frac{2\,f\,h\,x}{e\,h-f\,g}\right)}{h\,{\left(e\,h-f\,g\right)}^2} - \frac{b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{2\,h\,\left(g^2+2\,g\,h\,x+h^2\,x^2\right)} - \frac{\frac{a\,e\,h-a\,f\,g+b\,f\,g\,p\,q}{e\,h-f\,g}+\frac{b\,f\,h\,p\,q\,x}{e\,h-f\,g}}{2\,g^2\,h+4\,g\,h^2\,x+2\,h^3\,x^2}$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^3,x)
```

output
```
(b*f^2*p*q*atanh((2*e^2*h^3 - 2*f^2*g^2*h)/(2*h*(e*h - f*g)^2) + (2*f*h*x)
/(e*h - f*g)))/(h*(e*h - f*g)^2) - (b*log(c*(d*(e + f*x)^p)^q))/(2*h*(g^2
+ h^2*x^2 + 2*g*h*x)) - ((a*e*h - a*f*g + b*f*g*p*q)/(e*h - f*g) + (b*f*h*
p*q*x)/(e*h - f*g))/(2*g^2*h + 2*h^3*x^2 + 4*g*h^2*x)
```

3.427 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^4}\,dx$

3.427.1 Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \frac{bfpq}{6h(fg-eh)(g+hx)^2} + \frac{bf^2pq}{3h(fg-eh)^2(g+hx)} + \frac{bf^3pq\log(e+fx)}{3h(fg-eh)^3} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{3h(g+hx)^3} - \frac{bf^3pq\log(g+hx)}{3h(fg-eh)^3}$$

output
```
1/6*b*f*p*q/h/(-e*h+f*g)/(h*x+g)^2+1/3*b*f^2*p*q/h/(-e*h+f*g)^2/(h*x+g)+1/
3*b*f^3*p*q*ln(f*x+e)/h/(-e*h+f*g)^3+1/3*(-a-b*ln(c*(d*(f*x+e)^p)^q))/h/(h
*x+g)^3-1/3*b*f^3*p*q*ln(h*x+g)/h/(-e*h+f*g)^3
```

3.427.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \frac{-2a-2b\log\left(c(d(e+fx)^p)^q\right)+\frac{bfpq(g+hx)\left((fg-eh)(3fg-eh+2fhx)+2f^2(g+hx)^2\log(e+fx)-2f^2(g+hx)^2\log(g+hx)\right)}{(fg-eh)^3}}{6h(g+hx)^3}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^4,x]
```

output
```
(-2*a - 2*b*Log[c*(d*(e + f*x)^p)^q] + (b*f*p*q*(g + h*x)*((f*g - e*h)*(3*
f*g - e*h + 2*f*h*x) + 2*f^2*(g + h*x)^2*Log[e + f*x] - 2*f^2*(g + h*x)^2*
Log[g + h*x]))/(f*g - e*h)^3)/(6*h*(g + h*x)^3)
```

3.427.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2895, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^4}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^4}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{bfpq\int \frac{1}{(e+fx)(g+hx)^3}dx}{3h}-\frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{3h(g+hx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{bfpq\int \left(\frac{f^3}{(fg-eh)^3(e+fx)}-\frac{hf^2}{(fg-eh)^3(g+hx)}-\frac{hf}{(fg-eh)^2(g+hx)^2}-\frac{h}{(fg-eh)(g+hx)^3}\right)dx}{3h}-\frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{3h(g+hx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bfpq\left(\frac{f^2\log(e+fx)}{(fg-eh)^3}-\frac{f^2\log(g+hx)}{(fg-eh)^3}+\frac{f}{(g+hx)(fg-eh)^2}+\frac{1}{2(g+hx)^2(fg-eh)}\right)}{3h}-\frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{3h(g+hx)^3}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^4,x]
```

output
```
-1/3*(a + b*Log[c*(d*(e + f*x)^p)^q])/(h*(g + h*x)^3) + (b*f*p*q*(1/(2*(f*
g - e*h)*(g + h*x)^2) + f/((f*g - e*h)^2*(g + h*x)) + (f^2*Log[e + f*x])/(
f*g - e*h)^3 - (f^2*Log[g + h*x])/(f*g - e*h)^3))/(3*h)
```

3.427.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.427.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(142) = 284$.

Time = 11.92 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.27

method	result
parallelrisch	$-\frac{-6xbef^3gh^4pq-6\ln(c(d(fx+e)^p)^q)be^2f^2gh^4+6\ln(c(d(fx+e)^p)^q)bef^3g^2h^3+2\ln(c(d(fx+e)^p)^q)be^3fh^5-2\ln(c(d(fx+e$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-6*x*b*e*f^3*g*h^4*p*q-6*ln(c*(d*(f*x+e)^p)^q)*b*e^2*f^2*g*h^4+6*ln(
c*(d*(f*x+e)^p)^q)*b*e*f^3*g^2*h^3+2*ln(c*(d*(f*x+e)^p)^q)*b*e^3*f*h^5-2*l
n(c*(d*(f*x+e)^p)^q)*b*f^4*g^3*h^2+3*b*f^4*g^3*h^2*p*q+6*ln(f*x+e)*x^2*b*f
^4*g*h^4*p*q-6*ln(h*x+g)*x^2*b*f^4*g*h^4*p*q+6*ln(f*x+e)*x*b*f^4*g^2*h^3*p
*q-6*ln(h*x+g)*x*b*f^4*g^2*h^3*p*q+2*a*e^3*f*h^5-2*a*f^4*g^3*h^2-2*x^2*b*e
*f^3*h^5*p*q+2*x^2*b*f^4*g*h^4*p*q+2*ln(f*x+e)*x^3*b*f^4*h^5*p*q-2*ln(h*x+
g)*x^3*b*f^4*h^5*p*q+2*ln(f*x+e)*b*f^4*g^3*h^2*p*q-2*ln(h*x+g)*b*f^4*g^3*h
^2*p*q-6*a*e^2*f^2*g*h^4+6*a*e*f^3*g^2*h^3+b*e^2*f^2*g*h^4*p*q-4*b*e*f^3*g
^2*h^3*p*q+x*b*e^2*f^2*h^5*p*q+5*x*b*f^4*g^2*h^3*p*q)/(e^3*h^3-3*e^2*f*g*h
^2+3*e*f^2*g^2*h-f^3*g^3)/(h*x+g)^3/f/h^3
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(139) = 278$.

Time = 0.32 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.78

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = -\frac{2\,af^3g^3 - 6\,aef^2g^2h + 6\,ae^2fgh^2 - 2\,ae^3h^3 - 2\,(bf^3gh^2 - bef^2h^3)pqx^2 - (5\,bf^3g^2h - 6\,bef^2gh^2 + be^2f}{}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a*f^3*g^3 - 6*a*e*f^2*g^2*h + 6*a*e^2*f*g*h^2 - 2*a*e^3*h^3 - 2*(b
*f^3*g*h^2 - b*e*f^2*h^3)*p*q*x^2 - (5*b*f^3*g^2*h - 6*b*e*f^2*g*h^2 + b*e
^2*f*h^3)*p*q*x - (3*b*f^3*g^3 - 4*b*e*f^2*g^2*h + b*e^2*f*g*h^2)*p*q + 2*
(b*f^3*g^3 - 3*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*q*log(d) - 2*(
b*f^3*h^3*p*q*x^3 + 3*b*f^3*g*h^2*p*q*x^2 + 3*b*f^3*g^2*h*p*q*x + (3*b*e*f
^2*g^2*h - 3*b*e^2*f*g*h^2 + b*e^3*h^3)*p*q)*log(f*x + e) + 2*(b*f^3*h^3*p
*q*x^3 + 3*b*f^3*g*h^2*p*q*x^2 + 3*b*f^3*g^2*h*p*q*x + b*f^3*g^3*p*q)*log(
h*x + g) + 2*(b*f^3*g^3 - 3*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*l
og(c))/(f^3*g^6*h - 3*e*f^2*g^5*h^2 + 3*e^2*f*g^4*h^3 - e^3*g^3*h^4 + (f^3
*g^3*h^4 - 3*e*f^2*g^2*h^5 + 3*e^2*f*g*h^6 - e^3*h^7)*x^3 + 3*(f^3*g^4*h^3
 - 3*e*f^2*g^3*h^4 + 3*e^2*f*g^2*h^5 - e^3*g*h^6)*x^2 + 3*(f^3*g^5*h^2 - 3
*e*f^2*g^4*h^3 + 3*e^2*f*g^3*h^4 - e^3*g^2*h^5)*x)
```

3.427.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5673 vs. 2(133) = 266.

Time = 63.37 (sec) , antiderivative size = 5673, normalized size of antiderivative = 38.07

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**4,x)
```

output
```
Piecewise(((a*x + b*e*log(c*(d*(e + f*x)**p)**q)/f - b*p*q*x + b*x*log(c*(
d*(e + f*x)**p)**q))/g**4, Eq(h, 0)), (-3*a/(9*g**3*h + 27*g**2*h**2*x + 2
7*g*h**3*x**2 + 9*h**4*x**3) - b*p*q/(9*g**3*h + 27*g**2*h**2*x + 27*g*h**
3*x**2 + 9*h**4*x**3) - 3*b*log(c*(d*(f*g/h + f*x)**p)**q)/(9*g**3*h + 27*
g**2*h**2*x + 27*g*h**3*x**2 + 9*h**4*x**3), Eq(e, f*g/h)), (-2*a*e**3*h**
3/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h
**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h*
*5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**
3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h
- 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) +
6*a*e**2*f*g*h**2/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6
*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 5
4*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54
*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3
- 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g*
*3*h**4*x**3) - 6*a*e*f**2*g**2*h/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x
+ 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f
*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**
2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2
*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h*...
```

3.427.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(139) = 278.

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.05

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx$$

$$= \frac{1}{6}\left(\frac{2\,f^2\log(fx+e)}{f^3g^3h-3\,ef^2g^2h^2+3\,e^2fgh^3-e^3h^4} - \frac{2\,f^2\log(hx+g)}{f^3g^3h-3\,ef^2g^2h^2+3\,e^2fgh^3-e^3h^4} + \frac{}{f^2g^4h-2\,efg^3h^2+}\right.$$

$$- \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)\right)}{3\,(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h)} - \frac{a}{3\,(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h)}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x, algorithm="maxima")
```

output
```
1/6*(2*f^2*log(f*x + e)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f*g*h^3 - e^3
*h^4) - 2*f^2*log(h*x + g)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f*g*h^3 -
e^3*h^4) + (2*f*h*x + 3*f*g - e*h)/(f^2*g^4*h - 2*e*f*g^3*h^2 + e^2*g^2*h^
3 + (f^2*g^2*h^3 - 2*e*f*g*h^4 + e^2*h^5)*x^2 + 2*(f^2*g^3*h^2 - 2*e*f*g^2
*h^3 + e^2*g*h^4)*x))*b*f*p*q - 1/3*b*log(((f*x + e)^p*d)^q*c)/(h^4*x^3 +
3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*a/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*
h^2*x + g^3*h)
```

3.427.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(139) = 278$.

Time = 0.37 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.97

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \frac{bf^3pq\log\left(fx+e\right)}{3\left(f^3g^3h-3\,ef^2g^2h^2+3\,e^2fgh^3-e^3h^4\right)} - \frac{bf^3pq\log\left(hx+g\right)}{3\left(f^3g^3h-3\,ef^2g^2h^2+3\,e^2fgh^3-e^3h^4\right)} - \frac{bpq\log\left(fx+e\right)}{3\left(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h\right)} + \frac{2\,bf^2h^2pqx^2+5\,bf^2ghpqx-befh^2pqx+3\,bf^2g^2pq-befghpq-2\,bf^2g^2q\log\left(d\right)+4\,befghq\log\left(d\right)-2}{6\left(f^2g^2h^4x^3-2\,efgh^5x^3+e^2h^6x^3+3\,f^2g^3h^3x^2-6\,efg^2h^4x^2+3\,e^2gh^5x^2\right.}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x, algorithm="giac")
```

output
```
1/3*b*f^3*p*q*log(f*x + e)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f*g*h^3 -
e^3*h^4) - 1/3*b*f^3*p*q*log(h*x + g)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2
*f*g*h^3 - e^3*h^4) - 1/3*b*p*q*log(f*x + e)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^
2*h^2*x + g^3*h) + 1/6*(2*b*f^2*h^2*p*q*x^2 + 5*b*f^2*g*h*p*q*x - b*e*f*h^
2*p*q*x + 3*b*f^2*g^2*p*q - b*e*f*g*h*p*q - 2*b*f^2*g^2*q*log(d) + 4*b*e*f
*g*h*q*log(d) - 2*b*e^2*h^2*q*log(d) - 2*b*f^2*g^2*log(c) + 4*b*e*f*g*h*lo
g(c) - 2*b*e^2*h^2*log(c) - 2*a*f^2*g^2 + 4*a*e*f*g*h - 2*a*e^2*h^2)/(f^2*
g^2*h^4*x^3 - 2*e*f*g*h^5*x^3 + e^2*h^6*x^3 + 3*f^2*g^3*h^3*x^2 - 6*e*f*g^
2*h^4*x^2 + 3*e^2*g*h^5*x^2 + 3*f^2*g^4*h^2*x - 6*e*f*g^3*h^3*x + 3*e^2*g^
2*h^4*x + f^2*g^5*h - 2*e*f*g^4*h^2 + e^2*g^3*h^3)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.97

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \frac{2\,a\,e\,f\,g}{3\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}-\frac{a\,e^2\,h}{3\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}-\frac{b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{3\,h\,{\left(g+h\,x\right)}^3}-\frac{a\,f^2\,g^2}{3\,h\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}+\frac{b\,f^2\,h\,p\,q\,x^2}{3\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}-\frac{b\,e\,f\,g\,p\,q}{6\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}+\frac{b\,f^2\,g^2\,p\,q}{2\,h\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}+\frac{5\,b\,f^2\,g\,p\,q\,x}{6\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}-\frac{b\,e\,f\,h\,p\,q\,x}{6\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2}+\frac{b\,f^3\,p\,q\,\mathrm{atan}\left(\frac{e\,h\,1{}\mathrm{i}+f\,g\,1{}\mathrm{i}+f\,h\,x\,2{}\mathrm{i}}{e\,h-f\,g}\right)\,2{}\mathrm{i}}{3\,h\,{\left(e\,h-f\,g\right)}^3}$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^4,x)
```

output
```
(2*a*e*f*g)/(3*(g + h*x)^3*(e*h - f*g)^2) - (a*e^2*h)/(3*(g + h*x)^3*(e*h
- f*g)^2) - (b*log(c*(d*(e + f*x)^p)^q))/(3*h*(g + h*x)^3) - (a*f^2*g^2)/(
3*h*(g + h*x)^3*(e*h - f*g)^2) + (b*f^3*p*q*atan((e*h*1i + f*g*1i + f*h*x*
2i)/(e*h - f*g))*2i)/(3*h*(e*h - f*g)^3) + (b*f^2*h*p*q*x^2)/(3*(g + h*x)^
3*(e*h - f*g)^2) - (b*e*f*g*p*q)/(6*(g + h*x)^3*(e*h - f*g)^2) + (b*f^2*g^
2*p*q)/(2*h*(g + h*x)^3*(e*h - f*g)^2) + (5*b*f^2*g*p*q*x)/(6*(g + h*x)^3*
(e*h - f*g)^2) - (b*e*f*h*p*q*x)/(6*(g + h*x)^3*(e*h - f*g)^2)
```

3.428 $\int (g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.428.1 Optimal result

Integrand size = 28, antiderivative size = 409

$$
\begin{aligned}
&\int (g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx \\
&= \frac{2b^2(fg-eh)^3p^2q^2x}{f^3} + \frac{3b^2h(fg-eh)^2p^2q^2(e+fx)^2}{4f^4} + \frac{2b^2h^2(fg-eh)p^2q^2(e+fx)^3}{9f^4} \\
&+ \frac{b^2h^3p^2q^2(e+fx)^4}{32f^4} + \frac{b^2(fg-eh)^4p^2q^2\log^2(e+fx)}{4f^4h} \\
&- \frac{2b(fg-eh)^3pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^4} \\
&- \frac{3bh(fg-eh)^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4} \\
&- \frac{2bh^2(fg-eh)pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^4} \\
&- \frac{bh^3pq(e+fx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{8f^4} \\
&- \frac{b(fg-eh)^4pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4h} \\
&+ \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h}
\end{aligned}
$$

output
```
2*b^2*(-e*h+f*g)^3*p^2*q^2*x/f^3+3/4*b^2*h*(-e*h+f*g)^2*p^2*q^2*(f*x+e)^2/
f^4+2/9*b^2*h^2*(-e*h+f*g)*p^2*q^2*(f*x+e)^3/f^4+1/32*b^2*h^3*p^2*q^2*(f*x
+e)^4/f^4+1/4*b^2*(-e*h+f*g)^4*p^2*q^2*ln(f*x+e)^2/f^4/h-2*b*(-e*h+f*g)^3*
p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^4-3/2*b*h*(-e*h+f*g)^2*p*q*(f*x+
e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^4-2/3*b*h^2*(-e*h+f*g)*p*q*(f*x+e)^3*(a
+b*ln(c*(d*(f*x+e)^p)^q))/f^4-1/8*b*h^3*p*q*(f*x+e)^4*(a+b*ln(c*(d*(f*x+e)
^p)^q))/f^4-1/2*b*(-e*h+f*g)^4*p*q*ln(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f
^4/h+1/4*(h*x+g)^4*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h
```

3.428.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.98

$$\int (g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$=\frac{288(fg-eh)^3(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2+432h(fg-eh)^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right.}{}$$

input
```
Integrate[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(288*(f*g - e*h)^3*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 432*h*(f
*g - e*h)^2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 288*h^2*(f*g
- e*h)*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 72*h^3*(e + f*x)^4
*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 576*b*(f*g - e*h)^3*p*q*(f*(a - b*p*
q)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]) + 216*b*h*(f*g - e*h)^2*p*q*(
b*f*p*q*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])) +
64*b*h^2*(f*g - e*h)*p*q*(b*f*p*q*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f
*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])) + 9*b*h^3*p*q*(b*f*p*q*x*(4*e^3 +
6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - 4*(e + f*x)^4*(a + b*Log[c*(d*(e + f*
x)^p)^q])))/(288*f^4)
```

3.428.3 Rubi [A] (warning: unable to verify)

Time = 1.14 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2895, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{(g+hx)^4 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4h}-\frac{bfpq\int \frac{(g+hx)^4(a+b\log(c(d(e+fx)^p)^q))}{e+fx}dx}{2h}$$

$$\Big\downarrow \text{2858}$$

$$\frac{(g+hx)^4 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4h}-\frac{bpq\int \frac{\left(f\left(g-\frac{eh}{f}\right)+h(e+fx)\right)^4(a+b\log(cd^q(e+fx)^{pq}))}{f^4(e+fx)}d(e+fx)}{2h}$$

$$\Big\downarrow \text{27}$$

$$\frac{(g+hx)^4 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4h}-\frac{bpq\int \frac{(fg-eh+h(e+fx))^4(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)}{2f^4h}$$

$$\Big\downarrow \text{2772}$$

$$\frac{(g+hx)^4 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4h}-$$
$$bpq\Big(-bpq\int \Big(\tfrac{1}{4}(e+fx)^3h^4+\tfrac{4}{3}(fg-eh)(e+fx)^2h^3+3(fg-eh)^2(e+fx)h^2+4(fg-eh)^3h+\frac{(fg-eh)^4\log(e+fx}{e+fx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^4 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4h}-$$
$$bpq\left(\tfrac{4}{3}h^3(e+fx)^3(fg-eh)\left(a+b\log \left(cd^q(e+fx)^{pq}\right)\right)+3h^2(e+fx)^2(fg-eh)^2\left(a+b\log \left(cd^q(e+fx)^{pq}\right)\right)+(fg\right.$$

input
```
Int[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
-1/2*(b*p*q*(-(b*p*q*(4*h*(f*g - e*h)^3*(e + f*x) + (3*h^2*(f*g - e*h)^2*(
e + f*x)^2)/2 + (4*h^3*(f*g - e*h)*(e + f*x)^3)/9 + (h^4*(e + f*x)^4)/16 +
 ((f*g - e*h)^4*Log[e + f*x]^2)/2)) + 4*h*(f*g - e*h)^3*(e + f*x)*(a + b*L
og[c*d^q*(e + f*x)^(p*q)]) + 3*h^2*(f*g - e*h)^2*(e + f*x)^2*(a + b*Log[c*
d^q*(e + f*x)^(p*q)]) + (4*h^3*(f*g - e*h)*(e + f*x)^3*(a + b*Log[c*d^q*(e
 + f*x)^(p*q)]))/3 + (h^4*(e + f*x)^4*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/
4 + (f*g - e*h)^4*Log[e + f*x]*(a + b*Log[c*d^q*(e + f*x)^(p*q)])))/(f^4*h
) + ((g + h*x)^4*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(4*h)
```

3.428.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.428.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1536 vs. $2(391) = 782$.

Time = 16.24 (sec) , antiderivative size = 1537, normalized size of antiderivative = 3.76

method	result	size
parallelrisch	Expression too large to display	1537

input
```
int((h*x+g)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/288*(1632*ln(f*x+e)*b^2*e^3*f*g*h^2*p^2*q^2-864*x*ln(c*(d*(f*x+e)^p)^q)
*b^2*e*f^3*g^2*h*p*q+576*x*a*b*e^2*f^2*g*h^2*p*q-864*x*a*b*e*f^3*g^2*h*p*q
-576*ln(f*x+e)*a*b*e^3*f*g*h^2*p*q+864*ln(f*x+e)*a*b*e^2*f^2*g^2*h*p*q-288
*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*g*h^2*p*q-288*x^2*a*b*e*f^3*g*h^2*p*q
+576*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*g*h^2*p*q-2160*ln(f*x+e)*b^2*e^2*
f^2*g^2*h*p^2*q^2-1152*ln(f*x+e)*a*b*e*f^3*g^3*p*q-48*x^3*ln(c*(d*(f*x+e)^
p)^q)*b^2*e*f^3*h^3*p*q+192*x^3*ln(c*(d*(f*x+e)^p)^q)*b^2*f^4*g*h^2*p*q+24
0*x^2*b^2*e*f^3*g*h^2*p^2*q^2-48*x^3*a*b*e*f^3*h^3*p*q+192*x^3*a*b*f^4*g*h
^2*p*q+72*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*h^3*p*q+432*x^2*ln(c*(d*(f
*x+e)^p)^q)*b^2*f^4*g^2*h*p*q-1056*x*b^2*e^2*f^2*g*h^2*p^2*q^2+864*a*b*e^2
*f^2*g^2*h*p*q-576*a*b*e^3*f*g*h^2*p*q+1296*x*b^2*e*f^3*g^2*h*p^2*q^2+72*x
^2*a*b*e^2*f^2*h^3*p*q+432*x^2*a*b*f^4*g^2*h*p*q-144*x*ln(c*(d*(f*x+e)^p)^
q)*b^2*e^3*f*h^3*p*q-144*x*a*b*e^3*f*h^3*p*q-576*ln(c*(d*(f*x+e)^p)^q)*b^2
*e^3*f*g*h^2*p*q+864*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*g^2*h*p*q+36*x^4*ln
(c*(d*(f*x+e)^p)^q)*b^2*f^4*h^3*p*q+28*x^3*b^2*e*f^3*h^3*p^2*q^2-64*x^3*b^
2*f^4*g*h^2*p^2*q^2-576*a*b*e*f^3*g^3*p*q+1056*b^2*e^3*f*g*h^2*p^2*q^2-129
6*b^2*e^2*f^2*g^2*h*p^2*q^2+36*x^4*a*b*f^4*h^3*p*q-78*x^2*b^2*e^2*f^2*h^3*
p^2*q^2-216*x^2*b^2*f^4*g^2*h*p^2*q^2+300*x*b^2*e^3*f*h^3*p^2*q^2-576*x^3*
ln(c*(d*(f*x+e)^p)^q)*a*b*f^4*g*h^2+576*x*ln(c*(d*(f*x+e)^p)^q)*b^2*f^4*g^
3*p*q-864*x^2*ln(c*(d*(f*x+e)^p)^q)*a*b*f^4*g^2*h+576*x*a*b*f^4*g^3*p*q...
```

3.428.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(391) = 782.

Time = 0.35 (sec) , antiderivative size = 1742, normalized size of antiderivative = 4.26

$$\int (g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
1/288*(9*(b^2*f^4*h^3*p^2*q^2 - 4*a*b*f^4*h^3*p*q + 8*a^2*f^4*h^3)*x^4 + 4
*(72*a^2*f^4*g*h^2 + (16*b^2*f^4*g*h^2 - 7*b^2*e*f^3*h^3)*p^2*q^2 - 12*(4*
a*b*f^4*g*h^2 - a*b*e*f^3*h^3)*p*q)*x^3 + 6*(72*a^2*f^4*g^2*h + (36*b^2*f^
4*g^2*h - 40*b^2*e*f^3*g*h^2 + 13*b^2*e^2*f^2*h^3)*p^2*q^2 - 12*(6*a*b*f^4
*g^2*h - 4*a*b*e*f^3*g*h^2 + a*b*e^2*f^2*h^3)*p*q)*x^2 + 72*(b^2*f^4*h^3*p
^2*q^2*x^4 + 4*b^2*f^4*g*h^2*p^2*q^2*x^3 + 6*b^2*f^4*g^2*h*p^2*q^2*x^2 + 4
*b^2*f^4*g^3*p^2*q^2*x + (4*b^2*e*f^3*g^3 - 6*b^2*e^2*f^2*g^2*h + 4*b^2*e^
3*f*g*h^2 - b^2*e^4*h^3)*p^2*q^2)*log(f*x + e)^2 + 72*(b^2*f^4*h^3*x^4 + 4
*b^2*f^4*g*h^2*x^3 + 6*b^2*f^4*g^2*h*x^2 + 4*b^2*f^4*g^3*x)*log(c)^2 + 72*
(b^2*f^4*h^3*q^2*x^4 + 4*b^2*f^4*g*h^2*q^2*x^3 + 6*b^2*f^4*g^2*h*q^2*x^2 +
 4*b^2*f^4*g^3*q^2*x)*log(d)^2 + 12*(24*a^2*f^4*g^3 + (48*b^2*f^4*g^3 - 10
8*b^2*e*f^3*g^2*h + 88*b^2*e^2*f^2*g*h^2 - 25*b^2*e^3*f*h^3)*p^2*q^2 - 12*
(4*a*b*f^4*g^3 - 6*a*b*e*f^3*g^2*h + 4*a*b*e^2*f^2*g*h^2 - a*b*e^3*f*h^3)*
p*q)*x - 12*((48*b^2*e*f^3*g^3 - 108*b^2*e^2*f^2*g^2*h + 88*b^2*e^3*f*g*h^
2 - 25*b^2*e^4*h^3)*p^2*q^2 + 3*(b^2*f^4*h^3*p^2*q^2 - 4*a*b*f^4*h^3*p*q)*
x^4 - 4*(12*a*b*f^4*g*h^2*p*q - (4*b^2*f^4*g*h^2 - b^2*e*f^3*h^3)*p^2*q^2)
*x^3 - 12*(4*a*b*e*f^3*g^3 - 6*a*b*e^2*f^2*g^2*h + 4*a*b*e^3*f*g*h^2 - a*b
*e^4*h^3)*p*q - 6*(12*a*b*f^4*g^2*h*p*q - (6*b^2*f^4*g^2*h - 4*b^2*e*f^3*g
*h^2 + b^2*e^2*f^2*h^3)*p^2*q^2)*x^2 - 12*(4*a*b*f^4*g^3*p*q - (4*b^2*f^4*
g^3 - 6*b^2*e*f^3*g^2*h + 4*b^2*e^2*f^2*g*h^2 - b^2*e^3*f*h^3)*p^2*q^2)...
```

3.428.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. 2(394) = 788.

Time = 5.39 (sec) , antiderivative size = 1421, normalized size of antiderivative = 3.47

$$\int (g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)**3*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Piecewise((a**2*g**3*x + 3*a**2*g**2*h*x**2/2 + a**2*g*h**2*x**3 + a**2*h*
*3*x**4/4 - a*b*e**4*h**3*log(c*(d*(e + f*x)**p)**q)/(2*f**4) + 2*a*b*e**3
*g*h**2*log(c*(d*(e + f*x)**p)**q)/f**3 + a*b*e**3*h**3*p*q*x/(2*f**3) - 3
*a*b*e**2*g**2*h*log(c*(d*(e + f*x)**p)**q)/f**2 - 2*a*b*e**2*g*h**2*p*q*x
/f**2 - a*b*e**2*h**3*p*q*x**2/(4*f**2) + 2*a*b*e*g**3*log(c*(d*(e + f*x)*
*p)**q)/f + 3*a*b*e*g**2*h*p*q*x/f + a*b*e*g*h**2*p*q*x**2/f + a*b*e*h**3*
p*q*x**3/(6*f) - 2*a*b*g**3*p*q*x + 2*a*b*g**3*x*log(c*(d*(e + f*x)**p)**q
) - 3*a*b*g**2*h*p*q*x**2/2 + 3*a*b*g**2*h*x**2*log(c*(d*(e + f*x)**p)**q)
 - 2*a*b*g*h**2*p*q*x**3/3 + 2*a*b*g*h**2*x**3*log(c*(d*(e + f*x)**p)**q)
- a*b*h**3*p*q*x**4/8 + a*b*h**3*x**4*log(c*(d*(e + f*x)**p)**q)/2 + 25*b*
*2*e**4*h**3*p*q*log(c*(d*(e + f*x)**p)**q)/(24*f**4) - b**2*e**4*h**3*log
(c*(d*(e + f*x)**p)**q)**2/(4*f**4) - 11*b**2*e**3*g*h**2*p*q*log(c*(d*(e
+ f*x)**p)**q)/(3*f**3) + b**2*e**3*g*h**2*log(c*(d*(e + f*x)**p)**q)**2/f
**3 - 25*b**2*e**3*h**3*p**2*q**2*x/(24*f**3) + b**2*e**3*h**3*p*q*x*log(c
*(d*(e + f*x)**p)**q)/(2*f**3) + 9*b**2*e**2*g**2*h*p*q*log(c*(d*(e + f*x)
**p)**q)/(2*f**2) - 3*b**2*e**2*g**2*h*log(c*(d*(e + f*x)**p)**q)**2/(2*f*
*2) + 11*b**2*e**2*g*h**2*p**2*q**2*x/(3*f**2) - 2*b**2*e**2*g*h**2*p*q*x*
log(c*(d*(e + f*x)**p)**q)/f**2 + 13*b**2*e**2*h**3*p**2*q**2*x**2/(48*f**
2) - b**2*e**2*h**3*p*q*x**2*log(c*(d*(e + f*x)**p)**q)/(4*f**2) - 2*b**2*
e*g**3*p*q*log(c*(d*(e + f*x)**p)**q)/f + b**2*e*g**3*log(c*(d*(e + f*x...
```

3.428.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(391) = 782$.

Time = 0.23 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.19

$$\int (g + hx)^3 (a + b\log(c(d(e + fx)^p)^q))^2 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*h^3*x^4*log(((f*x + e)^p*d)^q*c)^2 + 1/2*a*b*h^3*x^4*log(((f*x + e
)^p*d)^q*c) + b^2*g*h^2*x^3*log(((f*x + e)^p*d)^q*c)^2 + 1/4*a^2*h^3*x^4 -
 2*a*b*f*g^3*p*q*(x/f - e*log(f*x + e)/f^2) - 1/24*a*b*f*h^3*p*q*(12*e^4*l
og(f*x + e)/f^5 + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/f^4)
+ 1/3*a*b*f*g*h^2*p*q*(6*e^3*log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6
*e^2*x)/f^3) - 3/2*a*b*f*g^2*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*
x)/f^2) + 2*a*b*g*h^2*x^3*log(((f*x + e)^p*d)^q*c) + 3/2*b^2*g^2*h*x^2*log
(((f*x + e)^p*d)^q*c)^2 + a^2*g*h^2*x^3 + 3*a*b*g^2*h*x^2*log(((f*x + e)^p
*d)^q*c) + b^2*g^3*x*log(((f*x + e)^p*d)^q*c)^2 + 3/2*a^2*g^2*h*x^2 + 2*a*
b*g^3*x*log(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log
(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*
q^2/f)*b^2*g^3 - 3/4*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^
2)*log(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x +
6*e^2*log(f*x + e))*p^2*q^2/f^2)*b^2*g^2*h + 1/18*(6*f*p*q*(6*e^3*log(f*x
+ e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3)*log(((f*x + e)^p*d)^q*c)
 + (4*f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3
*log(f*x + e))*p^2*q^2/f^3)*b^2*g*h^2 - 1/288*(12*f*p*q*(12*e^4*log(f*x +
e)/f^5 + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/f^4)*log(((f*x
 + e)^p*d)^q*c) - (9*f^4*x^4 - 28*e*f^3*x^3 + 78*e^2*f^2*x^2 + 72*e^4*log(
f*x + e)^2 - 300*e^3*f*x + 300*e^4*log(f*x + e))*p^2*q^2/f^4)*b^2*h^3 +...
```

3.428.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. $2(391) = 782$.

Time = 0.39 (sec) , antiderivative size = 3738, normalized size of antiderivative = 9.14

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
(f*x + e)*b^2*g^3*p^2*q^2*log(f*x + e)^2/f + 3/2*(f*x + e)^2*b^2*g^2*h*p^2
*q^2*log(f*x + e)^2/f^2 - 3*(f*x + e)*b^2*e*g^2*h*p^2*q^2*log(f*x + e)^2/f
^2 + (f*x + e)^3*b^2*g*h^2*p^2*q^2*log(f*x + e)^2/f^3 - 3*(f*x + e)^2*b^2*
e*g*h^2*p^2*q^2*log(f*x + e)^2/f^3 + 3*(f*x + e)*b^2*e^2*g*h^2*p^2*q^2*log
(f*x + e)^2/f^3 + 1/4*(f*x + e)^4*b^2*h^3*p^2*q^2*log(f*x + e)^2/f^4 - (f*
x + e)^3*b^2*e*h^3*p^2*q^2*log(f*x + e)^2/f^4 + 3/2*(f*x + e)^2*b^2*e^2*h^
3*p^2*q^2*log(f*x + e)^2/f^4 - (f*x + e)*b^2*e^3*h^3*p^2*q^2*log(f*x + e)^
2/f^4 - 2*(f*x + e)*b^2*g^3*p^2*q^2*log(f*x + e)/f - 3/2*(f*x + e)^2*b^2*g
^2*h*p^2*q^2*log(f*x + e)/f^2 + 6*(f*x + e)*b^2*e*g^2*h*p^2*q^2*log(f*x +
e)/f^2 - 2/3*(f*x + e)^3*b^2*g*h^2*p^2*q^2*log(f*x + e)/f^3 + 3*(f*x + e)^
2*b^2*e*g*h^2*p^2*q^2*log(f*x + e)/f^3 - 6*(f*x + e)*b^2*e^2*g*h^2*p^2*q^2
*log(f*x + e)/f^3 - 1/8*(f*x + e)^4*b^2*h^3*p^2*q^2*log(f*x + e)/f^4 + 2/3
*(f*x + e)^3*b^2*e*h^3*p^2*q^2*log(f*x + e)/f^4 - 3/2*(f*x + e)^2*b^2*e^2*
h^3*p^2*q^2*log(f*x + e)/f^4 + 2*(f*x + e)*b^2*e^3*h^3*p^2*q^2*log(f*x + e
)/f^4 + 2*(f*x + e)*b^2*g^3*p*q^2*log(f*x + e)*log(d)/f + 3*(f*x + e)^2*b^
2*g^2*h*p*q^2*log(f*x + e)*log(d)/f^2 - 6*(f*x + e)*b^2*e*g^2*h*p*q^2*log(
f*x + e)*log(d)/f^2 + 2*(f*x + e)^3*b^2*g*h^2*p*q^2*log(f*x + e)*log(d)/f^
3 - 6*(f*x + e)^2*b^2*e*g*h^2*p*q^2*log(f*x + e)*log(d)/f^3 + 6*(f*x + e)*
b^2*e^2*g*h^2*p*q^2*log(f*x + e)*log(d)/f^3 + 1/2*(f*x + e)^4*b^2*h^3*p*q^
2*log(f*x + e)*log(d)/f^4 - 2*(f*x + e)^3*b^2*e*h^3*p*q^2*log(f*x + e)*...
```

3.428. $\int (g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.428.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.82

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$$

$$= x^3 \left(\frac{h^2 \left(6\, a^2\, e\, h + 18\, a^2\, f\, g - b^2\, e\, h\, p^2\, q^2 + 4\, b^2\, f\, g\, p^2\, q^2 - 12\, a\, b\, f\, g\, p\, q\right)}{18\, f} - \frac{e\, h^3 \left(8\, a^2 - 4\, a\, b\, p\, q + b^2\, p^2\, q^2\right)}{24\, f} \right)$$

$$+ \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right) \left(\frac{x \left(\frac{e \left(\frac{e \left(\frac{4\, b\, h^2\, (a\, e\, h + 3\, a\, f\, g - b\, f\, g\, p\, q)}{f} - \frac{b\, e\, h^3\, (4\, a - b\, p\, q)}{f} \right)}{f} - \frac{6\, b\, g\, h\, (2\, a\, e\, h + 2\, a\, f\, g - b\, f\, g\, p\, q)}{f} \right)}{f} + \frac{4\, b\, g^2\, (3\, a\, e\, h + \cdots}{f} \cdots}{2} \cdots \right.$$

$$+ \frac{x^3 \left(\frac{4\, b\, h^2\, (a\, e\, h + 3\, a\, f\, g - b\, f\, g\, p\, q)}{3\, f} - \frac{b\, e\, h^3\, (4\, a - b\, p\, q)}{3\, f} \right)}{2}$$

$$- \frac{x^2 \left(\frac{e \left(\frac{4\, b\, h^2\, (a\, e\, h + 3\, a\, f\, g - b\, f\, g\, p\, q)}{f} - \frac{b\, e\, h^3\, (4\, a - b\, p\, q)}{f} \right)}{2\, f} - \frac{3\, b\, g\, h\, (2\, a\, e\, h + 2\, a\, f\, g - b\, f\, g\, p\, q)}{f} \right)}{2}$$

$$\left. + \frac{b\, h^3\, x^4\, (4\, a - b\, p\, q)}{8} \right)$$

$$+ \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2 \left(b^2\, g^3\, x - \frac{e \left(b^2\, e^3\, h^3 - 4\, b^2\, e^2\, f\, g\, h^2 + 6\, b^2\, e\, f^2\, g^2\, h - 4\, b^2\, f^3\, g^3\right)}{4\, f^4} + \frac{b^2\, h^3\, x^4}{4} + \frac{3\, b^2\, g^2\, h\, x^2}{2} + b^2\, g\, h^2\, x^3 \right)$$

$$+ x \left(\frac{72\, a^2\, e\, f^2\, g^2\, h + 24\, a^2\, f^3\, g^3 - 48\, a\, b\, f^3\, g^3\, p\, q - 12\, b^2\, e^3\, h^3\, p^2\, q^2 + 48\, b^2\, e^2\, f\, g\, h^2\, p^2\, q^2 - 72\, b^2\, e\, f^2\, g^2\, h \cdots}{24\, f^3} \cdots \right.$$

$$\left(\left(\frac{h^2 \left(6\, a^2\, e\, h + 18\, a^2\, f\, g - b^2\, e\, h\, p^2\, q^2 + 4\, b^2\, f\, g\, p^2\, q^2 - 12\, a\, b\, f\, g\, p\, q\right)}{\cdots} - \frac{e\, h^3 \left(8\, a^2 - 4\, a\, b\, p\, q + b^2\, p^2\, q^2\right)}{\cdots} \right) \cdots \right.$$

input
```
int((g + h*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
x^3*((h^2*(6*a^2*e*h + 18*a^2*f*g - b^2*e*h*p^2*q^2 + 4*b^2*f*g*p^2*q^2 -
12*a*b*f*g*p*q))/(18*f) - (e*h^3*(8*a^2 + b^2*p^2*q^2 - 4*a*b*p*q))/(24*f)
) + log(c*(d*(e + f*x)^p)^q)*((x*((e*((e*((4*b*h^2*(a*e*h + 3*a*f*g - b*f*
g*p*q))/f - (b*e*h^3*(4*a - b*p*q))/f))/f - (6*b*g*h*(2*a*e*h + 2*a*f*g -
b*f*g*p*q))/f))/f + (4*b*g^2*(3*a*e*h + a*f*g - b*f*g*p*q))/f))/2 + (x^3*(
(4*b*h^2*(a*e*h + 3*a*f*g - b*f*g*p*q))/(3*f) - (b*e*h^3*(4*a - b*p*q))/(3
*f)))/2 - (x^2*((e*((4*b*h^2*(a*e*h + 3*a*f*g - b*f*g*p*q))/f - (b*e*h^3*(
4*a - b*p*q))/f))/(2*f) - (3*b*g*h*(2*a*e*h + 2*a*f*g - b*f*g*p*q))/f))/2
+ (b*h^3*x^4*(4*a - b*p*q))/8) + log(c*(d*(e + f*x)^p)^q)^2*(b^2*g^3*x - (
e*(b^2*e^3*h^3 - 4*b^2*f^3*g^3 + 6*b^2*e*f^2*g^2*h - 4*b^2*e^2*f*g*h^2))/(
4*f^4) + (b^2*h^3*x^4)/4 + (3*b^2*g^2*h*x^2)/2 + b^2*g*h^2*x^3) + x*((24*a
^2*f^3*g^3 - 12*b^2*e^3*h^3*p^2*q^2 + 48*b^2*f^3*g^3*p^2*q^2 + 72*a^2*e*f^
2*g^2*h - 48*a*b*f^3*g^3*p*q - 72*b^2*e*f^2*g^2*h*p^2*q^2 + 48*b^2*e^2*f*g
*h^2*p^2*q^2)/(24*f^3) + (e*((e*((h^2*(6*a^2*e*h + 18*a^2*f*g - b^2*e*h*p^
2*q^2 + 4*b^2*f*g*p^2*q^2 - 12*a*b*f*g*p*q))/(6*f) - (e*h^3*(8*a^2 + b^2*p
^2*q^2 - 4*a*b*p*q))/(8*f)))/f - (h*(12*a^2*f^2*g^2 + b^2*e^2*h^2*p^2*q^2
+ 6*b^2*f^2*g^2*p^2*q^2 + 12*a^2*e*f*g*h - 12*a*b*f^2*g^2*p*q - 4*b^2*e*f*
g*h*p^2*q^2))/(4*f^2)))/f) - x^2*((e*((h^2*(6*a^2*e*h + 18*a^2*f*g - b^2*e
*h*p^2*q^2 + 4*b^2*f*g*p^2*q^2 - 12*a*b*f*g*p*q))/(6*f) - (e*h^3*(8*a^2 +
b^2*p^2*q^2 - 4*a*b*p*q))/(8*f)))/(2*f) - (h*(12*a^2*f^2*g^2 + b^2*e^2*...
```

3.428. $\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.429 $\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.429.1 Optimal result

Integrand size = 28, antiderivative size = 323

$$\begin{aligned}
&\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx \\
&= \frac{2b^2(fg-eh)^2p^2q^2x}{f^2} + \frac{b^2h(fg-eh)p^2q^2(e+fx)^2}{2f^3} + \frac{2b^2h^2p^2q^2(e+fx)^3}{27f^3} \\
&\quad + \frac{b^2(fg-eh)^3p^2q^2\log^2(e+fx)}{3f^3h} - \frac{2b(fg-eh)^2pq(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{f^3} \\
&\quad - \frac{bh(fg-eh)pq(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{f^3} \\
&\quad - \frac{2bh^2pq(e+fx)^3\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{9f^3} \\
&\quad - \frac{2b(fg-eh)^3pq\log(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{3f^3h} \\
&\quad + \frac{(g+hx)^3\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{3h}
\end{aligned}$$

output
```
2*b^2*(-e*h+f*g)^2*p^2*q^2*x/f^2+1/2*b^2*h*(-e*h+f*g)*p^2*q^2*(f*x+e)^2/f^
3+2/27*b^2*h^2*p^2*q^2*(f*x+e)^3/f^3+1/3*b^2*(-e*h+f*g)^3*p^2*q^2*ln(f*x+e
)^2/f^3/h-2*b*(-e*h+f*g)^2*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-b*h
*(-e*h+f*g)*p*q*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-2/9*b*h^2*p*q*(f
*x+e)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-2/3*b*(-e*h+f*g)^3*p*q*ln(f*x+e)*(
a+b*ln(c*(d*(f*x+e)^p)^q))/f^3/h+1/3*(h*x+g)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))
^2/h
```

3.429.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.86

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$$
$$= \frac{54(fg-eh)^2(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2+54h(fg-eh)(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{}$$

input
```
Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(54*(f*g - e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 54*h*(f*g
 - e*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 18*h^2*(e + f*x)^
3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 108*b*(f*g - e*h)^2*p*q*(f*(a - b*p
*q)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]) + 27*b*h*(f*g - e*h)*p*q*(b*
f*p*q*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])) + 4*
b*h^2*p*q*(b*f*p*q*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*(a + b*Lo
g[c*(d*(e + f*x)^p)^q])))/(54*f^3)
```

3.429.3 Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2895, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$$
$$\Big\downarrow \ 2895$$
$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 dx$$
$$\Big\downarrow \ 2845$$
$$\frac{(g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{3h} - \frac{2bfpq \int \frac{(g+hx)^3(a+b\log(c(d(e+fx)^p)^q))}{e+fx}dx}{3h}$$
$$\Big\downarrow \ 2858$$

$$\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{2bpq\int\frac{\left(f\left(g-\frac{eh}{f}\right)+h(e+fx)\right)^3(a+b\log(cd^q(e+fx)^{pq}))}{f^3(e+fx)}d(e+fx)}{3h}$$

$$\Big\downarrow \text{27}$$

$$\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{2bpq\int\frac{(fg-eh+h(e+fx))^3(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)}{3f^3h}$$

$$\Big\downarrow \text{2772}$$

$$\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$
$$2bpq\left(-bpq\int\left(\frac{1}{3}(e+fx)^2h^3+\frac{3}{2}(fg-eh)(e+fx)h^2+3(fg-eh)^2h+\frac{(fg-eh)^3\log(e+fx)}{e+fx}\right)d(e+fx)+\frac{3}{2}h^2(e+f$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$
$$2bpq\left(\frac{3}{2}h^2(e+fx)^2(fg-eh)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)+(fg-eh)^3\log(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)+3h\right.$$

input
```
Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(-2*b*p*q*(-(b*p*q*(3*h*(f*g - e*h)^2*(e + f*x) + (3*h^2*(f*g - e*h)*(e +
f*x)^2)/4 + (h^3*(e + f*x)^3)/9 + ((f*g - e*h)^3*Log[e + f*x]^2)/2)) + 3*h
*(f*g - e*h)^2*(e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)]) + (3*h^2*(f*g
- e*h)*(e + f*x)^2*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/2 + (h^3*(e + f*x)^
3*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/3 + (f*g - e*h)^3*Log[e + f*x]*(a +
b*Log[c*d^q*(e + f*x)^(p*q)])))/(3*f^3*h) + ((g + h*x)^3*(a + b*Log[c*(d*(
e + f*x)^p)^q])^2)/(3*h)
```

3.429.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.429.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(311) = 622$.

Time = 5.68 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.91

method	result
parallelrisch	$\frac{162\ln(fx+e)b^2e^3fghp^2q^2+108xabe^2f^2ghpq-108\ln(fx+e)abe^3fghpq+108x\ln(c(d(fx+e)^p)^q)abef^3g^2-108\ln(fx+e)b^2e^2\ldots}{}$

input
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

output
```
1/54*(162*ln(f*x+e)*b^2*e^3*f*g*h*p^2*q^2+108*x*a*b*e^2*f^2*g*h*p*q-108*ln
(f*x+e)*a*b*e^3*f*g*h*p*q+108*x*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f^3*g^2-108*ln
(f*x+e)*b^2*e^2*f^2*g^2*p^2*q^2+36*ln(f*x+e)*a*b*e^4*h^2*p*q+4*x^3*b^2*e*f
^3*h^2*p^2*q^2-15*x^2*b^2*e^2*f^2*h^2*p^2*q^2+66*x*b^2*e^3*f*h^2*p^2*q^2+1
08*x*b^2*e*f^3*g^2*p^2*q^2+36*x^3*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f^3*h^2+54*x
^2*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e*f^3*g*h+108*ln(f*x+e)*a*b*e^2*f^2*g^2*p*q
-12*x^3*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*h^2*p*q+27*x^2*b^2*e*f^3*g*h*p^2*q
^2-12*x^3*a*b*e*f^3*h^2*p*q+18*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*h^2*p
*q-108*a*b*e^3*f*g*h*p*q-54*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*g*h*p*q-54
*x^2*a*b*e*f^3*g*h*p*q+108*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*g*h*p*q-162
*x*b^2*e^2*f^2*g*h*p^2*q^2+18*x^2*a*b*e^2*f^2*h^2*p*q-36*x*ln(c*(d*(f*x+e)
^p)^q)*b^2*e^3*f*h^2*p*q-108*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*g^2*p*q+108
*x^2*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f^3*g*h-36*x*a*b*e^3*f*h^2*p*q-108*x*a*b*
e*f^3*g^2*p*q-108*b^2*e^2*f^2*g^2*p^2*q^2+36*a*b*e^4*h^2*p*q+54*x*a^2*e*f^
3*g^2+18*x^3*a^2*e*f^3*h^2+54*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^2*f^2*g^2+108*
a*b*e^2*f^2*g^2*p*q+18*x^3*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e*f^3*h^2+54*x*ln(c
*(d*(f*x+e)^p)^q)^2*b^2*e*f^3*g^2+54*x^2*a^2*e*f^3*g*h-54*ln(c*(d*(f*x+e)^
p)^q)^2*b^2*e^3*f*g*h-66*ln(f*x+e)*b^2*e^4*h^2*p^2*q^2-66*b^2*e^4*h^2*p^2*
q^2+162*b^2*e^3*f*g*h*p^2*q^2-54*a^2*e^2*f^2*g^2+18*ln(c*(d*(f*x+e)^p)^q)^
2*b^2*e^4*h^2)/e/f^3
```

3.429.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 1137, normalized size of antiderivative = 3.52

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
1/54*(2*(2*b^2*f^3*h^2*p^2*q^2 - 6*a*b*f^3*h^2*p*q + 9*a^2*f^3*h^2)*x^3 +
3*(18*a^2*f^3*g*h + (9*b^2*f^3*g*h - 5*b^2*e*f^2*h^2)*p^2*q^2 - 6*(3*a*b*f
^3*g*h - a*b*e*f^2*h^2)*p*q)*x^2 + 18*(b^2*f^3*h^2*p^2*q^2*x^3 + 3*b^2*f^3
*g*h*p^2*q^2*x^2 + 3*b^2*f^3*g^2*p^2*q^2*x + (3*b^2*e*f^2*g^2 - 3*b^2*e^2*
f*g*h + b^2*e^3*h^2)*p^2*q^2)*log(f*x + e)^2 + 18*(b^2*f^3*h^2*x^3 + 3*b^2
*f^3*g*h*x^2 + 3*b^2*f^3*g^2*x)*log(c)^2 + 18*(b^2*f^3*h^2*q^2*x^3 + 3*b^2
*f^3*g*h*q^2*x^2 + 3*b^2*f^3*g^2*q^2*x)*log(d)^2 + 6*(9*a^2*f^3*g^2 + (18*
b^2*f^3*g^2 - 27*b^2*e*f^2*g*h + 11*b^2*e^2*f*h^2)*p^2*q^2 - 6*(3*a*b*f^3*
g^2 - 3*a*b*e*f^2*g*h + a*b*e^2*f*h^2)*p*q)*x - 6*((18*b^2*e*f^2*g^2 - 27*
b^2*e^2*f*g*h + 11*b^2*e^3*h^2)*p^2*q^2 + 2*(b^2*f^3*h^2*p^2*q^2 - 3*a*b*f
^3*h^2*p*q)*x^3 - 6*(3*a*b*e*f^2*g^2 - 3*a*b*e^2*f*g*h + a*b*e^3*h^2)*p*q
- 3*(6*a*b*f^3*g*h*p*q - (3*b^2*f^3*g*h - b^2*e*f^2*h^2)*p^2*q^2)*x^2 - 6*
(3*a*b*f^3*g^2*p*q - (3*b^2*f^3*g^2 - 3*b^2*e*f^2*g*h + b^2*e^2*f*h^2)*p^2
*q^2)*x - 6*(b^2*f^3*h^2*p*q*x^3 + 3*b^2*f^3*g*h*p*q*x^2 + 3*b^2*f^3*g^2*p
*q*x + (3*b^2*e*f^2*g^2 - 3*b^2*e^2*f*g*h + b^2*e^3*h^2)*p*q)*log(c) - 6*(
b^2*f^3*h^2*p*q^2*x^3 + 3*b^2*f^3*g*h*p*q^2*x^2 + 3*b^2*f^3*g^2*p*q^2*x +
(3*b^2*e*f^2*g^2 - 3*b^2*e^2*f*g*h + b^2*e^3*h^2)*p*q^2)*log(d))*log(f*x +
 e) - 6*(2*(b^2*f^3*h^2*p*q - 3*a*b*f^3*h^2)*x^3 - 3*(6*a*b*f^3*g*h - (3*b
^2*f^3*g*h - b^2*e*f^2*h^2)*p*q)*x^2 - 6*(3*a*b*f^3*g^2 - (3*b^2*f^3*g^2 -
 3*b^2*e*f^2*g*h + b^2*e^2*f*h^2)*p*q)*x)*log(c) - 6*(2*(b^2*f^3*h^2*p*...
```

3.429.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(311) = 622$.

Time = 2.59 (sec) , antiderivative size = 894, normalized size of antiderivative = 2.77

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2\,dx$$
$$= \begin{cases} a^2g^2x + a^2ghx^2 + \frac{a^2h^2x^3}{3} + \frac{2abe^3h^2\log(c(d(e+fx)^p)^q)}{3f^3} - \frac{2abe^2gh\log(c(d(e+fx)^p)^q)}{f^2} - \frac{2abe^2h^2pqx}{3f^2} + \frac{2abeg^2\log(c(d(e+fx)^p}{f} \\ (a+b\log(c(de^p)^q))^2\left(g^2x + ghx^2 + \frac{h^2x^3}{3}\right) \end{cases}$$

input
```
integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Piecewise((a**2*g**2*x + a**2*g*h*x**2 + a**2*h**2*x**3/3 + 2*a*b*e**3*h**
2*log(c*(d*(e + f*x)**p)**q)/(3*f**3) - 2*a*b*e**2*g*h*log(c*(d*(e + f*x)*
*p)**q)/f**2 - 2*a*b*e**2*h**2*p*q*x/(3*f**2) + 2*a*b*e*g**2*log(c*(d*(e +
 f*x)**p)**q)/f + 2*a*b*e*g*h*p*q*x/f + a*b*e*h**2*p*q*x**2/(3*f) - 2*a*b*
g**2*p*q*x + 2*a*b*g**2*x*log(c*(d*(e + f*x)**p)**q) - a*b*g*h*p*q*x**2 +
2*a*b*g*h*x**2*log(c*(d*(e + f*x)**p)**q) - 2*a*b*h**2*p*q*x**3/9 + 2*a*b*
h**2*x**3*log(c*(d*(e + f*x)**p)**q)/3 - 11*b**2*e**3*h**2*p*q*log(c*(d*(e
 + f*x)**p)**q)/(9*f**3) + b**2*e**3*h**2*log(c*(d*(e + f*x)**p)**q)**2/(3
*f**3) + 3*b**2*e**2*g*h*p*q*log(c*(d*(e + f*x)**p)**q)/f**2 - b**2*e**2*g
*h*log(c*(d*(e + f*x)**p)**q)**2/f**2 + 11*b**2*e**2*h**2*p**2*q**2*x/(9*f
**2) - 2*b**2*e**2*h**2*p*q*x*log(c*(d*(e + f*x)**p)**q)/(3*f**2) - 2*b**2
*e*g**2*p*q*log(c*(d*(e + f*x)**p)**q)/f + b**2*e*g**2*log(c*(d*(e + f*x)*
*p)**q)**2/f - 3*b**2*e*g*h*p**2*q**2*x/f + 2*b**2*e*g*h*p*q*x*log(c*(d*(e
 + f*x)**p)**q)/f - 5*b**2*e*h**2*p**2*q**2*x**2/(18*f) + b**2*e*h**2*p*q*
x**2*log(c*(d*(e + f*x)**p)**q)/(3*f) + 2*b**2*g**2*p**2*q**2*x - 2*b**2*g
**2*p*q*x*log(c*(d*(e + f*x)**p)**q) + b**2*g**2*x*log(c*(d*(e + f*x)**p)*
*q)**2 + b**2*g*h*p**2*q**2*x**2/2 - b**2*g*h*p*q*x**2*log(c*(d*(e + f*x)*
*p)**q) + b**2*g*h*x**2*log(c*(d*(e + f*x)**p)**q)**2 + 2*b**2*h**2*p**2*q
**2*x**3/27 - 2*b**2*h**2*p*q*x**3*log(c*(d*(e + f*x)**p)**q)/9 + b**2*h**
2*x**3*log(c*(d*(e + f*x)**p)**q)**2/3, Ne(f, 0)), ((a + b*log(c*(d*e**...
```

3.429. $\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.429.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.87

$$
\begin{aligned}
&\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx \\
&= \frac{1}{3}\,b^2h^2x^3\log \left(\left(\left(fx+e\right)^pd\right)^q c\right)^2 - 2\,abfg^2pq\left(\frac{x}{f} - \frac{e\log \left(fx+e\right)}{f^2}\right) \\
&\quad + \frac{1}{9}\,abfh^2pq\left(\frac{6\,e^3\log \left(fx+e\right)}{f^4} - \frac{2\,f^2x^3 - 3\,efx^2 + 6\,e^2x}{f^3}\right) \\
&\quad - abfghpq\left(\frac{2\,e^2\log \left(fx+e\right)}{f^3} + \frac{fx^2 - 2\,ex}{f^2}\right) + \frac{2}{3}\,abh^2x^3\log \left(\left(\left(fx+e\right)^pd\right)^q c\right) \\
&\quad + b^2ghx^2\log \left(\left(\left(fx+e\right)^pd\right)^q c\right)^2 + \frac{1}{3}\,a^2h^2x^3 + 2\,abghx^2\log \left(\left(\left(fx+e\right)^pd\right)^q c\right) \\
&\quad + b^2g^2x\log \left(\left(\left(fx+e\right)^pd\right)^q c\right)^2 + a^2ghx^2 + 2\,abg^2x\log \left(\left(\left(fx+e\right)^pd\right)^q c\right) \\
&\quad - \left(2\,fpq\left(\frac{x}{f} - \frac{e\log \left(fx+e\right)}{f^2}\right)\log \left(\left(\left(fx+e\right)^pd\right)^q c\right) + \frac{\left(e\log \left(fx+e\right)^2 - 2\,fx + 2\,e\log \left(fx+e\right)\right)p^2q^2}{f}\right)b \\
&\quad - \frac{1}{2}\left(2\,fpq\left(\frac{2\,e^2\log \left(fx+e\right)}{f^3} + \frac{fx^2 - 2\,ex}{f^2}\right)\log \left(\left(\left(fx+e\right)^pd\right)^q c\right) - \frac{\left(f^2x^2 + 2\,e^2\log \left(fx+e\right)^2 - 6\,efx + \right.}{f^2}\right. \\
&\quad + \frac{1}{54}\left(6\,fpq\left(\frac{6\,e^3\log \left(fx+e\right)}{f^4} - \frac{2\,f^2x^3 - 3\,efx^2 + 6\,e^2x}{f^3}\right)\log \left(\left(\left(fx+e\right)^pd\right)^q c\right) + \frac{\left(4\,f^3x^3 - 15\,ef^2x^2 - \right.}{}\right. \\
&\quad + a^2g^2x
\end{aligned}
$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*h^2*x^3*log(((f*x + e)^p*d)^q*c)^2 - 2*a*b*f*g^2*p*q*(x/f - e*log(
f*x + e)/f^2) + 1/9*a*b*f*h^2*p*q*(6*e^3*log(f*x + e)/f^4 - (2*f^2*x^3 - 3
*e*f*x^2 + 6*e^2*x)/f^3) - a*b*f*g*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2
- 2*e*x)/f^2) + 2/3*a*b*h^2*x^3*log(((f*x + e)^p*d)^q*c) + b^2*g*h*x^2*log
(((f*x + e)^p*d)^q*c)^2 + 1/3*a^2*h^2*x^3 + 2*a*b*g*h*x^2*log(((f*x + e)^p
*d)^q*c) + b^2*g^2*x*log(((f*x + e)^p*d)^q*c)^2 + a^2*g*h*x^2 + 2*a*b*g^2*
x*log(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x
 + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)
*b^2*g^2 - 1/2*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2)*log
(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*
log(f*x + e))*p^2*q^2/f^2)*b^2*g*h + 1/54*(6*f*p*q*(6*e^3*log(f*x + e)/f^4
 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3)*log(((f*x + e)^p*d)^q*c) + (4*f^
3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x
 + e))*p^2*q^2/f^3)*b^2*h^2 + a^2*g^2*x
```

3.429.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2106 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 2106, normalized size of antiderivative = 6.52

$$\int (g+hx)^2 \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
(f*x + e)*b^2*g^2*p^2*q^2*log(f*x + e)^2/f + (f*x + e)^2*b^2*g*h*p^2*q^2*l
og(f*x + e)^2/f^2 - 2*(f*x + e)*b^2*e*g*h*p^2*q^2*log(f*x + e)^2/f^2 + 1/3
*(f*x + e)^3*b^2*h^2*p^2*q^2*log(f*x + e)^2/f^3 - (f*x + e)^2*b^2*e*h^2*p^
2*q^2*log(f*x + e)^2/f^3 + (f*x + e)*b^2*e^2*h^2*p^2*q^2*log(f*x + e)^2/f^
3 - 2*(f*x + e)*b^2*g^2*p^2*q^2*log(f*x + e)/f - (f*x + e)^2*b^2*g*h*p^2*q
^2*log(f*x + e)/f^2 + 4*(f*x + e)*b^2*e*g*h*p^2*q^2*log(f*x + e)/f^2 - 2/9
*(f*x + e)^3*b^2*h^2*p^2*q^2*log(f*x + e)/f^3 + (f*x + e)^2*b^2*e*h^2*p^2*
q^2*log(f*x + e)/f^3 - 2*(f*x + e)*b^2*e^2*h^2*p^2*q^2*log(f*x + e)/f^3 +
2*(f*x + e)*b^2*g^2*p*q^2*log(f*x + e)*log(d)/f + 2*(f*x + e)^2*b^2*g*h*p*
q^2*log(f*x + e)*log(d)/f^2 - 4*(f*x + e)*b^2*e*g*h*p*q^2*log(f*x + e)*log
(d)/f^2 + 2/3*(f*x + e)^3*b^2*h^2*p*q^2*log(f*x + e)*log(d)/f^3 - 2*(f*x +
 e)^2*b^2*e*h^2*p*q^2*log(f*x + e)*log(d)/f^3 + 2*(f*x + e)*b^2*e^2*h^2*p*
q^2*log(f*x + e)*log(d)/f^3 + 2*(f*x + e)*b^2*g^2*p^2*q^2/f + 1/2*(f*x + e
)^2*b^2*g*h*p^2*q^2/f^2 - 4*(f*x + e)*b^2*e*g*h*p^2*q^2/f^2 + 2/27*(f*x +
e)^3*b^2*h^2*p^2*q^2/f^3 - 1/2*(f*x + e)^2*b^2*e*h^2*p^2*q^2/f^3 + 2*(f*x
+ e)*b^2*e^2*h^2*p^2*q^2/f^3 + 2*(f*x + e)*b^2*g^2*p*q*log(f*x + e)*log(c)
/f + 2*(f*x + e)^2*b^2*g*h*p*q*log(f*x + e)*log(c)/f^2 - 4*(f*x + e)*b^2*e
*g*h*p*q*log(f*x + e)*log(c)/f^2 + 2/3*(f*x + e)^3*b^2*h^2*p*q*log(f*x + e
)*log(c)/f^3 - 2*(f*x + e)^2*b^2*e*h^2*p*q*log(f*x + e)*log(c)/f^3 + 2*(f*
x + e)*b^2*e^2*h^2*p*q*log(f*x + e)*log(c)/f^3 - 2*(f*x + e)*b^2*g^2*p*...
```

3.429.9 Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.02

$$\int (g+hx)^2 \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^2 \, dx$$
$$= \ln \left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2 \left(b^2\,g^2\,x+\frac{b^2\,h^2\,x^3}{3}+\frac{e\left(b^2\,e^2\,h^2-3\,b^2\,e\,f\,g\,h+3\,b^2\,f^2\,g^2\right)}{3\,f^3}+b^2\,g\,h\,x^2\right)$$
$$+\ln \left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(\frac{x^2\left(\frac{3\,b\,h\left(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q\right)}{f}-\frac{b\,e\,h^2\left(3\,a-b\,p\,q\right)}{f}\right)}{3} - \frac{x\left(\frac{e\left(\frac{6\,b\,h\left(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q\right)}{f}-\frac{2\,b\,e\,h^2\left(3\,a-b\,p\,q\right)}{f}\right)}{f}-\frac{6\,b\,g\left(2\,a\,e\,h+a\,f\,g-b\,f\,g\,p\,q\right)}{f}\right)}{3} + \frac{2\,b\,h^2\,x^3\left(3\,a-b\,p\,q\right)}{9}\right)$$
$$+x\left(\frac{18\,a^2\,e\,f\,g\,h+9\,a^2\,f^2\,g^2-18\,a\,b\,f^2\,g^2\,p\,q+6\,b^2\,e^2\,h^2\,p^2\,q^2-18\,b^2\,e\,f\,g\,h\,p^2\,q^2+18\,b^2\,f^2\,g^2\,p^2\,q^2}{9\,f^2} - \frac{e\left(\frac{h\left(3\,a^2\,e\,h+6\,a^2\,f\,g-b^2\,e\,h\,p^2\,q^2+3\,b^2\,f\,g\,p^2\,q^2-6\,a\,b\,f\,g\,p\,q\right)}{3\,f}-\frac{e\,h^2\left(9\,a^2-6\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)}{9\,f}\right)}{f}\right)$$
$$+x^2\left(\frac{h\left(3\,a^2\,e\,h+6\,a^2\,f\,g-b^2\,e\,h\,p^2\,q^2+3\,b^2\,f\,g\,p^2\,q^2-6\,a\,b\,f\,g\,p\,q\right)}{6\,f} - \frac{e\,h^2\left(9\,a^2-6\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)}{18\,f}\right)$$
$$-\frac{\ln\left(e+f\,x\right)\,\left(11\,b^2\,e^3\,h^2\,p^2\,q^2-27\,b^2\,e^2\,f\,g\,h\,p^2\,q^2+18\,b^2\,e\,f^2\,g^2\,p^2\,q^2-6\,a\,b\,e^3\,h^2\,p\,q+18\,a\,b\,e^2\,f\,g\,h\,p\,q}{9\,f^3}$$
$$+\frac{h^2\,x^3\left(9\,a^2-6\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)}{27}$$

input
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
log(c*(d*(e + f*x)^p)^q)^2*(b^2*g^2*x + (b^2*h^2*x^3)/3 + (e*(b^2*e^2*h^2
+ 3*b^2*f^2*g^2 - 3*b^2*e*f*g*h))/(3*f^3) + b^2*g*h*x^2) + log(c*(d*(e + f
*x)^p)^q)*((x^2*((3*b*h*(a*e*h + 2*a*f*g - b*f*g*p*q))/f - (b*e*h^2*(3*a -
 b*p*q))/f))/3 - (x*((e*((6*b*h*(a*e*h + 2*a*f*g - b*f*g*p*q))/f - (2*b*e*
h^2*(3*a - b*p*q))/f))/f - (6*b*g*(2*a*e*h + a*f*g - b*f*g*p*q))/f))/3 + (
2*b*h^2*x^3*(3*a - b*p*q))/9) + x*((9*a^2*f^2*g^2 + 6*b^2*e^2*h^2*p^2*q^2
+ 18*b^2*f^2*g^2*p^2*q^2 + 18*a^2*e*f*g*h - 18*a*b*f^2*g^2*p*q - 18*b^2*e*
f*g*h*p^2*q^2)/(9*f^2) - (e*((h*(3*a^2*e*h + 6*a^2*f*g - b^2*e*h*p^2*q^2 +
 3*b^2*f*g*p^2*q^2 - 6*a*b*f*g*p*q))/(3*f) - (e*h^2*(9*a^2 + 2*b^2*p^2*q^2
 - 6*a*b*p*q))/(9*f)))/f) + x^2*((h*(3*a^2*e*h + 6*a^2*f*g - b^2*e*h*p^2*q
^2 + 3*b^2*f*g*p^2*q^2 - 6*a*b*f*g*p*q))/(6*f) - (e*h^2*(9*a^2 + 2*b^2*p^2
*q^2 - 6*a*b*p*q))/(18*f)) - (log(e + f*x)*(11*b^2*e^3*h^2*p^2*q^2 - 6*a*b
*e^3*h^2*p*q + 18*b^2*e*f^2*g^2*p^2*q^2 - 27*b^2*e^2*f*g*h*p^2*q^2 - 18*a*
b*e*f^2*g^2*p*q + 18*a*b*e^2*f*g*h*p*q))/(9*f^3) + (h^2*x^3*(9*a^2 + 2*b^2
*p^2*q^2 - 6*a*b*p*q))/27
```

3.429. $\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.430 $\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

3.430.1 Optimal result

Integrand size = 26, antiderivative size = 211

$$\begin{aligned}\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx &= -\frac{2ab(fg-eh)pqx}{f}+\frac{2b^2(fg-eh)p^2q^2x}{f}+\frac{b^2hp^2q^2(e+fx)^2}{4f^2}\\&\quad-\frac{2b^2(fg-eh)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2}\\&\quad-\frac{bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2}\\&\quad+\frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2}\\&\quad+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2}\end{aligned}$$

output
```
-2*a*b*(-e*h+f*g)*p*q*x/f+2*b^2*(-e*h+f*g)*p^2*q^2*x/f+1/4*b^2*h*p^2*q^2*(
f*x+e)^2/f^2-2*b^2*(-e*h+f*g)*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^2-1/2*b*
h*p*q*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2+(-e*h+f*g)*(f*x+e)*(a+b*ln
(c*(d*(f*x+e)^p)^q))^2/f^2+1/2*h*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f
^2
```

3.430.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

$$\int (g+hx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$$
$$= \frac{4(fg-eh)(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2+2h(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2-8b(fg-e\ldots}{}$$

input
```
Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(4*(f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 2*h*(e + f*x
)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 8*b*(f*g - e*h)*p*q*(f*(a - b*p*q
)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]) + b*h*p*q*(b*f*p*q*x*(2*e + f*
x) - 2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])))/(4*f^2)
```

3.430.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$$
$$\Big\downarrow \ 2895$$
$$\int (g+hx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2dx$$
$$\Big\downarrow \ 2848$$
$$\int \left(\frac{(fg-eh)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{f}+\frac{h(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{f}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2}-\frac{bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2}+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2}-\frac{2abpqx(fg-eh)}{f}-\frac{2b^2pq(e+fx)(fg-eh)\log\left(c(d(e+fx)^p)^q\right)}{f^2}+\frac{b^2hp^2q^2(e+fx)^2}{4f^2}+\frac{2b^2p^2q^2x(fg-eh)}{f}$$

input
```
Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(-2*a*b*(f*g - e*h)*p*q*x)/f + (2*b^2*(f*g - e*h)*p^2*q^2*x)/f + (b^2*h*p^
2*q^2*(e + f*x)^2)/(4*f^2) - (2*b^2*(f*g - e*h)*p*q*(e + f*x)*Log[c*(d*(e
+ f*x)^p)^q])/f^2 - (b*h*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))
/(2*f^2) + ((f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/f^2
+ (h*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(2*f^2)
```

3.430.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.430.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(205) = 410.

Time = 1.53 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.56

method	result
parallelrisch	$-\frac{-4x\ln(c(d(fx+e)^p)^q)b^2efhpq-8\ln(c(d(fx+e)^p)^q)b^2efgpq+16\ln(fx+e)b^2efg\,p^2q^2-8abefgpq-6b^2e^2h\,p^2q^2-x^2b^2f^2h\,p^2}{}$

input
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f*h*p*q-8*ln(c*(d*(f*x+e)^p)^q)*b^2
*e*f*g*p*q+16*ln(f*x+e)*b^2*e*f*g*p^2*q^2-8*a*b*e*f*g*p*q-6*b^2*e^2*h*p^2*
q^2-x^2*b^2*f^2*h*p^2*q^2-8*x*b^2*f^2*g*p^2*q^2-4*x^2*ln(c*(d*(f*x+e)^p)^q
)*a*b*f^2*h+4*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*h*p*q-8*x*ln(c*(d*(f*x+e)^p)^q
)*a*b*f^2*g+8*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f*g-10*ln(f*x+e)*b^2*e^2*h*p^2*q
^2+4*a^2*e*f*g+4*a*b*e^2*h*p*q+8*b^2*e*f*g*p^2*q^2+8*x*a*b*f^2*g*p*q+4*ln(
f*x+e)*a*b*e^2*h*p*q+2*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*f^2*h*p*q+6*x*b^2*e*f
*h*p^2*q^2+2*x^2*a*b*f^2*h*p*q-4*x*a*b*e*f*h*p*q-16*ln(f*x+e)*a*b*e*f*g*p*
q+8*x*ln(c*(d*(f*x+e)^p)^q)*b^2*f^2*g*p*q-2*x^2*ln(c*(d*(f*x+e)^p)^q)^2*b^
2*f^2*h-4*x*ln(c*(d*(f*x+e)^p)^q)^2*b^2*f^2*g-4*ln(c*(d*(f*x+e)^p)^q)^2*b^
2*e*f*g-2*x^2*a^2*f^2*h+2*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^2*h-4*x*a^2*f^2*g)
/f^2
```

3.430.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(205) = 410.

Time = 0.33 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.95

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$=\frac{(b^2f^2hp^2q^2-2\,abf^2hpq+2\,a^2f^2h)x^2+2\left(b^2f^2hp^2q^2x^2+2\,b^2f^2gp^2q^2x+(2\,b^2efg-b^2e^2h)p^2q^2\right)\log\left(fx+\right.}{}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
1/4*((b^2*f^2*h*p^2*q^2 - 2*a*b*f^2*h*p*q + 2*a^2*f^2*h)*x^2 + 2*(b^2*f^2*
h*p^2*q^2*x^2 + 2*b^2*f^2*g*p^2*q^2*x + (2*b^2*e*f*g - b^2*e^2*h)*p^2*q^2)
*log(f*x + e)^2 + 2*(b^2*f^2*h*x^2 + 2*b^2*f^2*g*x)*log(c)^2 + 2*(b^2*f^2*
h*q^2*x^2 + 2*b^2*f^2*g*q^2*x)*log(d)^2 + 2*(2*a^2*f^2*g + (4*b^2*f^2*g -
3*b^2*e*f*h)*p^2*q^2 - 2*(2*a*b*f^2*g - a*b*e*f*h)*p*q)*x - 2*((4*b^2*e*f*
g - 3*b^2*e^2*h)*p^2*q^2 - 2*(2*a*b*e*f*g - a*b*e^2*h)*p*q + (b^2*f^2*h*p^
2*q^2 - 2*a*b*f^2*h*p*q)*x^2 - 2*(2*a*b*f^2*g*p*q - (2*b^2*f^2*g - b^2*e*f
*h)*p^2*q^2)*x - 2*(b^2*f^2*h*p*q*x^2 + 2*b^2*f^2*g*p*q*x + (2*b^2*e*f*g -
 b^2*e^2*h)*p*q)*log(c) - 2*(b^2*f^2*h*p*q^2*x^2 + 2*b^2*f^2*g*p*q^2*x + (
2*b^2*e*f*g - b^2*e^2*h)*p*q^2)*log(d))*log(f*x + e) - 2*((b^2*f^2*h*p*q -
 2*a*b*f^2*h)*x^2 - 2*(2*a*b*f^2*g - (2*b^2*f^2*g - b^2*e*f*h)*p*q)*x)*log
(c) - 2*((b^2*f^2*h*p*q^2 - 2*a*b*f^2*h*q)*x^2 - 2*(2*a*b*f^2*g*q - (2*b^2
*f^2*g - b^2*e*f*h)*p*q^2)*x - 2*(b^2*f^2*h*q*x^2 + 2*b^2*f^2*g*q*x)*log(c
))*log(d))/f^2
```

3.430.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(202) = 404$.

Time = 1.24 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.21

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$= \begin{cases} a^2gx + \frac{a^2hx^2}{2} - \frac{abe^2h\log\left(c(d(e+fx)^p)^q\right)}{f^2} + \frac{2abeg\log\left(c(d(e+fx)^p)^q\right)}{f} + \frac{abehpqx}{f} - 2abgpqx + 2abgx\log\left(c(d(e+fx)^p\right. \\ \left(a+b\log\left(c(de^p)^q\right)\right)^2\left(gx+\frac{hx^2}{2}\right) \end{cases}$$

input
```
integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Piecewise((a**2*g*x + a**2*h*x**2/2 - a*b*e**2*h*log(c*(d*(e + f*x)**p)**q
)/f**2 + 2*a*b*e*g*log(c*(d*(e + f*x)**p)**q)/f + a*b*e*h*p*q*x/f - 2*a*b*
g*p*q*x + 2*a*b*g*x*log(c*(d*(e + f*x)**p)**q) - a*b*h*p*q*x**2/2 + a*b*h*
x**2*log(c*(d*(e + f*x)**p)**q) + 3*b**2*e**2*h*p*q*log(c*(d*(e + f*x)**p)
**q)/(2*f**2) - b**2*e**2*h*log(c*(d*(e + f*x)**p)**q)**2/(2*f**2) - 2*b**
2*e*g*p*q*log(c*(d*(e + f*x)**p)**q)/f + b**2*e*g*log(c*(d*(e + f*x)**p)**
q)**2/f - 3*b**2*e*h*p**2*q**2*x/(2*f) + b**2*e*h*p*q*x*log(c*(d*(e + f*x)
**p)**q)/f + 2*b**2*g*p**2*q**2*x - 2*b**2*g*p*q*x*log(c*(d*(e + f*x)**p)*
*q) + b**2*g*x*log(c*(d*(e + f*x)**p)**q)**2 + b**2*h*p**2*q**2*x**2/4 - b
**2*h*p*q*x**2*log(c*(d*(e + f*x)**p)**q)/2 + b**2*h*x**2*log(c*(d*(e + f*
x)**p)**q)**2/2, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))**2*(g*x + h*x**2/2
), True))
```

3.430.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$= -2\,abfgpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)-\frac{1}{2}\,abfhpq\left(\frac{2\,e^2\log(fx+e)}{f^3}+\frac{fx^2-2\,ex}{f^2}\right)$$
$$+\frac{1}{2}\,b^2hx^2\log\left(((fx+e)^pd)^q\,c\right)^2+abhx^2\log\left(((fx+e)^pd)^q\,c\right)$$
$$+b^2gx\log\left(((fx+e)^pd)^q\,c\right)^2+\frac{1}{2}\,a^2hx^2+2\,abgx\log\left(((fx+e)^pd)^q\,c\right)$$
$$-\left(2\,fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(((fx+e)^pd)^q\,c\right)+\frac{\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)p^2q^2}{f}\right)b$$
$$-\frac{1}{4}\left(2\,fpq\left(\frac{2\,e^2\log(fx+e)}{f^3}+\frac{fx^2-2\,ex}{f^2}\right)\log\left(((fx+e)^pd)^q\,c\right)-\frac{\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+\right.}{f^2}$$
$$+a^2gx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
-2*a*b*f*g*p*q*(x/f - e*log(f*x + e)/f^2) - 1/2*a*b*f*h*p*q*(2*e^2*log(f*x
 + e)/f^3 + (f*x^2 - 2*e*x)/f^2) + 1/2*b^2*h*x^2*log(((f*x + e)^p*d)^q*c)^
2 + a*b*h*x^2*log(((f*x + e)^p*d)^q*c) + b^2*g*x*log(((f*x + e)^p*d)^q*c)^
2 + 1/2*a^2*h*x^2 + 2*a*b*g*x*log(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e
*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x +
2*e*log(f*x + e))*p^2*q^2/f)*b^2*g - 1/4*(2*f*p*q*(2*e^2*log(f*x + e)/f^3
+ (f*x^2 - 2*e*x)/f^2)*log(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x
 + e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))*p^2*q^2/f^2)*b^2*h + a^2*g*x
```

3.430.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(205) = 410$.

Time = 0.32 (sec) , antiderivative size = 939, normalized size of antiderivative = 4.45

$$\int (g + hx) \left(a + b \log \left(c(d(e + fx)^p)^q\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
(f*x + e)*b^2*g*p^2*q^2*log(f*x + e)^2/f + 1/2*(f*x + e)^2*b^2*h*p^2*q^2*l
og(f*x + e)^2/f^2 - (f*x + e)*b^2*e*h*p^2*q^2*log(f*x + e)^2/f^2 - 2*(f*x
+ e)*b^2*g*p^2*q^2*log(f*x + e)/f - 1/2*(f*x + e)^2*b^2*h*p^2*q^2*log(f*x
+ e)/f^2 + 2*(f*x + e)*b^2*e*h*p^2*q^2*log(f*x + e)/f^2 + 2*(f*x + e)*b^2*
g*p*q^2*log(f*x + e)*log(d)/f + (f*x + e)^2*b^2*h*p*q^2*log(f*x + e)*log(d
)/f^2 - 2*(f*x + e)*b^2*e*h*p*q^2*log(f*x + e)*log(d)/f^2 + 2*(f*x + e)*b^
2*g*p^2*q^2/f + 1/4*(f*x + e)^2*b^2*h*p^2*q^2/f^2 - 2*(f*x + e)*b^2*e*h*p^
2*q^2/f^2 + 2*(f*x + e)*b^2*g*p*q*log(f*x + e)*log(c)/f + (f*x + e)^2*b^2*
h*p*q*log(f*x + e)*log(c)/f^2 - 2*(f*x + e)*b^2*e*h*p*q*log(f*x + e)*log(c
)/f^2 - 2*(f*x + e)*b^2*g*p*q^2*log(d)/f - 1/2*(f*x + e)^2*b^2*h*p*q^2*log
(d)/f^2 + 2*(f*x + e)*b^2*e*h*p*q^2*log(d)/f^2 + (f*x + e)*b^2*g*q^2*log(d
)^2/f + 1/2*(f*x + e)^2*b^2*h*q^2*log(d)^2/f^2 - (f*x + e)*b^2*e*h*q^2*log
(d)^2/f^2 + 2*(f*x + e)*a*b*g*p*q*log(f*x + e)/f + (f*x + e)^2*a*b*h*p*q*l
og(f*x + e)/f^2 - 2*(f*x + e)*a*b*e*h*p*q*log(f*x + e)/f^2 - 2*(f*x + e)*b
^2*g*p*q*log(c)/f - 1/2*(f*x + e)^2*b^2*h*p*q*log(c)/f^2 + 2*(f*x + e)*b^2
*e*h*p*q*log(c)/f^2 + 2*(f*x + e)*b^2*g*q*log(c)*log(d)/f + (f*x + e)^2*b^
2*h*q*log(c)*log(d)/f^2 - 2*(f*x + e)*b^2*e*h*q*log(c)*log(d)/f^2 - 2*(f*x
 + e)*a*b*g*p*q/f - 1/2*(f*x + e)^2*a*b*h*p*q/f^2 + 2*(f*x + e)*a*b*e*h*p*
q/f^2 + (f*x + e)*b^2*g*log(c)^2/f + 1/2*(f*x + e)^2*b^2*h*log(c)^2/f^2 -
(f*x + e)*b^2*e*h*log(c)^2/f^2 + 2*(f*x + e)*a*b*g*q*log(d)/f + (f*x + ...
```

3.430.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.43

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$= x\left(\frac{2a^2eh+2a^2fg-2b^2ehp^2q^2+4b^2fgp^2q^2-4abfgpq}{2f} - \frac{eh(2a^2-2abpq+b^2p^2q^2)}{2f}\right) + \ln\left(c\left(d(e+fx)^p\right)^q\right)\left(\frac{bh(2a-bpq)x^2}{2} + \left(\frac{2b(aeh+afg-bfgpq)}{f} - \frac{beh(2a-bpq)}{f}\right)x\right) + \ln\left(c\left(d(e+fx)^p\right)^q\right)^2\left(\frac{b^2hx^2}{2} - \frac{e(b^2eh-2b^2fg)}{2f^2} + b^2gx\right) + \frac{\ln(e+fx)\left(3hb^2e^2p^2q^2-4fgb^2ep^2q^2-2ahbe^2pq+4afgbepq\right)}{2f^2} + \frac{hx^2(2a^2-2abpq+b^2p^2q^2)}{4}$$

input
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
x*((2*a^2*e*h + 2*a^2*f*g - 2*b^2*e*h*p^2*q^2 + 4*b^2*f*g*p^2*q^2 - 4*a*b*
f*g*p*q)/(2*f) - (e*h*(2*a^2 + b^2*p^2*q^2 - 2*a*b*p*q))/(2*f)) + log(c*(d
*(e + f*x)^p)^q)*(x*((2*b*(a*e*h + a*f*g - b*f*g*p*q))/f - (b*e*h*(2*a - b
*p*q))/f) + (b*h*x^2*(2*a - b*p*q))/2) + log(c*(d*(e + f*x)^p)^q)^2*((b^2*
h*x^2)/2 - (e*(b^2*e*h - 2*b^2*f*g))/(2*f^2) + b^2*g*x) + (log(e + f*x)*(3
*b^2*e^2*h*p^2*q^2 - 2*a*b*e^2*h*p*q - 4*b^2*e*f*g*p^2*q^2 + 4*a*b*e*f*g*p
*q))/(2*f^2) + (h*x^2*(2*a^2 + b^2*p^2*q^2 - 2*a*b*p*q))/4
```

3.431 $\int (a+b\log (c(d(e+fx)^p)^q))^2 \, dx$

3.431.1 Optimal result

Integrand size = 20, antiderivative size = 78

$$\int (a+b\log (c(d(e+fx)^p)^q))^2 \, dx = -2abpqx+2b^2p^2q^2x-\frac{2b^2pq(e+fx)\log (c(d(e+fx)^p)^q)}{f} + \frac{(e+fx)(a+b\log (c(d(e+fx)^p)^q))^2}{f}$$

output
```
-2*a*b*p*q*x+2*b^2*p^2*q^2*x-2*b^2*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f+(f*
x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f
```

3.431.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int (a+b\log (c(d(e+fx)^p)^q))^2 \, dx = \frac{(e+fx)(a+b\log (c(d(e+fx)^p)^q))^2}{f} - 2bpq\left(ax-bpqx+\frac{b(e+fx)\log (c(d(e+fx)^p)^q)}{f}\right)$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/f - 2*b*p*q*(a*x - b*p*q*x
+ (b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f)
```

3.431.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2895, 2836, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log(c(d(e+fx)^p)^q))^2dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(cd^q(e+fx)^{pq}))^2d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq\int (a+b\log(cd^q(e+fx)^{pq}))d(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq(a(e+fx)+b(e+fx)\log(cd^q(e+fx)^{pq})-bpq(e+fx))}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
((e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^2 - 2*b*p*q*(a*(e + f*x) - b
*p*q*(e + f*x) + b*(e + f*x)*Log[c*d^q*(e + f*x)^(p*q)]))/f
```

3.431.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.431.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(78) = 156$.

Time = 0.42 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.12

method	result
parallelrisch	$\frac{-2\ln(fx+e)b^2e^2p^2q^2+2x\,b^2ef\,p^2q^2-2x\ln(c(d(fx+e)^p)^q)b^2efpq+2\ln(fx+e)ab\,e^2pq+x\ln(c(d(fx+e)^p)^q)^2b^2ef-2xabefpq+}{ef}$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*ln(f*x+e)*b^2*e^2*p^2*q^2+2*x*b^2*e*f*p^2*q^2-2*x*ln(c*(d*(f*x+e)^p)^q
)*b^2*e*f*p*q+2*ln(f*x+e)*a*b*e^2*p*q+x*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e*f-2*
x*a*b*e*f*p*q+2*x*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f+ln(c*(d*(f*x+e)^p)^q)^2*b^
2*e^2+e*a^2*f*x)/e/f
```

3.431.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

Time = 0.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

$$\int (a + b\log (c(d(e + fx)^p)^q))^2 \, dx$$
$$= \frac{b^2fq^2x\log (d)^2 + b^2fx\log (c)^2 + (b^2fp^2q^2x + b^2ep^2q^2)\log (fx + e)^2 - 2\,(b^2fpq - abf)x\log (c) + (2\,b^2fp^2}{}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
(b^2*f*q^2*x*log(d)^2 + b^2*f*x*log(c)^2 + (b^2*f*p^2*q^2*x + b^2*e*p^2*q^
2)*log(f*x + e)^2 - 2*(b^2*f*p*q - a*b*f)*x*log(c) + (2*b^2*f*p^2*q^2 - 2*
a*b*f*p*q + a^2*f)*x - 2*(b^2*e*p^2*q^2 - a*b*e*p*q + (b^2*f*p^2*q^2 - a*b
*f*p*q)*x - (b^2*f*p*q*x + b^2*e*p*q)*log(c) - (b^2*f*p*q^2*x + b^2*e*p*q^
2)*log(d))*log(f*x + e) + 2*(b^2*f*q*x*log(c) - (b^2*f*p*q^2 - a*b*f*q)*x)
*log(d))/f
```

3.431.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.28

$$\int (a + b\log (c(d(e + fx)^p)^q))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{2abe\log (c(d(e+fx)^p)^q)}{f} - 2abpqx + 2abx\log (c(d(e + fx)^p)^q) - \frac{2b^2epq\log (c(d(e+fx)^p)^q)}{f} + \frac{b^2e\log (c(d(e+fx)^p)}{f} \\ x(a + b\log (c(de^p)^q))^2 \end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*e*log(c*(d*(e + f*x)**p)**q)/f - 2*a*b*p*q*x + 2
*a*b*x*log(c*(d*(e + f*x)**p)**q) - 2*b**2*e*p*q*log(c*(d*(e + f*x)**p)**q
)/f + b**2*e*log(c*(d*(e + f*x)**p)**q)**2/f + 2*b**2*p**2*q**2*x - 2*b**2
*p*q*x*log(c*(d*(e + f*x)**p)**q) + b**2*x*log(c*(d*(e + f*x)**p)**q)**2,
Ne(f, 0)), (x*(a + b*log(c*(d*e**p)**q))**2, True))
```

3.431.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int (a + b\log(c(d(e+fx)^p)^q))^2 \, dx$$
$$= -2\,abfpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right) + b^2x\log\left(((fx+e)^p d)^q\,c\right)^2 + 2\,abx\log\left(((fx+e)^p d)^q\,c\right) - \left(2\,fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log\left(((fx+e)^p d)^q\,c\right) + \frac{\left(e\log(fx+e)^2 - 2\,fx + 2\,e\log(fx+e)\right)p^2q^2}{f}\right)b + a^2x$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
-2*a*b*f*p*q*(x/f - e*log(f*x + e)/f^2) + b^2*x*log(((f*x + e)^p*d)^q*c)^2
 + 2*a*b*x*log(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log(f*x + e)/f^2)*
log(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p
^2*q^2/f)*b^2 + a^2*x
```

3.431.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(78) = 156.

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.63

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx = \frac{(fx+e)b^2p^2q^2\log(fx+e)^2}{f} - \frac{2(fx+e)b^2p^2q^2\log(fx+e)}{f} + \frac{2(fx+e)b^2pq^2\log(fx+e)\log(d)}{f} + \frac{2(fx+e)b^2p^2q^2}{f} + \frac{2(fx+e)b^2pq\log(fx+e)\log(c)}{f} - \frac{2(fx+e)b^2pq^2\log(d)}{f} + \frac{(fx+e)b^2q^2\log(d)^2}{f} + \frac{2(fx+e)abpq\log(fx+e)}{f} - \frac{2(fx+e)b^2pq\log(c)}{f} + \frac{2(fx+e)b^2q\log(c)\log(d)}{f} - \frac{2(fx+e)abpq}{f} + \frac{(fx+e)b^2\log(c)^2}{f} + \frac{2(fx+e)abq\log(d)}{f} + \frac{2(fx+e)ab\log(c)}{f} + \frac{(fx+e)a^2}{f}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
(f*x + e)*b^2*p^2*q^2*log(f*x + e)^2/f - 2*(f*x + e)*b^2*p^2*q^2*log(f*x +
 e)/f + 2*(f*x + e)*b^2*p*q^2*log(f*x + e)*log(d)/f + 2*(f*x + e)*b^2*p^2*
q^2/f + 2*(f*x + e)*b^2*p*q*log(f*x + e)*log(c)/f - 2*(f*x + e)*b^2*p*q^2*
log(d)/f + (f*x + e)*b^2*q^2*log(d)^2/f + 2*(f*x + e)*a*b*p*q*log(f*x + e)
/f - 2*(f*x + e)*b^2*p*q*log(c)/f + 2*(f*x + e)*b^2*q*log(c)*log(d)/f - 2*
(f*x + e)*a*b*p*q/f + (f*x + e)*b^2*log(c)^2/f + 2*(f*x + e)*a*b*q*log(d)/
f + 2*(f*x + e)*a*b*log(c)/f + (f*x + e)*a^2/f
```

3.431. $\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx$

3.431.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx = \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2\left(b^2\,x+\frac{b^2\,e}{f}\right) + x\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right) - \frac{\ln\left(e+f\,x\right)\,\left(2\,b^2\,e\,p^2\,q^2-2\,a\,b\,e\,p\,q\right)}{f} + 2\,b\,x\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\,\left(a-b\,p\,q\right)$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
log(c*(d*(e + f*x)^p)^q)^2*(b^2*x + (b^2*e)/f) + x*(a^2 + 2*b^2*p^2*q^2 -
2*a*b*p*q) - (log(e + f*x)*(2*b^2*e*p^2*q^2 - 2*a*b*e*p*q))/f + 2*b*x*log(
c*(d*(e + f*x)^p)^q)*(a - b*p*q)
```

3.432 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$

3.432.1 Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h+2*b*p*q*(a+b*ln(c
*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-2*b^2*p^2*q^2*polylo
g(3,-h*(f*x+e)/(-e*h+f*g))/h
```

3.432.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 324 vs. $2(123) = 246$.

Time = 0.14 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.63

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{g + hx}\,dx$$
$$= \frac{a^2\log(g+hx) - 2abpq\log(e+fx)\log(g+hx) + b^2p^2q^2\log^2(e+fx)\log(g+hx) + 2ab\log(c(d(e+fx)^p}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

output
```
(a^2*Log[g + h*x] - 2*a*b*p*q*Log[e + f*x]*Log[g + h*x] + b^2*p^2*q^2*Log[
e + f*x]^2*Log[g + h*x] + 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*
b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + b^2*Log[c*(d*
(e + f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*p*q*Log[e + f*x]*Log[(f*(g + h*x))/
(f*g - e*h)] - b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] +
 2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g -
e*h)] + 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e + f*x))/
(-(f*g) + e*h)] - 2*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)])/
h
```

3.432.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2895, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{g + hx}\,dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{g + hx}dx$$
$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{2bfpq\int\frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{2bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{2bpq\left(bpq\int\frac{\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))\right)}{h}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{2bpq\left(bpq\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))\right)}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])/h - (2
*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])*PolyLog[2, -((h*(e + f*x))/(f
*g - e*h))]) + b*p*q*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))]))/h
```

3.432.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.432.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

3.432.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c)
+ a^2)/(h*x + g), x)
```

3.432.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x), x)
```

3.432.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="maxima")
```

output
```
a^2*log(h*x + g)/h + integrate((b^2*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) +
 log(c))*a*b + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^2 + 2*((q*l
og(d) + log(c))*b^2 + a*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.432.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)^2}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x), x)
```

3.433 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx$

3.433.1 Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{(fg-eh)(g+hx)} - \frac{2bfpq(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{2b^2fp^2q^2\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

output
```
(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(-e*h+f*g)/(h*x+g)-2*b*f*p*q*(a+b*ln
(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)-2*b^2*f*p^2*q^2
*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g)
```

3.433.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx$$

$$= \frac{b^2fp^2q^2(g+hx)\log^2(e+fx) - 2bfpq(g+hx)\log(e+fx)(a+b\log(c(d(e+fx)^p)^q)) + (a+b\log(c(d(e}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^2,x]
```

output
```
(b^2*f*p^2*q^2*(g + h*x)*Log[e + f*x]^2 - 2*b*f*p*q*(g + h*x)*Log[e + f*x]
*(a + b*Log[c*(d*(e + f*x)^p)^q]) + (a + b*Log[c*(d*(e + f*x)^p)^q])*(a*(f
*g - e*h) + b*(f*g - e*h)*Log[c*(d*(e + f*x)^p)^q] + 2*b*f*p*q*(g + h*x)*L
og[(f*(g + h*x))/(f*g - e*h)]) + 2*b^2*f*p^2*q^2*(g + h*x)*PolyLog[2, (h*(
e + f*x))/(-(f*g) + e*h)])/(h*(-(f*g) + e*h)*(g + h*x))
```

3.433.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2895, 2844, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2844}$$

$$\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(fg-eh)}-\frac{2bfpq\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}dx}{fg-eh}$$

$$\Big\downarrow \text{2841}$$

$$\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(fg-eh)}-\frac{2bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfpq\int \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}\right)}{fg-eh}$$

$$\Big\downarrow \text{2840}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)(fg-eh)}-\frac{2bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bpq\int\frac{\log\left(\frac{h(e+fx)}{fg-eh}+1\right)}{e+fx}d(e+fx)}{h}\right)}{fg-eh}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)(fg-eh)}-\frac{2bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}+\frac{bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}\right)}{fg-eh}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^2,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/((f*g - e*h)*(g + h*x)) - (
2*b*f*p*q*(((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)
])/h + (b*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h))/(f*g - e*h)
```

3.433.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

3.433. $\int\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^2}\,dx$

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.433.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{\left(hx+g\right)^{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x)
```

3.433.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx=\int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c)
+ a^2)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

3.433.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**2, x)
```

3.433.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x, algorithm="maxima")
```

output
```
2*a*b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2))
- b^2*(log(((f*x + e)^p)^q)^2/(h^2*x + g*h) - integrate((e*h*q^2*log(d)^2
+ 2*e*h*q*log(c)*log(d) + e*h*log(c)^2 + (f*h*q^2*log(d)^2 + 2*f*h*q*log(c
)*log(d) + f*h*log(c)^2)*x + 2*(f*g*p*q + e*h*q*log(d) + e*h*log(c) + (f*h
*p*q + f*h*q*log(d) + f*h*log(c))*x)*log(((f*x + e)^p)^q))/(f*h^3*x^3 + e*
g^2*h + (2*f*g*h^2 + e*h^3)*x^2 + (f*g^2*h + 2*e*g*h^2)*x), x)) - 2*a*b*lo
g(((f*x + e)^p*d)^q*c)/(h^2*x + g*h) - a^2/(h^2*x + g*h)
```

3.433.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^2, x)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx=\int \frac{{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}{{\left(g+h\,x\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^2,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^2, x)
```

3.434 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx$

3.434.1 Optimal result

Integrand size = 28, antiderivative size = 222

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx = -\frac{bfpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))}{(fg-eh)^2(g+hx)} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2} + \frac{b^2f^2p^2q^2\log(g+hx)}{h(fg-eh)^2} - \frac{bf^2pq(a+b\log(c(d(e+fx)^p)^q))\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \frac{b^2f^2p^2q^2\operatorname{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}$$

output
```
-b*f*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/(-e*h+f*g)^2/(h*x+g)-1/2*(a+b
*ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^2+b^2*f^2*p^2*q^2*ln(h*x+g)/h/(-e*h+f*
g)^2-b*f^2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(1+(-e*h+f*g)/h/(f*x+e))/h/(-
e*h+f*g)^2+b^2*f^2*p^2*q^2*polylog(2,(e*h-f*g)/h/(f*x+e))/h/(-e*h+f*g)^2
```

3.434.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx =$$

$$-\frac{(a-bpq\log(e+fx)+b\log(c(d(e+fx)^p)^q))^2+\frac{2bpq(a-bpq\log(e+fx)+b\log(c(d(e+fx)^p)^q))\left(h(e+fx)(eh-f(2g+hx))\right.}{(fg-eh)^2}}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^3,x]
```

output
```
-1/2*((a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2 + (2*b*p*q*(
a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*(h*(e + f*x)*(e*h - f
*(2*g + h*x))*Log[e + f*x] + f*(g + h*x)*(h*(e + f*x) + f*(g + h*x)*Log[(f
*(g + h*x))/(f*g - e*h)])))/(f*g - e*h)^2 + (b^2*p^2*q^2*(h*(e + f*x)*(e*h
 - f*(2*g + h*x))*Log[e + f*x]^2 - 2*f^2*(g + h*x)^2*Log[(f*(g + h*x))/(f*
g - e*h)] + 2*f*(g + h*x)*Log[e + f*x]*(h*(e + f*x) + f*(g + h*x)*Log[(f*(
g + h*x))/(f*g - e*h)]) + 2f^2*(g + h*x)^2*PolyLog[2, (h*(e + f*x))/(-(f*
g) + e*h)]))/(f*g - e*h)^2)/(h*(g + h*x)^2)
```

3.434.3 Rubi [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2895, 2845, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{bfpq\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(e+fx)(g+hx)^2}dx}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2858}$$

$$\frac{bpq\int \frac{f^2(a+b\log(cd^q(e+fx)^{pq}))}{(e+fx)\left(f\left(g-\frac{eh}{f}\right)+h(e+fx)\right)^2}d(e+fx)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bf^2pq\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(fg-eh+h(e+fx))^2}d(e+fx)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2789}$$

$$\frac{bf^2pq\left(\frac{\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(fg-eh+h(e+fx))}d(e+fx)}{fg-eh}-\frac{h\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(fg-eh+h(e+fx))^2}d(e+fx)}{fg-eh}\right)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2751}$$

$$\frac{bf^2pq\left(\frac{\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(fg-eh+h(e+fx))}d(e+fx)}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))}{(fg-eh)(h(e+fx)-eh+fg)}-\frac{bpq\int \frac{1}{fg-eh+h(e+fx)}d(e+fx)}{fg-eh}\right)}{fg-eh}\right)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{bf^2pq\left(\frac{\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(fg-eh+h(e+fx))}d(e+fx)}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))}{(fg-eh)(h(e+fx)-eh+fg)}-\frac{bpq\log(h(e+fx)-eh+fg)}{h(fg-eh)}\right)}{fg-eh}\right)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2779}$$

$$\frac{bf^2pq\left(\frac{\frac{bpq\int \frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)}{e+fx}d(e+fx)}{fg-eh}-\frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(cd^q(e+fx)^{pq}))}{fg-eh}}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))}{(fg-eh)(h(e+fx)-eh+fg)}-\frac{bpq\log(h(e+fx)-eh+fg)}{h(fg-eh)}\right)}{fg-eh}\right)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2838}$$

3.434. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}dx$

$$\frac{bf^2pq\left(\frac{\frac{bpq\operatorname{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{fg-eh}-\frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(cd^q(e+fx)^{pq}))}{fg-eh}}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))}{(fg-eh)(h(e+fx)-eh+fg)}-\frac{bpq\log(h(e+fx)-eh+fg)}{h(fg-eh)}\right)}{fg-eh}\right)}{h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(h*(g + h*x)^2) + (b*f^2*p*q*(-((h
*(((e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/((f*g - e*h)*(f*g - e*h +
 h*(e + f*x))) - (b*p*q*Log[f*g - e*h + h*(e + f*x)])/(h*(f*g - e*h))))/(f
*g - e*h)) + (-(((a + b*Log[c*d^q*(e + f*x)^(p*q)])*Log[1 + (f*g - e*h)/(h
*(e + f*x))])/(f*g - e*h)) + (b*p*q*PolyLog[2, -((f*g - e*h)/(h*(e + f*x))
)])/(f*g - e*h))/(f*g - e*h)))/h
```

3.434.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.434.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{\left(hx+g\right)^{3}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x)
```

3.434.5 Fricas [F]

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx=\int \frac{\left(b\log \left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x, algorithm="fricas")
```

output
```
integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c)
+ a^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

3.434.6 Sympy [F]

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx=\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**3,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**3, x)
```

3.434.7 Maxima [F]

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx=\int \frac{\left(b\log \left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x, algorithm="maxima")
```

output
```
a*b*f*p*q*(f*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - f*log(h*x
+ g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) + 1/(f*g^2*h - e*g*h^2 + (f*g*h^2
 - e*h^3)*x)) - 1/2*b^2*(log(((f*x + e)^p)^q)^2/(h^3*x^2 + 2*g*h^2*x + g^2
*h) - 2*integrate((e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2
 + (f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*x + (f*g*p*q
+ 2*e*h*q*log(d) + 2*e*h*log(c) + (f*h*p*q + 2*f*h*q*log(d) + 2*f*h*log(c)
)*x)*log(((f*x + e)^p)^q))/(f*h^4*x^4 + e*g^3*h + (3*f*g*h^3 + e*h^4)*x^3
+ 3*(f*g^2*h^2 + e*g*h^3)*x^2 + (f*g^3*h + 3*e*g^2*h^2)*x), x)) - a*b*log(
((f*x + e)^p*d)^q*c)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*a^2/(h^3*x^2 + 2*
g*h^2*x + g^2*h)
```

3.434.8 Giac [F]

$$\int \frac{\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^3}\,dx=\int \frac{\left(b\log \left(((fx+e)^pd)^q\,c\right)+a\right)^2}{\left(hx+g\right)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^3, x)
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^3}\,dx=\int \frac{\left(a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}{{\left(g+h\,x\right)}^3}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^3,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^3, x)
```

3.435 $\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3\,dx$

3.435.1 Optimal result

Integrand size = 28, antiderivative size = 492

$$\begin{aligned}
&\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3\,dx \\
&= \frac{6ab^2(fg-eh)^2p^2q^2x}{f^2}-\frac{6b^3(fg-eh)^2p^3q^3x}{f^2}-\frac{3b^3h(fg-eh)p^3q^3(e+fx)^2}{4f^3} \\
&\quad -\frac{2b^3h^2p^3q^3(e+fx)^3}{27f^3}+\frac{6b^3(fg-eh)^2p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^3} \\
&\quad +\frac{3b^2h(fg-eh)p^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^3} \\
&\quad +\frac{2b^2h^2p^2q^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3} \\
&\quad -\frac{3b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^3} \\
&\quad -\frac{3bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^3} \\
&\quad -\frac{bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3f^3} \\
&\quad +\frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} \\
&\quad +\frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} \\
&\quad +\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3}
\end{aligned}$$

output
```
6*a*b^2*(-e*h+f*g)^2*p^2*q^2*x/f^2-6*b^3*(-e*h+f*g)^2*p^3*q^3*x/f^2-3/4*b^
3*h*(-e*h+f*g)*p^3*q^3*(f*x+e)^2/f^3-2/27*b^3*h^2*p^3*q^3*(f*x+e)^3/f^3+6*
b^3*(-e*h+f*g)^2*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^3+3/2*b^2*h*(-e*h
+f*g)*p^2*q^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3+2/9*b^2*h^2*p^2*q^
2*(f*x+e)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-3*b*(-e*h+f*g)^2*p*q*(f*x+e)*(
a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^3-3/2*b*h*(-e*h+f*g)*p*q*(f*x+e)^2*(a+b*ln(
c*(d*(f*x+e)^p)^q))^2/f^3-1/3*b*h^2*p*q*(f*x+e)^3*(a+b*ln(c*(d*(f*x+e)^p)^
q))^2/f^3+(-e*h+f*g)^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^3+h*(-e*h+f
*g)*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^3+1/3*h^2*(f*x+e)^3*(a+b*ln(
c*(d*(f*x+e)^p)^q))^3/f^3
```

3.435.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.77

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx$$

$$= \frac{108(fg-eh)^2(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3+108h(fg-eh)(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right.}{}$$

input
```
Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
(108*(f*g - e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 + 108*h*(f
*g - e*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 + 36*h^2*(e + f*x
)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 - 324*b*(f*g - e*h)^2*p*q*((e + f*x
)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f
*x)*Log[c*(d*(e + f*x)^p)^q])) - 81*b*h*(f*g - e*h)*p*q*(2*(e + f*x)^2*(a
+ b*Log[c*(d*(e + f*x)^p)^q])^2 + b*p*q*(b*f*p*q*x*(2*e + f*x) - 2*(e + f*
x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))) - 4*b*h^2*p*q*(9*(e + f*x)^3*(a +
b*Log[c*(d*(e + f*x)^p)^q])^2 + 2*b*p*q*(b*f*p*q*x*(3*e^2 + 3*e*f*x + f^2*
x^2) - 3*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))))/(108*f^3)
```

3.435.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(fg-eh)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2}+\frac{2h(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2}+\frac{h^2(e+fx)^2 (\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^2hp^2q^2(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^3}+\frac{2b^2h^2p^2q^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}+\frac{6ab^2p^2q^2x(fg-eh)^2}{f^2}-\frac{3bhpq(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^3}-\frac{3bpq(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^3}+\frac{h(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3}+\frac{(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3}-\frac{bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3f^3}+\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3}+\frac{6b^3p^2q^2(e+fx)(fg-eh)^2\log\left(c(d(e+fx)^p)^q\right)}{f^3}-\frac{3b^3hp^3q^3(e+fx)^2(fg-eh)}{4f^3}-\frac{2b^3h^2p^3q^3(e+fx)^3}{27f^3}-\frac{6b^3p^3q^3x(fg-eh)^2}{f^2}$$

input
```
Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
(6*a*b^2*(f*g - e*h)^2*p^2*q^2*x)/f^2 - (6*b^3*(f*g - e*h)^2*p^3*q^3*x)/f^
2 - (3*b^3*h*(f*g - e*h)*p^3*q^3*(e + f*x)^2)/(4*f^3) - (2*b^3*h^2*p^3*q^3
*(e + f*x)^3)/(27*f^3) + (6*b^3*(f*g - e*h)^2*p^2*q^2*(e + f*x)*Log[c*(d*(
e + f*x)^p)^q])/f^3 + (3*b^2*h*(f*g - e*h)*p^2*q^2*(e + f*x)^2*(a + b*Log[
c*(d*(e + f*x)^p)^q]))/(2*f^3) + (2*b^2*h^2*p^2*q^2*(e + f*x)^3*(a + b*Log
[c*(d*(e + f*x)^p)^q]))/(9*f^3) - (3*b*(f*g - e*h)^2*p*q*(e + f*x)*(a + b*
Log[c*(d*(e + f*x)^p)^q])^2)/f^3 - (3*b*h*(f*g - e*h)*p*q*(e + f*x)^2*(a +
 b*Log[c*(d*(e + f*x)^p)^q])^2)/(2*f^3) - (b*h^2*p*q*(e + f*x)^3*(a + b*Lo
g[c*(d*(e + f*x)^p)^q])^2)/(3*f^3) + ((f*g - e*h)^2*(e + f*x)*(a + b*Log[c
*(d*(e + f*x)^p)^q])^3)/f^3 + (h*(f*g - e*h)*(e + f*x)^2*(a + b*Log[c*(d*(
e + f*x)^p)^q])^3)/f^3 + (h^2*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])
^3)/(3*f^3)
```

3.435.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.435.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2033 vs. 2(478) = 956.

Time = 16.19 (sec) , antiderivative size = 2034, normalized size of antiderivative = 4.13

method	result	size
parallelrisch	Expression too large to display	2034

input
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*(648*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f^2*g*h*p*q-972*x*ln(c*(d*(f*x+
e)^p)^q)*b^3*e*f^2*g*h*p^2*q^2+108*x^2*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f^2*h
^2*p*q-2106*ln(f*x+e)*b^3*e^2*f*g*h*p^3*q^3-324*x^2*ln(c*(d*(f*x+e)^p)^q)*
a*b^2*f^3*g*h*p*q+324*x*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f^2*g*h*p*q-972*x*a*
b^2*e*f^2*g*h*p^2*q^2-216*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e^2*f*h^2*p*q+324*
x*a^2*b*e*f^2*g*h*p*q-648*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e^2*f*g*h*p*q+1620*l
n(f*x+e)*a*b^2*e^2*f*g*h*p^2*q^2-324*ln(f*x+e)*a^2*b*e^2*f*g*h*p*q+972*a*b
^2*e^2*f*g*h*p^2*q^2-324*a^2*b*e^2*f*g*h*p*q-108*x*a^2*b*e^2*f*h^2*p*q+648
*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f^2*g^2*p*q-90*x^2*ln(c*(d*(f*x+e)^p)^q)*b^
3*e*f^2*h^2*p^2*q^2+162*x^2*ln(c*(d*(f*x+e)^p)^q)*b^3*f^3*g*h*p^2*q^2+1134
*x*b^3*e*f^2*g*h*p^3*q^3-72*x^3*ln(c*(d*(f*x+e)^p)^q)*a*b^2*f^3*h^2*p*q+54
*x^2*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f^2*h^2*p*q-1296*ln(f*x+e)*a*b^2*e*f^2*
g^2*p^2*q^2+648*ln(f*x+e)*a^2*b*e*f^2*g^2*p*q-162*x^2*ln(c*(d*(f*x+e)^p)^q
)^2*b^3*f^3*g*h*p*q-90*x^2*a*b^2*e*f^2*h^2*p^2*q^2+162*x^2*a*b^2*f^3*g*h*p
^2*q^2+396*x*ln(c*(d*(f*x+e)^p)^q)*b^3*e^2*f*h^2*p^2*q^2-108*x*ln(c*(d*(f*
x+e)^p)^q)^2*b^3*e^2*f*h^2*p*q+396*x*a*b^2*e^2*f*h^2*p^2*q^2+972*ln(c*(d*(
f*x+e)^p)^q)*b^3*e^2*f*g*h*p^2*q^2+54*x^2*a^2*b*e*f^2*h^2*p*q-162*x^2*a^2*
b*f^3*g*h*p*q-648*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*f^3*g^2*p*q+486*ln(c*(d*(f
*x+e)^p)^q)^2*b^3*e^2*f*g*h*p*q+324*a^2*b*e*f^2*g^2*p*q-1134*b^3*e^2*f*g*h
*p^3*q^3-648*a*b^2*e*f^2*g^2*p^2*q^2-81*x^2*b^3*f^3*g*h*p^3*q^3-36*x^3*...
```

3.435.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3121 vs. $2(478) = 956$.

Time = 0.38 (sec) , antiderivative size = 3121, normalized size of antiderivative = 6.34

$$\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
-1/108*(4*(2*b^3*f^3*h^2*p^3*q^3 - 6*a*b^2*f^3*h^2*p^2*q^2 + 9*a^2*b*f^3*h
^2*p*q - 9*a^3*f^3*h^2)*x^3 - 36*(b^3*f^3*h^2*p^3*q^3*x^3 + 3*b^3*f^3*g*h*
p^3*q^3*x^2 + 3*b^3*f^3*g^2*p^3*q^3*x + (3*b^3*e*f^2*g^2 - 3*b^3*e^2*f*g*h
 + b^3*e^3*h^2)*p^3*q^3)*log(f*x + e)^3 - 36*(b^3*f^3*h^2*x^3 + 3*b^3*f^3*
g*h*x^2 + 3*b^3*f^3*g^2*x)*log(c)^3 - 36*(b^3*f^3*h^2*q^3*x^3 + 3*b^3*f^3*
g*h*q^3*x^2 + 3*b^3*f^3*g^2*q^3*x)*log(d)^3 - 3*(36*a^3*f^3*g*h - (27*b^3*
f^3*g*h - 19*b^3*e*f^2*h^2)*p^3*q^3 + 6*(9*a*b^2*f^3*g*h - 5*a*b^2*e*f^2*h
^2)*p^2*q^2 - 18*(3*a^2*b*f^3*g*h - a^2*b*e*f^2*h^2)*p*q)*x^2 + 18*((18*b^
3*e*f^2*g^2 - 27*b^3*e^2*f*g*h + 11*b^3*e^3*h^2)*p^3*q^3 - 6*(3*a*b^2*e*f^
2*g^2 - 3*a*b^2*e^2*f*g*h + a*b^2*e^3*h^2)*p^2*q^2 + 2*(b^3*f^3*h^2*p^3*q^
3 - 3*a*b^2*f^3*h^2*p^2*q^2)*x^3 - 3*(6*a*b^2*f^3*g*h*p^2*q^2 - (3*b^3*f^3
*g*h - b^3*e*f^2*h^2)*p^3*q^3)*x^2 - 6*(3*a*b^2*f^3*g^2*p^2*q^2 - (3*b^3*f
^3*g^2 - 3*b^3*e*f^2*g*h + b^3*e^2*f*h^2)*p^3*q^3)*x - 6*(b^3*f^3*h^2*p^2*
q^2*x^3 + 3*b^3*f^3*g*h*p^2*q^2*x^2 + 3*b^3*f^3*g^2*p^2*q^2*x + (3*b^3*e*f
^2*g^2 - 3*b^3*e^2*f*g*h + b^3*e^3*h^2)*p^2*q^2)*log(c) - 6*(b^3*f^3*h^2*p
^2*q^3*x^3 + 3*b^3*f^3*g*h*p^2*q^3*x^2 + 3*b^3*f^3*g^2*p^2*q^3*x + (3*b^3*
e*f^2*g^2 - 3*b^3*e^2*f*g*h + b^3*e^3*h^2)*p^2*q^3)*log(d))*log(f*x + e)^2
 + 18*(2*(b^3*f^3*h^2*p*q - 3*a*b^2*f^3*h^2)*x^3 - 3*(6*a*b^2*f^3*g*h - (3
*b^3*f^3*g*h - b^3*e*f^2*h^2)*p*q)*x^2 - 6*(3*a*b^2*f^3*g^2 - (3*b^3*f^3*g
^2 - 3*b^3*e*f^2*g*h + b^3*e^2*f*h^2)*p*q)*x)*log(c)^2 + 18*(2*(b^3*f^3...
```

3.435.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1846 vs. $2(481) = 962$.

Time = 5.38 (sec) , antiderivative size = 1846, normalized size of antiderivative = 3.75

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Piecewise((a**3*g**2*x + a**3*g*h*x**2 + a**3*h**2*x**3/3 + a**2*b*e**3*h*
*2*log(c*(d*(e + f*x)**p)**q)/f**3 - 3*a**2*b*e**2*g*h*log(c*(d*(e + f*x)*
*p)**q)/f**2 - a**2*b*e**2*h**2*p*q*x/f**2 + 3*a**2*b*e*g**2*log(c*(d*(e +
 f*x)**p)**q)/f + 3*a**2*b*e*g*h*p*q*x/f + a**2*b*e*h**2*p*q*x**2/(2*f) -
3*a**2*b*g**2*p*q*x + 3*a**2*b*g**2*x*log(c*(d*(e + f*x)**p)**q) - 3*a**2*
b*g*h*p*q*x**2/2 + 3*a**2*b*g*h*x**2*log(c*(d*(e + f*x)**p)**q) - a**2*b*h
**2*p*q*x**3/3 + a**2*b*h**2*x**3*log(c*(d*(e + f*x)**p)**q) - 11*a*b**2*e
**3*h**2*p*q*log(c*(d*(e + f*x)**p)**q)/(3*f**3) + a*b**2*e**3*h**2*log(c*
(d*(e + f*x)**p)**q)**2/f**3 + 9*a*b**2*e**2*g*h*p*q*log(c*(d*(e + f*x)**p
)**q)/f**2 - 3*a*b**2*e**2*g*h*log(c*(d*(e + f*x)**p)**q)**2/f**2 + 11*a*b
**2*e**2*h**2*p**2*q**2*x/(3*f**2) - 2*a*b**2*e**2*h**2*p*q*x*log(c*(d*(e
+ f*x)**p)**q)/f**2 - 6*a*b**2*e*g**2*p*q*log(c*(d*(e + f*x)**p)**q)/f + 3
*a*b**2*e*g**2*log(c*(d*(e + f*x)**p)**q)**2/f - 9*a*b**2*e*g*h*p**2*q**2*
x/f + 6*a*b**2*e*g*h*p*q*x*log(c*(d*(e + f*x)**p)**q)/f - 5*a*b**2*e*h**2*
p**2*q**2*x**2/(6*f) + a*b**2*e*h**2*p*q*x**2*log(c*(d*(e + f*x)**p)**q)/f
 + 6*a*b**2*g**2*p**2*q**2*x - 6*a*b**2*g**2*p*q*x*log(c*(d*(e + f*x)**p)*
*q) + 3*a*b**2*g**2*x*log(c*(d*(e + f*x)**p)**q)**2 + 3*a*b**2*g*h*p**2*q*
*2*x**2/2 - 3*a*b**2*g*h*p*q*x**2*log(c*(d*(e + f*x)**p)**q) + 3*a*b**2*g*
h*x**2*log(c*(d*(e + f*x)**p)**q)**2 + 2*a*b**2*h**2*p**2*q**2*x**3/9 - 2*
a*b**2*h**2*p*q*x**3*log(c*(d*(e + f*x)**p)**q)/3 + a*b**2*h**2*x**3*lo...
```

3.435.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. 2(478) = 956.

Time = 0.25 (sec) , antiderivative size = 1245, normalized size of antiderivative = 2.53

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^3 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*h^2*x^3*log(((f*x + e)^p*d)^q*c)^3 + a*b^2*h^2*x^3*log(((f*x + e)^
p*d)^q*c)^2 + b^3*g*h*x^2*log(((f*x + e)^p*d)^q*c)^3 - 3*a^2*b*f*g^2*p*q*(
x/f - e*log(f*x + e)/f^2) + 1/6*a^2*b*f*h^2*p*q*(6*e^3*log(f*x + e)/f^4 -
(2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3) - 3/2*a^2*b*f*g*h*p*q*(2*e^2*log(f*
x + e)/f^3 + (f*x^2 - 2*e*x)/f^2) + a^2*b*h^2*x^3*log(((f*x + e)^p*d)^q*c)
 + 3*a*b^2*g*h*x^2*log(((f*x + e)^p*d)^q*c)^2 + b^3*g^2*x*log(((f*x + e)^p
*d)^q*c)^3 + 1/3*a^3*h^2*x^3 + 3*a^2*b*g*h*x^2*log(((f*x + e)^p*d)^q*c) +
3*a*b^2*g^2*x*log(((f*x + e)^p*d)^q*c)^2 + a^3*g*h*x^2 + 3*a^2*b*g^2*x*log
(((f*x + e)^p*d)^q*c) - 3*(2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x +
e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*a*
b^2*g^2 - (3*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^2 -
 ((e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p^2*q
^2/f^2 - 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p*q*log(((f*x + e
)^p*d)^q*c)/f^2)*f*p*q)*b^3*g^2 - 3/2*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (
f*x^2 - 2*e*x)/f^2)*log(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x +
e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))*p^2*q^2/f^2)*a*b^2*g*h - 1/4*(6*f*p*q
*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2)*log(((f*x + e)^p*d)^q*c)^2
 + ((4*e^2*log(f*x + e)^3 + 3*f^2*x^2 + 18*e^2*log(f*x + e)^2 - 42*e*f*x +
 42*e^2*log(f*x + e))*p^2*q^2/f^3 - 6*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*
e*f*x + 6*e^2*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^3)*f*p*q)*b^...
```

3.435.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5146 vs. $2(478) = 956$.

Time = 0.42 (sec) , antiderivative size = 5146, normalized size of antiderivative = 10.46

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
1/108*(108*(f*x + e)*b^3*f^2*g^2*p^3*q^3*log(f*x + e)^3 + 108*(f*x + e)^2*
b^3*f*g*h*p^3*q^3*log(f*x + e)^3 - 216*(f*x + e)*b^3*e*f*g*h*p^3*q^3*log(f
*x + e)^3 + 36*(f*x + e)^3*b^3*h^2*p^3*q^3*log(f*x + e)^3 - 108*(f*x + e)^
2*b^3*e*h^2*p^3*q^3*log(f*x + e)^3 + 108*(f*x + e)*b^3*e^2*h^2*p^3*q^3*log
(f*x + e)^3 - 324*(f*x + e)*b^3*f^2*g^2*p^3*q^3*log(f*x + e)^2 - 162*(f*x
+ e)^2*b^3*f*g*h*p^3*q^3*log(f*x + e)^2 + 648*(f*x + e)*b^3*e*f*g*h*p^3*q^
3*log(f*x + e)^2 - 36*(f*x + e)^3*b^3*h^2*p^3*q^3*log(f*x + e)^2 + 162*(f*
x + e)^2*b^3*e*h^2*p^3*q^3*log(f*x + e)^2 - 324*(f*x + e)*b^3*e^2*h^2*p^3*
q^3*log(f*x + e)^2 + 324*(f*x + e)*b^3*f^2*g^2*p^2*q^3*log(f*x + e)^2*log(
d) + 324*(f*x + e)^2*b^3*f*g*h*p^2*q^3*log(f*x + e)^2*log(d) - 648*(f*x +
e)*b^3*e*f*g*h*p^2*q^3*log(f*x + e)^2*log(d) + 108*(f*x + e)^3*b^3*h^2*p^2
*q^3*log(f*x + e)^2*log(d) - 324*(f*x + e)^2*b^3*e*h^2*p^2*q^3*log(f*x + e
)^2*log(d) + 324*(f*x + e)*b^3*e^2*h^2*p^2*q^3*log(f*x + e)^2*log(d) + 648
*(f*x + e)*b^3*f^2*g^2*p^3*q^3*log(f*x + e) + 162*(f*x + e)^2*b^3*f*g*h*p^
3*q^3*log(f*x + e) - 1296*(f*x + e)*b^3*e*f*g*h*p^3*q^3*log(f*x + e) + 24*
(f*x + e)^3*b^3*h^2*p^3*q^3*log(f*x + e) - 162*(f*x + e)^2*b^3*e*h^2*p^3*q
^3*log(f*x + e) + 648*(f*x + e)*b^3*e^2*h^2*p^3*q^3*log(f*x + e) + 324*(f*
x + e)*b^3*f^2*g^2*p^2*q^2*log(f*x + e)^2*log(c) + 324*(f*x + e)^2*b^3*f*g
*h*p^2*q^2*log(f*x + e)^2*log(c) - 648*(f*x + e)*b^3*e*f*g*h*p^2*q^2*log(f
*x + e)^2*log(c) + 108*(f*x + e)^3*b^3*h^2*p^2*q^2*log(f*x + e)^2*log(c...
```

3.435.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 1400, normalized size of antiderivative = 2.85

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^3 \, dx = \text{Too large to display}$$

input
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

output

```
x*((18*a^3*f^2*g^2 - 66*b^3*e^2*h^2*p^3*q^3 - 108*b^3*f^2*g^2*p^3*q^3 + 36
*a^3*e*f*g*h + 36*a*b^2*e^2*h^2*p^2*q^2 + 108*a*b^2*f^2*g^2*p^2*q^2 - 54*a
^2*b*f^2*g^2*p*q + 162*b^3*e*f*g*h*p^3*q^3 - 108*a*b^2*e*f*g*h*p^2*q^2)/(1
8*f^2) - (e*((h*(6*a^3*e*h + 12*a^3*f*g + 5*b^3*e*h*p^3*q^3 - 9*b^3*f*g*p^
3*q^3 - 18*a^2*b*f*g*p*q - 6*a*b^2*e*h*p^2*q^2 + 18*a*b^2*f*g*p^2*q^2))/(6
*f) - (e*h^2*(9*a^3 - 2*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 9*a^2*b*p*q))/(9*f
)))/f) + log(c*(d*(e + f*x)^p)^q)^2*(x^2*((3*b^2*h*(a*e*h + 2*a*f*g - b*f*
g*p*q))/(2*f) - (b^2*e*h^2*(3*a - b*p*q))/(2*f)) - x*((e*((3*b^2*h*(a*e*h
+ 2*a*f*g - b*f*g*p*q))/f - (b^2*e*h^2*(3*a - b*p*q))/f))/f - (3*b^2*g*(2*
a*e*h + a*f*g - b*f*g*p*q))/f) + (e*(6*a*b^2*e^2*h^2 + 18*a*b^2*f^2*g^2 -
11*b^3*e^2*h^2*p*q - 18*b^3*f^2*g^2*p*q - 18*a*b^2*e*f*g*h + 27*b^3*e*f*g*
h*p*q))/(6*f^3) + (b^2*h^2*x^3*(3*a - b*p*q))/3) + log(c*(d*(e + f*x)^p)^q
)^3*(b^3*g^2*x + (b^3*h^2*x^3)/3 + (e*(b^3*e^2*h^2 + 3*b^3*f^2*g^2 - 3*b^3
*e*f*g*h))/(3*f^3) + b^3*g*h*x^2) + x^2*((h*(6*a^3*e*h + 12*a^3*f*g + 5*b^
3*e*h*p^3*q^3 - 9*b^3*f*g*p^3*q^3 - 18*a^2*b*f*g*p*q - 6*a*b^2*e*h*p^2*q^2
 + 18*a*b^2*f*g*p^2*q^2))/(12*f) - (e*h^2*(9*a^3 - 2*b^3*p^3*q^3 + 6*a*b^2
*p^2*q^2 - 9*a^2*b*p*q))/(18*f)) + (log(e + f*x)*(85*b^3*e^3*h^2*p^3*q^3 -
 66*a*b^2*e^3*h^2*p^2*q^2 + 108*b^3*e*f^2*g^2*p^3*q^3 + 18*a^2*b*e^3*h^2*p
*q - 108*a*b^2*e*f^2*g^2*p^2*q^2 + 54*a^2*b*e*f^2*g^2*p*q - 189*b^3*e^2*f*
g*h*p^3*q^3 + 162*a*b^2*e^2*f*g*h*p^2*q^2 - 54*a^2*b*e^2*f*g*h*p*q))/(1...
```

3.435. $\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx$

3.436 $\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx$

3.436.1 Optimal result

Integrand size = 26, antiderivative size = 306

$$\begin{aligned}
&\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx \\
&= \frac{6ab^2(fg-eh)p^2q^2x}{f} - \frac{6b^3(fg-eh)p^3q^3x}{f} - \frac{3b^3hp^3q^3(e+fx)^2}{8f^2} \\
&\quad + \frac{6b^3(fg-eh)p^2q^2(e+fx)\log \left(c(d(e+fx)^p)^q\right)}{f^2} \\
&\quad + \frac{3b^2hp^2q^2(e+fx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{4f^2} \\
&\quad - \frac{3b(fg-eh)pq(e+fx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} \\
&\quad - \frac{3bhpq(e+fx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2} \\
&\quad + \frac{(fg-eh)(e+fx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3}{f^2} \\
&\quad + \frac{h(e+fx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2}
\end{aligned}$$

output
```
6*a*b^2*(-e*h+f*g)*p^2*q^2*x/f-6*b^3*(-e*h+f*g)*p^3*q^3*x/f-3/8*b^3*h*p^3*
q^3*(f*x+e)^2/f^2+6*b^3*(-e*h+f*g)*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f
^2+3/4*b^2*h*p^2*q^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2-3*b*(-e*h+f
*g)*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2-3/4*b*h*p*q*(f*x+e)^2*(a
+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2+(-e*h+f*g)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)
^q))^3/f^2+1/2*h*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^2
```

3.436.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.75

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$$
$$=\frac{8(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3+4h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3-24b(fg-e}{}$$

input
```
Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
(8*(f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 + 4*h*(e + f*x
)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 - 24*b*(f*g - e*h)*p*q*((e + f*x)*(
a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)
*Log[c*(d*(e + f*x)^p)^q])) - 3*b*h*p*q*(2*(e + f*x)^2*(a + b*Log[c*(d*(e
+ f*x)^p)^q])^2 + b*p*q*(b*f*p*q*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[
c*(d*(e + f*x)^p)^q]))))/(8*f^2)
```

3.436.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3dx$$
$$\Big\downarrow \text{ 2848}$$
$$\int \left(\frac{(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f}+\frac{h(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{3b^2hp^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2}+\frac{6ab^2p^2q^2x(fg-eh)}{f}-\frac{3bpq(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2}+\frac{(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2}-\frac{3bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2}+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2}+\frac{6b^3p^2q^2(e+fx)(fg-eh)\log\left(c(d(e+fx)^p)^q\right)}{f^2}-\frac{3b^3hp^3q^3(e+fx)^2}{8f^2}-\frac{6b^3p^3q^3x(fg-eh)}{f}$$

input
```
Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
(6*a*b^2*(f*g - e*h)*p^2*q^2*x)/f - (6*b^3*(f*g - e*h)*p^3*q^3*x)/f - (3*b
^3*h*p^3*q^3*(e + f*x)^2)/(8*f^2) + (6*b^3*(f*g - e*h)*p^2*q^2*(e + f*x)*L
og[c*(d*(e + f*x)^p)^q])/f^2 + (3*b^2*h*p^2*q^2*(e + f*x)^2*(a + b*Log[c*(
d*(e + f*x)^p)^q]))/(4*f^2) - (3*b*(f*g - e*h)*p*q*(e + f*x)*(a + b*Log[c*
(d*(e + f*x)^p)^q])^2)/f^2 - (3*b*h*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e + f
*x)^p)^q])^2)/(4*f^2) + ((f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p
)^q])^3)/f^2 + (h*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(2*f^2)
```

3.436.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.436.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(298) = 596.

Time = 5.22 (sec) , antiderivative size = 1084, normalized size of antiderivative = 3.54

method	result	size
parallelrisch	Expression too large to display	1084

input
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-12*x^2*ln(c*(d*(f*x+e)^p)^q)*a^2*b*f^2*h-24*x*ln(c*(d*(f*x+e)^p)^q)
^2*a*b^2*f^2*g-24*x*ln(c*(d*(f*x+e)^p)^q)*a^2*b*f^2*g-24*ln(c*(d*(f*x+e)^p
)^q)^2*a*b^2*e*f*g+24*ln(c*(d*(f*x+e)^p)^q)*a^2*b*e*f*g+78*ln(f*x+e)*b^3*e
^2*h*p^3*q^3-36*ln(c*(d*(f*x+e)^p)^q)*b^3*e^2*h*p^2*q^2-12*x^2*ln(c*(d*(f*
x+e)^p)^q)^2*a*b^2*f^2*h+36*x*ln(c*(d*(f*x+e)^p)^q)*b^3*e*f*h*p^2*q^2+24*l
n(c*(d*(f*x+e)^p)^q)*a*b^2*e^2*h*p*q-96*ln(f*x+e)*b^3*e*f*g*p^3*q^3-60*ln(
f*x+e)*a*b^2*e^2*h*p^2*q^2+12*ln(f*x+e)*a^2*b*e^2*h*p*q-24*a^2*b*e*f*g*p*q
+48*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*f^2*g*p*q-12*x*a^2*b*e*f*h*p*q-48*ln(c*(
d*(f*x+e)^p)^q)*a*b^2*e*f*g*p*q+96*ln(f*x+e)*a*b^2*e*f*g*p^2*q^2-48*ln(f*x
+e)*a^2*b*e*f*g*p*q-24*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f*h*p*q+12*x^2*ln(c
*(d*(f*x+e)^p)^q)*a*b^2*f^2*h*p*q-12*x*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f*h*p
*q+36*x*a*b^2*e*f*h*p^2*q^2+48*a*b^2*e*f*g*p^2*q^2+42*b^3*e^2*h*p^3*q^3+8*
a^3*e*f*g-6*x^2*ln(c*(d*(f*x+e)^p)^q)*b^3*f^2*h*p^2*q^2-42*x*b^3*e*f*h*p^3
*q^3+6*x^2*ln(c*(d*(f*x+e)^p)^q)^2*b^3*f^2*h*p*q-6*x^2*a*b^2*f^2*h*p^2*q^2
-48*x*ln(c*(d*(f*x+e)^p)^q)*b^3*f^2*g*p^2*q^2+24*x*ln(c*(d*(f*x+e)^p)^q)^2
*b^3*f^2*g*p*q-48*x*a*b^2*f^2*g*p^2*q^2+48*ln(c*(d*(f*x+e)^p)^q)*b^3*e*f*g
*p^2*q^2+6*x^2*a^2*b*f^2*h*p*q+24*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f*g*p*q+24
*x*a^2*b*f^2*g*p*q+4*ln(c*(d*(f*x+e)^p)^q)^3*b^3*e^2*h-4*x^2*a^3*f^2*h-8*x
*a^3*f^2*g+3*x^2*b^3*f^2*h*p^3*q^3+48*x*b^3*f^2*g*p^3*q^3-18*ln(c*(d*(f*x+
e)^p)^q)^2*b^3*e^2*h*p*q-36*a*b^2*e^2*h*p^2*q^2+12*a^2*b*e^2*h*p*q-48*b...
```

3.436.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1692 vs. 2(298) = 596.

Time = 0.39 (sec) , antiderivative size = 1692, normalized size of antiderivative = 5.53

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
1/8*(4*(b^3*f^2*h*p^3*q^3*x^2 + 2*b^3*f^2*g*p^3*q^3*x + (2*b^3*e*f*g - b^3
*e^2*h)*p^3*q^3)*log(f*x + e)^3 + 4*(b^3*f^2*h*x^2 + 2*b^3*f^2*g*x)*log(c)
^3 + 4*(b^3*f^2*h*q^3*x^2 + 2*b^3*f^2*g*q^3*x)*log(d)^3 - (3*b^3*f^2*h*p^3
*q^3 - 6*a*b^2*f^2*h*p^2*q^2 + 6*a^2*b*f^2*h*p*q - 4*a^3*f^2*h)*x^2 - 6*((
4*b^3*e*f*g - 3*b^3*e^2*h)*p^3*q^3 - 2*(2*a*b^2*e*f*g - a*b^2*e^2*h)*p^2*q
^2 + (b^3*f^2*h*p^3*q^3 - 2*a*b^2*f^2*h*p^2*q^2)*x^2 - 2*(2*a*b^2*f^2*g*p^
2*q^2 - (2*b^3*f^2*g - b^3*e*f*h)*p^3*q^3)*x - 2*(b^3*f^2*h*p^2*q^2*x^2 +
2*b^3*f^2*g*p^2*q^2*x + (2*b^3*e*f*g - b^3*e^2*h)*p^2*q^2)*log(c) - 2*(b^3
*f^2*h*p^2*q^3*x^2 + 2*b^3*f^2*g*p^2*q^3*x + (2*b^3*e*f*g - b^3*e^2*h)*p^2
*q^3)*log(d))*log(f*x + e)^2 - 6*((b^3*f^2*h*p*q - 2*a*b^2*f^2*h)*x^2 - 2*
(2*a*b^2*f^2*g - (2*b^3*f^2*g - b^3*e*f*h)*p*q)*x)*log(c)^2 - 6*((b^3*f^2*
h*p*q^3 - 2*a*b^2*f^2*h*q^2)*x^2 - 2*(2*a*b^2*f^2*g*q^2 - (2*b^3*f^2*g - b
^3*e*f*h)*p*q^3)*x - 2*(b^3*f^2*h*q^2*x^2 + 2*b^3*f^2*g*q^2*x)*log(c))*log
(d)^2 - 2*(3*(8*b^3*f^2*g - 7*b^3*e*f*h)*p^3*q^3 - 4*a^3*f^2*g - 6*(4*a*b^
2*f^2*g - 3*a*b^2*e*f*h)*p^2*q^2 + 6*(2*a^2*b*f^2*g - a^2*b*e*f*h)*p*q)*x
+ 6*((8*b^3*e*f*g - 7*b^3*e^2*h)*p^3*q^3 - 2*(4*a*b^2*e*f*g - 3*a*b^2*e^2*
h)*p^2*q^2 + 2*(2*a^2*b*e*f*g - a^2*b*e^2*h)*p*q + (b^3*f^2*h*p^3*q^3 - 2*
a*b^2*f^2*h*p^2*q^2 + 2*a^2*b*f^2*h*p*q)*x^2 + 2*(b^3*f^2*h*p*q*x^2 + 2*b^
3*f^2*g*p*q*x + (2*b^3*e*f*g - b^3*e^2*h)*p*q)*log(c)^2 + 2*(b^3*f^2*h*p*q
^3*x^2 + 2*b^3*f^2*g*p*q^3*x + (2*b^3*e*f*g - b^3*e^2*h)*p*q^3)*log(d)^...
```

3.436.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. 2(299) = 598.

Time = 2.58 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.24

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Piecewise((a**3*g*x + a**3*h*x**2/2 - 3*a**2*b*e**2*h*log(c*(d*(e + f*x)**
p)**q)/(2*f**2) + 3*a**2*b*e*g*log(c*(d*(e + f*x)**p)**q)/f + 3*a**2*b*e*h
*p*q*x/(2*f) - 3*a**2*b*g*p*q*x + 3*a**2*b*g*x*log(c*(d*(e + f*x)**p)**q)
- 3*a**2*b*h*p*q*x**2/4 + 3*a**2*b*h*x**2*log(c*(d*(e + f*x)**p)**q)/2 + 9
*a*b**2*e**2*h*p*q*log(c*(d*(e + f*x)**p)**q)/(2*f**2) - 3*a*b**2*e**2*h*l
og(c*(d*(e + f*x)**p)**q)**2/(2*f**2) - 6*a*b**2*e*g*p*q*log(c*(d*(e + f*x
)**p)**q)/f + 3*a*b**2*e*g*log(c*(d*(e + f*x)**p)**q)**2/f - 9*a*b**2*e*h*
p**2*q**2*x/(2*f) + 3*a*b**2*e*h*p*q*x*log(c*(d*(e + f*x)**p)**q)/f + 6*a*
b**2*g*p**2*q**2*x - 6*a*b**2*g*p*q*x*log(c*(d*(e + f*x)**p)**q) + 3*a*b**
2*g*x*log(c*(d*(e + f*x)**p)**q)**2 + 3*a*b**2*h*p**2*q**2*x**2/4 - 3*a*b*
*2*h*p*q*x**2*log(c*(d*(e + f*x)**p)**q)/2 + 3*a*b**2*h*x**2*log(c*(d*(e +
 f*x)**p)**q)**2/2 - 21*b**3*e**2*h*p**2*q**2*log(c*(d*(e + f*x)**p)**q)/(
4*f**2) + 9*b**3*e**2*h*p*q*log(c*(d*(e + f*x)**p)**q)**2/(4*f**2) - b**3*
e**2*h*log(c*(d*(e + f*x)**p)**q)**3/(2*f**2) + 6*b**3*e*g*p**2*q**2*log(c
*(d*(e + f*x)**p)**q)/f - 3*b**3*e*g*p*q*log(c*(d*(e + f*x)**p)**q)**2/f +
 b**3*e*g*log(c*(d*(e + f*x)**p)**q)**3/f + 21*b**3*e*h*p**3*q**3*x/(4*f)
- 9*b**3*e*h*p**2*q**2*x*log(c*(d*(e + f*x)**p)**q)/(2*f) + 3*b**3*e*h*p*q
*x*log(c*(d*(e + f*x)**p)**q)**2/(2*f) - 6*b**3*g*p**3*q**3*x + 6*b**3*g*p
**2*q**2*x*log(c*(d*(e + f*x)**p)**q) - 3*b**3*g*p*q*x*log(c*(d*(e + f*x)*
*p)**q)**2 + b**3*g*x*log(c*(d*(e + f*x)**p)**q)**3 - 3*b**3*h*p**3*q**...
```

3.436.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(298) = 596$.

Time = 0.23 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.39

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$$
$$= \frac{1}{2}b^3hx^2\log\left(\left(\left((fx+e)^pd\right)^qc\right)^3 - 3a^2bfgpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\right.$$
$$-\frac{3}{4}a^2bfhpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right)+\frac{3}{2}ab^2hx^2\log\left(\left((fx+e)^pd\right)^qc\right)^2$$
$$+b^3gx\log\left(\left((fx+e)^pd\right)^qc\right)^3+\frac{3}{2}a^2bhx^2\log\left(\left((fx+e)^pd\right)^qc\right)$$
$$+3ab^2gx\log\left(\left((fx+e)^pd\right)^qc\right)^2+\frac{1}{2}a^3hx^2+3a^2bgx\log\left(\left((fx+e)^pd\right)^qc\right)$$
$$-3\left(2fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(\left((fx+e)^pd\right)^qc\right)+\frac{\left(e\log(fx+e)^2-2fx+2e\log(fx+e)\right)p^2q^2}{f}\right)$$
$$-\left(3fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(\left((fx+e)^pd\right)^qc\right)^2-\left(\frac{\left(e\log(fx+e)^3+3e\log(fx+e)^2-6fx+6\right.}{f^2}\right.\right.$$
$$-\frac{3}{4}\left(2fpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right)\log\left(\left((fx+e)^pd\right)^qc\right)-\frac{\left(f^2x^2+2e^2\log(fx+e)^2-6efx+\right.}{f^2}\right.$$
$$-\frac{1}{8}\left(6fpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right)\log\left(\left((fx+e)^pd\right)^qc\right)^2+\left(\frac{\left(4e^2\log(fx+e)^3+3f^2x^2+18\right.}{}\right.\right.$$
$$+a^3gx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

3.436. $\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$

output
```
1/2*b^3*h*x^2*log(((f*x + e)^p*d)^q*c)^3 - 3*a^2*b*f*g*p*q*(x/f - e*log(f*
x + e)/f^2) - 3/4*a^2*b*f*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/
f^2) + 3/2*a*b^2*h*x^2*log(((f*x + e)^p*d)^q*c)^2 + b^3*g*x*log(((f*x + e)
^p*d)^q*c)^3 + 3/2*a^2*b*h*x^2*log(((f*x + e)^p*d)^q*c) + 3*a*b^2*g*x*log(
((f*x + e)^p*d)^q*c)^2 + 1/2*a^3*h*x^2 + 3*a^2*b*g*x*log(((f*x + e)^p*d)^q
*c) - 3*(2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*
log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*a*b^2*g - (3*f*p*q*(
x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^2 - ((e*log(f*x + e)^3
+ 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p^2*q^2/f^2 - 3*(e*log(f*
x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^2)*f*p
*q)*b^3*g - 3/4*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2)*lo
g(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2
*log(f*x + e))*p^2*q^2/f^2)*a*b^2*h - 1/8*(6*f*p*q*(2*e^2*log(f*x + e)/f^3
 + (f*x^2 - 2*e*x)/f^2)*log(((f*x + e)^p*d)^q*c)^2 + ((4*e^2*log(f*x + e)^
3 + 3*f^2*x^2 + 18*e^2*log(f*x + e)^2 - 42*e*f*x + 42*e^2*log(f*x + e))*p^
2*q^2/f^3 - 6*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x +
e))*p*q*log(((f*x + e)^p*d)^q*c)/f^3)*f*p*q)*b^3*h + a^3*g*x
```

3.436.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2532 vs. $2(298) = 596$.

Time = 0.38 (sec) , antiderivative size = 2532, normalized size of antiderivative = 8.27

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^3 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
(f*x + e)*b^3*g*p^3*q^3*log(f*x + e)^3/f + 1/2*(f*x + e)^2*b^3*h*p^3*q^3*l
og(f*x + e)^3/f^2 - (f*x + e)*b^3*e*h*p^3*q^3*log(f*x + e)^3/f^2 - 3*(f*x
+ e)*b^3*g*p^3*q^3*log(f*x + e)^2/f - 3/4*(f*x + e)^2*b^3*h*p^3*q^3*log(f*
x + e)^2/f^2 + 3*(f*x + e)*b^3*e*h*p^3*q^3*log(f*x + e)^2/f^2 + 3*(f*x + e
)*b^3*g*p^2*q^3*log(f*x + e)^2*log(d)/f + 3/2*(f*x + e)^2*b^3*h*p^2*q^3*lo
g(f*x + e)^2*log(d)/f^2 - 3*(f*x + e)*b^3*e*h*p^2*q^3*log(f*x + e)^2*log(d
)/f^2 + 6*(f*x + e)*b^3*g*p^3*q^3*log(f*x + e)/f + 3/4*(f*x + e)^2*b^3*h*p
^3*q^3*log(f*x + e)/f^2 - 6*(f*x + e)*b^3*e*h*p^3*q^3*log(f*x + e)/f^2 + 3
*(f*x + e)*b^3*g*p^2*q^2*log(f*x + e)^2*log(c)/f + 3/2*(f*x + e)^2*b^3*h*p
^2*q^2*log(f*x + e)^2*log(c)/f^2 - 3*(f*x + e)*b^3*e*h*p^2*q^2*log(f*x + e
)^2*log(c)/f^2 - 6*(f*x + e)*b^3*g*p^2*q^3*log(f*x + e)*log(d)/f - 3/2*(f*
x + e)^2*b^3*h*p^2*q^3*log(f*x + e)*log(d)/f^2 + 6*(f*x + e)*b^3*e*h*p^2*q
^3*log(f*x + e)*log(d)/f^2 + 3*(f*x + e)*b^3*g*p*q^3*log(f*x + e)*log(d)^2
/f + 3/2*(f*x + e)^2*b^3*h*p*q^3*log(f*x + e)*log(d)^2/f^2 - 3*(f*x + e)*b
^3*e*h*p*q^3*log(f*x + e)*log(d)^2/f^2 - 6*(f*x + e)*b^3*g*p^3*q^3/f - 3/8
*(f*x + e)^2*b^3*h*p^3*q^3/f^2 + 6*(f*x + e)*b^3*e*h*p^3*q^3/f^2 + 3*(f*x
+ e)*a*b^2*g*p^2*q^2*log(f*x + e)^2/f + 3/2*(f*x + e)^2*a*b^2*h*p^2*q^2*lo
g(f*x + e)^2/f^2 - 3*(f*x + e)*a*b^2*e*h*p^2*q^2*log(f*x + e)^2/f^2 - 6*(f
*x + e)*b^3*g*p^2*q^2*log(f*x + e)*log(c)/f - 3/2*(f*x + e)^2*b^3*h*p^2*q^
2*log(f*x + e)*log(c)/f^2 + 6*(f*x + e)*b^3*e*h*p^2*q^2*log(f*x + e)*lo...
```

3.436. $\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$

3.436.9 Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.13

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$$
$$=x\left(\frac{4a^3eh+4a^3fg+18b^3ehp^3q^3-24b^3fgp^3q^3-12a^2bfgpq-12ab^2ehp^2q^2+24ab^2fgp^2q^2}{4f}\right.$$
$$\left.-\frac{eh\left(4a^3-6a^2bpq+6ab^2p^2q^2-3b^3p^3q^3\right)}{4f}\right)$$
$$+\ln\left(c\left(d\left(e+fx\right)^p\right)^q\right)^2\left(\frac{x\left(\frac{6b^2\left(aeh+afg-bfgpq\right)}{f}-\frac{3b^2eh\left(2a-bpq\right)}{f}\right)}{2}\right.$$
$$\left.-\frac{3e\left(2ab^2eh-4ab^2fg-3b^3ehpq+4b^3fgpq\right)}{4f^2}+\frac{3b^2hx^2\left(2a-bpq\right)}{4}\right)$$
$$+\ln\left(c\left(d\left(e+fx\right)^p\right)^q\right)^3\left(\frac{b^3hx^2}{2}-\frac{e\left(b^3eh-2b^3fg\right)}{2f^2}+b^3gx\right)$$
$$+\frac{\ln\left(c\left(d\left(e+fx\right)^p\right)^q\right)\left(x^2\left(6a^2bfg+\frac{3beh\left(2a^2-2abpq+b^2p^2q^2\right)}{2}-9b^3ehp^2q^2+12b^3fgp^2q^2+6ab^2eh\right.\right.}{2e}$$
$$+\frac{hx^2\left(4a^3-6a^2bpq+6ab^2p^2q^2-3b^3p^3q^3\right)}{8}$$
$$-\frac{\ln\left(e+fx\right)\left(6ha^2be^2pq-12fga^2bepq-18hab^2e^2p^2q^2+24fgab^2ep^2q^2+21hb^3e^2p^3q^3-24\right.}{4f^2}$$

input
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

output
```
x*((4*a^3*e*h + 4*a^3*f*g + 18*b^3*e*h*p^3*q^3 - 24*b^3*f*g*p^3*q^3 - 12*a
^2*b*f*g*p*q - 12*a*b^2*e*h*p^2*q^2 + 24*a*b^2*f*g*p^2*q^2)/(4*f) - (e*h*(
4*a^3 - 3*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 6*a^2*b*p*q))/(4*f)) + log(c*(d*
(e + f*x)^p)^q)^2*((x*((6*b^2*(a*e*h + a*f*g - b*f*g*p*q))/f - (3*b^2*e*h*
(2*a - b*p*q))/f))/2 - (3*e*(2*a*b^2*e*h - 4*a*b^2*f*g - 3*b^3*e*h*p*q + 4
*b^3*f*g*p*q))/(4*f^2) + (3*b^2*h*x^2*(2*a - b*p*q))/4) + log(c*(d*(e + f*
x)^p)^q)^3*((b^3*h*x^2)/2 - (e*(b^3*e*h - 2*b^3*f*g))/(2*f^2) + b^3*g*x) +
 (log(c*(d*(e + f*x)^p)^q)*(x^2*(6*a^2*b*f*g + (3*b*e*h*(2*a^2 + b^2*p^2*q
^2 - 2*a*b*p*q))/2 - 9*b^3*e*h*p^2*q^2 + 12*b^3*f*g*p^2*q^2 + 6*a*b^2*e*h*
p*q - 12*a*b^2*f*g*p*q) + (3*e*x*(2*a^2*b*f*g - 3*b^3*e*h*p^2*q^2 + 4*b^3*
f*g*p^2*q^2 + 2*a*b^2*e*h*p*q - 4*a*b^2*f*g*p*q))/f + (3*b*f*h*x^3*(2*a^2
+ b^2*p^2*q^2 - 2*a*b*p*q))/2))/(2*e + 2*f*x) + (h*x^2*(4*a^3 - 3*b^3*p^3*
q^3 + 6*a*b^2*p^2*q^2 - 6*a^2*b*p*q))/8 - (log(e + f*x)*(21*b^3*e^2*h*p^3*
q^3 - 18*a*b^2*e^2*h*p^2*q^2 + 6*a^2*b*e^2*h*p*q - 24*b^3*e*f*g*p^3*q^3 +
24*a*b^2*e*f*g*p^2*q^2 - 12*a^2*b*e*f*g*p*q))/(4*f^2)
```

3.436. $\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$

3.437 $\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$

3.437.1 Optimal result

Integrand size = 20, antiderivative size = 121

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx = 6ab^2p^2q^2x-6b^3p^3q^3x+\frac{6b^3p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{f}-\frac{3bpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f}+\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{f}$$

output
```
6*a*b^2*p^2*q^2*x-6*b^3*p^3*q^3*x+6*b^3*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)
^q)/f-3*b*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f+(f*x+e)*(a+b*ln(c*(d
*(f*x+e)^p)^q))^3/f
```

3.437.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$$
$$=\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3-3bpq\Big((e+fx)(a+b\log(c(d(e+fx)^p)^q))^2-2bpq(f(a-bpq)x-}{f}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 - 3*b*p*q*((e + f*x)*(a + b*
Log[c*(d*(e + f*x)^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)*Log[c
*(d*(e + f*x)^p)^q])))/f
```

3.437.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2895, 2836, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(cd^q(e+fx)^{pq}))^3\,d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^3-3bpq\int (a+b\log(cd^q(e+fx)^{pq}))^2\,d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^3-3bpq\Big((e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq\int (a+b\log(cd^q(e+fx)\ldots}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^3-3bpq\Big((e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq(a(e+fx)+b(e+fx)\mathrm{l}\ldots}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
((e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^3 - 3*b*p*q*((e + f*x)*(a +
b*Log[c*d^q*(e + f*x)^(p*q)])^2 - 2*b*p*q*(a*(e + f*x) - b*p*q*(e + f*x) +
 b*(e + f*x)*Log[c*d^q*(e + f*x)^(p*q)])))/f
```

3.437.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(121) = 242$.

Time = 1.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
parallelrisch	$\frac{6\ln(fx+e)b^3e^2p^3q^3-6x\,b^3ef\,p^3q^3+6x\ln(c(d(fx+e)^p)^q)b^3ef\,p^2q^2+6b^3e^2p^3q^3-6\ln(fx+e)a\,b^2e^2p^2q^2-3x\ln(c(d(fx+e)^p)^q)^2}{}$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3,x,method=_RETURNVERBOSE)
```

output
```
(6*ln(f*x+e)*b^3*e^2*p^3*q^3-6*x*b^3*e*f*p^3*q^3+6*x*ln(c*(d*(f*x+e)^p)^q)
*b^3*e*f*p^2*q^2+6*b^3*e^2*p^3*q^3-6*ln(f*x+e)*a*b^2*e^2*p^2*q^2-3*x*ln(c*
(d*(f*x+e)^p)^q)^2*b^3*e*f*p*q+6*x*a*b^2*e*f*p^2*q^2+x*ln(c*(d*(f*x+e)^p)^
q)^3*b^3*e*f-6*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f*p*q-3*ln(c*(d*(f*x+e)^p)^
q)^2*b^3*e^2*p*q-6*a*b^2*e^2*p^2*q^2+3*ln(f*x+e)*a^2*b*e^2*p*q+3*x*ln(c*(d
*(f*x+e)^p)^q)^2*a*b^2*e*f-3*x*a^2*b*e*f*p*q+ln(c*(d*(f*x+e)^p)^q)^3*b^3*e
^2+3*x*ln(c*(d*(f*x+e)^p)^q)*a^2*b*e*f+3*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*e^2
+3*a^2*b*e^2*p*q+x*a^3*e*f-a^3*e^2)/e/f
```

3.437.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(121) = 242$.

Time = 0.30 (sec) , antiderivative size = 639, normalized size of antiderivative = 5.28

$$\int (a + b\log(c(d(e+fx)^p)^q))^3\,dx$$
$$= \frac{b^3fq^3x\log(d)^3 + b^3fx\log(c)^3 + (b^3fp^3q^3x + b^3ep^3q^3)\log(fx+e)^3 - 3(b^3fpq - ab^2f)x\log(c)^2 - 3(b^3ep}{}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
(b^3*f*q^3*x*log(d)^3 + b^3*f*x*log(c)^3 + (b^3*f*p^3*q^3*x + b^3*e*p^3*q^
3)*log(f*x + e)^3 - 3*(b^3*f*p*q - a*b^2*f)*x*log(c)^2 - 3*(b^3*e*p^3*q^3
- a*b^2*e*p^2*q^2 + (b^3*f*p^3*q^3 - a*b^2*f*p^2*q^2)*x - (b^3*f*p^2*q^2*x
 + b^3*e*p^2*q^2)*log(c) - (b^3*f*p^2*q^3*x + b^3*e*p^2*q^3)*log(d))*log(f
*x + e)^2 + 3*(2*b^3*f*p^2*q^2 - 2*a*b^2*f*p*q + a^2*b*f)*x*log(c) + 3*(b^
3*f*q^2*x*log(c) - (b^3*f*p*q^3 - a*b^2*f*q^2)*x)*log(d)^2 - (6*b^3*f*p^3*
q^3 - 6*a*b^2*f*p^2*q^2 + 3*a^2*b*f*p*q - a^3*f)*x + 3*(2*b^3*e*p^3*q^3 -
2*a*b^2*e*p^2*q^2 + a^2*b*e*p*q + (b^3*f*p*q*x + b^3*e*p*q)*log(c)^2 + (b^
3*f*p*q^3*x + b^3*e*p*q^3)*log(d)^2 + (2*b^3*f*p^3*q^3 - 2*a*b^2*f*p^2*q^2
 + a^2*b*f*p*q)*x - 2*(b^3*e*p^2*q^2 - a*b^2*e*p*q + (b^3*f*p^2*q^2 - a*b^
2*f*p*q)*x)*log(c) - 2*(b^3*e*p^2*q^3 - a*b^2*e*p*q^2 + (b^3*f*p^2*q^3 - a
*b^2*f*p*q^2)*x - (b^3*f*p*q^2*x + b^3*e*p*q^2)*log(c))*log(d))*log(f*x +
e) + 3*(b^3*f*q*x*log(c)^2 - 2*(b^3*f*p*q^2 - a*b^2*f*q)*x*log(c) + (2*b^3
*f*p^2*q^3 - 2*a*b^2*f*p*q^2 + a^2*b*f*q)*x)*log(d))/f
```

3.437.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(117) = 234$.

Time = 1.02 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.98

$$\int (a + b\log (c(d(e + fx)^p)^q))^3 \, dx$$
$$= \begin{cases} a^3x + \frac{3a^2be\log (c(d(e+fx)^p)^q)}{f} - 3a^2bpqx + 3a^2bx\log (c(d(e + fx)^p)^q) - \frac{6ab^2epq\log (c(d(e+fx)^p)^q)}{f} + \frac{3ab^2e\log (c(d(e\cdot}{f} \\ x(a + b\log (c(de^p)^q))^3 \end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*e*log(c*(d*(e + f*x)**p)**q)/f - 3*a**2*b*p*q
*x + 3*a**2*b*x*log(c*(d*(e + f*x)**p)**q) - 6*a*b**2*e*p*q*log(c*(d*(e +
f*x)**p)**q)/f + 3*a*b**2*e*log(c*(d*(e + f*x)**p)**q)**2/f + 6*a*b**2*p**
2*q**2*x - 6*a*b**2*p*q*x*log(c*(d*(e + f*x)**p)**q) + 3*a*b**2*x*log(c*(d
*(e + f*x)**p)**q)**2 + 6*b**3*e*p**2*q**2*log(c*(d*(e + f*x)**p)**q)/f -
3*b**3*e*p*q*log(c*(d*(e + f*x)**p)**q)**2/f + b**3*e*log(c*(d*(e + f*x)**
p)**q)**3/f - 6*b**3*p**3*q**3*x + 6*b**3*p**2*q**2*x*log(c*(d*(e + f*x)**
p)**q) - 3*b**3*p*q*x*log(c*(d*(e + f*x)**p)**q)**2 + b**3*x*log(c*(d*(e +
 f*x)**p)**q)**3, Ne(f, 0)), (x*(a + b*log(c*(d*e**p)**q))**3, True))
```

3.437.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(121) = 242$.

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.62

$$\int (a + b\log (c(d(e + fx)^p)^q))^3 \, dx = -3\,a^2bfpq\left(\frac{x}{f} - \frac{e\log (fx + e)}{f^2}\right)$$
$$+ b^3x\log \left(\left((fx + e)^pd\right)^q c\right)^3 + 3\,ab^2x\log \left(\left((fx + e)^pd\right)^q c\right)^2 + 3\,a^2bx\log \left(\left((fx + e)^pd\right)^q c\right)$$
$$-3\left(2\,fpq\left(\frac{x}{f} - \frac{e\log (fx + e)}{f^2}\right)\log \left(\left((fx + e)^pd\right)^q c\right) + \frac{(e\log (fx + e)^2 - 2\,fx + 2\,e\log (fx + e))p^2q^2}{f}\right)$$
$$-\left(3\,fpq\left(\frac{x}{f} - \frac{e\log (fx + e)}{f^2}\right)\log \left(\left((fx + e)^pd\right)^q c\right)^2 - \left(\frac{(e\log (fx + e)^3 + 3\,e\log (fx + e)^2 - 6\,fx + 6\,\cdots}{f^2}\right.\right.$$
$$+ a^3x$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

output
```
-3*a^2*b*f*p*q*(x/f - e*log(f*x + e)/f^2) + b^3*x*log(((f*x + e)^p*d)^q*c)
^3 + 3*a*b^2*x*log(((f*x + e)^p*d)^q*c)^2 + 3*a^2*b*x*log(((f*x + e)^p*d)^
q*c) - 3*(2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e
*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*a*b^2 - (3*f*p*q*(x
/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^2 - ((e*log(f*x + e)^3 +
 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p^2*q^2/f^2 - 3*(e*log(f*x
 + e)^2 - 2*f*x + 2*e*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^2)*f*p*
q)*b^3 + a^3*x
```

3.437.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 772, normalized size of antiderivative = 6.38

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx = \frac{(fx+e)b^3p^3q^3\log(fx+e)^3}{f} - \frac{3(fx+e)b^3p^3q^3\log(fx+e)^2}{f} + \frac{3(fx+e)b^3p^2q^3\log(fx+e)^2\log(d)}{f} + \frac{6(fx+e)b^3p^3q^3\log(fx+e)}{f} + \frac{3(fx+e)b^3p^2q^2\log(fx+e)^2\log(c)}{f} - \frac{6(fx+e)b^3p^2q^3\log(fx+e)\log(d)}{f} + \frac{3(fx+e)b^3pq^3\log(fx+e)\log(d)^2}{f} - \frac{6(fx+e)b^3p^3q^3}{f} + \frac{3(fx+e)ab^2p^2q^2\log(fx+e)^2}{f} - \frac{6(fx+e)b^3p^2q^2\log(fx+e)\log(c)}{f} + \frac{6(fx+e)b^3p^2q^3\log(d)}{f} + \frac{6(fx+e)b^3pq^2\log(fx+e)\log(c)\log(d)}{f} - \frac{3(fx+e)b^3pq^3\log(d)^2}{f} + \frac{(fx+e)b^3q^3\log(d)^3}{f} - \frac{6(fx+e)ab^2p^2q^2\log(fx+e)}{f} + \frac{6(fx+e)b^3p^2q^2\log(c)}{f} + \frac{3(fx+e)b^3pq\log(fx+e)\log(c)^2}{f} + \frac{6(fx+e)ab^2pq^2\log(fx+e)\log(d)}{f} - \frac{6(fx+e)b^3pq^2\log(c)\log(d)}{f} + \frac{3(fx+e)b^3q^2\log(c)\log(d)^2}{f} + \frac{6(fx+e)ab^2p^2q^2}{f} + \frac{6(fx+e)ab^2pq\log(fx+e)\log(c)}{f} - \frac{3(fx+e)b^3pq\log(c)^2}{f} - \frac{6(fx+e)ab^2pq^2\log(d)}{f} + \frac{3(fx+e)b^3q\log(c)^2\log(d)}{f} + \frac{3(fx+e)ab^2q^2\log(d)^2}{}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
(f*x + e)*b^3*p^3*q^3*log(f*x + e)^3/f - 3*(f*x + e)*b^3*p^3*q^3*log(f*x +
 e)^2/f + 3*(f*x + e)*b^3*p^2*q^3*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^
3*p^3*q^3*log(f*x + e)/f + 3*(f*x + e)*b^3*p^2*q^2*log(f*x + e)^2*log(c)/f
 - 6*(f*x + e)*b^3*p^2*q^3*log(f*x + e)*log(d)/f + 3*(f*x + e)*b^3*p*q^3*l
og(f*x + e)*log(d)^2/f - 6*(f*x + e)*b^3*p^3*q^3/f + 3*(f*x + e)*a*b^2*p^2
*q^2*log(f*x + e)^2/f - 6*(f*x + e)*b^3*p^2*q^2*log(f*x + e)*log(c)/f + 6*
(f*x + e)*b^3*p^2*q^3*log(d)/f + 6*(f*x + e)*b^3*p*q^2*log(f*x + e)*log(c)
*log(d)/f - 3*(f*x + e)*b^3*p*q^3*log(d)^2/f + (f*x + e)*b^3*q^3*log(d)^3/
f - 6*(f*x + e)*a*b^2*p^2*q^2*log(f*x + e)/f + 6*(f*x + e)*b^3*p^2*q^2*log
(c)/f + 3*(f*x + e)*b^3*p*q*log(f*x + e)*log(c)^2/f + 6*(f*x + e)*a*b^2*p*
q^2*log(f*x + e)*log(d)/f - 6*(f*x + e)*b^3*p*q^2*log(c)*log(d)/f + 3*(f*x
 + e)*b^3*q^2*log(c)*log(d)^2/f + 6*(f*x + e)*a*b^2*p^2*q^2/f + 6*(f*x + e
)*a*b^2*p*q*log(f*x + e)*log(c)/f - 3*(f*x + e)*b^3*p*q*log(c)^2/f - 6*(f*
x + e)*a*b^2*p*q^2*log(d)/f + 3*(f*x + e)*b^3*q*log(c)^2*log(d)/f + 3*(f*x
 + e)*a*b^2*q^2*log(d)^2/f + 3*(f*x + e)*a^2*b*p*q*log(f*x + e)/f - 6*(f*x
 + e)*a*b^2*p*q*log(c)/f + (f*x + e)*b^3*log(c)^3/f + 6*(f*x + e)*a*b^2*q*
log(c)*log(d)/f - 3*(f*x + e)*a^2*b*p*q/f + 3*(f*x + e)*a*b^2*log(c)^2/f +
 3*(f*x + e)*a^2*b*q*log(d)/f + 3*(f*x + e)*a^2*b*log(c)/f + (f*x + e)*a^3
/f
```

3.437.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.00

$$\int (a+b\log (c(d(e+fx)^p)^q))^3\,dx = x\left(a^3-3\,a^2\,b\,p\,q+6\,a\,b^2\,p^2\,q^2-6\,b^3\,p^3\,q^3\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2\left(\frac{3\left(a\,b^2\,e-b^3\,e\,p\,q\right)}{f}+3\,b^2\,x\left(a-b\,p\,q\right)\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^3\left(b^3\,x+\frac{b^3\,e}{f}\right)$$
$$+\frac{\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(3\,b\,f\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)\,x^2+3\,b\,e\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)\,x\right)}{e+f\,x}$$
$$+\frac{\ln\left(e+f\,x\right)\left(3\,e\,a^2\,b\,p\,q-6\,e\,a\,b^2\,p^2\,q^2+6\,e\,b^3\,p^3\,q^3\right)}{f}$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

output
```
x*(a^3 - 6*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 3*a^2*b*p*q) + log(c*(d*(e + f*
x)^p)^q)^2*((3*(a*b^2*e - b^3*e*p*q))/f + 3*b^2*x*(a - b*p*q)) + log(c*(d*
(e + f*x)^p)^q)^3*(b^3*x + (b^3*e)/f) + (log(c*(d*(e + f*x)^p)^q)*(3*b*e*x
*(a^2 + 2*b^2*p^2*q^2 - 2*a*b*p*q) + 3*b*f*x^2*(a^2 + 2*b^2*p^2*q^2 - 2*a*
b*p*q)))/(e + f*x) + (log(e + f*x)*(6*b^3*e*p^3*q^3 - 6*a*b^2*e*p^2*q^2 +
3*a^2*b*e*p*q))/f
```

3.437. $\int (a+b\log (c(d(e+fx)^p)^q))^3 \, dx$

3.438 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

3.438.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h+3*b*p*q*(a+b*ln(c
*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-6*b^2*p^2*q^2*(a+b
*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h+6*b^3*p^3*q^3*p
olylog(4,-h*(f*x+e)/(-e*h+f*g))/h
```

3.438.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 646 vs. 2(177) = 354.

Time = 0.23 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.65

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$
$$= \frac{a^3\log(g+hx)-3a^2bpq\log(e+fx)\log(g+hx)+3ab^2p^2q^2\log^2(e+fx)\log(g+hx)-b^3p^3q^3\log^3(e+f\cdots}{\cdots}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

output
```
(a^3*Log[g + h*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*p^2*q^
2*Log[e + f*x]^2*Log[g + h*x] - b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] +
3*a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*p*q*Log[e + f*x]*L
og[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*
(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g
 + h*x] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] +
 b^3*Log[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] + 3*a^2*b*p*q*Log[e + f*x]*Lo
g[(f*(g + h*x))/(f*g - e*h)] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g +
h*x))/(f*g - e*h)] + b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e
*h)] + 6*a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))
/(f*g - e*h)] - 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^
q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)
^q])^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 6*b^2*p^2*q^2*(a + b*Log
[c*(d*(e + f*x)^p)^q])*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] + 6*b^3*p^
3*q^3*PolyLog[4, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

3.438.3 Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2895, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}dx$$

$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bfpq\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))\right.}{h}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\left(\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\int\frac{\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)\right)-\operatorname{PolyLog}\right.}{h}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\left(\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{h(e+}{fg-}\right.\right.}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

3.438. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/h - (3
*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])^2*PolyLog[2, -((h*(e + f*x))/
(f*g - e*h))]) + 2*b*p*q*((a + b*Log[c*d^q*(e + f*x)^(p*q)])*PolyLog[3, -(
(h*(e + f*x))/(f*g - e*h))] - b*p*q*PolyLog[4, -((h*(e + f*x))/(f*g - e*h)
)])))/h
```

3.438.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.438. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.438.4 Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^3}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

3.438.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^3}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c
)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*x + g), x)
```

3.438.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x), x)
```

3.438.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="maxima")
```

output
```
a^3*log(h*x + g)/h + integrate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) +
 log(c))*a^2*b + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (
q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3
 + 3*((q*log(d) + log(c))*b^3 + a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*lo
g(d) + log(c))*a*b^2 + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 +
 a^2*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.438.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^3}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x), x)
```

3.439 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$

3.439.1 Optimal result

Integrand size = 28, antiderivative size = 209

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{(fg-eh)(g+hx)} - \frac{3bfpq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{6b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \frac{6b^3fp^3q^3\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

output
```
(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(-e*h+f*g)/(h*x+g)-3*b*f*p*q*(a+b*ln
(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)-6*b^2*f*p^2*q
^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*
g)+6*b^3*f*p^3*q^3*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g)
```

3.439.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 444 vs. 2(209) = 418.

Time = 0.34 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$$
$$= \frac{-3b(fg-eh)pq\log(e+fx)\left(a-bpq\log(e+fx)+b\log(c(d(e+fx)^p)^q)\right)^2+3bfpq(g+hx)\log(e+fx)}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^2,x]
```

output
```
(-3*b*(f*g - e*h)*p*q*Log[e + f*x]*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e
 + f*x)^p)^q])^2 + 3*b*f*p*q*(g + h*x)*Log[e + f*x]*(a - b*p*q*Log[e + f*x
] + b*Log[c*(d*(e + f*x)^p)^q])^2 - (f*g - e*h)*(a - b*p*q*Log[e + f*x] +
b*Log[c*(d*(e + f*x)^p)^q])^3 - 3*b*f*p*q*(g + h*x)*(a - b*p*q*Log[e + f*x
] + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[g + h*x] + 3*b^2*p^2*q^2*(a - b*p*q*
Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*(Log[e + f*x]*(h*(e + f*x)*Log[
e + f*x] - 2*f*(g + h*x)*Log[(f*(g + h*x))/(f*g - e*h)]) - 2*f*(g + h*x)*P
olyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) + b^3*p^3*q^3*(Log[e + f*x]^2*(h*
(e + f*x)*Log[e + f*x] - 3*f*(g + h*x)*Log[(f*(g + h*x))/(f*g - e*h)]) - 6
*f*(g + h*x)*Log[e + f*x]*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] + 6*f*(
g + h*x)*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)]))/(h*(f*g - e*h)*(g + h*
x))
```

3.439.3 Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2895, 2844, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2844}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(fg-eh)}-\frac{3bfpq\int\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}dx}{fg-eh}$$

$$\Big\downarrow \text{2843}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(fg-eh)}-\frac{3bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bfpq\int\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}\right)}{fg-eh}$$

$$\Big\downarrow \text{2881}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(fg-eh)}-\frac{3bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bpq\int\frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}\right)}{fg-eh}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(fg-eh)}-\frac{3bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bpq\left(bpq\int\frac{\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right.\right.\right.}{h}}{fg-eh}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(fg-eh)}-\frac{3bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bpq\left(bpq\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\right)}{h}\right)}{fg-eh}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^2,x]
```

3.439. $\int\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)^2}dx$

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/((f*g - e*h)*(g + h*x)) - (
3*b*f*p*q*(((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*
h)])/h - (2*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])*PolyLog[2, -((h*(e
 + f*x))/(f*g - e*h))]) + b*p*q*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))]))
/h))/(f*g - e*h)
```

3.439.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.439. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.439.4 Maple [F]

$$\int \frac{(a + b \ln (c(d(fx+e)^p)^q))^3}{(hx+g)^2} dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x)
```

3.439.5 Fricas [F]

$$\int \frac{(a + b \log (c(d(e+fx)^p)^q))^3}{(g+hx)^2} dx = \int \frac{(b \log (((fx+e)^p d)^q c) + a)^3}{(hx+g)^2} dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c
)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h^2*x^2 + 2*g*h*x + g^2), x
)
```

3.439.6 Sympy [F]

$$\int \frac{(a + b \log (c(d(e+fx)^p)^q))^3}{(g+hx)^2} dx = \int \frac{(a + b \log (c(d(e+fx)^p)^q))^3}{(g+hx)^2} dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x)**2, x)
```

3.439.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x, algorithm="maxima")
```

output
```
3*a^2*b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2)
) - b^3*log(((f*x + e)^p)^q)^3/(h^2*x + g*h) - 3*a^2*b*log(((f*x + e)^p*d)
^q*c)/(h^2*x + g*h) - a^3/(h^2*x + g*h) + integrate((3*(e*h*q^2*log(d)^2 +
 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*a*b^2 + (e*h*q^3*log(d)^3 + 3*e*h*q
^2*log(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*b^3 + 3*(a*b^
2*e*h + (f*g*p*q + e*h*q*log(d) + e*h*log(c))*b^3 + (a*b^2*f*h + (f*h*p*q
+ f*h*q*log(d) + f*h*log(c))*b^3)*x)*log(((f*x + e)^p)^q)^2 + (3*(f*h*q^2*
log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*a*b^2 + (f*h*q^3*log(d)^3
 + 3*f*h*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2*log(d) + f*h*log(c)^3)*b^3
)*x + 3*(2*(e*h*q*log(d) + e*h*log(c))*a*b^2 + (e*h*q^2*log(d)^2 + 2*e*h*q
*log(c)*log(d) + e*h*log(c)^2)*b^3 + (2*(f*h*q*log(d) + f*h*log(c))*a*b^2
+ (f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*b^3)*x)*log(((
f*x + e)^p)^q))/(f*h^3*x^3 + e*g^2*h + (2*f*g*h^2 + e*h^3)*x^2 + (f*g^2*h
+ 2*e*g*h^2)*x), x)
```

3.439.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g)^2, x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^3}{\left(g+h\,x\right)^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^2,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^2, x)
```

3.440 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$

3.440.1 Optimal result

Integrand size = 28, antiderivative size = 376

$$\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx \\
&= -\frac{3bfpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{2(fg-eh)^2(g+hx)} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2} \\
&\quad + \frac{3b^2f^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)^2} \\
&\quad - \frac{3bf^2pq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{2h(fg-eh)^2} \\
&\quad + \frac{3b^2f^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} \\
&\quad + \frac{3b^3f^2p^3q^3\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)^2} + \frac{3b^3f^2p^3q^3\operatorname{PolyLog}\left(3,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}
\end{aligned}$$

output
```
-3/2*b*f*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(-e*h+f*g)^2/(h*x+g)-1/
2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/h/(h*x+g)^2+3*b^2*f^2*p^2*q^2*(a+b*ln(c*(d
*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)^2-3/2*b*f^2*p*q*(a+b
*ln(c*(d*(f*x+e)^p)^q))^2*ln(1+(-e*h+f*g)/h/(f*x+e))/h/(-e*h+f*g)^2+3*b^2*
f^2*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,(e*h-f*g)/h/(f*x+e))/h/(
-e*h+f*g)^2+3*b^3*f^2*p^3*q^3*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g
)^2+3*b^3*f^2*p^3*q^3*polylog(3,(e*h-f*g)/h/(f*x+e))/h/(-e*h+f*g)^2
```

3.440.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx =$$
$$-\frac{-3bf(fg-eh)pq(g+hx)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)^2+3b(fg-eh)^2pq\log(e+fx}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^3,x]
```

output
```
-1/2*(-3*b*f*(f*g - e*h)*p*q*(g + h*x)*(a - b*p*q*Log[e + f*x] + b*Log[c*(
d*(e + f*x)^p)^q])^2 + 3*b*(f*g - e*h)^2*p*q*Log[e + f*x]*(a - b*p*q*Log[e
 + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2 - 3*b*f^2*p*q*(g + h*x)^2*Log[e +
f*x]*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2 + (f*g - e*h)
^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^3 + 3*b*f^2*p*q*(
g + h*x)^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[g +
 h*x] + 3*b^2*p^2*q^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q]
)*(h*(e + f*x)*(e*h - f*(2*g + h*x))*Log[e + f*x]^2 - 2*f^2*(g + h*x)^2*Lo
g[(f*(g + h*x))/(f*g - e*h)] + 2*f*(g + h*x)*Log[e + f*x]*(h*(e + f*x) + f
*(g + h*x)*Log[(f*(g + h*x))/(f*g - e*h)]) + 2*f^2*(g + h*x)^2*PolyLog[2,
(h*(e + f*x))/(-(f*g) + e*h)]) + b^3*p^3*q^3*(h*(e + f*x)*(e*h - f*(2*g +
h*x))*Log[e + f*x]^3 + 3*f*(g + h*x)*Log[e + f*x]^2*(h*(e + f*x) + f*(g +
h*x)*Log[(f*(g + h*x))/(f*g - e*h)]) - 6*f^2*(g + h*x)^2*Log[e + f*x]*(Log
[(f*(g + h*x))/(f*g - e*h)] - PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) -
6*f^2*(g + h*x)^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 6*f^2*(g + h*
x)^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)]))/(h*(f*g - e*h)^2*(g + h*x)
^2)
```

3.440.3 Rubi [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2895, 2845, 2858, 27, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{3bfpq\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(e+fx)(g+hx)^2}dx}{2h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2858}$$

$$\frac{3bpq\int \frac{f^2(a+b\log(cd^q(e+fx)^{pq}))^2}{(e+fx)\left(f\left(g-\frac{eh}{f}\right)+h(e+fx)\right)^2}d(e+fx)}{2h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3bf^2pq\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(e+fx)(fg-eh+h(e+fx))^2}d(e+fx)}{2h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2789}$$

$$\frac{3bf^2pq\left(\frac{\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(e+fx)(fg-eh+h(e+fx))}d(e+fx)}{fg-eh}-\frac{h\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(fg-eh+h(e+fx))^2}d(e+fx)}{fg-eh}\right)}{2h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2755}$$

$$\frac{3bf^2pq\left(\frac{\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(e+fx)(fg-eh+h(e+fx))}d(e+fx)}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2}{(fg-eh)(h(e+fx)-eh+fg)}-\frac{2bpq\int \frac{a+b\log(cd^q(e+fx)^{pq})}{fg-eh+h(e+fx)}d(e+fx)}{fg-eh}\right)}{fg-eh}\right)}{2h}-\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2754}$$

3.440. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$

$$\frac{3bf^2pq\left(\frac{\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(e+fx)(fg-eh+h(e+fx))}d(e+fx)}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2}{(fg-eh)(h(e+fx)-eh+fg)}-\frac{2bpq\left(\frac{\log\left(\frac{h(e+fx)}{fg-eh}+1\right)(a+b\log(cd^q(e+fx)^{pq}))}{h}-\frac{bpq\int \frac{\log\left(\frac{h(e\cdots}{fg\cdots}\right)}{e-\cdots}\cdots}{\cdots}\right)}{fg-eh}\cdots\right)}{fg-eh}\cdots\right)}{2h}\cdots$$

$$\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$\downarrow$ 2779

$$\frac{3bf^2pq\left(\frac{\frac{2bpq\int \frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{fg-eh}{h(e+fx)}+1\right)}{e+fx}d(e+fx)}{fg-eh}-\frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(cd^q(e+fx)^{pq}))^2}{fg-eh}}{fg-eh}-\frac{h\left(\frac{(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^2}{(fg-eh)(h(e+fx)-eh+fg)}\cdots\right)}{\cdots}\right)}{2h}\cdots$$

$$\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$\downarrow$ 2821

3.440. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$

$$\frac{3bf^2pq\left(\frac{\frac{2bpq\left(\text{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\int\frac{\text{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fg-eh}-\frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(cd^q(e+fx)^{pq}))^2}{fg-eh}}{fg-eh}-\right.}{2h}$$

$$\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$\downarrow$ 2838

$$\frac{3bf^2pq\left(\frac{\frac{2bpq\left(\text{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\int\frac{\text{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{e+fx}d(e+fx)\right)}{fg-eh}-\frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(cd^q(e+fx)^{pq}))^2}{fg-eh}}{fg-eh}-\right.}{2h}$$

$$\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

$\downarrow$ 7143

$$\frac{3bf^2pq\left(\frac{\frac{2bpq\left(\text{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)(a+b\log(cd^q(e+fx)^{pq}))+bpq\,\text{PolyLog}\left(3,-\frac{fg-eh}{h(e+fx)}\right)\right)}{fg-eh}-\frac{\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(cd^q(e+fx)^{pq}))^2}{fg-eh}}{fg-eh}-\frac{h\left(\frac{(e+fx)}{(fg-}\right.}{}\right.}{2h}$$

$$\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2}$$

3.440. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^3,x]
```

output
```
-1/2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(h*(g + h*x)^2) + (3*b*f^2*p*q*(-(
(h*(((e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^2)/((f*g - e*h)*(f*g - e
*h + h*(e + f*x))) - (2*b*p*q*(((a + b*Log[c*d^q*(e + f*x)^(p*q)])*Log[1 +
 (h*(e + f*x))/(f*g - e*h)])/h + (b*p*q*PolyLog[2, -((h*(e + f*x))/(f*g -
e*h))])/h))/(f*g - e*h)))/(f*g - e*h)) + (-(((a + b*Log[c*d^q*(e + f*x)^(p
*q)])^2*Log[1 + (f*g - e*h)/(h*(e + f*x))])/(f*g - e*h)) + (2*b*p*q*((a +
b*Log[c*d^q*(e + f*x)^(p*q)])*PolyLog[2, -((f*g - e*h)/(h*(e + f*x)))] + b
*p*q*PolyLog[3, -((f*g - e*h)/(h*(e + f*x)))]))/(f*g - e*h))/(f*g - e*h)))
/(2*h)
```

3.440.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.440. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$

3.440.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}{\left(hx+g\right)^{3}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x)
```

3.440.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{3}}{\left(hx+g\right)^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c
)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g
^2*h*x + g^3), x)
```

3.440.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx = \int \frac{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{3}}{\left(g+hx\right)^{3}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)**3,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x)**3, x)
```

3.440.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx=\int \frac{(b\log(((fx+e)^pd)^qc)+a)^3}{(hx+g)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x, algorithm="maxima")
```

output
```
3/2*a^2*b*f*p*q*(f*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - f*lo
g(h*x + g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) + 1/(f*g^2*h - e*g*h^2 + (f
*g*h^2 - e*h^3)*x)) - 1/2*b^3*log(((f*x + e)^p)^q)^3/(h^3*x^2 + 2*g*h^2*x
+ g^2*h) - 3/2*a^2*b*log(((f*x + e)^p*d)^q*c)/(h^3*x^2 + 2*g*h^2*x + g^2*h
) - 1/2*a^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(6*(e*h*q^2*log(
d)^2 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*a*b^2 + 2*(e*h*q^3*log(d)^3 +
 3*e*h*q^2*log(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*b^3 +
 3*(2*a*b^2*e*h + (f*g*p*q + 2*e*h*q*log(d) + 2*e*h*log(c))*b^3 + (2*a*b^2
*f*h + (f*h*p*q + 2*f*h*q*log(d) + 2*f*h*log(c))*b^3)*x)*log(((f*x + e)^p)
^q)^2 + 2*(3*(f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*a*b
^2 + (f*h*q^3*log(d)^3 + 3*f*h*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2*log(
d) + f*h*log(c)^3)*b^3)*x + 6*(2*(e*h*q*log(d) + e*h*log(c))*a*b^2 + (e*h*
q^2*log(d)^2 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*b^3 + (2*(f*h*q*log(d
) + f*h*log(c))*a*b^2 + (f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*lo
g(c)^2)*b^3)*x)*log(((f*x + e)^p)^q))/(f*h^4*x^4 + e*g^3*h + (3*f*g*h^3 +
e*h^4)*x^3 + 3*(f*g^2*h^2 + e*g*h^3)*x^2 + (f*g^3*h + 3*e*g^2*h^2)*x), x)
```

3.440.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx=\int \frac{(b\log(((fx+e)^pd)^qc)+a)^3}{(hx+g)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g)^3, x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx=\int \frac{{\left(a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^3}{{\left(g+h\,x\right)}^3}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^3,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^3, x)
```

3.441 $\int (a+b\log (c(d(e+fx)^p)^q))^4\,dx$

3.441.1 Optimal result

Integrand size = 20, antiderivative size = 160

$$\begin{aligned}\int (a+b\log (c(d(e+fx)^p)^q))^4\,dx =& -24ab^3p^3q^3x+24b^4p^4q^4x \\ &-\frac{24b^4p^3q^3(e+fx)\log (c(d(e+fx)^p)^q)}{f} \\ &+\frac{12b^2p^2q^2(e+fx)\,(a+b\log (c(d(e+fx)^p)^q))^2}{f} \\ &-\frac{4bpq(e+fx)\,(a+b\log (c(d(e+fx)^p)^q))^3}{f} \\ &+\frac{(e+fx)\,(a+b\log (c(d(e+fx)^p)^q))^4}{f}\end{aligned}$$

output
```
-24*a*b^3*p^3*q^3*x+24*b^4*p^4*q^4*x-24*b^4*p^3*q^3*(f*x+e)*ln(c*(d*(f*x+e
)^p)^q)/f+12*b^2*p^2*q^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f-4*b*p*q*(
f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))
^4/f
```

3.441.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int (a+b\log (c(d(e+fx)^p)^q))^4 \, dx$$
$$= \frac{(e+fx)\,(a+b\log (c(d(e+fx)^p)^q))^4-4bpq\Big((e+fx)\,(a+b\log (c(d(e+fx)^p)^q))^3-3bpq\Big((e+fx)\,(a+}{f}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^4,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^4 - 4*b*p*q*((e + f*x)*(a + b*
Log[c*(d*(e + f*x)^p)^q])^3 - 3*b*p*q*((e + f*x)*(a + b*Log[c*(d*(e + f*x)
^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q
]))))/f
```

3.441.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2895, 2836, 2733, 2733, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d(e+fx)^p)^q))^4 \, dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int (a+b\log (c(d(e+fx)^p)^q))^4 dx$$
$$\Big\downarrow \text{ 2836}$$
$$\frac{\int (a+b\log (cd^q(e+fx)^{pq}))^4 d(e+fx)}{f}$$
$$\Big\downarrow \text{ 2733}$$
$$\frac{(e+fx)\,(a+b\log (cd^q(e+fx)^{pq}))^4-4bpq\int (a+b\log (cd^q(e+fx)^{pq}))^3 d(e+fx)}{f}$$
$$\Big\downarrow \text{ 2733}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^4-4bpq\Big((e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3-3bpq\int\left(a+b\log\left(cd^q(e+fx)\right.\right.}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^4-4bpq\Big((e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3-3bpq\Big((e+fx)\left(a+b\log\left(cd^q\right.\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^4-4bpq\Big((e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3-3bpq\Big((e+fx)\left(a+b\log\left(cd^q\right.\right.}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^4,x]
```

output
```
((e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^4 - 4*b*p*q*((e + f*x)*(a +
b*Log[c*d^q*(e + f*x)^(p*q)])^3 - 3*b*p*q*((e + f*x)*(a + b*Log[c*d^q*(e +
 f*x)^(p*q)])^2 - 2*b*p*q*(a*(e + f*x) - b*p*q*(e + f*x) + b*(e + f*x)*Log
[c*d^q*(e + f*x)^(p*q)]))))/f
```

3.441.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.441.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(160) = 320$.

Time = 5.17 (sec) , antiderivative size = 642, normalized size of antiderivative = 4.01

method	result
parallelrisch	$\frac{24x\,b^4ef\,p^4q^4-12\ln(c(d(fx+e)^p)^q)^2a\,b^3e^2pq+24\ln(fx+e)a\,b^3e^2p^3q^3-24b^4e^2p^4q^4+12\ln(c(d(fx+e)^p)^q)^2b^4e^2p^2q^2-4\ln(c(d(}{}$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^4,x,method=_RETURNVERBOSE)
```

output
```
(24*x*b^4*e*f*p^4*q^4-12*ln(c*(d*(f*x+e)^p)^q)^2*a*b^3*e^2*p*q+24*ln(f*x+e
)*a*b^3*e^2*p^3*q^3-24*b^4*e^2*p^4*q^4+12*ln(c*(d*(f*x+e)^p)^q)^2*b^4*e^2*
p^2*q^2-4*ln(c*(d*(f*x+e)^p)^q)^3*b^4*e^2*p*q-24*ln(f*x+e)*b^4*e^2*p^4*q^4
+x*ln(c*(d*(f*x+e)^p)^q)^4*b^4*e*f+24*x*ln(c*(d*(f*x+e)^p)^q)*a*b^3*e*f*p^
2*q^2+ln(c*(d*(f*x+e)^p)^q)^4*b^4*e^2-12*x*ln(c*(d*(f*x+e)^p)^q)^2*a*b^3*e
*f*p*q-12*x*ln(c*(d*(f*x+e)^p)^q)*a^2*b^2*e*f*p*q-a^4*e^2+4*ln(c*(d*(f*x+e
)^p)^q)^3*a*b^3*e^2+6*ln(c*(d*(f*x+e)^p)^q)^2*a^2*b^2*e^2+x*a^4*e*f+24*a*b
^3*e^2*p^3*q^3-12*a^2*b^2*e^2*p^2*q^2+4*a^3*b*e^2*p*q-24*x*ln(c*(d*(f*x+e)
^p)^q)*b^4*e*f*p^3*q^3+12*x*ln(c*(d*(f*x+e)^p)^q)^2*b^4*e*f*p^2*q^2-24*x*a
*b^3*e*f*p^3*q^3-4*x*ln(c*(d*(f*x+e)^p)^q)^3*b^4*e*f*p*q+12*x*a^2*b^2*e*f*
p^2*q^2-4*x*a^3*b*e*f*p*q-12*ln(f*x+e)*a^2*b^2*e^2*p^2*q^2+4*ln(f*x+e)*a^3
*b*e^2*p*q+4*x*ln(c*(d*(f*x+e)^p)^q)*a^3*b*e*f+4*x*ln(c*(d*(f*x+e)^p)^q)^3
*a*b^3*e*f+6*x*ln(c*(d*(f*x+e)^p)^q)^2*a^2*b^2*e*f)/e/f
```

3.441.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 1409, normalized size of antiderivative = 8.81

$$\int (a + b\log (c(d(e + fx)^p)^q))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4,x, algorithm="fricas")
```

output
```
(b^4*f*q^4*x*log(d)^4 + b^4*f*x*log(c)^4 + (b^4*f*p^4*q^4*x + b^4*e*p^4*q^
4)*log(f*x + e)^4 - 4*(b^4*f*p*q - a*b^3*f)*x*log(c)^3 - 4*(b^4*e*p^4*q^4
- a*b^3*e*p^3*q^3 + (b^4*f*p^4*q^4 - a*b^3*f*p^3*q^3)*x - (b^4*f*p^3*q^3*x
 + b^4*e*p^3*q^3)*log(c) - (b^4*f*p^3*q^4*x + b^4*e*p^3*q^4)*log(d))*log(f
*x + e)^3 + 6*(2*b^4*f*p^2*q^2 - 2*a*b^3*f*p*q + a^2*b^2*f)*x*log(c)^2 + 4
*(b^4*f*q^3*x*log(c) - (b^4*f*p*q^4 - a*b^3*f*q^3)*x)*log(d)^3 + 6*(2*b^4*
e*p^4*q^4 - 2*a*b^3*e*p^3*q^3 + a^2*b^2*e*p^2*q^2 + (b^4*f*p^2*q^2*x + b^4
*e*p^2*q^2)*log(c)^2 + (b^4*f*p^2*q^4*x + b^4*e*p^2*q^4)*log(d)^2 + (2*b^4
*f*p^4*q^4 - 2*a*b^3*f*p^3*q^3 + a^2*b^2*f*p^2*q^2)*x - 2*(b^4*e*p^3*q^3 -
 a*b^3*e*p^2*q^2 + (b^4*f*p^3*q^3 - a*b^3*f*p^2*q^2)*x)*log(c) - 2*(b^4*e*
p^3*q^4 - a*b^3*e*p^2*q^3 + (b^4*f*p^3*q^4 - a*b^3*f*p^2*q^3)*x - (b^4*f*p
^2*q^3*x + b^4*e*p^2*q^3)*log(c))*log(d))*log(f*x + e)^2 - 4*(6*b^4*f*p^3*
q^3 - 6*a*b^3*f*p^2*q^2 + 3*a^2*b^2*f*p*q - a^3*b*f)*x*log(c) + 6*(b^4*f*q
^2*x*log(c)^2 - 2*(b^4*f*p*q^3 - a*b^3*f*q^2)*x*log(c) + (2*b^4*f*p^2*q^4
- 2*a*b^3*f*p*q^3 + a^2*b^2*f*q^2)*x)*log(d)^2 + (24*b^4*f*p^4*q^4 - 24*a*
b^3*f*p^3*q^3 + 12*a^2*b^2*f*p^2*q^2 - 4*a^3*b*f*p*q + a^4*f)*x - 4*(6*b^4
*e*p^4*q^4 - 6*a*b^3*e*p^3*q^3 + 3*a^2*b^2*e*p^2*q^2 - a^3*b*e*p*q - (b^4*
f*p*q*x + b^4*e*p*q)*log(c)^3 - (b^4*f*p*q^4*x + b^4*e*p*q^4)*log(d)^3 + 3
*(b^4*e*p^2*q^2 - a*b^3*e*p*q + (b^4*f*p^2*q^2 - a*b^3*f*p*q)*x)*log(c)^2
+ 3*(b^4*e*p^2*q^4 - a*b^3*e*p*q^3 + (b^4*f*p^2*q^4 - a*b^3*f*p*q^3)*x ...
```

3.441.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(155) = 310$.

Time = 2.07 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.81

$$\int (a + b\log (c(d(e + fx)^p)^q))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3be\log (c(d(e+fx)^p)^q)}{f} - 4a^3bpqx + 4a^3bx\log (c(d(e + fx)^p)^q) - \frac{12a^2b^2epq\log (c(d(e+fx)^p)^q)}{f} + \frac{6a^2b^2e\log (c(\\ x(a + b\log (c(de^p)^q))^4 \end{cases}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*e*log(c*(d*(e + f*x)**p)**q)/f - 4*a**3*b*p*q
*x + 4*a**3*b*x*log(c*(d*(e + f*x)**p)**q) - 12*a**2*b**2*e*p*q*log(c*(d*(
e + f*x)**p)**q)/f + 6*a**2*b**2*e*log(c*(d*(e + f*x)**p)**q)**2/f + 12*a*
*2*b**2*p**2*q**2*x - 12*a**2*b**2*p*q*x*log(c*(d*(e + f*x)**p)**q) + 6*a*
*2*b**2*x*log(c*(d*(e + f*x)**p)**q)**2 + 24*a*b**3*e*p**2*q**2*log(c*(d*(
e + f*x)**p)**q)/f - 12*a*b**3*e*p*q*log(c*(d*(e + f*x)**p)**q)**2/f + 4*a
*b**3*e*log(c*(d*(e + f*x)**p)**q)**3/f - 24*a*b**3*p**3*q**3*x + 24*a*b**
3*p**2*q**2*x*log(c*(d*(e + f*x)**p)**q) - 12*a*b**3*p*q*x*log(c*(d*(e + f
*x)**p)**q)**2 + 4*a*b**3*x*log(c*(d*(e + f*x)**p)**q)**3 - 24*b**4*e*p**3
*q**3*log(c*(d*(e + f*x)**p)**q)/f + 12*b**4*e*p**2*q**2*log(c*(d*(e + f*x
)**p)**q)**2/f - 4*b**4*e*p*q*log(c*(d*(e + f*x)**p)**q)**3/f + b**4*e*log
(c*(d*(e + f*x)**p)**q)**4/f + 24*b**4*p**4*q**4*x - 24*b**4*p**3*q**3*x*l
og(c*(d*(e + f*x)**p)**q) + 12*b**4*p**2*q**2*x*log(c*(d*(e + f*x)**p)**q)
**2 - 4*b**4*p*q*x*log(c*(d*(e + f*x)**p)**q)**3 + b**4*x*log(c*(d*(e + f*
x)**p)**q)**4, Ne(f, 0)), (x*(a + b*log(c*(d*e**p)**q))**4, True))
```

3.441.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(160) = 320$.

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.49

$$\int (a + b\log(c(d(e+fx)^p)^q))^4\,dx$$
$$= b^4x\log\left(\left(\left((fx+e)^pd\right)^q c\right)^4 - 4a^3bfpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right) + 4ab^3x\log\left(\left(\left((fx+e)^pd\right)^q c\right)^3$$
$$+ 6a^2b^2x\log\left(\left(\left((fx+e)^pd\right)^q c\right)^2 + 4a^3bx\log\left(\left(\left((fx+e)^pd\right)^q c\right)$$
$$-6\left(2fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log\left(\left(\left((fx+e)^pd\right)^q c\right) + \frac{\left(e\log(fx+e)^2 - 2fx + 2e\log(fx+e)\right)p^2q^2}{f}\right)$$
$$-4\left(3fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log\left(\left(\left((fx+e)^pd\right)^q c\right)^2 - \left(\left(\frac{\left(e\log(fx+e)^3 + 3e\log(fx+e)^2 - 6fx + \right.}{f^2}\right.\right.\right.$$
$$-\left(4fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log\left(\left(\left((fx+e)^pd\right)^q c\right)^3 + \left(\left(\left(\frac{\left(e\log(fx+e)^4 + 4e\log(fx+e)^3 + 12e\log\right.}{f^3}\right.\right.\right.\right.$$
$$+ a^4x$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(((f*x + e)^p*d)^q*c)^4 - 4*a^3*b*f*p*q*(x/f - e*log(f*x + e)/f^2
) + 4*a*b^3*x*log(((f*x + e)^p*d)^q*c)^3 + 6*a^2*b^2*x*log(((f*x + e)^p*d)
^q*c)^2 + 4*a^3*b*x*log(((f*x + e)^p*d)^q*c) - 6*(2*f*p*q*(x/f - e*log(f*x
 + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(
f*x + e))*p^2*q^2/f)*a^2*b^2 - 4*(3*f*p*q*(x/f - e*log(f*x + e)/f^2)*log((
(f*x + e)^p*d)^q*c)^2 - ((e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x +
6*e*log(f*x + e))*p^2*q^2/f^2 - 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x
+ e))*p*q*log(((f*x + e)^p*d)^q*c)/f^2)*f*p*q)*a*b^3 - (4*f*p*q*(x/f - e*l
og(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^3 + (((e*log(f*x + e)^4 + 4*e*lo
g(f*x + e)^3 + 12*e*log(f*x + e)^2 - 24*f*x + 24*e*log(f*x + e))*p^2*q^2/f
^3 - 4*(e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*
p*q*log(((f*x + e)^p*d)^q*c)/f^3)*f*p*q + 6*(e*log(f*x + e)^2 - 2*f*x + 2*
e*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)^2/f^2)*f*p*q)*b^4 + a^4*x
```

3.441.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(160) = 320$.

Time = 0.36 (sec) , antiderivative size = 1697, normalized size of antiderivative = 10.61

$$\int (a + b\log(c(d(e + fx)^p)^q))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4,x, algorithm="giac")
```

output
```
(f*x + e)*b^4*p^4*q^4*log(f*x + e)^4/f - 4*(f*x + e)*b^4*p^4*q^4*log(f*x +
 e)^3/f + 4*(f*x + e)*b^4*p^3*q^4*log(f*x + e)^3*log(d)/f + 12*(f*x + e)*b
^4*p^4*q^4*log(f*x + e)^2/f + 4*(f*x + e)*b^4*p^3*q^3*log(f*x + e)^3*log(c
)/f - 12*(f*x + e)*b^4*p^3*q^4*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^4*p
^2*q^4*log(f*x + e)^2*log(d)^2/f - 24*(f*x + e)*b^4*p^4*q^4*log(f*x + e)/f
 + 4*(f*x + e)*a*b^3*p^3*q^3*log(f*x + e)^3/f - 12*(f*x + e)*b^4*p^3*q^3*l
og(f*x + e)^2*log(c)/f + 24*(f*x + e)*b^4*p^3*q^4*log(f*x + e)*log(d)/f +
12*(f*x + e)*b^4*p^2*q^3*log(f*x + e)^2*log(c)*log(d)/f - 12*(f*x + e)*b^4
*p^2*q^4*log(f*x + e)*log(d)^2/f + 4*(f*x + e)*b^4*p*q^4*log(f*x + e)*log(
d)^3/f + 24*(f*x + e)*b^4*p^4*q^4/f - 12*(f*x + e)*a*b^3*p^3*q^3*log(f*x +
 e)^2/f + 24*(f*x + e)*b^4*p^3*q^3*log(f*x + e)*log(c)/f + 6*(f*x + e)*b^4
*p^2*q^2*log(f*x + e)^2*log(c)^2/f - 24*(f*x + e)*b^4*p^3*q^4*log(d)/f + 1
2*(f*x + e)*a*b^3*p^2*q^3*log(f*x + e)^2*log(d)/f - 24*(f*x + e)*b^4*p^2*q
^3*log(f*x + e)*log(c)*log(d)/f + 12*(f*x + e)*b^4*p^2*q^4*log(d)^2/f + 12
*(f*x + e)*b^4*p*q^3*log(f*x + e)*log(c)*log(d)^2/f - 4*(f*x + e)*b^4*p*q^
4*log(d)^3/f + (f*x + e)*b^4*q^4*log(d)^4/f + 24*(f*x + e)*a*b^3*p^3*q^3*l
og(f*x + e)/f - 24*(f*x + e)*b^4*p^3*q^3*log(c)/f + 12*(f*x + e)*a*b^3*p^2
*q^2*log(f*x + e)^2*log(c)/f - 12*(f*x + e)*b^4*p^2*q^2*log(f*x + e)*log(c
)^2/f - 24*(f*x + e)*a*b^3*p^2*q^3*log(f*x + e)*log(d)/f + 24*(f*x + e)*b^
4*p^2*q^3*log(c)*log(d)/f + 12*(f*x + e)*b^4*p*q^2*log(f*x + e)*log(c)^...
```

3.441.9 Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.38

$$\int (a+b\log (c(d(e+fx)^p)^q))^4\,dx$$
$$= \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^3\left(\frac{4\left(a\,b^3\,e-b^4\,e\,p\,q\right)}{f}+4\,b^3\,x\left(a-b\,p\,q\right)\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^4\left(b^4\,x+\frac{b^4\,e}{f}\right)$$
$$+x\left(a^4-4\,a^3\,b\,p\,q+12\,a^2\,b^2\,p^2\,q^2-24\,a\,b^3\,p^3\,q^3+24\,b^4\,p^4\,q^4\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2\left(\frac{6\left(e\,a^2\,b^2-2\,e\,a\,b^3\,p\,q+2\,e\,b^4\,p^2\,q^2\right)}{f}+6\,b^2\,x\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)\right)$$
$$+\frac{\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(4\,b\,f\left(a^3-3\,a^2\,b\,p\,q+6\,a\,b^2\,p^2\,q^2-6\,b^3\,p^3\,q^3\right)x^2+4\,b\,e\left(a^3-3\,a^2\,b\,p\,q+6\,a\,b^2\,p^2\,q^2\right.}{e+f\,x}$$
$$-\frac{\ln\left(e+f\,x\right)\left(-4\,e\,a^3\,b\,p\,q+12\,e\,a^2\,b^2\,p^2\,q^2-24\,e\,a\,b^3\,p^3\,q^3+24\,e\,b^4\,p^4\,q^4\right)}{f}$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^4,x)
```

output
```
log(c*(d*(e + f*x)^p)^q)^3*((4*(a*b^3*e - b^4*e*p*q))/f + 4*b^3*x*(a - b*p
*q)) + log(c*(d*(e + f*x)^p)^q)^4*(b^4*x + (b^4*e)/f) + x*(a^4 + 24*b^4*p^
4*q^4 - 24*a*b^3*p^3*q^3 - 4*a^3*b*p*q + 12*a^2*b^2*p^2*q^2) + log(c*(d*(e
 + f*x)^p)^q)^2*((6*(a^2*b^2*e + 2*b^4*e*p^2*q^2 - 2*a*b^3*e*p*q))/f + 6*b
^2*x*(a^2 + 2*b^2*p^2*q^2 - 2*a*b*p*q)) + (log(c*(d*(e + f*x)^p)^q)*(4*b*e
*x*(a^3 - 6*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 3*a^2*b*p*q) + 4*b*f*x^2*(a^3
- 6*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 3*a^2*b*p*q)))/(e + f*x) - (log(e + f*
x)*(24*b^4*e*p^4*q^4 - 24*a*b^3*e*p^3*q^3 - 4*a^3*b*e*p*q + 12*a^2*b^2*e*p
^2*q^2))/f
```

3.441. $\int (a+b\log(c(d(e+fx)^p)^q))^4 \, dx$

3.442 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$

3.442.1 Optimal result

Integrand size = 28, antiderivative size = 231

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx &= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} \\ &+ \frac{4bpq(a+b\log(c(d(e+fx)^p)^q))^3\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} \\ &- \frac{12b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h} \\ &+ \frac{24b^3p^3q^3(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h} \\ &- \frac{24b^4p^4q^4\operatorname{PolyLog}\left(5,-\frac{h(e+fx)}{fg-eh}\right)}{h}\end{aligned}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^4*ln(f*(h*x+g)/(-e*h+f*g))/h+4*b*p*q*(a+b*ln(c
*(d*(f*x+e)^p)^q))^3*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-12*b^2*p^2*q^2*(a+
b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h+24*b^3*p^3*q
^3*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h-24*b^4*p
^4*q^4*polylog(5,-h*(f*x+e)/(-e*h+f*g))/h
```

3.442.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1095 vs. $2(231) = 462$.

Time = 0.33 (sec) , antiderivative size = 1095, normalized size of antiderivative = 4.74

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$
$$= \frac{a^4\log(g+hx) - 4a^3bpq\log(e+fx)\log(g+hx) + 6a^2b^2p^2q^2\log^2(e+fx)\log(g+hx) - 4ab^3p^3q^3\log^3(e-}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x),x]
```

output
```
(a^4*Log[g + h*x] - 4*a^3*b*p*q*Log[e + f*x]*Log[g + h*x] + 6*a^2*b^2*p^2*
q^2*Log[e + f*x]^2*Log[g + h*x] - 4*a*b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h
*x] + b^4*p^4*q^4*Log[e + f*x]^4*Log[g + h*x] + 4*a^3*b*Log[c*(d*(e + f*x)
^p)^q]*Log[g + h*x] - 12*a^2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]
*Log[g + h*x] + 12*a*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*L
og[g + h*x] - 4*b^4*p^3*q^3*Log[e + f*x]^3*Log[c*(d*(e + f*x)^p)^q]*Log[g
 + h*x] + 6*a^2*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] - 12*a*b^3*p*q*
Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 6*b^4*p^2*q^2*Log[e
 + f*x]^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 4*a*b^3*Log[c*(d*(e +
f*x)^p)^q]^3*Log[g + h*x] - 4*b^4*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q
]^3*Log[g + h*x] + b^4*Log[c*(d*(e + f*x)^p)^q]^4*Log[g + h*x] + 4*a^3*b*p
*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)] - 6*a^2*b^2*p^2*q^2*Log[e +
 f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 4*a*b^3*p^3*q^3*Log[e + f*x]^3*Lo
g[(f*(g + h*x))/(f*g - e*h)] - b^4*p^4*q^4*Log[e + f*x]^4*Log[(f*(g + h*x)
)/(f*g - e*h)] + 12*a^2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] - 12*a*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e +
 f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 4*b^4*p^3*q^3*Log[e + f*x]^3*
Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a*b^3*p*q*Log
[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] - 6*b^
4*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*(g + h*x))/(...
```

3.442.3 Rubi [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2895, 2843, 2881, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}dx$$

$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h}-\frac{4bfpq\int\frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h}-\frac{4bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^3\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h}-$$
$$\frac{4bpq\left(3bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq})\right.}{h}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h}-$$
$$\frac{4bpq\left(3bpq\left(\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(\right.\right.}{h}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h}-$$
$$\frac{4bpq\left(3bpq\left(\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq\left(\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e\right.\right.\right.}{h}$$

$\downarrow$ 7143

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h}-$$
$$\frac{4bpq\left(3bpq\left(\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))^2-2bpq\left(\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+\right.\right.\right.}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^4*Log[(f*(g + h*x))/(f*g - e*h)])/h - (4
*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])^3*PolyLog[2, -((h*(e + f*x))/
(f*g - e*h))]) + 3*b*p*q*((a + b*Log[c*d^q*(e + f*x)^(p*q)])^2*PolyLog[3,
-((h*(e + f*x))/(f*g - e*h))] - 2*b*p*q*((a + b*Log[c*d^q*(e + f*x)^(p*q)]
)*PolyLog[4, -((h*(e + f*x))/(f*g - e*h))] - b*p*q*PolyLog[5, -((h*(e + f*
x))/(f*g - e*h))]))))/h
```

3.442.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.442.4 Maple [F]

$$\int \frac{(a + b\ln(c(d(fx + e)^p)^q))^4}{hx + g} dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x)
```

3.442.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^pd)^qc)+a)^4}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b^4*log(((f*x + e)^p*d)^q*c)^4 + 4*a*b^3*log(((f*x + e)^p*d)^q*c
)^3 + 6*a^2*b^2*log(((f*x + e)^p*d)^q*c)^2 + 4*a^3*b*log(((f*x + e)^p*d)^q
*c) + a^4)/(h*x + g), x)
```

3.442.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**4/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**4/(g + h*x), x)
```

3.442.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^pd)^qc)+a)^4}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x, algorithm="maxima")
```

output
```
a^4*log(h*x + g)/h + integrate((b^4*log(((f*x + e)^p)^q)^4 + 4*(q*log(d) +
 log(c))*a^3*b + 6*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a^2*b^2 +
 4*(q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)
*a*b^3 + (q^4*log(d)^4 + 4*q^3*log(c)*log(d)^3 + 6*q^2*log(c)^2*log(d)^2 +
 4*q*log(c)^3*log(d) + log(c)^4)*b^4 + 4*((q*log(d) + log(c))*b^4 + a*b^3)
*log(((f*x + e)^p)^q)^3 + 6*(2*(q*log(d) + log(c))*a*b^3 + (q^2*log(d)^2 +
 2*q*log(c)*log(d) + log(c)^2)*b^4 + a^2*b^2)*log(((f*x + e)^p)^q)^2 + 4*(
3*(q*log(d) + log(c))*a^2*b^2 + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(
c)^2)*a*b^3 + (q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d)
+ log(c)^3)*b^4 + a^3*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.442.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^p d)^q c)+a)^4}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^4/(h*x + g), x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(a+b\ln(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x), x)
```

3.443 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx$

3.443.1 Optimal result

Integrand size = 28, antiderivative size = 274

$$\begin{aligned}
\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx &= \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^4}{(fg-eh)(g+hx)} - \frac{4bfpq(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} \\
&\quad - \frac{12b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} \\
&\quad + \frac{24b^3fp^3q^3(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} \\
&\quad - \frac{24b^4fp^4q^4\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}
\end{aligned}$$

output

```
(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^4/(-e*h+f*g)/(h*x+g)-4*b*f*p*q*(a+b*ln
(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)-12*b^2*f*p^2*
q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h
+f*g)+24*b^3*f*p^3*q^3*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-
e*h+f*g))/h/(-e*h+f*g)-24*b^4*f*p^4*q^4*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h
/(-e*h+f*g)
```

3.443.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1301 vs. $2(274) = 548$.

Time = 0.38 (sec) , antiderivative size = 1301, normalized size of antiderivative = 4.75

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^4}{(g + hx)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x)^2,x]
```

output
```
(a^4*f*g - a^4*e*h - 4*a^3*b*f*g*p*q*Log[e + f*x] - 4*a^3*b*f*h*p*q*x*Log[
e + f*x] + 6*a^2*b^2*f*g*p^2*q^2*Log[e + f*x]^2 + 6*a^2*b^2*f*h*p^2*q^2*x*
Log[e + f*x]^2 - 4*a*b^3*f*g*p^3*q^3*Log[e + f*x]^3 - 4*a*b^3*f*h*p^3*q^3*
x*Log[e + f*x]^3 + b^4*f*g*p^4*q^4*Log[e + f*x]^4 + b^4*f*h*p^4*q^4*x*Log[
e + f*x]^4 + 4*a^3*b*f*g*Log[c*(d*(e + f*x)^p)^q] - 4*a^3*b*e*h*Log[c*(d*(
e + f*x)^p)^q] - 12*a^2*b^2*f*g*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]
- 12*a^2*b^2*f*h*p*q*x*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] + 12*a*b^3*f*
g*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q] + 12*a*b^3*f*h*p^2*q^2*x
*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q] - 4*b^4*f*g*p^3*q^3*Log[e + f*x]^
3*Log[c*(d*(e + f*x)^p)^q] - 4*b^4*f*h*p^3*q^3*x*Log[e + f*x]^3*Log[c*(d*(
e + f*x)^p)^q] + 6*a^2*b^2*f*g*Log[c*(d*(e + f*x)^p)^q]^2 - 6*a^2*b^2*e*h*
Log[c*(d*(e + f*x)^p)^q]^2 - 12*a*b^3*f*g*p*q*Log[e + f*x]*Log[c*(d*(e + f
*x)^p)^q]^2 - 12*a*b^3*f*h*p*q*x*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2 +
 6*b^4*f*g*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]^2 + 6*b^4*f*h*p
^2*q^2*x*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]^2 + 4*a*b^3*f*g*Log[c*(d*
(e + f*x)^p)^q]^3 - 4*a*b^3*e*h*Log[c*(d*(e + f*x)^p)^q]^3 - 4*b^4*f*g*p*q
*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^3 - 4*b^4*f*h*p*q*x*Log[e + f*x]*Lo
g[c*(d*(e + f*x)^p)^q]^3 + b^4*f*g*Log[c*(d*(e + f*x)^p)^q]^4 - b^4*e*h*Lo
g[c*(d*(e + f*x)^p)^q]^4 + 4*a^3*b*f*g*p*q*Log[(f*(g + h*x))/(f*g - e*h)]
+ 4*a^3*b*f*h*p*q*x*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a^2*b^2*f*g*p*q...
```

3.443.3 Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2895, 2844, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^4}{(g+hx)^2}dx$$

$\Big\downarrow$ 2895

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^4}{(g+hx)^2}dx$$

$\Big\downarrow$ 2844

$$\frac{(e+fx) (a+b\log (c(d(e+fx)^p)^q))^4}{(g+hx)(fg-eh)}-\frac{4bfpq\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}dx}{fg-eh}$$

$\Big\downarrow$ 2843

$$\frac{(e+fx) (a+b\log (c(d(e+fx)^p)^q))^4}{(g+hx)(fg-eh)}-\frac{4bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bfpq\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}\right)}{fg-eh}$$

$\Big\downarrow$ 2881

$$\frac{(e+fx) (a+b\log (c(d(e+fx)^p)^q))^4}{(g+hx)(fg-eh)}-\frac{4bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}\right)}{fg-eh}$$

$\Big\downarrow$ 2821

$$\frac{(e+fx) (a+b\log (c(d(e+fx)^p)^q))^4}{(g+hx)(fg-eh)}-\frac{4bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\int \frac{(a+b\log(cd^q(e+fx)^{pq}))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right.}{h}\right.}{fg-eh}$$

$\Big\downarrow$ 2830

3.443. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx$

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{(g+hx)(fg-eh)} - \frac{4bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h} - \frac{3bpq\left(2bpq\left(\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\int\frac{\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}\right.\right.}{h}\right.}{fg-eh}$$

$\big\downarrow$ 7143

$$\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{(g+hx)(fg-eh)} - \frac{4bfpq\left(\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h} - \frac{3bpq\left(2bpq\left(\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\,\text{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)\right)-\right.}{h}\right.}{fg-eh}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x)^2,x]
```

output
```
((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^4)/((f*g - e*h)*(g + h*x)) - (
4*b*f*p*q*(((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*
h)])/h - (3*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])^2*PolyLog[2, -((h*
(e + f*x))/(f*g - e*h))]) + 2*b*p*q*((a + b*Log[c*d^q*(e + f*x)^(p*q)])*Po
lyLog[3, -((h*(e + f*x))/(f*g - e*h))] - b*p*q*PolyLog[4, -((h*(e + f*x))/
(f*g - e*h))])))/h))/(f*g - e*h)
```

3.443.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.443.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)\right)^{4}}{\left(hx+g\right)^{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x)
```

3.443.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^4}{\left(hx+g\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral((b^4*log(((f*x + e)^p*d)^q*c)^4 + 4*a*b^3*log(((f*x + e)^p*d)^q*c
)^3 + 6*a^2*b^2*log(((f*x + e)^p*d)^q*c)^2 + 4*a^3*b*log(((f*x + e)^p*d)^q
*c) + a^4)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

3.443.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^4}{\left(g+hx\right)^2}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**4/(h*x+g)**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**4/(g + h*x)**2, x)
```

3.443.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^4}{\left(hx+g\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x, algorithm="maxima")
```

output
```
4*a^3*b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2)
) - b^4*log(((f*x + e)^p)^q)^4/(h^2*x + g*h) - 4*a^3*b*log(((f*x + e)^p*d)
^q*c)/(h^2*x + g*h) - a^4/(h^2*x + g*h) + integrate((6*(e*h*q^2*log(d)^2 +
 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*a^2*b^2 + 4*(e*h*q^3*log(d)^3 + 3*e
*h*q^2*log(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*a*b^3 + (
e*h*q^4*log(d)^4 + 4*e*h*q^3*log(c)*log(d)^3 + 6*e*h*q^2*log(c)^2*log(d)^2
 + 4*e*h*q*log(c)^3*log(d) + e*h*log(c)^4)*b^4 + 4*(a*b^3*e*h + (f*g*p*q +
 e*h*q*log(d) + e*h*log(c))*b^4 + (a*b^3*f*h + (f*h*p*q + f*h*q*log(d) + f
*h*log(c))*b^4)*x)*log(((f*x + e)^p)^q)^3 + 6*(a^2*b^2*e*h + 2*(e*h*q*log(
d) + e*h*log(c))*a*b^3 + (e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*log(d) + e*h*l
og(c)^2)*b^4 + (a^2*b^2*f*h + 2*(f*h*q*log(d) + f*h*log(c))*a*b^3 + (f*h*q
^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*b^4)*x)*log(((f*x + e)
^p)^q)^2 + (6*(f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*a^
2*b^2 + 4*(f*h*q^3*log(d)^3 + 3*f*h*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2
*log(d) + f*h*log(c)^3)*a*b^3 + (f*h*q^4*log(d)^4 + 4*f*h*q^3*log(c)*log(d
)^3 + 6*f*h*q^2*log(c)^2*log(d)^2 + 4*f*h*q*log(c)^3*log(d) + f*h*log(c)^4
)*b^4)*x + 4*(3*(e*h*q*log(d) + e*h*log(c))*a^2*b^2 + 3*(e*h*q^2*log(d)^2
+ 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*a*b^3 + (e*h*q^3*log(d)^3 + 3*e*h*
q^2*log(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*b^4 + (3*(f*
h*q*log(d) + f*h*log(c))*a^2*b^2 + 3*(f*h*q^2*log(d)^2 + 2*f*h*q*log(c)...
```

3.443.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^4}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^4/(h*x + g)^2, x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^4}{\left(g+h\,x\right)^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x)^2,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x)^2, x)
```

3.444 $\int \log \left(c\left(d(e+fx)^p\right)^q\right) \, dx$

3.444.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \log \left(c\left(d(e+fx)^p\right)^q\right) \, dx = -pqx + \frac{(e+fx)\log \left(c\left(d(e+fx)^p\right)^q\right)}{f}$$

output
```
-p*q*x+(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f
```

3.444.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(d(e+fx)^p\right)^q\right) \, dx = -pqx + \frac{(e+fx)\log \left(c\left(d(e+fx)^p\right)^q\right)}{f}$$

input
```
Integrate[Log[c*(d*(e + f*x)^p)^q],x]
```

output
```
-(p*q*x) + ((e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

3.444.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2895, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(d(e+fx)^p)^q\right) dx$$

$$\Big\downarrow \text{2895}$$

$$\int \log \left(c(d(e+fx)^p)^q\right) dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log \left(cd^q(e+fx)^{pq}\right) d(e+fx)}{f}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(e+fx)\log \left(cd^q(e+fx)^{pq}\right)-pq(e+fx)}{f}$$

input
```
Int[Log[c*(d*(e + f*x)^p)^q],x]
```

output
```
(-(p*q*(e + f*x)) + (e + f*x)*Log[c*d^q*(e + f*x)^(p*q)])/f
```

3.444.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.444.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

method	result	size
default	$\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)x-qpf\left(\frac{x}{f}-\frac{e\ln\left(fx+e\right)}{f^{2}}\right)$	41
parts	$\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)x-qpf\left(\frac{x}{f}-\frac{e\ln\left(fx+e\right)}{f^{2}}\right)$	41
parallelrisch	$\frac{2\ln\left(fx+e\right)e^{2}pq-xefpq+x\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)ef-\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)e^{2}}{ef}$	66

input
```
int(ln(c*(d*(f*x+e)^p)^q),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(d*(f*x+e)^p)^q)*x-q*p*f*(x/f-e/f^2*ln(f*x+e))
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \log\left(c(d(e+fx)^{p})^{q}\right)\,dx=-\frac{fpqx-fqx\log\left(d\right)-fx\log\left(c\right)-\left(fpqx+epq\right)\log\left(fx+e\right)}{f}$$

input
```
integrate(log(c*(d*(f*x+e)^p)^q),x, algorithm="fricas")
```

output
```
-(f*p*q*x - f*q*x*log(d) - f*x*log(c) - (f*p*q*x + e*p*q)*log(f*x + e))/f
```

3.444.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \log \left(c(d(e+fx)^p)^q\right) dx = \begin{cases} \frac{e\log\left(c(d(e+fx)^p)^q\right)}{f} - pqx + x\log\left(c(d(e+fx)^p)^q\right) & \text{for } f \neq 0 \\ x\log\left(c(de^p)^q\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(d*(f*x+e)**p)**q),x)
```

output
```
Piecewise((e*log(c*(d*(e + f*x)**p)**q)/f - p*q*x + x*log(c*(d*(e + f*x)**
p)**q), Ne(f, 0)), (x*log(c*(d*e**p)**q), True))
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \log \left(c(d(e+fx)^p)^q\right) dx = -fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right) + x\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)$$

input
```
integrate(log(c*(d*(f*x+e)^p)^q),x, algorithm="maxima")
```

output
```
-f*p*q*(x/f - e*log(f*x + e)/f^2) + x*log(((f*x + e)^p*d)^q*c)
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \log \left(c(d(e+fx)^p)^q\right) dx = \frac{(fx+e)pq\log(fx+e)}{f} - \frac{(fx+e)pq}{f} + \frac{(fx+e)q\log(d)}{f} + \frac{(fx+e)\log(c)}{f}$$

input
```
integrate(log(c*(d*(f*x+e)^p)^q),x, algorithm="giac")
```

output
```
(f*x + e)*p*q*log(f*x + e)/f - (f*x + e)*p*q/f + (f*x + e)*q*log(d)/f + (f
*x + e)*log(c)/f
```

3.444.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \log \left(c(d(e+fx)^p)^q\right) \, dx = x \ln \left(c \left(d \left(e+f x\right)^p\right)^q\right)+\frac{p\, q \left(e \ln \left(e+f x\right)-f x\right)}{f}$$

input
```
int(log(c*(d*(e + f*x)^p)^q),x)
```

output
```
x*log(c*(d*(e + f*x)^p)^q) + (p*q*(e*log(e + f*x) - f*x))/f
```

3.445 $\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.445.1 Optimal result

Integrand size = 28, antiderivative size = 279

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^3pq} + \frac{2e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq} + \frac{e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}$$

output
```
(-e*h+f*g)^2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(a/b/p/q)/
f^3/p/q/((c*(d*(f*x+e)^p)^q)^(1/p/q))+2*h*(-e*h+f*g)*(f*x+e)^2*Ei(2*(a+b*l
n(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(2*a/b/p/q)/f^3/p/q/((c*(d*(f*x+e)^p)^q)
^(2/p/q))+h^2*(f*x+e)^3*Ei(3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(3*a/
b/p/q)/f^3/p/q/((c*(d*(f*x+e)^p)^q)^(3/p/q))
```

3.445.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$= \frac{e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(e^{\frac{2a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\right.}{}$$

input
```
Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
((e + f*x)*(E^((2*a)/(b*p*q))*(f*g - e*h)^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q)
)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)] - h*(e + f*x)*(-
2*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[
(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)] - h*(e + f*x)*ExpIntegralEi[
(3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])))/(b*E^((3*a)/(b*p*q))*f^3*
p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

3.445.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2895, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$\Big\downarrow \; 2895$$
$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$
$$\Big\downarrow \; 2846$$
$$\int \left(\frac{(fg-eh)^2}{f^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}+\frac{2h(e+fx)(fg-eh)}{f^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}+\frac{h^2(e+fx)^2}{f^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{2h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}+\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^3pq}+\frac{h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}$$

input
```
Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
((f*g - e*h)^2*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b
*p*q)])/(b*E^(a/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (2*h*(
f*g - e*h)*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/
(b*p*q)])/(b*E^((2*a)/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) +
(h^2*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q
)])/(b*E^((3*a)/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

3.445.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.445. $\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.445.4 Maple [F]

$$\int \frac{(hx+g)^2}{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

input
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{\left(h^2\operatorname{log\_integral}\left((f^3x^3+3\,ef^2x^2+3\,e^2fx+e^3)e^{\left(\frac{3\,(bq\log(d)+b\log(c)+a)}{bpq}\right)}\right)+2\,(fgh-eh^2)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right.}{}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
(h^2*log_integral((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*e^(3*(b*q*log(
d) + b*log(c) + a)/(b*p*q))) + 2*(f*g*h - e*h^2)*e^((b*q*log(d) + b*log(c)
 + a)/(b*p*q))*log_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b
*log(c) + a)/(b*p*q))) + (f^2*g^2 - 2*e*f*g*h + e^2*h^2)*e^(2*(b*q*log(d)
+ b*log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c)
+ a)/(b*p*q))))*e^(-3*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b*f^3*p*q)
```

3.445.6 Sympy [F]

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(g+hx)^2}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

input
```
integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.445.7 Maxima [F]

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(hx+g)^2}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx &= \frac{g^2\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}fpq} \\ &- \frac{2\,egh\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}f^2pq} \\ &+ \frac{e^2h^2\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}f^3pq} \\ &+ \frac{2\,gh\mathrm{Ei}\left(\frac{2\,\log(d)}{p}+\frac{2\,\log(c)}{pq}+\frac{2\,a}{bpq}+2\,\log\left(fx+e\right)\right)e^{\left(-\frac{2\,a}{bpq}\right)}}{bc^{\frac{2}{pq}}d^{\frac{2}{p}}f^2pq} \\ &- \frac{2\,eh^2\mathrm{Ei}\left(\frac{2\,\log(d)}{p}+\frac{2\,\log(c)}{pq}+\frac{2\,a}{bpq}+2\,\log\left(fx+e\right)\right)e^{\left(-\frac{2\,a}{bpq}\right)}}{bc^{\frac{2}{pq}}d^{\frac{2}{p}}f^3pq} \\ &+ \frac{h^2\mathrm{Ei}\left(\frac{3\,\log(d)}{p}+\frac{3\,\log(c)}{pq}+\frac{3\,a}{bpq}+3\,\log\left(fx+e\right)\right)e^{\left(-\frac{3\,a}{bpq}\right)}}{bc^{\frac{3}{pq}}d^{\frac{3}{p}}f^3pq}\end{aligned}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
g^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/
(b*c^(1/(p*q))*d^(1/p)*f*p*q) - 2*e*g*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*
p*q) + log(f*x + e))*e^(-a/(b*p*q))/(b*c^(1/(p*q))*d^(1/p)*f^2*p*q) + e^2*
h^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/
(b*c^(1/(p*q))*d^(1/p)*f^3*p*q) + 2*g*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2
*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/(b*c^(2/(p*q))*d^(2/p)*f^2*p
*q) - 2*e*h^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e
))*e^(-2*a/(b*p*q))/(b*c^(2/(p*q))*d^(2/p)*f^3*p*q) + h^2*Ei(3*log(d)/p +
3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))/(b*c^(3/(p
*q))*d^(3/p)*f^3*p*q)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{{\left(g+h\,x\right)}^2}{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

input
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.446 $\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.446.1 Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq} + \frac{e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^2pq}$$

output
```
(-e*h+f*g)*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(a/b/p/q)/f^
2/p/q/((c*(d*(f*x+e)^p)^q)^(1/p/q))+h*(f*x+e)^2*Ei(2*(a+b*ln(c*(d*(f*x+e)^
p)^q))/b/p/q)/b/exp(2*a/b/p/q)/f^2/p/q/((c*(d*(f*x+e)^p)^q)^(2/p/q))
```

3.446.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\right.}{bf^2pq}$$

input
```
Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
((e + f*x)*(E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpI
ntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)] + h*(e + f*x)*ExpInteg
ralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]))/(b*E^((2*a)/(b*p*q))
*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

3.446.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2895, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$
$$\Big\downarrow \text{2846}$$
$$\int \left(\frac{fg-eh}{f\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}+\frac{h(e+fx)}{f\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq}+\frac{h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^2pq}$$

input
```
Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p
*q)])/(b*E^(a/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*(e +
f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(b*E^(
(2*a)/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

3.446.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.446.4 Maple [F]

$$\int \frac{hx+g}{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.78

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \frac{\left((fg-eh)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\operatorname{log\_integral}\left((fx+e)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right)+h\operatorname{log\_integral}\left((f^{2}x^{2}+2\,efx\right.\right.}{bf^{2}pq}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
((f*g - e*h)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e
)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) + h*log_integral((f^2*x^2 + 2*e
*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-2*(b*q*log(d)
+ b*log(c) + a)/(b*p*q))/(b*f^2*p*q)
```

3.446.6 Sympy [F]

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{g+hx}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

input
```
integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.446.7 Maxima [F]

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{hx+g}{b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=&\frac{g\text{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}fpq}\\&-\frac{eh\text{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}f^2pq}\\&+\frac{h\text{Ei}\left(\frac{2\log(d)}{p}+\frac{2\log(c)}{pq}+\frac{2a}{bpq}+2\log\left(fx+e\right)\right)e^{\left(-\frac{2a}{bpq}\right)}}{bc^{\frac{2}{pq}}d^{\frac{2}{p}}f^2pq}\end{aligned}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
g*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(b
*c^(1/(p*q))*d^(1/p)*f*p*q) - e*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) +
 log(f*x + e))*e^(-a/(b*p*q))/(b*c^(1/(p*q))*d^(1/p)*f^2*p*q) + h*Ei(2*log
(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/(b
*c^(2/(p*q))*d^(2/p)*f^2*p*q)
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{g+h\,x}{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}\,dx$$

input
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.447 $\int \frac{1}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.447.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bfpq}$$

output
```
(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(a/b/p/q)/f/p/q/((c*(d*
(f*x+e)^p)^q)^(1/p/q))
```

3.447.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bfpq}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-1),x]
```

output
```
((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(b*E^(
a/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q)))
```

3.447.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2895, 2836, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log\left(cd^q(e+fx)^{pq}\right)}{fpq}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{bfpq}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-1),x]
```

output
```
((e + f*x)*ExpIntegralEi[(a + b*Log[c*d^q*(e + f*x)^(p*q)])/(b*p*q)])/(b*E
^(a/(b*p*q))*f*p*q*(c*d^q*(e + f*x)^(p*q))^(1/(p*q)))
```

3.447.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.447.4 Maple [F]

$$\int \frac{1}{a + b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{\left(-\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\,\text{log\_integral}\left((fx+e)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right)}{bfpq}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
e^(-(b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*lo
g(d) + b*log(c) + a)/(b*p*q)))/(b*f*p*q)
```

3.447.6 Sympy [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{1}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.447.7 Maxima [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{1}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate(1/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.447.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}fpq}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(b*c
^(1/(p*q))*d^(1/p)*f*p*q)
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{1}{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.448 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.448.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.448.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.448.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$
$$\Big\downarrow \text{ 2896}$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.448.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.448.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.448.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(((fx+e)^pd)^q\,c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
integral(1/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

3.448.6 Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

3.448.7 Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(((fx+e)^pd)^q\,c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.448.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.448.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.449 $\int \frac{1}{(g+hx)^2(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.449.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\text{Int}\left(\frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

output
```
Unintegrable(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.449.2 Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

input
```
Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.449.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)^2 (a+b\log (c (d(e+fx)^p)^q))} \, dx$$

$$\Big\downarrow \text{ 2896}$$

$$\int \frac{1}{(g+hx)^2 (a+b\log (c (d(e+fx)^p)^q))}dx$$

input
```
Int[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.449.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.449.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(hx+g\right)^{2} \left(a+b\ln \left(c \left(d \left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

input
```
int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.449.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
integral(1/(a*h^2*x^2 + 2*a*g*h*x + a*g^2 + (b*h^2*x^2 + 2*b*g*h*x + b*g^2
)*log(((f*x + e)^p*d)^q*c)), x)
```

3.449.6 Sympy [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))(g+hx)^2}\,dx$$

input
```
integrate(1/(h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**2), x)
```

3.449.7 Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.449.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)^2\left(b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.449.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.450 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.450.1 Optimal result

Integrand size = 28, antiderivative size = 326

$$\begin{aligned}\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx &= \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^3p^2q^2}\\ &+ \frac{4e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2}\\ &+ \frac{3e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2}\\ &- \frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\end{aligned}$$

output
```
(-e*h+f*g)^2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(a/b/p/q
)/f^3/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))+4*h*(-e*h+f*g)*(f*x+e)^2*Ei(2*
(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(2*a/b/p/q)/f^3/p^2/q^2/((c*(d*(
f*x+e)^p)^q)^(2/p/q))+3*h^2*(f*x+e)^3*Ei(3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p
/q)/b^2/exp(3*a/b/p/q)/f^3/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(3/p/q))-(f*x+e)*(
h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

3.450.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1310 vs. $2(326) = 652$.

Time = 0.41 (sec) , antiderivative size = 1310, normalized size of antiderivative = 4.02

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(-(b*e*E^((3*a)/(b*p*q))*f^2*g^2*p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) - b*
E^((3*a)/(b*p*q))*f^3*g^2*p*q*x*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - 2*b*e*E^
((3*a)/(b*p*q))*f^2*g*h*p*q*x*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - 2*b*E^((3*
a)/(b*p*q))*f^3*g*h*p*q*x^2*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - b*e*E^((3*a)
/(b*p*q))*f^2*h^2*p*q*x^2*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - b*E^((3*a)/(b*
p*q))*f^3*h^2*p*q*x^3*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) + a*E^((2*a)/(b*p*q)
)*f^2*g^2*(e + f*x)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*L
og[c*(d*(e + f*x)^p)^q])/(b*p*q)] - 2*a*e*E^((2*a)/(b*p*q))*f*g*h*(e + f*x
)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^
p)^q])/(b*p*q)] + a*e^2*E^((2*a)/(b*p*q))*h^2*(e + f*x)*(c*(d*(e + f*x)^p)
^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)] + 4*
a*E^(a/(b*p*q))*f*g*h*(e + f*x)^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpInteg
ralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)] - 4*a*e*E^(a/(b*p*q))*
h^2*(e + f*x)^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a + b*Lo
g[c*(d*(e + f*x)^p)^q]))/(b*p*q)] + 3*a*h^2*(e + f*x)^3*ExpIntegralEi[(3*(
a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)] + b*E^((2*a)/(b*p*q))*f^2*g^2*(e
 + f*x)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e +
 f*x)^p)^q])/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q] - 2*b*e*E^((2*a)/(b*p*q))*f
*g*h*(e + f*x)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*
(d*(e + f*x)^p)^q])/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q] + b*e^2*E^((2*a)/...
```

3.450.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2895, 2847, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.450. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$$
$$\Big\downarrow \text{2847}$$
$$-\frac{2(fg-eh)\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{3\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}$$
$$\Big\downarrow \text{2846}$$
$$\frac{3\int \left(\frac{(fg-eh)^2}{f^2(a+b\log(c(d(e+fx)^p)^q))}+\frac{2h(e+fx)(fg-eh)}{f^2(a+b\log(c(d(e+fx)^p)^q))}+\frac{h^2(e+fx)^2}{f^2(a+b\log(c(d(e+fx)^p)^q))}\right)dx}{bpq}-$$
$$\frac{2(fg-eh)\int \left(\frac{fg-eh}{f(a+b\log(c(d(e+fx)^p)^q))}+\frac{h(e+fx)}{f(a+b\log(c(d(e+fx)^p)^q))}\right)dx}{bfpq}-$$
$$\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}$$
$$\Big\downarrow \text{2009}$$
$$\frac{3\left(\frac{2h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}+\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\text{ExpI}\ldots}{bf^3pq}\right.}{bpq}$$
$$\frac{2(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq}+\frac{h(e+fx)^2e^{-\frac{2a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\text{ExpI}\ldots}{bf^2pq}\right.}{bfpq}$$
$$\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}$$

input
```
Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

3.450. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$

output
```
(-2*(f*g - e*h)*(((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e +
 f*x)^p)^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(1/(
p*q))) + (h*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))
/(b*p*q)])/(b*E^((2*a)/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))))
/(b*f*p*q) + (3*(((f*g - e*h)^2*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e
 + f*x)^p)^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(1
/(p*q))) + (2*h*(f*g - e*h)*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(
e + f*x)^p)^q]))/(b*p*q)])/(b*E^((2*a)/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)
^q)^(2/(p*q))) + (h^2*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*(e + f*
x)^p)^q]))/(b*p*q)])/(b*E^((3*a)/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(3
/(p*q)))))/(b*p*q) - ((e + f*x)*(g + h*x)^2)/(b*f*p*q*(a + b*Log[c*(d*(e +
 f*x)^p)^q]))
```

3.450.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.450. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.450.4 Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^2}dx$$

input
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.76

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$
$$= \frac{\left(4\left(afgh-aeh^2+(bfgh-beh^2)pq\log(fx+e)+(bfgh-beh^2)q\log(d)+(bfgh-beh^2)\log(c)\right)e^{\left(\frac{bq\log(}{}\right.}\right.}{}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
(4*(a*f*g*h - a*e*h^2 + (b*f*g*h - b*e*h^2)*p*q*log(f*x + e) + (b*f*g*h -
b*e*h^2)*q*log(d) + (b*f*g*h - b*e*h^2)*log(c))*e^((b*q*log(d) + b*log(c)
+ a)/(b*p*q))*log_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*
log(c) + a)/(b*p*q))) + (a*f^2*g^2 - 2*a*e*f*g*h + a*e^2*h^2 + (b*f^2*g^2
- 2*b*e*f*g*h + b*e^2*h^2)*p*q*log(f*x + e) + (b*f^2*g^2 - 2*b*e*f*g*h + b
*e^2*h^2)*q*log(d) + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))*e^(2*(b
*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) +
 b*log(c) + a)/(b*p*q))) - (b*f^3*h^2*p*q*x^3 + b*e*f^2*g^2*p*q + (2*b*f^3
*g*h + b*e*f^2*h^2)*p*q*x^2 + (b*f^3*g^2 + 2*b*e*f^2*g*h)*p*q*x)*e^(3*(b*q
*log(d) + b*log(c) + a)/(b*p*q)) + 3*(b*h^2*p*q*log(f*x + e) + b*h^2*q*log
(d) + b*h^2*log(c) + a*h^2)*log_integral((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*
x + e^3)*e^(3*(b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-3*(b*q*log(d) + b
*log(c) + a)/(b*p*q))/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(
d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)
```

3.450.6 Sympy [F]

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2}\,dx$$

input
```
integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**2, x)
```

3.450.7 Maxima [F]

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{(hx+g)^2}{\left(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a\right)^2}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
-(f*h^2*x^3 + e*g^2 + (2*f*g*h + e*h^2)*x^2 + (f*g^2 + 2*e*g*h)*x)/(b^2*f*
p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) + f*p*q*log(c))*b^2
) + integrate((3*f*h^2*x^2 + f*g^2 + 2*e*g*h + 2*(2*f*g*h + e*h^2)*x)/(b^2
*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) + f*p*q*log(c))*
b^2), x)
```

3.450.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3975 vs. $2(328) = 656$.

Time = 0.46 (sec) , antiderivative size = 3975, normalized size of antiderivative = 12.19

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
-(f*x + e)*b*f^2*g^2*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*l
og(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2) - 2*(f*x + e)^2*b*f*g*
h*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2
*q^2*log(c) + a*b^2*f^3*p^2*q^2) + 2*(f*x + e)*b*e*f*g*h*p*q/(b^3*f^3*p^3*
q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2
*f^3*p^2*q^2) - (f*x + e)^3*b*h^2*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*
f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2) + 2*(f*x
+ e)^2*b*e*h^2*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d)
+ b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2) - (f*x + e)*b*e^2*h^2*p*q/(b
^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log
(c) + a*b^2*f^3*p^2*q^2) + b*f^2*g^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b
*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*
x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q
^2)*c^(1/(p*q))*d^(1/p)) - 2*b*e*f*g*h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/
(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(
f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2
*q^2)*c^(1/(p*q))*d^(1/p)) + b*e^2*h^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/
(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(
f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2
*q^2)*c^(1/(p*q))*d^(1/p)) + 4*b*f*g*h*p*q*Ei(2*log(d)/p + 2*log(c)/(p*...
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{\left(g+h\,x\right)^2}{\left(a+b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)\right)^2}\,dx$$

input
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

3.451 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.451.1 Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^2p^2q^2} + \frac{2e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^2p^2q^2} - \frac{(e+fx)(g+hx)}{bfpq\,(a+b\log(c(d(e+fx)^p)^q))}$$

output
```
(-e*h+f*g)*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(a/b/p/q)/
f^2/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))+2*h*(f*x+e)^2*Ei(2*(a+b*ln(c*(d*
(f*x+e)^p)^q))/b/p/q)/b^2/exp(2*a/b/p/q)/f^2/p^2/q^2/((c*(d*(f*x+e)^p)^q)^
(2/p/q))-(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

3.451.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.20

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx =$$
$$-\frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(be^{\frac{2a}{bpq}}fpq(c(d(e+fx)^p)^q)^{\frac{2}{pq}}(g+hx)-e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p\right.\right.}{}$$

input
```
Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
-(((e + f*x)*(b*E^((2*a)/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(g
 + h*x) - E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpInt
egralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*(e + f*x
)^p)^q]) - 2*h*(e + f*x)*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q])
)/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])))/(b^2*E^((2*a)/(b*p*q))*f^2*p
^2*q^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q])))
```

3.451.3 Rubi [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2895, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$$
$$\Big\downarrow \text{ 2847}$$
$$-\frac{(fg-eh)\int \frac{1}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq\,(a+b\log(c(d(e+fx)^p)^q))}$$
$$\Big\downarrow \text{ 2836}$$

$$-\frac{(fg-eh)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{bf^2pq}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$\Big\downarrow \text{2737}$$

$$-\frac{(e+fx)(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{\left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log\left(cd^q(e+fx)^{pq}\right)}{bf^2p^2q^2}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$\Big\downarrow \text{2846}$$

$$\frac{2\int \left(\frac{fg-eh}{f(a+b\log(c(d(e+fx)^p)^q))}+\frac{h(e+fx)}{f(a+b\log(c(d(e+fx)^p)^q))}\right)dx}{bpq}-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}+\frac{2\left(\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq}+\frac{h(e+fx)^2e^{-\frac{2a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(}{}\right.}{bf^2pq}\right.}{bpq}-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

input
```
Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

3.451. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

output
```
-(((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*d^q*(e + f*x)^(p*q)])/
(b*p*q)])/(b^2*E^(a/(b*p*q))*f^2*p^2*q^2*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))
)) + (2*(((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)
^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) +
 (h*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)
])/(b*E^((2*a)/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))))/(b*p*q)
 - ((e + f*x)*(g + h*x))/(b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q]))
```

3.451.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.451. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.451.4 Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

input
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.46

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2}}\,dx = \frac{\left(((bfg-beh)pq\log\left(fx+e\right)+afg-aeh+(bfg-beh)q\log\left(d\right)+(bfg-beh)\log\left(c\right))e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right.}{}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
(((b*f*g - b*e*h)*p*q*log(f*x + e) + a*f*g - a*e*h + (b*f*g - b*e*h)*q*log
(d) + (b*f*g - b*e*h)*log(c))*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_
integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) - (b*f^2*h*p*q
*x^2 + b*e*f*g*p*q + (b*f^2*g + b*e*f*h)*p*q*x)*e^(2*(b*q*log(d) + b*log(c
) + a)/(b*p*q)) + 2*(b*h*p*q*log(f*x + e) + b*h*q*log(d) + b*h*log(c) + a*
h)*log_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)
/(b*p*q))))*e^(-2*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^3*f^2*p^3*q^3*lo
g(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p
^2*q^2)
```

3.451.6 Sympy [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2} dx$$

input
```
integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**2, x)
```

3.451.7 Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{hx+g}{(b\log(((fx+e)^p d)^q c)+a)^2} dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
-(f*h*x^2 + e*g + (f*g + e*h)*x)/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p
*q + (f*p*q^2*log(d) + f*p*q*log(c))*b^2) + integrate((2*f*h*x + f*g + e*h
)/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) + f*p*q*lo
g(c))*b^2), x)
```

3.451.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1930 vs. 2(225) = 450.

Time = 0.39 (sec) , antiderivative size = 1930, normalized size of antiderivative = 8.62

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2} dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
-(f*x + e)*b*f*g*p*q/(b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d
) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2) - (f*x + e)^2*b*h*p*q/(b^3
*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c
) + a*b^2*f^2*p^2*q^2) + (f*x + e)*b*e*h*p*q/(b^3*f^2*p^3*q^3*log(f*x + e)
 + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2) +
b*f*g*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*
p*q))*log(f*x + e)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d)
 + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) - b*e*
h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q)
)*log(f*x + e)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b
^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + 2*b*h*p*
q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(
b*p*q))*log(f*x + e)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(
d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(2/(p*q))*d^(2/p)) + b*
f*g*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q)
)*log(d)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2
*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) - b*e*h*q*Ei(log
(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b
^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log
(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + b*f*g*Ei(log(d)/p + log...
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{g+h\,x}{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^2}\,dx$$

input
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

3.452 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.452.1 Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2fp^2q^2} - \frac{e+fx}{bfpq\,(a+b\log(c(d(e+fx)^p)^q))}$$

output
```
(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(a/b/p/q)/f/p^2/q^2/(
(c*(d*(f*x+e)^p)^q)^(1/p/q))+(-f*x-e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

3.452.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = -\frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(be^{\frac{a}{bpq}}pq(c(d(e+fx)^p)^q)^{\frac{1}{pq}} - \operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a\right.}{b^2fp^2q^2\,(a+b\log(c(d(e+fx)^p)^q))}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-2),x]
```

output
```
-(((e + f*x)*(b*E^(a/(b*p*q))*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q)) - ExpInt
egralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*(e + f*x
)^p)^q])))/(b^2*E^(a/(b*p*q))*f*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(a
 + b*Log[c*(d*(e + f*x)^p)^q])))
```

3.452.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2895, 2836, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^2}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{bpq}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(e+fx)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log(cd^q(e+fx)^{pq})}{bp^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}}{f}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{(e+fx)e^{-\frac{a}{bpq}}(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-2),x]
```

output
```
(((e + f*x)*ExpIntegralEi[(a + b*Log[c*d^q*(e + f*x)^(p*q)])/(b*p*q)])/(b^
2*E^(a/(b*p*q))*p^2*q^2*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))) - (e + f*x)/(b*
p*q*(a + b*Log[c*d^q*(e + f*x)^(p*q)])))/f
```

3.452.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.452. $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.452.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2}}\,dx = -\frac{\left((bfpqx+bepq)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)} - (bpq\log\left(fx+e\right)+bq\log\left(d\right)+b\log\left(c\right)+a)\,\mathrm{log\_integral}\left((fx+\right.\right.}{b^{3}fp^{3}q^{3}\log\left(fx+e\right)+b^{3}fp^{2}q^{3}\log\left(d\right)+b^{3}fp^{2}q^{2}\log\left(c\right)+ab^{2}fp^{2}}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
-((b*f*p*q*x + b*e*p*q)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q)) - (b*p*q*l
og(f*x + e) + b*q*log(d) + b*log(c) + a)*log_integral((f*x + e)*e^((b*q*lo
g(d) + b*log(c) + a)/(b*p*q))))*e^(-(b*q*log(d) + b*log(c) + a)/(b*p*q))/(
b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log(c) +
 a*b^2*f*p^2*q^2)
```

3.452.6 Sympy [F]

$$\int \frac{1}{\left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2}}\,dx = \int \frac{1}{\left(a+b\log \left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-2), x)
```

3.452.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
-(f*x + e)/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) +
 f*p*q*log(c))*b^2) + integrate(1/(b^2*p*q*log(((f*x + e)^p)^q) + a*b*p*q
+ (p*q^2*log(d) + p*q*log(c))*b^2), x)
```

3.452.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(123) = 246$.

Time = 0.34 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.73

$$\begin{aligned}
&\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx \\
&= -\frac{(fx+e)bpq}{b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2} \\
&\quad+\frac{bpq\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}\log(fx+e)}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}} \\
&\quad+\frac{bq\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}\log(d)}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}} \\
&\quad+\frac{b\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}\log(c)}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}} \\
&\quad+\frac{a\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}}
\end{aligned}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
-(f*x + e)*b*p*q/(b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d) + b^3*
f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2) + b*p*q*Ei(log(d)/p + log(c)/(p*q) + a
/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f*p^3*q^3*log(f
*x + e) + b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2)*c
^(1/(p*q))*d^(1/p)) + b*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x
 + e))*e^(-a/(b*p*q))*log(d)/((b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*
log(d) + b^3*f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + b*
Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(
c)/((b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log
(c) + a*b^2*f*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + a*Ei(log(d)/p + log(c)/(p*q)
 + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/((b^3*f*p^3*q^3*log(f*x + e) +
 b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2)*c^(1/(p*q)
)*d^(1/p))
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

3.453 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx$

3.453.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx = \text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}, x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.453.2 Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

3.453.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$
$$\Big\downarrow \ 2896$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
$Aborted
```

3.453.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.453.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^2}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.453.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*h*x + a^2*g + (b^2*h*x + b^2*g)*log(((f*x + e)^p*d)^q*c)^2
 + 2*(a*b*h*x + a*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

3.453.6 Sympy [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)), x)
```

3.453.7 Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
(f*g - e*h)*integrate(1/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) + f*g^2*p*q*l
og(c))*b^2 + (a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*log(c))*b^2)
*x^2 + 2*(a*b*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c))*b^2)*x +
 (b^2*f*h^2*p*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*x + e)^p)
^q)), x) - (f*x + e)/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q*log(c))*b^
2 + (a*b*f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x + (b^2*f*h*p
*q*x + b^2*f*g*p*q)*log(((f*x + e)^p)^q))
```

3.453.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

3.453.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{\left(g+h\,x\right)\,{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

3.454 $\int \frac{1}{(g+hx)^2(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.454.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)^2 \left(a+b\log \left(c \left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(g+hx)^2 \left(a+b\log \left(c \left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.454.2 Mathematica [N/A]

Not integrable

Time = 8.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 \left(a+b\log \left(c \left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)^2 \left(a+b\log \left(c \left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

input
```
Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

3.454.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} \, dx$$

$$\Big\downarrow \text{ 2896}$$

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$

input
```
Int[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
$Aborted
```

3.454.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.454.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(hx+g\right)^{2} \left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

input
```
int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.454.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.00

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{1}{(hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)^2} dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas"
)
```

output
```
integral(1/(a^2*h^2*x^2 + 2*a^2*g*h*x + a^2*g^2 + (b^2*h^2*x^2 + 2*b^2*g*h
*x + b^2*g^2)*log(((f*x + e)^p*d)^q*c)^2 + 2*(a*b*h^2*x^2 + 2*a*b*g*h*x +
a*b*g^2)*log(((f*x + e)^p*d)^q*c)), x)
```

3.454.6 Sympy [N/A]

Not integrable

Time = 31.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2 (g+hx)^2} dx$$

input
```
integrate(1/(h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)**2), x)
```

3.454.7 Maxima [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 14.50

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{1}{(hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)^2} dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima"
)
```

output
```
-(f*x + e)/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) + f*g^2*p*q*log(c))*b^2 +
(a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*log(c))*b^2)*x^2 + 2*(a*b
*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c))*b^2)*x + (b^2*f*h^2*p
*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*x + e)^p)^q)) - integr
ate((f*h*x - f*g + 2*e*h)/(a*b*f*g^3*p*q + (a*b*f*h^3*p*q + (f*h^3*p*q^2*l
og(d) + f*h^3*p*q*log(c))*b^2)*x^3 + (f*g^3*p*q^2*log(d) + f*g^3*p*q*log(c
))*b^2 + 3*(a*b*f*g*h^2*p*q + (f*g*h^2*p*q^2*log(d) + f*g*h^2*p*q*log(c))*
b^2)*x^2 + 3*(a*b*f*g^2*h*p*q + (f*g^2*h*p*q^2*log(d) + f*g^2*h*p*q*log(c)
)*b^2)*x + (b^2*f*h^3*p*q*x^3 + 3*b^2*f*g*h^2*p*q*x^2 + 3*b^2*f*g^2*h*p*q*
x + b^2*f*g^3*p*q)*log(((f*x + e)^p)^q)), x)
```

3.454.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)^2(b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

3.454.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^2\,{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}\,dx$$

input
```
int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

output
```
int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

3.455 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3} \, dx$

3.455.1 Optimal result

Integrand size = 28, antiderivative size = 432

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3} \, dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}} \text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^3p^3q^3} + \frac{4e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}} \text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^3p^3q^3} + \frac{9e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}} \text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{2b^3f^3p^3q^3} - \frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} - \frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

output
```
1/2*(-e*h+f*g)^2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(a/b
/p/q)/f^3/p^3/q^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))+4*h*(-e*h+f*g)*(f*x+e)^2*E
i(2*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(2*a/b/p/q)/f^3/p^3/q^3/((c*
(d*(f*x+e)^p)^q)^(2/p/q))+9/2*h^2*(f*x+e)^3*Ei(3*(a+b*ln(c*(d*(f*x+e)^p)^q
))/b/p/q)/b^3/exp(3*a/b/p/q)/f^3/p^3/q^3/((c*(d*(f*x+e)^p)^q)^(3/p/q))-1/2
*(f*x+e)*(h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^2+(-e*h+f*g)*(f*x+e
)*(h*x+g)/b^2/f^2/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))-3/2*(f*x+e)*(h*x+g)^
2/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

3.455.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.01

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$
$$= \frac{e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(e^{\frac{2a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\right.}{}$$

input
```
Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
((e + f*x)*(E^((2*a)/(b*p*q))*(f*g - e*h)^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q)
)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*
(e + f*x)^p)^q])^2 - 8*E^(a/(b*p*q))*h*(-(f*g) + e*h)*(e + f*x)*(c*(d*(e +
 f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(
b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 9*h^2*(e + f*x)^2*ExpIntegral
Ei[(3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)
^p)^q])^2 - b*E^((3*a)/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q))*(g +
 h*x)*(b*f*p*q*(g + h*x) + a*(f*g + 2*e*h + 3*f*h*x) + b*(2*e*h + f*(g + 3
*h*x))*Log[c*(d*(e + f*x)^p)^q])))/(2*b^3*E^((3*a)/(b*p*q))*f^3*p^3*q^3*(c
*(d*(e + f*x)^p)^q)^(3/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)
```

3.455.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 951 vs. 2(432) = 864.

Time = 3.25 (sec) , antiderivative size = 951, normalized size of antiderivative = 2.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2895, 2847, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}dx$$
$$\Big\downarrow \text{2847}$$

$$-\frac{(fg-eh)\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{bfpq}+\frac{3\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{2bpq}-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$\Big\downarrow$ 2847

$$-\frac{(fg-eh)\left(-\frac{(fg-eh)\int \frac{1}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\right)}{bfpq}+\frac{3\left(-\frac{2(fg-eh)\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{3\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\right)}{2bpq}-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$\Big\downarrow$ 2836

$$-\frac{(fg-eh)\left(-\frac{(fg-eh)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{bf^2pq}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\right)}{bfpq}+\frac{3\left(-\frac{2(fg-eh)\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{3\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\right)}{2bpq}-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$\Big\downarrow$ 2737

$$-\frac{(fg-eh)\left(-\frac{(e+fx)(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log(cd^q(e+fx)^{pq})}{bf^2p^2q^2}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+\cdots}{bfpq(a+b\,\mathrm{lo}\cdots}\cdots\right.}{bfpq}+\frac{3\left(-\frac{2(fg-eh)\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{3\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\right)}{2bpq}-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$\Big\downarrow$ 2609

3.455. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

$$-\frac{(fg-eh)\left(\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}-\frac{(e+}{bfpq(a+b\log}\right.}{bfpq}$$

$$\frac{3\left(-\frac{2(fg-eh)\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{3\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}\right)}{2bpq}-$$

$$\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2846}$$

$$-\frac{(fg-eh)\left(\frac{2\int\left(\frac{fg-eh}{f(a+b\log(c(d(e+fx)^p)^q))}+\frac{h(e+fx)}{f(a+b\log(c(d(e+fx)^p)^q))}\right)dx}{bpq}-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log}{}\right.}{b^2f^2p^2q^2}\right.}{bfpq}$$

$$\frac{3\left(\frac{3\int\left(\frac{(fg-eh)^2}{f^2(a+b\log(c(d(e+fx)^p)^q))}+\frac{2h(e+fx)(fg-eh)}{f^2(a+b\log(c(d(e+fx)^p)^q))}+\frac{h^2(e+fx)^2}{f^2(a+b\log(c(d(e+fx)^p)^q))}\right)dx}{bpq}-\frac{2(fg-eh)\int\left(\frac{fg-eh}{f(a+b\log(c(d(e+fx)^p)^q))}+\frac{h(}{f(a+b\log(}\right.}{bfpq}\right.}{2bpq}$$

$$\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}-$$

$$\frac{(fg-eh)\left(-\frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}}{b^2f^2p^2q^2}+\frac{2\left(\frac{e^{-\frac{2a}{bpq}}h(e+fx)^2\text{ExpIntegralEi}\left(\frac{2(a+b\log(}{}\right.}{bf^2pq}\right.}{}\right.}{}$$

$$\frac{3\left(-\frac{(e+fx)(g+hx)^2}{bfpq(a+b\log(c(d(e+fx)^p)^q))}-\frac{2(fg-eh)\left(\frac{e^{-\frac{2a}{bpq}}h(e+fx)^2\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}}{bf^2pq}+\frac{e^{-\frac{a}{bpq}}(fg-eh)(e}{}\right.}{bfpq}\right.}{}$$

input
```
Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

3.455. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

output
```
-1/2*((e + f*x)*(g + h*x)^2)/(b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)
- ((f*g - e*h)*(-(((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*d^q*(e
 + f*x)^(p*q)])/(b*p*q)])/(b^2*E^(a/(b*p*q))*f^2*p^2*q^2*(c*d^q*(e + f*x)^
(p*q))^(1/(p*q)))) + (2*(((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c
*(d*(e + f*x)^p)^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)
^q)^(1/(p*q))) + (h*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)
^p)^q]))/(b*p*q)])/(b*E^((2*a)/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(
p*q)))))/(b*p*q) - ((e + f*x)*(g + h*x))/(b*f*p*q*(a + b*Log[c*(d*(e + f*x
)^p)^q]))))/(b*f*p*q) + (3*((-2*(f*g - e*h)*(((f*g - e*h)*(e + f*x)*ExpInt
egralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f^2*p*
q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*(e + f*x)^2*ExpIntegralEi[(2*(a +
b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(b*E^((2*a)/(b*p*q))*f^2*p*q*(c*(d*
(e + f*x)^p)^q)^(2/(p*q)))))/(b*f*p*q) + (3*(((f*g - e*h)^2*(e + f*x)*ExpI
ntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f^3*
p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (2*h*(f*g - e*h)*(e + f*x)^2*ExpInt
egralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(b*E^((2*a)/(b*p*q)
)*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) + (h^2*(e + f*x)^3*ExpIntegralE
i[(3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(b*E^((3*a)/(b*p*q))*f^3*
p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))))/(b*p*q) - ((e + f*x)*(g + h*x)^2)/(
b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q]))))/(2*b*p*q)
```

3.455.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.455. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.455.4 Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^3}dx$$

input
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

output
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.455.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(426) = 852$.

Time = 0.37 (sec) , antiderivative size = 1682, normalized size of antiderivative = 3.89

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
1/2*(8*((b^2*f*g*h - b^2*e*h^2)*p^2*q^2*log(f*x + e)^2 + a^2*f*g*h - a^2*e
*h^2 + (b^2*f*g*h - b^2*e*h^2)*q^2*log(d)^2 + (b^2*f*g*h - b^2*e*h^2)*log(
c)^2 + 2*((b^2*f*g*h - b^2*e*h^2)*p*q^2*log(d) + (b^2*f*g*h - b^2*e*h^2)*p
*q*log(c) + (a*b*f*g*h - a*b*e*h^2)*p*q)*log(f*x + e) + 2*(a*b*f*g*h - a*b
*e*h^2)*log(c) + 2*((b^2*f*g*h - b^2*e*h^2)*q*log(c) + (a*b*f*g*h - a*b*e*
h^2)*q)*log(d))*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f^2*
x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))) + ((b^2*f^
2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*p^2*q^2*log(f*x + e)^2 + a^2*f^2*g^2
- 2*a^2*e*f*g*h + a^2*e^2*h^2 + (b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2
)*q^2*log(d)^2 + (b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*log(c)^2 + 2*
((b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*p*q^2*log(d) + (b^2*f^2*g^2 -
 2*b^2*e*f*g*h + b^2*e^2*h^2)*p*q*log(c) + (a*b*f^2*g^2 - 2*a*b*e*f*g*h +
a*b*e^2*h^2)*p*q)*log(f*x + e) + 2*(a*b*f^2*g^2 - 2*a*b*e*f*g*h + a*b*e^2*
h^2)*log(c) + 2*((b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*q*log(c) + (a
*b*f^2*g^2 - 2*a*b*e*f*g*h + a*b*e^2*h^2)*q)*log(d))*e^(2*(b*q*log(d) + b*
log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)
/(b*p*q))) - (b^2*e*f^2*g^2*p^2*q^2 + (b^2*f^3*h^2*p^2*q^2 + 3*a*b*f^3*h^2
*p*q)*x^3 + (a*b*e*f^2*g^2 + 2*a*b*e^2*f*g*h)*p*q + ((2*b^2*f^3*g*h + b^2*
e*f^2*h^2)*p^2*q^2 + (4*a*b*f^3*g*h + 5*a*b*e*f^2*h^2)*p*q)*x^2 + ((b^2*f^
3*g^2 + 2*b^2*e*f^2*g*h)*p^2*q^2 + (a*b*f^3*g^2 + 6*a*b*e*f^2*g*h + 2*a...
```

3.455.6 Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$$

input
```
integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**3, x)
```

3.455.7 Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^3}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

output
```
-1/2*((3*a*f^2*h^2 + (f^2*h^2*p*q + 3*f^2*h^2*q*log(d) + 3*f^2*h^2*log(c))
*b)*x^3 + ((4*f^2*g*h + 5*e*f*h^2)*a + (2*f^2*g*h*p*q + e*f*h^2*p*q + (4*f
^2*g*h + 5*e*f*h^2)*log(c) + (4*f^2*g*h*q + 5*e*f*h^2*q)*log(d))*b)*x^2 +
(e*f*g^2 + 2*e^2*g*h)*a + (e*f*g^2*p*q + (e*f*g^2 + 2*e^2*g*h)*log(c) + (e
*f*g^2*q + 2*e^2*g*h*q)*log(d))*b + ((f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*a +
 (f^2*g^2*p*q + 2*e*f*g*h*p*q + (f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*log(c) +
 (f^2*g^2*q + 6*e*f*g*h*q + 2*e^2*h^2*q)*log(d))*b)*x + (3*b*f^2*h^2*x^3 +
 (4*f^2*g*h + 5*e*f*h^2)*b*x^2 + (f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*b*x + (
e*f*g^2 + 2*e^2*g*h)*b)*log(((f*x + e)^p)^q))/(b^4*f^2*p^2*q^2*log(((f*x +
 e)^p)^q)^2 + a^2*b^2*f^2*p^2*q^2 + 2*(f^2*p^2*q^3*log(d) + f^2*p^2*q^2*lo
g(c))*a*b^3 + (f^2*p^2*q^4*log(d)^2 + 2*f^2*p^2*q^3*log(c)*log(d) + f^2*p^
2*q^2*log(c)^2)*b^4 + 2*(a*b^3*f^2*p^2*q^2 + (f^2*p^2*q^3*log(d) + f^2*p^2
*q^2*log(c))*b^4)*log(((f*x + e)^p)^q)) + integrate(1/2*(9*f^2*h^2*x^2 + f
^2*g^2 + 6*e*f*g*h + 2*e^2*h^2 + 2*(4*f^2*g*h + 5*e*f*h^2)*x)/(b^3*f^2*p^2
*q^2*log(((f*x + e)^p)^q) + a*b^2*f^2*p^2*q^2 + (f^2*p^2*q^3*log(d) + f^2*
p^2*q^2*log(c))*b^3), x)
```

3.455.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5889 vs. $2(426) = 852$.

Time = 0.54 (sec) , antiderivative size = 5889, normalized size of antiderivative = 13.63

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
-1/2*((f*x + e)*b^2*f^2*g^2*p^2*q^2*log(f*x + e) + 4*(f*x + e)^2*b^2*f*g*h
*p^2*q^2*log(f*x + e) - 2*(f*x + e)*b^2*e*f*g*h*p^2*q^2*log(f*x + e) + 3*(
f*x + e)^3*b^2*h^2*p^2*q^2*log(f*x + e) - 4*(f*x + e)^2*b^2*e*h^2*p^2*q^2*
log(f*x + e) + (f*x + e)*b^2*e^2*h^2*p^2*q^2*log(f*x + e) - b^2*f^2*g^2*p^
2*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q)
)*log(f*x + e)^2/(c^(1/(p*q))*d^(1/p)) + 2*b^2*e*f*g*h*p^2*q^2*Ei(log(d)/p
 + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/
(c^(1/(p*q))*d^(1/p)) - b^2*e^2*h^2*p^2*q^2*Ei(log(d)/p + log(c)/(p*q) + a
/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/(c^(1/(p*q))*d^(1/p
)) + (f*x + e)*b^2*f^2*g^2*p^2*q^2 + 2*(f*x + e)^2*b^2*f*g*h*p^2*q^2 - 2*(
f*x + e)*b^2*e*f*g*h*p^2*q^2 + (f*x + e)^3*b^2*h^2*p^2*q^2 - 2*(f*x + e)^2
*b^2*e*h^2*p^2*q^2 + (f*x + e)*b^2*e^2*h^2*p^2*q^2 - 8*b^2*f*g*h*p^2*q^2*E
i(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p
*q))*log(f*x + e)^2/(c^(2/(p*q))*d^(2/p)) + 8*b^2*e*h^2*p^2*q^2*Ei(2*log(d
)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(
f*x + e)^2/(c^(2/(p*q))*d^(2/p)) + (f*x + e)*b^2*f^2*g^2*p*q^2*log(d) + 4*
(f*x + e)^2*b^2*f*g*h*p*q^2*log(d) - 2*(f*x + e)*b^2*e*f*g*h*p*q^2*log(d)
+ 3*(f*x + e)^3*b^2*h^2*p*q^2*log(d) - 4*(f*x + e)^2*b^2*e*h^2*p*q^2*log(d
) + (f*x + e)*b^2*e^2*h^2*p*q^2*log(d) - 2*b^2*f^2*g^2*p*q^2*Ei(log(d)/p +
 log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*l...
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{{\left(g+h\,x\right)}^2}{{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^3}\,dx$$

input
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

output
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^3, x)
```

3.456 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

3.456.1 Optimal result

Integrand size = 26, antiderivative size = 322

$$\begin{aligned}\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx &= \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^2p^3q^3}\\ &+\frac{2e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^2p^3q^3}\\ &-\frac{(e+fx)(g+hx)}{2bfpq\,(a+b\log(c(d(e+fx)^p)^q))^2}+\frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\,(a+b\log(c(d(e+fx)^p)^q))}\\ &-\frac{(e+fx)(g+hx)}{b^2fp^2q^2\,(a+b\log(c(d(e+fx)^p)^q))}\end{aligned}$$

output
```
1/2*(-e*h+f*g)*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(a/b/p
/q)/f^2/p^3/q^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))+2*h*(f*x+e)^2*Ei(2*(a+b*ln(c
*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(2*a/b/p/q)/f^2/p^3/q^3/((c*(d*(f*x+e)^p)
^q)^(2/p/q))-1/2*(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^2+1/2
*(-e*h+f*g)*(f*x+e)/b^2/f^2/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))-(f*x+e)*(h
*x+g)/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

3.456.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = -\frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(-e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)}{bpq}\right.\right.}{}$$

input
```
Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
-1/2*((e + f*x)*(-(E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q
))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d
*(e + f*x)^p)^q])^2) - 4*h*(e + f*x)*ExpIntegralEi[(2*(a + b*Log[c*(d*(e +
 f*x)^p)^q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + b*E^((2*a)/(b*
p*q))*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(b*f*p*q*(g + h*x) + a*(f*g + e*
h + 2*f*h*x) + b*(e*h + f*(g + 2*h*x))*Log[c*(d*(e + f*x)^p)^q])))/(b^3*E^
((2*a)/(b*p*q))*f^2*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(a + b*Log[c*(
d*(e + f*x)^p)^q])^2)
```

3.456.3 Rubi [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.65, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2895, 2847, 2836, 2734, 2737, 2609, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$$
$$\Big\downarrow \; 2895$$
$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}dx$$
$$\Big\downarrow \; 2847$$

$$-\frac{(fg-eh)\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{2bfpq}+\frac{\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{bpq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{(fg-eh)\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^2}d(e+fx)}{2bf^2pq}+\frac{\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{bpq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2734}$$

$$-\frac{(fg-eh)\left(\frac{\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{bpq}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}+\frac{\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{bpq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2737}$$

$$-\frac{(fg-eh)\left(\frac{(e+fx)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log(cd^q(e+fx)^{pq})}{bp^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}+\frac{\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{bpq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}dx}{bpq}-\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2847}$$

3.456. $\displaystyle\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

$$\frac{-\frac{(fg-eh)\int \frac{1}{a+b\log(c(d(e+fx)^p)^q)}dx}{bfpq}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq(a+b\log(c(d(e+fx)^p)^q))}}{bpq}-\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \; 2836$$

$$\frac{-\frac{(fg-eh)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{bf^2pq}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq(a+b\log(c(d(e+fx)^p)^q))}}{bpq}-\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \; 2737$$

$$\frac{-\frac{(e+fx)(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{\left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log(cd^q(e+fx)^{pq})}{bf^2p^2q^2}+\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)(g+hx)}{bfpq(a+b\log(c(d(e+fx)^p)^q))}}{bpq}-\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \; 2609$$

$$\frac{\frac{2\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}dx}{bpq}-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}-\frac{(e+fx)(g+hx)}{bfpq(a+b\log(c(d(e+fx)^p)^q))}}{bpq}-\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \; 2846$$

3.456. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

$$\frac{\frac{2\int\left(\frac{fg-eh}{f(a+b\log(c(d(e+fx)^p)^q))}+\frac{h(e+fx)}{f(a+b\log(c(d(e+fx)^p)^q))}\right)dx}{bpq}-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}-}{bpq}$$

$$\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}-$$

$$\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(fg-eh)\left(\frac{(e+fx)e^{-\frac{a}{bpq}}(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}\right)}{2bf^2pq}+$$

$$\frac{-\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2f^2p^2q^2}+\frac{2\left(\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a}{}\right.}{bf^2pq}\right.}{bpq}}{}$$

$$\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

input
```
Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

output
```
-1/2*((f*g - e*h)*(((e + f*x)*ExpIntegralEi[(a + b*Log[c*d^q*(e + f*x)^(p*
q)])/(b*p*q)])/(b^2*E^(a/(b*p*q))*p^2*q^2*(c*d^q*(e + f*x)^(p*q))^(1/(p*q)
)) - (e + f*x)/(b*p*q*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))))/(b*f^2*p*q) -
((e + f*x)*(g + h*x))/(2*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2) + (-(
((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*d^q*(e + f*x)^(p*q)])/(b
*p*q)])/(b^2*E^(a/(b*p*q))*f^2*p^2*q^2*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))))
 + (2*(((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q
])/(b*p*q)])/(b*E^(a/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (
h*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])
/(b*E^((2*a)/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))))/(b*p*q) -
 ((e + f*x)*(g + h*x))/(b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])))/(b*p*q)
```

3.456. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

3.456.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

3.456. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.456.4 Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}dx$$

input
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

output
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(317) = 634$.

Time = 0.39 (sec) , antiderivative size = 931, normalized size of antiderivative = 2.89

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
1/2*(((b^2*f*g - b^2*e*h)*p^2*q^2*log(f*x + e)^2 + (b^2*f*g - b^2*e*h)*q^2
*log(d)^2 + a^2*f*g - a^2*e*h + (b^2*f*g - b^2*e*h)*log(c)^2 + 2*((b^2*f*g
 - b^2*e*h)*p*q^2*log(d) + (b^2*f*g - b^2*e*h)*p*q*log(c) + (a*b*f*g - a*b
*e*h)*p*q)*log(f*x + e) + 2*(a*b*f*g - a*b*e*h)*log(c) + 2*((b^2*f*g - b^2
*e*h)*q*log(c) + (a*b*f*g - a*b*e*h)*q)*log(d))*e^((b*q*log(d) + b*log(c)
+ a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q
))) - (b^2*e*f*g*p^2*q^2 + (a*b*e*f*g + a*b*e^2*h)*p*q + (b^2*f^2*h*p^2*q^
2 + 2*a*b*f^2*h*p*q)*x^2 + ((b^2*f^2*g + b^2*e*f*h)*p^2*q^2 + (a*b*f^2*g +
 3*a*b*e*f*h)*p*q)*x + (2*b^2*f^2*h*p^2*q^2*x^2 + (b^2*f^2*g + 3*b^2*e*f*h
)*p^2*q^2*x + (b^2*e*f*g + b^2*e^2*h)*p^2*q^2)*log(f*x + e) + (2*b^2*f^2*h
*p*q*x^2 + (b^2*f^2*g + 3*b^2*e*f*h)*p*q*x + (b^2*e*f*g + b^2*e^2*h)*p*q)*
log(c) + (2*b^2*f^2*h*p*q^2*x^2 + (b^2*f^2*g + 3*b^2*e*f*h)*p*q^2*x + (b^2
*e*f*g + b^2*e^2*h)*p*q^2)*log(d))*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q
)) + 4*(b^2*h*p^2*q^2*log(f*x + e)^2 + b^2*h*q^2*log(d)^2 + b^2*h*log(c)^2
 + 2*a*b*h*log(c) + a^2*h + 2*(b^2*h*p*q^2*log(d) + b^2*h*p*q*log(c) + a*b
*h*p*q)*log(f*x + e) + 2*(b^2*h*q*log(c) + a*b*h*q)*log(d))*log_integral((
f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-2
*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + b^
5*f^2*p^3*q^5*log(d)^2 + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^3*lo
g(c) + a^2*b^3*f^2*p^3*q^3 + 2*(b^5*f^2*p^4*q^5*log(d) + b^5*f^2*p^4*q^...
```

3.456.6 Sympy [F]

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{g+hx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^3}\,dx$$

input
```
integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**3, x)
```

3.456.7 Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^p d)^q c)+a)^3}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a*f^2*h + (f^2*h*p*q + 2*f^2*h*q*log(d) + 2*f^2*h*log(c))*b)*x^2
+ (e*f*g + e^2*h)*a + (e*f*g*p*q + (e*f*g + e^2*h)*log(c) + (e*f*g*q + e^2
*h*q)*log(d))*b + ((f^2*g + 3*e*f*h)*a + (f^2*g*p*q + e*f*h*p*q + (f^2*g +
 3*e*f*h)*log(c) + (f^2*g*q + 3*e*f*h*q)*log(d))*b)*x + (2*b*f^2*h*x^2 + (
f^2*g + 3*e*f*h)*b*x + (e*f*g + e^2*h)*b)*log(((f*x + e)^p)^q))/(b^4*f^2*p
^2*q^2*log(((f*x + e)^p)^q)^2 + a^2*b^2*f^2*p^2*q^2 + 2*(f^2*p^2*q^3*log(d
) + f^2*p^2*q^2*log(c))*a*b^3 + (f^2*p^2*q^4*log(d)^2 + 2*f^2*p^2*q^3*log(
c)*log(d) + f^2*p^2*q^2*log(c)^2)*b^4 + 2*(a*b^3*f^2*p^2*q^2 + (f^2*p^2*q^
3*log(d) + f^2*p^2*q^2*log(c))*b^4)*log(((f*x + e)^p)^q)) + integrate(1/2*
(4*f*h*x + f*g + 3*e*h)/(b^3*f*p^2*q^2*log(((f*x + e)^p)^q) + a*b^2*f*p^2*
q^2 + (f*p^2*q^3*log(d) + f*p^2*q^2*log(c))*b^3), x)
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11278 vs. $2(317) = 634$.

Time = 0.55 (sec) , antiderivative size = 11278, normalized size of antiderivative = 35.02

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
-1/2*(f*x + e)*b^2*f*g*p^2*q^2*log(f*x + e)/(b^5*f^2*p^5*q^5*log(f*x + e)^
2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)
*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*
b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3
*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) - (f*x + e
)^2*b^2*h*p^2*q^2*log(f*x + e)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2
*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5
*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q
^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) +
 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*(f*x + e)*b^2*e*h
*p^2*q^2*log(f*x + e)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*
log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*
q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)
*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*
f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*b^2*f*g*p^2*q^2*Ei(log(d)/
p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2
/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d)
+ 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b
^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^
3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c...
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{g+h\,x}{\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)^3}\,dx$$

input
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

output
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^3, x)
```

3.457 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

3.457.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3fp^3q^3} - \frac{e+fx}{2bfpq(a+b\log(c(d(e+fx)^p)^q))^2} - \frac{e+fx}{2b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))}$$

output
```
1/2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(a/b/p/q)/f/p^3/q
^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))+1/2*(-f*x-e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)
^p)^q))^2+1/2*(-f*x-e)/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

3.457.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = -\frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(-\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log(c(d(e+fx)^p)^q))^2\right.}{2b^3fp^3q^3(a+b\log(c(d(e+fx)^p)^q))^2}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3),x]
```

output
```
-1/2*((e + f*x)*(-(ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]
*(a + b*Log[c*(d*(e + f*x)^p)^q])^2) + b*E^(a/(b*p*q))*p*q*(c*(d*(e + f*x)
^p)^q)^(1/(p*q))*(a + b*p*q + b*Log[c*(d*(e + f*x)^p)^q])))/(b^3*E^(a/(b*p
*q))*f*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(a + b*Log[c*(d*(e + f*x)^p
)^q])^2)
```

3.457.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2895, 2836, 2734, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^3}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^2}d(e+fx)}{2bpq}-\frac{e+fx}{2bpq(a+b\log(cd^q(e+fx)^{pq}))^2}}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\frac{\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}d(e+fx)}{bpq}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}}{2bpq}-\frac{e+fx}{2bpq(a+b\log(cd^q(e+fx)^{pq}))^2}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{\frac{(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{\left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}}{a+b\log(cd^q(e+fx)^{pq})}d\log(cd^q(e+fx)^{pq})}{bp^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}}{2bpq}-\frac{e+fx}{2bpq(a+b\log(cd^q(e+fx)^{pq}))^2}}{f}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)}{b^2p^2q^2}-\frac{e+fx}{bpq(a+b\log(cd^q(e+fx)^{pq}))}}{2bpq}-\frac{e+fx}{2bpq(a+b\log(cd^q(e+fx)^{pq}))^2}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3),x]
```

output
```
(-1/2*(e + f*x)/(b*p*q*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^2) + (((e + f*x)
*ExpIntegralEi[(a + b*Log[c*d^q*(e + f*x)^(p*q)])/(b*p*q)])/(b^2*E^(a/(b*p
*q))*p^2*q^2*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))) - (e + f*x)/(b*p*q*(a + b*
Log[c*d^q*(e + f*x)^(p*q)])))/(2*b*p*q))/f
```

3.457.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.457. $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.457.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.457.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(163) = 326.

Time = 0.32 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.63

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3}}\,dx =$$
$$-\frac{\left((b^{2}ep^{2}q^{2}+abepq+(b^{2}fp^{2}q^{2}+abfpq)x+(b^{2}fp^{2}q^{2}x+b^{2}ep^{2}q^{2})\log\left(fx+e\right)+(b^{2}fpqx+b^{2}epq)\log\left(\right.\right.\right.}{2\left(b^{5}fp^{5}q^{5}\log\left(fx\right.\right.}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*e*p^2*q^2 + a*b*e*p*q + (b^2*f*p^2*q^2 + a*b*f*p*q)*x + (b^2*f*
p^2*q^2*x + b^2*e*p^2*q^2)*log(f*x + e) + (b^2*f*p*q*x + b^2*e*p*q)*log(c)
 + (b^2*f*p*q^2*x + b^2*e*p*q^2)*log(d))*e^((b*q*log(d) + b*log(c) + a)/(b
*p*q)) - (b^2*p^2*q^2*log(f*x + e)^2 + b^2*q^2*log(d)^2 + b^2*log(c)^2 + 2
*a*b*log(c) + a^2 + 2*(b^2*p*q^2*log(d) + b^2*p*q*log(c) + a*b*p*q)*log(f*
x + e) + 2*(b^2*q*log(c) + a*b*q)*log(d))*log_integral((f*x + e)*e^((b*q*l
og(d) + b*log(c) + a)/(b*p*q))))*e^(-(b*q*log(d) + b*log(c) + a)/(b*p*q))/
(b^5*f*p^5*q^5*log(f*x + e)^2 + b^5*f*p^3*q^5*log(d)^2 + b^5*f*p^3*q^3*log
(c)^2 + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3 + 2*(b^5*f*p^4*q^5*lo
g(d) + b^5*f*p^4*q^4*log(c) + a*b^4*f*p^4*q^4)*log(f*x + e) + 2*(b^5*f*p^3
*q^4*log(c) + a*b^4*f*p^3*q^4)*log(d))
```

3.457.6 Sympy [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx=\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-3), x)
```

3.457.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx=\int \frac{1}{(b\log(((fx+e)^pd)^qc)+a)^3}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

output
```
-1/2*((e*p*q + e*q*log(d) + e*log(c))*b + a*e + ((f*p*q + f*q*log(d) + f*l
og(c))*b + a*f)*x + (b*f*x + b*e)*log(((f*x + e)^p)^q))/(b^4*f*p^2*q^2*log
(((f*x + e)^p)^q)^2 + a^2*b^2*f*p^2*q^2 + 2*(f*p^2*q^3*log(d) + f*p^2*q^2*
log(c))*a*b^3 + (f*p^2*q^4*log(d)^2 + 2*f*p^2*q^3*log(c)*log(d) + f*p^2*q^
2*log(c)^2)*b^4 + 2*(a*b^3*f*p^2*q^2 + (f*p^2*q^3*log(d) + f*p^2*q^2*log(c
))*b^4)*log(((f*x + e)^p)^q)) + integrate(1/2/(b^3*p^2*q^2*log(((f*x + e)^
p)^q) + a*b^2*p^2*q^2 + (p^2*q^3*log(d) + p^2*q^2*log(c))*b^3), x)
```

3.457.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3401 vs. 2(163) = 326.

Time = 0.40 (sec) , antiderivative size = 3401, normalized size of antiderivative = 20.12

$$\int \frac{1}{(a + b\log(c(d(e + fx)^p)^q))^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
-1/2*(f*x + e)*b^2*p^2*q^2*log(f*x + e)/(b^5*f*p^5*q^5*log(f*x + e)^2 + 2*
b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) +
b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*
log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^
4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3) + 1/2*b^2*p^2*q^2*Ei(log(d)/p + lo
g(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/((b^5
*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*
p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*l
og(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a
*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3)*c^(1
/(p*q))*d^(1/p)) - 1/2*(f*x + e)*b^2*p^2*q^2/(b^5*f*p^5*q^5*log(f*x + e)^2
 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(
c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3
*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2
*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3) - 1/2*(f*x + e)*b^2*p*q^2*log
(d)/(b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) +
2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p
^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)
^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q
^3) + b^2*p*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*...
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\log(c(d(e + fx)^p)^q))^3} \, dx = \int \frac{1}{\left(a + b\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)\right)^3} \, dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^3, x)
```

3.458 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

3.458.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.458.2 Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3), x]
```

3.458.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
```

output
```
$Aborted
```

3.458.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.458.4 Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.458.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.82

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(hx+g)\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*h*x + a^3*g + (b^3*h*x + b^3*g)*log(((f*x + e)^p*d)^q*c)^3
 + 3*(a*b^2*h*x + a*b^2*g)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*h*x + a^2
*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

3.458.6 Sympy [N/A]

Not integrable

Time = 17.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^3(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**3*(g + h*x)), x)
```

3.458.7 Maxima [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 1116, normalized size of antiderivative = 39.86

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(hx+g)\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

output
```
-1/2*(b*f^2*h*p*q*x^2 + (e*f*g - e^2*h)*a + (e*f*g*p*q + (e*f*g - e^2*h)*l
og(c) + (e*f*g*q - e^2*h*q)*log(d))*b + ((f^2*g - e*f*h)*a + (f^2*g*p*q +
e*f*h*p*q + (f^2*g - e*f*h)*log(c) + (f^2*g*q - e*f*h*q)*log(d))*b)*x + ((
f^2*g - e*f*h)*b*x + (e*f*g - e^2*h)*b)*log(((f*x + e)^p)^q))/(a^2*b^2*f^2
*g^2*p^2*q^2 + 2*(f^2*g^2*p^2*q^3*log(d) + f^2*g^2*p^2*q^2*log(c))*a*b^3 +
 (f^2*g^2*p^2*q^4*log(d)^2 + 2*f^2*g^2*p^2*q^3*log(c)*log(d) + f^2*g^2*p^2
*q^2*log(c)^2)*b^4 + (a^2*b^2*f^2*h^2*p^2*q^2 + 2*(f^2*h^2*p^2*q^3*log(d)
+ f^2*h^2*p^2*q^2*log(c))*a*b^3 + (f^2*h^2*p^2*q^4*log(d)^2 + 2*f^2*h^2*p^
2*q^3*log(c)*log(d) + f^2*h^2*p^2*q^2*log(c)^2)*b^4)*x^2 + (b^4*f^2*h^2*p^
2*q^2*x^2 + 2*b^4*f^2*g*h*p^2*q^2*x + b^4*f^2*g^2*p^2*q^2)*log(((f*x + e)^
p)^q)^2 + 2*(a^2*b^2*f^2*g*h*p^2*q^2 + 2*(f^2*g*h*p^2*q^3*log(d) + f^2*g*h
*p^2*q^2*log(c))*a*b^3 + (f^2*g*h*p^2*q^4*log(d)^2 + 2*f^2*g*h*p^2*q^3*log
(c)*log(d) + f^2*g*h*p^2*q^2*log(c)^2)*b^4)*x + 2*(a*b^3*f^2*g^2*p^2*q^2 +
 (f^2*g^2*p^2*q^3*log(d) + f^2*g^2*p^2*q^2*log(c))*b^4 + (a*b^3*f^2*h^2*p^
2*q^2 + (f^2*h^2*p^2*q^3*log(d) + f^2*h^2*p^2*q^2*log(c))*b^4)*x^2 + 2*(a*
b^3*f^2*g*h*p^2*q^2 + (f^2*g*h*p^2*q^3*log(d) + f^2*g*h*p^2*q^2*log(c))*b^
4)*x)*log(((f*x + e)^p)^q)) + integrate(1/2*(f^2*g^2 - 3*e*f*g*h + 2*e^2*h
^2 - (f^2*g*h - e*f*h^2)*x)/(a*b^2*f^2*g^3*p^2*q^2 + (f^2*g^3*p^2*q^3*log(
d) + f^2*g^3*p^2*q^2*log(c))*b^3 + (a*b^2*f^2*h^3*p^2*q^2 + (f^2*h^3*p^2*q
^3*log(d) + f^2*h^3*p^2*q^2*log(c))*b^3)*x^3 + 3*(a*b^2*f^2*g*h^2*p^2*q...
```

3.458.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^3}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^3), x)
```

3.458.9 Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(g+h\,x)\ \left(a+b\ln\left(c\left(d\,(e+f\,x)^p\right)^q\right)\right)^3}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3), x)
```

3.459 $\int \frac{1}{(g+hx)^2(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$

3.459.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \text{Int}\left(\frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3},x\right)$$

output
```
Unintegrable(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.459.2 Mathematica [N/A]

Not integrable

Time = 23.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$

input
```
Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
```

output
```
Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3), x]
```

3.459.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}dx$$
$$\Big\downarrow \text{ 2896}$$
$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}dx$$

input
```
Int[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
```

output
```
$Aborted
```

3.459.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.459.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)^2 (a+b\ln(c(d(fx+e)^p)^q))^3}dx$$

input
```
int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

output
```
int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

3.459.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.89

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(hx+g)^2\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas"
)
```

output
```
integral(1/(a^3*h^2*x^2 + 2*a^3*g*h*x + a^3*g^2 + (b^3*h^2*x^2 + 2*b^3*g*h
*x + b^3*g^2)*log(((f*x + e)^p*d)^q*c)^3 + 3*(a*b^2*h^2*x^2 + 2*a*b^2*g*h*
x + a*b^2*g^2)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*h^2*x^2 + 2*a^2*b*g*h
*x + a^2*b*g^2)*log(((f*x + e)^p*d)^q*c)), x)
```

3.459.6 Sympy [N/A]

Not integrable

Time = 135.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^3\left(g+hx\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**3*(g + h*x)**2), x)
```

3.459.7 Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 1486, normalized size of antiderivative = 53.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(hx+g)^2\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima"
)
```

output
```
1/2*((a*f^2*h - (f^2*h*p*q - f^2*h*q*log(d) - f^2*h*log(c))*b)*x^2 - (e*f*
g - 2*e^2*h)*a - (e*f*g*p*q + (e*f*g - 2*e^2*h)*log(c) + (e*f*g*q - 2*e^2*
h*q)*log(d))*b - ((f^2*g - 3*e*f*h)*a + (f^2*g*p*q + e*f*h*p*q + (f^2*g -
3*e*f*h)*log(c) + (f^2*g*q - 3*e*f*h*q)*log(d))*b)*x + (b*f^2*h*x^2 - (f^2
*g - 3*e*f*h)*b*x - (e*f*g - 2*e^2*h)*b)*log(((f*x + e)^p)^q))/(a^2*b^2*f^
2*g^3*p^2*q^2 + 2*(f^2*g^3*p^2*q^3*log(d) + f^2*g^3*p^2*q^2*log(c))*a*b^3
+ (f^2*g^3*p^2*q^4*log(d)^2 + 2*f^2*g^3*p^2*q^3*log(c)*log(d) + f^2*g^3*p^
2*q^2*log(c)^2)*b^4 + (a^2*b^2*f^2*h^3*p^2*q^2 + 2*(f^2*h^3*p^2*q^3*log(d)
 + f^2*h^3*p^2*q^2*log(c))*a*b^3 + (f^2*h^3*p^2*q^4*log(d)^2 + 2*f^2*h^3*p
^2*q^3*log(c)*log(d) + f^2*h^3*p^2*q^2*log(c)^2)*b^4)*x^3 + 3*(a^2*b^2*f^2
*g*h^2*p^2*q^2 + 2*(f^2*g*h^2*p^2*q^3*log(d) + f^2*g*h^2*p^2*q^2*log(c))*a
*b^3 + (f^2*g*h^2*p^2*q^4*log(d)^2 + 2*f^2*g*h^2*p^2*q^3*log(c)*log(d) + f
^2*g*h^2*p^2*q^2*log(c)^2)*b^4)*x^2 + (b^4*f^2*h^3*p^2*q^2*x^3 + 3*b^4*f^2
*g*h^2*p^2*q^2*x^2 + 3*b^4*f^2*g^2*h*p^2*q^2*x + b^4*f^2*g^3*p^2*q^2)*log(
((f*x + e)^p)^q)^2 + 3*(a^2*b^2*f^2*g^2*h*p^2*q^2 + 2*(f^2*g^2*h*p^2*q^3*l
og(d) + f^2*g^2*h*p^2*q^2*log(c))*a*b^3 + (f^2*g^2*h*p^2*q^4*log(d)^2 + 2*
f^2*g^2*h*p^2*q^3*log(c)*log(d) + f^2*g^2*h*p^2*q^2*log(c)^2)*b^4)*x + 2*(
a*b^3*f^2*g^3*p^2*q^2 + (f^2*g^3*p^2*q^3*log(d) + f^2*g^3*p^2*q^2*log(c))*
b^4 + (a*b^3*f^2*h^3*p^2*q^2 + (f^2*h^3*p^2*q^3*log(d) + f^2*h^3*p^2*q^2*l
og(c))*b^4)*x^3 + 3*(a*b^3*f^2*g*h^2*p^2*q^2 + (f^2*g*h^2*p^2*q^3*log(d...
```

3.459.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(hx+g)^2\left(b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)^3}\,dx$$

input
```
integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^3), x)
```

3.459.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3} dx = \int \frac{1}{(g+h\,x)^2 \,(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^3} \,dx$$

input
```
int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3),x)
```

output
```
int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3), x)
```

3.460 $\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$

3.460.1 Optimal result

Integrand size = 30, antiderivative size = 488

$$\begin{aligned}
&\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx \\
&= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3} \\
&\quad-\frac{\sqrt{b}e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{p}\sqrt{\frac{\pi}{2}}\sqrt{q}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3} \\
&\quad-\frac{\sqrt{b}e^{-\frac{3a}{bpq}}h^2\sqrt{p}\sqrt{\frac{\pi}{3}}\sqrt{q}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{6f^3} \\
&\quad+\frac{(fg-eh)^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3} \\
&\quad+\frac{h(fg-eh)(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3} \\
&\quad+\frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}
\end{aligned}$$

output
```
-1/18*h^2*(f*x+e)^3*erfi(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)
/p^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*3^(1/2)*Pi^(1/2)*q^(1/2)/exp(3*a/b/p/q)/
f^3/((c*(d*(f*x+e)^p)^q)^(3/p/q))-1/4*h*(-e*h+f*g)*(f*x+e)^2*erfi(2^(1/2)*
(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)
*2^(1/2)*Pi^(1/2)*q^(1/2)/exp(2*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(2/p/q))
-1/2*(-e*h+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p
^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*Pi^(1/2)*q^(1/2)/exp(a/b/p/q)/f^3/((c*(d*(
f*x+e)^p)^q)^(1/p/q))+(-e*h+f*g)^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/
2)/f^3+h*(-e*h+f*g)*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3+1/3*h^
2*(f*x+e)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3
```

3.460.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.94

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{(e+fx)\left(-18\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{p}\sqrt{\pi}\sqrt{q}(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+9\sqrt{b}e^{-\frac{2a}{bpq}}h(-\right.}{}$$

input
```
Integrate[(g + h*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
((e + f*x)*((-18*Sqrt[b]*(f*g - e*h)^2*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqrt[
a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^(a/(b*p*q))
*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (9*Sqrt[b]*h*(-(f*g) + e*h)*Sqrt[p]*Sq
rt[2*Pi]*Sqrt[q]*(e + f*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^
q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^
(2/(p*q))) - (2*Sqrt[b]*h^2*Sqrt[p]*Sqrt[3*Pi]*Sqrt[q]*(e + f*x)^2*Erfi[(S
qrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(
E^((3*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) + 36*(f*g - e*h)^2*Sqrt
[a + b*Log[c*(d*(e + f*x)^p)^q]] + 36*h*(f*g - e*h)*(e + f*x)*Sqrt[a + b*L
og[c*(d*(e + f*x)^p)^q]] + 12*h^2*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x
)^p)^q]]))/(36*f^3)
```

3.460.3 Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

$$\Big\downarrow \text{ 2848}$$

$$\int \left(\frac{(fg-eh)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2}+\frac{2h(e+fx)(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2}+\frac{h^2(e+fx)^2\sqrt{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}h\sqrt{p}\sqrt{q}(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3}-\frac{\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}h^2\sqrt{p}\sqrt{q}(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{6f^3}+\frac{h(e+fx)^2(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}+\frac{(e+fx)(fg-eh)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}+\frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}$$

input
```
Int[(g + h*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
-1/2*(Sqrt[b]*(f*g - e*h)^2*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a
 + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*
f^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - (Sqrt[b]*h*(f*g - e*h)*Sqrt[p]*Sqrt
[Pi/2]*Sqrt[q]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^
q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(2*E^((2*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^
p)^q)^(2/(p*q))) - (Sqrt[b]*h^2*Sqrt[p]*Sqrt[Pi/3]*Sqrt[q]*(e + f*x)^3*Erf
i[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])
])/(6*E^((3*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) + ((f*g - e*h
)^2*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f^3 + (h*(f*g - e*h)*(
e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f^3 + (h^2*(e + f*x)^3*Sq
rt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(3*f^3)
```

3.460.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.460.4 Maple [F]

$$\int (hx+g)^2 \sqrt{a+b\ln (c\,(d\,(fx+e)^p)^q)}dx$$

input
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.460.5 Fricas [F(-2)]

Exception generated.

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.460.6 Sympy [F]

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}(g+hx)^2\,dx$$

input
```
integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**2, x)
```

3.460.7 Maxima [F]

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (hx+g)^2\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^2*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.460.8 Giac [F]

$$\int (g+hx)^2\sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}\,dx=\int (hx+g)^2\sqrt{b\log \left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^2*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2\sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}\,dx=\int (g+h\,x)^2\,\sqrt{a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

input
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.461 $\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$

3.461.1 Optimal result

Integrand size = 28, antiderivative size = 311

$$\begin{aligned}\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx &= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^2}\\ &\quad-\frac{\sqrt{b}e^{-\frac{2a}{bpq}}h\sqrt{p}\sqrt{\frac{\pi}{2}}\sqrt{q}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2}\\ &\quad+\frac{(fg-eh)(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2}\\ &\quad+\frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}\end{aligned}$$

output
```
-1/8*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^
(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*2^(1/2)*Pi^(1/2)*q^(1/2)/exp(2*a/b/p/q)/f^2
/((c*(d*(f*x+e)^p)^q)^(2/p/q))-1/2*(-e*h+f*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f
*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*Pi^(1/2)*q^(1/
2)/exp(a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))+(-e*h+f*g)*(f*x+e)*(a+b*
ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^2+1/2*h*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q
))^(1/2)/f^2
```

3.461.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx =$$
$$-\frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(4\sqrt{b}e^{\frac{a}{bpq}}(fg-eh)\sqrt{p}\sqrt{\pi}\sqrt{q}(c(d(e+fx)^p)^q)^{\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.\right.}{}$$

input
```
Integrate[(g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
-1/8*((e + f*x)*(4*Sqrt[b]*E^(a/(b*p*q))*(f*g - e*h)*Sqrt[p]*Sqrt[Pi]*Sqrt
[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^
q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])] + Sqrt[b]*h*Sqrt[p]*Sqrt[2*Pi]*Sqrt[q]*(e +
 f*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]
*Sqrt[q])] - 4*E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(2*f*g -
e*h + f*h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]))/(E^((2*a)/(b*p*q))*f^2
*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

3.461.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$\Big\downarrow \text{2895}$$
$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f}+\frac{h(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}h\sqrt{p}\sqrt{q}(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2}+\frac{(e+fx)(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2}+\frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$

input
```
Int[(g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
-1/2*(Sqrt[b]*(f*g - e*h)*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a +
 b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*f^
2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - (Sqrt[b]*h*Sqrt[p]*Sqrt[Pi/2]*Sqrt[q]
*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*
Sqrt[p]*Sqrt[q])])/(4*E^((2*a)/(b*p*q))*f^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q)
)) + ((f*g - e*h)*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f^2 + (h
*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(2*f^2)
```

3.461.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.461.4 Maple [F]

$$\int (hx+g)\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.461.5 Fricas [F(-2)]

Exception generated.

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.461.6 Sympy [F]

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}(g+hx)\,dx$$

input
```
integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x), x)
```

3.461.7 Maxima [F]

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (hx+g)\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}\,dx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.461.8 Giac [F]

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (hx+g)\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}\,dx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
```

output
```
integrate((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (g+h\,x)\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

input
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.462 $\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$

3.462.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bpq}}\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f} + \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f}$$

output
```
-1/2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2
))*b^(1/2)*p^(1/2)*Pi^(1/2)*q^(1/2)/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1
/p/q))+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f
```

3.462.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{(e+fx)\left(-\sqrt{b}e^{-\frac{a}{bpq}}\sqrt{p}\sqrt{\pi}\sqrt{q}\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right.\right.}}{2f}$$

input
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
((e + f*x)*(-((Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqrt[a + b*Log[c*(d*(
e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*(d*(e + f*x)
^p)^q)^(1/(p*q)))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]))/(2*f)
```

3.462.3 Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2895, 2836, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}-\frac{1}{2}bpq\int \frac{1}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}-\frac{1}{2}b(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{\left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}d\log\left(cd^q(e+fx)^{pq}\right)}{f}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}-(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log\left(\right.}}{f}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
(-1/2*(Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^
q*(e + f*x)^(p*q)]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*d^q*(e +
 f*x)^(p*q))^(1/(p*q))) + (e + f*x)*Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*q)]]
)/f
```

3.462.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.462.4 Maple [F]

$$\int \sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.462.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.462.6 Sympy [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.462.7 Maxima [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.462.8 Giac [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.463 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{g+hx}\,dx$

3.463.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx = \text{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx},x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x)
```

3.463.2 Mathematica [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x),x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x), x]
```

3.463.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}dx$$
$$\Big\downarrow \text{ 2896}$$
$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}dx$$

input
```
Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x),x]
```

output
```
$Aborted
```

3.463.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.463.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x)
```

3.463.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}}{g + hx}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.463.6 Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}}{g + hx}\, dx = \int \frac{\sqrt{a + b\log\left(c\left(d\left(e + fx\right)^p\right)^q\right)}}{g + hx}\, dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x), x)
```

3.463.7 Maxima [N/A]

Not integrable

Time = 11.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}}{g + hx}\, dx = \int \frac{\sqrt{b\log\left(\left((fx + e)^p d\right)^q c\right) + a}}{hx + g}\, dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.463.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx=\int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x, algorithm="giac")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.463.9 Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x), x)
```

3.464 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{(g+hx)^2} dx$

3.464.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2} dx = \text{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}, x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x)
```

3.464.2 Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2} dx = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2} dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2,x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2, x]
```

3.464.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}dx$$
$$\Big\downarrow \text{ 2896}$$
$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}dx$$

input
```
Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2,x]
```

output
```
$Aborted
```

3.464.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.464.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{\left(hx+g\right)^{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x)
```

3.464.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.464.6 Sympy [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}}{\left(g+hx\right)^{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g)**2,x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**2, x)
```

3.464.7 Maxima [N/A]

Not integrable

Time = 10.69 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx = \int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^2, x)
```

3.464.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x, algorithm="giac"
)
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^2, x)
```

3.464.9 Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}{(g+h\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^2,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^2, x)
```

3.465 $\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx$

3.465.1 Optimal result

Integrand size = 30, antiderivative size = 625

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \frac{3b^{3/2}e^{-\frac{a}{bpq}}(fg-eh)^2p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(c\cdots}{\sqrt{b}\sqrt{\cdots}}\right.}{4f^3}$$

$$+\frac{3b^{3/2}e^{-\frac{2a}{bpq}}h(fg-eh)p^{3/2}\sqrt{\frac{\pi}{2}}q^{3/2}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{8f^3}$$

$$+\frac{b^{3/2}e^{-\frac{3a}{bpq}}h^2p^{3/2}\sqrt{\frac{\pi}{3}}q^{3/2}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{12f^3}$$

$$-\frac{3b(fg-eh)^2pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3}$$

$$-\frac{3bh(fg-eh)pq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3}$$

$$-\frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3}$$

$$+\frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$

$$+\frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$

$$+\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3}$$

output
```
(-e*h+f*g)^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^3+h*(-e*h+f*g)*(f
*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^3+1/3*h^2*(f*x+e)^3*(a+b*ln(c*
(d*(f*x+e)^p)^q))^(3/2)/f^3+1/36*b^(3/2)*h^2*p^(3/2)*q^(3/2)*(f*x+e)^3*erf
i(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*3^(1/
2)*Pi^(1/2)/exp(3*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(3/p/q))+3/16*b^(3/2)*
h*(-e*h+f*g)*p^(3/2)*q^(3/2)*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p
)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^3/(
(c*(d*(f*x+e)^p)^q)^(2/p/q))+3/4*b^(3/2)*(-e*h+f*g)^2*p^(3/2)*q^(3/2)*(f*x
+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/
2)/exp(a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))-3/2*b*(-e*h+f*g)^2*p*q*(
f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3-3/4*b*h*(-e*h+f*g)*p*q*(f*x+e
)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3-1/6*b*h^2*p*q*(f*x+e)^3*(a+b*ln(
c*(d*(f*x+e)^p)^q))^(1/2)/f^3
```

3.465.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.87

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^{3/2}\, dx = \frac{(e+fx)\left(144(fg-eh)^2 (a+b\log (c(d(e+fx)^p)^q))^{3/2}+144h(fg-eh)(e\cdots\right.}{}$$

input
```
Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
((e + f*x)*(144*(f*g - e*h)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 144
*h*(f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 48*h^2*(
e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 4*b*h^2*p*q*(e + f*x)^
2*((Sqrt[b]*Sqrt[p]*Sqrt[3*Pi]*Sqrt[q]*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(
e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^((3*a)/(b*p*q))*(c*(d*(e +
 f*x)^p)^q)^(3/(p*q))) - 6*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]) + 27*b*h*
(f*g - e*h)*p*q*(e + f*x)*((Sqrt[b]*Sqrt[p]*Sqrt[2*Pi]*Sqrt[q]*Erfi[(Sqrt[
2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^((
2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - 4*Sqrt[a + b*Log[c*(d*(e
+ f*x)^p)^q]]) + 108*b*(f*g - e*h)^2*p*q*((Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q
]*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E
^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - 2*Sqrt[a + b*Log[c*(d*(e +
 f*x)^p)^q]])))/(144*f^3)
```

3.465.3 Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2848}$$

$$\int \left(\frac{(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}+\frac{2h(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}+\frac{h^2(e+fx\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}hp^{3/2}q^{3/2}(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{8f^3}+\frac{3\sqrt{\pi}b^{3/2}p^{3/2}q^{3/2}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}h^2p^{3/2}q^{3/2}(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{12f^3}+\frac{h(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}+\frac{(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}-\frac{3bhpq(e+fx)^2(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3}-\frac{3bpq(e+fx)(fg-eh)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3}+\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3}-\frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3}$$

input
```
Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

3.465. $\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx$

output
```
(3*b^(3/2)*(f*g - e*h)^2*p^(3/2)*Sqrt[Pi]*q^(3/2)*(e + f*x)*Erfi[Sqrt[a +
b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(4*E^(a/(b*p*q))*f
^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (3*b^(3/2)*h*(f*g - e*h)*p^(3/2)*Sqr
t[Pi/2]*q^(3/2)*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)
^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(8*E^((2*a)/(b*p*q))*f^3*(c*(d*(e + f*x)
^p)^q)^(2/(p*q))) + (b^(3/2)*h^2*p^(3/2)*Sqrt[Pi/3]*q^(3/2)*(e + f*x)^3*Er
fi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q]
)])/(12*E^((3*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) - (3*b*(f*g
 - e*h)^2*p*q*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(2*f^3) - (3
*b*h*(f*g - e*h)*p*q*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(4*
f^3) - (b*h^2*p*q*(e + f*x)^3*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(6*f^3
) + ((f*g - e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f^3 +
 (h*(f*g - e*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f^3 +
(h^2*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/(3*f^3)
```

3.465.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.465.4 Maple [F]

$$\int (hx+g)^2 \left(a+b\ln \left(c{\left(d{\left(fx+e\right)}^{p}\right)}^{q}\right)\right)^{\frac{3}{2}}dx$$

input
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.465.5 Fricas [F(-2)]

Exception generated.

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.465.6 Sympy [F]

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx = \int \left(a+b\log \left(c{\left(d{\left(e+fx\right)}^{p}\right)}^{q}\right)\right)^{\frac{3}{2}} (g+hx)^2\,dx$$

input
```
integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)*(g + h*x)**2, x)
```

3.465.7 Maxima [F]

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \int (hx+g)^2 (b\log \left(((fx+e)^p d)^q c\right)+a)^{\frac{3}{2}} dx$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.465.8 Giac [F]

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \int (hx+g)^2 (b\log \left(((fx+e)^p d)^q c\right)+a)^{\frac{3}{2}} dx$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \int (g+h\,x)^2\,\left(a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{3/2} dx$$

input
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.466 $\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx$

3.466.1 Optimal result

Integrand size = 28, antiderivative size = 396

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \frac{3b^{3/2}e^{-\frac{a}{bpq}}(fg-eh)p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(\cdots}}{\sqrt{b}\sqrt{p}\cdots}\right.}{4f^2}$$
$$+\frac{3b^{3/2}e^{-\frac{2a}{bpq}}hp^{3/2}\sqrt{\frac{\pi}{2}}q^{3/2}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{16f^2}$$
$$-\frac{3b(fg-eh)pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$
$$-\frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2}$$
$$+\frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}$$
$$+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}$$

output
```
(-e*h+f*g)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^2+1/2*h*(f*x+e)^2*(
a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^2+3/32*b^(3/2)*h*p^(3/2)*q^(3/2)*(f*x+e
)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2)
)*2^(1/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(2/p/q))+3/4*b^
(3/2)*(-e*h+f*g)*p^(3/2)*q^(3/2)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^
(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/b/p/q)/f^2/((c*(d*(f*x+e)^p)
^q)^(1/p/q))-3/2*b*(-e*h+f*g)*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2
)/f^2-3/8*b*h*p*q*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^2
```

3.466.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.88

$$\int (g+hx) \left(a+b \log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} \, dx = \frac{(e+fx) \left(32(fg-eh) \left(a+b \log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} + 16h(e+fx) \left(a+b \mathrm{l}\right.\right.}{}$$

input
```
Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
((e + f*x)*(32*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 16*h*(
e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 3*b*h*p*q*(e + f*x)*((Sq
rt[b]*Sqrt[p]*Sqrt[2*Pi]*Sqrt[q]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*
x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^
p)^q)^(2/(p*q))) - 4*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]) + 24*b*(f*g - e
*h)*p*q*((Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqrt[a + b*Log[c*(d*(e + f
*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q
)^(1/(p*q))) - 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])))/(32*f^2)
```

3.466.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{2848}$$

$$\int \left(\frac{(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f}+\frac{h(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}b^{3/2}p^{3/2}q^{3/2}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}hp^{3/2}q^{3/2}(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{16f^2}+\frac{(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}-\frac{3bpq(e+fx)(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}-\frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2}$$

input
```
Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
(3*b^(3/2)*(f*g - e*h)*p^(3/2)*Sqrt[Pi]*q^(3/2)*(e + f*x)*Erfi[Sqrt[a + b*
Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(4*E^(a/(b*p*q))*f^2
*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (3*b^(3/2)*h*p^(3/2)*Sqrt[Pi/2]*q^(3/2
)*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]
*Sqrt[p]*Sqrt[q])])/(16*E^((2*a)/(b*p*q))*f^2*(c*(d*(e + f*x)^p)^q)^(2/(p*
q))) - (3*b*(f*g - e*h)*p*q*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
)/(2*f^2) - (3*b*h*p*q*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(
8*f^2) + ((f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f^
2 + (h*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/(2*f^2)
```

3.466. $\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx$

3.466.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.466.4 Maple [F]

$$\int (hx+g)\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{\frac{3}{2}}dx$$

input
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.466.5 Fricas [F(-2)]

Exception generated.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.466.6 Sympy [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{\frac{3}{2}}(g+hx)\,dx$$

input
```
integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)*(g + h*x), x)
```

3.466.7 Maxima [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int (hx+g)(b\log\left(((fx+e)^pd)^q\,c\right)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.466.8 Giac [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int (hx+g)(b\log\left(((fx+e)^pd)^q\,c\right)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
```

output
```
integrate((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int (g+h\,x)\,\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)^{3/2}\,dx$$

input
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.467 $\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx$

3.467.1 Optimal result

Integrand size = 22, antiderivative size = 176

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx=\frac{3b^{3/2}e^{-\frac{a}{bpq}}p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f}-\frac{3bpq(e+fx)\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{2f}+\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{f}$$

output
```
(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f+3/4*b^(3/2)*p^(3/2)*q^(3/2)*(f
*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(
1/2)/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p/q))-3/2*b*p*q*(f*x+e)*(a+b*l
n(c*(d*(f*x+e)^p)^q))^(1/2)/f
```

3.467.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx=\frac{(e+fx)\left(3b^{3/2}e^{-\frac{a}{bpq}}p^{3/2}\sqrt{\pi}q^{3/2}(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)\right.}{4f}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
((e + f*x)*((3*b^(3/2)*p^(3/2)*Sqrt[Pi]*q^(3/2)*Erfi[Sqrt[a + b*Log[c*(d*(
e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*(d*(e + f*x)
^p)^q)^(1/(p*q))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]*(2*a - 3*b*p*q
+ 2*b*Log[c*(d*(e + f*x)^p)^q])))/(4*f)
```

3.467.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2895, 2836, 2733, 2733, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2} dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log (cd^q(e+fx)^{pq}))^{3/2} d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx) (a+b\log (cd^q(e+fx)^{pq}))^{3/2}-\frac{3}{2}bpq\int \sqrt{a+b\log (cd^q(e+fx)^{pq})}d(e+fx)}{f}$$

$$\Big\downarrow \text{2733}$$

$$\frac{(e+fx) (a+b\log (cd^q(e+fx)^{pq}))^{3/2}-\frac{3}{2}bpq\left((e+fx)\sqrt{a+b\log (cd^q(e+fx)^{pq})}-\frac{1}{2}bpq\int \frac{1}{\sqrt{a+b\log (cd^q(e+fx)^{pq})}}d$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx) (a+b\log (cd^q(e+fx)^{pq}))^{3/2}-\frac{3}{2}bpq\Bigg((e+fx)\sqrt{a+b\log (cd^q(e+fx)^{pq})}-\frac{1}{2}b(e+fx) (cd^q(e+fx)^{pq})^{-}}{f}$$

$\Big\downarrow$ 2611

$$\frac{(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}-\frac{3}{2}bpq\Big((e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}-(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}}{f}$$

$\Big\downarrow$ 2633

$$\frac{(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}-\frac{3}{2}bpq\Big((e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
((e + f*x)*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^(3/2) - (3*b*p*q*(-1/2*(Sqrt
[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)
^(p*q)]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*d^q*(e + f*x)^(p*q)
)^(1/(p*q))) + (e + f*x)*Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*q)]]))/2)/f
```

3.467.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.467.4 Maple [F]

$$\int \left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.467.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.467.6 Sympy [F]

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx=\int (a+b\log (c(d(e+fx)^p)^q))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2), x)
```

3.467.7 Maxima [F]

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx=\int (b\log (((fx+e)^pd)^q\,c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.467.8 Giac [F]

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx=\int (b\log (((fx+e)^pd)^q\,c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2} \, dx = \int (a+b\,\ln (c\,(d\,(e+f\,x)^p)^q))^{3/2} \, dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.468 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$

3.468.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\text{Int}\left(\frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx},x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x)
```

3.468.2 Mathematica [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x),x]
```

output
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x), x]
```

3.468.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$$
$$\Big\downarrow \; 2896$$
$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}dx$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x),x]
```

output
```
$Aborted
```

3.468.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.468.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x)
```

3.468.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^{3/2}}{g + hx} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.468.6 Sympy [N/A]

Not integrable

Time = 52.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^{3/2}}{g + hx} dx = \int \frac{(a + b\log(c(d(e + fx)^p)^q))^{\frac{3}{2}}}{g + hx} dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2)/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)/(g + h*x), x)
```

3.468.7 Maxima [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^{3/2}}{g + hx} dx = \int \frac{(b\log(((fx + e)^p d)^q c) + a)^{\frac{3}{2}}}{hx + g} dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x, algorithm="maxima"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g), x)
```

3.468.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^{\frac{3}{2}}}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g), x)
```

3.468.9 Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x), x)
```

3.469 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx$

3.469.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\text{Int}\left(\frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2},x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x)
```

3.469.2 Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2,x]
```

output
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2, x]
```

3.469.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2} \, dx$$

$$\Big\downarrow \text{2896}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2} dx$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2,x]
```

output
```
$Aborted
```

3.469.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.469.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}{(hx+g)^2} dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x)
```

3.469.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\log (c(d(e + fx)^p)^q))^{3/2}}{(g + hx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.469.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\log (c(d(e + fx)^p)^q))^{3/2}}{(g + hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2)/(h*x+g)**2,x)
```

output
```
Timed out
```

3.469.7 Maxima [N/A]

Not integrable

Time = 10.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\log (c(d(e + fx)^p)^q))^{3/2}}{(g + hx)^2}\,dx = \int \frac{(b\log (((fx + e)^p d)^q\,c) + a)^{\frac{3}{2}}}{(hx + g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x, algorithm="maxim
a")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g)^2, x)
```

3.469.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\int \frac{(b\log\left(((fx+e)^pd)^q\,c\right)+a)^{\frac{3}{2}}}{(hx+g)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g)^2, x)
```

3.469.9 Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)\right)^{3/2}}{(g+h\,x)^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x)^2,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x)^2, x)
```

3.470 $\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.470.1 Optimal result

Integrand size = 30, antiderivative size = 355

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{3a}{bpq}}h^2\sqrt{\frac{\pi}{3}}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

output
```
1/3*h^2*(f*x+e)^3*erfi(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p
^(1/2)/q^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(
3/p/q))/b^(1/2)/p^(1/2)/q^(1/2)+(-e*h+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*
x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/b/p/q)/f^3/((c*(
d*(f*x+e)^p)^q)^(1/p/q))/b^(1/2)/p^(1/2)/q^(1/2)+h*(-e*h+f*g)*(f*x+e)^2*er
fi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1
/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(2/p/q))/b^(1/2)/p^(1
/2)/q^(1/2)
```

3.470.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.89

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$
$$= \frac{e^{-\frac{3a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(3e^{\frac{2a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+3\sqrt{}\right.}{}$$

input
```
Integrate[(g + h*x)^2/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
(Sqrt[Pi]*(e + f*x)*(3*E^((2*a)/(b*p*q))*(f*g - e*h)^2*(c*(d*(e + f*x)^p)^
q)^(2/(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sq
rt[q])] + 3*Sqrt[2]*E^(a/(b*p*q))*h*(f*g - e*h)*(e + f*x)*(c*(d*(e + f*x)^
p)^q)^(1/(p*q))*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[
b]*Sqrt[p]*Sqrt[q])] + Sqrt[3]*h^2*(e + f*x)^2*Erfi[(Sqrt[3]*Sqrt[a + b*Lo
g[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]))/(3*Sqrt[b]*E^((3*a)/
(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

3.470.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$
$$\Big\downarrow \ 2895$$
$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$
$$\Big\downarrow \ 2848$$
$$\int \left(\frac{(fg-eh)^2}{f^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}+\frac{2h(e+fx)(fg-eh)}{f^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}+\frac{h^2(e+fx)^2}{f^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}+\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}+\frac{\sqrt{\frac{\pi}{3}}h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

input
```
Int[(g + h*x)^2/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
((f*g - e*h)^2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]
]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*(f*g - e*h)*Sqrt[2*Pi]*(e + f*x)^2*Er
fi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q]
)])/(Sqrt[b]*E^((2*a)/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(
2/(p*q))) + (h^2*Sqrt[Pi/3]*(e + f*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*
(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((3*a)/(b*p*q))*f
^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

3.470.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.470. $\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.470.4 Maple [F]

$$\int \frac{(hx+g)^2}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}}dx$$

input
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.470.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.470.6 Sympy [F]

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

input
```
integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral((g + h*x)**2/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.470.7 Maxima [F]

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(hx+g)^2}{\sqrt{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^2/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.470.8 Giac [F]

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(hx+g)^2}{\sqrt{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^2/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(g+h\,x)^2}{\sqrt{a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)}}\,dx$$

input
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.471 $\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.471.1 Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{2a}{bpq}}h\sqrt{\frac{\pi}{2}}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

output
```
1/2*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(
1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(2/
p/q))/b^(1/2)/p^(1/2)/q^(1/2)+(-e*h+f*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)
^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/b/p/q)/f^2/((c*(d*(f
*x+e)^p)^q)^(1/p/q))/b^(1/2)/p^(1/2)/q^(1/2)
```

3.471.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.91

$$\int \frac{g+hx}{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{2a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(2e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+\sqrt{2}\right.}{2\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

input
```
Integrate[(g + h*x)/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
(Sqrt[Pi]*(e + f*x)*(2*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/
(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])
] + Sqrt[2]*h*(e + f*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
)/(Sqrt[b]*Sqrt[p]*Sqrt[q])]))/(2*Sqrt[b]*E^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sq
rt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

3.471.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

$$\Big\downarrow \; 2895$$

$$\int \frac{g+hx}{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

$$\Big\downarrow \; 2848$$

$$\int \left(\frac{fg-eh}{f\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}+\frac{h(e+fx)}{f\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}+\frac{\sqrt{\frac{\pi}{2}}h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

input
```
Int[(g + h*x)/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
((f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/
(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*
(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*Sqrt[Pi/2]*(e + f*x)^2*Erfi[(Sqrt[2]*Sq
rt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E
^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

3.471.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.471. $\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.471.4 Maple [F]

$$\int \frac{hx+g}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

input
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.471.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.471.6 Sympy [F]

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}}\,dx = \int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}}\,dx$$

input
```
integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral((g + h*x)/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.471.7 Maxima [F]

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{hx+g}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.471.8 Giac [F]

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{hx+g}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
```

output
```
integrate((h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{g+h\,x}{\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

input
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.472 $\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.472.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

output
```
(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi
^(1/2)/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p/q))/b^(1/2)/p^(1/2)/q^(1/2
)
```

3.472.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

input
```
Integrate[1/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
(Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqr
t[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p
)^q)^(1/(p*q)))
```

3.472.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2895, 2836, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{\left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}d\log\left(cd^q(e+fx)^{pq}\right)}{fpq}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{bfpq}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

input
```
Int[1/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
(Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*q)]]/(Sqrt[b]*S
qrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f*Sqrt[p]*Sqrt[q]*(c*d^q*(e + f*x
)^(p*q))^(1/(p*q)))
```

3.472.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.472. $\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.472.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.472.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.472.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.472.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.472.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.473 $\int \frac{1}{(g+hx)\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.473.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Int}\left(\frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.473.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

input
```
Integrate[1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

output
```
Integrate[1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

3.473.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

$$\Big\downarrow \text{2896}$$

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

input
```
Int[1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

output
```
$Aborted
```

3.473.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.473.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.473.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.473.6 Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

3.473.7 Maxima [N/A]

Not integrable

Time = 10.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(hx+g)\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.473.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(hx+g)\sqrt{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right.}}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.473.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(g+h\,x)\,\sqrt{a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)}}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)), x)
```

3.474 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

3.474.1 Optimal result

Integrand size = 30, antiderivative size = 404

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.}{b^{3/2}f^3p^{3/2}q^{3/2}}$$

$$+\frac{4e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$

$$+\frac{2e^{-\frac{3a}{bpq}}h^2\sqrt{3\pi}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$

$$-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

output
```
2*(-e*h+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1
/2)/q^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b/p/q)/f^3/p^(3/2)/q^(3/2)/((c*(d*(f*x
+e)^p)^q)^(1/p/q))+4*h*(-e*h+f*g)*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x
+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/exp(2*a
/b/p/q)/f^3/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))+2*h^2*(f*x+e)^3*
erfi(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*3^
(1/2)*Pi^(1/2)/b^(3/2)/exp(3*a/b/p/q)/f^3/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p
)^q)^(3/p/q))-2*(f*x+e)*(h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2
)
```

3.474.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1040 vs. $2(404) = 808$.

Time = 1.52 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.57

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2\Big(-\sqrt{b}ef^2g^2\sqrt{p}\sqrt{q}-\sqrt{b}f^3g^2\sqrt{p}\sqrt{q}x-2\sqrt{b}ef^2gh\sqrt{p}\sqrt{q}x-2\sqrt{b}f^3}{}$$

input
```
Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
(2*(-(Sqrt[b]*e*f^2*g^2*Sqrt[p]*Sqrt[q]) - Sqrt[b]*f^3*g^2*Sqrt[p]*Sqrt[q]
*x - 2*Sqrt[b]*e*f^2*g*h*Sqrt[p]*Sqrt[q]*x - 2*Sqrt[b]*f^3*g*h*Sqrt[p]*Sqr
t[q]*x^2 - Sqrt[b]*e*f^2*h^2*Sqrt[p]*Sqrt[q]*x^2 - Sqrt[b]*f^3*h^2*Sqrt[p]
*Sqrt[q]*x^3 - (4*e*f*g*h*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e +
 f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q
]])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (e^2*h^2*Sqrt[Pi]*(e
 + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q]
)]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)
^q)^(1/(p*q))) + (2*f*g*h*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*
Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*
(e + f*x)^p)^q]])/(E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - (2
*e*h^2*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^
p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(
E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) + (h^2*Sqrt[3*Pi]*(e +
f*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p
]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(E^((3*a)/(b*p*q))*(c*(d
*(e + f*x)^p)^q)^(3/(p*q))) + (Sqrt[b]*f^2*g^2*Sqrt[p]*Sqrt[q]*(e + f*x)*G
amma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*Sqrt[-((a + b*Log[c
*(d*(e + f*x)^p)^q])/(b*p*q))])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p
*q))) + (2*Sqrt[b]*e*f*g*h*Sqrt[p]*Sqrt[q]*(e + f*x)*Gamma[1/2, -((a + ...
```

3.474.3 Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2895, 2847, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.474. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 2895

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 2847

$$-\frac{4(fg-eh)\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2848

$$\frac{6\int \left(\frac{(fg-eh)^2}{f^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{2h(e+fx)(fg-eh)}{f^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{h^2(e+fx)^2}{f^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)dx}{bpq}-\frac{4(fg-eh)\int \left(\frac{fg-eh}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{h(e+fx)}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)dx}{bfpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2009

$$\frac{6\left(\frac{\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}+\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\text{erfi}\left(\cdots\right.}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}\right.}{bpq}$$

$$\frac{4(fg-eh)\left(\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}+\frac{\sqrt{\frac{\pi}{2}}h(e+fx)^2e^{-\frac{2a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\text{erfi}\left(\cdots\right.}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}\right.}{bfpq}$$

$$\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

input

```
Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

3.474. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx$

output
```
(-4*(f*g - e*h)*(((f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*
(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^2*Sq
rt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*Sqrt[Pi/2]*(e + f*x)^2
*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt
[q])])/(Sqrt[b]*E^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q
)^(2/(p*q)))))/(b*f*p*q) + (6*(((f*g - e*h)^2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt
[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a
/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*(f*g -
 e*h)*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p
)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((2*a)/(b*p*q))*f^3*Sqrt[p]*
Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) + (h^2*Sqrt[Pi/3]*(e + f*x)^3*Erf
i[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])
])/(Sqrt[b]*E^((3*a)/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(3
/(p*q)))))/(b*p*q) - (2*(e + f*x)*(g + h*x)^2)/(b*f*p*q*Sqrt[a + b*Log[c*(
d*(e + f*x)^p)^q]])
```

3.474.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.474. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

3.474.4 Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.474.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.474.6 Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**(3/2), x)
```

3.474.7 Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.474.8 Giac [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(g+h\,x)^2}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}}\,dx$$

input
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.475 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

3.475.1 Optimal result

Integrand size = 28, antiderivative size = 275

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+f}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.}{b^{3/2}f^2p^{3/2}q^{3/2}}$$
$$+\frac{2e^{-\frac{2a}{bpq}}h\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}$$
$$-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

output
```
2*(-e*h+f*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2
)/q^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b/p/q)/f^2/p^(3/2)/q^(3/2)/((c*(d*(f*x+e
)^p)^q)^(1/p/q))+2*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1
/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/exp(2*a/b/p/q)/f^2/p
^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))-2*(f*x+e)*(h*x+g)/b/f/p/q/(a+
b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

3.475.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.58

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(-2ee^{\frac{a}{bpq}}h\sqrt{\pi}(c(d(e+fx)^p)^q)^{\frac{1}{pq}}\right.}{}$$

input
```
Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
(2*(e + f*x)*(-2*e*E^(a/(b*p*q))*h*Sqrt[Pi]*(c*(d*(e + f*x)^p)^q)^(1/(p*q)
)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqr
t[a + b*Log[c*(d*(e + f*x)^p)^q]] + h*Sqrt[2*Pi]*(e + f*x)*Erfi[(Sqrt[2]*S
qrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b
*Log[c*(d*(e + f*x)^p)^q]] + Sqrt[b]*E^(a/(b*p*q))*Sqrt[p]*Sqrt[q]*(c*(d*(
e + f*x)^p)^q)^(1/(p*q))*(-(E^(a/(b*p*q))*f*(c*(d*(e + f*x)^p)^q)^(1/(p*q)
)*(g + h*x)) + (f*g + e*h)*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(
b*p*q))]*Sqrt[-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))])))/(b^(3/2)*E^(
(2*a)/(b*p*q))*f^2*p^(3/2)*q^(3/2)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*Sqrt[a
+ b*Log[c*(d*(e + f*x)^p)^q]])
```

3.475.3 Rubi [A] (warning: unable to verify)

Time = 1.82 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2895, 2847, 2836, 2737, 2611, 2633, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx$$

$$\Big\downarrow \text{ 2847}$$

$$-\frac{2(fg-eh)\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2836

$$-\frac{2(fg-eh)\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d(e+fx)}{bf^2pq}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2737

$$-\frac{2(e+fx)(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d\log\left(cd^q(e+fx)^{pq}\right)}{bf^2p^2q^2}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2611

$$-\frac{4(e+fx)(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{b^2f^2p^2q^2}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2633

$$\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$\Big\downarrow$ 2848

$$\frac{4\int \left(\frac{fg-eh}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{h(e+fx)}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)dx}{bpq}-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

3.475. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}+\frac{4\left(\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}+\frac{\sqrt{\frac{\pi}{2}}h(e+fx)^2e^{-\frac{2a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c\ldots}}{\sqrt{b}\ldots}\right.}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}\right.}{bpq}$$

$$\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

input
```
Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
(-2*(f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*
q)]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*f^2*p^(3/2)*q^(3/2
)*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))) + (4*(((f*g - e*h)*Sqrt[Pi]*(e + f*x)
*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sq
rt[b]*E^(a/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) +
 (h*Sqrt[Pi/2]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^
q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sq
rt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))))/(b*p*q) - (2*(e + f*x)*(g + h*x))
/(b*f*p*q*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
```

3.475.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.475.4 Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.475.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.475.6 Sympy [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**(3/2), x)
```

3.475.7 Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.475.8 Giac [F]

$$\int \frac{g+hx}{(a+b\log (c(d(e+fx)^p)^q))^{3/2}}\,dx=\int \frac{hx+g}{(b\log (((fx+e)^pd)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
```

output
```
integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+b\log (c(d(e+fx)^p)^q))^{3/2}}\,dx=\int \frac{g+h\,x}{(a+b\,\ln (c\,(d\,(e+f\,x)^p)^q))^{3/2}}\,dx$$

input
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.476 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

3.476.1 Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=\frac{2e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}fp^{3/2}q^{3/2}}-\frac{2(e+fx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

output
```
2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*
Pi^(1/2)/b^(3/2)/exp(a/b/p/q)/f/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(1/p/
q))-2*(f*x+e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

3.476.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=-\frac{2e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(e^{\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}-\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\sqrt{-\frac{a+b\log(c(d(e+f}{bpq}}}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3/2),x]
```

output
```
(-2*(e + f*x)*(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q)) - Gamma[1/2,
-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*Sqrt[-((a + b*Log[c*(d*(e + f
*x)^p)^q])/(b*p*q))]))/(b*E^(a/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*
q))*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
```

3.476.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2895, 2836, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^{3/2}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d(e+fx)}{bpq}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{2(e+fx)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d\log(cd^q(e+fx)^{pq})}{bp^2q^2}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}}{f}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{4(e+fx)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{b^2p^2q^2}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}}{f}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3/2),x]
```

output
```
((2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*q)]]/(Sqrt[b
]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*p^(3/2)*q^(3/2)*(c*d^q*(e + f*
x)^(p*q))^(1/(p*q))) - (2*(e + f*x))/(b*p*q*Sqrt[a + b*Log[c*d^q*(e + f*x)
^(p*q)]]))/f
```

3.476.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.476.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.476.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.476.6 Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-3/2), x)
```

3.476.7 Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-3/2), x)
```

3.476.8 Giac [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-3/2), x)
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.477 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

3.477.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.477.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)), x]
```

3.477.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx$$
$$\Big\downarrow \; 2896$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)),x]
```

output
```
$Aborted
```

3.477.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.477.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.477.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.477.6 Sympy [N/A]

Not integrable

Time = 24.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^{\frac{3}{2}}(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)*(g + h*x)), x)
```

3.477.7 Maxima [N/A]

Not integrable

Time = 10.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)), x)
```

3.477.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)), x)
```

3.477.9 Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(g+h\,x\right)\,{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^{3/2}}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)), x)
```

3.478 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

3.478.1 Optimal result

Integrand size = 30, antiderivative size = 514

$$\begin{aligned}\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx &= \frac{4e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}\\
&+\frac{16e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}\\
&+\frac{4e^{-\frac{3a}{bpq}}h^2\sqrt{3\pi}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{5/2}f^3p^{5/2}q^{5/2}}\\
&-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}+\frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\\
&-\frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\end{aligned}$$

output
```
-2/3*(f*x+e)*(h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)+4/3*(-e*h
+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(
1/2))*Pi^(1/2)/b^(5/2)/exp(a/b/p/q)/f^3/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^
q)^(1/p/q))+16/3*h*(-e*h+f*g)*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^
p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/exp(2*a/b/p
/q)/f^3/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))+4*h^2*(f*x+e)^3*erfi
(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*3^(1/2
)*Pi^(1/2)/b^(5/2)/exp(3*a/b/p/q)/f^3/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)
^(3/p/q))+8/3*(-e*h+f*g)*(f*x+e)*(h*x+g)/b^2/f^2/p^2/q^2/(a+b*ln(c*(d*(f*x
+e)^p)^q))^(1/2)-4*(f*x+e)*(h*x+g)^2/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)
^q))^(1/2)
```

3.478.2 Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.27

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx =$$

$$-\frac{2e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(2ee^{\frac{2a}{bpq}}h(8fg+eh)\sqrt{\pi}(c(d(e+fx)^p)^q)^{\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)(c\right.}{}$$

input
```
Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

output
```
(-2*(e + f*x)*(2*e*E^((2*a)/(b*p*q))*h*(8*f*g + e*h)*Sqrt[Pi]*(c*(d*(e + f
*x)^p)^q)^(2/(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqr
t[p]*Sqrt[q])]*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 8*E^(a/(b*p*q))*h*
(-(f*g) + e*h)*Sqrt[2*Pi]*(e + f*x)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*Erfi[(
Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(
a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) - 6*h^2*Sqrt[3*Pi]*(e + f*x)^2*Erfi[
(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*
(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + Sqrt[b]*E^((2*a)/(b*p*q))*Sqrt[p]
*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(2*b*(f^2*g^2 + 6*e*f*g*h + 2*e^2
*h^2)*p*q*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*(-((a +
b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^(3/2) + E^(a/(b*p*q))*f*(c*(d*(e + f
*x)^p)^q)^(1/(p*q))*(g + h*x)*(b*f*p*q*(g + h*x) + 2*a*(f*g + 2*e*h + 3*f*
h*x) + 2*b*(2*e*h + f*(g + 3*h*x))*Log[c*(d*(e + f*x)^p)^q]))))/(3*b^(5/2)
*E^((3*a)/(b*p*q))*f^3*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)^p)^q)^(3/(p*q))*(a
+ b*Log[c*(d*(e + f*x)^p)^q])^(3/2))
```

3.478.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1154 vs. 2(514) = 1028.

Time = 5.18 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2895, 2847, 2847, 2836, 2737, 2611, 2633, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}dx$$

$$\Big\downarrow \text{2847}$$

$$-\frac{4(fg-eh)\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bfpq}+\frac{2\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{3bfpq(a+b\log(c(d(e+fx)^p)^q))^{3/2}}$$

$$\Big\downarrow \text{2847}$$

$$-\frac{4(fg-eh)\left(-\frac{2(fg-eh)\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{3bfpq}+\frac{2\left(-\frac{4(fg-eh)\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{bpq}-\frac{2(e+fx)(g+hx)^2}{3bfpq(a+b\log(c(d(e+fx)^p)^q))^{3/2}}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{4(fg-eh)\left(-\frac{2(fg-eh)\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d(e+fx)}{bf^2pq}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{3bfpq}+\frac{2\left(-\frac{4(fg-eh)\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{bpq}-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$\downarrow$ 2737

$$-\frac{4(fg-eh)\left(-\frac{2(e+fx)(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d\log(cd^q(e+fx)^{pq})}{bf^2p^2q^2}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{}{bfpq}\right.}{3bfpq}$$

$$\frac{2\left(-\frac{4(fg-eh)\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{bpq}-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$\downarrow$ 2611

$$-\frac{4(fg-eh)\left(-\frac{4(e+fx)(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{b^2f^2p^2q^2}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)}}}{bpq}\right.}{3bfpq}$$

$$\frac{2\left(-\frac{4(fg-eh)\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{bpq}-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$\downarrow$ 2633

3.478. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

$$-\frac{4(fg-eh)\left(\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}-\frac{2(e+}{bfpq\sqrt{a+b}}\right.}{3bfpq}$$

$$\frac{2\left(-\frac{4(fg-eh)\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{bpq}-$$

$$\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$\Big\downarrow$ 2848

$$-\frac{4(fg-eh)\left(\frac{4\int \left(\frac{fg-eh}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{h(e+fx)}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)dx}{bpq}-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(}}{\sqrt{b}}\right.}{b^{3/2}f^2p^{3/2}q^{3/2}}\right.}{3bfpq}$$

$$\frac{2\left(\frac{6\int \left(\frac{(fg-eh)^2}{f^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{2h(e+fx)(fg-eh)}{f^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{h^2(e+fx)^2}{f^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)dx}{bpq}-\frac{4(fg-eh)\int \left(\frac{fg-eh}{f\sqrt{a+b\log(c(d(e+fx)^p)^q)}}+\frac{}{f\sqrt{a+b\log}}\right.}{bfpq}\right.}{bpq}$$

$$\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}-$$

$$\frac{4(fg-eh)\left(-\frac{2e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}}{b^{3/2}f^2p^{3/2}q^{3/2}}+\frac{4\left(\frac{e^{-\frac{2a}{bpq}}h\sqrt{\frac{\pi}{2}}(e+fx)^2\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}\right.}{}\right.}{3bf}$$

$$2\left(-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}-\frac{4(fg-eh)\left(\frac{e^{-\frac{2a}{bpq}}h\sqrt{\frac{\pi}{2}}(e+fx)^2\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}+\frac{e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e}{}\right.}{bfpq}\right.$$

input
```
Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

3.478. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

output
```
(-2*(e + f*x)*(g + h*x)^2)/(3*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/
2)) - (4*(f*g - e*h)*((-2*(f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*L
og[c*d^q*(e + f*x)^(p*q)]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*
q))*f^2*p^(3/2)*q^(3/2)*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))) + (4*(((f*g - e
*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*
Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f
*x)^p)^q)^(1/(p*q))) + (h*Sqrt[Pi/2]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*
Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((2*a)/(
b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))))/(b*p*q) - (
2*(e + f*x)*(g + h*x))/(b*f*p*q*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])))/(3
*b*f*p*q) + (2*((-4*(f*g - e*h)*(((f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt
[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a
/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*Sqrt[P
i/2]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt
[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(
d*(e + f*x)^p)^q)^(2/(p*q)))))/(b*f*p*q) + (6*((((f*g - e*h)^2*Sqrt[Pi]*(e
 + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])
])/(Sqrt[b]*E^(a/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*
q))) + (h*(f*g - e*h)*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[
c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((2*a)/(b...
```

3.478.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

3.478. $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.478.4 Maple [F]

$$\int \frac{(hx+g)^2}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^{\frac{5}{2}}}dx$$

input
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

output
```
int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

3.478.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.478.6 Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{5}{2}}}\,dx$$

input
```
integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
```

output
```
Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**(5/2), x)
```

3.478.7 Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q\,c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

3.478.8 Giac [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^pd)^q c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(g+h\,x)^2}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{5/2}}\,dx$$

input
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2),x)
```

output
```
int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2), x)
```

3.479 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

3.479.1 Optimal result

Integrand size = 28, antiderivative size = 380

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \frac{4e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+f\cdots}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.}{3b^{5/2}f^2p^{5/2}q^{5/2}}$$
$$+\frac{8e^{-\frac{2a}{bpq}}h\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^2p^{5/2}q^{5/2}}$$
$$-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}+\frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$-\frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

output
```
-2/3*(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)+4/3*(-e*h+f
*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2)
)*Pi^(1/2)/b^(5/2)/exp(a/b/p/q)/f^2/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(
1/p/q))+8/3*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(
1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/exp(2*a/b/p/q)/f^2/p^(5/2)/
q^(5/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))+4/3*(-e*h+f*g)*(f*x+e)/b^2/f^2/p^2/q
^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)-8/3*(f*x+e)*(h*x+g)/b^2/f/p^2/q^2/(a+
b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

3.479.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.29

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx =$$

$$-\frac{2e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(8ee^{\frac{a}{bpq}}h\sqrt{\pi}\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)(a+b\log(c(}{}$$

input
```
Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

output
```
(-2*(e + f*x)*(8*e*E^(a/(b*p*q))*h*Sqrt[Pi]*(c*(d*(e + f*x)^p)^q)^(1/(p*q)
)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(a
+ b*Log[c*(d*(e + f*x)^p)^q])^(3/2) - 4*h*Sqrt[2*Pi]*(e + f*x)*Erfi[(Sqrt[
2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(a + b
*Log[c*(d*(e + f*x)^p)^q])^(3/2) + Sqrt[b]*E^(a/(b*p*q))*Sqrt[p]*Sqrt[q]*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))*(2*b*(f*g + 3*e*h)*p*q*Gamma[1/2, -((a + b*
Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b
*p*q)))^(3/2) + E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(b*f*p*q*(g
+ h*x) + 2*a*(f*g + e*h + 2*f*h*x) + 2*b*(e*h + f*(g + 2*h*x))*Log[c*(d*(e
 + f*x)^p)^q]))))/(3*b^(5/2)*E^((2*a)/(b*p*q))*f^2*p^(5/2)*q^(5/2)*(c*(d*(
e + f*x)^p)^q)^(2/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))
```

3.479.3 Rubi [A] (warning: unable to verify)

Time = 3.14 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.66, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2895, 2847, 2836, 2734, 2737, 2611, 2633, 2847, 2836, 2737, 2611, 2633, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2847}$$

$$-\frac{2(fg-eh)\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bfpq}+\frac{4\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bpq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{2(fg-eh)\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^{3/2}}d(e+fx)}{3bf^2pq}+\frac{4\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bpq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2734}$$

$$-\frac{2(fg-eh)\left(\frac{2\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d(e+fx)}{bpq}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}+\frac{4\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bpq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2737}$$

$$-\frac{2(fg-eh)\left(\frac{2(e+fx)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d\log(cd^q(e+fx)^{pq})}{bp^2q^2}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}+\frac{4\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bpq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2(fg-eh)\left(\frac{4(e+fx)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{b^2p^2q^2}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}+\frac{4\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bpq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2633}$$

3.479. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

$$\frac{4\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx}{3bpq}-\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \; 2847$$

$$\frac{4\left(-\frac{2(fg-eh)\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bfpq}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{3bpq}-\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \; 2836$$

$$\frac{4\left(-\frac{2(fg-eh)\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d(e+fx)}{bf^2pq}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{3bpq}-\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \; 2737$$

$$\frac{4\left(-\frac{2(e+fx)(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\int \frac{(cd^q(e+fx)^{pq})^{\frac{1}{pq}}}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}d\log(cd^q(e+fx)^{pq})}{bf^2p^2q^2}+\frac{4\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}dx}{bpq}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\right)}{3bpq}-\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}-\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

3.479. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

$$\Big\downarrow \text{2611}$$

$$\frac{4\left(-\frac{4(e+fx)(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{b^2f^2p^2q^2}+\frac{4\int\frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx}{bpq}-\frac{\cdots}{bfp\cdots}\cdots\right.}{3bpq}$$

$$\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right)}{3bf^2pq}-$$

$$\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{4\left(\frac{4\int\frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx}{bpq}-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\cdots\right.\right.}}\cdots\right.}{3bpq}$$

$$\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right)}{3bf^2pq}-$$

$$\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2848}$$

$$\frac{4\left(\frac{4\int\left(\frac{fg-eh}{f\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}+\frac{h(e+fx)}{f\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\right)dx}{bpq}-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}\cdots\right.}{3bpq}$$

$$\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right)}{3bf^2pq}-$$

$$\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

3.479. $\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$

$$-\frac{2(fg-eh)\left(\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)}{3bf^2pq}+\frac{4\left(-\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(cd^q(e+fx)^{pq})^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}+\frac{4\left(\frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\ldots}}{\ldots}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}\ldots\right.}{\ldots}\right.}{3bpq}$$

$$\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

input
```
Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

output
```
(-2*(f*g - e*h)*((2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)
^(p*q)]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*p^(3/2)*q^(3/2
)*(c*d^q*(e + f*x)^(p*q))^(1/(p*q))) - (2*(e + f*x))/(b*p*q*Sqrt[a + b*Log
[c*d^q*(e + f*x)^(p*q)]])))/(3*b*f^2*p*q) - (2*(e + f*x)*(g + h*x))/(3*b*f
*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)) + (4*((-2*(f*g - e*h)*Sqrt[Pi
]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*q)]]/(Sqrt[b]*Sqrt[p]*S
qrt[q])])/(b^(3/2)*E^(a/(b*p*q))*f^2*p^(3/2)*q^(3/2)*(c*d^q*(e + f*x)^(p*q
))^(1/(p*q))) + (4*(((f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*
(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^2
*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*Sqrt[Pi/2]*(e + f*x
)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*S
qrt[q])])/(Sqrt[b]*E^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p
)^q)^(2/(p*q)))))/(b*p*q) - (2*(e + f*x)*(g + h*x))/(b*f*p*q*Sqrt[a + b*Lo
g[c*(d*(e + f*x)^p)^q]])))/(3*b*p*q)
```

3.479.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.479. $\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.479. $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

3.479.4 Maple [F]

$$\int \frac{hx+g}{(a+b\ln(c(d(fx+e)^{p})^{q}))^{\frac{5}{2}}}dx$$

input
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

output
```
int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

3.479.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^{p})^{q}))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.479.6 Sympy [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^{p})^{q}))^{5/2}}\,dx = \int \frac{g+hx}{(a+b\log(c(d(e+fx)^{p})^{q}))^{\frac{5}{2}}}\,dx$$

input
```
integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
```

output
```
Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**(5/2), x)
```

3.479.7 Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

3.479.8 Giac [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac")
```

output
```
integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{g+h\,x}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{5/2}}\,dx$$

input
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2),x)
```

output
```
int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2), x)
```

3.480 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

3.480.1 Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\,(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}fp^{5/2}q^{5/2}} - \frac{2(e+fx)}{3bfpq\,(a+b\log(c(d(e+fx)^p)^q))^{3/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

output
```
-2/3*(f*x+e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)+4/3*(f*x+e)*erfi((a
+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/b^(5/2)/
exp(a/b/p/q)/f/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(1/p/q))-4/3*(f*x+e)/b
^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

3.480.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = -\frac{2e^{-\frac{a}{bpq}}(e+fx)\,(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\left(2bpq\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{3/2} + e^{\frac{a}{bpq}}(c(d(e}{3b^2fp^2q^2\,(a+b\log(c(d(e+fx)^p)^q))^{3/2}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-5/2),x]
```

output
```
(-2*(e + f*x)*(2*b*p*q*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*
q))]*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^(3/2) + E^(a/(b*p*q))*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))*(2*a + b*p*q + 2*b*Log[c*(d*(e + f*x)^p)^q]
)))/(3*b^2*E^(a/(b*p*q))*f*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(a + b*
Log[c*(d*(e + f*x)^p)^q])^(3/2))
```

3.480.3 Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2895, 2836, 2734, 2734, 2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{5/2}}d(e+fx)}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\int \frac{1}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}d(e+fx)}{3bpq}-\frac{2(e+fx)}{3bpq\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}d(e+fx)}{bpq}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right)}{3bpq}-\frac{2(e+fx)}{3bpq\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{2\left(\frac{\frac{2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \frac{\left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}d\log\left(cd^q(e+fx)^{pq}\right)}{bp^2q^2}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}}{3bpq}\right)-\frac{2(e+fx)}{3bpq\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{2\left(\frac{\frac{4(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \exp\left(\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{bpq}-\frac{a}{bpq}\right)d\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{b^2p^2q^2}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}}{3bpq}\right)-\frac{2(e+fx)}{3bpq\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{2\left(\frac{\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}p^{3/2}q^{3/2}}-\frac{2(e+fx)}{bpq\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}}{3bpq}\right)-\frac{2(e+fx)}{3bpq\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-5/2),x]
```

output
```
((-2*(e + f*x))/(3*b*p*q*(a + b*Log[c*d^q*(e + f*x)^(p*q)])^(3/2)) + (2*((
2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*d^q*(e + f*x)^(p*q)]]/(Sqrt[b]*
Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*p^(3/2)*q^(3/2)*(c*d^q*(e + f*x)
^(p*q))^(1/(p*q))) - (2*(e + f*x))/(b*p*q*Sqrt[a + b*Log[c*d^q*(e + f*x)^(
p*q)]])))/(3*b*p*q))/f
```

3.480.3.1 Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

3.480. $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.480.4 Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

3.480.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.480.6 Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-5/2), x)
```

3.480.7 Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-5/2), x)
```

3.480.8 Giac [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-5/2), x)
```

3.480.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2),x)
```

output
```
int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2), x)
```

3.481 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$

3.481.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

3.481.2 Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)), x]
```

3.481.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$$
$$\Big\downarrow \ 2896$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)),x]
```

output
```
$Aborted
```

3.481.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.481.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

3.481.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.481.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
```

output
```
Timed out
```

3.481.7 Maxima [N/A]

Not integrable

Time = 12.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \int \frac{1}{(hx+g)(b\log\left(((fx+e)^pd)^qc\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2)), x)
```

3.481.8 Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(((fx+e)^p d\right)^q c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac"
)
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2)), x)
```

3.481.9 Mupad [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{5/2}}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2)),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2)), x)
```

3.482 $\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q))\, dx$

3.482.1 Optimal result

Integrand size = 28, antiderivative size = 171

$$\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q))\, dx = -\frac{4b(fg-eh)^2pq\sqrt{g+hx}}{5f^2h} - \frac{4b(fg-eh)pq(g+hx)^{3/2}}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h} + \frac{4b(fg-eh)^{5/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5f^{5/2}h} + \frac{2(g+hx)^{5/2}(a+b\log (c(d(e+fx)^p)^q))}{5h}$$

output
```
-4/15*b*(-e*h+f*g)*p*q*(h*x+g)^(3/2)/f/h-4/25*b*p*q*(h*x+g)^(5/2)/h+4/5*b*
(-e*h+f*g)^(5/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/f^(5/
2)/h+2/5*(h*x+g)^(5/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))/h-4/5*b*(-e*h+f*g)^2*p*
q*(h*x+g)^(1/2)/f^2/h
```

3.482.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q))\, dx = \frac{2\left(\frac{1}{5}a(g+hx)^{5/2} - \frac{2}{75}bpq\left(3(g+hx)^{5/2} + \frac{5(fg-eh)\left(\sqrt{f}\sqrt{g+hx}(4fg-3eh+fhx)-3(fg-eh\right.}{f^{5/2}}\right.\right.}{h}$$

input
```
Integrate[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
(2*((a*(g + h*x)^(5/2))/5 - (2*b*p*q*(3*(g + h*x)^(5/2) + (5*(f*g - e*h)*(
Sqrt[f]*Sqrt[g + h*x]*(4*f*g - 3*e*h + f*h*x) - 3*(f*g - e*h)^(3/2)*ArcTan
h[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]))/f^(5/2)))/75 + (b*(g + h*x)^(
5/2)*Log[c*(d*(e + f*x)^p)^q])/5))/h
```

3.482.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2895, 2842, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}-\frac{2bfpq\int \frac{(g+hx)^{5/2}}{e+fx}dx}{5h}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}-\frac{2bfpq\left(\frac{(fg-eh)\int \frac{(g+hx)^{3/2}}{e+fx}dx}{f}+\frac{2(g+hx)^{5/2}}{5f}\right)}{5h}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}-\frac{2bfpq\left(\frac{(fg-eh)\left(\frac{(fg-eh)\int \frac{\sqrt{g+hx}}{e+fx}dx}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{f}+\frac{2(g+hx)^{5/2}}{5f}\right)}{5h}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}-\frac{2bfpq\left(\frac{(fg-eh)\left(\frac{(fg-eh)\left(\frac{(fg-eh)\int\frac{1}{(e+fx)\sqrt{g+hx}}dx}{f}+\frac{2\sqrt{g+hx}}{f}\right)}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{f}+\frac{2(g+hx)^{5/2}}{5f}\right)}{5h}$$

$\downarrow$ 73

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}-\frac{2bfpq\left(\frac{(fg-eh)\left(\frac{(fg-eh)\left(\frac{2(fg-eh)\int\frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{fh}+\frac{2\sqrt{g+hx}}{f}\right)}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{f}+\frac{2(g+hx)^{5/2}}{5f}\right)}{5h}$$

$\downarrow$ 221

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}-\frac{2bfpq\left(\frac{(fg-eh)\left(\frac{(fg-eh)\left(\frac{2\sqrt{g+hx}}{f}-\frac{2\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{f^{3/2}}\right)}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{f}+\frac{2(g+hx)^{5/2}}{5f}\right)}{5h}$$

input
```
Int[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

3.482. $\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx$

output
```
(-2*b*f*p*q*((2*(g + h*x)^(5/2))/(5*f) + ((f*g - e*h)*((2*(g + h*x)^(3/2))
/(3*f) + ((f*g - e*h)*((2*Sqrt[g + h*x])/f - (2*Sqrt[f*g - e*h]*ArcTanh[(S
qrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/f^(3/2)))/f))/f))/(5*h) + (2*(g +
h*x)^(5/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(5*h)
```

3.482.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.482.4 Maple [F]

$$\int (hx+g)^{\frac{3}{2}} \left(a+b\ln \left(c{\left(d{\left(fx+e\right)}^{p}\right)}^{q}\right)\right)dx$$

input
```
int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.482.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(143) = 286$.

Time = 0.34 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.65

$$\int (g+hx)^{3/2} (a+b\log \left(c(d(e+fx)^p)^q\right))\, dx = \left[\frac{2\left(15\,(bf^2g^2-2\,befgh+be^2h^2)pq\sqrt{\frac{fg-eh}{f}}\log\left(\frac{fhx+2\,fg-eh+2\,\sqrt{hx+g}f\sqrt{\frac{fg-eh}{f}}}{fx+e}\right)\right.}{}\right.$$

input
```
integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas"
)
```

output
```
[2/75*(15*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*sqrt((f*g - e*h)/f)*lo
g((f*h*x + 2*f*g - e*h + 2*sqrt(h*x + g)*f*sqrt((f*g - e*h)/f))/(f*x + e))
 + (15*a*f^2*g^2 - 2*(23*b*f^2*g^2 - 35*b*e*f*g*h + 15*b*e^2*h^2)*p*q - 3*
(2*b*f^2*h^2*p*q - 5*a*f^2*h^2)*x^2 + 2*(15*a*f^2*g*h - (11*b*f^2*g*h - 5*
b*e*f*h^2)*p*q)*x + 15*(b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + b*f^2*g^2*
p*q)*log(f*x + e) + 15*(b*f^2*h^2*x^2 + 2*b*f^2*g*h*x + b*f^2*g^2)*log(c)
+ 15*(b*f^2*h^2*q*x^2 + 2*b*f^2*g*h*q*x + b*f^2*g^2*q)*log(d))*sqrt(h*x +
g))/(f^2*h), 2/75*(30*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*sqrt(-(f*g
 - e*h)/f)*arctan(-sqrt(h*x + g)*f*sqrt(-(f*g - e*h)/f)/(f*g - e*h)) + (15
*a*f^2*g^2 - 2*(23*b*f^2*g^2 - 35*b*e*f*g*h + 15*b*e^2*h^2)*p*q - 3*(2*b*f
^2*h^2*p*q - 5*a*f^2*h^2)*x^2 + 2*(15*a*f^2*g*h - (11*b*f^2*g*h - 5*b*e*f*
h^2)*p*q)*x + 15*(b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + b*f^2*g^2*p*q)*l
og(f*x + e) + 15*(b*f^2*h^2*x^2 + 2*b*f^2*g*h*x + b*f^2*g^2)*log(c) + 15*(
b*f^2*h^2*q*x^2 + 2*b*f^2*g*h*q*x + b*f^2*g^2*q)*log(d))*sqrt(h*x + g))/(f
^2*h)]
```

3.482.6 Sympy [F]

$$\int (g+hx)^{3/2} \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\,dx = \int \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)(g+hx)^{\frac{3}{2}}\,dx$$

input
```
integrate((h*x+g)**(3/2)*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**(3/2), x)
```

3.482.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^{3/2} \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.482.8 Giac [F]

$$\int (g+hx)^{3/2} \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\,dx = \int (hx+g)^{\frac{3}{2}}\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)dx$$

input
```
integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx=\int (g+h\,x)^{3/2}\,\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)\,dx$$

input
```
int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.483 $\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx$

3.483.1 Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{4b(fg-eh)pq\sqrt{g+hx}}{3fh}-\frac{4bpq(g+hx)^{3/2}}{9h}+\frac{4b(fg-eh)^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3f^{3/2}h}+\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{3h}$$

output
```
-4/9*b*p*q*(h*x+g)^(3/2)/h+4/3*b*(-e*h+f*g)^(3/2)*p*q*arctanh(f^(1/2)*(h*x
+g)^(1/2)/(-e*h+f*g)^(1/2))/f^(3/2)/h+2/3*(h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+
e)^p)^q))/h-4/3*b*(-e*h+f*g)*p*q*(h*x+g)^(1/2)/f/h
```

3.483.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{2\left(6b(fg-eh)^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)+\sqrt{f}\sqrt{g+hx}(3af(g+hx)-2bpq(4fg-3eh+fhx)+3bf(g+\right.}{9f^{3/2}h}$$

input
```
Integrate[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
(2*(6*b*(f*g - e*h)^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e
*h]] + Sqrt[f]*Sqrt[g + h*x]*(3*a*f*(g + h*x) - 2*b*p*q*(4*f*g - 3*e*h + f
*h*x) + 3*b*f*(g + h*x)*Log[c*(d*(e + f*x)^p)^q])))/(9*f^(3/2)*h)
```

3.483.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2895, 2842, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{3h}-\frac{2bfpq\int \frac{(g+hx)^{3/2}}{e+fx}dx}{3h}$$

$$\Big\downarrow \text{60}$$

$$\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{3h}-\frac{2bfpq\left(\frac{(fg-eh)\int \frac{\sqrt{g+hx}}{e+fx}dx}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{3h}$$

$$\Big\downarrow \text{60}$$

$$\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{3h}-\frac{2bfpq\left(\frac{(fg-eh)\left(\frac{(fg-eh)\int \frac{1}{(e+fx)\sqrt{g+hx}}dx}{f}+\frac{2\sqrt{g+hx}}{f}\right)}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{3h}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h}-2bfpq\left(\frac{(fg-eh)\left(\frac{2(fg-eh)\int\frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{fh}+\frac{2\sqrt{g+hx}}{f}\right)}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{3h}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h}-2bfpq\left(\frac{(fg-eh)\left(\frac{2\sqrt{g+hx}}{f}-\frac{2\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{f^{3/2}}\right)}{f}+\frac{2(g+hx)^{3/2}}{3f}\right)}{3h}$$

input
```
Int[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
(-2*b*f*p*q*((2*(g + h*x)^(3/2))/(3*f) + ((f*g - e*h)*((2*Sqrt[g + h*x]])/f
 - (2*Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/f^
(3/2)))/f))/(3*h) + (2*(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(
3*h)
```

3.483.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.483.4 Maple [F]

$$\int \sqrt{hx+g}\,(a+b\ln(c(d(fx+e)^{p})^{q}))\,dx$$

input
```
int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.54

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^{p})^{q}))\,dx$$
$$= \left[-\frac{2\left(3\,(bfg-beh)pq\sqrt{\frac{fg-eh}{f}}\log\left(\frac{fhx+2\,fg-eh-2\,\sqrt{hx+g}f\sqrt{\frac{fg-eh}{f}}}{fx+e}\right)-(3\,afg-2\,(4\,bfg-3\,beh)pq-(2\,bfh\right.}{}\right.$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas"
)
```

output
```
[-2/9*(3*(b*f*g - b*e*h)*p*q*sqrt((f*g - e*h)/f)*log((f*h*x + 2*f*g - e*h
- 2*sqrt(h*x + g)*f*sqrt((f*g - e*h)/f))/(f*x + e)) - (3*a*f*g - 2*(4*b*f*
g - 3*b*e*h)*p*q - (2*b*f*h*p*q - 3*a*f*h)*x + 3*(b*f*h*p*q*x + b*f*g*p*q)
*log(f*x + e) + 3*(b*f*h*x + b*f*g)*log(c) + 3*(b*f*h*q*x + b*f*g*q)*log(d
))*sqrt(h*x + g))/(f*h), 2/9*(6*(b*f*g - b*e*h)*p*q*sqrt(-(f*g - e*h)/f)*a
rctan(-sqrt(h*x + g)*f*sqrt(-(f*g - e*h)/f)/(f*g - e*h)) + (3*a*f*g - 2*(4
*b*f*g - 3*b*e*h)*p*q - (2*b*f*h*p*q - 3*a*f*h)*x + 3*(b*f*h*p*q*x + b*f*g
*p*q)*log(f*x + e) + 3*(b*f*h*x + b*f*g)*log(c) + 3*(b*f*h*q*x + b*f*g*q)*
log(d))*sqrt(h*x + g))/(f*h)]
```

3.483.6 Sympy [F]

$$\int \sqrt{g+hx}(a+b\log \left(c(d(e+fx)^p)^q\right))\,dx = \int \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)\sqrt{g+hx}\,dx$$

input
```
integrate((h*x+g)**(1/2)*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x), x)
```

3.483.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{g+hx}(a+b\log \left(c(d(e+fx)^p)^q\right))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.483.8 Giac [F]

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = \int \sqrt{hx+g}(b\log(((fx+e)^pd)^qc)+a)\,dx$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = \int \sqrt{g+h\,x}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)\,dx$$

input
```
int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.484 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx}}\,dx$

3.484.1 Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx = -\frac{4bpq\sqrt{g+hx}}{h}+\frac{4b\sqrt{fg-eh}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} + \frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}$$

output
```
4*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(-e*h+f*g)^(1/2)/h
/f^(1/2)-4*b*p*q*(h*x+g)^(1/2)/h+2*(a+b*ln(c*(d*(f*x+e)^p)^q))*(h*x+g)^(1/
2)/h
```

3.484.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx = \frac{2\left(\frac{2b\sqrt{fg-eh}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}}+\sqrt{g+hx}\left(a-2bpq+b\log\left(c(d(e+fx)^p)^q\right)\right)\right)}{h}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x],x]
```

output
```
(2*((2*b*Sqrt[f*g - e*h]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*
h]])/Sqrt[f] + Sqrt[g + h*x]*(a - 2*b*p*q + b*Log[c*(d*(e + f*x)^p)^q])))/
h
```

3.484.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2895, 2842, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{2bfpq\int \frac{\sqrt{g+hx}}{e+fx}dx}{h}$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{2bfpq\left(\frac{(fg-eh)\int \frac{1}{(e+fx)\sqrt{g+hx}}dx}{f}+\frac{2\sqrt{g+hx}}{f}\right)}{h}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{2bfpq\left(\frac{2(fg-eh)\int \frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{fh}+\frac{2\sqrt{g+hx}}{f}\right)}{h}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{2bfpq\left(\frac{2\sqrt{g+hx}}{f}-\frac{2\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{f^{3/2}}\right)}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x],x]
```

output
```
(-2*b*f*p*q*((2*Sqrt[g + h*x])/f - (2*Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqr
t[g + h*x])/Sqrt[f*g - e*h]])/f^(3/2)))/h + (2*Sqrt[g + h*x]*(a + b*Log[c*
(d*(e + f*x)^p)^q]))/h
```

3.484.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.484. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx}}\,dx$

3.484.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{2\sqrt{hx+g}\,a+2b\left(\ln\left(c\left(d\left(\frac{f(hx+g)+eh-fg}{h}\right)^{p}\right)^{q}\right)\sqrt{hx+g}-2qpf\left(\frac{\sqrt{hx+g}}{f}+\frac{(-eh+fg)\arctan\left(\frac{f\sqrt{hx+g}}{\sqrt{(eh-fg)f}}\right)}{f\sqrt{(eh-fg)f}}\right)\right)}{h}$	118
default	$\frac{2\sqrt{hx+g}\,a+2b\left(\ln\left(c\left(d\left(\frac{f(hx+g)+eh-fg}{h}\right)^{p}\right)^{q}\right)\sqrt{hx+g}-2qpf\left(\frac{\sqrt{hx+g}}{f}+\frac{(-eh+fg)\arctan\left(\frac{f\sqrt{hx+g}}{\sqrt{(eh-fg)f}}\right)}{f\sqrt{(eh-fg)f}}\right)\right)}{h}$	118
parts	$\frac{2a\sqrt{hx+g}}{h}+\frac{2b\left(\ln\left(c\left(d\left(\frac{f(hx+g)+eh-fg}{h}\right)^{p}\right)^{q}\right)\sqrt{hx+g}-2qpf\left(\frac{\sqrt{hx+g}}{f}+\frac{(-eh+fg)\arctan\left(\frac{f\sqrt{hx+g}}{\sqrt{(eh-fg)f}}\right)}{f\sqrt{(eh-fg)f}}\right)\right)}{h}$	121

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/h*((h*x+g)^(1/2)*a+b*(ln(c*(d*((f*(h*x+g)+e*h-f*g)/h)^p)^q)*(h*x+g)^(1/2
)-2*q*p*f*((h*x+g)^(1/2)/f+(-e*h+f*g)/f/((e*h-f*g)*f)^(1/2)*arctan(f*(h*x+
g)^(1/2)/((e*h-f*g)*f)^(1/2)))))
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.95

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx$$
$$= \left[\frac{2\left(bpq\sqrt{\frac{fg-eh}{f}}\log\left(\frac{fhx+2\,fg-eh+2\,\sqrt{hx+g}f\sqrt{\frac{fg-eh}{f}}}{fx+e}\right)+(bpq\log\left(fx+e\right)-2\,bpq+bq\log\left(d\right)+b\log\left(c\right)+a)\right.}{h}\right.$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="fricas"
)
```

output
```
[2*(b*p*q*sqrt((f*g - e*h)/f)*log((f*h*x + 2*f*g - e*h + 2*sqrt(h*x + g)*f
*sqrt((f*g - e*h)/f))/(f*x + e)) + (b*p*q*log(f*x + e) - 2*b*p*q + b*q*log
(d) + b*log(c) + a)*sqrt(h*x + g))/h, 2*(2*b*p*q*sqrt(-(f*g - e*h)/f)*arct
an(-sqrt(h*x + g)*f*sqrt(-(f*g - e*h)/f)/(f*g - e*h)) + (b*p*q*log(f*x + e
) - 2*b*p*q + b*q*log(d) + b*log(c) + a)*sqrt(h*x + g))/h]
```

3.484.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(g + h*x), x)
```

3.484.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx=-\frac{2\left(\left(\left(2\,f\left(\frac{(fg-eh)\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{\sqrt{-f^2g+efhf}}+\frac{\sqrt{hx+g}}{f}\right)-\sqrt{hx+g}\log\left(fx+e\right)\right)bpq-\sqrt{hx+g}bq\log\left(d\right)-\sqrt{h\cdots}\right.\right.}{h}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
-2*((2*f*((f*g - e*h)*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/(sqrt(-
f^2*g + e*f*h)*f) + sqrt(h*x + g)/f) - sqrt(h*x + g)*log(f*x + e))*b*p*q -
 sqrt(h*x + g)*b*q*log(d) - sqrt(h*x + g)*b*log(c) - sqrt(h*x + g)*a)/h
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx = \int \frac{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\sqrt{g+h\,x}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(1/2), x)
```

3.485 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{3/2}}\,dx$

3.485.1 Optimal result

Integrand size = 28, antiderivative size = 86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=-\frac{4b\sqrt{f}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}$$

output
```
-4*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*f^(1/2)/h/(-e*h+f
*g)^(1/2)-2*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^(1/2)
```

3.485.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=\frac{-\frac{4b\sqrt{f}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{g+hx}}}{h}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(3/2),x]
```

output
```
((-4*b*Sqrt[f]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/Sqrt[
f*g - e*h] - (2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/Sqrt[g + h*x])/h
```

3.485.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2895, 2842, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx$$

$$\Big\downarrow \ 2895$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}dx$$

$$\Big\downarrow \ 2842$$

$$\frac{2bfpq\int \frac{1}{(e+fx)\sqrt{g+hx}}dx}{h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}$$

$$\Big\downarrow \ 73$$

$$\frac{4bfpq\int \frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{h^2}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}$$

$$\Big\downarrow \ 221$$

$$-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}-\frac{4b\sqrt{f}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(3/2),x]
```

output
```
(-4*b*Sqrt[f]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(h*Sqr
t[f*g - e*h]) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(h*Sqrt[g + h*x])
```

3.485.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.485.4 Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)}{(hx+g)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x)
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.79

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \left[\frac{2\left((bhpqx + bgpq)\sqrt{\frac{f}{fg-eh}}\log\left(\frac{fhx+2\,fg-eh-2\,(fg-eh)\sqrt{hx+g}\sqrt{\frac{f}{fg-eh}}}{fx+e}\right) - (bp\right.}{h^2x+gh}\right.$$

$$-\frac{2\left(2\,(bhpqx + bgpq)\sqrt{-\frac{f}{fg-eh}}\arctan\left(-\frac{(fg-eh)\sqrt{hx+g}\sqrt{-\frac{f}{fg-eh}}}{fhx+fg}\right) + (bpq\log(fx+e) + bq\log(d) + b\log(c)\right.}{h^2x+gh}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x, algorithm="fricas"
)
```

output
```
[2*((b*h*p*q*x + b*g*p*q)*sqrt(f/(f*g - e*h))*log((f*h*x + 2*f*g - e*h - 2
*(f*g - e*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)))/(f*x + e)) - (b*p*q*log(f*
x + e) + b*q*log(d) + b*log(c) + a)*sqrt(h*x + g))/(h^2*x + g*h), -2*(2*(b
*h*p*q*x + b*g*p*q)*sqrt(-f/(f*g - e*h))*arctan(-(f*g - e*h)*sqrt(h*x + g)
*sqrt(-f/(f*g - e*h))/(f*h*x + f*g)) + (b*p*q*log(f*x + e) + b*q*log(d) +
b*log(c) + a)*sqrt(h*x + g))/(h^2*x + g*h)]
```

3.485.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**(3/2), x)
```

3.485.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.485.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \frac{4\,bfpq\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{\sqrt{-f^2g+efh}h} - \frac{2\,bpq\log\left((hx+g)f - fg + eh\right)}{\sqrt{hx+g}h} + \frac{2\,(bpq\log\left(h\right) - bq\log\left(d\right) - b\log\left(c\right) - a)}{\sqrt{hx+g}h}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x, algorithm="giac")
```

output
```
4*b*f*p*q*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/(sqrt(-f^2*g + e*f*
h)*h) - 2*b*p*q*log((h*x + g)*f - f*g + e*h)/(sqrt(h*x + g)*h) + 2*(b*p*q*
log(h) - b*q*log(d) - b*log(c) - a)/(sqrt(h*x + g)*h)
```

3.485.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{{\left(g+h\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(3/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(3/2), x)
```

3.486 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{5/2}}\,dx$

3.486.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \frac{4bfpq}{3h(fg-eh)\sqrt{g+hx}} - \frac{4bf^{3/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} - \frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h(g+hx)^{3/2}}$$

output
```
-4/3*b*f^(3/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/(-e*h
+f*g)^(3/2)-2/3*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^(3/2)+4/3*b*f*p*q/h/
(-e*h+f*g)/(h*x+g)^(1/2)
```

3.486.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \frac{-4bfpq(g+hx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{f(g+hx)}{fg-eh}\right)+2(fg-eh)\,(a}{3h(-fg+eh)(g+hx)^{3/2}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(5/2),x]
```

output
```
(-4*b*f*p*q*(g + h*x)*Hypergeometric2F1[-1/2, 1, 1/2, (f*(g + h*x))/(f*g -
 e*h)] + 2*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(3*h*(-(f*g) + e*
h)*(g + h*x)^(3/2))
```

3.486.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2895, 2842, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}dx$$

$$\Big\downarrow \ 2895$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}dx$$

$$\Big\downarrow \ 2842$$

$$\frac{2bfpq\int \frac{1}{(e+fx)(g+hx)^{3/2}}dx}{3h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{3h(g+hx)^{3/2}}$$

$$\Big\downarrow \ 61$$

$$\frac{2bfpq\left(\frac{f\int \frac{1}{(e+fx)\sqrt{g+hx}}dx}{fg-eh}+\frac{2}{\sqrt{g+hx}(fg-eh)}\right)}{3h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{3h(g+hx)^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{2bfpq\left(\frac{2f\int \frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{h(fg-eh)}+\frac{2}{\sqrt{g+hx}(fg-eh)}\right)}{3h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{3h(g+hx)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{2bfpq\left(\frac{2}{\sqrt{g+hx}(fg-eh)}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}\right)}{3h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{3h(g+hx)^{3/2}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(5/2),x]
```

output
```
(2*b*f*p*q*(2/((f*g - e*h)*Sqrt[g + h*x]) - (2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sq
rt[g + h*x])/Sqrt[f*g - e*h]])/(f*g - e*h)^(3/2)))/(3*h) - (2*(a + b*Log[c
*(d*(e + f*x)^p)^q]))/(3*h*(g + h*x)^(3/2))
```

3.486.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.486.4 Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^p\right)^q\right)}{(hx+g)^{\frac{5}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x)
```

3.486.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(100) = 200$.

Time = 0.35 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.89

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{5/2}}\,dx = \left[-\frac{2\left(\left(bfh^2pqx^2+2\,bfghpqx+bfg^2pq\right)\sqrt{\frac{f}{fg-eh}}\log\left(\frac{fhx+2\,fg-eh+2\,(fg-eh}{fx+e}\right.\right.}{}\right.$$

$$-\frac{2\left(2\left(bfh^2pqx^2+2\,bfghpqx+bfg^2pq\right)\sqrt{-\frac{f}{fg-eh}}\arctan\left(-\frac{(fg-eh)\sqrt{hx+g}\sqrt{-\frac{f}{fg-eh}}}{fhx+fg}\right)-(2\,bfhpqx+2\,bfgpq\right.}{3\left(fg^3h-eg^2h^2+(fgh^3-eh^4\right.}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x, algorithm="fricas"
)
```

output
```
[-2/3*((b*f*h^2*p*q*x^2 + 2*b*f*g*h*p*q*x + b*f*g^2*p*q)*sqrt(f/(f*g - e*h
))*log((f*h*x + 2*f*g - e*h + 2*(f*g - e*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*
h)))/(f*x + e)) - (2*b*f*h*p*q*x + 2*b*f*g*p*q - (b*f*g - b*e*h)*p*q*log(f
*x + e) - a*f*g + a*e*h - (b*f*g - b*e*h)*q*log(d) - (b*f*g - b*e*h)*log(c
))*sqrt(h*x + g))/(f*g^3*h - e*g^2*h^2 + (f*g*h^3 - e*h^4)*x^2 + 2*(f*g^2*
h^2 - e*g*h^3)*x), -2/3*(2*(b*f*h^2*p*q*x^2 + 2*b*f*g*h*p*q*x + b*f*g^2*p*
q)*sqrt(-f/(f*g - e*h))*arctan(-(f*g - e*h)*sqrt(h*x + g)*sqrt(-f/(f*g - e
*h))/(f*h*x + f*g)) - (2*b*f*h*p*q*x + 2*b*f*g*p*q - (b*f*g - b*e*h)*p*q*l
og(f*x + e) - a*f*g + a*e*h - (b*f*g - b*e*h)*q*log(d) - (b*f*g - b*e*h)*l
og(c))*sqrt(h*x + g))/(f*g^3*h - e*g^2*h^2 + (f*g*h^3 - e*h^4)*x^2 + 2*(f*
g^2*h^2 - e*g*h^3)*x)]
```

3.486.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{5/2}}\,dx = \int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(5/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**(5/2), x)
```

3.486.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.486.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.53

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{5/2}}\,dx = \frac{4\,bf^2pq\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{3\sqrt{-f^2g + efh}(fgh - eh^2)} - \frac{2\,bpq\log\left((hx + g)f - fg + eh\right)}{3\,(hx + g)^{\frac{3}{2}}h} + \frac{2\,(bfgpq\log\left(h\right) - behpq\log\left(h\right) + 2\,(hx + g)bfpq - bfgq\log\left(d\right) + behq\log\left(d\right) - bfg\log\left(c\right) + beh\log\left(c\right)}{3\left((hx + g)^{\frac{3}{2}}fgh - (hx + g)^{\frac{3}{2}}eh^2\right)}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x, algorithm="giac")
```

output
```
4/3*b*f^2*p*q*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/(sqrt(-f^2*g +
e*f*h)*(f*g*h - e*h^2)) - 2/3*b*p*q*log((h*x + g)*f - f*g + e*h)/((h*x + g
)^(3/2)*h) + 2/3*(b*f*g*p*q*log(h) - b*e*h*p*q*log(h) + 2*(h*x + g)*b*f*p*
q - b*f*g*q*log(d) + b*e*h*q*log(d) - b*f*g*log(c) + b*e*h*log(c) - a*f*g
+ a*e*h)/((h*x + g)^(3/2)*f*g*h - (h*x + g)^(3/2)*e*h^2)
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \int \frac{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\left(g+h\,x\right)^{5/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(5/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(5/2), x)
```

3.487 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{7/2}}\,dx$

3.487.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{7/2}}\,dx=\frac{4bfpq}{15h(fg-eh)(g+hx)^{3/2}}+\frac{4bf^2pq}{5h(fg-eh)^2\sqrt{g+hx}}-\frac{4bf^{5/2}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5h(fg-eh)^{5/2}}-\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h(g+hx)^{5/2}}$$

output
```
4/15*b*f*p*q/h/(-e*h+f*g)/(h*x+g)^(3/2)-4/5*b*f^(5/2)*p*q*arctanh(f^(1/2)*
(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/(-e*h+f*g)^(5/2)-2/5*(a+b*ln(c*(d*(f*x+e
)^p)^q))/h/(h*x+g)^(5/2)+4/5*b*f^2*p*q/h/(-e*h+f*g)^2/(h*x+g)^(1/2)
```

3.487.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{7/2}}\,dx=\frac{-4bfpq(g+hx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{f(g+hx)}{fg-eh}\right)+6(fg-eh)\,(}{15h(-fg+eh)(g+hx)^{5/2}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(7/2),x]
```

output
```
(-4*b*f*p*q*(g + h*x)*Hypergeometric2F1[-3/2, 1, -1/2, (f*(g + h*x))/(f*g
- e*h)] + 6*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(15*h*(-(f*g) +
e*h)*(g + h*x)^(5/2))
```

3.487.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2895, 2842, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{7/2}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{7/2}}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2bfpq\int \frac{1}{(e+fx)(g+hx)^{5/2}}dx}{5h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2bfpq\left(\frac{f\int \frac{1}{(e+fx)(g+hx)^{3/2}}dx}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{5h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2bfpq\left(\frac{f\left(\frac{f\int \frac{1}{(e+fx)\sqrt{g+hx}}dx}{fg-eh}+\frac{2}{\sqrt{g+hx}(fg-eh)}\right)}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{5h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2bfpq\left(\frac{f\left(\frac{2f\int \frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{h(fg-eh)}+\frac{2}{\sqrt{g+hx}(fg-eh)}\right)}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{5h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{2bfpq\left(\frac{f\left(\frac{2}{\sqrt{g+hx}(fg-eh)}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}\right)}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{5h}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{5h(g+hx)^{5/2}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(7/2),x]
```

output
```
(2*b*f*p*q*(2/(3*(f*g - e*h)*(g + h*x)^(3/2)) + (f*(2/((f*g - e*h)*Sqrt[g
+ h*x]) - (2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(f*
g - e*h)^(3/2)))/(f*g - e*h)))/(5*h) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q])
)/(5*h*(g + h*x)^(5/2))
```

3.487.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.487.4 Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)}{(hx+g)^{\frac{7}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x)
```

3.487.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(128) = 256$.

Time = 0.37 (sec) , antiderivative size = 863, normalized size of antiderivative = 5.68

$$\int \frac{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}{(g+hx)^{7/2}}\,dx = \left[\frac{2\left(3\left(bf^2h^3pqx^3+3\,bf^2gh^2pqx^2+3\,bf^2g^2hpqx+bf^2g^3pq\right)\sqrt{\frac{f}{fg-eh}}\log\left(\right.\right.}{}\right.$$

$$-\frac{2\left(6\left(bf^2h^3pqx^3+3\,bf^2gh^2pqx^2+3\,bf^2g^2hpqx+bf^2g^3pq\right)\sqrt{-\frac{f}{fg-eh}}\arctan\left(-\frac{(fg-eh)\sqrt{hx+g}\sqrt{-\frac{f}{fg-eh}}}{fhx+fg}\right)-\left(\right.\right.}{15\left(f^2\right.}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x, algorithm="fricas"
)
```

output
```
[2/15*(3*(b*f^2*h^3*p*q*x^3 + 3*b*f^2*g*h^2*p*q*x^2 + 3*b*f^2*g^2*h*p*q*x
+ b*f^2*g^3*p*q)*sqrt(f/(f*g - e*h))*log((f*h*x + 2*f*g - e*h - 2*(f*g - e
*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)))/(f*x + e)) + (6*b*f^2*h^2*p*q*x^2 -
 3*a*f^2*g^2 + 6*a*e*f*g*h - 3*a*e^2*h^2 + 2*(7*b*f^2*g*h - b*e*f*h^2)*p*q
*x - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*log(f*x + e) + 2*(4*b*f^2
*g^2 - b*e*f*g*h)*p*q - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*q*log(d) -
 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))*sqrt(h*x + g))/(f^2*g^5*h
 - 2*e*f*g^4*h^2 + e^2*g^3*h^3 + (f^2*g^2*h^4 - 2*e*f*g*h^5 + e^2*h^6)*x^3
 + 3*(f^2*g^3*h^3 - 2*e*f*g^2*h^4 + e^2*g*h^5)*x^2 + 3*(f^2*g^4*h^2 - 2*e*
f*g^3*h^3 + e^2*g^2*h^4)*x), -2/15*(6*(b*f^2*h^3*p*q*x^3 + 3*b*f^2*g*h^2*p
*q*x^2 + 3*b*f^2*g^2*h*p*q*x + b*f^2*g^3*p*q)*sqrt(-f/(f*g - e*h))*arctan(
-(f*g - e*h)*sqrt(h*x + g)*sqrt(-f/(f*g - e*h))/(f*h*x + f*g)) - (6*b*f^2*
h^2*p*q*x^2 - 3*a*f^2*g^2 + 6*a*e*f*g*h - 3*a*e^2*h^2 + 2*(7*b*f^2*g*h - b
*e*f*h^2)*p*q*x - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*log(f*x + e)
 + 2*(4*b*f^2*g^2 - b*e*f*g*h)*p*q - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^
2)*q*log(d) - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))*sqrt(h*x + g
))/(f^2*g^5*h - 2*e*f*g^4*h^2 + e^2*g^3*h^3 + (f^2*g^2*h^4 - 2*e*f*g*h^5 +
 e^2*h^6)*x^3 + 3*(f^2*g^3*h^3 - 2*e*f*g^2*h^4 + e^2*g*h^5)*x^2 + 3*(f^2*g
^4*h^2 - 2*e*f*g^3*h^3 + e^2*g^2*h^4)*x)]
```

3.487.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(7/2),x)
```

output
```
Timed out
```

3.487.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.487.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(128) = 256$.

Time = 0.34 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.13

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{7/2}}\,dx = \frac{4\,bf^3hpq\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{5\,(f^2g^2h^2 - 2\,efgh^3 + e^2h^4)\sqrt{-f^2g + efh}} - \frac{2\,bpq\log\left((hx + g)f - fg + eh\right)}{5\,(hx + g)^{\frac{5}{2}}h} + \frac{2\left(3\,bf^2g^2pq\log\left(h\right) - 6\,befghpq\log\left(h\right) + 3\,be^2h^2pq\log\left(h\right) + 6\,(hx + g)^2bf^2pq + 2\,(hx + g)bf^2gpq - 2\,(hx\right.}{15\left((hx\right.}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x, algorithm="giac")
```

output
```
4/5*b*f^3*h*p*q*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/((f^2*g^2*h^2
 - 2*e*f*g*h^3 + e^2*h^4)*sqrt(-f^2*g + e*f*h)) - 2/5*b*p*q*log((h*x + g)*
f - f*g + e*h)/((h*x + g)^(5/2)*h) + 2/15*(3*b*f^2*g^2*p*q*log(h) - 6*b*e*
f*g*h*p*q*log(h) + 3*b*e^2*h^2*p*q*log(h) + 6*(h*x + g)^2*b*f^2*p*q + 2*(h
*x + g)*b*f^2*g*p*q - 2*(h*x + g)*b*e*f*h*p*q - 3*b*f^2*g^2*q*log(d) + 6*b
*e*f*g*h*q*log(d) - 3*b*e^2*h^2*q*log(d) - 3*b*f^2*g^2*log(c) + 6*b*e*f*g*
h*log(c) - 3*b*e^2*h^2*log(c) - 3*a*f^2*g^2 + 6*a*e*f*g*h - 3*a*e^2*h^2)/(
(h*x + g)^(5/2)*f^2*g^2*h - 2*(h*x + g)^(5/2)*e*f*g*h^2 + (h*x + g)^(5/2)*
e^2*h^3)
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{7/2}}\,dx = \int \frac{a + b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\left(g+h\,x\right)^{7/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(7/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(7/2), x)
```

3.488 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{9/2}}\,dx$

3.488.1 Optimal result

Integrand size = 28, antiderivative size = 184

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx=\frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}}+\frac{4bf^2pq}{21h(fg-eh)^2(g+hx)^{3/2}}+\frac{4bf^3pq}{7h(fg-eh)^3\sqrt{g+hx}}-\frac{4bf^{7/2}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{7h(fg-eh)^{7/2}}-\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{7h(g+hx)^{7/2}}$$

output
```
4/35*b*f*p*q/h/(-e*h+f*g)/(h*x+g)^(5/2)+4/21*b*f^2*p*q/h/(-e*h+f*g)^2/(h*x
+g)^(3/2)-4/7*b*f^(7/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)
)/h/(-e*h+f*g)^(7/2)-2/7*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^(7/2)+4/7*b
*f^3*p*q/h/(-e*h+f*g)^3/(h*x+g)^(1/2)
```

3.488.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.49

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx=\frac{-4bfpq(g+hx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{f(g+hx)}{fg-eh}\right)+10(fg-eh)}{35h(-fg+eh)(g+hx)^{7/2}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(9/2),x]
```

output
```
(-4*b*f*p*q*(g + h*x)*Hypergeometric2F1[-5/2, 1, -3/2, (f*(g + h*x))/(f*g
- e*h)] + 10*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(35*h*(-(f*g) +
 e*h)*(g + h*x)^(7/2))
```

3.488.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2895, 2842, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{9/2}}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{9/2}}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{2bfpq\int \frac{1}{(e+fx)(g+hx)^{7/2}}dx}{7h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{7h(g+hx)^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2bfpq\left(\frac{f\int \frac{1}{(e+fx)(g+hx)^{5/2}}dx}{fg-eh}+\frac{2}{5(g+hx)^{5/2}(fg-eh)}\right)}{7h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{7h(g+hx)^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2bfpq\left(\frac{f\left(\frac{f\int \frac{1}{(e+fx)(g+hx)^{3/2}}dx}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{fg-eh}+\frac{2}{5(g+hx)^{5/2}(fg-eh)}\right)}{7h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{7h(g+hx)^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2bfpq\left(\frac{f\left(\frac{f\left(\frac{f\int\frac{1}{(e+fx)\sqrt{g+hx}}dx}{fg-eh}+\frac{2}{\sqrt{g+hx}(fg-eh)}\right)}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{fg-eh}+\frac{2}{5(g+hx)^{5/2}(fg-eh)}\right)}{7h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}}$$

$\downarrow$ 73

$$\frac{2bfpq\left(\frac{f\left(\frac{f\left(\frac{2f\int\frac{1}{e+\frac{f(g+hx)}{h}-\frac{fg}{h}}d\sqrt{g+hx}}{h(fg-eh)}+\frac{2}{\sqrt{g+hx}(fg-eh)}\right)}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{fg-eh}+\frac{2}{5(g+hx)^{5/2}(fg-eh)}\right)}{7h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}}$$

$\downarrow$ 221

$$\frac{2bfpq\left(\frac{f\left(\frac{f\left(\frac{2}{\sqrt{g+hx}(fg-eh)}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}\right)}{fg-eh}+\frac{2}{3(g+hx)^{3/2}(fg-eh)}\right)}{fg-eh}+\frac{2}{5(g+hx)^{5/2}(fg-eh)}\right)}{7h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(9/2),x]
```

3.488. $\int\frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{9/2}}dx$

output
```
(2*b*f*p*q*(2/(5*(f*g - e*h)*(g + h*x)^(5/2)) + (f*(2/(3*(f*g - e*h)*(g +
h*x)^(3/2)) + (f*(2/((f*g - e*h)*Sqrt[g + h*x]) - (2*Sqrt[f]*ArcTanh[(Sqrt
[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(f*g - e*h)^(3/2)))/(f*g - e*h)))/(f*
g - e*h)))/(7*h) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(7*h*(g + h*x)^(7/
2))
```

3.488.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.488. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{9/2}}\,dx$

3.488.4 Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)}{(hx+g)^{\frac{9}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x)
```

3.488.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(156) = 312$.

Time = 0.41 (sec) , antiderivative size = 1362, normalized size of antiderivative = 7.40

$$\int \frac{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}{(g+hx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x, algorithm="fricas"
)
```

output
```
[-2/105*(15*(b*f^3*h^4*p*q*x^4 + 4*b*f^3*g*h^3*p*q*x^3 + 6*b*f^3*g^2*h^2*p
*q*x^2 + 4*b*f^3*g^3*h*p*q*x + b*f^3*g^4*p*q)*sqrt(f/(f*g - e*h))*log((f*h
*x + 2*f*g - e*h + 2*(f*g - e*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)))/(f*x +
 e)) - (30*b*f^3*h^3*p*q*x^3 - 15*a*f^3*g^3 + 45*a*e*f^2*g^2*h - 45*a*e^2*
f*g*h^2 + 15*a*e^3*h^3 + 10*(10*b*f^3*g*h^2 - b*e*f^2*h^3)*p*q*x^2 + 2*(58
*b*f^3*g^2*h - 16*b*e*f^2*g*h^2 + 3*b*e^2*f*h^3)*p*q*x - 15*(b*f^3*g^3 - 3
*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*p*q*log(f*x + e) + 2*(23*b*f
^3*g^3 - 11*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2)*p*q - 15*(b*f^3*g^3 - 3*b*e*f
^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*q*log(d) - 15*(b*f^3*g^3 - 3*b*e*f
^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*log(c))*sqrt(h*x + g))/(f^3*g^7*h
 - 3*e*f^2*g^6*h^2 + 3*e^2*f*g^5*h^3 - e^3*g^4*h^4 + (f^3*g^3*h^5 - 3*e*f^2
*g^2*h^6 + 3*e^2*f*g*h^7 - e^3*h^8)*x^4 + 4*(f^3*g^4*h^4 - 3*e*f^2*g^3*h^5
 + 3*e^2*f*g^2*h^6 - e^3*g*h^7)*x^3 + 6*(f^3*g^5*h^3 - 3*e*f^2*g^4*h^4 + 3
*e^2*f*g^3*h^5 - e^3*g^2*h^6)*x^2 + 4*(f^3*g^6*h^2 - 3*e*f^2*g^5*h^3 + 3*e
^2*f*g^4*h^4 - e^3*g^3*h^5)*x), -2/105*(30*(b*f^3*h^4*p*q*x^4 + 4*b*f^3*g*
h^3*p*q*x^3 + 6*b*f^3*g^2*h^2*p*q*x^2 + 4*b*f^3*g^3*h*p*q*x + b*f^3*g^4*p*
q)*sqrt(-f/(f*g - e*h))*arctan(-(f*g - e*h)*sqrt(h*x + g)*sqrt(-f/(f*g - e
*h))/(f*h*x + f*g)) - (30*b*f^3*h^3*p*q*x^3 - 15*a*f^3*g^3 + 45*a*e*f^2*g^
2*h - 45*a*e^2*f*g*h^2 + 15*a*e^3*h^3 + 10*(10*b*f^3*g*h^2 - b*e*f^2*h^3)*
p*q*x^2 + 2*(58*b*f^3*g^2*h - 16*b*e*f^2*g*h^2 + 3*b*e^2*f*h^3)*p*q*x -...
```

3.488.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \log (c(d(e + fx)^p)^q)}{(g + hx)^{9/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(9/2),x)
```

output
```
Timed out
```

3.488.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \log (c(d(e + fx)^p)^q)}{(g + hx)^{9/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.488.8 Giac [F]

$$\int \frac{a + b \log (c(d(e + fx)^p)^q)}{(g + hx)^{9/2}} \, dx = \int \frac{b \log (((fx + e)^p d)^q c) + a}{(hx + g)^{\frac{9}{2}}} \, dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^(9/2), x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx=\int \frac{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\left(g+h\,x\right)^{9/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(9/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(9/2), x)
```

3.489 $\int (g+hx)^{3/2} \left(a + b \log \left(c(d(e + fx)^p)^q\right)\right)^2 dx$

3.489.1 Optimal result

Integrand size = 30, antiderivative size = 635

$$\int (g+hx)^{3/2} \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2 \, dx = \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} + \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh} + \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{368b^2(fg-eh)^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h} - \frac{8b^2(fg-eh)^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h} - \frac{8b(fg-eh)^2pq\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{5f^2h} - \frac{8b(fg-eh)pq(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{15fh} - \frac{8bpq(g+hx)^{5/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{25h} + \frac{8b(fg-eh)^{5/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{5f^{5/2}h} + \frac{2(g+hx)^{5/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h} + \frac{16b^2(fg-eh)^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h} + \frac{8b^2(fg-eh)^{5/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h}$$

output
```
128/225*b^2*(-e*h+f*g)*p^2*q^2*(h*x+g)^(3/2)/f/h+16/125*b^2*p^2*q^2*(h*x+g
)^(5/2)/h-368/75*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2
)/(-e*h+f*g)^(1/2))/f^(5/2)/h-8/5*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*arctanh(f^(
1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2/f^(5/2)/h-8/15*b*(-e*h+f*g)*p*q*(h*
x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f/h-8/25*b*p*q*(h*x+g)^(5/2)*(a+b*l
n(c*(d*(f*x+e)^p)^q))/h+8/5*b*(-e*h+f*g)^(5/2)*p*q*arctanh(f^(1/2)*(h*x+g)
^(1/2)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^(5/2)/h+2/5*(h*x+g)
^(5/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h+16/5*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*a
rctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/
2)/(-e*h+f*g)^(1/2)))/f^(5/2)/h+8/5*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*polylog(2
,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/f^(5/2)/h+368/75*b^2*(-e*
h+f*g)^2*p^2*q^2*(h*x+g)^(1/2)/f^2/h-8/5*b*(-e*h+f*g)^2*p*q*(a+b*ln(c*(d*(
f*x+e)^p)^q))*(h*x+g)^(1/2)/f^2/h
```

3.489.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4354 vs. $2(635) = 1270$.

Time = 18.56 (sec) , antiderivative size = 4354, normalized size of antiderivative = 6.86

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(4*b*g*p*q*((6*(f*g - e*h)^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e +
 f*x))/f])/Sqrt[f*g - e*h]])/Sqrt[f] - Sqrt[(f*g - e*h + h*(e + f*x))/f]*(
h*(e + f*x)*(2 - 3*Log[e + f*x]) + (f*g - e*h)*(8 - 3*Log[e + f*x])))*(a +
 b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x])
+ Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + L
og[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) +
Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e
 + f*x)^p]))/Log[d*(e + f*x)^p])]))))/(9*f*h) - (4*b*p*q*(30*(f*g - e*h)^(3
/2)*(2*f*g + 3*e*h)*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sq
rt[f*g - e*h]] + Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(9*h^2*(e + f*x
)^2*(2 - 5*Log[e + f*x]) + (f*g - e*h)*(3*e*h*(-46 + 15*Log[e + f*x]) + 2*
f*g*(-31 + 15*Log[e + f*x])) + h*(e + f*x)*(f*g*(16 - 15*Log[e + f*x]) + 6
*e*h*(-11 + 15*Log[e + f*x]))))*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f
*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*
x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]
) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(
q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))))/(
225*f^(5/2)*h) + Sqrt[g + h*x]*((2*g^2*(a + b*q*(-(p*Log[e + f*x]) + Log[d
*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*
(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e...
```

3.489.3 Rubi [A] (warning: unable to verify)

Time = 6.41 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.63, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.967, Rules used = {2895, 2845, 2858, 2788, 2756, 60, 60, 60, 73, 221, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx$$

$$\Big\downarrow \text{2895}$$

$$\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2(g+hx)^{5/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-\frac{4bfpq\int \frac{(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{e+fx}dx}{5h}$$

$\downarrow$ 2858

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-\frac{4bpq\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)}{5h}$$

$\downarrow$ 2788

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-\frac{4bpq\left(\frac{h\int\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))d(e+fx)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)\right.}{5h}$$

$\downarrow$ 2756

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h}-\frac{2bfpq\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}{e+fx}d(e+fx)}{5h}\right)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}(a+b}{e+fx}\right.}{5h}$$

$\downarrow$ 60

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}{e+fx}d(e+fx)+\frac{2}{5}\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}\right)}{5h}\right)}{f}+\left(g-\right.\right.}{5h}$$

$\downarrow$ 60

3.489. $\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{e+fx}d(e+fx)+\frac{2}{3}\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\right)+\frac{2}{5}\left(\frac{h(\cdots}{}\right.\right.}{5h}\right)}{f}\right.}{5h}$$

$\downarrow$ 60

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+2\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}\cdots}\right.\right.\right.}{5h}\right)}{f}\right.}{5h}$$

$\downarrow$ 73

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{2f\left(g-\frac{eh}{f}\right)\int\frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h}+\cdots\right.\right.\right.}{5h}\right)}{f}\right.$$

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$\downarrow$ 221

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)+\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h}-\frac{2bfpq\left(\right.}{}\right.}{}\right.$$

5

$\downarrow$ 2788

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\int\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}(a+b\log(cd^q(e+fx)^{pq}))d(e+fx)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e\right.\right.$$

$\downarrow$ 2756

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}{e+fx}d(e+fx)}{3h}\right)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h}{}}}{}\right.\right.$$

$$\Big\downarrow \; 60$$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{e+fx}d(e+fx)+\frac{2}{3}\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\right)}{3h}\right)}{f}\right.\right.$$

$$\Big\downarrow \; 60$$

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+2\sqrt{\frac{h(e+fx)}{f}\cdots}\right.\right.}{3h}\right.}{f}\right.\right.$$

$\downarrow$ 73

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{2f\left(g-\frac{eh}{f}\right)\int\frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e\cdots}{}}}{h}\right.\right.}{3h}\right.}{f}\right.\right.$$

$\downarrow$ 221

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\left(a+b\log(cd^q(e+fx)^{pq})\right)}{e+fx}d(e+fx)+\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log(cd^q(e+fx)^{pq}\right.}{3h}\right.}{}\right.\right.$$

$\downarrow$ 2788

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\int\frac{a+b\log(cd^q(e+fx)^{pq})}{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)\right)+\frac{h\left(\frac{2f\left(\frac{h}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2756

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{e+fx}d(e+fx)}{h}\right)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{a+}{(e+f}\right.\right.\right.$$

$\downarrow$ 60

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+2\sqrt{\frac{h(e+f}{f}}\right.}{h}\right.}{f}\right.\right.\right.$$

$\downarrow$ 73

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\,(a+b\log(cd^q(e+fx)^{pq}))}{h}-\frac{2bfpq\left(\frac{2f\left(g-\frac{eh}{f}\right)\int\frac{1}{\frac{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(}{}}}{h}}{h}\right)}{f}\right.\right.\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\,(a+b\log(cd^q(e+fx)^{pq}))}{h}-\frac{2b}{}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{2790}$$

3.489. $\int(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h}-\frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right.}{f}\right.$$

$\downarrow$ 27

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h}-\frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right.}{f}\right.$$

$\downarrow$ 7267

3.489. $\int (g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$
$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h}-\frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right.}{f}\right.\cdots$$

$\downarrow$ 2092

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$
$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h}-\frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right.}{f}\right.\cdots$$

$\downarrow$ 6546

3.489. $\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \frac{4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h} - \frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right)}{f}\cdots\right)}{\cdots}$$

$\downarrow$ 6470

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \frac{4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}(a+b\log(cd^q(e+fx)^{pq}))}{5h} - \frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right)}{f}\cdots\right)}{\cdots}$$

$\downarrow$ 2849

3.489. $\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} -$$

$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h} - \frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\cdots}\right.\right.\right.}{5h}\right.}{f}\right.$$

$\downarrow$ 2752

3.489. $\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\, dx$

$$\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h}-$$

$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h}-\frac{2bfpq\left(\frac{2}{5}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}+\left(g-\frac{eh}{f}\right)\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\right.\right.\right.}{5h}\right.}{f}\right.$$

input
```
Int[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

3.489. $\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

output
```
(2*(g + h*x)^(5/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(5*h) - (4*b*p*q*((
h*((-2*b*f*p*q*((2*(g - (e*h)/f + (h*(e + f*x))/f)^(5/2))/5 + (g - (e*h)/f
)*((2*(g - (e*h)/f + (h*(e + f*x))/f)^(3/2))/3 + (g - (e*h)/f)*(2*Sqrt[g -
 (e*h)/f + (h*(e + f*x))/f] - (2*Sqrt[f]*(g - (e*h)/f)*ArcTanh[(Sqrt[f]*Sq
rt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]])/Sqrt[f*g - e*h]))))/(
5*h) + (2*f*(g - (e*h)/f + (h*(e + f*x))/f)^(5/2)*(a + b*Log[c*d^q*(e + f*
x)^(p*q)]))/(5*h)))/f + (g - (e*h)/f)*((h*((-2*b*f*p*q*((2*(g - (e*h)/f +
(h*(e + f*x))/f)^(3/2))/3 + (g - (e*h)/f)*(2*Sqrt[g - (e*h)/f + (h*(e + f*
x))/f] - (2*Sqrt[f]*(g - (e*h)/f)*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(
e + f*x))/f])/Sqrt[f*g - e*h]])/Sqrt[f*g - e*h])))/(3*h) + (2*f*(g - (e*h)
/f + (h*(e + f*x))/f)^(3/2)*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/(3*h)))/f
+ (g - (e*h)/f)*((h*((-2*b*f*p*q*(2*Sqrt[g - (e*h)/f + (h*(e + f*x))/f] -
(2*Sqrt[f]*(g - (e*h)/f)*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))
/f])/Sqrt[f*g - e*h]])/Sqrt[f*g - e*h]))/h + (2*f*Sqrt[g - (e*h)/f + (h*(e
 + f*x))/f]*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/h))/f + (g - (e*h)/f)*((-2
*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g -
e*h]]*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/Sqrt[f*g - e*h] + (4*b*f^(3/2)*p
*q*(ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]
^2/(2*f) - ((Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f
*x))/f])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e ...
```

3.489.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

3.489. $\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.489. $\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$

3.489.4 Maple [F]

$$\int (hx+g)^{\frac{3}{2}} \left(a+b\ln \left(c(d(fx+e)^{p})^{q}\right)\right)^{2}dx$$

input
```
int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.489.5 Fricas [F]

$$\int (g+hx)^{3/2} \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \int (hx+g)^{\frac{3}{2}}(b\log \left(((fx+e)^{p}d)^{q}\, c\right)+a)^{2}\, dx$$

input
```
integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="frica
s")
```

output
```
integral((b^2*h*x + b^2*g)*sqrt(h*x + g)*log(((f*x + e)^p*d)^q*c)^2 + 2*(a
*b*h*x + a*b*g)*sqrt(h*x + g)*log(((f*x + e)^p*d)^q*c) + (a^2*h*x + a^2*g)
*sqrt(h*x + g), x)
```

3.489.6 Sympy [F]

$$\int (g+hx)^{3/2} \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \int \left(a+b\log \left(c(d(e+fx)^{p})^{q}\right)\right)^{2} (g+hx)^{\frac{3}{2}}\, dx$$

input
```
integrate((h*x+g)**(3/2)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)**(3/2), x)
```

3.489.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^{3/2} \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.489.8 Giac [F]

$$\int (g+hx)^{3/2} \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \int (hx+g)^{\frac{3}{2}} \left(b\log \left(((fx+e)^p d)^q c\right)+a\right)^2 dx$$

input
```
integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a)^2, x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{3/2} \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \int (g+h\,x)^{3/2} \left(a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2 dx$$

input
```
int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

3.490 $\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$

3.490.1 Optimal result

Integrand size = 30, antiderivative size = 547

$$\begin{aligned}
&\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx \\
&= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh}+\frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} \\
&\quad-\frac{64b^2(fg-eh)^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h}-\frac{8b^2(fg-eh)^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h} \\
&\quad-\frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} \\
&\quad-\frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} \\
&\quad+\frac{8b(fg-eh)^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} \\
&\quad+\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} \\
&\quad+\frac{16b^2(fg-eh)^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h} \\
&\quad+\frac{8b^2(fg-eh)^{3/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h}
\end{aligned}$$

output
```
16/27*b^2*p^2*q^2*(h*x+g)^(3/2)/h-64/9*b^2*(-e*h+f*g)^(3/2)*p^2*q^2*arctan
h(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/f^(3/2)/h-8/3*b^2*(-e*h+f*g)^(3/
2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2/f^(3/2)/h-8/9
*b*p*q*(h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))/h+8/3*b*(-e*h+f*g)^(3/2)*
p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p
)^q))/f^(3/2)/h+2/3*(h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h+16/3*b^2
*(-e*h+f*g)^(3/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*
ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/f^(3/2)/h+8/3*b^2*(-e*h+f
*g)^(3/2)*p^2*q^2*polylog(2,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))
)/f^(3/2)/h+64/9*b^2*(-e*h+f*g)*p^2*q^2*(h*x+g)^(1/2)/f/h-8/3*b*(-e*h+f*g)
*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*(h*x+g)^(1/2)/f/h
```

3.490.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1323 vs. $2(547) = 1094$.

Time = 11.77 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.42

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$$
$$= \frac{2\left(-\frac{6bpq\left(6(fg-eh)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)+\sqrt{f}\sqrt{g+hx}(6eh-2f(4g+hx)+3f(g+hx)\log(e+fx))\right)(-a+bpq\log(e+fx)-b\log(c(d(e+fx)^p}{f^{3/2}}\right.}{}$$

input
```
Integrate[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(2*((-6*b*p*q*(6*(f*g - e*h)^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*
g - e*h]] + Sqrt[f]*Sqrt[g + h*x]*(6*e*h - 2*f*(4*g + h*x) + 3*f*(g + h*x)
*Log[e + f*x]))*(-a + b*p*q*Log[e + f*x] - b*Log[c*(d*(e + f*x)^p)^q]))/f^
(3/2) + 9*(g + h*x)^(3/2)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p
)^q])^2 - (b^2*p^2*q^2*(96*f^2*g^2*h^(3/2)*Sqrt[f*g - e*h]*(e + f*x)^(3/2)
*((f*(g + h*x))/(h*(e + f*x)))^(3/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sq
rt[e + f*x])] - 192*e*f*g*h^(5/2)*Sqrt[f*g - e*h]*(e + f*x)^(3/2)*((f*(g +
 h*x))/(h*(e + f*x)))^(3/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*
x])] + 96*e^2*h^(7/2)*Sqrt[f*g - e*h]*(e + f*x)^(3/2)*((f*(g + h*x))/(h*(e
 + f*x)))^(3/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])] - f^2*S
qrt[-(f*g - e*h)^2]*(g + h*x)^2*(8*(13*f*g - 12*e*h + f*h*x) - 12*(4*f*g -
 3*e*h + f*h*x)*Log[e + f*x] + 9*f*(g + h*x)*Log[e + f*x]^2) - 9*f^3*g^2*S
qrt[-(f*g) + e*h]*(g + h*x)*(4*Sqrt[f*(g + h*x)]*ArcTanh[Sqrt[f*(g + h*x)]
/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)]) - Sqr
t[f*g - e*h]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*(Log[(h*(e + f*x))/(-(f*g) +
e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) + e*h)]*Log[(1 + Sqrt[(f*(g + h*x))/
(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2]^2 - 4*Po
lyLog[2, 1/2 - Sqrt[(f*(g + h*x))/(f*g - e*h)]/2])) + 18*e*f^2*g*h*Sqrt[-(
f*g) + e*h]*(g + h*x)*(4*Sqrt[f*(g + h*x)]*ArcTanh[Sqrt[f*(g + h*x)]/Sqrt[
f*g - e*h]]*(Log[e + f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)]) - Sqrt[f...
```

3.490.3 Rubi [A] (warning: unable to verify)

Time = 4.85 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.44, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {2895, 2845, 2858, 2788, 2756, 60, 60, 73, 221, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h}-\frac{4bfpq\int \frac{(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{e+fx}dx}{3h}$$

$\big\downarrow$ 2858

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{4bpq\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)}{3h}$$

$\big\downarrow$ 2788

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{4bpq\left(\frac{h\int\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}(a+b\log(cd^q(e+fx)^{pq}))d(e+fx)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)\right)}{3h}$$

$\big\downarrow$ 2756

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{3h}-\frac{2bfpq\int\frac{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}{e+fx}d(e+fx)}{3h}\right)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}(a+b\log}{e+fx}\right.}{3h}$$

$\big\downarrow$ 60

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{3h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{e+fx}d(e+fx)+\frac{2}{3}\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\right)}{3h}\right)}{f}+\left(g-\frac{eh}{f}\right.\right.}{3h}$$

$\big\downarrow$ 60

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+2\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\right)+\frac{2}{3}\left(h\cdots\right.\right.}{3h}\right)}{f}\cdots\right)}{3h}$$

$\downarrow$ 73

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-\frac{4bpq\left(\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{2f\left(g-\frac{eh}{f}\right)\int\frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h}+2\sqrt{\frac{h(e+f\cdots}{f}}\cdots\right.\right.}{3h}\right)}{f}\cdots\right)}{3h}$$

$\downarrow$ 221

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\,(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)+\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{3h}-\frac{2bfpq\left(\left(g-\right.\right.}{}\right.}{}\right.$$

$$3h$$

$\downarrow$ 2788

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\int\frac{a+b\log(cd^q(e+fx)^{pq})}{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)\right)+\frac{h\left(\frac{2f\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)}{}\right.}{}\right.$$

$$3h$$

$\downarrow$ 2756

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{e+fx}d(e+fx)}{h}\right)}{f}+\left(g-\frac{eh}{f}\right)\int\frac{a+b\log(cd^q(e+fx}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(}{}}}\right.\right.$$

$$\Big\downarrow \; 60$$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+2\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\right)}{h}\right)}{f}-\right.\right.$$

$$\Big\downarrow \; 73$$

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$\frac{4bpq\left(g-\frac{eh}{f}\right)\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\left(\frac{2f\left(g-\frac{eh}{f}\right)\int\frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h}+2\sqrt{\frac{h(e+}{f}}\right.}{h}\right.}{f}\right.}{}$$

$$\downarrow 221$$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$\frac{4bpq\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\int\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\left(2\sqrt{\frac{h(e+fx}{f}}\right.}{}\right.}{f}\right.}{}$$

$$\downarrow 2790$$

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(-bpq\int-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}(e+fx)}d(e+fx)-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}\right.\right.\right.$$

$$\downarrow 27$$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{2b\sqrt{f}pq\int\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{e+fx}d(e+fx)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(}{\sqrt{fg-eh}}\right.\right.\right.$$

$$\downarrow 7267$$

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{eh-f\left(\frac{eh}{f}-\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 2092

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{-fg+eh+f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 6546

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\left(\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\int\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}\right.\right.\right.$$

$\downarrow$ 6470

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}-\frac{2\sqrt{f}\left(g-\frac{eh}{f}\right)\text{arctanh}\left(\right.}{\sqrt{fg-}}\right.\right.}{3h}\right.}{f}\right.$$

$\downarrow$ 2849

3.490. $\int\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h} -$$

$$4bpq\left(\frac{h\left(\frac{2f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{3h}-\frac{2bfpq\left(\frac{2}{3}\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}+\left(g-\frac{eh}{f}\right)\left(2\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}-\frac{2\sqrt{f}\left(g-\frac{eh}{f}\right)\text{arctanh}\left(\right.}{\sqrt{fg-}}\right.\right.}{3h}\right)}{f}\right.$$

$$\Big\downarrow \; 2752$$

3.490. $\int \sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

$$\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{}{1-\frac{\sqrt{f}\sqrt{h(e}}{\sqrt{}}}\right.}{\sqrt{f}}}{\sqrt{}}\right.}{\sqrt{fg-eh}}\right.\right.\right.$$

input
```
Int[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
(2*(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(3*h) - (4*b*p*q*((
h*((-2*b*f*p*q*((2*(g - (e*h)/f + (h*(e + f*x))/f)^(3/2))/3 + (g - (e*h)/f
)*(2*Sqrt[g - (e*h)/f + (h*(e + f*x))/f] - (2*Sqrt[f]*(g - (e*h)/f)*ArcTan
h[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]])/Sqrt[f*g
 - e*h])))/(3*h) + (2*f*(g - (e*h)/f + (h*(e + f*x))/f)^(3/2)*(a + b*Log[c
*d^q*(e + f*x)^(p*q)]))/(3*h)))/f + (g - (e*h)/f)*((h*((-2*b*f*p*q*(2*Sqrt
[g - (e*h)/f + (h*(e + f*x))/f] - (2*Sqrt[f]*(g - (e*h)/f)*ArcTanh[(Sqrt[f
]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]])/Sqrt[f*g - e*h]))
/h + (2*f*Sqrt[g - (e*h)/f + (h*(e + f*x))/f]*(a + b*Log[c*d^q*(e + f*x)^(
p*q)]))/h))/f + (g - (e*h)/f)*((-2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)
/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]*(a + b*Log[c*d^q*(e + f*x)^(p*q)])
)/Sqrt[f*g - e*h] + (4*b*f^(3/2)*p*q*(ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f +
(h*(e + f*x))/f])/Sqrt[f*g - e*h]]^2/(2*f) - ((Sqrt[f*g - e*h]*ArcTanh[(Sq
rt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sq
rt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h])])/Sqrt[f] + (S
qrt[f*g - e*h]*PolyLog[2, 1 - 2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f
*x))/f])/Sqrt[f*g - e*h])])/(2*Sqrt[f]))/(Sqrt[f]*Sqrt[f*g - e*h])))/Sqrt[
f*g - e*h]))))/(3*h)
```

3.490. $\int \sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$

3.490.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.490. $\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.490. $\int \sqrt{g+hx}(a+b\log (c(d(e+fx)^p)^q))^2\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.490.4 Maple [F]

$$\int \sqrt{hx+g}\,(a+b\ln(c(d(fx+e)^p)^q))^2 dx$$

input
```
int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.490.5 Fricas [F]

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx = \int \sqrt{hx+g}(b\log(((fx+e)^pd)^q\,c)+a)^2\,dx$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*
b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2, x)
```

3.490.6 Sympy [F]

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx=\int\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\sqrt{g+hx}\,dx$$

input
```
integrate((h*x+g)**(1/2)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*sqrt(g + h*x), x)
```

3.490.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.490.8 Giac [F]

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx=\int\sqrt{hx+g}\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2\,dx$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac"
)
```

output
```
integrate(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2, x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2 \, dx = \int \sqrt{g+h\,x}\,\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)^2 dx$$

input
```
int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

3.491 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$

3.491.1 Optimal result

Integrand size = 30, antiderivative size = 447

$$\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx \\
&= \frac{16b^2p^2q^2\sqrt{g+hx}}{h}-\frac{16b^2\sqrt{fg-eh}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} \\
&\quad-\frac{8b^2\sqrt{fg-eh}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h}-\frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} \\
&\quad+\frac{8b\sqrt{fg-eh}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} \\
&\quad+\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} \\
&\quad+\frac{16b^2\sqrt{fg-eh}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h} \\
&\quad+\frac{8b^2\sqrt{fg-eh}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h}
\end{aligned}$$

output
```
-16*b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(-e*h+f*g)
^(1/2)/h/f^(1/2)-8*b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1
/2))^2*(-e*h+f*g)^(1/2)/h/f^(1/2)+8*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-
e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))*(-e*h+f*g)^(1/2)/h/f^(1/2)+16*
b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2
)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))*(-e*h+f*g)^(1/2)/h/f^(1/2)+8*b^2*p^2*q^
2*polylog(2,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))*(-e*h+f*g)^(1/
2)/h/f^(1/2)+16*b^2*p^2*q^2*(h*x+g)^(1/2)/h-8*b*p*q*(a+b*ln(c*(d*(f*x+e)^p
)^q))*(h*x+g)^(1/2)/h+2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*(h*x+g)^(1/2)/h
```

3.491.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. $2(447) = 894$.

Time = 12.63 (sec) , antiderivative size = 1407, normalized size of antiderivative = 3.15

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{\sqrt{g + hx}}\, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/Sqrt[g + h*x],x]
```

output
```
(2*b*p*q*((4*Sqrt[f]*Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*
(e + f*x))/f])/Sqrt[f*g - e*h]])/h + (2*f*Sqrt[(f*g - e*h + h*(e + f*x))/f
]*(-2 + Log[e + f*x]))/h)*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]
) + b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*
(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Lo
g[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q
*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])])))/f + (2*
Sqrt[g + h*x]*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(
-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p
*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(
p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e
+ f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/h + (b^2*p^2*q^2*
((-16*f^2*g*Sqrt[e + f*x]*Sqrt[1 + (f*g - e*h)/(h*(e + f*x))]*Sqrt[(f*g -
e*h + h*(e + f*x))/f]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/
(Sqrt[h]*Sqrt[-(f*g) + e*h]*(f*g - e*h + h*(e + f*x))) + (16*e*f*Sqrt[h]*S
qrt[e + f*x]*Sqrt[1 + (f*g - e*h)/(h*(e + f*x))]*Sqrt[(f*g - e*h + h*(e +
f*x))/f]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(Sqrt[-(f*g)
+ e*h]*(f*g - e*h + h*(e + f*x))) + (2*f*Sqrt[(f*g - e*h + h*(e + f*x))/f]
*(8 - 4*Log[e + f*x] + Log[e + f*x]^2))/h + (2*e*f*Sqrt[(f*g - e*h + h*(e
+ f*x))/f]*((-4*ArcTanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*...
```

3.491.3 Rubi [A] (warning: unable to verify)

Time = 3.46 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.28, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2895, 2845, 2858, 2788, 2756, 60, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2845}$$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{4bfpq\int \frac{\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{4bpq\int \frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2788}$$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{4bpq\left(\frac{h\int \frac{a+b\log(cd^q(e+fx)^{pq})}{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{f}+\left(g-\frac{eh}{f}\right)\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)\right)}{h}$$

$$\Big\downarrow \text{2756}$$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{4bpq\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}(a+b\log(cd^q(e+fx)^{pq}))}{h}-\frac{2bfpq\int \frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{e+fx}d(e+fx)}{h}\right)}{f}+\left(g-\frac{eh}{f}\right)\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+f\right.}{h}$$

$$\downarrow 60$$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{4bpq\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\left(\left(g-\frac{eh}{f}\right)\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+2\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\right)}{h}\right)}{f}+\left(g-\frac{eh}{f}\right)\cdots\right)}{h}$$

$$\downarrow 73$$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-\frac{4bpq\left(\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}-\frac{2bfpq\left(\frac{2f\left(g-\frac{eh}{f}\right)\int\frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h}+2\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}\right)}{h}\right)}{f}\cdots\right)}{h}$$

$$\downarrow 221$$

3.491. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-$$

$$\frac{4bpq\left(\left(g-\frac{eh}{f}\right)\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)+\frac{h\left(\frac{2f\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}(a+b\log(cd^q(e+fx)^{pq}))}{h}-\frac{2bfpq\left(2\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}-\frac{2\sqrt{f}(\cdots}{\cdots}\right)}{\cdots}\right)}{f}\right)}{h}$$

$\downarrow$ 2790

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-$$

$$\frac{4bpq\left(\left(g-\frac{eh}{f}\right)\left(-bpq\int-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}(e+fx)}d(e+fx)-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(\cdots}{\sqrt{fg-eh}}\right)\cdots\right)}{h}$$

$\downarrow$ 27

3.491. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}dx$

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-$$

$$\frac{4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{2b\sqrt{f}pq\int\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{e+fx}d(e+fx)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(e+fx)^{pq}))}{\sqrt{fg-eh}}\right.\right.}{h}$$

$\downarrow$ 7267

$$\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{eh-f\left(\frac{eh}{f}-\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}}}{\sqrt{fg-eh}}\right.}{\sqrt{fg-e}}\right.\right.$$

$\downarrow$ 2092

3.491. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{-fg+eh+f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}}}{\sqrt{fg-e}}\right.}{\sqrt{fg-e}}\right.\right.$$

$\downarrow$ 6546

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{h}-$$

$$4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\left(\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\int\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arcta}}{}\right.\right.$$

$\downarrow$ 6470

3.491. $\int\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{\sqrt{g+hx}}dx$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{h} - \frac{4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}}{\sqrt{f}\sqrt{fg-eh}}\right.}{\sqrt{fg-eh}}\right.\right.}{}$$

$$\Big\downarrow \ 2849$$

3.491. $\displaystyle\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{\sqrt{g+hx}}\,dx$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{h}-$$

$$\frac{4bpq\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}}d\frac{1}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\quad\sqrt{fg-}}{\sqrt{f}}+\frac{}{\sqrt{f}\sqrt{fg-}}}{}\right)}{\sqrt{fg-eh}}}{}$$

$\downarrow$ 2752

3.491. $\int\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{\sqrt{g+hx}}\,dx$

$$\frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h} - \frac{4bpq\left(\left(g-\frac{eh}{f}\right)\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}\right)\right)}{}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/Sqrt[g + h*x],x]
```

output
```
(2*Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/h - (4*b*p*q*((h*((-2
*b*f*p*q*(2*Sqrt[g - (e*h)/f + (h*(e + f*x))/f] - (2*Sqrt[f]*(g - (e*h)/f)
*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]])/S
qrt[f*g - e*h]))/h + (2*f*Sqrt[g - (e*h)/f + (h*(e + f*x))/f]*(a + b*Log[c
*d^q*(e + f*x)^(p*q)]))/h))/f + (g - (e*h)/f)*((-2*Sqrt[f]*ArcTanh[(Sqrt[f
]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]*(a + b*Log[c*d^q*(
e + f*x)^(p*q)]))/Sqrt[f*g - e*h] + (4*b*f^(3/2)*p*q*(ArcTanh[(Sqrt[f]*Sqr
t[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]^2/(2*f) - ((Sqrt[f*g -
e*h]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]
]*Log[2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]
)])/Sqrt[f] + (Sqrt[f*g - e*h]*PolyLog[2, 1 - 2/(1 - (Sqrt[f]*Sqrt[g - (e*
h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h])])/(2*Sqrt[f]))/(Sqrt[f]*Sqrt[f*g
 - e*h])))/Sqrt[f*g - e*h])))/h
```

3.491. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$

3.491.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.491. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.491. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.491.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{2}}{\sqrt{hx+g}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x)
```

3.491.5 Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{2}}{\sqrt{g+hx}}\,dx=\int \frac{\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{2}}{\sqrt{hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a
*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h*x + g), x)
```

3.491.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/sqrt(g + h*x), x)
```

3.491.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.491.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{\sqrt{hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/sqrt(h*x + g), x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q))^2}}{\sqrt{g+hx}}\,dx=\int \frac{{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}{\sqrt{g+h\,x}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(1/2), x)
```

3.492 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

3.492.1 Optimal result

Integrand size = 30, antiderivative size = 330

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx &= \frac{8b^2\sqrt{f}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} \\ &- \frac{8b\sqrt{f}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} \\ &- \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \frac{16b^2\sqrt{f}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}} \\ &- \frac{8b^2\sqrt{f}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}}\end{aligned}$$

output
```
8*b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2*f^(1/2)/h/
(-e*h+f*g)^(1/2)-8*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(
a+b*ln(c*(d*(f*x+e)^p)^q))*f^(1/2)/h/(-e*h+f*g)^(1/2)-16*b^2*p^2*q^2*arcta
nh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(
-e*h+f*g)^(1/2)))*f^(1/2)/h/(-e*h+f*g)^(1/2)-8*b^2*p^2*q^2*polylog(2,1-2/(
1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))*f^(1/2)/h/(-e*h+f*g)^(1/2)-2*(a
+b*ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^(1/2)
```

3.492.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 785 vs. 2(330) = 660.

Time = 2.54 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.38

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx =$$
$$-\frac{2\left(a^2\sqrt{fg-eh}\sqrt{f(g+hx)}+4b^2fgp^2q^2\text{arctanh}\left(\frac{\sqrt{f(g+hx)}}{\sqrt{fg-eh}}\right)\log(e+fx)+4b^2fhp^2q^2x\text{arctanh}\left(\frac{\sqrt{f(g+hx)}}{\sqrt{fg-eh}}\right)\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(3/2),x]
```

output
```
(-2*(a^2*Sqrt[f*g - e*h]*Sqrt[f*(g + h*x)] + 4*b^2*f*g*p^2*q^2*ArcTanh[Sqr
t[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[e + f*x] + 4*b^2*f*h*p^2*q^2*x*ArcTanh
[Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[e + f*x] - 4*b^2*f*g*p^2*q^2*ArcTa
nh[Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[(h*(e + f*x))/(-(f*g) + e*h)] -
4*b^2*f*h*p^2*q^2*x*ArcTanh[Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[(h*(e +
 f*x))/(-(f*g) + e*h)] - b^2*Sqrt[f*g - e*h]*p^2*q^2*Sqrt[f*(g + h*x)]*Sqr
t[(f*(g + h*x))/(f*g - e*h)]*Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 + 2*a*b*S
qrt[f*g - e*h]*Sqrt[f*(g + h*x)]*Log[c*(d*(e + f*x)^p)^q] + b^2*Sqrt[f*g -
 e*h]*Sqrt[f*(g + h*x)]*Log[c*(d*(e + f*x)^p)^q]^2 + 4*b*Sqrt[f]*p*q*Sqrt[
g + h*x]*Sqrt[f*(g + h*x)]*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]
]*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q]) + 4*b^2*Sqrt[f*g -
 e*h]*p^2*q^2*Sqrt[f*(g + h*x)]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Log[(h*(e
+ f*x))/(-(f*g) + e*h)]*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2] - 2*b
^2*Sqrt[f*g - e*h]*p^2*q^2*Sqrt[f*(g + h*x)]*Sqrt[(f*(g + h*x))/(f*g - e*h
)]*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2]^2 + 4*b^2*Sqrt[f*g - e*h]*
p^2*q^2*Sqrt[f*(g + h*x)]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*PolyLog[2, 1/2 -
 Sqrt[(f*(g + h*x))/(f*g - e*h)]/2]))/(h*Sqrt[f*g - e*h]*Sqrt[g + h*x]*Sqr
t[f*(g + h*x)])
```

3.492.3 Rubi [A] (warning: unable to verify)

Time = 2.42 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2895, 2845, 2858, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.492. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}dx$$

$$\Big\downarrow \; 2895$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}dx$$

$$\Big\downarrow \; 2845$$

$$\frac{4bfpq\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(e+fx)\sqrt{g+hx}}dx}{h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \; 2858$$

$$\frac{4bpq\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \; 2790$$

$$\frac{4bpq\left(-bpq\int -\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}(e+fx)}d(e+fx)-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(e+fx)^{pq}))}{\sqrt{fg-eh}}\right)}{h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \; 27$$

$$\frac{4bpq\left(\frac{2b\sqrt{f}pq\int \frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{e+fx}d(e+fx)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(e+fx)^{pq}))}{\sqrt{fg-eh}}\right)}{h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \; 7267$$

3.492. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}dx$

$$\frac{4bpq\left(\frac{4bf^{3/2}pq\int \frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{eh-f\left(\frac{eh}{f}-\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(c\dots}{\sqrt{fg-eh}}\dots\right.}{h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \ \textcolor{blue}{2092}$$

$$\frac{4bpq\left(\frac{4bf^{3/2}pq\int \frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{-fg+eh+f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(c\dots}{\sqrt{fg-eh}}\dots\right.}{h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \ \textcolor{blue}{6546}$$

$$\frac{4bpq\left(\frac{4bf^{3/2}pq\left(\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\int \frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+\dots}{f}}}{\sqrt{f\dots}}\dots\right.}{\dots}\dots\right.}{h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$$\Big\downarrow \ \textcolor{blue}{6470}$$

3.492. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

$$4bpq\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}-\int\frac{\log\left(\frac{}{1-\frac{\sqrt{f}\sqrt{}}{}}\right)}{1-\frac{f\left(g-\right)}{}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}\right.$$

$$h$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}}$$

$\Big\downarrow$ 2849

3.492. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

$$\frac{4bpq\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\sqrt{fg-eh}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}}d\frac{1}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}}{\sqrt{f}}+\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\cdots\right.}{\sqrt{f}\sqrt{fg-eh}}\cdots\right.}{\sqrt{fg-eh}}\cdots\right.}{h}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{h\sqrt{g+hx}}$$

$\downarrow$ 2752

3.492. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

$$\frac{4bpq\left(\frac{4bf^{3/2}pq\left(\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}+\frac{\sqrt{fg-eh}\,\text{PolyL}}{\ }}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}\right)}{h}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{h\sqrt{g+hx}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(3/2),x]
```

output
```
(-2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(h*Sqrt[g + h*x]) + (4*b*p*q*((-2*
Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e
*h]]*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/Sqrt[f*g - e*h] + (4*b*f^(3/2)*p*
q*(ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]^
2/(2*f) - ((Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*
x))/f])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*
x))/f])/Sqrt[f*g - e*h])])/Sqrt[f] + (Sqrt[f*g - e*h]*PolyLog[2, 1 - 2/(1
- (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h])])/(2*Sqrt
[f]))/(Sqrt[f]*Sqrt[f*g - e*h])))/Sqrt[f*g - e*h]))/h
```

3.492.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

3.492. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

3.492. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.492.4 Maple [F]

$$\int \frac{\left(a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{\left(hx+g\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x)
```

3.492.5 Fricas [F]

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx=\int \frac{\left(b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{2}}{\left(hx+g\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a
*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^2*x^2 + 2*g*h*x + g^2)
, x)
```

3.492.6 Sympy [F]

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{3/2}}\,dx=\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(3/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**(3/2), x)
```

3.492.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.492.8 Giac [F]

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{3/2}}\,dx=\int \frac{\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{(hx+g)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(3/2), x)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^2}{\left(g+h\,x\right)^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(3/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(3/2), x)
```

3.493 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

3.493.1 Optimal result

Integrand size = 30, antiderivative size = 449

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \frac{16b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \frac{16b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}} - \frac{8b^2f^{3/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}}$$

output
```
16/3*b^2*f^(3/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h
/(-e*h+f*g)^(3/2)+8/3*b^2*f^(3/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-
e*h+f*g)^(1/2))^2/h/(-e*h+f*g)^(3/2)-8/3*b*f^(3/2)*p*q*arctanh(f^(1/2)*(h*
x+g)^(1/2)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^(3/2
)-2/3*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^(3/2)-16/3*b^2*f^(3/2)*p^2*q
^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)
^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(3/2)-8/3*b^2*f^(3/2)*p^2*q^2*polyl
og(2,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(3/2)+8/
3*b*f*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)/(h*x+g)^(1/2)
```

3.493.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1289 vs. $2(449) = 898$.

Time = 12.81 (sec) , antiderivative size = 1289, normalized size of antiderivative = 2.87

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \frac{4abf^{3/2}pq\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2h(e+fx)-f}{(fg-e}\right.}{3h}$$

$$+\frac{4b^2f^{3/2}pq^2\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2h(e+fx)-fg(-2+\log(e+fx))+eh(-2+\log(e+fx)))}{(fg-eh)(fg+fhx)^2}\right)(-}{3h}$$

$$+\frac{4b^2f^{3/2}pq\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2h(e+fx)-fg(-2+\log(e+fx))+eh(-2+\log(e+fx)))}{(fg-eh)(fg+fhx)^2}\right)\left(-\right.}{}$$

$$-\frac{2\left(a+bq(-p\log(e+fx)+\log(d(e+fx)^p))+b\left(-q(-p\log(e+fx)+\log(d(e+fx)^p))-\log(d(e+f\right.\right.}{3h(}$$

$$+\frac{2b^2f^2p^2q^2\sqrt{\frac{fg-eh+h(e+fx)}{f}}\left(-\frac{8\arcsin\left(\frac{\sqrt{-fg+eh}}{\sqrt{h}\sqrt{e+fx}}\right)}{(-fg+eh)^{3/2}\sqrt{e+fx}\sqrt{\frac{fg+fhx}{h(e+fx)}}}-\frac{\sqrt{h}(4h(e+fx)-fg(-4+\log(e+fx))+eh(-4+\log(e+fx)))\log(e+fx}{(-fg+eh)(fg+fhx)^2}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(5/2),x]
```

output
```
(4*a*b*f^(3/2)*p*q*((-2*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]
)/Sqrt[f*g - e*h]])/(f*g - e*h)^(3/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e +
f*x))/f]*(2*h*(e + f*x) - f*g*(-2 + Log[e + f*x]) + e*h*(-2 + Log[e + f*x]
)))/((f*g - e*h)*(f*g + f*h*x)^2)))/(3*h) + (4*b^2*f^(3/2)*p*q^2*((-2*ArcT
anh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e
*h)^(3/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(2*h*(e + f*x) - f*
g*(-2 + Log[e + f*x]) + e*h*(-2 + Log[e + f*x])))/((f*g - e*h)*(f*g + f*h*
x)^2))*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/(3*h) + (4*b^2*f^(3/2)*p*
q*((-2*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]
])/(f*g - e*h)^(3/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(2*h*(e
+ f*x) - f*g*(-2 + Log[e + f*x]) + e*h*(-2 + Log[e + f*x])))/((f*g - e*h)*
(f*g + f*h*x)^2))*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(
e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f
*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x
)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])
]))/(3*h) - (2*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*
(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(
p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-
(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e
 + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/(3*h*(g + h*x...
```

3.493.3 Rubi [A] (warning: unable to verify)

Time = 3.52 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.22, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2895, 2845, 2858, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{4bfpq\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(e+fx)(g+hx)^{3/2}}dx}{3h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 2858

$$\frac{4bpq\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{3h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 2789

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\int \frac{a+b\log(cd^q(e+fx)^{pq})}{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}\right)}{3h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 2756

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\int \frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{h}-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}\right)}{3h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 73

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{4bf^2pq\int \frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h^2}-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}\right)}{3h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 221

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}dx$

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(-\frac{2f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{4bf^{3/2}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}\right)}{fg-eh}\right)}{3h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 2790

$$\frac{4bpq\left(\frac{f\left(-bpq\int -\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}(e+fx)}d(e+fx)-\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{\sqrt{fg-eh}}\right)}{fg-eh}-\frac{h\left(-\frac{2f\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{3h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 27

$$\frac{4bpq\left(\frac{f\left(\frac{2b\sqrt{f}pq\int \frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{e+fx}d(e+fx)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{\sqrt{fg-eh}}\right)}{fg-eh}-\frac{h\left(-\frac{2f(a+b\cdots}{h\sqrt{h\cdots}}\cdots\right.}{\cdots}\cdots\right.}{3h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 7267

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

$$\frac{4bpq\left(\frac{f\left(\frac{4bf^{3/2}pq\int \frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{eh-f\left(\frac{eh}{f}-\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(}{\sqrt{fg-eh}}\right.}{fg-eh}\right.}{3h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 2092

$$\frac{4bpq\left(\frac{f\left(\frac{4bf^{3/2}pq\int \frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{-fg+eh+f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(}{\sqrt{fg-eh}}\right.}{fg-eh}\right.}{3h}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 6546

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

$$\frac{4bpq\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+\cdots}{f}}}{\sqrt{fg\cdots}}\right.}{\cdots}\right)}{fg-eh}\cdots\right)}{3h}\cdots$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 6470

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}-\int\frac{\log\left(\frac{\cdots}{1-\cdots}\right)}{1-\frac{f(\cdots}{\cdots}}}{\sqrt{f}\sqrt{fg-eh}}\cdots}{\sqrt{fg-eh}}\cdots\right)}{fg-eh}\cdots\right)}{\cdots}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{3h(g+hx)^{3/2}}$$

$\downarrow$ 2849

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\sqrt{fg-eh}\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}}\,d\frac{1}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}}{\sqrt{f}}+\frac{\sqrt{fg-eh}\operatorname{arctanh}}{\sqrt{f}\sqrt{fg-eh}}\right.}{\sqrt{fg-eh}}\right.}{fg-eh}\right.$$

3.493. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$ $\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{3h(g+hx)^{3/2}}$

$\downarrow$ 2752

$$4bpq\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}+\frac{\sqrt{fg-eh}\,\mathrm{Po}\ldots}{\ldots}}{\sqrt{f}\sqrt{fg-eh}}\ldots\right)}{\sqrt{fg-eh}}\ldots\right)}{fg-eh}\ldots\right)$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{3h(g+hx)^{3/2}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(5/2),x]
```

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

output
```
(-2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(3*h*(g + h*x)^(3/2)) + (4*b*p*q*(
-((h*((-4*b*f^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/
f])/Sqrt[f*g - e*h]])/(h*Sqrt[f*g - e*h]) - (2*f*(a + b*Log[c*d^q*(e + f*x
)^(p*q)]))/(h*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])))/(f*g - e*h)) + (f*((-
2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g -
 e*h]]*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/Sqrt[f*g - e*h] + (4*b*f^(3/2)*
p*q*(ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]
]^2/(2*f) - ((Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e +
f*x))/f])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e +
f*x))/f])/Sqrt[f*g - e*h])])/Sqrt[f] + (Sqrt[f*g - e*h]*PolyLog[2, 1 - 2/(
1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h])])/(2*Sq
rt[f]))/(Sqrt[f]*Sqrt[f*g - e*h])))/Sqrt[f*g - e*h]))/(f*g - e*h)))/(3*h)
```

3.493.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.493. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.493.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{2}}{(hx+g)^{\frac{5}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x)
```

3.493.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a
*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^3*x^3 + 3*g*h^2*x^2 +
3*g^2*h*x + g^3), x)
```

3.493.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(5/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**(5/2), x)
```

3.493.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.493.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^2}{(hx+g)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(5/2), x)
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}{(g+h\,x)^{5/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(5/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(5/2), x)
```

3.494 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

3.494.1 Optimal result

Integrand size = 30, antiderivative size = 537

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{64b^2f^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} + \frac{8b^2f^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{8bf^{5/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} - \frac{16b^2f^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}} - \frac{8b^2f^{5/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}}$$

output
```
64/15*b^2*f^(5/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/
h/(-e*h+f*g)^(5/2)+8/5*b^2*f^(5/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(
-e*h+f*g)^(1/2))^2/h/(-e*h+f*g)^(5/2)+8/15*b*f*p*q*(a+b*ln(c*(d*(f*x+e)^p)
^q))/h/(-e*h+f*g)/(h*x+g)^(3/2)-8/5*b*f^(5/2)*p*q*arctanh(f^(1/2)*(h*x+g)^
(1/2)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^(5/2)-2/5
*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^(5/2)-16/5*b^2*f^(5/2)*p^2*q^2*ar
ctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2
)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(5/2)-8/5*b^2*f^(5/2)*p^2*q^2*polylog(2,
1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(5/2)-16/15*b
^2*f^2*p^2*q^2/h/(-e*h+f*g)^2/(h*x+g)^(1/2)+8/5*b*f^2*p*q*(a+b*ln(c*(d*(f*
x+e)^p)^q))/h/(-e*h+f*g)^2/(h*x+g)^(1/2)
```

3.494.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1349 vs. $2(537) = 1074$.

Time = 9.70 (sec) , antiderivative size = 1349, normalized size of antiderivative = 2.51

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{(g + hx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(7/2),x]
```

output
```
(4*a*b*f^(5/2)*p*q*((-6*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]
)/Sqrt[f*g - e*h]])/(f*g - e*h)^(5/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e +
f*x))/f]*(2*(f*g - e*h)*(f*g + f*h*x) + 6*(f*g + f*h*x)^2 - 3*(f*g - e*h)^
2*Log[e + f*x]))/((f*g - e*h)^2*(f*g + f*h*x)^3)))/(15*h) + (4*b^2*f^(5/2)
*p*q^2*((-6*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g -
 e*h]])/(f*g - e*h)^(5/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(2*
(f*g - e*h)*(f*g + f*h*x) + 6*(f*g + f*h*x)^2 - 3*(f*g - e*h)^2*Log[e + f*
x]))/((f*g - e*h)^2*(f*g + f*h*x)^3))*(-(p*Log[e + f*x]) + Log[d*(e + f*x)
^p]))/(15*h) + (4*b^2*f^(5/2)*p*q*((-6*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h +
h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(5/2) + (Sqrt[f]*Sqrt[(f*g
- e*h + h*(e + f*x))/f]*(2*(f*g - e*h)*(f*g + f*h*x) + 6*(f*g + f*h*x)^2 -
 3*(f*g - e*h)^2*Log[e + f*x]))/((f*g - e*h)^2*(f*g + f*h*x)^3))*(-(q*(-(p
*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Lo
g[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*L
og[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f
*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))/(15*h) - (2*(a + b*q*(-
(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Log[d
*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e
 + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(
e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f...
```

3.494.3 Rubi [A] (warning: unable to verify)

Time = 4.76 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.36, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2895, 2845, 2858, 2789, 2756, 61, 73, 221, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}dx$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{4bfpq\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(e+fx)(g+hx)^{5/2}}dx}{5h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 2858

$$\frac{4bpq\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{5h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 2789

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\int \frac{a+b\log(cd^q(e+fx)^{pq})}{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}\right)}{5h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 2756

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\int \frac{1}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{3h}-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{3h\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}}\right)}{fg-eh}\right)}{5h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 61

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{f\int \frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}+\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{3h}-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{3h\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}}\right)}{fg-eh}\right)}{5h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 73

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{2f^2\int \frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h(fg-eh)}+\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{3h}\right.}{fg-eh}\right.}{5h}$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 221

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{2f^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}\right)}{3h}-\frac{2f(a+b\log}{3h\left(\frac{h(e+f}{f}\right.}\right.}{fg-eh}\right.}{5h}$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 2789

3.494. $\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(g+hx)^{7/2}}\,dx$

$$\frac{4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{2f^{3/2}\text{arctanh}\left(\cdots\right.}{(fg-\cdots}\right.}{3h}\cdots}{fg-e\cdots}\right.}{5h}\cdots$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \; 2756$$

$$\frac{4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\int \frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{h}-\frac{2f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{\cdots}{(fg-eh)\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{5h}\cdots$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \; 73$$

3.494. $\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(g+hx)^{7/2}}\,dx$

$$\frac{4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{4bf^2pq\int \frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h^2}-\frac{2f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h(\cdots}{\cdots}}{5h}$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 221

$$\frac{4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(-\frac{2f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{4bf^{3/2}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\cdots\right.}{(f\cdots}\right.}{\cdots}}{5h}$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 2790

3.494. $\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(g+hx)^{7/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(-bpq\int -\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}(e+fx)}d(e+fx)-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(e+fx)^{pq}))}{\sqrt{fg-eh}}\right)}{fg-eh}-\frac{h\left(-\frac{2f(a\cdots}{h\cdots}\right.}{\cdots}\right)}{fg-eh}\cdots\right.$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \; 27$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$\frac{4bpq\left(\frac{f\left(\frac{f\left(\frac{2b\sqrt{f}pq\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{e+fx}d(e+fx)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(e+fx)^{pq}))}{\sqrt{fg-eh}}\right)}{fg-eh}-\frac{h\left(-\frac{2f(a+b\,l\ldots}{h\sqrt{h\ldots}}\right.}{\ldots}\right)}{fg-eh}\ldots\right)}{\ldots}$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\big\downarrow$ 7267

3.494. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{eh-f\left(\frac{eh}{f}-\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(c}{\sqrt{fg-eh}}\right)}{fg-eh}}{fg-eh}\right.$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{5h(g+hx)^{5/2}}$$

$$\Big\downarrow \text{ 2092}$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{-fg+eh+f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(c}{\sqrt{fg-eh}}\right.}{fg-eh}\right.}{fg-eh}\right.$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$\downarrow$ 6546

3.494. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(\cdots}{}}}{\sqrt{\cdots}}\right.\cdots}{\cdots}\right.\cdots}{fg-eh}\right.\cdots}{fg-eh}\right.\cdots$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

3.494.

$$\downarrow 6470$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$4bpq \Bigg(f \Bigg(f \Bigg(4bf^{3/2}pq \Bigg(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f} - \frac{-\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}} - \int \frac{\log\Big(- \cdots}{1 \cdots}}{\sqrt{f}\sqrt{fg-eh}} \quad 1- \cdots}{\sqrt{fg-eh}} \cdots}{fg-eh} \cdots$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

3.494. $\downarrow$ 2849

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$4bpq \Bigg(f \Bigg(f \Bigg(4bf^{3/2}pq \Bigg(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)^2}{2f} - \frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}}{\sqrt{f}\sqrt{fg-eh}} + \frac{\sqrt{fg-eh}\cdots}{} \cdots}{\sqrt{fg-eh}} \cdots}{fg-eh} \cdots$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$\Big\downarrow \; 2752$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}+\frac{\sqrt{fg-eh}\ \cdots}{\sqrt{f}\sqrt{fg-eh}}\cdots\right)}{\sqrt{fg-eh}}\cdots\right)}{fg-eh}\cdots\right)}{\cdots}\cdots\right.$$

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(7/2),x]
```

output
```
(-2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(5*h*(g + h*x)^(5/2)) + (4*b*p*q*(
-((h*((2*b*f*p*q*((2*f)/((f*g - e*h)*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])
- (2*f^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*
g - e*h]])/(f*g - e*h)^(3/2)))/(3*h) - (2*f*(a + b*Log[c*d^q*(e + f*x)^(p*
q)]))/(3*h*(g - (e*h)/f + (h*(e + f*x))/f)^(3/2))))/(f*g - e*h)) + (f*(-((
h*((-4*b*f^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])
/Sqrt[f*g - e*h]])/(h*Sqrt[f*g - e*h]) - (2*f*(a + b*Log[c*d^q*(e + f*x)^(
p*q)]))/(h*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])))/(f*g - e*h)) + (f*((-2*S
qrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*
h]]*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/Sqrt[f*g - e*h] + (4*b*f^(3/2)*p*q
*(ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]]^2
/(2*f) - ((Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x
))/f])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x
))/f])/Sqrt[f*g - e*h])])/Sqrt[f] + (Sqrt[f*g - e*h]*PolyLog[2, 1 - 2/(1 -
 (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h])])/(2*Sqrt[
f]))/(Sqrt[f]*Sqrt[f*g - e*h])))/Sqrt[f*g - e*h]))/(f*g - e*h)))/(f*g - e*
h)))/(5*h)
```

3.494.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.494. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$

3.494.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{\left(hx+g\right)^{\frac{7}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x)
```

3.494.5 Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{2}}{(g+hx)^{7/2}}\,dx=\int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{2}}{\left(hx+g\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a
*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^4*x^4 + 4*g*h^3*x^3 +
6*g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)
```

3.494.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{2}}{(g+hx)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(7/2),x)
```

output
```
Timed out
```

3.494.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{(g + hx)^{7/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.494.8 Giac [F]

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{(g + hx)^{7/2}}\, dx = \int \frac{(b\log(((fx + e)^p d)^q c) + a)^2}{(hx + g)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(7/2), x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{(g + hx)^{7/2}}\, dx = \int \frac{(a + b\,\ln(c\,(d\,(e + f\,x)^p)^q))^2}{(g + h\,x)^{7/2}}\, dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(7/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(7/2), x)
```

3.495 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

3.495.1 Optimal result

Integrand size = 30, antiderivative size = 625

$$\begin{aligned}
\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = &-\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} \\
&-\frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}}+\frac{368b^2f^{7/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} \\
&+\frac{8b^2f^{7/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}}+\frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} \\
&+\frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}}+\frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} \\
&-\frac{8bf^{7/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} \\
&-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} \\
&-\frac{16b^2f^{7/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} \\
&-\frac{8b^2f^{7/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}}
\end{aligned}$$

output
```
-16/105*b^2*f^2*p^2*q^2/h/(-e*h+f*g)^2/(h*x+g)^(3/2)+368/105*b^2*f^(7/2)*p
^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/(-e*h+f*g)^(7/2)+
8/7*b^2*f^(7/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2/
h/(-e*h+f*g)^(7/2)+8/35*b*f*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)/(
h*x+g)^(5/2)+8/21*b*f^2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^2/(h*
x+g)^(3/2)-8/7*b*f^(7/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2
))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^(7/2)-2/7*(a+b*ln(c*(d*(f*x+e)
^p)^q))^2/h/(h*x+g)^(7/2)-16/7*b^2*f^(7/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)
^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/
h/(-e*h+f*g)^(7/2)-8/7*b^2*f^(7/2)*p^2*q^2*polylog(2,1-2/(1-f^(1/2)*(h*x+g
)^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(7/2)-128/105*b^2*f^3*p^2*q^2/h/(-
e*h+f*g)^3/(h*x+g)^(1/2)+8/7*b*f^3*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h
+f*g)^3/(h*x+g)^(1/2)
```

3.495.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1457 vs. $2(625) = 1250$.

Time = 11.04 (sec) , antiderivative size = 1457, normalized size of antiderivative = 2.33

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{(g + hx)^{9/2}}\, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(9/2),x]
```

output
```
(4*a*b*f^(7/2)*p*q*((-30*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f
])/Sqrt[f*g - e*h]])/(f*g - e*h)^(7/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e +
 f*x))/f]*(6*(f*g - e*h)^2*(f*g + f*h*x) + 10*(f*g - e*h)*(f*g + f*h*x)^2
+ 30*(f*g + f*h*x)^3 - 15*(f*g - e*h)^3*Log[e + f*x]))/((f*g - e*h)^3*(f*g
 + f*h*x)^4)))/(105*h) + (4*b^2*f^(7/2)*p*q^2*((-30*ArcTanh[(Sqrt[f]*Sqrt[
(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(7/2) + (Sqrt[
f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(6*(f*g - e*h)^2*(f*g + f*h*x) + 10*(
f*g - e*h)*(f*g + f*h*x)^2 + 30*(f*g + f*h*x)^3 - 15*(f*g - e*h)^3*Log[e +
 f*x]))/((f*g - e*h)^3*(f*g + f*h*x)^4))*(-(p*Log[e + f*x]) + Log[d*(e + f
*x)^p]))/(105*h) + (4*b^2*f^(7/2)*p*q*((-30*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e
*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(7/2) + (Sqrt[f]*Sqrt[
(f*g - e*h + h*(e + f*x))/f]*(6*(f*g - e*h)^2*(f*g + f*h*x) + 10*(f*g - e*
h)*(f*g + f*h*x)^2 + 30*(f*g + f*h*x)^3 - 15*(f*g - e*h)^3*Log[e + f*x]))/
((f*g - e*h)^3*(f*g + f*h*x)^4))*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)
^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]
))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p
]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[
d*(e + f*x)^p])]))/(105*h) - (2*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f
*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*
x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)...
```

3.495.3 Rubi [A] (warning: unable to verify)

Time = 6.40 (sec) , antiderivative size = 969, normalized size of antiderivative = 1.55, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {2895, 2845, 2858, 2789, 2756, 61, 61, 73, 221, 2789, 2756, 61, 73, 221, 2789, 2756, 73, 221, 2790, 27, 7267, 2092, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$$

$$\Big\downarrow \ 2895$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}dx$$

$$\Big\downarrow \ 2845$$

$$\frac{4bfpq\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(e+fx)(g+hx)^{7/2}}dx}{7h}-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}}$$

$\Big\downarrow$ 2858

$$\frac{4bpq\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{7/2}}d(e+fx)}{7h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{7h(g+hx)^{7/2}}$$

$\Big\downarrow$ 2789

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}-\frac{h\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{7/2}}d(e+fx)}{fg-eh}\right)}{7h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{7h(g+hx)^{7/2}}$$

$\Big\downarrow$ 2756

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\int \frac{1}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{5h}-\frac{2f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{5h\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{5/2}}\right)}{fg-eh}\right)}{7h}-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{7h(g+hx)^{7/2}}$$

$\Big\downarrow$ 61

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{f\int \frac{1}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}+\frac{2f}{3(fg-eh)\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}}\right)}{5h}-\frac{2f(a\cdots}{5h\left(\cdots\right.}\cdots\right.}{fg-eh}\cdots\right.}{7h}\cdots$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{7h(g+hx)^{7/2}}$$

3.495. $\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(g+hx)^{9/2}}\,dx$

$$\Big\downarrow \; 61$$

$$\frac{4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{f\left(\frac{f\int \frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}+\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}+\frac{\cdots}{3(fg-eh)\left(\frac{h\cdots}{\cdots}\right)}\cdots\right)}{5h}\cdots\right)}{fg-eh}\cdots\right)}{7h}\cdots$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{7h(g+hx)^{7/2}}$$

$$\Big\downarrow \; 73$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$2f^2\int \frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}\,d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}$$

$$2bfpq\,\frac{f\left(\frac{2f^2\int \frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}\,d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h(fg-eh)}+\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+}}\right.}{fg-eh}$$

$$h\,\frac{\left(2bfpq\,\cdots\right.}{5h}$$

$$4bpq\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}\,d(e+fx)}{fg-eh}-\frac{h\left(\cdots\right.}{fg-eh}\right.$$

$$7h$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 221

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{f\left(\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{2f^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}\right)}{fg-eh}+\frac{\cdots}{3(fg-e\cdots}\right)}{5h}\cdots\right)}{fg-eh}\cdots\right)}{7h}\cdots$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 2789

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\frac{4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{5/2}}d(e+fx)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{f\left(\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{2f^{3/2}\text{arct}\cdots}{\cdots}\right)}{fg-eh}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{7h}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$$\Big\downarrow \; 2756$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\int\frac{1}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{3h}-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{3h\left(\frac{h(e+fx)}{f}-\frac{eh}{f}+g\right)^{3/2}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\cdots}{\cdots}\right)}{\cdots}\right)$$

$$\overline{7h}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 61

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{f\int \frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}+\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{3h}-\frac{2f(a+b\log(cd^q(\ldots}{3h\left(\frac{h(e+fx)}{f}-\ldots\right.}\ldots\right.}{fg-eh}\ldots\right.}{fg-eh}\ldots\right.$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 73

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{2f^2\int \frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h(fg-eh)}+\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{3h}\right.}{fg-eh}\right.}{fg-eh}\right.$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$$\Big\downarrow \; 221$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{2f^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}\right)}{3h}-\frac{2f(a+b\log(\cdots}{3h\left(\frac{h(e+f\cdots}{f}\cdots\right.}\cdots\right)}{fg-eh}\cdots\right)}{fg-eh}\cdots\right)}{\cdots}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 2789

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\int\frac{a+b\log(cd^q(e+fx)^{pq})}{\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)^{3/2}}d(e+fx)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{2f}{(fg-eh)\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{2f^{3/2}\text{arctanh}\left(\cdots\right.}{(fg\cdots}\right.}{3h}\cdots\right.}{fg-e\cdots}\right.}{fg-eh}\cdots\right.$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 2756

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\int\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{2bfpq\int\frac{1}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{h}-\frac{2f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\frac{2bfpq\left(\frac{\cdots}{(fg-eh)\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots}{fg-eh}\cdots\right)$$

$$\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 73

3.495. $\int\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(\frac{4bf^2pq\int\frac{1}{e+\frac{f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}{h}-\frac{fg}{h}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{h^2}-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h\left(\cdots\right.}{\cdots}}{fg-eh}\cdots\right.$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 221

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\int\frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}d(e+fx)}{fg-eh}-\frac{h\left(-\frac{2f(a+b\log(cd^q(e+fx)^{pq}))}{h\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}-\frac{4bf^{3/2}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}\right)}{fg-eh}\right)}{fg-eh}-\frac{h\left(2bfpq\left(\cdots\right.\right.}{\cdots}\right.}{fg-eh}\cdots\right. \text{(cut off)}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$$\downarrow \; 2790$$

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\left(-bpq\int -\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}(e+fx)}d(e+fx)-\frac{2\sqrt{f}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)(a+b\log(cd^q(e+fx)^{pq}))}{\sqrt{fg-eh}}\right)}{fg-eh}-\frac{h\left(-\frac{2f}{}\right.}{}}{fg-eh}}{}\right.}{}\right.$$

$$\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 27

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\left(\frac{2b\sqrt{f}pq\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{e+fx}d(e+fx)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{h(e+fx)}{f}-\frac{eh}{f}+g}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{\sqrt{fg-eh}}\right)}{fg-eh}-\frac{h\left(-\frac{2f(a\cdots}{h\cdots}\cdots\right.}{\cdots}\cdots\right)}{fg-eh}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{7h(g+hx)^{7/2}}$$

$\downarrow$ 7267

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(\frac{f\left(\frac{f\left(\frac{f\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{eh-f\left(\frac{eh}{f}-\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)(a+b\,\mathrm{lo}}{\sqrt{fg-eh}}}{fg-eh}}{fg-eh}}{\ldots}\right.$$

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

3.494. $\downarrow$ 2092

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$4bpq\left(f\left(\frac{f\left(\frac{f\left(\frac{4bf^{3/2}pq\int\frac{\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{-fg+eh+f\left(g-\frac{eh}{f}+\frac{h(e+fx)}{f}\right)}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)(a+b\log\cdots}{\sqrt{fg-eh}}\cdots\right)}{fg-eh}\cdots\right)}{fg-eh}\cdots\right)\cdots\right.$$

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\downarrow 6546$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\left(f\left(f\left(f\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}d\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.}{fg-eh}\right.}{fg-eh}\right.$$

3.495. $\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

3.495. $\downarrow$ 6470

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$f\left(f\left(f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}-\int \cdots \log}{\sqrt{f}\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}\right)\Big/(fg-eh)\right)\right)$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\downarrow 2849$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\frac{\frac{\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}+\frac{\sqrt{fg\cdots}}{\sqrt{f}\sqrt{fg-eh}}}{\cdots}\right)}{\sqrt{fg-eh}}}{f\cdots}}{f\cdots}$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\downarrow 2752$$

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

$$\left(f\left(f\left(\frac{f\left(\frac{4bf^{3/2}pq\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)^2}{2f}-\frac{\frac{\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{h(e+fx)}{f}}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}}+\frac{\sqrt{fg\cdots}}{\cdots}}{\sqrt{f}\sqrt{fg-eh}}\cdots\right.}{\sqrt{fg-eh}}\cdots\right.}{fg-eh}\cdots\right.\right.$$

3.495. $\displaystyle\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(9/2),x]
```

output
```
(-2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(7*h*(g + h*x)^(7/2)) + (4*b*p*q*(
-((h*((2*b*f*p*q*((2*f)/(3*(f*g - e*h)*(g - (e*h)/f + (h*(e + f*x))/f)^(3/
2)) + (f*((2*f)/((f*g - e*h)*Sqrt[g - (e*h)/f + (h*(e + f*x))/f]) - (2*f^(
3/2)*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]
])/(f*g - e*h)^(3/2)))/(f*g - e*h)))/(5*h) - (2*f*(a + b*Log[c*d^q*(e + f*
x)^(p*q)]))/(5*h*(g - (e*h)/f + (h*(e + f*x))/f)^(5/2))))/(f*g - e*h)) + (
f*(-((h*((2*b*f*p*q*((2*f)/((f*g - e*h)*Sqrt[g - (e*h)/f + (h*(e + f*x))/f
]) - (2*f^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt
[f*g - e*h]])/(f*g - e*h)^(3/2)))/(3*h) - (2*f*(a + b*Log[c*d^q*(e + f*x)^
(p*q)]))/(3*h*(g - (e*h)/f + (h*(e + f*x))/f)^(3/2))))/(f*g - e*h)) + (f*(
-((h*((-4*b*f^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/
f])/Sqrt[f*g - e*h]])/(h*Sqrt[f*g - e*h]) - (2*f*(a + b*Log[c*d^q*(e + f*x
)^(p*q)]))/(h*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])))/(f*g - e*h)) + (f*((-
2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g -
 e*h]]*(a + b*Log[c*d^q*(e + f*x)^(p*q)]))/Sqrt[f*g - e*h] + (4*b*f^(3/2)*
p*q*(ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h]
]^2/(2*f) - ((Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e +
f*x))/f])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e +
f*x))/f])/Sqrt[f*g - e*h])])/Sqrt[f] + (Sqrt[f*g - e*h]*PolyLog[2, 1 - 2/(
1 - (Sqrt[f]*Sqrt[g - (e*h)/f + (h*(e + f*x))/f])/Sqrt[f*g - e*h])])/(2...
```

3.495.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2790
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d, e, n
, r}, x] && IntegerQ[q - 1/2]
```

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}} \, dx$

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.495. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.495.4 Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)^{\frac{9}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x)
```

3.495.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a
*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^5*x^5 + 5*g*h^4*x^4 +
10*g^2*h^3*x^3 + 10*g^3*h^2*x^2 + 5*g^4*h*x + g^5), x)
```

3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(9/2),x)
```

output
```
Timed out
```

3.495.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for m
ore detail
```

3.495.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(9/2), x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = \int \frac{(a+b\ln(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(9/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(9/2), x)
```

3.496 $\int \frac{(g+hx)^{3/2}}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.496.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Int}\left(\frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

output
```
Unintegrable((h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.496.2 Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

input
```
Integrate[(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
Integrate[(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

3.496.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$\Big\downarrow \; 2896$$
$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

input
```
Int[(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
$Aborted
```

3.496.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.496.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(hx+g)^{\frac{3}{2}}}{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int((h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.496.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(hx+g)^{\frac{3}{2}}}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas"
)
```

output
```
integral((h*x + g)^(3/2)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.496.6 Sympy [N/A]

Not integrable

Time = 33.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(g+hx)^{\frac{3}{2}}}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

input
```
integrate((h*x+g)**(3/2)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((g + h*x)**(3/2)/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.496.7 Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(hx+g)^{\frac{3}{2}}}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima"
)
```

output
```
integrate((h*x + g)^(3/2)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.496.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^{\frac{3}{2}}}{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate((h*x + g)^(3/2)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.496.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+h\,x)^{3/2}}{a+b\,\ln\left(c\,{\left(d\,{(e+f\,x)}^p\right)}^q\right)}\,dx$$

input
```
int((g + h*x)^(3/2)/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^(3/2)/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.497 $\int \frac{\sqrt{g+hx}}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.497.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \operatorname{Int}\left(\frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

output
```
Unintegrable((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.497.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

input
```
Integrate[Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
Integrate[Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

3.497.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$\Big\downarrow \text{ 2896}$$
$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

input
```
Int[Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
$Aborted
```

3.497.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.497.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{hx+g}}{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.497.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{hx+g}}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas"
)
```

output
```
integral(sqrt(h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.497.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

input
```
integrate((h*x+g)**(1/2)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(sqrt(g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.497.7 Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{hx+g}}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.497.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{\sqrt{hx+g}}{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate(sqrt(h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.497.9 Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{\sqrt{g+h\,x}}{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

input
```
int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.498 $\int \frac{1}{\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.498.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))},x\right)$$

output
```
Unintegrable(1/(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x)
```

3.498.2 Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))}\,dx$$

input
```
Integrate[1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.498.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$

input
```
Int[1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.498.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.498.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)\sqrt{hx+g}}dx$$

input
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x)
```

output
```
int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x)
```

3.498.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))}\,dx=\int \frac{1}{\sqrt{hx+g}(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a)}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(h*x + g)/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*
c)), x)
```

3.498.6 Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(1/2),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x)), x)
```

3.498.7 Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c(d(e+fx)^p)^q))}\,dx=\int \frac{1}{\sqrt{hx+g}(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a)}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.498.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx = \int \frac{1}{\sqrt{hx+g}(b\log\left(\left(((fx+e)^p d\right)^q c\right)+a)}\,dx$$

input
```
integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.498.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx = \int \frac{1}{\sqrt{g+h\,x}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.499 $\int \frac{1}{(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.499.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\text{Int}\left(\frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

output
```
Unintegrable(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.499.2 Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

input
```
Integrate[1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.499.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)^{3/2} (a+b\log(c(d(e+fx)^p)^q))} dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{(g+hx)^{3/2} (a+b\log(c(d(e+fx)^p)^q))} dx$$

input
```
Int[1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.499.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.499.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)^{\frac{3}{2}} (a+b\ln(c(d(fx+e)^p)^q))}dx$$

input
```
int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.499.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.23

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="frica
s")
```

output
```
integral(sqrt(h*x + g)/(a*h^2*x^2 + 2*a*g*h*x + a*g^2 + (b*h^2*x^2 + 2*b*g
*h*x + b*g^2)*log(((f*x + e)^p*d)^q*c)), x)
```

3.499.6 Sympy [N/A]

Not integrable

Time = 15.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\left(g+hx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(h*x+g)**(3/2)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**(3/2)), x)
```

3.499.7 Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxim
a")
```

output
```
integrate(1/((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.499.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\left(b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac"
)
```

output
```
integrate(1/((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.499.9 Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^{3/2}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.500 $\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$

3.500.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Int}\left(\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

output
```
Unintegrable((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.500.2 Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

input
```
Integrate[Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
Integrate[Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

3.500.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

$$\Big\downarrow \text{2896}$$

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

input
```
Int[Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
$Aborted
```

3.500.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.500.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{hx+g}\,\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

input
```
int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.500.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="f
ricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.500.6 Sympy [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\sqrt{g+hx}\,dx$$

input
```
integrate((h*x+g)**(1/2)*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x), x)
```

3.500.7 Maxima [N/A]

Not integrable

Time = 11.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{hx+g}\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.500.8 Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{hx+g}\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.500.9 Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{g+h\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

input
```
int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.501 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{g+hx}}\,dx$

3.501.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\text{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}},x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x)
```

3.501.2 Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x],x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x], x]
```

3.501.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}} \, dx$$

$$\Big\downarrow \ 2896$$

$$\int \frac{\sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}dx$$

input
```
Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x],x]
```

output
```
$Aborted
```

3.501.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.501.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\ln \left(c \left(d \left(fx+e\right)^{p}\right)^{q}\right)}}{\sqrt{hx+g}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x)
```

3.501.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.501.6 Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}}{\sqrt{g+hx}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(g + h*x), x)
```

3.501.7 Maxima [N/A]

Not integrable

Time = 11.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}}{\sqrt{hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x + g), x)
```

3.501.8 Giac [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}{\sqrt{hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x + g), x)
```

3.501.9 Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}{\sqrt{g+h\,x}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(1/2), x)
```

3.502 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{(g+hx)^{3/2}}\,dx$

3.502.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x)
```

3.502.2 Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2),x]
```

output
```
Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2), x]
```

3.502.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx$$

$$\Big\downarrow \; 2896$$

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}dx$$

input
```
Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2),x]
```

output
```
$Aborted
```

3.502.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.502.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{\left(hx+g\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x)
```

3.502.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x, algorithm="f
ricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.502.6 Sympy [N/A]

Not integrable

Time = 17.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}{(g+hx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g)**(3/2),x)
```

output
```
Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**(3/2), x)
```

3.502.7 Maxima [N/A]

Not integrable

Time = 11.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \int \frac{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}{(hx+g)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^(3/2), x)
```

3.502.8 Giac [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}{(hx+g)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^(3/2), x)
```

3.502.9 Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}{\left(g+h\,x\right)^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(3/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(3/2), x)
```

3.503 $\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.503.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Int}\left(\frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

output
```
Unintegrable((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.503.2 Mathematica [N/A]

Not integrable

Time = 4.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

input
```
Integrate[Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
Integrate[Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

3.503.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$
$$\Big\downarrow \ 2896$$
$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

input
```
Int[Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
$Aborted
```

3.503.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.503.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{hx+g}}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

input
```
int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.503.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="f
ricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.503.6 Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

input
```
integrate((h*x+g)**(1/2)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(sqrt(g + h*x)/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.503.7 Maxima [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{\sqrt{hx+g}}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.503.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{hx+g}}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.503.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{g+h\,x}}{\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

input
```
int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.504 $\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.504.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Int}\left(\frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

output
```
Unintegrable(1/(h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.504.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

input
```
Integrate[1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

output
```
Integrate[1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

3.504.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$
$$\Big\downarrow \ 2896$$
$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

input
```
Int[1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

output
```
$Aborted
```

3.504.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.504.4 Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{hx+g}\,\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

input
```
int(1/(h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int(1/(h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.504.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm=
"fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.504.6 Sympy [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(h*x+g)**(1/2)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x)), x)
```

3.504.7 Maxima [N/A]

Not integrable

Time = 11.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{hx+g}\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate(1/(h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(1/(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.504.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{hx+g}\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate(1/(h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm=
"giac")
```

output
```
integrate(1/(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.504.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{g+h\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

input
```
int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)),x)
```

output
```
int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)), x)
```

3.505 $\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.505.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Int}\left(\frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

output
```
Unintegrable(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.505.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

input
```
Integrate[1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

output
```
Integrate[1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

3.505.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$
$$\Big\downarrow \ 2896$$
$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

input
```
Int[1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

output
```
$Aborted
```

3.505.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.505.4 Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{(hx+g)^{\frac{3}{2}}\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

input
```
int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.505.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm=
"fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.505.6 Sympy [N/A]

Not integrable

Time = 51.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\left(g+hx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(h*x+g)**(3/2)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**(3/2)), x)
```

3.505.7 Maxima [N/A]

Not integrable

Time = 11.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(hx+g)^{\frac{3}{2}}\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((h*x + g)^(3/2)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.505.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\sqrt{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}}\,dx$$

input
```
integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm=
"giac")
```

output
```
integrate(1/((h*x + g)^(3/2)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.505.9 Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^{3/2}\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

input
```
int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)),x)
```

output
```
int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)), x)
```

3.506 $\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))\,dx$

3.506.1 Optimal result

Integrand size = 26, antiderivative size = 99

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{bfpq(g+hx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)(1+m)(2+m)} + \frac{(g+hx)^{1+m}(a+b\log(c(d(e+fx)^p)^q))}{h(1+m)}$$

output
```
b*f*p*q*(h*x+g)^(2+m)*hypergeom([1, 2+m],[3+m],f*(h*x+g)/(-e*h+f*g))/h/(-e
*h+f*g)/(1+m)/(2+m)+(h*x+g)^(1+m)*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(1+m)
```

3.506.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{(g+hx)^{1+m}\left(a+\frac{bfpq(g+hx)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{f(g+hx)}{fg-eh}\right)}{(fg-eh)(2+m)}+b\log(c(d(e+fx)^p)^q)\right)}{h(1+m)}$$

input
```
Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
((g + h*x)^(1 + m)*(a + (b*f*p*q*(g + h*x)*Hypergeometric2F1[1, 2 + m, 3 +
 m, (f*(g + h*x))/(f*g - e*h)])/((f*g - e*h)*(2 + m)) + b*Log[c*(d*(e + f*
x)^p)^q]))/(h*(1 + m))
```

3.506.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2895, 2842, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right) dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right) dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{(g+hx)^{m+1}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{h(m+1)}-\frac{bfpq\int \frac{(g+hx)^{m+1}}{e+fx}dx}{h(m+1)}$$

$$\Big\downarrow \text{ 78}$$

$$\frac{(g+hx)^{m+1}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{h(m+1)}+\frac{bfpq(g+hx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{f(g+hx)}{fg-eh}\right)}{h(m+1)(m+2)(fg-eh)}$$

input
```
Int[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
(b*f*p*q*(g + h*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (f*(g + h*x)
)/(f*g - e*h)])/(h*(f*g - e*h)*(1 + m)*(2 + m)) + ((g + h*x)^(1 + m)*(a +
b*Log[c*(d*(e + f*x)^p)^q]))/(h*(1 + m))
```

3.506.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.506.4 Maple [F]

$$\int (hx+g)^m (a+b\ln (c(d(fx+e)^p)^q))\, dx$$

input
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.506.5 Fricas [F]

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\, dx = \int (b\log (((fx+e)^p d)^q c)+a)(hx+g)^m\, dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
integral((h*x + g)^m*b*log(((f*x + e)^p*d)^q*c) + (h*x + g)^m*a, x)
```

3.506.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^m \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.506.7 Maxima [F]

$$\int (g+hx)^m \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) dx = \int (b\log \left(((fx+e)^p d)^q c\right)+a)(hx+g)^m \, dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
b*((h*x + g)*(h*x + g)^m*log(((f*x + e)^p)^q)/(h*(m + 1)) + integrate(-(f*
g*p*q - e*h*(m + 1)*log(c) - (m*q + q)*e*h*log(d) + (f*h*p*q - f*h*(m + 1)
*log(c) - (m*q + q)*f*h*log(d))*x)*(h*x + g)^m/(f*h*(m + 1)*x + e*h*(m + 1
)), x)) + (h*x + g)^(m + 1)*a/(h*(m + 1))
```

3.506.8 Giac [F]

$$\int (g+hx)^m \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) dx = \int (b\log \left(((fx+e)^p d)^q c\right)+a)(hx+g)^m \, dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx=\int (g+h\,x)^m\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)\,dx$$

input
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.507 $\int \frac{(g+hx)^m}{a+b\log(c(d(e+fx)^p)^q)}\,dx$

3.507.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Int}\left(\frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

output
```
Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.507.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

input
```
Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

3.507.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$\Big\downarrow \ 2896$$
$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

input
```
Int[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

output
```
$Aborted
```

3.507.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.507.4 Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(hx+g)^m}{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

input
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.507.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^m}{b\log\left(\left(\left(fx+e\right)^pd\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
integral((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.507.6 Sympy [N/A]

Not integrable

Time = 13.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^m}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

input
```
integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((g + h*x)**m/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.507.7 Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^m}{b\log\left(\left(\left(fx+e\right)^pd\right)^q c\right)+a}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.507.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^m}{b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.507.9 Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+h\,x)^m}{a+b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)}\,dx$$

input
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

output
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

3.508 $\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.508.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx=\text{Int}\left(\frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2},x\right)$$

output
```
Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.508.2 Mathematica [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx=\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$

input
```
Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2, x]
```

3.508.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$

$$\Big\downarrow \; 2896$$

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}dx$$

input
```
Int[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

output
```
$Aborted
```

3.508.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.508.4 Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(hx+g)^m}{(a+b\ln(c(d(fx+e)^p)^q))^2}dx$$

input
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.508.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{(hx+g)^m}{(b\log(((fx+e)^pd)^qc)+a)^2}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
integral((h*x + g)^m/(b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e
)^p*d)^q*c) + a^2), x)
```

3.508.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.508.7 Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 6.29

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{(hx+g)^m}{(b\log(((fx+e)^pd)^qc)+a)^2}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
-(f*x + e)*(h*x + g)^m/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*
q^2*log(d) + f*p*q*log(c))*b^2) + integrate((f*h*(m + 1)*x + e*h*m + f*g)*
(h*x + g)^m/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q*log(c))*b^2 + (a*b*
f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x + (b^2*f*h*p*q*x + b^
2*f*g*p*q)*log(((f*x + e)^p)^q)), x)
```

3.508.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{(hx+g)^m}{(b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a)^2, x)
```

3.508.9 Mupad [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{(g+h\,x)^m}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}\,dx$$

input
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

output
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

3.509 $\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx$

3.509.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int (g+hx)^m (a+b\log\left(c(d(e+fx)^p)^q\right))^{3/2}\,dx = \text{Int}\left((g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2},x\right)$$

output
```
Unintegrable((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.509.2 Mathematica [N/A]

Not integrable

Time = 15.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx = \int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx$$

input
```
Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2), x]
```

3.509.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} \, dx$$
$$\Big\downarrow \text{ 2896}$$
$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} dx$$

input
```
Int[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
$Aborted
```

3.509.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.509.4 Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (hx+g)^m \left(a+b\ln\left(c\left(d(fx+e)^p\right)^q\right)\right)^{\frac{3}{2}}dx$$

input
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.509.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int (g+hx)^m \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} dx = \int \left(b\log \left(\left(\left(fx+e\right)^pd\right)^q c\right)+a\right)^{\frac{3}{2}} (hx+g)^m dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="frica
s")
```

output
```
integral(((h*x + g)^m*b*log(((f*x + e)^p*d)^q*c) + (h*x + g)^m*a)*sqrt(b*l
og(((f*x + e)^p*d)^q*c) + a), x)
```

3.509.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Timed out
```

3.509.7 Maxima [N/A]

Not integrable

Time = 11.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} dx = \int \left(b\log \left(\left(\left(fx+e\right)^pd\right)^q c\right)+a\right)^{\frac{3}{2}} (hx+g)^m dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)*(h*x + g)^m, x)
```

3.509.8 Giac [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int \left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{3}{2}}(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)*(h*x + g)^m, x)
```

3.509.9 Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int (g+h\,x)^m\,\left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^{3/2}\,dx$$

input
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.510 $\int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$

3.510.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Int}\left((g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

output
```
Unintegrable((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.510.2 Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

input
```
Integrate[(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
Integrate[(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

3.510.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

$$\Big\downarrow \text{2896}$$

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}dx$$

input
```
Int[(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
$Aborted
```

3.510.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.510.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (hx+g)^m\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

input
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.510.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right.}(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

3.510.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.510.7 Maxima [N/A]

Not integrable

Time = 11.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right.}(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

3.510.8 Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}\, dx = \int \sqrt{b\log \left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}(hx+g)^m \, dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

3.510.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}\, dx = \int (g+h\,x)^m\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)} \,dx$$

input
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.511 $\int \frac{(g+hx)^m}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$

3.511.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\mathrm{Int}\left(\frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

output
```
Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.511.2 Mathematica [N/A]

Not integrable

Time = 10.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

input
```
Integrate[(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
Integrate[(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

3.511.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$
$$\Big\downarrow \; 2896$$
$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}dx$$

input
```
Int[(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

output
```
$Aborted
```

3.511.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.511.4 Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(hx+g)^m}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}}dx$$

input
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

output
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

3.511.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(hx+g)^m}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="frica
s")
```

output
```
integral((h*x + g)^m/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.511.6 Sympy [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

input
```
integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

output
```
Integral((g + h*x)**m/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

3.511.7 Maxima [N/A]

Not integrable

Time = 10.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(hx+g)^m}{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^m/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.511.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(hx+g)^m}{\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^m/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

3.511.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+h\,x)^m}{\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

input
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

output
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

3.512 $\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$

3.512.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Int}\left(\frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}},x\right)$$

output
```
Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.512.2 Mathematica [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$$

input
```
Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2), x]
```

3.512.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$$

$$\Big\downarrow \; 2896$$

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}dx$$

input
```
Int[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

output
```
$Aborted
```

3.512.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.512.4 Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(hx+g)^m}{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

output
```
int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

3.512.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(hx+g)^m}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m/(b^2*log(((f*x +
 e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) + a^2), x)
```

3.512.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.512.7 Maxima [N/A]

Not integrable

Time = 11.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(hx+g)^m}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.512.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=\int \frac{(hx+g)^m}{(b\log(((fx+e)^pd)^qc)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

3.512.9 Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=\int \frac{(g+h\,x)^m}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}}\,dx$$

input
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
```

output
```
int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

3.513 $\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n \, dx$

3.513.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \text{Int}\left((g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n, x\right)$$

output
```
Unintegrable((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

3.513.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n \, dx$$

input
```
Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n, x]
```

3.513.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))^n \, dx$$

$$\Big\downarrow \; 2896$$

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))^n \, dx$$

input
```
Int[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
$Aborted
```

3.513.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.513.4 Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (hx+g)^m (a+b\ln (c(d(fx+e)^p)^q))^n dx$$

input
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

output
```
int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

3.513.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n \, dx = \int (hx+g)^m (b\log\left(((fx+e)^p d)^q c\right)+a)^n \, dx$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")
```

output
```
integral((h*x + g)^m*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

3.513.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.513.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^m \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.513.8 Giac [F(-2)]

Exception generated.

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,7,4,0,5,0,3,3,3,0,2,0,0,0]%%%}+%%%{5,[0,0,6,4,0,4,1,3,3,3,0,2,0,0,0]%%%
}+%%%{2,[0,0
```

3.513.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \int (g+h\,x)^m \left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^n dx$$

input
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
```

output
```
int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

3.514 $\int (g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n\,dx$

3.514.1 Optimal result

Integrand size = 28, antiderivative size = 432

$$\int (g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n\,dx$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{3}{pq}}\Gamma\left(1+n,-\frac{3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{bpq}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n}{f^3}$$
$$+\frac{2^{-n}e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{2}{pq}}\Gamma\left(1+n,-\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{bpq}\right)\left(a+b\log(c(d(e+f\right.}{f^3}$$
$$+\frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{bpq}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{f^3}$$

output
```
3^(-1-n)*h^2*(f*x+e)^3*GAMMA(1+n,-3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+
b*ln(c*(d*(f*x+e)^p)^q))^n/exp(3*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(3/p/q)
)/(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)+h*(-e*h+f*g)*(f*x+e)^2*GAMMA(1+
n,-2*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n/(2^n
)/exp(2*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(2/p/q))/(((-a-b*ln(c*(d*(f*x+e)
^p)^q))/b/p/q)^n)+(-e*h+f*g)^2*(f*x+e)*GAMMA(1+n,(-a-b*ln(c*(d*(f*x+e)^p)^
q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n/exp(a/b/p/q)/f^3/((c*(d*(f*x+e)^p
)^q)^(1/p/q))/(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)
```

3.514.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.75

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^n \, dx$$
$$= \frac{2^{-n}3^{-1-n}e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(2^n h^2(e+fx)^2\Gamma\left(1+n,-\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)+3^{1+n}e^{\frac{a}{bpq}}(f\right.}{}$$

input
```
Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
(3^(-1 - n)*(e + f*x)*(2^n*h^2*(e + f*x)^2*Gamma[1 + n, (-3*(a + b*Log[c*(
d*(e + f*x)^p)^q]))/(b*p*q)] + 3^(1 + n)*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(
e + f*x)^p)^q)^(1/(p*q))*(h*(e + f*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e
+ f*x)^p)^q]))/(b*p*q)] + 2^n*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)
^q)^(1/(p*q))*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]))*
(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(2^n*E^((3*a)/(b*p*q))*f^3*(c*(d*(e +
f*x)^p)^q)^(3/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

3.514.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^n \, dx$$
$$\Big\downarrow \text{2895}$$
$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^n dx$$
$$\Big\downarrow \text{2848}$$
$$\int \left(\frac{(fg-eh)^2 (a+b\log (c(d(e+fx)^p)^q))^n}{f^2}+\frac{2h(e+fx)(fg-eh) (a+b\log (c(d(e+fx)^p)^q))^n}{f^2}+\frac{h^2(e+fx)^2 (}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{h2^{-n}(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n+}{f^3}$$

$$\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n+1,-\frac{a}{}}{f^3}$$

$$\frac{h^23^{-n-1}(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n+1,-\frac{3(a}{}}{f^3}$$

input
```
Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
(3^(-1 - n)*h^2*(e + f*x)^3*Gamma[1 + n, (-3*(a + b*Log[c*(d*(e + f*x)^p)^
q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^((3*a)/(b*p*q))*f^3*(
c*(d*(e + f*x)^p)^q)^(3/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)
))^n) + (h*(f*g - e*h)*(e + f*x)^2*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f
*x)^p)^q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(2^n*E^((2*a)/(b*
p*q))*f^3*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^
q])/(b*p*q)))^n) + ((f*g - e*h)^2*(e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d
*(e + f*x)^p)^q])/(b*p*q))]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p
*q))*f^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q
])/(b*p*q)))^n)
```

3.514.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.514.4 Maple [F]

$$\int (hx+g)^2 \left(a+b\ln \left(c\left(d(fx+e)^p\right)^q\right)\right)^n dx$$

input
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

output
```
int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

3.514.5 Fricas [F]

$$\int (g+hx)^2 \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^n \, dx = \int (hx+g)^2 \left(b\log \left(\left((fx+e)^p d\right)^q c\right)+a\right)^n dx$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")
```

output
```
integral((h^2*x^2 + 2*g*h*x + g^2)*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

3.514.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^n \, dx = \text{Timed out}$$

input
```
integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
```

output
```
Timed out
```

3.514.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.514.8 Giac [F]

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx = \int (hx+g)^2 (b\log\left(((fx+e)^p d)^q c\right)+a)^n\,dx$$

input
```
integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
```

output
```
integrate((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx = \int (g+h\,x)^2 \left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^n\,dx$$

input
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
```

output
```
int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

3.515 $\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n \, dx$

3.515.1 Optimal result

Integrand size = 26, antiderivative size = 281

$$\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n \, dx = \frac{2^{-1-n} e^{-\frac{2a}{bpq}} h(e+fx)^2 \left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}} \Gamma\left(1+n, -\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n \left(\right.}{f^2} + \frac{e^{-\frac{a}{bpq}} (fg-eh)(e+fx) \left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}} \Gamma\left(1+n, -\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n}{f^2}$$

output
```
2^(-1-n)*h*(f*x+e)^2*GAMMA(1+n,-2*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*
ln(c*(d*(f*x+e)^p)^q))^n/exp(2*a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(2/p/q))/
(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)+(-e*h+f*g)*(f*x+e)*GAMMA(1+n,(-a-
b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n/exp(a/b/p/q)
/f^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))/(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n
)
```

3.515.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.81

$$\int (g+hx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n\,dx$$
$$=\frac{2^{-1-n}e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(h(e+fx)\Gamma\left(1+n,-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)+2^{1+n}e^{\frac{a}{bpq}}(fg-eh)\right.}{f}$$

input
```
Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
(2^(-1 - n)*(e + f*x)*(h*(e + f*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f
*x)^p)^q]))/(b*p*q)] + 2^(1 + n)*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)
^p)^q)^(1/(p*q))*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]
)*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^((2*a)/(b*p*q))*f^2*(c*(d*(e + f*
x)^p)^q)^(2/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

3.515.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2895, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n\,dx$$
$$\Big\downarrow \ 2895$$
$$\int (g+hx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n dx$$
$$\Big\downarrow \ 2848$$
$$\int \left(\frac{(fg-eh)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n}{f}+\frac{h(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^n}{f}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n+1,-\frac{a+}{}}{f^2}$$

$$\frac{h2^{-n-1}(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n+1,-\frac{2(a+}{}}{f^2}$$

input
```
Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
(2^(-1 - n)*h*(e + f*x)^2*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f*x)^p)^q]
))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^((2*a)/(b*p*q))*f^2*(c*
(d*(e + f*x)^p)^q)^(2/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))
^n) + ((f*g - e*h)*(e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q
])/(b*p*q))]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p*q))*f^2*(c*(d*
(e + f*x)^p)^q)^(1/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

3.515.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.515.4 Maple [F]

$$\int (hx+g)\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{n}dx$$

input
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

output
```
int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

3.515.5 Fricas [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{n}\,dx=\int (hx+g)(b\log\left(((fx+e)^{p}d)^{q}\,c\right)+a)^{n}\,dx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")
```

output
```
integral((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

3.515.6 Sympy [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{n}\,dx=\int \left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{n}(g+hx)\,dx$$

input
```
integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**n*(g + h*x), x)
```

3.515.7 Maxima [F(-2)]

Exception generated.

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.515.8 Giac [F]

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^n \, dx = \int (hx + g)(b \log (((fx + e)^p d)^q c) + a)^n \, dx$$

input
```
integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
```

output
```
integrate((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^n \, dx = \int (g + h\,x)\,\left(a + b\,\ln \left(c\,{\left(d\,{\left(e + f\,x\right)}^p\right)}^q\right)\right)^n \, dx$$

input
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
```

output
```
int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

3.516 $\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx$

3.516.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log (c(d(e+fx)^p)^q))^n \left(-\frac{a+b\log(c(\cdots}{b\cdots}\right.}{f}$$

output
```
(f*x+e)*GAMMA(1+n,(-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)
^p)^q))^n/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p/q))/(((-a-b*ln(c*(d*(f*
x+e)^p)^q))/b/p/q)^n)
```

3.516.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log (c(d(e+fx)^p)^q))^n \left(-\frac{a+b\log(c(\cdots}{b\cdots}\right.}{f}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
((e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*(a +
b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p*q))*f*(c*(d*(e + f*x)^p)^q)^(1/(
p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

3.516.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2895, 2836, 2737, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int (a+b\log(cd^q(e+fx)^{pq}))^n\,d(e+fx)}{f}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\int \left(cd^q(e+fx)^{pq}\right)^{\frac{1}{pq}}(a+b\log(cd^q(e+fx)^{pq}))^n\,d\log(cd^q(e+fx)^{pq})}{fpq}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(e+fx)e^{-\frac{a}{bpq}}\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}(a+b\log(cd^q(e+fx)^{pq}))^n\left(-\frac{a+b\log(cd^q(e+fx)^{pq})}{bpq}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\log(cd^q(e+fx)}{bpq}\right.}{f}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

output
```
((e + f*x)*Gamma[1 + n, -((a + b*Log[c*d^q*(e + f*x)^(p*q)])/(b*p*q))]*(a
+ b*Log[c*d^q*(e + f*x)^(p*q)])^n)/(E^(a/(b*p*q))*f*(c*d^q*(e + f*x)^(p*q)
)^(1/(p*q))*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])/(b*p*q)))^n)
```

3.516.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.516.4 Maple [F]

$$\int (a+b\ln (c(d(fx+e)^p)^q))^n dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{e^{\left(-\frac{bnpq\log\left(-\frac{1}{bpq}\right)+bq\log(d)+b\log(c)+a}{bpq}\right)}\Gamma\left(n+1,-\frac{bpq\log(fx+e)+bq\log(d)+b\log(c)+a}{bpq}\right)}{f}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")
```

output
```
e^(-(b*n*p*q*log(-1/(b*p*q)) + b*q*log(d) + b*log(c) + a)/(b*p*q))*gamma(n
 + 1, -(b*p*q*log(f*x + e) + b*q*log(d) + b*log(c) + a)/(b*p*q))/f
```

3.516.6 Sympy [F]

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \int (a+b\log (c(d(e+fx)^p)^q))^n \, dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**n, x)
```

3.516.7 Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.516.8 Giac [F]

$$\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx=\int (b\log(((fx+e)^pd)^q\,c)+a)^n\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx=\int \left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^n\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

3.517 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx$

3.517.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \operatorname{Int}\left(\frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx},x\right)$$

output
```
Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x)
```

3.517.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x),x]
```

output
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x), x]
```

3.517.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}dx$$

$$\Big\downarrow \text{2896}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}dx$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x),x]
```

output
```
$Aborted
```

3.517.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.517.4 Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^n}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x)
```

3.517.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^n}{g+hx} \, dx = \int \frac{(b\log (((fx+e)^p d)^q c)+a)^n}{hx+g} \, dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)^n/(h*x + g), x)
```

3.517.6 Sympy [N/A]

Not integrable

Time = 6.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^n}{g+hx} \, dx = \int \frac{(a+b\log (c(d(e+fx)^p)^q))^n}{g+hx} \, dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**n/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**n/(g + h*x), x)
```

3.517.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^n}{g+hx} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

3.517.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^n}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^n/(h*x + g), x)
```

3.517.9 Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^n}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^n/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^n/(g + h*x), x)
```

3.518 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx^2}\,dx$

3.518.1 Optimal result

Integrand size = 28, antiderivative size = 249

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx = \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f\left(\sqrt{-g}-\sqrt{h}x\right)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} - \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f\left(\sqrt{-g}+\sqrt{h}x\right)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} + \frac{bpq\operatorname{PolyLog}\left(2,\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}$$

output
```
1/2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*((-g)^(1/2)-x*h^(1/2))/(f*(-g)^(1/2)+
e*h^(1/2)))/(-g)^(1/2)/h^(1/2)-1/2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*((-g)^
(1/2)+x*h^(1/2))/(f*(-g)^(1/2)-e*h^(1/2)))/(-g)^(1/2)/h^(1/2)-1/2*b*p*q*po
lylog(2,-(f*x+e)*h^(1/2)/(f*(-g)^(1/2)-e*h^(1/2)))/(-g)^(1/2)/h^(1/2)+1/2*
b*p*q*polylog(2,(f*x+e)*h^(1/2)/(f*(-g)^(1/2)+e*h^(1/2)))/(-g)^(1/2)/h^(1/
2)
```

3.518.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx$$
$$=\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\left(\log\left(\frac{f\left(\sqrt{-g}-\sqrt{h}x\right)}{f\sqrt{-g}+e\sqrt{h}}\right)-\log\left(\frac{f\left(\sqrt{-g}+\sqrt{h}x\right)}{f\sqrt{-g}-e\sqrt{h}}\right)\right)-bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)+}{2\sqrt{-g}\sqrt{h}}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x^2),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*(Log[(f*(Sqrt[-g] - Sqrt[h]*x))/(f*Sqrt[
-g] + e*Sqrt[h])] - Log[(f*(Sqrt[-g] + Sqrt[h]*x))/(f*Sqrt[-g] - e*Sqrt[h]
)]) - b*p*q*PolyLog[2, -((Sqrt[h]*(e + f*x))/(f*Sqrt[-g] - e*Sqrt[h]))] +
b*p*q*PolyLog[2, (Sqrt[h]*(e + f*x))/(f*Sqrt[-g] + e*Sqrt[h])])/(2*Sqrt[-g
]*Sqrt[h])
```

3.518.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2895, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}dx$$
$$\Big\downarrow \text{2856}$$
$$\int \left(\frac{\sqrt{-g}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2g\left(\sqrt{-g}-\sqrt{h}x\right)}+\frac{\sqrt{-g}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2g\left(\sqrt{-g}+\sqrt{h}x\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(\frac{f(\sqrt{-g}-\sqrt{h}x)}{e\sqrt{h}+f\sqrt{-g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{2\sqrt{-g}\sqrt{h}} - \frac{\log\left(\frac{f(\sqrt{-g}+\sqrt{h}x)}{f\sqrt{-g}-e\sqrt{h}}\right)(a+b\log(c(d(e+fx)^p)^q))}{2\sqrt{-g}\sqrt{h}} - \frac{bpq\,\text{PolyLog}\left(2,-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} + \frac{bpq\,\text{PolyLog}\left(2,\frac{\sqrt{h}(e+fx)}{\sqrt{h}e+f\sqrt{-g}}\right)}{2\sqrt{-g}\sqrt{h}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x^2),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(Sqrt[-g] - Sqrt[h]*x))/(f*Sqrt[-
g] + e*Sqrt[h])])/(2*Sqrt[-g]*Sqrt[h]) - ((a + b*Log[c*(d*(e + f*x)^p)^q])
*Log[(f*(Sqrt[-g] + Sqrt[h]*x))/(f*Sqrt[-g] - e*Sqrt[h])])/(2*Sqrt[-g]*Sqr
t[h]) - (b*p*q*PolyLog[2, -((Sqrt[h]*(e + f*x))/(f*Sqrt[-g] - e*Sqrt[h]))]
)/(2*Sqrt[-g]*Sqrt[h]) + (b*p*q*PolyLog[2, (Sqrt[h]*(e + f*x))/(f*Sqrt[-g]
 + e*Sqrt[h])])/(2*Sqrt[-g]*Sqrt[h])
```

3.518.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.518.4 Maple [F]

$$\int \frac{a+b\ln \left(c{\left(d{\left(fx+e\right)}^{p}\right)}^{q}\right)}{h\,x^{2}+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x)
```

3.518.5 Fricas [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^{p})^{q}\right)}{g+hx^{2}}\,dx=\int \frac{b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}{hx^{2}+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x, algorithm="fricas")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x^2 + g), x)
```

3.518.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d(e+fx)^{p})^{q}\right)}{g+hx^{2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x**2+g),x)
```

output
```
Timed out
```

3.518.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{g + hx^2}\,dx = \int \frac{b\log\left(\left(\left((fx + e)^p d\right)^q c\right) + a}{hx^2 + g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x, algorithm="maxima")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*x^2 + g), x) + a
*arctan(h*x/sqrt(g*h))/sqrt(g*h)
```

3.518.8 Giac [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{g + hx^2}\,dx = \int \frac{b\log\left(\left(\left((fx + e)^p d\right)^q c\right) + a}{hx^2 + g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x^2 + g), x)
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{g + hx^2}\,dx = \int \frac{a + b\,\ln\left(c\,\left(d\,(e + f\,x)^p\right)^q\right)}{h\,x^2 + g}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2), x)
```

3.519 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2+hx^2}}\,dx$

3.519.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx = \frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}} - \frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} - \frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

output
```
1/2*b*p*q*arcsinh(1/2*x*h^(1/2)*2^(1/2))^2/h^(1/2)+arcsinh(1/2*x*h^(1/2)*2
^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h^(1/2)-b*p*q*arcsinh(1/2*x*h^(1/2)*2^
(1/2))*ln(1+(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(
1/2)-(e^2*h+2*f^2)^(1/2)))/h^(1/2)-b*p*q*arcsinh(1/2*x*h^(1/2)*2^(1/2))*ln
(1+(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(1/2)+(e^2
*h+2*f^2)^(1/2)))/h^(1/2)-b*p*q*polylog(2,-(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h
*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(1/2)-(e^2*h+2*f^2)^(1/2)))/h^(1/2)-b*p*q*po
lylog(2,-(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(1/2
)+(e^2*h+2*f^2)^(1/2)))/h^(1/2)
```

3.519.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx = \frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(2a+bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)-2bpq\log\left(1+\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)-2bpq\log\left(1+\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right.\right.}{2}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[2 + h*x^2],x]
```

output
```
(ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*(2*a + b*p*q*ArcSinh[(Sqrt[h]*x)/Sqrt[2]] -
2*b*p*q*Log[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] - Sq
rt[2*f^2 + e^2*h])] - 2*b*p*q*Log[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[
2]]*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h])] + 2*b*Log[c*(d*(e + f*x)^p)^q])
- 2*b*p*q*PolyLog[2, (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(-(e*Sqrt[
h]) + Sqrt[2*f^2 + e^2*h])] - 2*b*p*q*PolyLog[2, -((Sqrt[2]*E^ArcSinh[(Sqr
t[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h]))])/(2*Sqrt[h])
```

3.519.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2895, 2851, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{hx^2+2}}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{hx^2+2}}dx$$

$$\Big\downarrow \text{2851}$$

$$\frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}}-bfpq\int \frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{\sqrt{h}(e+fx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}}-\frac{bfpq\int \frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{e+fx}dx}{\sqrt{h}}$$

$$\Big\downarrow \text{6242}$$

$$\frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}}-\frac{bfpq\int \frac{\sqrt{\frac{hx^2}{2}+1}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{\frac{\sqrt{h}e}{\sqrt{2}}+\frac{f\sqrt{h}x}{\sqrt{2}}}d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{\sqrt{h}}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}}-$$

$$\frac{bfpq\left(\int \frac{e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{\frac{\sqrt{h}e}{\sqrt{2}}+e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f-\frac{\sqrt{he^2+2f^2}}{\sqrt{2}}}d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)+\int \frac{e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{\frac{\sqrt{h}e}{\sqrt{2}}+e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f+\frac{\sqrt{he^2+2f^2}}{\sqrt{2}}}d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)-\frac{\text{arcsinh}(}{2f}\right.}{\sqrt{h}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}}-$$

$$\frac{bfpq\left(-\frac{\int \log\left(\frac{\sqrt{2}e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{he^2+2f^2}}+1\right)d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{f}-\frac{\int \log\left(\frac{\sqrt{2}e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{\sqrt{h}e+\sqrt{he^2+2f^2}}+1\right)d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{f}+\frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(\right.}{f}\right.}{\sqrt{h}}$$

$$\Big\downarrow \text{2715}$$

3.519. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2+hx^2}}dx$

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bfpq\left(-\frac{\int e^{-\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}\log\left(\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{he^2+2f^2}}+1\right)de^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}}{f}-\frac{\int e^{-\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}\log\left(\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{\sqrt{h}e+\sqrt{he^2+2f^2}}+1\right)de^{\operatorname{arcsinh}\left(\right.}}{f}\right.}{\sqrt{h}}$$

$\big\downarrow$ 2838

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bfpq\left(\frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{he^2+2f^2}}\right)}{f}+\frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{\sqrt{h}e+\sqrt{he^2+2f^2}}\right)}{f}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}fe^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}}{e\sqrt{h}-\sqrt{e^2h+2f^2}}+1\right)}{f}+\frac{\mathrm{ar}}{}\right.}{\sqrt{h}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[2 + h*x^2],x]
```

output
```
(ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/Sqrt[h] -
(b*f*p*q*(-1/2*ArcSinh[(Sqrt[h]*x)/Sqrt[2]]^2/f + (ArcSinh[(Sqrt[h]*x)/Sqr
t[2]]*Log[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] - Sqrt
[2*f^2 + e^2*h])])/f + (ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*Log[1 + (Sqrt[2]*E^Ar
cSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h])])/f + Poly
Log[2, -((Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] - Sqrt[2*f^
2 + e^2*h]))]/f + PolyLog[2, -((Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/
(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h]))]/f))/Sqrt[h]
```

3.519. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2+hx^2}}\,dx$

3.519.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.519. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2+hx^2}}\,dx$

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.519.4 Maple [F]

$$\int \frac{a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}{\sqrt{h\,x^{2}+2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x)
```

3.519.5 Fricas [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)}{\sqrt{2+hx^{2}}}\,dx=\int \frac{b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}{\sqrt{hx^{2}+2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x^2 + 2)*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x^2 + 2)*a)/
(h*x^2 + 2), x)
```

3.519.6 Sympy [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)}{\sqrt{2+hx^{2}}}\,dx=\int \frac{a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)}{\sqrt{hx^{2}+2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x**2+2)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(h*x**2 + 2), x)
```

3.519.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}{\sqrt{hx^2+2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x, algorithm="maxim
a")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/sqrt(h*x^2 + 2), x)
 + a*arcsinh(1/2*sqrt(2)*sqrt(h)*x)/sqrt(h)
```

3.519.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}{\sqrt{hx^2+2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x^2 + 2), x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)}{\sqrt{h\,x^2+2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(h*x^2 + 2)^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(h*x^2 + 2)^(1/2), x)
```

3.520 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx^2}}\,dx$

3.520.1 Optimal result

Integrand size = 30, antiderivative size = 515

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx &= \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\,\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\,\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\,\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &+ \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\,\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\,\mathrm{PolyLog}\left(2,-\frac{e^{\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\,\mathrm{PolyLog}\left(2,-\frac{e^{\mathrm{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}}\end{aligned}$$

output
```
1/2*b*p*q*arcsinh(x*h^(1/2)/g^(1/2))^2*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(
h*x^2+g)^(1/2)+arcsinh(x*h^(1/2)/g^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))*g^(1
/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)^(1/2)-b*p*q*arcsinh(x*h^(1/2)/g^(1
/2))*ln(1+(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(1/2)/(e*h^(1/2)-(e^2*
h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)^(1/2)-b*p*q*a
rcsinh(x*h^(1/2)/g^(1/2))*ln(1+(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(
1/2)/(e*h^(1/2)+(e^2*h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h
*x^2+g)^(1/2)-b*p*q*polylog(2,-(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(
1/2)/(e*h^(1/2)-(e^2*h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h
*x^2+g)^(1/2)-b*p*q*polylog(2,-(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(
1/2)/(e*h^(1/2)+(e^2*h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h
*x^2+g)^(1/2)
```

3.520.2 Mathematica [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x^2],x]
```

output
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x^2], x]
```

3.520.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2895, 2853, 2851, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx$$
$$\Big\downarrow \; 2895$$
$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}dx$$

$$\Big\downarrow \; 2853$$

$$\frac{\sqrt{\frac{hx^2}{g}+1}\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{\frac{hx^2}{g}+1}}dx}{\sqrt{g+hx^2}}$$

$$\Big\downarrow \; 2851$$

$$\frac{\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-bfpq\int \frac{\sqrt{g}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{\sqrt{h}(e+fx)}dx\right)}{\sqrt{g+hx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bf\sqrt{g}pq\int \frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{e+fx}dx}{\sqrt{h}}\right)}{\sqrt{g+hx^2}}$$

$$\Big\downarrow \; 6242$$

$$\frac{\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bf\sqrt{g}pq\int \frac{\sqrt{\frac{hx^2}{g}+1}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{\frac{\sqrt{h}e}{\sqrt{g}}+\frac{f\sqrt{h}x}{\sqrt{g}}}d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{\sqrt{h}}\right)}{\sqrt{g+hx^2}}$$

$$\Big\downarrow \; 6095$$

$$\frac{\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bf\sqrt{g}pq\left(\int \frac{e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{\frac{\sqrt{h}e}{\sqrt{g}}+e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f-\frac{\sqrt{he^2+f^2g}}{\sqrt{g}}}d\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)+\int \frac{e^{\text{arcsinh}(}}{\frac{\sqrt{h}e}{\sqrt{g}}+e^{\text{arc}}}\right.}{\sqrt{h}}\right.}{\sqrt{g+hx^2}}$$

$$\Big\downarrow \; 2620$$

3.520. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx^2}}\,dx$

$$\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bf\sqrt{g}pq\left(-\frac{\int\log\left(\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}\sqrt{g}f}{e\sqrt{h}-\sqrt{he^2+f^2g}}+1\right)d\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{f}-\frac{\int\log\left(\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{\cdot}}{\cdot}\right.}}{\sqrt{h}e+\sqrt{he\cdots}}\right.}{\cdots}\right.}{\cdots}\right.$$

$\sqrt{\cdots}$

$\downarrow$ 2715

$$\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bf\sqrt{g}pq\left(-\frac{\int e^{-\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}\log\left(\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}\sqrt{g}f}{e\sqrt{h}-\sqrt{he^2+f^2g}}+1\right)de^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}}{f}-\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2838

$$\sqrt{\frac{hx^2}{g}+1}\left(\frac{\sqrt{g}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}-\frac{bf\sqrt{g}pq\left(\frac{\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{he^2+f^2g}}\right)}{f}+\frac{\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{\sqrt{h}e+\sqrt{he^2+f^2g}}\right)}{f}\cdots\right.}{\cdots}\right.$$

$$\sqrt{g+hx^2}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x^2],x]
```

3.520. $\int\frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx^2}}\,dx$

output
```
(Sqrt[1 + (h*x^2)/g]*((Sqrt[g]*ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*(a + b*Log[c*(
d*(e + f*x)^p)^q]))/Sqrt[h] - (b*f*Sqrt[g]*p*q*(-1/2*ArcSinh[(Sqrt[h]*x)/S
qrt[g]]^2/f + (ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*Log[1 + (E^ArcSinh[(Sqrt[h]*x)
/Sqrt[g]]*f*Sqrt[g])/(e*Sqrt[h] - Sqrt[f^2*g + e^2*h])])/f + (ArcSinh[(Sqr
t[h]*x)/Sqrt[g]]*Log[1 + (E^ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*f*Sqrt[g])/(e*Sqr
t[h] + Sqrt[f^2*g + e^2*h])])/f + PolyLog[2, -((E^ArcSinh[(Sqrt[h]*x)/Sqrt
[g]]*f*Sqrt[g])/(e*Sqrt[h] - Sqrt[f^2*g + e^2*h]))]/f + PolyLog[2, -((E^Ar
cSinh[(Sqrt[h]*x)/Sqrt[g]]*f*Sqrt[g])/(e*Sqrt[h] + Sqrt[f^2*g + e^2*h]))]/
f))/Sqrt[h]))/Sqrt[g + h*x^2]
```

3.520.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 2853
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> Simp[Sqrt[1 + (g/f)*x^2]/Sqrt[f + g*x^2]   Int[(a + b
*Log[c*(d + e*x)^n])/Sqrt[1 + (g/f)*x^2], x], x] /; FreeQ[{a, b, c, d, e, f
, g, n}, x] &&  !GtQ[f, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.520.4 Maple [F]

$$\int \frac{a+b\ln \left(c{\left(d{\left(fx+e\right)}^{p}\right)}^{q}\right)}{\sqrt{h\,x^{2}+g}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x)
```

3.520.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}{\sqrt{hx^2+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x^2 + g)*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x^2 + g)*a)/
(h*x^2 + g), x)
```

3.520.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x**2+g)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(g + h*x**2), x)
```

3.520.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}{\sqrt{hx^2+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x, algorithm="maxim
a")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/sqrt(h*x^2 + g), x)
 + a*arcsinh(h*x/sqrt(g*h))/sqrt(h)
```

3.520.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}{\sqrt{hx^2+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x^2 + g), x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{a+b\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\sqrt{h\,x^2+g}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2)^(1/2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2)^(1/2), x)
```

3.521 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx$

3.521.1 Optimal result

Integrand size = 38, antiderivative size = 287

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx &= \frac{ibpq\arcsin\left(\frac{hx}{2}\right)^2}{2h}\\
&-\frac{bpq\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h}\\
&-\frac{bpq\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h}\\
&+\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}\\
&+\frac{ibpq\,\mathrm{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h}\\
&+\frac{ibpq\,\mathrm{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h}\end{aligned}$$

output
```
1/2*I*b*p*q*arcsin(1/2*h*x)^2/h+arcsin(1/2*h*x)*(a+b*ln(c*(d*(f*x+e)^p)^q)
)/h-b*p*q*arcsin(1/2*h*x)*ln(1+2*(1/2*I*h*x+1/2*(-h^2*x^2+4)^(1/2))*f/(I*e
*h-(-e^2*h^2+4*f^2)^(1/2)))/h-b*p*q*arcsin(1/2*h*x)*ln(1+2*(1/2*I*h*x+1/2*
(-h^2*x^2+4)^(1/2))*f/(I*e*h+(-e^2*h^2+4*f^2)^(1/2)))/h+I*b*p*q*polylog(2,
-2*(1/2*I*h*x+1/2*(-h^2*x^2+4)^(1/2))*f/(I*e*h-(-e^2*h^2+4*f^2)^(1/2)))/h+
I*b*p*q*polylog(2,-2*(1/2*I*h*x+1/2*(-h^2*x^2+4)^(1/2))*f/(I*e*h+(-e^2*h^2
+4*f^2)^(1/2)))/h
```

3.521.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx = \frac{\arcsin\left(\frac{hx}{2}\right)\left(2a+ibpq\arcsin\left(\frac{hx}{2}\right)-2bpq\log\left(1-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{-ieh+\sqrt{4f^2-e^2h^2}}\right)-2bpq\log\left(1+\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)+2b}{2h}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[2 - h*x]*Sqrt[2 + h*x]),x
]
```

output
```
(ArcSin[(h*x)/2]*(2*a + I*b*p*q*ArcSin[(h*x)/2] - 2*b*p*q*Log[1 - (2*E^(I*
ArcSin[(h*x)/2])*f)/((-I)*e*h + Sqrt[4*f^2 - e^2*h^2])] - 2*b*p*q*Log[1 +
(2*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h + Sqrt[4*f^2 - e^2*h^2])] + 2*b*Log[c*(
d*(e + f*x)^p)^q]) + (2*I)*b*p*q*PolyLog[2, (2*E^(I*ArcSin[(h*x)/2])*f)/((
-I)*e*h + Sqrt[4*f^2 - e^2*h^2])] + (2*I)*b*p*q*PolyLog[2, (-2*E^(I*ArcSin
[(h*x)/2])*f)/(I*e*h + Sqrt[4*f^2 - e^2*h^2])])/(2*h)
```

3.521.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {2895, 2852, 27, 5240, 5032, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2-hx}\sqrt{hx+2}}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2-hx}\sqrt{hx+2}}dx$$

$$\Big\downarrow \text{2852}$$

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{h}-bfpq\int \frac{\arcsin\left(\frac{hx}{2}\right)}{h(e+fx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\int\frac{\arcsin\left(\frac{hx}{2}\right)}{e+fx}dx}{h}$$

$\Big\downarrow$ 5240

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\int\frac{\sqrt{1-\frac{h^2x^2}{4}}\arcsin\left(\frac{hx}{2}\right)}{\frac{eh}{2}+\frac{fxh}{2}}d\arcsin\left(\frac{hx}{2}\right)}{h}$$

$\Big\downarrow$ 5032

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\left(i\int\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}\arcsin\left(\frac{hx}{2}\right)}{2e^{i\arcsin\left(\frac{hx}{2}\right)}f+ieh-\sqrt{4f^2-e^2h^2}}d\arcsin\left(\frac{hx}{2}\right)+i\int\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}\arcsin\left(\frac{hx}{2}\right)}{2e^{i\arcsin\left(\frac{hx}{2}\right)}f+ieh+\sqrt{4f^2-e^2h^2}}d\arcsin\left(\frac{hx}{2}\right)-\frac{i\arcsin\left(\frac{hx}{2}\right)^2}{2f}\right)}{h}$$

$\Big\downarrow$ 27

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\left(2i\int\frac{e^{i\arcsin\left(\frac{hx}{2}\right)}\arcsin\left(\frac{hx}{2}\right)}{2e^{i\arcsin\left(\frac{hx}{2}\right)}f+ieh-\sqrt{4f^2-e^2h^2}}d\arcsin\left(\frac{hx}{2}\right)+2i\int\frac{e^{i\arcsin\left(\frac{hx}{2}\right)}\arcsin\left(\frac{hx}{2}\right)}{2e^{i\arcsin\left(\frac{hx}{2}\right)}f+ieh+\sqrt{4f^2-e^2h^2}}d\arcsin\left(\frac{hx}{2}\right)-\frac{i\arcsin\left(\frac{hx}{2}\right)^2}{2f}\right)}{h}$$

$\Big\downarrow$ 2620

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\left(2i\left(\frac{i\int\log\left(\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}+1\right)d\arcsin\left(\frac{hx}{2}\right)}{2f}-\frac{i\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2fe^{i\arcsin\left(\frac{hx}{2}\right)}}{-\sqrt{4f^2-e^2h^2}+ieh}\right)}{2f}\right)+2i\left(\frac{i\int\log\left(\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}+1\right)d}{2f}\right)\right)}{h}$$

$\Big\downarrow$ 2715

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\left(2i\left(\frac{\int e^{-i\arcsin\left(\frac{hx}{2}\right)}\log\left(\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}+1\right)de^{i\arcsin\left(\frac{hx}{2}\right)}}{2f}-\frac{i\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2fe^{i\arcsin\left(\frac{hx}{2}\right)}}{-\sqrt{4f^2-e^2h^2}+ieh}\right)}{2f}\right)+2i\left(\frac{\int e^{-i\arcsin\left(\frac{hx}{2}\right)}}{}\right)\right)}{h}$$

$\Big\downarrow$ 2838

3.521. $\int\frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2-hx}\sqrt{2+hx}}dx$

$$\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-$$

$$\frac{bfpq\left(2i\left(-\frac{\text{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{2f}-\frac{i\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2fe^{i\arcsin\left(\frac{hx}{2}\right)}}{-\sqrt{4f^2-e^2h^2}+ieh}\right)}{2f}\right)+2i\left(-\frac{\text{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{2f}-\right.\right.}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[2 - h*x]*Sqrt[2 + h*x]),x]
```

output
```
(ArcSin[(h*x)/2]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/h - (b*f*p*q*(((-1/2*I)
*ArcSin[(h*x)/2]^2)/f + (2*I)*(((-1/2*I)*ArcSin[(h*x)/2]*Log[1 + (2*E^(I*A
rcSin[(h*x)/2])*f)/(I*e*h - Sqrt[4*f^2 - e^2*h^2])]])/f - PolyLog[2, (-2*E^
(I*ArcSin[(h*x)/2])*f)/(I*e*h - Sqrt[4*f^2 - e^2*h^2])]/(2*f)) + (2*I)*(((
-1/2*I)*ArcSin[(h*x)/2]*Log[1 + (2*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h + Sqrt[
4*f^2 - e^2*h^2])]])/f - PolyLog[2, (-2*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h + S
qrt[4*f^2 - e^2*h^2])]/(2*f))))/h
```

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2852
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> With[{u = IntHide[1/Sqrt[
f1*f2 + g1*g2*x^2], x]}, Simp[u*(a + b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n
   Int[SimplifyIntegrand[u/(d + e*x), x], x], x]] /; FreeQ[{a, b, c, d, e,
f1, g1, f2, g2, n}, x] && EqQ[f2*g1 + f1*g2, 0] && GtQ[f1, 0] && GtQ[f2, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 5032
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2,
2] + b*E^(I*(c + d*x)))), x], x] + Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x)
)/(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

rule 5240
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.521.4 Maple [F]

$$\int \frac{a+b\ln \left(c \left(d \left(f x+e\right)^{p}\right)^{q}\right)}{\sqrt{-h x+2}\, \sqrt{h x+2}}d x$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x)
```

3.521.5 Fricas [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{\sqrt{2 - hx}\sqrt{2 + hx}}\,dx = \int \frac{b\log\left(\left(((fx + e)^p d)^q c\right) + a\right.}{\sqrt{hx + 2}\sqrt{-hx + 2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x, alg
orithm="fricas")
```

output
```
integral(-(sqrt(h*x + 2)*sqrt(-h*x + 2)*b*log(((f*x + e)^p*d)^q*c) + sqrt(
h*x + 2)*sqrt(-h*x + 2)*a)/(h^2*x^2 - 4), x)
```

3.521.6 Sympy [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{\sqrt{2 - hx}\sqrt{2 + hx}}\,dx = \int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{\sqrt{-hx + 2}\sqrt{hx + 2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(-h*x+2)**(1/2)/(h*x+2)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(sqrt(-h*x + 2)*sqrt(h*x + 2))
, x)
```

3.521.7 Maxima [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{\sqrt{2 - hx}\sqrt{2 + hx}}\,dx = \int \frac{b\log\left(\left(((fx + e)^p d)^q c\right) + a\right.}{\sqrt{hx + 2}\sqrt{-hx + 2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x, alg
orithm="maxima")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(sqrt(h*x + 2)*sqrt
(-h*x + 2)), x) + a*arcsin(1/2*h*x)/h
```

3.521.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}{\sqrt{hx+2}\sqrt{-hx+2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(sqrt(h*x + 2)*sqrt(-h*x + 2)),
 x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\int \frac{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\sqrt{2-h\,x}\,\sqrt{h\,x+2}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((2 - h*x)^(1/2)*(h*x + 2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((2 - h*x)^(1/2)*(h*x + 2)^(1/2)), x)
```

3.522 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$

3.522.1 Optimal result

Integrand size = 38, antiderivative size = 519

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx &= \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}} \\ &\quad - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} \\ &\quad - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} \\ &\quad + \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}} \\ &\quad + \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} \\ &\quad + \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}\end{aligned}$$

output
```
1/2*I*b*g*p*q*arcsin(h*x/g)^2*(1-h^2*x^2/g^2)^(1/2)/h/(-h*x+g)^(1/2)/(h*x+
g)^(1/2)+g*arcsin(h*x/g)*(a+b*ln(c*(d*(f*x+e)^p)^q))*(1-h^2*x^2/g^2)^(1/2)
/h/(-h*x+g)^(1/2)/(h*x+g)^(1/2)-b*g*p*q*arcsin(h*x/g)*ln(1+(I*h*x/g+(1-h^2
*x^2/g^2)^(1/2))*f*g/(I*e*h-(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/
2)/h/(-h*x+g)^(1/2)/(h*x+g)^(1/2)-b*g*p*q*arcsin(h*x/g)*ln(1+(I*h*x/g+(1-h
^2*x^2/g^2)^(1/2))*f*g/(I*e*h+(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(
1/2)/h/(-h*x+g)^(1/2)/(h*x+g)^(1/2)+I*b*g*p*q*polylog(2,-(I*h*x/g+(1-h^2*x
^2/g^2)^(1/2))*f*g/(I*e*h-(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/2)
/h/(-h*x+g)^(1/2)/(h*x+g)^(1/2)+I*b*g*p*q*polylog(2,-(I*h*x/g+(1-h^2*x^2/g
^2)^(1/2))*f*g/(I*e*h+(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/2)/h/(
-h*x+g)^(1/2)/(h*x+g)^(1/2)
```

3.522.2 Mathematica [A] (warning: unable to verify)

Time = 10.98 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.23

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$$
$$= \frac{\arctan\left(\frac{hx}{\sqrt{g-hx}\sqrt{g+hx}}\right)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}$$
$$- \frac{bpq\sqrt{g-hx}\left(2gh(e+fx)\sqrt{\frac{g+hx}{g-hx}}\arctan\left(\frac{1}{\sqrt{\frac{g+hx}{g-hx}}}\right)\log(e+fx)+(g+hx)\left(eh+fg\cos\left(2\arctan\left(\frac{}{\sqrt{}}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[g - h*x]*Sqrt[g + h*x]),x
]
```

output
```
(ArcTan[(h*x)/(Sqrt[g - h*x]*Sqrt[g + h*x])]*(a - b*p*q*Log[e + f*x] + b*L
og[c*(d*(e + f*x)^p)^q]))/h - (b*p*q*Sqrt[g - h*x]*(2*g*h*(e + f*x)*Sqrt[(
g + h*x)/(g - h*x)]*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]]*Log[e + f*x] + (g
+ h*x)*(e*h + f*g*Cos[2*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]]])*Csc[2*ArcTan
[1/Sqrt[(g + h*x)/(g - h*x)]]]*((2*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]]^
2 - (4*I)*ArcSin[Sqrt[1 + (e*h)/(f*g)]/Sqrt[2]]*ArcTan[(-(f*g) + e*h)/(Sqr
t[-(f^2*g^2) + e^2*h^2]*Sqrt[(g + h*x)/(g - h*x)])] - 2*(ArcSin[Sqrt[1 + (
e*h)/(f*g)]/Sqrt[2]] + ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*Log[1 + (E^((2
*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*(e*h - Sqrt[-(f^2*g^2) + e^2*h^2]
))/(f*g)] + 2*(ArcSin[Sqrt[1 + (e*h)/(f*g)]/Sqrt[2]] - ArcTan[1/Sqrt[(g +
h*x)/(g - h*x)]])*Log[1 + (E^((2*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*(
e*h + Sqrt[-(f^2*g^2) + e^2*h^2]))/(f*g)] + I*(PolyLog[2, (E^((2*I)*ArcTan
[1/Sqrt[(g + h*x)/(g - h*x)]])*(-(e*h) + Sqrt[-(f^2*g^2) + e^2*h^2]))/(f*g
)] + PolyLog[2, -((E^((2*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*(e*h + Sq
rt[-(f^2*g^2) + e^2*h^2]))/(f*g))])))/(g*h^2*(e + f*x)*Sqrt[g + h*x])
```

3.522.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.66, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {2895, 2854, 2851, 27, 5240, 5032, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2854}$$

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{1-\frac{h^2x^2}{g^2}}}dx}{\sqrt{g-hx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{2851}$$

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-bfpq\int \frac{g\arcsin\left(\frac{hx}{g}\right)}{h(e+fx)}dx\right)}{\sqrt{g-hx}\sqrt{g+hx}}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfgpq\int\frac{\arcsin\left(\frac{hx}{g}\right)}{e+fx}dx}{h}\right)}{\sqrt{g-hx}\sqrt{g+hx}}$$

$\downarrow$ 5240

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfgpq\int\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)}{\frac{fxh}{g}+\frac{eh}{g}}d\arcsin\left(\frac{hx}{g}\right)}{h}\right)}{\sqrt{g-hx}\sqrt{g+hx}}$$

$\downarrow$ 5032

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfgpq\left(i\int\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}\arcsin\left(\frac{hx}{g}\right)}{e^{i\arcsin\left(\frac{hx}{g}\right)}f+\frac{ieh-\sqrt{f^2g^2-e^2h^2}}{g}}d\arcsin\left(\frac{hx}{g}\right)+i\int\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}\arcsin}{e^{i\arcsin\left(\frac{hx}{g}\right)}f+\frac{ieh+\sqrt{}}{}}\right.}{h}\right.}{\sqrt{g-hx}\sqrt{g+hx}}$$

$\downarrow$ 2620

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfgpq\left(i\left(\left(\frac{i\int\log\left(\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}+1\right)d\arcsin\left(\frac{hx}{g}\right)}{f}-\frac{i\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{fge^{i\arcsin(}}{-\sqrt{f^2g^2-e^2h}}\right.}{f}\right.\right.\right.}{}\right.}{\sqrt{g-hx}\sqrt{g+hx}}$$

$\downarrow$ 2715

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfgpq\left(i\left(\left(\frac{\int e^{-i\arcsin\left(\frac{hx}{g}\right)}\log\left(\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}+1\right)de^{i\arcsin\left(\frac{hx}{g}\right)}}{f}-\frac{i\arcsin\left(\frac{hx}{g}\right)\log\left(\right.}{}\right.\right.\right.}{}\right.}{\sqrt{g-h}}$$

3.522. $\int\frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g-hx}\sqrt{g+hx}}dx$

$$\Big\downarrow \textbf{2838}$$

$$\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\left(\frac{g\arcsin\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}-\frac{bfgpq\left(i\left(-\frac{\text{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{f}-\frac{i\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{fge^{i\arcsin\left(\frac{hx}{g}\right)}}{-\sqrt{f^2g^2-e^2h^2}+ieh}\right)}{f}\right.\right.}{}\right.}{\sqrt{g-hx}\sqrt{g+hx}}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[g - h*x]*Sqrt[g + h*x]),x]
```

output
```
(Sqrt[1 - (h^2*x^2)/g^2]*((g*ArcSin[(h*x)/g]*(a + b*Log[c*(d*(e + f*x)^p)^
q]))/h - (b*f*g*p*q*(((-1/2*I)*ArcSin[(h*x)/g]^2)/f + I*(((-I)*ArcSin[(h*x
)/g]*Log[1 + (E^(I*ArcSin[(h*x)/g])*f*g)/(I*e*h - Sqrt[f^2*g^2 - e^2*h^2])
])/f - PolyLog[2, -((E^(I*ArcSin[(h*x)/g])*f*g)/(I*e*h - Sqrt[f^2*g^2 - e^
2*h^2]))]/f) + I*(((-I)*ArcSin[(h*x)/g]*Log[1 + (E^(I*ArcSin[(h*x)/g])*f*g
)/(I*e*h + Sqrt[f^2*g^2 - e^2*h^2])])/f - PolyLog[2, -((E^(I*ArcSin[(h*x)/
g])*f*g)/(I*e*h + Sqrt[f^2*g^2 - e^2*h^2]))]/f)))/h))/(Sqrt[g - h*x]*Sqrt[
g + h*x])
```

3.522.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 2854
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> Simp[Sqrt[1 + g1*(g2/(f1*
f2))*x^2]/(Sqrt[f1 + g1*x]*Sqrt[f2 + g2*x])   Int[(a + b*Log[c*(d + e*x)^n]
)/Sqrt[1 + g1*(g2/(f1*f2))*x^2], x], x] /; FreeQ[{a, b, c, d, e, f1, g1, f2
, g2, n}, x] && EqQ[f2*g1 + f1*g2, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 5032
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2,
2] + b*E^(I*(c + d*x)))), x], x] + Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x)
)/(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

rule 5240
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.522. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$

3.522.4 Maple [F]

$$\int \frac{a+b\ln\left(c(d(fx+e)^{p})^{q}\right)}{\sqrt{-hx+g}\,\sqrt{hx+g}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x)
```

3.522.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{b\log\left(((fx+e)^{p}d)^{q}\,c\right)+a}{\sqrt{hx+g}\sqrt{-hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
integral(-(sqrt(h*x + g)*sqrt(-h*x + g)*b*log(((f*x + e)^p*d)^q*c) + sqrt(
h*x + g)*sqrt(-h*x + g)*a)/(h^2*x^2 - g^2), x)
```

3.522.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(-h*x+g)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(sqrt(g - h*x)*sqrt(g + h*x)),
 x)
```

3.522.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a}{\sqrt{hx+g}\sqrt{-hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(sqrt(h*x + g)*sqrt
(-h*x + g)), x) + a*arcsin(h*x/g)/h
```

3.522.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a}{\sqrt{hx+g}\sqrt{-hx+g}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(sqrt(h*x + g)*sqrt(-h*x + g)),
 x)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx=\int \frac{a+b\,\ln\left(c\left(d\,(e+f\,x)^p\right)^q\right)}{\sqrt{g+h\,x}\,\sqrt{g-h\,x}}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)^(1/2)*(g - h*x)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)^(1/2)*(g - h*x)^(1/2)), x)
```

3.523 $\int \frac{(i+jx)^3(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$

3.523.1 Optimal result

Integrand size = 33, antiderivative size = 427

$$\begin{aligned}
\int \frac{(i+jx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx
&= \frac{aj(hi-gj)^2x}{h^3}-\frac{bj(fi-ej)^2pqx}{3f^2h}-\frac{bj(fi-ej)(hi-gj)pqx}{2fh^2}-\frac{bj(hi-gj)^2pqx}{h^3}\\
&-\frac{b(fi-ej)pq(i+jx)^2}{6fh}-\frac{b(hi-gj)pq(i+jx)^2}{4h^2}-\frac{bpq(i+jx)^3}{9h}\\
&-\frac{b(fi-ej)^3pq\log(e+fx)}{3f^3h}-\frac{b(fi-ej)^2(hi-gj)pq\log(e+fx)}{2f^2h^2}\\
&+\frac{bj(hi-gj)^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^3}\\
&+\frac{(hi-gj)(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h^2}+\frac{(i+jx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h}\\
&+\frac{(hi-gj)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^4}\\
&+\frac{b(hi-gj)^3pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^4}
\end{aligned}$$

output
```
a*j*(-g*j+h*i)^2*x/h^3-1/3*b*j*(-e*j+f*i)^2*p*q*x/f^2/h-1/2*b*j*(-e*j+f*i)
*(-g*j+h*i)*p*q*x/f/h^2-b*j*(-g*j+h*i)^2*p*q*x/h^3-1/6*b*(-e*j+f*i)*p*q*(j
*x+i)^2/f/h-1/4*b*(-g*j+h*i)*p*q*(j*x+i)^2/h^2-1/9*b*p*q*(j*x+i)^3/h-1/3*b
*(-e*j+f*i)^3*p*q*ln(f*x+e)/f^3/h-1/2*b*(-e*j+f*i)^2*(-g*j+h*i)*p*q*ln(f*x
+e)/f^2/h^2+b*j*(-g*j+h*i)^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^3+1/2*(-g*j
+h*i)*(j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/h^2+1/3*(j*x+i)^3*(a+b*ln(c*(d
*(f*x+e)^p)^q))/h+(-g*j+h*i)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-
e*h+f*g))/h^4+b*(-g*j+h*i)^3*p*q*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^4
```

3.523.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.90

$$\int \frac{(i+jx)^3 (a+b\log (c(d(e+fx)^p)^q))}{g+hx}\,dx$$
$$= \frac{6be^2h^2j^2(-9fhi+3fgj+2ehj)pq\log(e+fx)+f\Big(hjx(6af^2(6g^2j^2-3ghj(6i+jx)+h^2(18i^2+9ijx+}{}$$

input
```
Integrate[((i + j*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

output
```
(6*b*e^2*h^2*j^2*(-9*f*h*i + 3*f*g*j + 2*e*h*j)*p*q*Log[e + f*x] + f*(h*j*
x*(6*a*f^2*(6*g^2*j^2 - 3*g*h*j*(6*i + j*x) + h^2*(18*i^2 + 9*i*j*x + 2*j^
2*x^2)) - b*p*q*(12*e^2*h^2*j^2 - 6*e*f*h*j*(9*h*i - 3*g*j + h*j*x) + f^2*
(36*g^2*j^2 - 9*g*h*j*(12*i + j*x) + h^2*(108*i^2 + 27*i*j*x + 4*j^2*x^2))
)) + 36*a*f^2*(h*i - g*j)^3*Log[(f*(g + h*x))/(f*g - e*h)] + 6*b*f*Log[c*(
d*(e + f*x)^p)^q]*(h*j*(6*e*(3*h^2*i^2 - 3*g*h*i*j + g^2*j^2) + f*x*(6*g^2
*j^2 - 3*g*h*j*(6*i + j*x) + h^2*(18*i^2 + 9*i*j*x + 2*j^2*x^2))) + 6*f*(h
*i - g*j)^3*Log[(f*(g + h*x))/(f*g - e*h)])) + 36*b*f^3*(h*i - g*j)^3*p*q*
PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/(36*f^3*h^4)
```

3.523.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.523. $\int \frac{(i+jx)^3(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx} dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx} dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{(hi-gj)^3 (a+b\log(c(d(e+fx)^p)^q))}{h^3(g+hx)} + \frac{j(hi-gj)^2 (a+b\log(c(d(e+fx)^p)^q))}{h^3} + \frac{j(i+jx)(hi-gj) (a+b\log(c(d(e+fx)^p)^q))}{h^2}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{(hi-gj)^3 \log\left(\frac{f(g+hx)}{fg-eh}\right) (a+b\log(c(d(e+fx)^p)^q))}{h^4} + \frac{(i+jx)^2(hi-gj) (a+b\log(c(d(e+fx)^p)^q))}{2h^2} + \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{3h} + \frac{ajx(hi-gj)^2}{h^3} + \frac{bj(e+fx)(hi-gj)^2 \log(c(d(e+fx)^p)^q)}{fh^3} - \frac{bpq(fi-ej)^3 \log(e+fx)}{3f^3h} - \frac{bpq(fi-ej)^2 \log(e+fx)(hi-gj)}{2f^2h^2} - \frac{bjpqx(fi-ej)^2}{3f^2h} + \frac{bpq(hi-gj)^3 \text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^4} - \frac{bjpqx(fi-ej)(hi-gj)}{2fh^2} - \frac{bpq(i+jx)^2(fi-ej)}{6fh} - \frac{bjpqx(hi-gj)^2}{h^3} - \frac{bpq(i+jx)^2(hi-gj)}{4h^2} - \frac{bpq(i+jx)^3}{9h}$$

input
```
Int[((i + j*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

output
```
(a*j*(h*i - g*j)^2*x)/h^3 - (b*j*(f*i - e*j)^2*p*q*x)/(3*f^2*h) - (b*j*(f*
i - e*j)*(h*i - g*j)*p*q*x)/(2*f*h^2) - (b*j*(h*i - g*j)^2*p*q*x)/h^3 - (b
*(f*i - e*j)*p*q*(i + j*x)^2)/(6*f*h) - (b*(h*i - g*j)*p*q*(i + j*x)^2)/(4
*h^2) - (b*p*q*(i + j*x)^3)/(9*h) - (b*(f*i - e*j)^3*p*q*Log[e + f*x])/(3*
f^3*h) - (b*(f*i - e*j)^2*(h*i - g*j)*p*q*Log[e + f*x])/(2*f^2*h^2) + (b*j
*(h*i - g*j)^2*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h^3) + ((h*i - g*j)*
(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(2*h^2) + ((i + j*x)^3*(a +
b*Log[c*(d*(e + f*x)^p)^q]))/(3*h) + ((h*i - g*j)^3*(a + b*Log[c*(d*(e + f
*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h^4 + (b*(h*i - g*j)^3*p*q*Poly
Log[2, -((h*(e + f*x))/(f*g - e*h))])/h^4
```

3.523. $\int \frac{(i+jx)^3(a+b\log(c(d(e+fx)^p)^q))}{g+hx} dx$

3.523.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.523.4 Maple [F]

$$\int \frac{(jx+i)^3 (a+b\ln (c(d(fx+e)^p)^q))}{hx+g}dx$$

input
```
int((j*x+i)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

output
```
int((j*x+i)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

3.523.5 Fricas [F]

$$\int \frac{(i+jx)^3 (a+b\log (c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^3(b\log (((fx+e)^p d)^q c)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)^3*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fri
cas")
```

output
```
integral((a*j^3*x^3 + 3*a*i*j^2*x^2 + 3*a*i^2*j*x + a*i^3 + (b*j^3*x^3 + 3
*b*i*j^2*x^2 + 3*b*i^2*j*x + b*i^3)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x
)
```

3.523.6 Sympy [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))(i+jx)^3}{g+hx}\,dx$$

input
```
integrate((j*x+i)**3*(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(i + j*x)**3/(g + h*x), x)
```

3.523.7 Maxima [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^3(b\log(((fx+e)^p d)^q c)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)^3*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="max
ima")
```

output
```
3*a*i^2*j*(x/h - g*log(h*x + g)/h^2) - 1/6*a*j^3*(6*g^3*log(h*x + g)/h^4 -
 (2*h^2*x^3 - 3*g*h*x^2 + 6*g^2*x)/h^3) + 3/2*a*i*j^2*(2*g^2*log(h*x + g)/
h^3 + (h*x^2 - 2*g*x)/h^2) + a*i^3*log(h*x + g)/h + integrate(((j^3*q*log(
d) + j^3*log(c))*b*x^3 + 3*(i*j^2*q*log(d) + i*j^2*log(c))*b*x^2 + 3*(i^2*
j*q*log(d) + i^2*j*log(c))*b*x + (i^3*q*log(d) + i^3*log(c))*b + (b*j^3*x^
3 + 3*b*i*j^2*x^2 + 3*b*i^2*j*x + b*i^3)*log(((f*x + e)^p)^q))/(h*x + g),
x)
```

3.523.8 Giac [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^3(b\log(((fx+e)^p d)^q c)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)^3*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="gia
c")
```

output
```
integrate((j*x + i)^3*(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(i+j\,x)^3\,(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))}{g+h\,x}\,dx$$

input
```
int(((i + j*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x),x)
```

output
```
int(((i + j*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x), x)
```

3.524 $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$

3.524.1 Optimal result

Integrand size = 33, antiderivative size = 258

$$\begin{aligned}\int \frac{(i+jx)^2\,(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx &= \frac{aj(hi-gj)x}{h^2}-\frac{bj(fi-ej)pqx}{2fh}-\frac{bj(hi-gj)pqx}{h^2}-\frac{bpq(i+jx)^2}{4h}\\&\quad-\frac{b(fi-ej)^2pq\log(e+fx)}{2f^2h}+\frac{bj(hi-gj)(e+fx)\log(c(d(e+fx)^p)^q)}{fh^2}\\&\quad+\frac{(i+jx)^2\,(a+b\log(c(d(e+fx)^p)^q))}{2h}\\&\quad+\frac{(hi-gj)^2\,(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3}\\&\quad+\frac{b(hi-gj)^2pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}\end{aligned}$$

output
```
a*j*(-g*j+h*i)*x/h^2-1/2*b*j*(-e*j+f*i)*p*q*x/f/h-b*j*(-g*j+h*i)*p*q*x/h^2
-1/4*b*p*q*(j*x+i)^2/h-1/2*b*(-e*j+f*i)^2*p*q*ln(f*x+e)/f^2/h+b*j*(-g*j+h*
i)*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^2+1/2*(j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^
p)^q))/h+(-g*j+h*i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))
/h^3+b*(-g*j+h*i)^2*p*q*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^3
```

3.524.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.90

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))}{g+hx} dx$$
$$= \frac{-2be^2h^2j^2pq\log(e+fx)+f\Big(hjx(2af(4hi-2gj+hjx)+bpq(2ehj-f(8hi-4gj+hjx)))+4af(hi-}{}$$

input
```
Integrate[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

output
```
(-2*b*e^2*h^2*j^2*p*q*Log[e + f*x] + f*(h*j*x*(2*a*f*(4*h*i - 2*g*j + h*j*
x) + b*p*q*(2*e*h*j - f*(8*h*i - 4*g*j + h*j*x))) + 4*a*f*(h*i - g*j)^2*Lo
g[(f*(g + h*x))/(f*g - e*h)] + 2*b*Log[c*(d*(e + f*x)^p)^q]*(h*j*(e*(4*h*i
 - 2*g*j) + f*x*(4*h*i - 2*g*j + h*j*x)) + 2*f*(h*i - g*j)^2*Log[(f*(g + h
*x))/(f*g - e*h)])) + 4*b*f^2*(h*i - g*j)^2*p*q*PolyLog[2, (h*(e + f*x))/(
-(f*g) + e*h)])/(4*f^2*h^3)
```

3.524.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))}{g+hx} dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))}{g+hx}dx$$
$$\Big\downarrow \text{2865}$$
$$\int \bigg(\frac{(hi-gj)^2 (a+b\log (c(d(e+fx)^p)^q))}{h^2(g+hx)}+\frac{j(hi-gj) (a+b\log (c(d(e+fx)^p)^q))}{h^2}+\frac{j(i+jx) (a+b\log (c(d(e-}{h}$$
$$\Big\downarrow \text{2009}$$

$$\frac{(hi-gj)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^3}+\frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}+\frac{ajx(hi-gj)}{h^2}+\frac{bj(e+fx)(hi-gj)\log\left(c(d(e+fx)^p)^q\right)}{fh^2}-\frac{bpq(fi-ej)^2\log(e+fx)}{2f^2h}+\frac{bpq(hi-gj)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}-\frac{bjpqx(fi-ej)}{2fh}-\frac{bjpqx(hi-gj)}{h^2}-\frac{bpq(i+jx)^2}{4h}$$

input
```
Int[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

output
```
(a*j*(h*i - g*j)*x)/h^2 - (b*j*(f*i - e*j)*p*q*x)/(2*f*h) - (b*j*(h*i - g*
j)*p*q*x)/h^2 - (b*p*q*(i + j*x)^2)/(4*h) - (b*(f*i - e*j)^2*p*q*Log[e + f
*x])/(2*f^2*h) + (b*j*(h*i - g*j)*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h
^2) + ((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(2*h) + ((h*i - g*j)^
2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h^3 + (
b*(h*i - g*j)^2*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^3
```

3.524.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.524. $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$

3.524.4 Maple [F]

$$\int \frac{(jx+i)^2 \left(a+b\ln \left(c\left(d(fx+e)^p\right)^q\right)\right)}{hx+g}dx$$

input
```
int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

output
```
int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

3.524.5 Fricas [F]

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^2 (b\log \left(\left((fx+e)^p d\right)^q c\right)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fri
cas")
```

output
```
integral((a*j^2*x^2 + 2*a*i*j*x + a*i^2 + (b*j^2*x^2 + 2*b*i*j*x + b*i^2)*
log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

3.524.6 Sympy [F]

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(a+b\log (c(d(e+fx)^p)^q)) (i+jx)^2}{g+hx}\,dx$$

input
```
integrate((j*x+i)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(i + j*x)**2/(g + h*x), x)
```

3.524.7 Maxima [F]

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log(((fx+e)^p d)^q c)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="max
ima")
```

output
```
2*a*i*j*(x/h - g*log(h*x + g)/h^2) + 1/2*a*j^2*(2*g^2*log(h*x + g)/h^3 + (
h*x^2 - 2*g*x)/h^2) + a*i^2*log(h*x + g)/h + integrate(((j^2*q*log(d) + j^
2*log(c))*b*x^2 + 2*(i*j*q*log(d) + i*j*log(c))*b*x + (i^2*q*log(d) + i^2*
log(c))*b + (b*j^2*x^2 + 2*b*i*j*x + b*i^2)*log(((f*x + e)^p)^q))/(h*x + g
), x)
```

3.524.8 Giac [F]

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log(((fx+e)^p d)^q c)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="gia
c")
```

output
```
integrate((j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(i+j\,x)^2\,(a+b\ln(c\,(d\,(e+f\,x)^p)^q))}{g+h\,x}\,dx$$

input
```
int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x),x)
```

output
```
int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x), x)
```

3.525 $\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$

3.525.1 Optimal result

Integrand size = 31, antiderivative size = 129

$$\int \frac{(i+jx)\,(a+b\log\left(c(d(e+fx)^p)^q\right))}{g+hx}\,dx = \frac{ajx}{h}-\frac{bjpqx}{h}+\frac{bj(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh} + \frac{(hi-gj)\,(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{b(hi-gj)pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$

output
```
a*j*x/h-b*j*p*q*x/h+b*j*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h+(-g*j+h*i)*(a+b*
ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h^2+b*(-g*j+h*i)*p*q*polyl
og(2,-h*(f*x+e)/(-e*h+f*g))/h^2
```

3.525.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int \frac{(i+jx)\,(a+b\log\left(c(d(e+fx)^p)^q\right))}{g+hx}\,dx = \frac{ahjx-bhjpqx+\frac{bhj(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}+(hi-gj)\,(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)+b(hi-g}{h^2}$$

input
```
Integrate[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

output
```
(a*h*j*x - b*h*j*p*q*x + (b*h*j*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f + (h
*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)]
+ b*(h*i - g*j)*p*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/h^2
```

3.525.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h(g+hx)}+\frac{j\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(hi-gj)\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^2}+\frac{ajx}{h}+\frac{bj(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh}+\frac{bpq(hi-gj)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}-\frac{bjpqx}{h}$$

input
```
Int[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

output
```
(a*j*x)/h - (b*j*p*q*x)/h + (b*j*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h)
 + ((h*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g -
e*h)])/h^2 + (b*(h*i - g*j)*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/
h^2
```

3.525.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.525.4 Maple [F]

$$\int \frac{(jx+i)\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)}{hx+g}dx$$

input
```
int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

output
```
int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

3.525.5 Fricas [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)}{g+hx}\,dx=\int \frac{(jx+i)(b\log\left(((fx+e)^{p}d)^{q}c\right)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="frica
s")
```

output
```
integral((a*j*x + a*i + (b*j*x + b*i)*log(((f*x + e)^p*d)^q*c))/(h*x + g),
 x)
```

3.525.6 Sympy [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx = \int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)(i+jx)}{g+hx}\,dx$$

input
```
integrate((j*x+i)*(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(i + j*x)/(g + h*x), x)
```

3.525.7 Maxima [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx = \int \frac{(jx+i)(b\log\left(((fx+e)^pd)^qc\right)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxim
a")
```

output
```
a*j*(x/h - g*log(h*x + g)/h^2) + a*i*log(h*x + g)/h + integrate(((j*q*log(
d) + j*log(c))*b*x + (i*q*log(d) + i*log(c))*b + (b*j*x + b*i)*log(((f*x +
 e)^p)^q))/(h*x + g), x)
```

3.525.8 Giac [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx = \int \frac{(jx+i)(b\log\left(((fx+e)^pd)^qc\right)+a)}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac"
)
```

output
```
integrate((j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)\,(a+b\log\left(c(d(e+fx)^p)^q\right))}{g+hx}\,dx = \int \frac{(i+j\,x)\,\left(a+b\,\ln\left(c\,\left(d\,{(e+f\,x)}^p\right)^q\right)\right)}{g+h\,x}\,dx$$

input
```
int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x),x)
```

output
```
int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x), x)
```

3.526 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}\,dx$

3.526.1 Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h+b*p*q*polylog(2,-h*
(f*x+e)/(-e*h+f*g))/h
```

3.526.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\,\mathrm{PolyLog}\left(2,\frac{h(e+fx)}{-fg+eh}\right)}{h}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p
*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

3.526.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2895, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}dx$$

$$\Big\downarrow \text{2841}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bfpq\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2840}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}-\frac{bpq\int\frac{\log\left(\frac{h(e+fx)}{fg-eh}+1\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}+\frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p
*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h
```

3.526.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.526.4 Maple [F]

$$\int \frac{a + b\ln \left(c(d(fx+e)^{p})^{q}\right)}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

3.526.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.526.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x), x)
```

3.526.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxima")
```

output
```
b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*x + g), x) + a*l
og(h*x + g)/h
```

3.526.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx = \int \frac{b\log\left(\left(((fx+e)^pd)^q\,c\right)+a\right.}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx = \int \frac{a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x), x)
```

3.527 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)}\,dx$

3.527.1 Optimal result

Integrand size = 33, antiderivative size = 165

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx = \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

output

```
(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)-(a+b*ln(c*
(d*(f*x+e)^p)^q))*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)+b*p*q*polylog(2,-h*(
f*x+e)/(-e*h+f*g))/(-g*j+h*i)-b*p*q*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j
+h*i)
```

3.527.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.71

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx = \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\left(\log\left(\frac{f(g+hx)}{fg-eh}\right)-\log\left(\frac{f(i+jx)}{fi-ej}\right)\right)+bpq\operatorname{PolyLog}\left(2,\frac{h(e+fx)}{-fg+eh}\right)-bpq\operatorname{PolyLog}\left(2 \right.}{hi-gj}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*(Log[(f*(g + h*x))/(f*g - e*h)] - Log[(f
*(i + j*x))/(f*i - e*j)]) + b*p*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]
 - b*p*q*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)])/(h*i - g*j)
```

3.527.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))}{(g+hx)(hi-gj)}-\frac{j(a+b\log\left(c(d(e+fx)^p)^q\right))}{(i+jx)(hi-gj)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{hi-gj}-\frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{hi-gj}+\frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}-\frac{bpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i - g
*j) - ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)])/(h
*i - g*j) + (b*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) -
 (b*p*q*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)
```

3.527.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.527.4 Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)}{(hx+g)\,(jx+i)}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x)
```

3.527.5 Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{(g+hx)(i+jx)}\,dx=\int \frac{b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x, algorithm="frica
s")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*j*x^2 + g*i + (h*i + g*j)*x),
 x)
```

3.527.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)\,(i+jx)}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)/(j*x+i),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/((g + h*x)*(i + j*x)), x)
```

3.527.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx=\int \frac{b\log\left(((fx+e)^p d)^q\,c\right)+a}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x, algorithm="maxim
a")
```

output
```
a*(log(h*x + g)/(h*i - g*j) - log(j*x + i)/(h*i - g*j)) + b*integrate((q*l
og(d) + log(((f*x + e)^p)^q) + log(c))/(h*j*x^2 + g*i + (h*i + g*j)*x), x)
```

3.527.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx=\int \frac{b\log\left(((fx+e)^p d)^q\,c\right)+a}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x, algorithm="giac"
)
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/((h*x + g)*(j*x + i)), x)
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx = \int \frac{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\left(g+h\,x\right)\,\left(i+j\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)), x)
```

3.528 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^2}\,dx$

3.528.1 Optimal result

Integrand size = 33, antiderivative size = 268

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx = -\frac{bfpq\log(e+fx)}{(fi-ej)(hi-gj)}+\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{bfpq\log(i+jx)}{(fi-ej)(hi-gj)} - \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} + \frac{bhpq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{bhpq\,\mathrm{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

output
```
-b*f*p*q*ln(f*x+e)/(-e*j+f*i)/(-g*j+h*i)+(a+b*ln(c*(d*(f*x+e)^p)^q))/(-g*j
+h*i)/(j*x+i)+h*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j
+h*i)^2+b*f*p*q*ln(j*x+i)/(-e*j+f*i)/(-g*j+h*i)-h*(a+b*ln(c*(d*(f*x+e)^p)^
q))*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)^2+b*h*p*q*polylog(2,-h*(f*x+e)/(-e
*h+f*g))/(-g*j+h*i)^2-b*h*p*q*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^
2
```

3.528.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx$$
$$= \frac{\frac{a(hi-gj)}{i+jx}+\frac{b(hi-gj)\log(c(d(e+fx)^p)^q)}{i+jx}+h(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)-\frac{bf(hi-gj)pq(\log(e+fx)-\log(i+jx}{fi-ej}}{(hi-}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^2),x]
```

output
```
((a*(h*i - g*j))/(i + j*x) + (b*(h*i - g*j)*Log[c*(d*(e + f*x)^p)^q])/(i +
 j*x) + h*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)]
- (b*f*(h*i - g*j)*p*q*(Log[e + f*x] - Log[i + j*x]))/(f*i - e*j) - h*(a +
 b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)] + b*h*p*q*Poly
Log[2, (h*(e + f*x))/(-(f*g) + e*h)] - b*h*p*q*PolyLog[2, (j*(e + f*x))/(-
(f*i) + e*j)])/(h*i - g*j)^2
```

3.528.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx$$
$$\Big\downarrow \; 2895$$
$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}dx$$
$$\Big\downarrow \; 2865$$
$$\int \left(\frac{h^2(a+b\log \left(c(d(e+fx)^p)^q\right))}{(g+hx)(hi-gj)^2}-\frac{hj(a+b\log \left(c(d(e+fx)^p)^q\right))}{(i+jx)(hi-gj)^2}-\frac{j(a+b\log \left(c(d(e+fx)^p)^q\right))}{(i+jx)^2(hi-gj)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(i+jx)(hi-gj)}+\frac{h\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^2}-\frac{h\log\left(\frac{f(i+jx)}{fi-ej}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^2}+\frac{bhpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}-\frac{bhpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}-\frac{bfpq\log(e+fx)}{(fi-ej)(hi-gj)}+\frac{bfpq\log(i+jx)}{(fi-ej)(hi-gj)}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^2),x]
```

output
```
-((b*f*p*q*Log[e + f*x])/((f*i - e*j)*(h*i - g*j))) + (a + b*Log[c*(d*(e +
 f*x)^p)^q])/((h*i - g*j)*(i + j*x)) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q])
*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i - g*j)^2 + (b*f*p*q*Log[i + j*x])/((
f*i - e*j)*(h*i - g*j)) - (h*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i +
j*x))/(f*i - e*j)])/(h*i - g*j)^2 + (b*h*p*q*PolyLog[2, -((h*(e + f*x))/(f
*g - e*h))])/(h*i - g*j)^2 - (b*h*p*q*PolyLog[2, -((j*(e + f*x))/(f*i - e*
j))])/(h*i - g*j)^2
```

3.528.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.528.4 Maple [F]

$$\int \frac{a+b\ln \left(c \left(d \left(f x+e\right)^{p}\right)^{q}\right)}{\left(h x+g\right) \left(j x+i\right)^{2}}d x$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x)
```

3.528.5 Fricas [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\int \frac{b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}{(hx+g)(jx+i)^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x, algorithm="fri
cas")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*j^2*x^3 + g*i^2 + (2*h*i*j +
g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

3.528.6 Sympy [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\int \frac{a+b\log \left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}{\left(g+hx\right)\left(i+jx\right)^{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)/(j*x+i)**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/((g + h*x)*(i + j*x)**2), x)
```

3.528.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx = \int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x, algorithm="max
ima")
```

output
```
a*(h*log(h*x + g)/(h^2*i^2 - 2*g*h*i*j + g^2*j^2) - h*log(j*x + i)/(h^2*i^
2 - 2*g*h*i*j + g^2*j^2) + 1/(h*i^2 - g*i*j + (h*i*j - g*j^2)*x)) + b*inte
grate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*j^2*x^3 + g*i^2 + (2*h
*i*j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

3.528.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx = \int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x, algorithm="gia
c")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/((h*x + g)*(j*x + i)^2), x)
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx = \int \frac{a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^2), x)
```

3.529 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx$

3.529.1 Optimal result

Integrand size = 33, antiderivative size = 425

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx =& -\frac{bfpq}{2(fi-ej)(hi-gj)(i+jx)}-\frac{bfhpq\log(e+fx)}{(fi-ej)(hi-gj)^2}\\&-\frac{bf^2pq\log(e+fx)}{2(fi-ej)^2(hi-gj)}+\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2(hi-gj)(i+jx)^2}\\&+\frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^2(i+jx)}\\&+\frac{h^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^3}\\&+\frac{bfhpq\log(i+jx)}{(fi-ej)(hi-gj)^2}+\frac{bf^2pq\log(i+jx)}{2(fi-ej)^2(hi-gj)}\\&-\frac{h^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^3}\\&+\frac{bh^2pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^3}\\&-\frac{bh^2pq\,\mathrm{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^3}\end{aligned}$$

output
```
-1/2*b*f*p*q/(-e*j+f*i)/(-g*j+h*i)/(j*x+i)-b*f*h*p*q*ln(f*x+e)/(-e*j+f*i)/
(-g*j+h*i)^2-1/2*b*f^2*p*q*ln(f*x+e)/(-e*j+f*i)^2/(-g*j+h*i)+1/2*(a+b*ln(c
*(d*(f*x+e)^p)^q))/(-g*j+h*i)/(j*x+i)^2+h*(a+b*ln(c*(d*(f*x+e)^p)^q))/(-g*
j+h*i)^2/(j*x+i)+h^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/
(-g*j+h*i)^3+b*f*h*p*q*ln(j*x+i)/(-e*j+f*i)/(-g*j+h*i)^2+1/2*b*f^2*p*q*ln(
j*x+i)/(-e*j+f*i)^2/(-g*j+h*i)-h^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(j*x+i
)/(-e*j+f*i))/(-g*j+h*i)^3+b*h^2*p*q*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*
j+h*i)^3-b*h^2*p*q*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^3
```

3.529.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx$$
$$= \frac{\frac{a(hi-gj)^2}{(i+jx)^2}+\frac{2ah(hi-gj)}{i+jx}+\frac{b(hi-gj)^2\log(c(d(e+fx)^p)^q)}{(i+jx)^2}+\frac{2bh(hi-gj)\log(c(d(e+fx)^p)^q)}{i+jx}+2h^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\mathrm{l}}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^3),x]
```

output
```
((a*(h*i - g*j)^2)/(i + j*x)^2 + (2*a*h*(h*i - g*j))/(i + j*x) + (b*(h*i -
 g*j)^2*Log[c*(d*(e + f*x)^p)^q])/(i + j*x)^2 + (2*b*h*(h*i - g*j)*Log[c*(
d*(e + f*x)^p)^q])/(i + j*x) + 2*h^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[
(f*(g + h*x))/(f*g - e*h)] - (2*b*f*h*(h*i - g*j)*p*q*(Log[e + f*x] - Log[
i + j*x]))/(f*i - e*j) - (b*f*(h*i - g*j)^2*p*q*(f*i - e*j + f*(i + j*x)*L
og[e + f*x] - f*(i + j*x)*Log[i + j*x]))/((f*i - e*j)^2*(i + j*x)) - 2*h^2
*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)] + 2*b*h^2
*p*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*b*h^2*p*q*PolyLog[2, (j*
(e + f*x))/(-(f*i) + e*j)])/(2*(h*i - g*j)^3)
```

3.529.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.529. $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx$

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)(i+jx)^3}\,dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)(i+jx)^3}dx$$
$$\Big\downarrow \text{2865}$$
$$\int \left(\frac{h^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(g+hx)(hi-gj)^3}-\frac{h^2j\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(i+jx)(hi-gj)^3}-\frac{hj\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(i+jx)^2(hi-gj)^2}-\frac{j(a+b\log}{(i+}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{h^2\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(hi-gj)^3}-\frac{h^2\log\left(\frac{f(i+jx)}{fi-ej}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(hi-gj)^3}+\frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(i+jx)(hi-gj)^2}+\frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{2(i+jx)^2(hi-gj)}-\frac{bf^2pq\log(e+fx)}{2(fi-ej)^2(hi-gj)}+\frac{bf^2pq\log(i+jx)}{2(fi-ej)^2(hi-gj)}+\frac{bh^2pq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^3}-\frac{bh^2pq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^3}-\frac{bfpq}{2(i+jx)(fi-ej)(hi-gj)}-\frac{bfhpq\log(e+fx)}{(fi-ej)(hi-gj)^2}+\frac{bfhpq\log(i+jx)}{(fi-ej)(hi-gj)^2}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^3),x]
```

output
```
-1/2*(b*f*p*q)/((f*i - e*j)*(h*i - g*j)*(i + j*x)) - (b*f*h*p*q*Log[e + f*
x])/((f*i - e*j)*(h*i - g*j)^2) - (b*f^2*p*q*Log[e + f*x])/(2*(f*i - e*j)^
2*(h*i - g*j)) + (a + b*Log[c*(d*(e + f*x)^p)^q])/(2*(h*i - g*j)*(i + j*x)
^2) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q]))/((h*i - g*j)^2*(i + j*x)) + (h^
2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i -
g*j)^3 + (b*f*h*p*q*Log[i + j*x])/((f*i - e*j)*(h*i - g*j)^2) + (b*f^2*p*q
*Log[i + j*x])/(2*(f*i - e*j)^2*(h*i - g*j)) - (h^2*(a + b*Log[c*(d*(e + f
*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i - g*j)^3 + (b*h^2*p*q*Poly
Log[2, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j)^3 - (b*h^2*p*q*PolyLog[2
, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)^3
```

3.529. $\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)(i+jx)^3}\,dx$

3.529.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.529.4 Maple [F]

$$\int \frac{a+b\ln \left(c{\left(d{\left(fx+e\right)}^{p}\right)}^{q}\right)}{\left(hx+g\right){\left(jx+i\right)}^{3}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x)
```

3.529.5 Fricas [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}{(hx+g)(jx+i)^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x, algorithm="fri
cas")
```

output
```
integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*j^3*x^4 + g*i^3 + (3*h*i*j^2
+ g*j^3)*x^3 + 3*(h*i^2*j + g*i*j^2)*x^2 + (h*i^3 + 3*g*i^2*j)*x), x)
```

3.529.6 Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)\left(i+jx\right)^3}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)/(j*x+i)**3,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))/((g + h*x)*(i + j*x)**3), x)
```

3.529.7 Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{b\log\left(\left((fx+e)^pd\right)^qc\right)+a}{(hx+g)(jx+i)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x, algorithm="max
ima")
```

output
```
1/2*(2*h^2*log(h*x + g)/(h^3*i^3 - 3*g*h^2*i^2*j + 3*g^2*h*i*j^2 - g^3*j^3
) - 2*h^2*log(j*x + i)/(h^3*i^3 - 3*g*h^2*i^2*j + 3*g^2*h*i*j^2 - g^3*j^3)
 + (2*h*j*x + 3*h*i - g*j)/(h^2*i^4 - 2*g*h*i^3*j + g^2*i^2*j^2 + (h^2*i^2
*j^2 - 2*g*h*i*j^3 + g^2*j^4)*x^2 + 2*(h^2*i^3*j - 2*g*h*i^2*j^2 + g^2*i*j
^3)*x))*a + b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*j^3*
x^4 + g*i^3 + (3*h*i*j^2 + g*j^3)*x^3 + 3*(h*i^2*j + g*i*j^2)*x^2 + (h*i^3
 + 3*g*i^2*j)*x), x)
```

3.529.8 Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{b\log\left(\left((fx+e)^pd\right)^qc\right)+a}{(hx+g)(jx+i)^3}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x, algorithm="gia
c")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)/((h*x + g)*(j*x + i)^3), x)
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^3}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^3),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^3), x)
```

3.530 $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$

3.530.1 Optimal result

Integrand size = 35, antiderivative size = 519

$$\begin{aligned}
&\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx \\
&= -\frac{2abj(fi-ej)pqx}{fh}-\frac{2abj(hi-gj)pqx}{h^2}+\frac{2b^2j(fi-ej)p^2q^2x}{fh}+\frac{2b^2j(hi-gj)p^2q^2x}{h^2} \\
&+\frac{b^2j^2p^2q^2(e+fx)^2}{4f^2h}-\frac{2b^2j(fi-ej)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2h} \\
&-\frac{2b^2j(hi-gj)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} \\
&-\frac{bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2h} \\
&+\frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} \\
&+\frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} \\
&+\frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2h} \\
&+\frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} \\
&+\frac{2b(hi-gj)^2pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&-\frac{2b^2(hi-gj)^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}
\end{aligned}$$

output
```
-2*a*b*j*(-e*j+f*i)*p*q*x/f/h-2*a*b*j*(-g*j+h*i)*p*q*x/h^2+2*b^2*j*(-e*j+f
*i)*p^2*q^2*x/f/h+2*b^2*j*(-g*j+h*i)*p^2*q^2*x/h^2+1/4*b^2*j^2*p^2*q^2*(f*
x+e)^2/f^2/h-2*b^2*j*(-e*j+f*i)*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^2/h-2*
b^2*j*(-g*j+h*i)*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^2-1/2*b*j^2*p*q*(f*
x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2/h+j*(-e*j+f*i)*(f*x+e)*(a+b*ln(c*(d
*(f*x+e)^p)^q))^2/f^2/h+j*(-g*j+h*i)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2
/f/h^2+1/2*j^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2/h+(-g*j+h*i)^2*
(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h^3+2*b*(-g*j+h*i)^
2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^3-2*b
^2*(-g*j+h*i)^2*p^2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^3
```

3.530.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.79

$$\int \frac{(i+jx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx} \, dx$$
$$= \frac{4f^2hj(2hi-gj)x(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right))^2+2f^2h^2j^2x^2(a-bpq\log(e+fx)+b\log}{}$$

input
```
Integrate[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

output
```
(4*f^2*h*j*(2*h*i - g*j)*x*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^
p)^q])^2 + 2*f^2*h^2*j^2*x^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x
)^p)^q])^2 + 4*f^2*(h*i - g*j)^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e +
 f*x)^p)^q])^2*Log[g + h*x] - 8*b*f^2*h^2*i^2*p*q*(-a + b*p*q*Log[e + f*x]
 - b*Log[c*(d*(e + f*x)^p)^q])*(Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)
] + PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) - 16*b*f*h*i*j*p*q*(-a + b*p
*q*Log[e + f*x] - b*Log[c*(d*(e + f*x)^p)^q])*(-(h*(e + f*x)) + Log[e + f*
x]*(e*h + f*h*x - f*g*Log[(f*(g + h*x))/(f*g - e*h)]) - f*g*PolyLog[2, (h*
(e + f*x))/(-(f*g) + e*h)]) + 2*b*j^2*p*q*(-a + b*p*q*Log[e + f*x] - b*Log
[c*(d*(e + f*x)^p)^q])*(f*h*(f*x*(-4*g + h*x) - 2*e*(2*g + h*x)) + 2*Log[e
 + f*x]*(h*(e + f*x)*(2*f*g + e*h - f*h*x) - 2*f^2*g^2*Log[(f*(g + h*x))/(
f*g - e*h)]) - 4*f^2*g^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) + 8*b^2
*f*h*i*j*p^2*q^2*(h*(2*f*x - 2*(e + f*x)*Log[e + f*x] + (e + f*x)*Log[e +
f*x]^2) - f*g*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f
*x]*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/
(-(f*g) + e*h)])) - b^2*j^2*p^2*q^2*(4*f*g*h*(2*f*x - 2*(e + f*x)*Log[e +
f*x] + (e + f*x)*Log[e + f*x]^2) + h^2*(f*x*(6*e - f*x) + (-6*e^2 - 4*e*f*
x + 2*f^2*x^2)*Log[e + f*x] + 2*(e^2 - f^2*x^2)*Log[e + f*x]^2) - 4*f^2*g^
2*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*x]*PolyLog[
2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-(f*g) +...
```

3.530.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{j(hi-gj)(a+b\log(c(d(e+fx)^p)^q))^2}{h^2}+\frac{(hi-gj)^2(a+b\log(c(d(e+fx)^p)^q))^2}{h^2(g+hx)}+\frac{j(i+jx)(a+b\log(c(d(\right.$$

$$\downarrow 2009$$

$$\frac{j(e+fx)(fi-ej)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h}-\frac{bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2h}+\frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2h}+\frac{2bpq(hi-gj)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^3}+\frac{(hi-gj)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h^3}+\frac{j(e+fx)(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2}-\frac{2abjpqx(fi-ej)}{fh}-\frac{2abjpqx(hi-gj)}{h^2}-\frac{2b^2jpq(e+fx)(fi-ej)\log\left(c(d(e+fx)^p)^q\right)}{f^2h}-\frac{2b^2jpq(e+fx)(hi-gj)\log\left(c(d(e+fx)^p)^q\right)}{fh^2}+\frac{b^2j^2p^2q^2(e+fx)^2}{4f^2h}-\frac{2b^2p^2q^2(hi-gj)^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}+\frac{2b^2jp^2q^2x(fi-ej)}{fh}+\frac{2b^2jp^2q^2x(hi-gj)}{h^2}$$

input
```
Int[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

output
```
(-2*a*b*j*(f*i - e*j)*p*q*x)/(f*h) - (2*a*b*j*(h*i - g*j)*p*q*x)/h^2 + (2*
b^2*j*(f*i - e*j)*p^2*q^2*x)/(f*h) + (2*b^2*j*(h*i - g*j)*p^2*q^2*x)/h^2 +
 (b^2*j^2*p^2*q^2*(e + f*x)^2)/(4*f^2*h) - (2*b^2*j*(f*i - e*j)*p*q*(e + f
*x)*Log[c*(d*(e + f*x)^p)^q])/(f^2*h) - (2*b^2*j*(h*i - g*j)*p*q*(e + f*x)
*Log[c*(d*(e + f*x)^p)^q])/(f*h^2) - (b*j^2*p*q*(e + f*x)^2*(a + b*Log[c*(
d*(e + f*x)^p)^q]))/(2*f^2*h) + (j*(f*i - e*j)*(e + f*x)*(a + b*Log[c*(d*(
e + f*x)^p)^q])^2)/(f^2*h) + (j*(h*i - g*j)*(e + f*x)*(a + b*Log[c*(d*(e +
 f*x)^p)^q])^2)/(f*h^2) + (j^2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]
)^2)/(2*f^2*h) + ((h*i - g*j)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*
(g + h*x))/(f*g - e*h)])/h^3 + (2*b*(h*i - g*j)^2*p*q*(a + b*Log[c*(d*(e +
 f*x)^p)^q])*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^3 - (2*b^2*(h*i -
 g*j)^2*p^2*q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h^3
```

3.530. $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$

3.530.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.530.4 Maple [F]

$$\int \frac{(jx+i)^2 (a+b\ln(c(d(fx+e)^p)^q))^2}{hx+g}dx$$

input
```
int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

output
```
int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

3.530.5 Fricas [F]

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log(((fx+e)^pd)^q\,c)+a)^2}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="f
ricas")
```

output
```
integral((a^2*j^2*x^2 + 2*a^2*i*j*x + a^2*i^2 + (b^2*j^2*x^2 + 2*b^2*i*j*x
 + b^2*i^2)*log(((f*x + e)^p*d)^q*c)^2 + 2*(a*b*j^2*x^2 + 2*a*b*i*j*x + a*
b*i^2)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

3.530.6 Sympy [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2(i+jx)^2}{g+hx}\,dx$$

input
```
integrate((j*x+i)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*(i + j*x)**2/(g + h*x), x)
```

3.530.7 Maxima [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="m
axima")
```

output
```
2*a^2*i*j*(x/h - g*log(h*x + g)/h^2) + 1/2*a^2*j^2*(2*g^2*log(h*x + g)/h^3
 + (h*x^2 - 2*g*x)/h^2) + a^2*i^2*log(h*x + g)/h + integrate((2*(i^2*q*log
(d) + i^2*log(c))*a*b + (i^2*q^2*log(d)^2 + 2*i^2*q*log(c)*log(d) + i^2*lo
g(c)^2)*b^2 + (2*(j^2*q*log(d) + j^2*log(c))*a*b + (j^2*q^2*log(d)^2 + 2*j
^2*q*log(c)*log(d) + j^2*log(c)^2)*b^2)*x^2 + (b^2*j^2*x^2 + 2*b^2*i*j*x +
 b^2*i^2)*log(((f*x + e)^p)^q)^2 + 2*(2*(i*j*q*log(d) + i*j*log(c))*a*b +
(i*j*q^2*log(d)^2 + 2*i*j*q*log(c)*log(d) + i*j*log(c)^2)*b^2)*x + 2*(a*b*
i^2 + (i^2*q*log(d) + i^2*log(c))*b^2 + (a*b*j^2 + (j^2*q*log(d) + j^2*log
(c))*b^2)*x^2 + 2*(a*b*i*j + (i*j*q*log(d) + i*j*log(c))*b^2)*x)*log(((f*x
 + e)^p)^q))/(h*x + g), x)
```

3.530.8 Giac [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="g
iac")
```

output
```
integrate((j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{{\left(i+j\,x\right)}^2\,{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}{g+h\,x}\,dx$$

input
```
int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x),x)
```

output
```
int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x), x)
```

3.531 $\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$

3.531.1 Optimal result

Integrand size = 33, antiderivative size = 240

$$\begin{aligned}\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx &= -\frac{2abjpqx}{h}+\frac{2b^2jp^2q^2x}{h}-\frac{2b^2jpq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh}\\ &\quad+\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh}\\ &\quad+\frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2}\\ &\quad+\frac{2b(hi-gj)pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}\\ &\quad-\frac{2b^2(hi-gj)p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}\end{aligned}$$

output
```
-2*a*b*j*p*q*x/h+2*b^2*j*p^2*q^2*x/h-2*b^2*j*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p
)^q)/f/h+j*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h+(-g*j+h*i)*(a+b*ln(c*
(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h^2+2*b*(-g*j+h*i)*p*q*(a+b*l
n(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^2-2*b^2*(-g*j+h*i
)*p^2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^2
```

3.531.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 852 vs. $2(240) = 480$.

Time = 0.23 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.55

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx$$
$$= \frac{-2abehjpq + a^2fhjx - 2abfhjpqx + 2b^2fhjp^2q^2x + 2abehjpq\log(e+fx) - b^2ehjp^2q^2\log^2(e+fx) - 2}{}$$

input
```
Integrate[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

output
```
(-2*a*b*e*h*j*p*q + a^2*f*h*j*x - 2*a*b*f*h*j*p*q*x + 2*b^2*f*h*j*p^2*q^2*
x + 2*a*b*e*h*j*p*q*Log[e + f*x] - b^2*e*h*j*p^2*q^2*Log[e + f*x]^2 - 2*b^
2*e*h*j*p*q*Log[c*(d*(e + f*x)^p)^q] + 2*a*b*f*h*j*x*Log[c*(d*(e + f*x)^p)
^q] - 2*b^2*f*h*j*p*q*x*Log[c*(d*(e + f*x)^p)^q] + 2*b^2*e*h*j*p*q*Log[e +
 f*x]*Log[c*(d*(e + f*x)^p)^q] + b^2*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]^2 +
a^2*f*h*i*Log[g + h*x] - a^2*f*g*j*Log[g + h*x] - 2*a*b*f*h*i*p*q*Log[e +
f*x]*Log[g + h*x] + 2*a*b*f*g*j*p*q*Log[e + f*x]*Log[g + h*x] + b^2*f*h*i*
p^2*q^2*Log[e + f*x]^2*Log[g + h*x] - b^2*f*g*j*p^2*q^2*Log[e + f*x]^2*Log
[g + h*x] + 2*a*b*f*h*i*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*a*b*f*g*
j*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*b^2*f*h*i*p*q*Log[e + f*x]*Log
[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 2*b^2*f*g*j*p*q*Log[e + f*x]*Log[c*(d
*(e + f*x)^p)^q]*Log[g + h*x] + b^2*f*h*i*Log[c*(d*(e + f*x)^p)^q]^2*Log[g
 + h*x] - b^2*f*g*j*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*f*h*i*
p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)] - 2*a*b*f*g*j*p*q*Log[e +
f*x]*Log[(f*(g + h*x))/(f*g - e*h)] - b^2*f*h*i*p^2*q^2*Log[e + f*x]^2*Log
[(f*(g + h*x))/(f*g - e*h)] + b^2*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[(f*(g +
 h*x))/(f*g - e*h)] + 2*b^2*f*h*i*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q
]*Log[(f*(g + h*x))/(f*g - e*h)] - 2*b^2*f*g*j*p*q*Log[e + f*x]*Log[c*(d*(
e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 2*b*f*(h*i - g*j)*p*q*(a +
 b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] +...
```

3.531.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h(g+hx)}+\frac{j\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bpq(hi-gj)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^2}+\frac{(hi-gj)\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h^2}+\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh}-\frac{2abjpqx}{h}-\frac{2b^2jpq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh}-\frac{2b^2p^2q^2(hi-gj)\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}+\frac{2b^2jp^2q^2x}{h}$$

input
```
Int[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

output
```
(-2*a*b*j*p*q*x)/h + (2*b^2*j*p^2*q^2*x)/h - (2*b^2*j*p*q*(e + f*x)*Log[c*
(d*(e + f*x)^p)^q])/(f*h) + (j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^
2)/(f*h) + ((h*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x
))/(f*g - e*h)])/h^2 + (2*b*(h*i - g*j)*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q
])*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^2 - (2*b^2*(h*i - g*j)*p^2*
q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h^2
```

3.531.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.531.4 Maple [F]

$$\int \frac{(jx+i)\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{2}}{hx+g}dx$$

input
```
int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

output
```
int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

3.531.5 Fricas [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{2}}{g+hx}\,dx=\int \frac{(jx+i)\left(b\log\left(((fx+e)^{p}d)^{q}c\right)+a\right)^{2}}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fri
cas")
```

output
```
integral((a^2*j*x + a^2*i + (b^2*j*x + b^2*i)*log(((f*x + e)^p*d)^q*c)^2 +
 2*(a*b*j*x + a*b*i)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

3.531.6 Sympy [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2(i+jx)}{g+hx}\,dx$$

input
```
integrate((j*x+i)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*(i + j*x)/(g + h*x), x)
```

3.531.7 Maxima [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="max
ima")
```

output
```
a^2*j*(x/h - g*log(h*x + g)/h^2) + a^2*i*log(h*x + g)/h + integrate((2*(i*
q*log(d) + i*log(c))*a*b + (i*q^2*log(d)^2 + 2*i*q*log(c)*log(d) + i*log(c
)^2)*b^2 + (b^2*j*x + b^2*i)*log(((f*x + e)^p)^q)^2 + (2*(j*q*log(d) + j*l
og(c))*a*b + (j*q^2*log(d)^2 + 2*j*q*log(c)*log(d) + j*log(c)^2)*b^2)*x +
2*((i*q*log(d) + i*log(c))*b^2 + a*b*i + ((j*q*log(d) + j*log(c))*b^2 + a*
b*j)*x)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.531.8 Giac [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="gia
c")
```

output
```
integrate((j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)\,(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{g+hx}\,dx = \int \frac{(i+j\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}{g+h\,x}\,dx$$

input
```
int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x),x)
```

output
```
int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x), x)
```

3.532 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$

3.532.1 Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h+2*b*p*q*(a+b*ln(c
*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-2*b^2*p^2*q^2*polylo
g(3,-h*(f*x+e)/(-e*h+f*g))/h
```

3.532.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 324 vs. 2(123) = 246.

Time = 0.05 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.63

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$
$$= \frac{a^2\log(g+hx)-2abpq\log(e+fx)\log(g+hx)+b^2p^2q^2\log^2(e+fx)\log(g+hx)+2ab\log(c(d(e+fx)^p}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

output
```
(a^2*Log[g + h*x] - 2*a*b*p*q*Log[e + f*x]*Log[g + h*x] + b^2*p^2*q^2*Log[
e + f*x]^2*Log[g + h*x] + 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*
b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + b^2*Log[c*(d*
(e + f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*p*q*Log[e + f*x]*Log[(f*(g + h*x))/
(f*g - e*h)] - b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] +
 2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g -
e*h)] + 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e + f*x))/
(-(f*g) + e*h)] - 2*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)])/
h
```

3.532.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2895, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}dx$$
$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bfpq\int\frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bpq\left(bpq\int\frac{\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\right)}{h}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h}-\frac{2bpq\left(bpq\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)-\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\right)}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])/h - (2
*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])*PolyLog[2, -((h*(e + f*x))/(f
*g - e*h))]) + b*p*q*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))]))/h
```

3.532.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.532.4 Maple [F]

$$\int \frac{(a+b\ln (c(d(fx+e)^p)^q))^2}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

3.532.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c)
+ a^2)/(h*x + g), x)
```

3.532.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x), x)
```

3.532.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="maxima")
```

output
```
a^2*log(h*x + g)/h + integrate((b^2*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) +
 log(c))*a*b + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^2 + 2*((q*l
og(d) + log(c))*b^2 + a*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.532.8 Giac [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{g+hx}\,dx=\int \frac{\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{g+hx}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^2}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x), x)
```

3.533 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$

3.533.1 Optimal result

Integrand size = 35, antiderivative size = 288

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx &= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj}\\
&+ \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}\\
&- \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}\\
&- \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} + \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}\end{aligned}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)-(a+b*ln(
c*(d*(f*x+e)^p)^q))^2*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)+2*b*p*q*(a+b*ln(
c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)-2*b*p*q*(a
+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)-2*b^
2*p^2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)+2*b^2*p^2*q^2*polylo
g(3,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)
```

3.533.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 652 vs. 2(288) = 576.

Time = 0.18 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.26

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$$
$$= \frac{a^2\log(g+hx) - 2abpq\log(e+fx)\log(g+hx) + b^2p^2q^2\log^2(e+fx)\log(g+hx) + 2ab\log(c(d(e+fx)^p}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)),x]
```

output
```
(a^2*Log[g + h*x] - 2*a*b*p*q*Log[e + f*x]*Log[g + h*x] + b^2*p^2*q^2*Log[
e + f*x]^2*Log[g + h*x] + 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*
b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + b^2*Log[c*(d*
(e + f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*p*q*Log[e + f*x]*Log[(f*(g + h*x))/
(f*g - e*h)] - b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] +
 2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g -
e*h)] - a^2*Log[i + j*x] + 2*a*b*p*q*Log[e + f*x]*Log[i + j*x] - b^2*p^2*q
^2*Log[e + f*x]^2*Log[i + j*x] - 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*
x] + 2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] - b^2*Lo
g[c*(d*(e + f*x)^p)^q]^2*Log[i + j*x] - 2*a*b*p*q*Log[e + f*x]*Log[(f*(i +
 j*x))/(f*i - e*j)] + b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(i + j*x))/(f*i -
e*j)] - 2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(i + j*x))/
(f*i - e*j)] + 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e +
 f*x))/(-(f*g) + e*h)] - 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[
2, (j*(e + f*x))/(-(f*i) + e*j)] - 2*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/
(-(f*g) + e*h)] + 2*b^2*p^2*q^2*PolyLog[3, (j*(e + f*x))/(-(f*i) + e*j)])/
(h*i - g*j)
```

3.533.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}dx$$
$$\Big\downarrow \text{ 2865}$$
$$\int \left(\frac{h(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(hi-gj)}-\frac{j(a+b\log(c(d(e+fx)^p)^q))^2}{(i+jx)(hi-gj)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}-\frac{2bpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}+\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}-\frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}-\frac{2b^2p^2q^2\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}+\frac{2b^2p^2q^2\,\text{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i -
 g*j) - ((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(i + j*x))/(f*i - e*j)]
)/(h*i - g*j) + (2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((h*
(e + f*x))/(f*g - e*h))])/(h*i - g*j) - (2*b*p*q*(a + b*Log[c*(d*(e + f*x)
^p)^q])*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j) - (2*b^2*p^2
*q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) + (2*b^2*p^2*q^
2*PolyLog[3, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)
```

3.533. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$

3.533.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.533.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{\left(hx+g\right)\left(jx+i\right)}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x)
```

3.533.5 Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{(g+hx)(i+jx)}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{2}}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x, algorithm="fri
cas")
```

output
```
integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c)
+ a^2)/(h*j*x^2 + g*i + (h*i + g*j)*x), x)
```

3.533.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)/(j*x+i),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/((g + h*x)*(i + j*x)), x)
```

3.533.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^2}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x, algorithm="max
ima")
```

output
```
a^2*(log(h*x + g)/(h*i - g*j) - log(j*x + i)/(h*i - g*j)) + integrate((b^2
*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) + log(c))*a*b + (q^2*log(d)^2 + 2*q*
log(c)*log(d) + log(c)^2)*b^2 + 2*((q*log(d) + log(c))*b^2 + a*b)*log(((f*
x + e)^p)^q))/(h*j*x^2 + g*i + (h*i + g*j)*x), x)
```

3.533.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^2}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x, algorithm="gia
c")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/((h*x + g)*(j*x + i)), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)(i+jx)}\,dx = \int \frac{{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}{\left(g+h\,x\right)\,\left(i+j\,x\right)}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)), x)
```

3.534 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx$

3.534.1 Optimal result

Integrand size = 35, antiderivative size = 463

$$
\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx \\
&= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fi-ej)(hi-gj)(i+jx)}+\frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} \\
&\quad+\frac{2bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} \\
&\quad-\frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} \\
&\quad+\frac{2bhpq(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} \\
&\quad+\frac{2b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} \\
&\quad-\frac{2bhpq(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} \\
&\quad-\frac{2b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}+\frac{2b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}
\end{aligned}
$$

output
```
-j*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(-e*j+f*i)/(-g*j+h*i)/(j*x+i)+h*(
a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)^2+2*b*f*p
*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(j*x+i)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h
*i)-h*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)^2+
2*b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g
*j+h*i)^2+2*b^2*f*p^2*q^2*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-e*j+f*i)/(-g*
j+h*i)-2*b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-j*(f*x+e)/(-e*j+f*
i))/(-g*j+h*i)^2-2*b^2*h*p^2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*
i)^2+2*b^2*h*p^2*q^2*polylog(3,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2
```

3.534.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.41

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx$$

$$= \frac{(fi-ej)(hi-gj)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)^2+h(fi-ej)(i+jx)\left(a-bpq\log(e+f\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)^2),x]
```

output
```
((f*i - e*j)*(h*i - g*j)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)
^q])^2 + h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e +
 f*x)^p)^q])^2*Log[g + h*x] - h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f
*x] + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[i + j*x] - 2*b*p*q*(a - b*p*q*Log[
e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*((h*i - g*j)*(j*(e + f*x)*Log[e + f
*x] - f*(i + j*x)*Log[i + j*x]) - h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]*Lo
g[(f*(g + h*x))/(f*g - e*h)] + PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) +
 h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]*Log[(f*(i + j*x))/(f*i - e*j)] + Po
lyLog[2, (j*(e + f*x))/(-(f*i) + e*j)])) - b^2*p^2*q^2*((h*i - g*j)*(Log[e
 + f*x]*(j*(e + f*x)*Log[e + f*x] - 2*f*(i + j*x)*Log[(f*(i + j*x))/(f*i -
 e*j)]) - 2*f*(i + j*x)*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)]) - h*(f*i
 - e*j)*(i + j*x)*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e
 + f*x]*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*
x))/(-(f*g) + e*h)]) + h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]^2*Log[(f*(i +
 j*x))/(f*i - e*j)] + 2*Log[e + f*x]*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*
j)] - 2*PolyLog[3, (j*(e + f*x))/(-(f*i) + e*j)])))/((f*i - e*j)*(h*i - g*
j)^2*(i + j*x))
```

3.534.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{h^2(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(hi-gj)^2}-\frac{hj(a+b\log(c(d(e+fx)^p)^q))^2}{(i+jx)(hi-gj)^2}-\frac{j(a+b\log(c(d(e+fx)^p)^q))^2}{(i+jx)^2(hi-gj)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bhpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2}-\frac{2bhpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2}+\frac{2bfpq\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(fi-ej)(hi-gj)}-\frac{j(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{(i+jx)(fi-ej)(hi-gj)}+\frac{h\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2}-\frac{h\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2}+\frac{2b^2fp^2q^2\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}-\frac{2b^2hp^2q^2\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}+\frac{2b^2hp^2q^2\,\text{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)^2),x]
```

output
```
-((j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/((f*i - e*j)*(h*i - g*j
)*(i + j*x))) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f
*g - e*h)])/(h*i - g*j)^2 + (2*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*Lo
g[(f*(i + j*x))/(f*i - e*j)])/((f*i - e*j)*(h*i - g*j)) - (h*(a + b*Log[c*
(d*(e + f*x)^p)^q])^2*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i - g*j)^2 + (2*b
*h*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((h*(e + f*x))/(f*g -
e*h))])/(h*i - g*j)^2 + (2*b^2*f*p^2*q^2*PolyLog[2, -((j*(e + f*x))/(f*i -
 e*j))])/((f*i - e*j)*(h*i - g*j)) - (2*b*h*p*q*(a + b*Log[c*(d*(e + f*x)^
p)^q])*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)^2 - (2*b^2*h*
p^2*q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j)^2 + (2*b^2*h
*p^2*q^2*PolyLog[3, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)^2
```

3.534.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.534.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}{\left(hx+g\right)\left(jx+i\right)^{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x)
```

3.534.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c)
+ a^2)/(h*j^2*x^3 + g*i^2 + (2*h*i*j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x),
x)
```

3.534.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)\,(i+jx)^2}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)/(j*x+i)**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/((g + h*x)*(i + j*x)**2), x
)
```

3.534.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x, algorithm="m
axima")
```

output
```
a^2*(h*log(h*x + g)/(h^2*i^2 - 2*g*h*i*j + g^2*j^2) - h*log(j*x + i)/(h^2*
i^2 - 2*g*h*i*j + g^2*j^2) + 1/(h*i^2 - g*i*j + (h*i*j - g*j^2)*x)) + inte
grate((b^2*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) + log(c))*a*b + (q^2*log(d
)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^2 + 2*((q*log(d) + log(c))*b^2 + a*b
)*log(((f*x + e)^p)^q))/(h*j^2*x^3 + g*i^2 + (2*h*i*j + g*j^2)*x^2 + (h*i^
2 + 2*g*i*j)*x), x)
```

3.534.8 Giac [F]

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx=\int \frac{(b\log \left(\left(\left(fx+e\right)^p d\right)^q c\right)+a)^2}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x, algorithm="g
iac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/((h*x + g)*(j*x + i)^2), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)\right)^2}{\left(g+h\,x\right)\,\left(i+j\,x\right)^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)^2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)^2), x)
```

3.535 $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

3.535.1 Optimal result

Integrand size = 35, antiderivative size = 742

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$
$$= \frac{6ab^2j(fi-ej)p^2q^2x}{fh} + \frac{6ab^2j(hi-gj)p^2q^2x}{h^2} - \frac{6b^3j(fi-ej)p^3q^3x}{fh} - \frac{6b^3j(hi-gj)p^3q^3x}{h^2}$$
$$- \frac{3b^3j^2p^3q^3(e+fx)^2}{8f^2h} + \frac{6b^3j(fi-ej)p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{f^2h}$$
$$+ \frac{6b^3j(hi-gj)p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{fh^2}$$
$$+ \frac{3b^2j^2p^2q^2(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))}{4f^2h}$$
$$- \frac{3bj(fi-ej)pq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f^2h}$$
$$- \frac{3bj(hi-gj)pq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{fh^2}$$
$$- \frac{3bj^2pq(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{4f^2h}$$
$$+ \frac{j(fi-ej)(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{f^2h}$$
$$+ \frac{j(hi-gj)(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{fh^2}$$
$$+ \frac{j^2(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{2f^2h}$$
$$+ \frac{(hi-gj)^2(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3}$$
$$+ \frac{3b(hi-gj)^2pq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$
$$- \frac{6b^2(hi-gj)^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$
$$+ \frac{6b^3(hi-gj)^2p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$

output
```
6*a*b^2*j*(-e*j+f*i)*p^2*q^2*x/f/h+6*a*b^2*j*(-g*j+h*i)*p^2*q^2*x/h^2-6*b^
3*j*(-e*j+f*i)*p^3*q^3*x/f/h-6*b^3*j*(-g*j+h*i)*p^3*q^3*x/h^2-3/8*b^3*j^2*
p^3*q^3*(f*x+e)^2/f^2/h+6*b^3*j*(-e*j+f*i)*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)
^p)^q)/f^2/h+6*b^3*j*(-g*j+h*i)*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^
2+3/4*b^2*j^2*p^2*q^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2/h-3*b*j*(-
e*j+f*i)*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2/h-3*b*j*(-g*j+h*i)*
p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h^2-3/4*b*j^2*p*q*(f*x+e)^2*(a
+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2/h+j*(-e*j+f*i)*(f*x+e)*(a+b*ln(c*(d*(f*x+e
)^p)^q))^3/f^2/h+j*(-g*j+h*i)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f/h^2+
1/2*j^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^2/h+(-g*j+h*i)^2*(a+b*ln
(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h^3+3*b*(-g*j+h*i)^2*p*q*(
a+b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^3-6*b^2*(-
g*j+h*i)^2*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+
f*g))/h^3+6*b^3*(-g*j+h*i)^2*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h^3
```

3.535.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4056 vs. 2(742) = 1484.

Time = 0.89 (sec) , antiderivative size = 4056, normalized size of antiderivative = 5.47

$$\int \frac{(i+jx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

output
```
(-48*a^2*b*e*f*h^2*i*j*p*q + 24*a^2*b*e*f*g*h*j^2*p*q + 16*a^3*f^2*h^2*i*j
*x - 8*a^3*f^2*g*h*j^2*x - 48*a^2*b*f^2*h^2*i*j*p*q*x + 24*a^2*b*f^2*g*h*j
^2*p*q*x + 12*a^2*b*e*f*h^2*j^2*p*q*x + 96*a*b^2*f^2*h^2*i*j*p^2*q^2*x - 4
8*a*b^2*f^2*g*h*j^2*p^2*q^2*x - 36*a*b^2*e*f*h^2*j^2*p^2*q^2*x - 96*b^3*f^
2*h^2*i*j*p^3*q^3*x + 48*b^3*f^2*g*h*j^2*p^3*q^3*x + 42*b^3*e*f*h^2*j^2*p^
3*q^3*x + 4*a^3*f^2*h^2*j^2*x^2 - 6*a^2*b*f^2*h^2*j^2*p*q*x^2 + 6*a*b^2*f^
2*h^2*j^2*p^2*q^2*x^2 - 3*b^3*f^2*h^2*j^2*p^3*q^3*x^2 + 48*a^2*b*e*f*h^2*i
*j*p*q*Log[e + f*x] - 24*a^2*b*e*f*g*h*j^2*p*q*Log[e + f*x] - 12*a^2*b*e^2
*h^2*j^2*p*q*Log[e + f*x] + 36*a*b^2*e^2*h^2*j^2*p^2*q^2*Log[e + f*x] + 96
*b^3*e*f*h^2*i*j*p^3*q^3*Log[e + f*x] - 48*b^3*e*f*g*h*j^2*p^3*q^3*Log[e +
 f*x] - 42*b^3*e^2*h^2*j^2*p^3*q^3*Log[e + f*x] - 48*a*b^2*e*f*h^2*i*j*p^2
*q^2*Log[e + f*x]^2 + 24*a*b^2*e*f*g*h*j^2*p^2*q^2*Log[e + f*x]^2 + 12*a*b
^2*e^2*h^2*j^2*p^2*q^2*Log[e + f*x]^2 - 18*b^3*e^2*h^2*j^2*p^3*q^3*Log[e +
 f*x]^2 + 16*b^3*e*f*h^2*i*j*p^3*q^3*Log[e + f*x]^3 - 8*b^3*e*f*g*h*j^2*p^
3*q^3*Log[e + f*x]^3 - 4*b^3*e^2*h^2*j^2*p^3*q^3*Log[e + f*x]^3 - 96*a*b^2
*e*f*h^2*i*j*p*q*Log[c*(d*(e + f*x)^p)^q] + 48*a*b^2*e*f*g*h*j^2*p*q*Log[c
*(d*(e + f*x)^p)^q] + 48*a^2*b*f^2*h^2*i*j*x*Log[c*(d*(e + f*x)^p)^q] - 24
*a^2*b*f^2*g*h*j^2*x*Log[c*(d*(e + f*x)^p)^q] - 96*a*b^2*f^2*h^2*i*j*p*q*x
*Log[c*(d*(e + f*x)^p)^q] + 48*a*b^2*f^2*g*h*j^2*p*q*x*Log[c*(d*(e + f*x)^
p)^q] + 24*a*b^2*e*f*h^2*j^2*p*q*x*Log[c*(d*(e + f*x)^p)^q] + 96*b^3*f^...
```

3.535.3 Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(i+jx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(i+jx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{j(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h^2}+\frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h^2(g+hx)}+\frac{j(i+jx)\left(a+b\log\left(c(d(\right.\right.}{h}\right.$$

$$\downarrow 2009$$

$$\frac{3b^2j^2p^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2h} - \frac{6b^2p^2q^2(hi-gj)^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^3} + \frac{6ab^2jp^2q^2x(fi-ej)}{fh} + \frac{6ab^2jp^2q^2x(hi-gj)}{h^2} - \frac{3bjpq(e+fx)(fi-ej)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} + \frac{j(e+fx)(fi-ej)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2h} - \frac{3bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2h} + \frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2h} + \frac{3bpq(hi-gj)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h^3} + \frac{(hi-gj)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h^3} - \frac{3bjpq(e+fx)(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} + \frac{j(e+fx)(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2} + \frac{6b^3jp^2q^2(e+fx)(fi-ej)\log\left(c(d(e+fx)^p)^q\right)}{f^2h} + \frac{6b^3jp^2q^2(e+fx)(hi-gj)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} - \frac{3b^3j^2p^3q^3(e+fx)^2}{8f^2h} + \frac{6b^3p^3q^3(hi-gj)^2\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \frac{6b^3jp^3q^3x(fi-ej)}{fh} - \frac{6b^3jp^3q^3x(hi-gj)}{h^2}$$

input
```
Int[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

3.535. $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

output
```
(6*a*b^2*j*(f*i - e*j)*p^2*q^2*x)/(f*h) + (6*a*b^2*j*(h*i - g*j)*p^2*q^2*x
)/h^2 - (6*b^3*j*(f*i - e*j)*p^3*q^3*x)/(f*h) - (6*b^3*j*(h*i - g*j)*p^3*q
^3*x)/h^2 - (3*b^3*j^2*p^3*q^3*(e + f*x)^2)/(8*f^2*h) + (6*b^3*j*(f*i - e*
j)*p^2*q^2*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f^2*h) + (6*b^3*j*(h*i - g
*j)*p^2*q^2*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h^2) + (3*b^2*j^2*p^2*q
^2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(4*f^2*h) - (3*b*j*(f*i -
 e*j)*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(f^2*h) - (3*b*j*(
h*i - g*j)*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(f*h^2) - (3*
b*j^2*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(4*f^2*h) + (j*(
f*i - e*j)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(f^2*h) + (j*(h*i
 - g*j)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(f*h^2) + (j^2*(e +
f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(2*f^2*h) + ((h*i - g*j)^2*(a +
 b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/h^3 + (3*b*
(h*i - g*j)^2*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(e +
f*x))/(f*g - e*h))])/h^3 - (6*b^2*(h*i - g*j)^2*p^2*q^2*(a + b*Log[c*(d*(e
 + f*x)^p)^q])*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h^3 + (6*b^3*(h*i
 - g*j)^2*p^3*q^3*PolyLog[4, -((h*(e + f*x))/(f*g - e*h))])/h^3
```

3.535.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.535. $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

3.535.4 Maple [F]

$$\int \frac{(jx+i)^2 \left(a+b\ln \left(c(d(fx+e)^p)^q\right)\right)^3}{hx+g}dx$$

input
```
int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

output
```
int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

3.535.5 Fricas [F]

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(jx+i)^2 (b\log \left(((fx+e)^p d)^q\,c\right)+a)^3}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="f
ricas")
```

output
```
integral((a^3*j^2*x^2 + 2*a^3*i*j*x + a^3*i^2 + (b^3*j^2*x^2 + 2*b^3*i*j*x
 + b^3*i^2)*log(((f*x + e)^p*d)^q*c)^3 + 3*(a*b^2*j^2*x^2 + 2*a*b^2*i*j*x
+ a*b^2*i^2)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*j^2*x^2 + 2*a^2*b*i*j*x
 + a^2*b*i^2)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

3.535.6 Sympy [F]

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(a+b\log (c(d(e+fx)^p)^q))^3 (i+jx)^2}{g+hx}\,dx$$

input
```
integrate((j*x+i)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3*(i + j*x)**2/(g + h*x), x)
```

3.535.7 Maxima [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx=\int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^qc\right)+a\right)^3}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="m
axima")
```

output
```
2*a^3*i*j*(x/h - g*log(h*x + g)/h^2) + 1/2*a^3*j^2*(2*g^2*log(h*x + g)/h^3
 + (h*x^2 - 2*g*x)/h^2) + a^3*i^2*log(h*x + g)/h + integrate((3*(i^2*q*log
(d) + i^2*log(c))*a^2*b + 3*(i^2*q^2*log(d)^2 + 2*i^2*q*log(c)*log(d) + i^
2*log(c)^2)*a*b^2 + (i^2*q^3*log(d)^3 + 3*i^2*q^2*log(c)*log(d)^2 + 3*i^2*
q*log(c)^2*log(d) + i^2*log(c)^3)*b^3 + (b^3*j^2*x^2 + 2*b^3*i*j*x + b^3*i
^2)*log(((f*x + e)^p)^q)^3 + (3*(j^2*q*log(d) + j^2*log(c))*a^2*b + 3*(j^2
*q^2*log(d)^2 + 2*j^2*q*log(c)*log(d) + j^2*log(c)^2)*a*b^2 + (j^2*q^3*log
(d)^3 + 3*j^2*q^2*log(c)*log(d)^2 + 3*j^2*q*log(c)^2*log(d) + j^2*log(c)^3
)*b^3)*x^2 + 3*(a*b^2*i^2 + (i^2*q*log(d) + i^2*log(c))*b^3 + (a*b^2*j^2 +
 (j^2*q*log(d) + j^2*log(c))*b^3)*x^2 + 2*(a*b^2*i*j + (i*j*q*log(d) + i*j
*log(c))*b^3)*x)*log(((f*x + e)^p)^q)^2 + 2*(3*(i*j*q*log(d) + i*j*log(c))
*a^2*b + 3*(i*j*q^2*log(d)^2 + 2*i*j*q*log(c)*log(d) + i*j*log(c)^2)*a*b^2
 + (i*j*q^3*log(d)^3 + 3*i*j*q^2*log(c)*log(d)^2 + 3*i*j*q*log(c)^2*log(d)
 + i*j*log(c)^3)*b^3)*x + 3*(a^2*b*i^2 + 2*(i^2*q*log(d) + i^2*log(c))*a*b
^2 + (i^2*q^2*log(d)^2 + 2*i^2*q*log(c)*log(d) + i^2*log(c)^2)*b^3 + (a^2*
b*j^2 + 2*(j^2*q*log(d) + j^2*log(c))*a*b^2 + (j^2*q^2*log(d)^2 + 2*j^2*q*
log(c)*log(d) + j^2*log(c)^2)*b^3)*x^2 + 2*(a^2*b*i*j + 2*(i*j*q*log(d) +
i*j*log(c))*a*b^2 + (i*j*q^2*log(d)^2 + 2*i*j*q*log(c)*log(d) + i*j*log(c)
^2)*b^3)*x)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.535.8 Giac [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx=\int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^qc\right)+a\right)^3}{hx+g}\,dx$$

input
```
integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="g
iac")
```

output
```
integrate((j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(i+j\,x)^2\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^3}{g+h\,x}\,dx$$

input
```
int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x),x)
```

output
```
int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x), x)
```

3.536 $\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

3.536.1 Optimal result

Integrand size = 33, antiderivative size = 349

$$\begin{aligned}
&\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx \\
&= \frac{6ab^2jp^2q^2x}{h} - \frac{6b^3jp^3q^3x}{h} + \frac{6b^3jp^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh} \\
&\quad - \frac{3bjpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh} + \frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh} \\
&\quad + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} \\
&\quad + \frac{3b(hi-gj)pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2} \\
&\quad - \frac{6b^2(hi-gj)p^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^2} \\
&\quad + \frac{6b^3(hi-gj)p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}
\end{aligned}$$

output
```
6*a*b^2*j*p^2*q^2*x/h-6*b^3*j*p^3*q^3*x/h+6*b^3*j*p^2*q^2*(f*x+e)*ln(c*(d*
(f*x+e)^p)^q)/f/h-3*b*j*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h+j*(f
*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f/h+(-g*j+h*i)*(a+b*ln(c*(d*(f*x+e)^p)
^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h^2+3*b*(-g*j+h*i)*p*q*(a+b*ln(c*(d*(f*x+e
)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^2-6*b^2*(-g*j+h*i)*p^2*q^2*(
a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^2+6*b^3*(-g*
j+h*i)*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h^2
```

3.536.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1769 vs. $2(349) = 698$.

Time = 0.48 (sec) , antiderivative size = 1769, normalized size of antiderivative = 5.07

$$\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \text{Too large to display}$$

input
```
Integrate[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

output
```
(-3*a^2*b*e*h*j*p*q + a^3*f*h*j*x - 3*a^2*b*f*h*j*p*q*x + 6*a*b^2*f*h*j*p^
2*q^2*x - 6*b^3*f*h*j*p^3*q^3*x + 3*a^2*b*e*h*j*p*q*Log[e + f*x] + 6*b^3*e
*h*j*p^3*q^3*Log[e + f*x] - 3*a*b^2*e*h*j*p^2*q^2*Log[e + f*x]^2 + b^3*e*h
*j*p^3*q^3*Log[e + f*x]^3 - 6*a*b^2*e*h*j*p*q*Log[c*(d*(e + f*x)^p)^q] + 3
*a^2*b*f*h*j*x*Log[c*(d*(e + f*x)^p)^q] - 6*a*b^2*f*h*j*p*q*x*Log[c*(d*(e
+ f*x)^p)^q] + 6*b^3*f*h*j*p^2*q^2*x*Log[c*(d*(e + f*x)^p)^q] + 6*a*b^2*e*
h*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] - 3*b^3*e*h*j*p^2*q^2*Log[e
+ f*x]^2*Log[c*(d*(e + f*x)^p)^q] - 3*b^3*e*h*j*p*q*Log[c*(d*(e + f*x)^p)^
q]^2 + 3*a*b^2*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]^2 - 3*b^3*f*h*j*p*q*x*Log[
c*(d*(e + f*x)^p)^q]^2 + 3*b^3*e*h*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p
)^q]^2 + b^3*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]^3 + a^3*f*h*i*Log[g + h*x] -
 a^3*f*g*j*Log[g + h*x] - 3*a^2*b*f*h*i*p*q*Log[e + f*x]*Log[g + h*x] + 3*
a^2*b*f*g*j*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*f*h*i*p^2*q^2*Log[e +
f*x]^2*Log[g + h*x] - 3*a*b^2*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[g + h*x] -
b^3*f*h*i*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] + b^3*f*g*j*p^3*q^3*Log[e +
f*x]^3*Log[g + h*x] + 3*a^2*b*f*h*i*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x]
- 3*a^2*b*f*g*j*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*f*h*i*p*q*
Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 6*a*b^2*f*g*j*p*q*Log
[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*f*h*i*p^2*q^2*Log[
e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 3*b^3*f*g*j*p^2*q^2*...
```

3.536.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h(g+hx)}+\frac{j\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^2p^2q^2(hi-gj)\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^2}+\frac{6ab^2jp^2q^2x}{h}+\frac{3bpq(hi-gj)\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h^2}+\frac{(hi-gj)\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h^2}-\frac{3bjpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh}+\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh}+\frac{6b^3jp^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh}+\frac{6b^3p^3q^3(hi-gj)\text{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}-\frac{6b^3jp^3q^3x}{h}$$

input
```
Int[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

output
```
(6*a*b^2*j*p^2*q^2*x)/h - (6*b^3*j*p^3*q^3*x)/h + (6*b^3*j*p^2*q^2*(e + f*
x)*Log[c*(d*(e + f*x)^p)^q])/(f*h) - (3*b*j*p*q*(e + f*x)*(a + b*Log[c*(d*
(e + f*x)^p)^q])^2)/(f*h) + (j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^
3)/(f*h) + ((h*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x
))/(f*g - e*h)])/h^2 + (3*b*(h*i - g*j)*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q
])^2*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^2 - (6*b^2*(h*i - g*j)*p^
2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h*(e + f*x))/(f*g - e
*h))])/h^2 + (6*b^3*(h*i - g*j)*p^3*q^3*PolyLog[4, -((h*(e + f*x))/(f*g -
e*h))])/h^2
```

3.536.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.536.4 Maple [F]

$$\int \frac{(jx+i)\left(a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)\right)^{3}}{hx+g}dx$$

input
```
int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

output
```
int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

3.536.5 Fricas [F]

$$\int \frac{(i + jx)\left(a + b\log\left(c(d(e + fx)^p)^q\right)\right)^3}{g + hx}\,dx = \int \frac{(jx + i)(b\log\left(((fx + e)^p d)^q\,c\right) + a)^3}{hx + g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fri
cas")
```

output
```
integral((a^3*j*x + a^3*i + (b^3*j*x + b^3*i)*log(((f*x + e)^p*d)^q*c)^3 +
 3*(a*b^2*j*x + a*b^2*i)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*j*x + a^2*b
*i)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

3.536.6 Sympy [F]

$$\int \frac{(i + jx)\left(a + b\log\left(c(d(e + fx)^p)^q\right)\right)^3}{g + hx}\,dx = \int \frac{\left(a + b\log\left(c(d(e + fx)^p)^q\right)\right)^3 (i + jx)}{g + hx}\,dx$$

input
```
integrate((j*x+i)*(a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3*(i + j*x)/(g + h*x), x)
```

3.536.7 Maxima [F]

$$\int \frac{(i + jx)\left(a + b\log\left(c(d(e + fx)^p)^q\right)\right)^3}{g + hx}\,dx = \int \frac{(jx + i)(b\log\left(((fx + e)^p d)^q\,c\right) + a)^3}{hx + g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="max
ima")
```

output
```
a^3*j*(x/h - g*log(h*x + g)/h^2) + a^3*i*log(h*x + g)/h + integrate((3*(i*
q*log(d) + i*log(c))*a^2*b + 3*(i*q^2*log(d)^2 + 2*i*q*log(c)*log(d) + i*l
og(c)^2)*a*b^2 + (i*q^3*log(d)^3 + 3*i*q^2*log(c)*log(d)^2 + 3*i*q*log(c)^
2*log(d) + i*log(c)^3)*b^3 + (b^3*j*x + b^3*i)*log(((f*x + e)^p)^q)^3 + 3*
((i*q*log(d) + i*log(c))*b^3 + a*b^2*i + ((j*q*log(d) + j*log(c))*b^3 + a*
b^2*j)*x)*log(((f*x + e)^p)^q)^2 + (3*(j*q*log(d) + j*log(c))*a^2*b + 3*(j
*q^2*log(d)^2 + 2*j*q*log(c)*log(d) + j*log(c)^2)*a*b^2 + (j*q^3*log(d)^3
+ 3*j*q^2*log(c)*log(d)^2 + 3*j*q*log(c)^2*log(d) + j*log(c)^3)*b^3)*x + 3
*(2*(i*q*log(d) + i*log(c))*a*b^2 + (i*q^2*log(d)^2 + 2*i*q*log(c)*log(d)
+ i*log(c)^2)*b^3 + a^2*b*i + (2*(j*q*log(d) + j*log(c))*a*b^2 + (j*q^2*lo
g(d)^2 + 2*j*q*log(c)*log(d) + j*log(c)^2)*b^3 + a^2*b*j)*x)*log(((f*x + e
)^p)^q))/(h*x + g), x)
```

3.536.8 Giac [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx=\int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^3}{hx+g}\,dx$$

input
```
integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="gia
c")
```

output
```
integrate((j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx=\int \frac{(i+j\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^3}{g+h\,x}\,dx$$

input
```
int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x),x)
```

output
```
int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x), x)
```

3.537 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

3.537.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx &= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}\\ &+ \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}\\ &- \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}\\ &+ \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h}\end{aligned}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h+3*b*p*q*(a+b*ln(c
*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-6*b^2*p^2*q^2*(a+b
*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h+6*b^3*p^3*q^3*p
olylog(4,-h*(f*x+e)/(-e*h+f*g))/h
```

3.537.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 646 vs. $2(177) = 354$.

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.65

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$

$$= \frac{a^3\log(g+hx) - 3a^2bpq\log(e+fx)\log(g+hx) + 3ab^2p^2q^2\log^2(e+fx)\log(g+hx) - b^3p^3q^3\log^3(e+f}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

output
```
(a^3*Log[g + h*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*p^2*q^
2*Log[e + f*x]^2*Log[g + h*x] - b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] +
3*a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*p*q*Log[e + f*x]*L
og[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*
(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g
 + h*x] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] +
 b^3*Log[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] + 3*a^2*b*p*q*Log[e + f*x]*Lo
g[(f*(g + h*x))/(f*g - e*h)] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g +
h*x))/(f*g - e*h)] + b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e
*h)] + 6*a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))
/(f*g - e*h)] - 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^
q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)
^q])^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 6*b^2*p^2*q^2*(a + b*Log
[c*(d*(e + f*x)^p)^q])*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] + 6*b^3*p^
3*q^3*PolyLog[4, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

3.537.3 Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2895, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}dx$$

$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bfpq\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f\left(g-\frac{eh}{f}\right)+h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)}{h}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)-\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))\right.}{h}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\left(\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\int\frac{\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{e+fx}d(e+fx)\right)-\operatorname{PolyLog}\right.}{h}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}-\frac{3bpq\left(2bpq\left(\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(cd^q(e+fx)^{pq}))-bpq\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{h(e+}{fg-}\right.\right.}{h}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

3.537. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/h - (3
*b*p*q*(-((a + b*Log[c*d^q*(e + f*x)^(p*q)])^2*PolyLog[2, -((h*(e + f*x))/
(f*g - e*h))]) + 2*b*p*q*((a + b*Log[c*d^q*(e + f*x)^(p*q)])*PolyLog[3, -(
(h*(e + f*x))/(f*g - e*h))] - b*p*q*PolyLog[4, -((h*(e + f*x))/(f*g - e*h)
)])))/h
```

3.537.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.537.4 Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^3}{hx+g}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

3.537.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c
)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*x + g), x)
```

3.537.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x), x)
```

3.537.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="maxima")
```

output
```
a^3*log(h*x + g)/h + integrate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) +
 log(c))*a^2*b + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (
q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3
 + 3*((q*log(d) + log(c))*b^3 + a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*lo
g(d) + log(c))*a*b^2 + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 +
 a^2*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

3.537.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{hx+g}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(a+b\ln(c\,(d\,(e+f\,x)^p)^q))^3}{g+h\,x}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x), x)
```

3.538 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx$

3.538.1 Optimal result

Integrand size = 35, antiderivative size = 410

$$\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx \\
&= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} \\
&+ \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} \\
&- \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} \\
&- \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} \\
&+ \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} \\
&+ \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}
\end{aligned}$$

output
```
(a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)-(a+b*ln(
c*(d*(f*x+e)^p)^q))^3*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)+3*b*p*q*(a+b*ln(
c*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)-3*b*p*q*
(a+b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)-
6*b^2*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))
/(-g*j+h*i)+6*b^2*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-j*(f*x+e)
/(-e*j+f*i))/(-g*j+h*i)+6*b^3*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/(-g
*j+h*i)-6*b^3*p^3*q^3*polylog(4,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)
```

3.538.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1350 vs. $2(410) = 820$.

Time = 0.31 (sec) , antiderivative size = 1350, normalized size of antiderivative = 3.29

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)),x]
```

output
```
(a^3*Log[g + h*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*p^2*q^
2*Log[e + f*x]^2*Log[g + h*x] - b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] +
3*a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*p*q*Log[e + f*x]*L
og[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*
(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g
 + h*x] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] +
 b^3*Log[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] + 3*a^2*b*p*q*Log[e + f*x]*Lo
g[(f*(g + h*x))/(f*g - e*h)] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g +
h*x))/(f*g - e*h)] + b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e
*h)] + 6*a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))
/(f*g - e*h)] - 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^
q]^2*Log[(f*(g + h*x))/(f*g - e*h)] - a^3*Log[i + j*x] + 3*a^2*b*p*q*Log[e
 + f*x]*Log[i + j*x] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[i + j*x] + b^3*p
^3*q^3*Log[e + f*x]^3*Log[i + j*x] - 3*a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[
i + j*x] + 6*a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x]
- 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] - 3*a
*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[i + j*x] + 3*b^3*p*q*Log[e + f*x]*Log[
c*(d*(e + f*x)^p)^q]^2*Log[i + j*x] - b^3*Log[c*(d*(e + f*x)^p)^q]^3*Log[i
 + j*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[(f*(i + j*x))/(f*i - e*j)] + 3*a...
```

3.538.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{h(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(hi-gj)}-\frac{j(a+b\log(c(d(e+fx)^p)^q))^3}{(i+jx)(hi-gj)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}+\frac{6b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}+\frac{3bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}-\frac{3bpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}+\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{hi-gj}-\frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{hi-gj}+\frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}-\frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)),x]
```

output
```
((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i -
 g*j) - ((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(i + j*x))/(f*i - e*j)]
)/(h*i - g*j) + (3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((
h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) - (3*b*p*q*(a + b*Log[c*(d*(e + f*
x)^p)^q])^2*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j) - (6*b^2
*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h*(e + f*x))/(f*g
- e*h))])/(h*i - g*j) + (6*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Po
lyLog[3, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j) + (6*b^3*p^3*q^3*PolyL
og[4, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) - (6*b^3*p^3*q^3*PolyLog[
4, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)
```

3.538.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.538.4 Maple [F]

$$\int \frac{(a + b\ln(c(d(fx+e)^p)^q))^3}{(hx+g)(jx+i)}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x)
```

3.538.5 Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx=\int \frac{(b\log(((fx+e)^pd)^q c)+a)^3}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x, algorithm="fri
cas")
```

output
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c
)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*j*x^2 + g*i + (h*i + g*j)
*x), x)
```

3.538.6 Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)\,(i+jx)}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)/(j*x+i),x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/((g + h*x)*(i + j*x)), x)
```

3.538.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx=\int \frac{(b\log(((fx+e)^pd)^q c)+a)^3}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x, algorithm="max
ima")
```

output
```
a^3*(log(h*x + g)/(h*i - g*j) - log(j*x + i)/(h*i - g*j)) + integrate((b^3
*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) + log(c))*a^2*b + 3*(q^2*log(d)^2 +
2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (q^3*log(d)^3 + 3*q^2*log(c)*log(d)^
2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3 + 3*((q*log(d) + log(c))*b^3 + a*b
^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*log(d) + log(c))*a*b^2 + (q^2*log(d)^
2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 + a^2*b)*log(((f*x + e)^p)^q))/(h*j*
x^2 + g*i + (h*i + g*j)*x), x)
```

3.538.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx=\int \frac{(b\log(((fx+e)^pd)^qc)+a)^3}{(hx+g)(jx+i)}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x, algorithm="gia
c")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/((h*x + g)*(j*x + i)), x)
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx=\int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^3}{(g+h\,x)\,(i+j\,x)}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)), x)
```

3.539 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$

3.539.1 Optimal result

Integrand size = 35, antiderivative size = 659

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(fi-ej)(hi-gj)(i+jx)} + \frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+ \frac{3bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$- \frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$+ \frac{3bhpq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+ \frac{6b^2fp^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$- \frac{3bhpq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$- \frac{6b^2hp^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$- \frac{6b^3fp^3q^3\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$+ \frac{6b^2hp^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$+ \frac{6b^3hp^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{6b^3hp^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

output
```
-j*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(-e*j+f*i)/(-g*j+h*i)/(j*x+i)+h*(
a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)^2+3*b*f*p
*q*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(j*x+i)/(-e*j+f*i))/(-e*j+f*i)/(-g*j
+h*i)-h*(a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)^
2+3*b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))
/(-g*j+h*i)^2+6*b^2*f*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-j*(f*
x+e)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h*i)-3*b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q
))^2*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2-6*b^2*h*p^2*q^2*(a+b*ln
(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)^2-6*b^3*f
*p^3*q^3*polylog(3,-j*(f*x+e)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h*i)+6*b^2*h*p^
2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h
*i)^2+6*b^3*h*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)^2-6*b^3*
h*p^3*q^3*polylog(4,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2
```

3.539.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$$
$$= \frac{(fi-ej)(hi-gj)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)^3+h(fi-ej)(i+jx)\left(a-bpq\log(e+f\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)^2),x]
```

output
```
((f*i - e*j)*(h*i - g*j)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)
^q])^3 + h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e +
 f*x)^p)^q])^3*Log[g + h*x] - h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f
*x] + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[i + j*x] - 3*b*p*q*(a - b*p*q*Log[
e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2*((h*i - g*j)*(j*(e + f*x)*Log[e +
 f*x] - f*(i + j*x)*Log[i + j*x]) - h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]*
Log[(f*(g + h*x))/(f*g - e*h)] + PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])
 + h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]*Log[(f*(i + j*x))/(f*i - e*j)] +
PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)])) - 3*b^2*p^2*q^2*(a - b*p*q*Log[
e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*((h*i - g*j)*(Log[e + f*x]*(j*(e +
f*x)*Log[e + f*x] - 2*f*(i + j*x)*Log[(f*(i + j*x))/(f*i - e*j)]) - 2*f*(i
 + j*x)*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)]) - h*(f*i - e*j)*(i + j*x
)*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*x]*PolyLog[
2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*
h)]) + h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]^2*Log[(f*(i + j*x))/(f*i - e*
j)] + 2*Log[e + f*x]*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)] - 2*PolyLog[
3, (j*(e + f*x))/(-(f*i) + e*j)])) - b^3*p^3*q^3*((h*i - g*j)*(Log[e + f*x
]^2*(j*(e + f*x)*Log[e + f*x] - 3*f*(i + j*x)*Log[(f*(i + j*x))/(f*i - e*j
)]) - 6*f*(i + j*x)*Log[e + f*x]*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)]
+ 6*f*(i + j*x)*PolyLog[3, (j*(e + f*x))/(-(f*i) + e*j)]) - h*(f*i - e*...
```

3.539.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2895, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}dx$$

$$\Big\downarrow \text{2865}$$

$$\int \left(\frac{h^2(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(hi-gj)^2}-\frac{hj(a+b\log(c(d(e+fx)^p)^q))^3}{(i+jx)(hi-gj)^2}-\frac{j(a+b\log(c(d(e+fx)^p)^q))^3}{(i+jx)^2(hi-gj)}\right)dx$$

$$\downarrow 2009$$

$$\frac{6b^2fp^2q^2 \operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(fi-ej)(hi-gj)} - \frac{6b^2hp^2q^2 \operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2} + \frac{6b^2hp^2q^2 \operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2} + \frac{3bhpq \operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2} - \frac{3bhpq \operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2} + \frac{3bfpq\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(fi-ej)(hi-gj)} - \frac{j(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{(i+jx)(fi-ej)(hi-gj)} + \frac{h\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{(hi-gj)^2} - \frac{h\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{(hi-gj)^2} - \frac{6b^3fp^3q^3 \operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} + \frac{6b^3hp^3q^3 \operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{6b^3hp^3q^3 \operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

input
```
Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)^2),x]
```

3.539. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$

output
```
-((j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/((f*i - e*j)*(h*i - g*j
)*(i + j*x))) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f
*g - e*h)])/(h*i - g*j)^2 + (3*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*
Log[(f*(i + j*x))/(f*i - e*j)])/((f*i - e*j)*(h*i - g*j)) - (h*(a + b*Log[
c*(d*(e + f*x)^p)^q])^3*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i - g*j)^2 + (3
*b*h*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(e + f*x))/(f*
g - e*h))])/(h*i - g*j)^2 + (6*b^2*f*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^
q])*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/((f*i - e*j)*(h*i - g*j)) -
(3*b*h*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((j*(e + f*x))/(
f*i - e*j))])/(h*i - g*j)^2 - (6*b^2*h*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p
)^q])*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j)^2 - (6*b^3*f*p
^3*q^3*PolyLog[3, -((j*(e + f*x))/(f*i - e*j))])/((f*i - e*j)*(h*i - g*j))
 + (6*b^2*h*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((j*(e +
f*x))/(f*i - e*j))])/(h*i - g*j)^2 + (6*b^3*h*p^3*q^3*PolyLog[4, -((h*(e +
 f*x))/(f*g - e*h))])/(h*i - g*j)^2 - (6*b^3*h*p^3*q^3*PolyLog[4, -((j*(e
+ f*x))/(f*i - e*j))])/(h*i - g*j)^2
```

3.539.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

3.539. $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$

3.539.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}{\left(hx+g\right)\left(jx+i\right)^{2}}dx$$

input
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x)
```

output
```
int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x)
```

3.539.5 Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{(g+hx)(i+jx)^{2}}\,dx=\int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{3}}{(hx+g)(jx+i)^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x, algorithm="f
ricas")
```

output
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c
)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*j^2*x^3 + g*i^2 + (2*h*i*
j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

3.539.6 Sympy [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{(g+hx)(i+jx)^{2}}\,dx=\int \frac{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{3}}{\left(g+hx\right)\left(i+jx\right)^{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)/(j*x+i)**2,x)
```

output
```
Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/((g + h*x)*(i + j*x)**2), x
)
```

3.539.7 Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x, algorithm="m
axima")
```

output
```
a^3*(h*log(h*x + g)/(h^2*i^2 - 2*g*h*i*j + g^2*j^2) - h*log(j*x + i)/(h^2*
i^2 - 2*g*h*i*j + g^2*j^2) + 1/(h*i^2 - g*i*j + (h*i*j - g*j^2)*x)) + inte
grate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) + log(c))*a^2*b + 3*(q^2*l
og(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (q^3*log(d)^3 + 3*q^2*log(
c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3 + 3*((q*log(d) + log(c))
*b^3 + a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*log(d) + log(c))*a*b^2 + (q
^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 + a^2*b)*log(((f*x + e)^p)
^q))/(h*j^2*x^3 + g*i^2 + (2*h*i*j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

3.539.8 Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)(jx+i)^2}\,dx$$

input
```
integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x, algorithm="g
iac")
```

output
```
integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/((h*x + g)*(j*x + i)^2), x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^3}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^2}\,dx$$

input
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)^2),x)
```

output
```
int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)^2), x)
```

3.540 $\int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.540.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\text{Int}\left(\frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

output
```
Unintegrable((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.540.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

input
```
Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.540.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$
$$\Big\downarrow \; 2896$$
$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$

input
```
Int[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.540.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.540.4 Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{jx+i}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

input
```
int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.540.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{jx+i}{(hx+g)\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="frica
s")
```

output
```
integral((j*x + i)/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c)),
 x)
```

3.540.6 Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{i+jx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)(g+hx)}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral((i + j*x)/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

3.540.7 Maxima [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{jx+i}{(hx+g)\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxim
a")
```

output
```
integrate((j*x + i)/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.540.8 Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac"
)
```

output
```
integrate((j*x + i)/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.540.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{i+j\,x}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

input
```
int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.541 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.541.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.541.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.541.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$
$$\Big\downarrow \; 2896$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.541.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.541.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.541.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

output
```
integral(1/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

3.541.6 Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

3.541.7 Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.541.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.541.9 Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.542 $\int \frac{1}{(g+hx)(i+jx)(a+b\log(c(d(e+fx)^p)^q))} dx$

3.542.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx = \text{Int}\left(\frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}, x\right)$$

output
```
Unintegrable(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.542.2 Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx$$

input
```
Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.542.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$

input
```
Int[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.542.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.542.4 Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)(jx+i)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

input
```
int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.542.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(((fx+e)^p d)^q c\right)+a)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fri
cas")
```

output
```
integral(1/(a*h*j*x^2 + a*g*i + (a*h*i + a*g*j)*x + (b*h*j*x^2 + b*g*i + (
b*h*i + b*g*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

3.542.6 Sympy [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\left(g+hx\right)\left(i+jx\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)*(i + j*x)), x)
```

3.542.7 Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(((fx+e)^p d)^q c\right)+a)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="max
ima")
```

output
```
integrate(1/((h*x + g)*(j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.542.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(((fx+e)^p d)^q c\right)+a)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="gia
c")
```

output
```
integrate(1/((h*x + g)*(j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.542.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+h\,x)\,(i+j\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.543 $\int \frac{1}{(g+hx)(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}\,dx$

3.543.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \text{Int}\left(\frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.543.2 Mathematica [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

input
```
Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

3.543.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}dx$$

input
```
Int[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

output
```
$Aborted
```

3.543.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.543.4 Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\,(jx+i)^2\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)}dx$$

input
```
int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

output
```
int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

3.543.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.29

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)^2(b\log\left(((fx+e)^pd)^q\,c\right)+a)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="f
ricas")
```

output
```
integral(1/(a*h*j^2*x^3 + a*g*i^2 + (2*a*h*i*j + a*g*j^2)*x^2 + (a*h*i^2 +
 2*a*g*i*j)*x + (b*h*j^2*x^3 + b*g*i^2 + (2*b*h*i*j + b*g*j^2)*x^2 + (b*h*
i^2 + 2*b*g*i*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

3.543.6 Sympy [N/A]

Not integrable

Time = 12.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\left(g+hx\right)\left(i+jx\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)**2/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)*(i + j*x)**2), x)
```

3.543.7 Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)^2(b\log\left(((fx+e)^pd)^q\,c\right)+a)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="m
axima")
```

output
```
integrate(1/((h*x + g)*(j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.543.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)^2(b\log\left(((fx+e)^pd)^q\,c\right)+a)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="g
iac")
```

output
```
integrate(1/((h*x + g)*(j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

3.543.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+h\,x)\,\left(i+j\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

input
```
int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
```

output
```
int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

3.544 $\int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.544.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\text{Int}\left(\frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

output
```
Unintegrable((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.544.2 Mathematica [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

input
```
Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

3.544.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

$$\Big\downarrow \; 2896$$

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$

input
```
Int[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
$Aborted
```

3.544.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.544.4 Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{jx+i}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

input
```
int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.544.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \frac{i+jx}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))^2}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a)^2}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fri
cas")
```

output
```
integral((j*x + i)/(a^2*h*x + a^2*g + (b^2*h*x + b^2*g)*log(((f*x + e)^p*d
)^q*c)^2 + 2*(a*b*h*x + a*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

3.544.6 Sympy [N/A]

Not integrable

Time = 16.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{i+jx}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))^2}\,dx=\int \frac{i+jx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2(g+hx)}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral((i + j*x)/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)), x)
```

3.544.7 Maxima [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 302, normalized size of antiderivative = 9.15

$$\int \frac{i+jx}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))^2}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a)^2}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="max
ima")
```

output
```
-(f*j*x^2 + e*i + (f*i + e*j)*x)/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*
q*log(c))*b^2 + (a*b*f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x
+ (b^2*f*h*p*q*x + b^2*f*g*p*q)*log(((f*x + e)^p)^q)) + integrate((f*h*j*x
^2 + 2*f*g*j*x + f*g*i - (h*i - g*j)*e)/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(
d) + f*g^2*p*q*log(c))*b^2 + (a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*
p*q*log(c))*b^2)*x^2 + 2*(a*b*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*
log(c))*b^2)*x + (b^2*f*h^2*p*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*l
og(((f*x + e)^p)^q)), x)
```

3.544.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a)^2}\,dx$$

input
```
integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="gia
c")
```

output
```
integrate((j*x + i)/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

3.544.9 Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{i+j\,x}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}\,dx$$

input
```
int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

output
```
int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

3.545 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.545.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.545.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

input
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

3.545.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$

input
```
Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
$Aborted
```

3.545.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.545.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

input
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.545.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*h*x + a^2*g + (b^2*h*x + b^2*g)*log(((f*x + e)^p*d)^q*c)^2
 + 2*(a*b*h*x + a*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

3.545.6 Sympy [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2(g+hx)}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)), x)
```

3.545.7 Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

output
```
(f*g - e*h)*integrate(1/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) + f*g^2*p*q*l
og(c))*b^2 + (a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*log(c))*b^2)
*x^2 + 2*(a*b*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c))*b^2)*x +
 (b^2*f*h^2*p*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*x + e)^p)
^q)), x) - (f*x + e)/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q*log(c))*b^
2 + (a*b*f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x + (b^2*f*h*p
*q*x + b^2*f*g*p*q)*log(((f*x + e)^p)^q))
```

3.545.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

output
```
integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

3.545.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,\left(d\,{(e+f\,x)}^p\right)^q\right)\right)^2}\,dx$$

input
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

output
```
int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

3.546 $\int \frac{1}{(g+hx)(i+jx)(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.546.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.546.2 Mathematica [N/A]

Not integrable

Time = 13.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

input
```
Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

3.546.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$
$$\Big\downarrow \text{2896}$$
$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}dx$$

input
```
Int[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
$Aborted
```

3.546.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.546.4 Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)(jx+i)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^2}dx$$

input
```
int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.546.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.60

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(a^2*h*j*x^2 + a^2*g*i + (b^2*h*j*x^2 + b^2*g*i + (b^2*h*i + b^
2*g*j)*x)*log(((f*x + e)^p*d)^q*c)^2 + (a^2*h*i + a^2*g*j)*x + 2*(a*b*h*j*
x^2 + a*b*g*i + (a*b*h*i + a*b*g*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

3.546.6 Sympy [N/A]

Not integrable

Time = 16.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2\left(g+hx\right)\left(i+jx\right)}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)*(i + j*x)), x)
```

3.546.7 Maxima [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 701, normalized size of antiderivative = 20.03

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="m
axima")
```

output
```
-(f*x + e)/(a*b*f*g*i*p*q + (f*g*i*p*q^2*log(d) + f*g*i*p*q*log(c))*b^2 +
(a*b*f*h*j*p*q + (f*h*j*p*q^2*log(d) + f*h*j*p*q*log(c))*b^2)*x^2 + ((h*i*
p*q + g*j*p*q)*a*b*f + ((h*i*p*q + g*j*p*q)*f*log(c) + (h*i*p*q^2 + g*j*p*
q^2)*f*log(d))*b^2)*x + (b^2*f*h*j*p*q*x^2 + b^2*f*g*i*p*q + (h*i*p*q + g*
j*p*q)*b^2*f*x)*log(((f*x + e)^p)^q)) - integrate((f*h*j*x^2 + 2*e*h*j*x -
 f*g*i + (h*i + g*j)*e)/(a*b*f*g^2*i^2*p*q + (a*b*f*h^2*j^2*p*q + (f*h^2*j
^2*p*q^2*log(d) + f*h^2*j^2*p*q*log(c))*b^2)*x^4 + 2*((h^2*i*j*p*q + g*h*j
^2*p*q)*a*b*f + ((h^2*i*j*p*q + g*h*j^2*p*q)*f*log(c) + (h^2*i*j*p*q^2 + g
*h*j^2*p*q^2)*f*log(d))*b^2)*x^3 + (f*g^2*i^2*p*q^2*log(d) + f*g^2*i^2*p*q
*log(c))*b^2 + ((h^2*i^2*p*q + 4*g*h*i*j*p*q + g^2*j^2*p*q)*a*b*f + ((h^2*
i^2*p*q + 4*g*h*i*j*p*q + g^2*j^2*p*q)*f*log(c) + (h^2*i^2*p*q^2 + 4*g*h*i
*j*p*q^2 + g^2*j^2*p*q^2)*f*log(d))*b^2)*x^2 + 2*((g*h*i^2*p*q + g^2*i*j*p
*q)*a*b*f + ((g*h*i^2*p*q + g^2*i*j*p*q)*f*log(c) + (g*h*i^2*p*q^2 + g^2*i
*j*p*q^2)*f*log(d))*b^2)*x + (b^2*f*h^2*j^2*p*q*x^4 + b^2*f*g^2*i^2*p*q +
2*(h^2*i*j*p*q + g*h*j^2*p*q)*b^2*f*x^3 + (h^2*i^2*p*q + 4*g*h*i*j*p*q + g
^2*j^2*p*q)*b^2*f*x^2 + 2*(g*h*i^2*p*q + g^2*i*j*p*q)*b^2*f*x)*log(((f*x +
 e)^p)^q)), x)
```

3.546.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((h*x + g)*(j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

3.546.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+h\,x)\,\left(i+j\,x\right)\,\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^2}\,dx$$

input
```
int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

output
```
int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

3.547 $\int \frac{1}{(g+hx)(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$

3.547.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.547.2 Mathematica [N/A]

Not integrable

Time = 19.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

input
```
Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

3.547.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2896}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(g+hx)(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^2} dx$$

$$\Big\downarrow \text{2896}$$

$$\int \frac{1}{(g+hx)(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^2} dx$$

input
```
Int[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

output
```
$Aborted
```

3.547.3.1 Defintions of rubi rules used

rule 2896
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(AFx_), x_Symbol] :> Unintegrable[AFx*(a + b*Log[c*(d*(e + f*x)^m)^n])^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && AlgebraicFunctionQ[AFx, x,
True]
```

3.547.4 Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)(jx+i)^2 (a+b\ln(c(d(fx+e)^p)^q))^2} dx$$

input
```
int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

output
```
int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

3.547.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 6.03

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$
$$=\int \frac{1}{(hx+g)(jx+i)^2\left(b\log\left(\left((fx+e)^pd\right)^q c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(a^2*h*j^2*x^3 + a^2*g*i^2 + (2*a^2*h*i*j + a^2*g*j^2)*x^2 + (b
^2*h*j^2*x^3 + b^2*g*i^2 + (2*b^2*h*i*j + b^2*g*j^2)*x^2 + (b^2*h*i^2 + 2*
b^2*g*i*j)*x)*log(((f*x + e)^p*d)^q*c)^2 + (a^2*h*i^2 + 2*a^2*g*i*j)*x + 2
*(a*b*h*j^2*x^3 + a*b*g*i^2 + (2*a*b*h*i*j + a*b*g*j^2)*x^2 + (a*b*h*i^2 +
 2*a*b*g*i*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

3.547.6 Sympy [N/A]

Not integrable

Time = 112.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$
$$=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2\left(g+hx\right)\left(i+jx\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

output
```
Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)*(i + j*x)**2),
 x)
```

3.547.7 Maxima [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 1039, normalized size of antiderivative = 29.69

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)^2\left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm=
"maxima")
```

output
```
-(f*x + e)/(a*b*f*g*i^2*p*q + (a*b*f*h*j^2*p*q + (f*h*j^2*p*q^2*log(d) + f
*h*j^2*p*q*log(c))*b^2)*x^3 + (f*g*i^2*p*q^2*log(d) + f*g*i^2*p*q*log(c))*
b^2 + ((2*h*i*j*p*q + g*j^2*p*q)*a*b*f + ((2*h*i*j*p*q + g*j^2*p*q)*f*log(
c) + (2*h*i*j*p*q^2 + g*j^2*p*q^2)*f*log(d))*b^2)*x^2 + ((h*i^2*p*q + 2*g*
i*j*p*q)*a*b*f + ((h*i^2*p*q + 2*g*i*j*p*q)*f*log(c) + (h*i^2*p*q^2 + 2*g*
i*j*p*q^2)*f*log(d))*b^2)*x + (b^2*f*h*j^2*p*q*x^3 + b^2*f*g*i^2*p*q + (2*
h*i*j*p*q + g*j^2*p*q)*b^2*f*x^2 + (h*i^2*p*q + 2*g*i*j*p*q)*b^2*f*x)*log(
((f*x + e)^p)^q)) - integrate((2*f*h*j*x^2 - f*g*i + (h*i + 2*g*j)*e + (f*
g*j + 3*e*h*j)*x)/(a*b*f*g^2*i^3*p*q + (a*b*f*h^2*j^3*p*q + (f*h^2*j^3*p*q
^2*log(d) + f*h^2*j^3*p*q*log(c))*b^2)*x^5 + ((3*h^2*i*j^2*p*q + 2*g*h*j^3
*p*q)*a*b*f + ((3*h^2*i*j^2*p*q + 2*g*h*j^3*p*q)*f*log(c) + (3*h^2*i*j^2*p
*q^2 + 2*g*h*j^3*p*q^2)*f*log(d))*b^2)*x^4 + ((3*h^2*i^2*j*p*q + 6*g*h*i*j
^2*p*q + g^2*j^3*p*q)*a*b*f + ((3*h^2*i^2*j*p*q + 6*g*h*i*j^2*p*q + g^2*j^
3*p*q)*f*log(c) + (3*h^2*i^2*j*p*q^2 + 6*g*h*i*j^2*p*q^2 + g^2*j^3*p*q^2)*
f*log(d))*b^2)*x^3 + (f*g^2*i^3*p*q^2*log(d) + f*g^2*i^3*p*q*log(c))*b^2 +
 ((h^2*i^3*p*q + 6*g*h*i^2*j*p*q + 3*g^2*i*j^2*p*q)*a*b*f + ((h^2*i^3*p*q
+ 6*g*h*i^2*j*p*q + 3*g^2*i*j^2*p*q)*f*log(c) + (h^2*i^3*p*q^2 + 6*g*h*i^2
*j*p*q^2 + 3*g^2*i*j^2*p*q^2)*f*log(d))*b^2)*x^2 + ((2*g*h*i^3*p*q + 3*g^2
*i^2*j*p*q)*a*b*f + ((2*g*h*i^3*p*q + 3*g^2*i^2*j*p*q)*f*log(c) + (2*g*h*i
^3*p*q^2 + 3*g^2*i^2*j*p*q^2)*f*log(d))*b^2)*x + (b^2*f*h^2*j^3*p*q*x^5...
```

3.547.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$
$$=\int \frac{1}{(hx+g)(jx+i)^2(b\log\left(((fx+e)^pd)^qc\right)+a)^2}\,dx$$

input
```
integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((h*x + g)*(j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

3.547.9 Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$
$$=\int \frac{1}{(g+h\,x)\,\left(i+j\,x\right)^2\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^2}\,dx$$

input
```
int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

output
```
int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
                finalresult={"A"," "}
            ,(*ELSE*)
                finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```